Institute of
Applied Physics
Friedrich-Schiller-Universität Jena

Advanced Lens Design

Lecture 10: Special topics

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Contents

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Correction Effectiveness

- Effectiveness of correction features on aberration types

	Makes a good impact.
	Makes a smaller impact.
	Makes a negligible impact.
	Zero influence.

		Aberration									
		Primary Aberration					5th	Chromatic			
Action		Spherical Aberration	Coma	Astigmatism	Petzval Curvature	Distortion	5th Order Spherical	Axial Color	Lateral Color	Secondary Spectrum	Spherochromatism
Lens Parameters	Lens Bending	(a)	(c)		e	(f)					
	Power Splitting										
	Power Combination	a	c			f		i	j		(k)
	Distances				(e)						k
	Stop Position										
Material	Refractive Index	(b)	(d)			(g)	(h)				
	Dispersion							(i)	(j)		(l)
	Relative Partial Disp.										
	GRIN										
Special Surfaces	Cemented Surface	b	d			g	h	i	j		l
	Aplanatic Surface										
	Aspherical Surface										
	Mirror										
	Diffractive Surface										
Struc	Symmetry Principle										
	Field Lens										

Ref : H. Zügge

Sensitivity of a System

- Sensitivity/relaxation:
  Average of weighted surface contributions of all aberrations

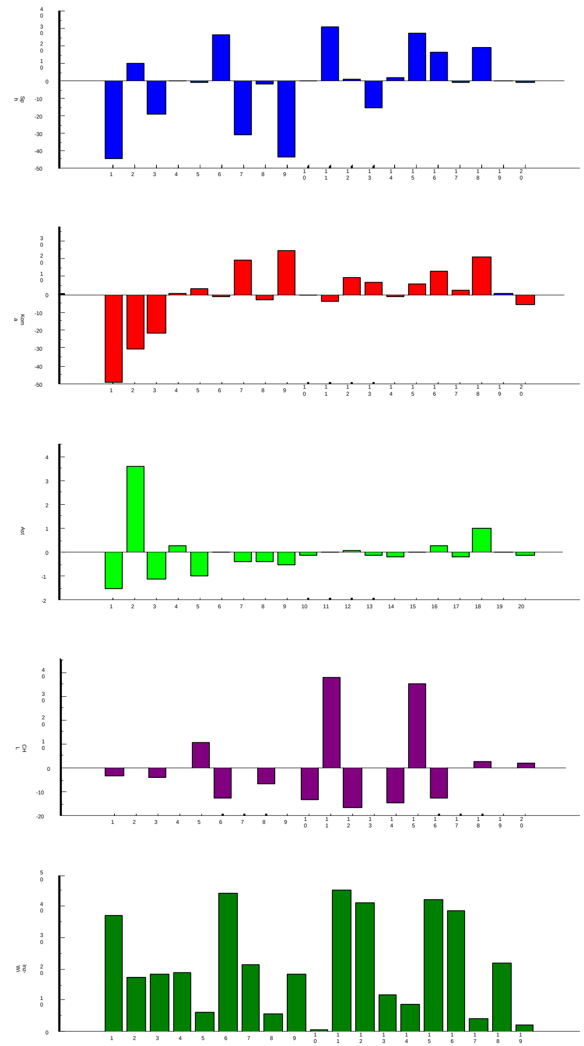
- Correctability:
  Average of all total aberration values

- Total refractive power

$$F = F_1 + \sum_{j=2}^{k} \omega_j F_j$$

- Important weighting factor:
  ratio of marginal ray heights

$$\omega_j = \frac{h_j}{h_1}$$



Sph

Coma

Ast

CHL

incidence angle

Sensitivity of a System

- Quantitative measure for relaxation

$$A_j = \omega_j \cdot \frac{F_j}{F} = \frac{h_j \cdot F_j}{h_1 \cdot F}$$

with normalization $$\sum_{j=1}^{k} A_j = 1$$

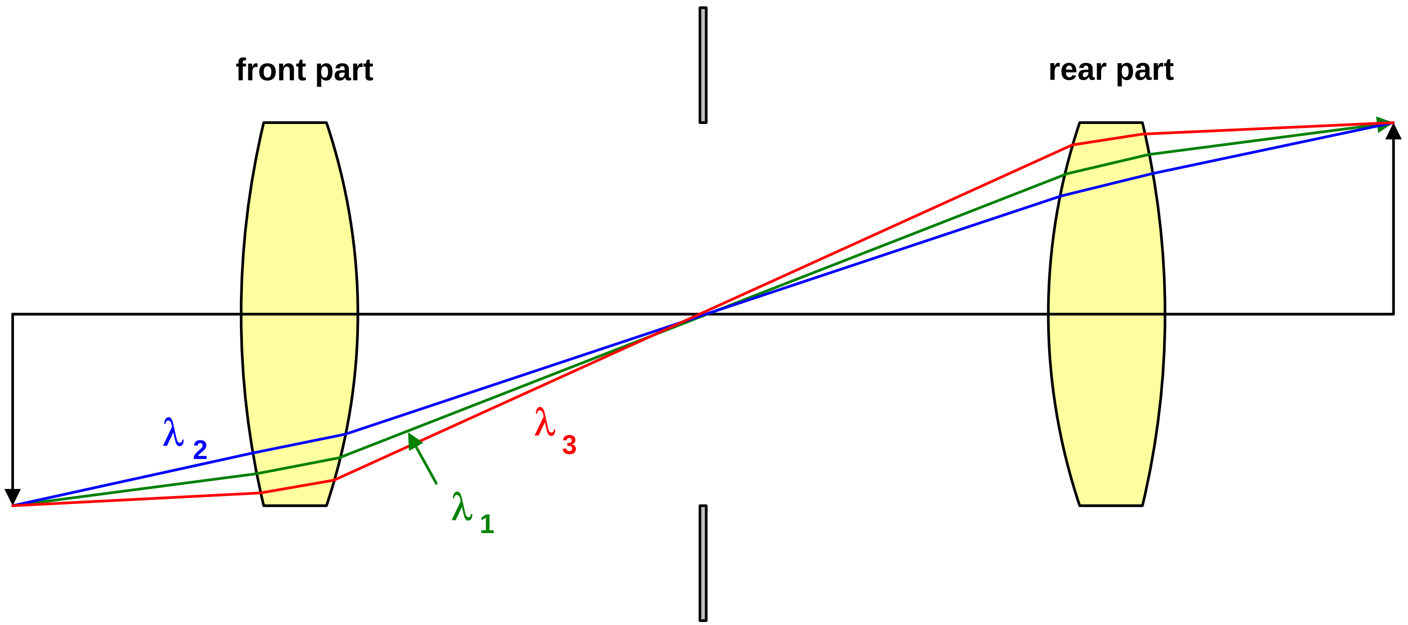
- Non-relaxed surfaces:
  1. Large incidence angles
  2. Large ray bending
  3. Large surface contributions of aberrations
  4. Significant occurence of higher aberration orders
  5. Large sensitivity for centering

- Internal relaxation can not be easily recognized in the total performance

- Large sensitivities can be avoided by incorporating surface contribution of aberrations into merit function during optimization

Principle of Symmetry

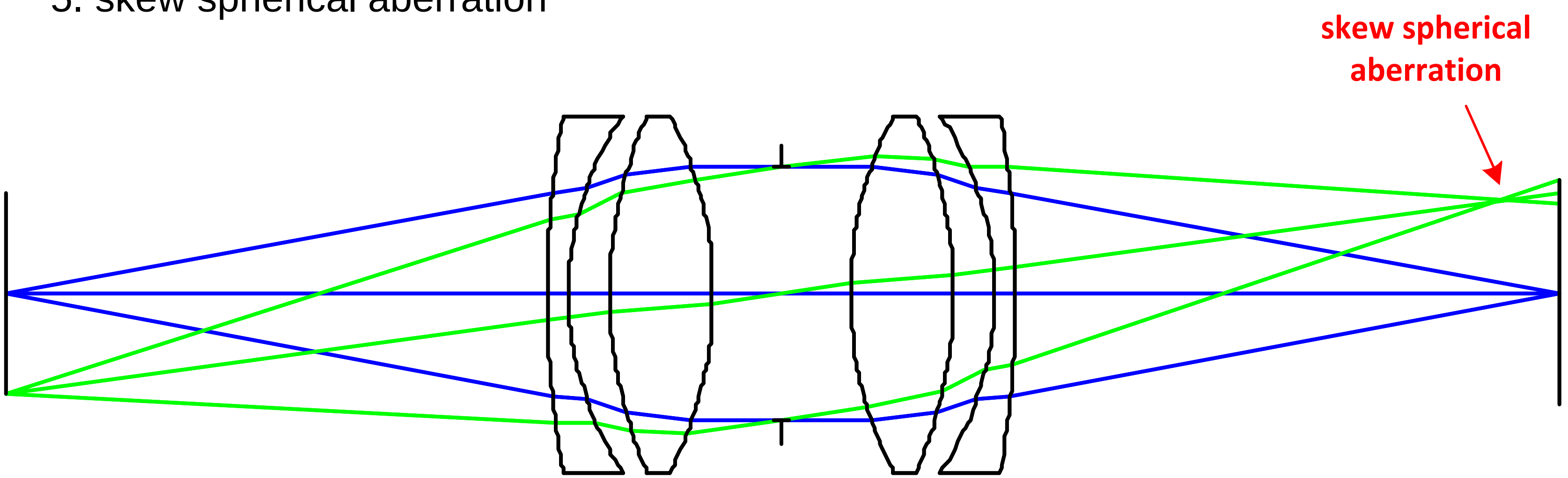
- Perfect symmetrical system: magnification m = -1
- Stop in centre of symmetry
- Symmetrical contributions of wave aberrations are doubled (spherical)
- Asymmetrical contributions of wave aberration vanishes W(-x) = -W(x)
- Easy correction of:
  coma, distortion, chromatical change of magnification



Symmetrical Systems

Ideal symmetrical systems:

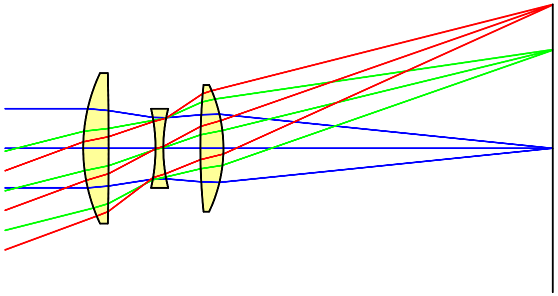
- Vanishing coma, distortion, lateral color aberration
- Remaining residual aberrations:
  1. spherical aberration
  2. astigmatism
  3. field curvature
  4. axial chromatical aberration
  5. skew spherical aberration



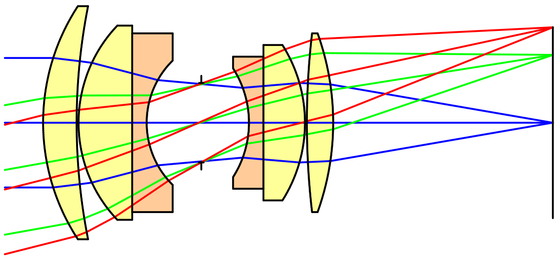
Symmetry Principle

- Application of symmetry principle: photographic lenses
- Especially field dominant aberrations can be corrected
- Also approximate fulfillment of symmetry condition helps significantly:
  quasi symmetry
- Realization of quasi-symmetric setups in nearly all photographic systems

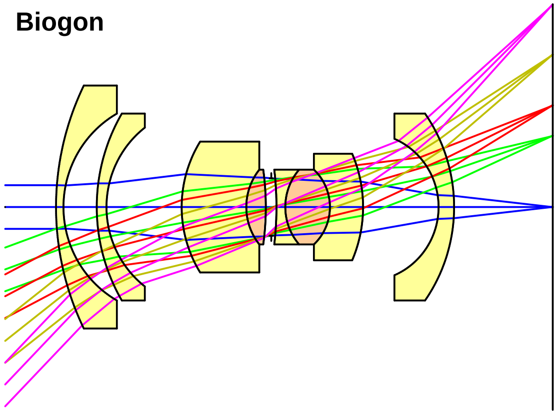
Triplet



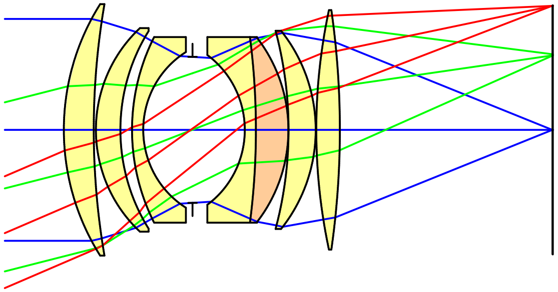
Double Gauss (6 elements)



Biogon



Double Gauss (7 elements)



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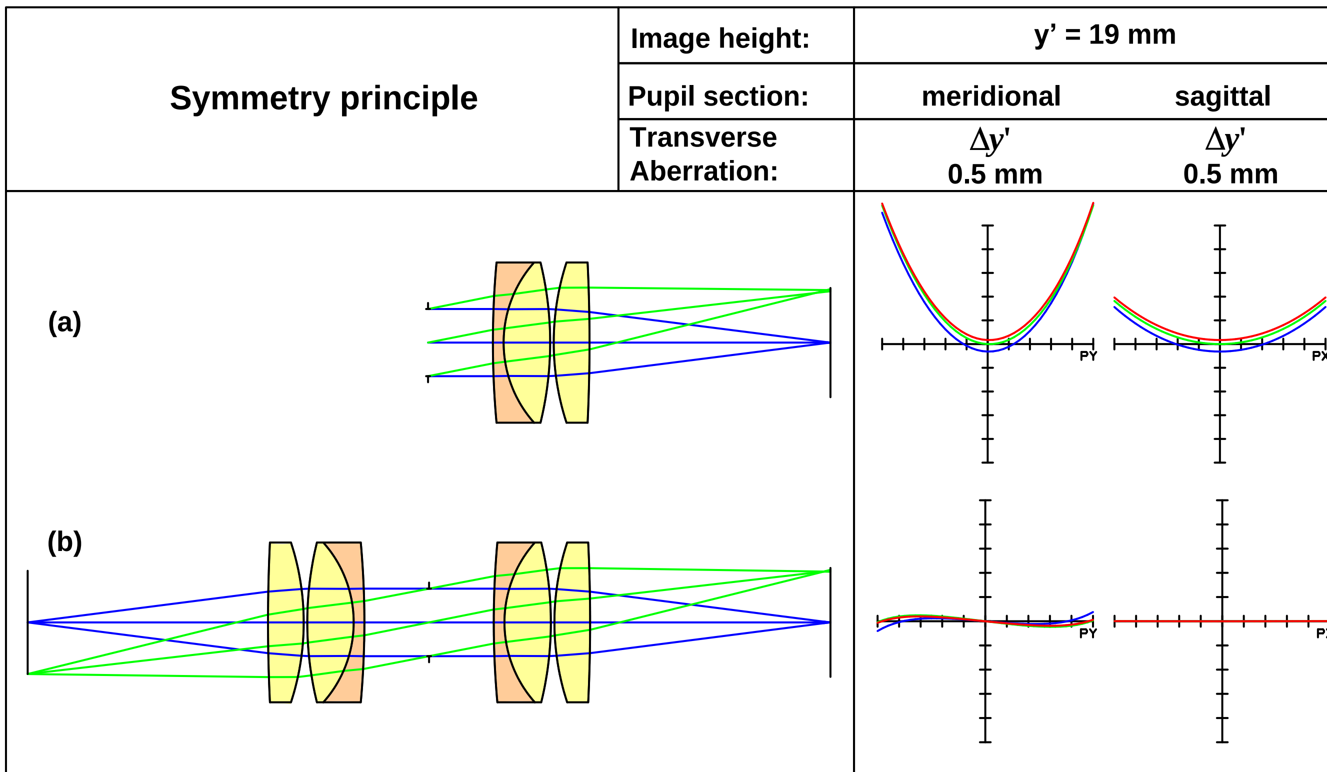
Coma Correction: Symmetry Principle

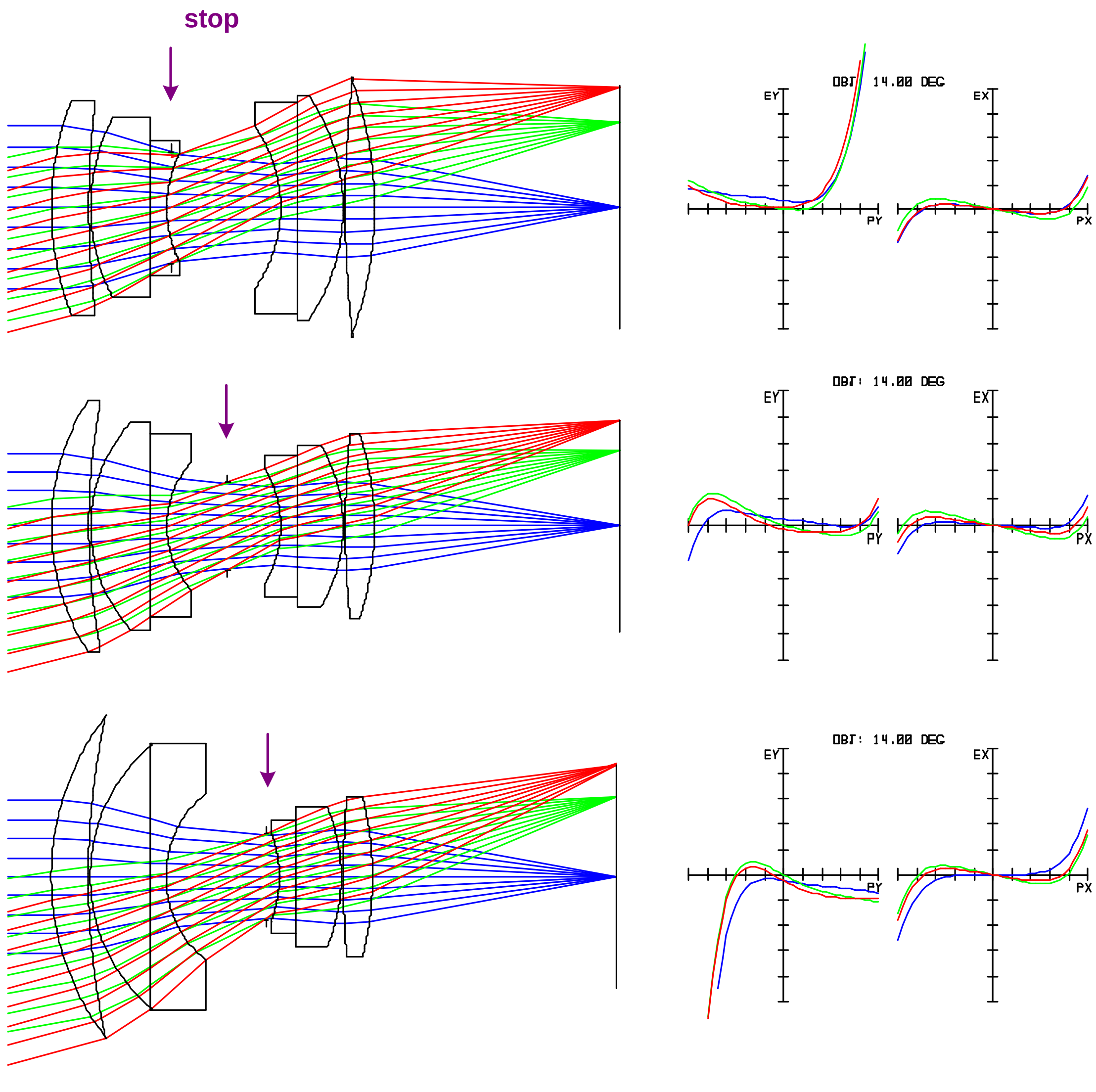
- Perfect coma correction in the case of symmetry
- But magnification m = -1 not useful in most practical cases



From : H. Zügge

Effect of Stop Position

- Example photographic lens
- Small axial shift of stop changes tranverse aberrations
- In particular coma is strongly influenced



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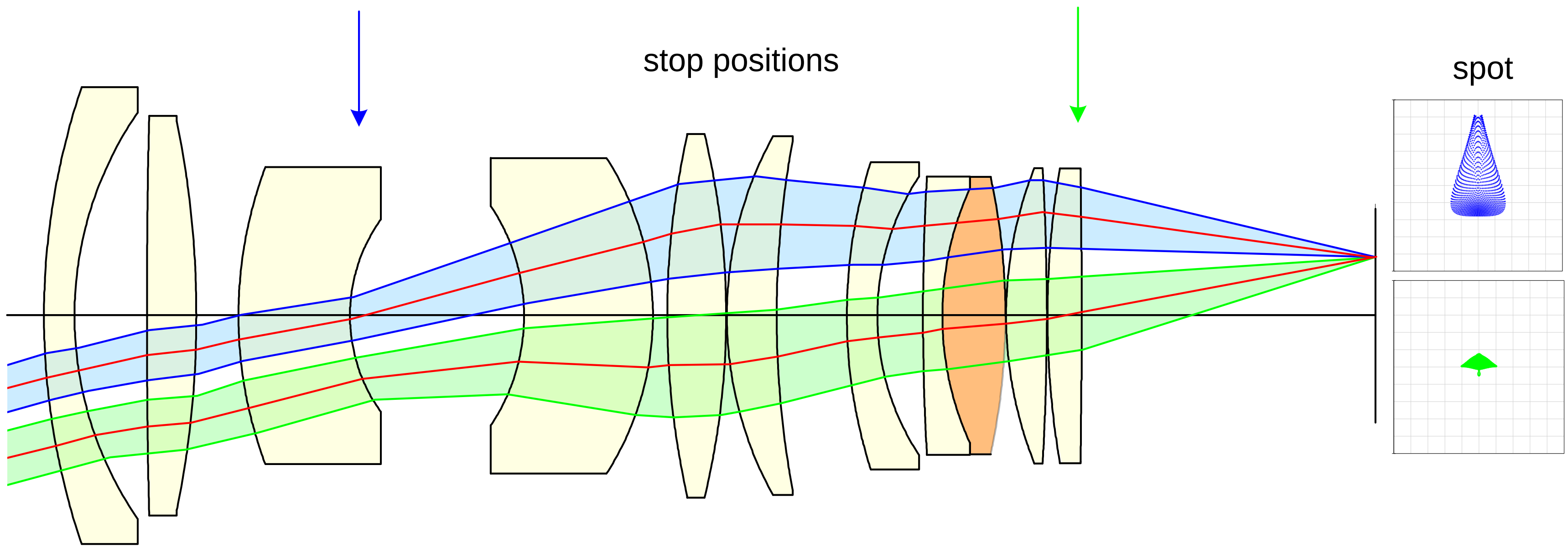
Influence of Stop Position on Performance

- Ray path of chief ray depends on stop position



Telecentricity

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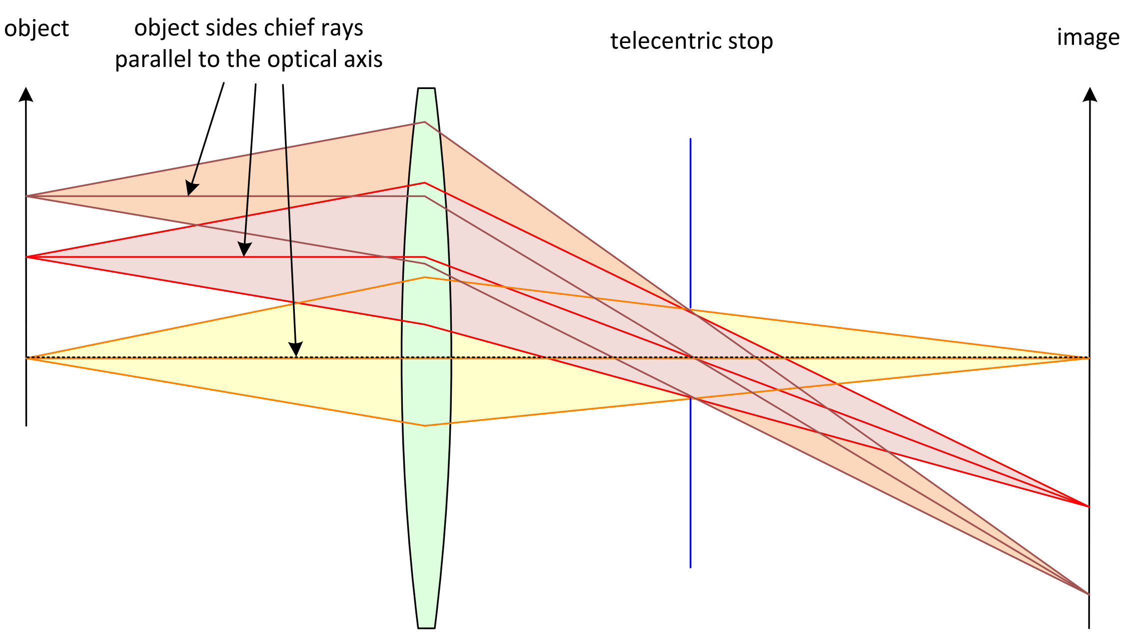
- Special stop positions:
  1. stop in back focal plane: object sided telecentricity
  2. stop in front focal plane: image sided telecentricity
  3. stop in intermediate focal plane: both-sided telecentricity

- Telecentricity:
  1. pupil in infinity
  2. chief ray parallel to the optical axis

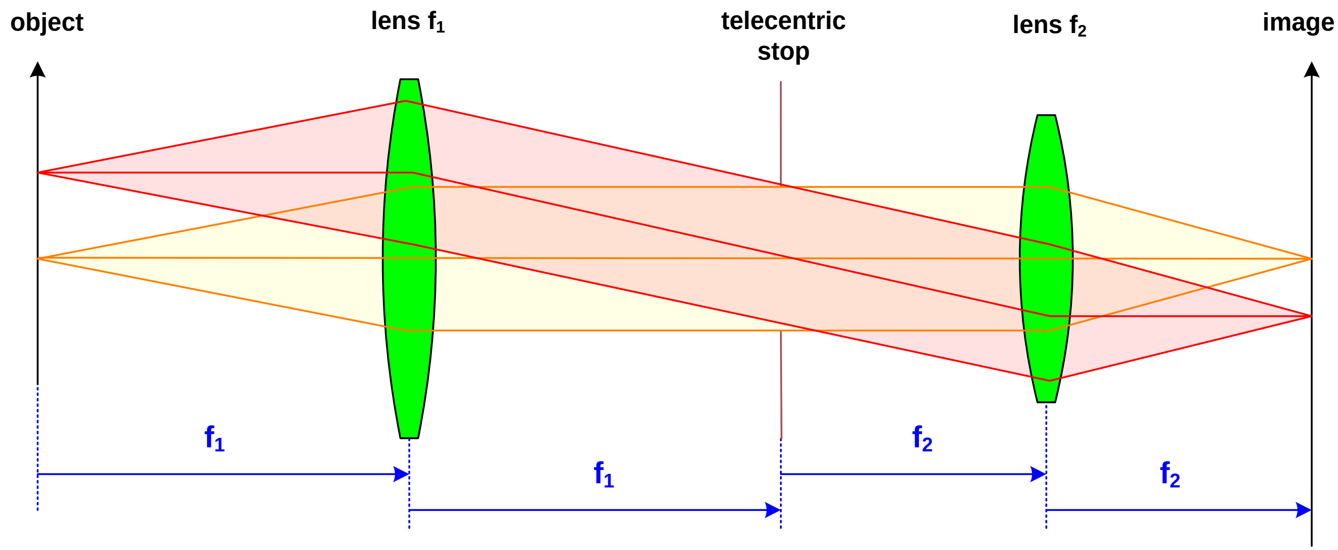
- Problem in practical systems:
  large diameters necessary

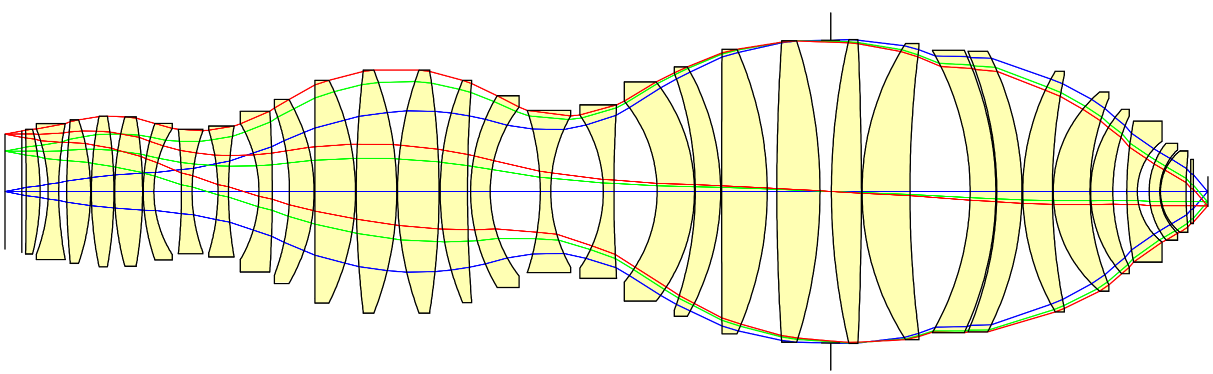
$$D > 2 \cdot \left(y_{\max} + f \cdot NA \right)$$



Telecentricity

- Double telecentric system: stop in intermediate focus
- Realization in lithographic projection systems





Telecentric Systems

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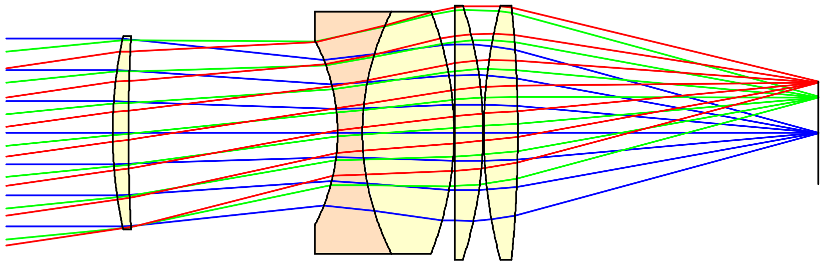
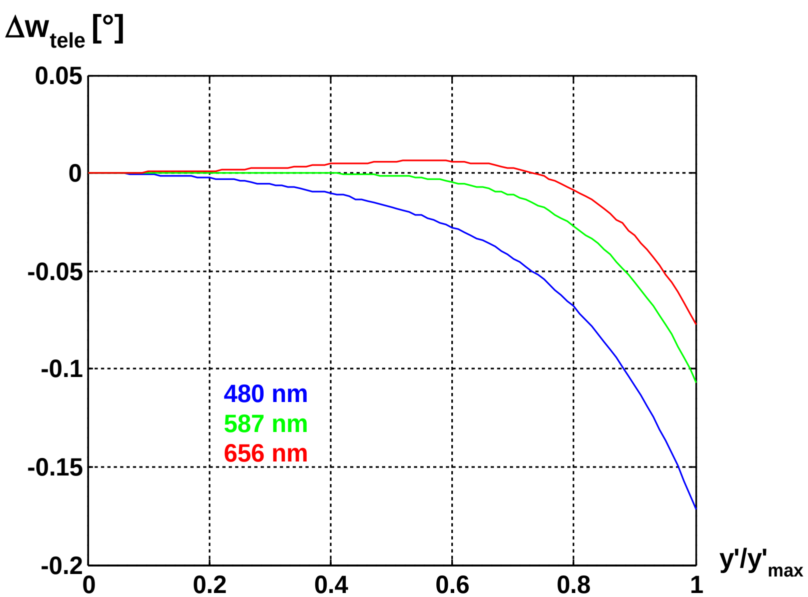
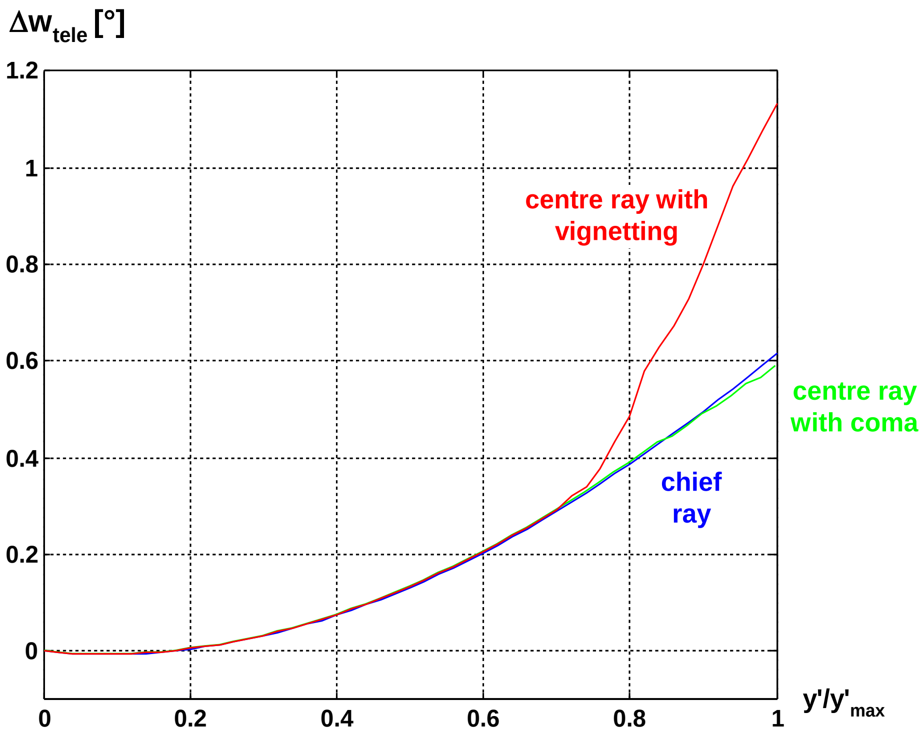
- Example system
- Problem : coma and vignetting disturbe telecentricity
- Definition of telecentricity deviation:
  range of telecentricity for accepted lateral deviation Δy' for finite cheif ray angle w



$$\Delta s = \frac{\Delta y'}{m \cdot \tan w}$$



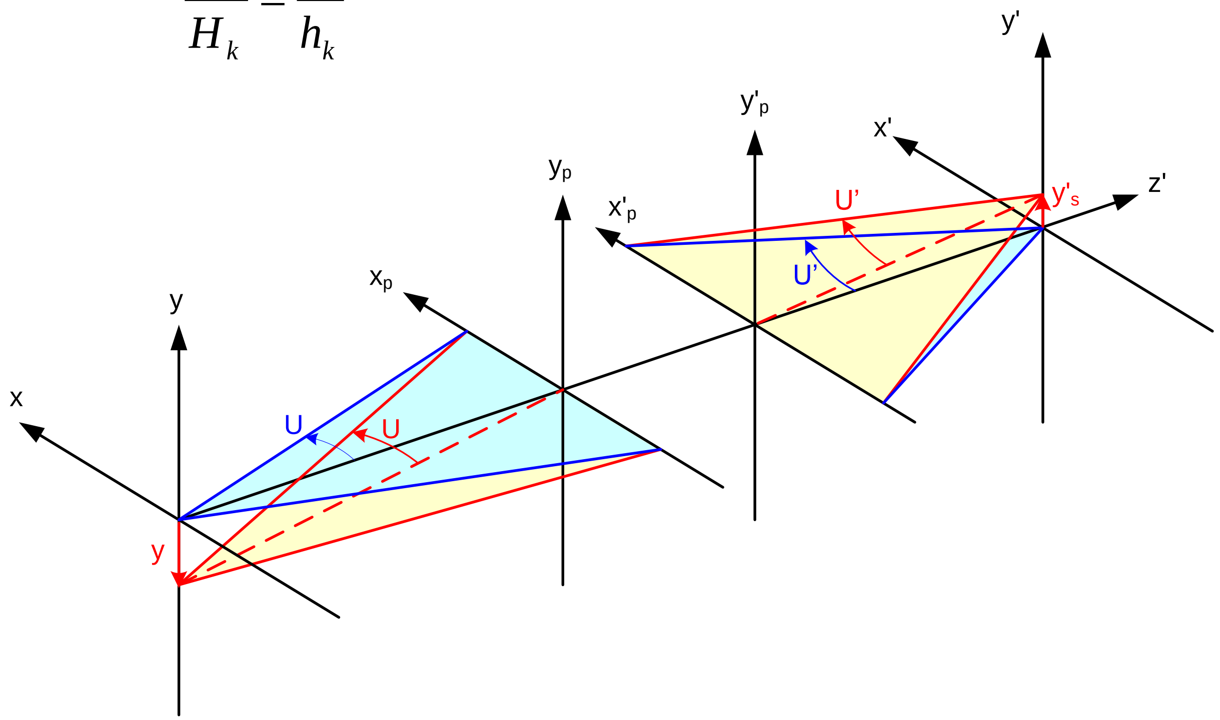


Optical Sine Condition

- Condition for finite angles $$m = \frac{nu}{n'u'} = \frac{n \sin U}{n' \sin U'}$$
- Condition for object at infinity $$f' = -\frac{h}{u'} = -\frac{h}{\sin U'}$$
- Condition for afocal system $$\frac{H_1}{H_k} = \frac{h_1}{h_k}$$
- In the formulation

$$m_s = \frac{y'_s}{y} = \frac{n \sin U}{n' \sin U'}$$

the sagittal magnification is used



Abbe Sine Condition

- If for example a small field area and a widespread ray bundle is considered, a perfect imaging is possible
- The eikonal with the expression $\delta L = n'\vec{s}'\cdot d\vec{r}' - n\vec{s}\cdot d\vec{r}$ can be written for δL=0 as

$$n\cdot\vec{s}\cdot d\vec{r} = n'\cdot\vec{s}'\cdot d\vec{r}'$$

$$n\cdot dr\cdot\cos\theta = n'\cdot dr'\cdot\cos\theta'$$

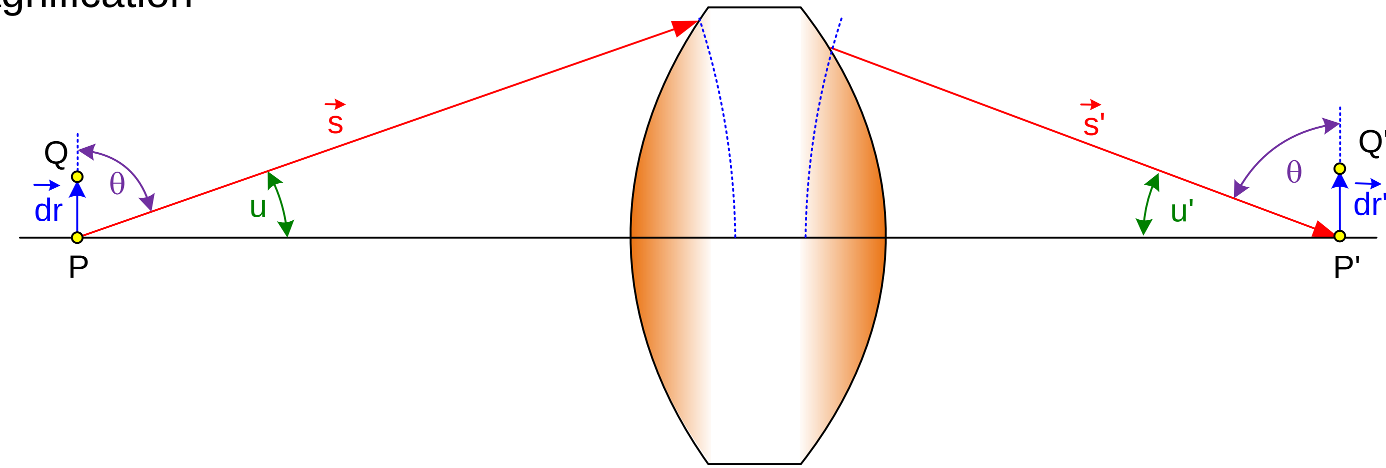
$$n\cdot\cos\theta = n'\cdot\beta\cdot\cos\theta'$$

- In the special case of an angle 90° we get with cos(θ)=sin(u) the Abbe sine condition

$$m = \frac{n\sin u}{n'\sin u'}$$

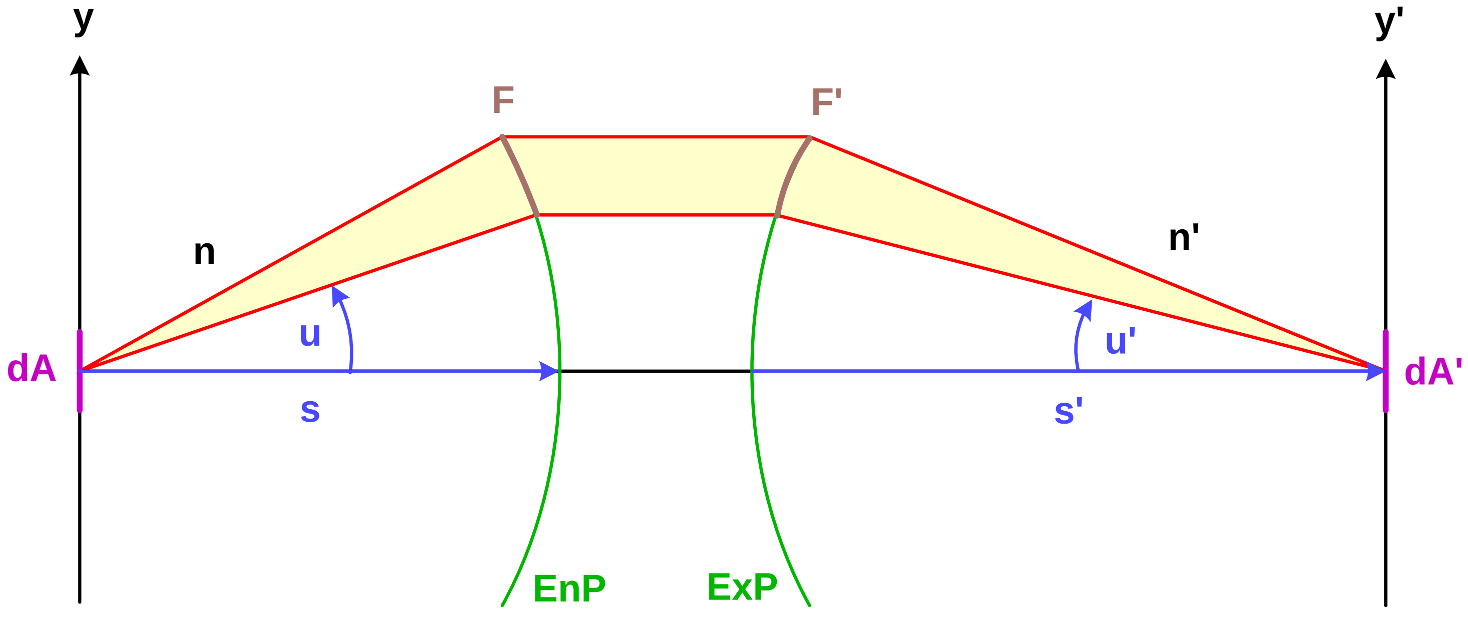
with the lateral magnification

$$m = \frac{d\vec{r}'}{d\vec{r}}$$



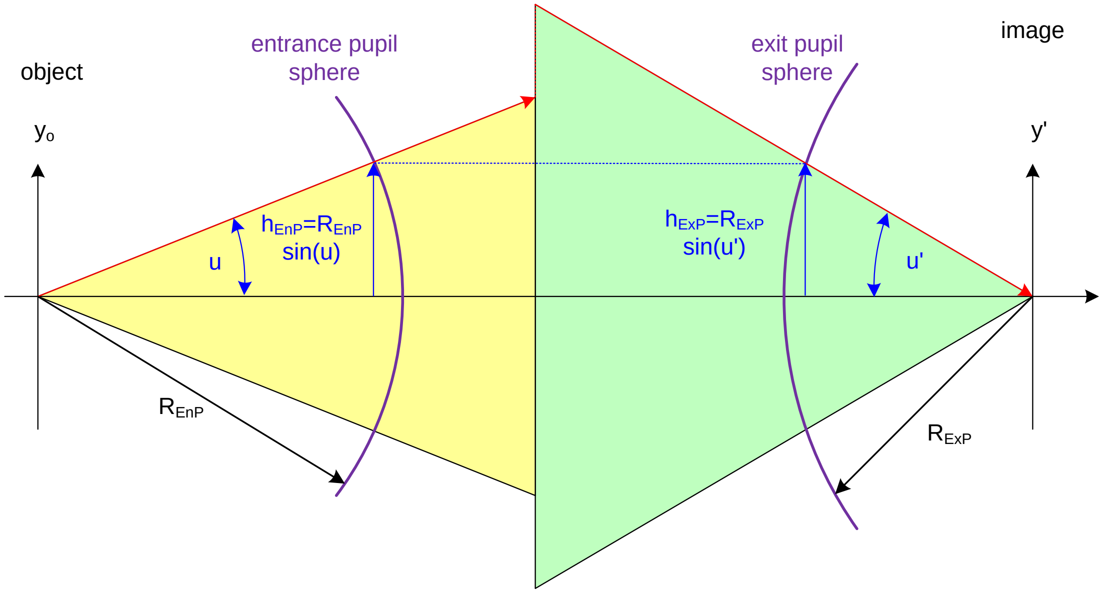
Transfer of Energy in Optical Systems

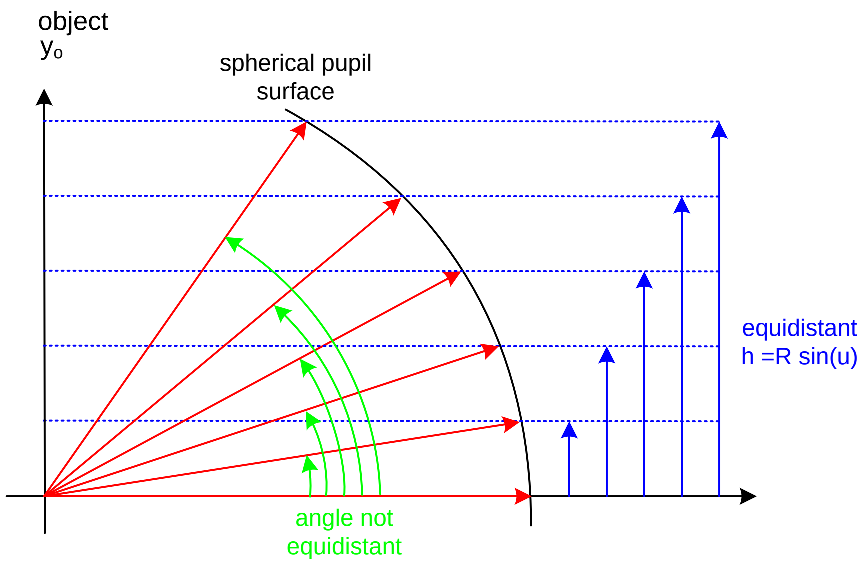
- Conservation of energy $$d^2P = d^2P'$$
- Invariant local differential flux $$d^2P = L \cdot \sin u \cdot \cos u \cdot dA \cdot du\, d\varphi$$
- Assumption: no absorption $$T = 1$$
- Delivers the sine condition $$n y \cdot \sin u = n' y' \cdot \sin u'$$



Pupil Sphere

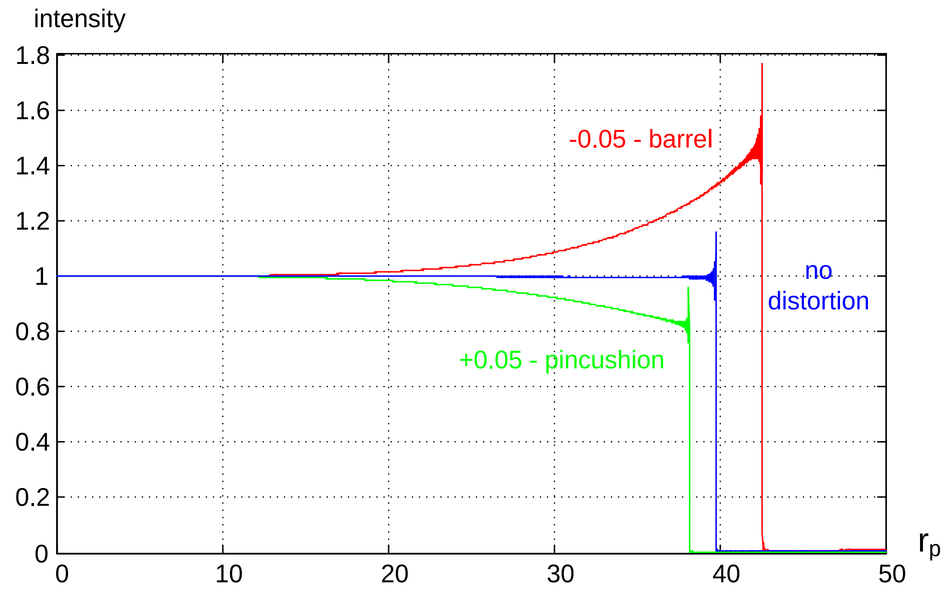
- Sine condition fulfilled: linear scaling from entrance to exit pupil
- Pupil surface must be sperical
- The pupil height scales with the sine of the angle

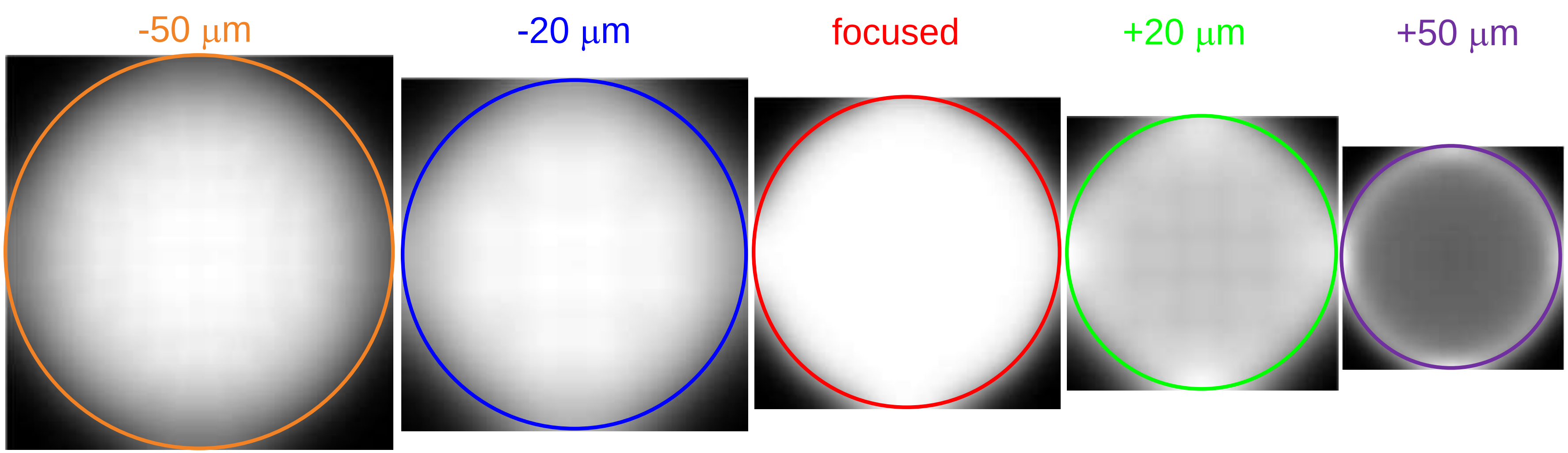




OSC and Apodization

- Photometric effect of pupil distortion: illumination changes at pupil boundary
- Effect induces apodization
- Sign of distortion determines the effect: outer zone of pupil brighter / darker
- Additional effect: absolute diameter of pupil changes

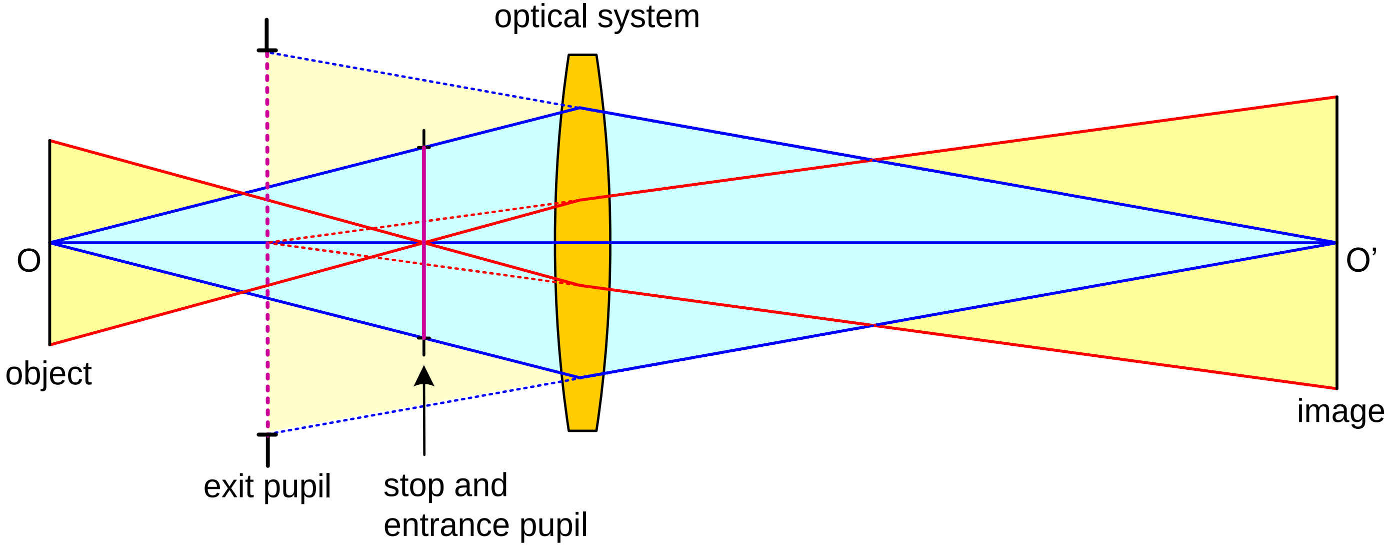




Pupil Aberration

- Interlinked imaging of field and pupil
- Distortion of object imaging corresponds to spherical aberration of the pupil imaging
- Corrected spherical pupil aberration: tangent condition

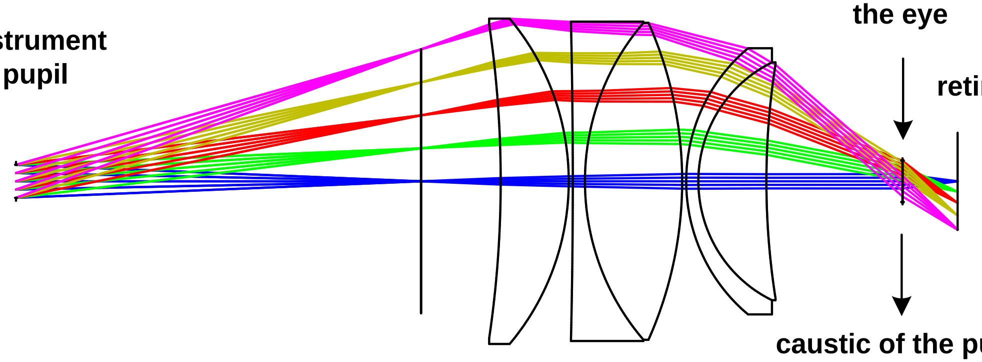
$$\frac{\tan w'}{\tan w} = const.$$



	Object imaging	Pupil imaging
Blue rays	Marginal rays	Chief rays
Red rays	Chief rays	Marginal rays

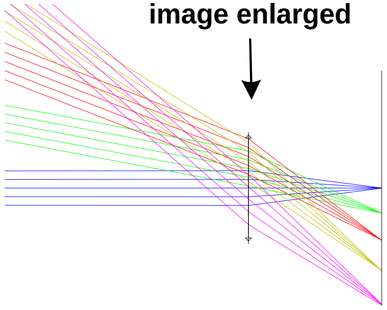
Pupil Aberration

- Eyepiece with pupil aberration

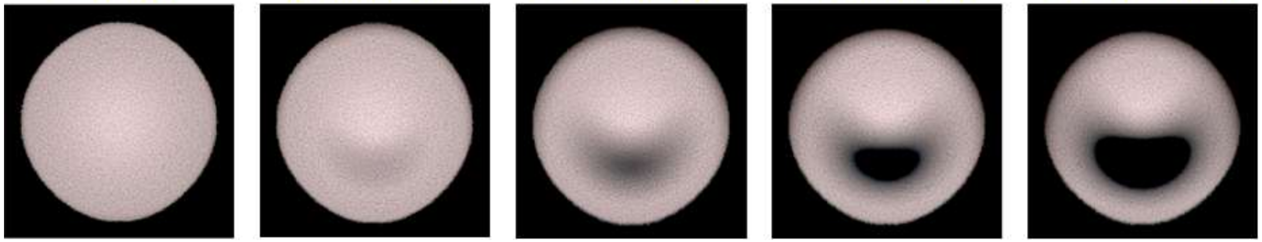


- Illumination for decentered pupil : dark zones due to vignetting

caustic of the pupil image enlarged



centered	decenter 3 mm	decenter 5 mm	decenter 6 mm	decenter 7 mm



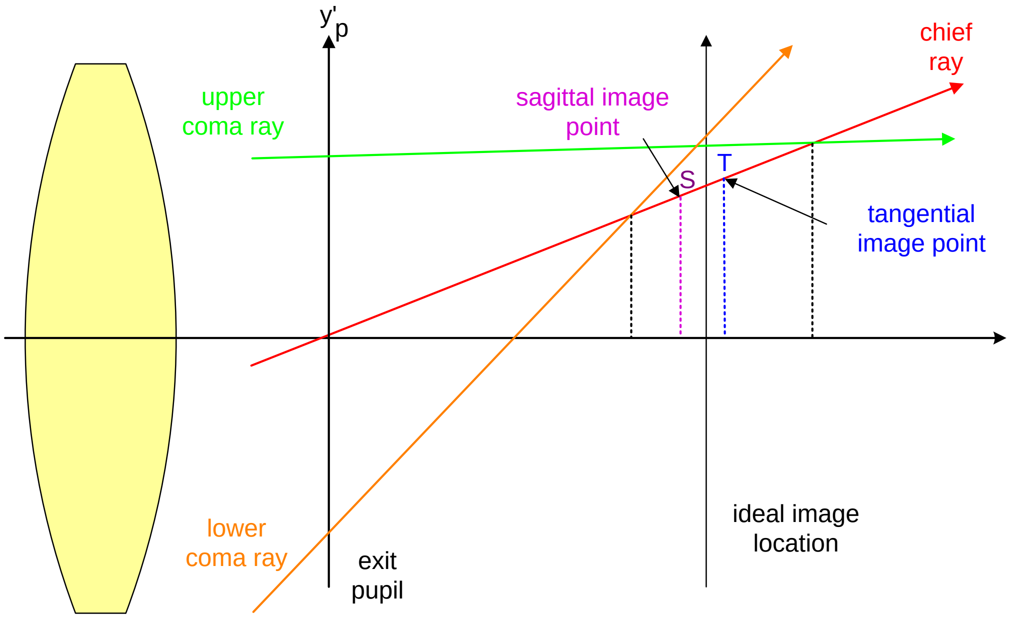
Skew Spherical aberration

- Decomposition of coma:
  1. part symmetrical around chief ray: skew spherical aberration

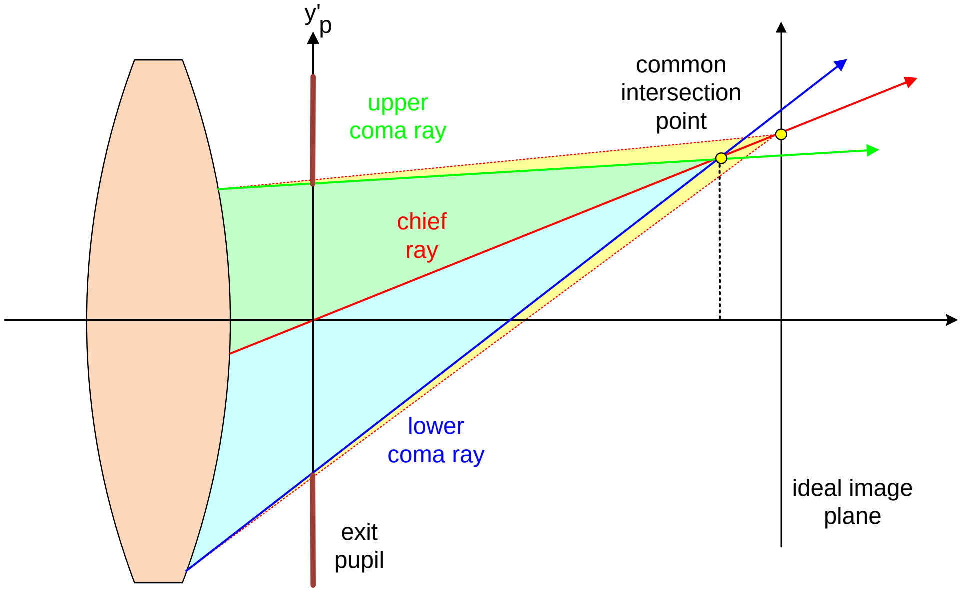
$$\Delta y_{skewsph} = \frac{\Delta y_{upcom} + \Delta y_{lowcom}}{2}$$

  2. asymmetrical part: tangential coma

$$\Delta y_{tangcoma} = \frac{\Delta y_{upcom} - \Delta y_{lowcom}}{2}$$



- Skew spherical aberration:
  - higher order aberration
  - caustic symmetric around chief ray



Isoplanatism

- General definition of isoplanatism:
  - Invariance of performance for small lateral shifts of the field position
  - spherical aberration not necessarily corrected
- Usual simple case: near to axis
- Consequences:
  - vanishing linear growing coma
  - caustic symmetrical arounf chief ray

Isoplanatism Condition of Staeble-Lihotzky

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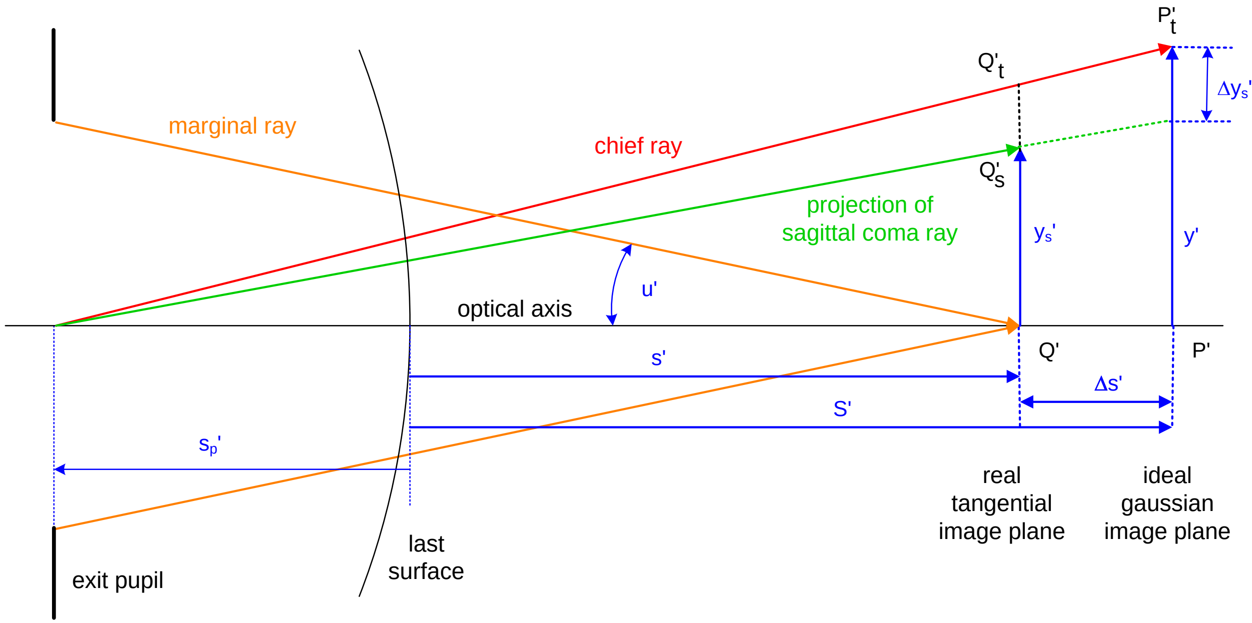
- Sagittal coma aberration:
  from the geometry of the figure and Lagrange invariant

$$\Delta y'_s = \frac{y'}{m} \cdot \left[\frac{n \sin u}{n' \sin u'} \cdot \frac{S' - s_p'}{S' - s_p' + \Delta s_{sph}'} - m \right]$$

- Condition of Staeble-Lihotzky

$$s' - s_p' = \frac{S' - s_p'}{m} \cdot \left(\frac{n \sin u}{n' \sin u'} - m \right)$$

- Problems:
  - no quantitative measure
  - only tangential rays are considered
  - integral criterion



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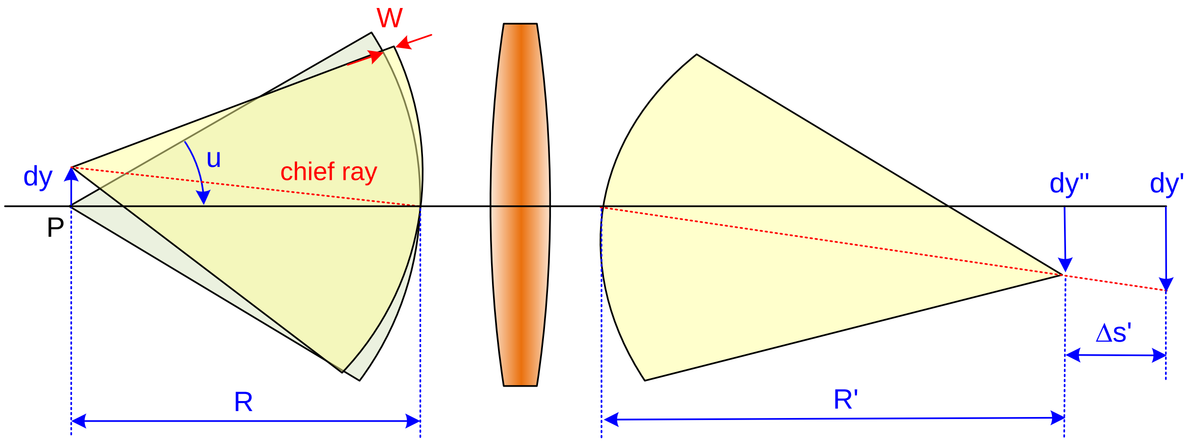
Isoplanatism from Wave Aberrations

- Lateral shift of object point

$$dW = dy \cdot n \cdot \sin u$$



- Change in image

$$dy'' = dy' \cdot \frac{R' + ds'}{R'} = m \cdot dy \cdot \frac{R' + ds'}{R'}$$

- Change of wave aberration must be equal

$$dW' = -dy'' \cdot n' \cdot \sin u' = -m \cdot dy \cdot \frac{R' + ds'}{R'} n' \sin u'$$

- Isoplanatism

$$n \cdot \sin u = -m \cdot \frac{R' + ds'}{R'} n' \sin u'$$

$$\frac{1}{m} \cdot \frac{n \cdot \sin u}{n' \cdot \sin u'} \cdot \frac{R' + ds'}{R'} - 1 = 0$$

Piecewise Isoplanatism

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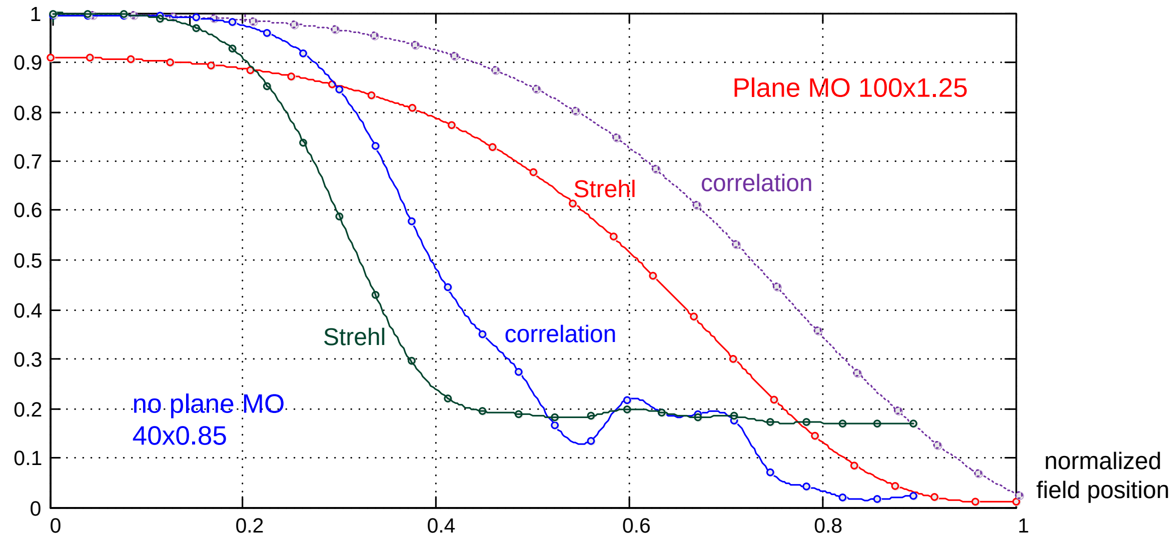
- Invariance of PSF: to be defined
- Possible options:
  1. relative change of Strehl
  2. correlation of PSF's
- Examples for microscopic lenses with and without flattening correction
- In medium field size: small isoplanatic patches
- On axis: large isoplanatic area
- Criteria not useful at the edge for low performance

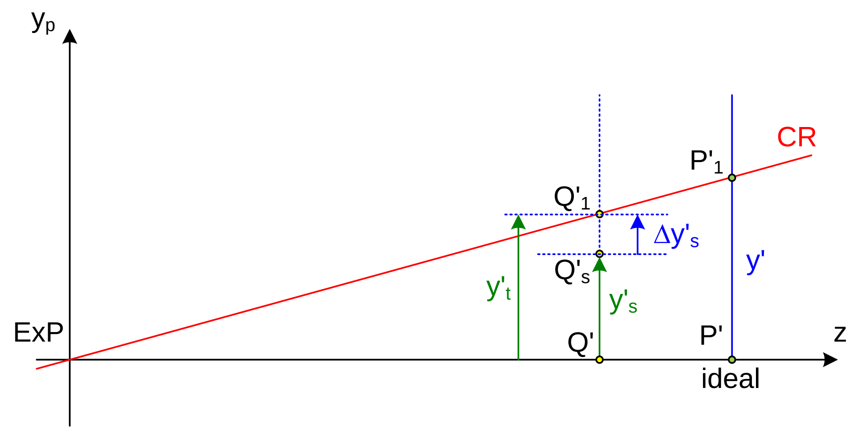
System	MO plane 100x1.25 isoplanatic patch size in µm		MO not plane 40x0.85 isoplanatic patch size in µm	
	Strehl 1%	Psf correlation 0.5%	Strehl 1%	Psf correlation 0.5%
on axis	70	72	81	100
half field	3.8	3.8	27	3.1
field zone	2.5	2.5	29	39
full field	45	3.8	117	62



Offence Against the Sine Condition

- Conradys OSC (offense against sine condition):
  - measurement of deviation of sagittal coma
  - quantitative validation of the sine condition

$$\Delta_{OSC} = \frac{y_t' - y_s'}{y_t'} = 1 - \frac{n \sin u}{m \cdot n' \sin u'} \cdot \frac{S' - s_p'}{s' - s_p'}$$



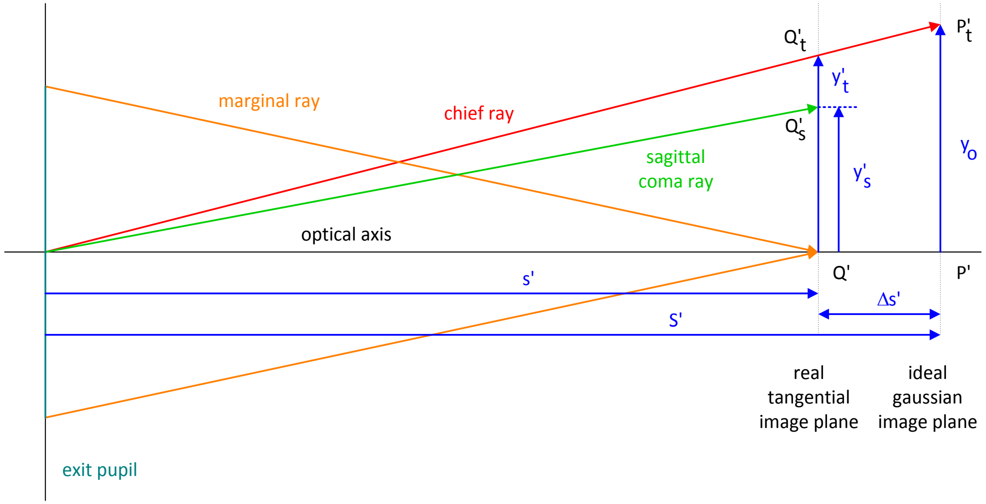
- Only sagittal coma considered
  in case of OSC=0 the Staeble-Lihotzky-condition is automatically fulfilled

$$W_{coma}(y, r_p, 0) = r_p \cdot y_t \cdot \Delta_{OSC}$$

$$\Delta y_t' = -3y \cdot \left(m - \frac{n \sin u}{n' \sin u'} \right)$$

- OSC allows for the definition of surface contribution

$$\Delta_{OSC} = \frac{\sin w_1}{\sin u_1} \cdot \sum_k \frac{(Q_k - Q'_k) \cdot n_k i_k^{(CR)}}{h'_k \, n'_k \, u'_k}$$



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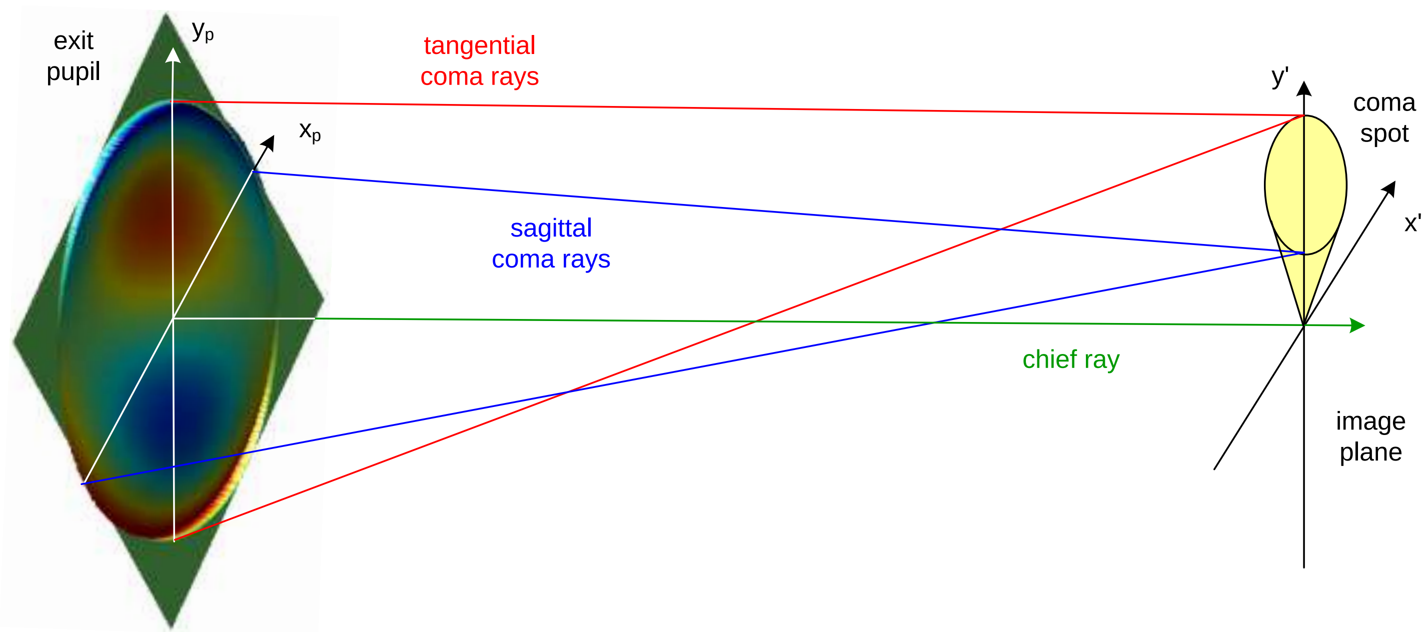
- Coma and isoplanatism are strongly connected



- Vectorial OSC:
  linear scaling of spatial frequencies: $$\vec{v}'_{apl} = \frac{1}{m_p} \cdot \vec{v}_{apl}$$

  perturbation of the linearity $$\Delta\vec{v} = \vec{v}' - \vec{v}'_{apl} = -\frac{1}{\lambda} \cdot \nabla_{x,y} W$$

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Overview Aplanatism-Isoplanatism

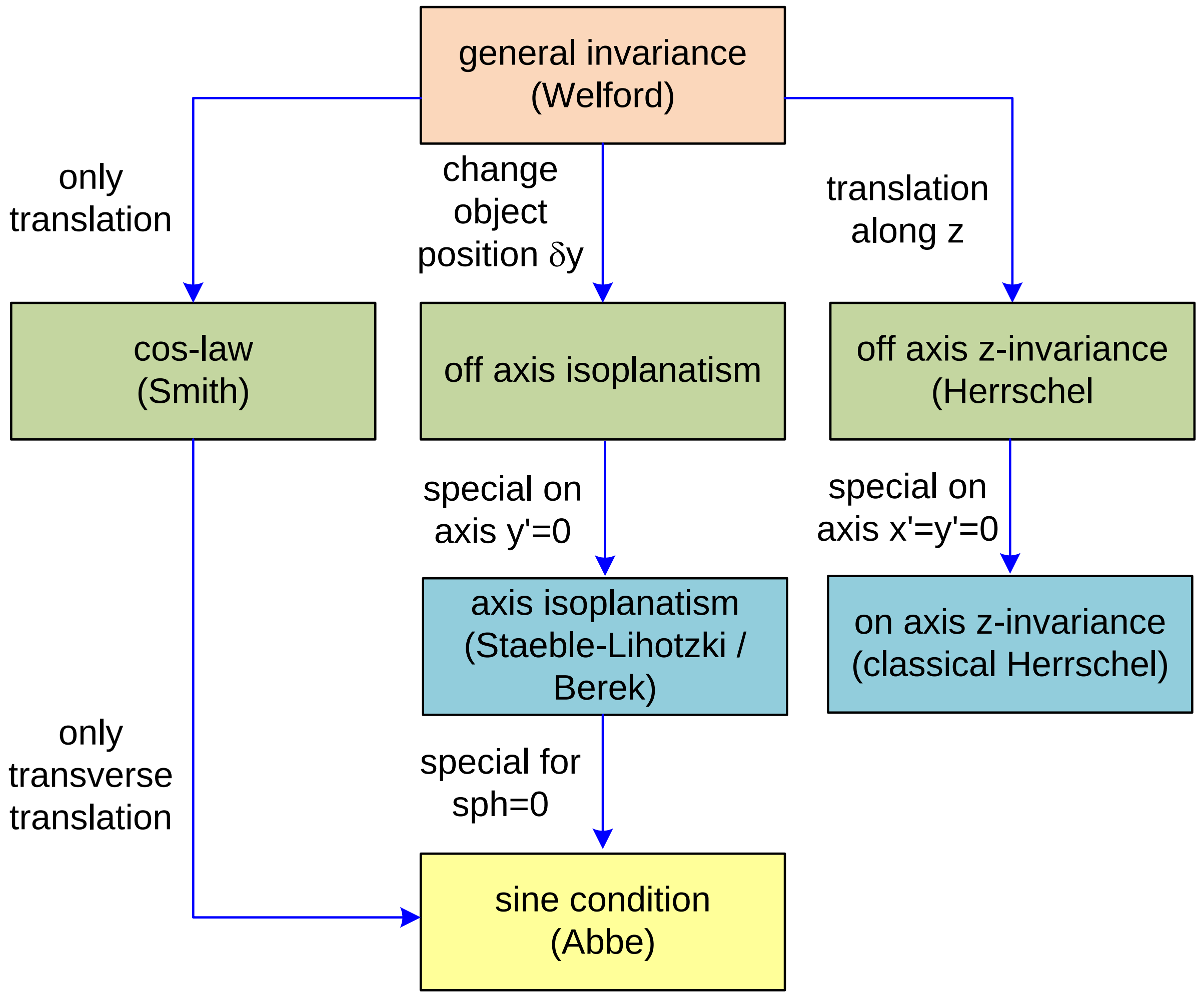
- Overview on conditions for aberrations and aplanatism-isoplanatism

Nr	Sine cond.	Iso-planat cond.	Isoplanatism condition	Spherical aberration	Sagittal coma	Tangential coma	Imaging system
1	#	#		#	#	#	general
2a	#	✓	OSC=0, Conrady	#	0	#	isoplanatic-I
2b	#	✓	Staeble-Lihotzky / Berek	#	0	0	isoplanatic-II
3a	✓	✓		0	0		axial aplanatic
3b	✓	✓		0 (skew)	0	0	off-axis aplanatic

	Isoplanatism Conrady OSC	Isoplanatism Staeble-Lihotzky	sine condition off-axis Aplanatism	sine condition axial Aplanatism
Tangential coma		0	0	
Sagittal coma	0	0	0	0
Spherical aberration				0
Skew Spherical aberration			0	0

Overview

- Overview on invariants and conditions



Telephoto Systems

- Combination of a positiv and a negative lens:
  Shift of the first principal plane in front of the system
- The intersection length is smaller than the focal length: reduction factor k
- Typical values: k = 0.6...0.9
- Focal lengths:

$$f_a = \frac{f'\cdot d}{f'\cdot(1-k)+d}$$

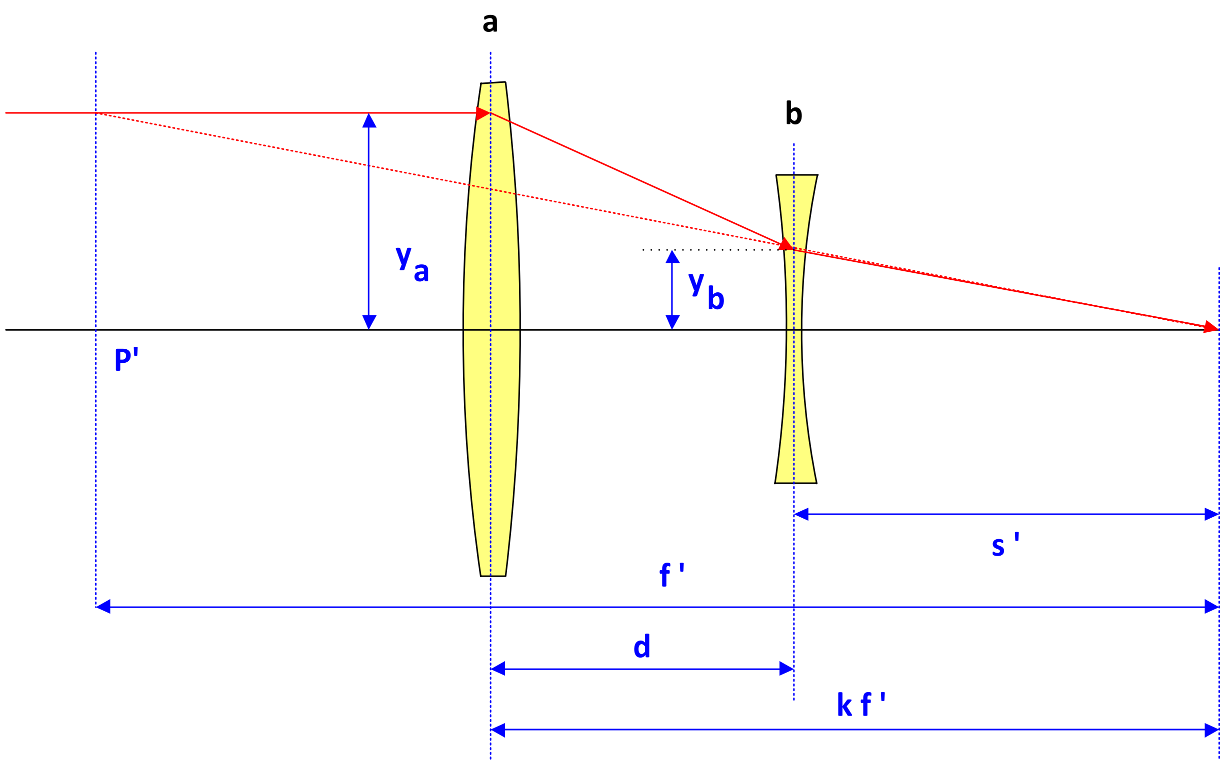
$$f_b = \frac{(f_a-d)(kf'-d)}{f_a-kf'}$$

- Overall length

$$L = k\cdot f'$$

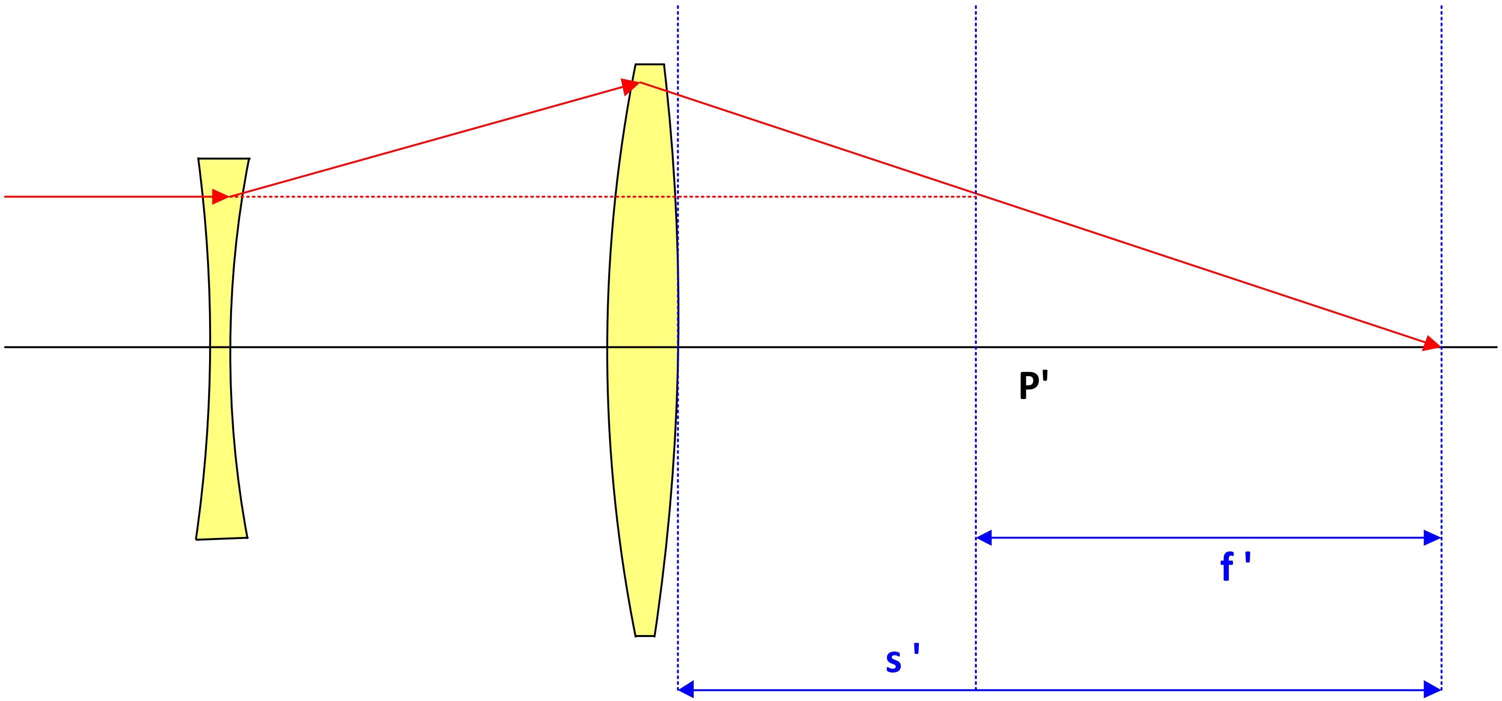
- Free intersection length

$$s_f = k\cdot f' - d$$



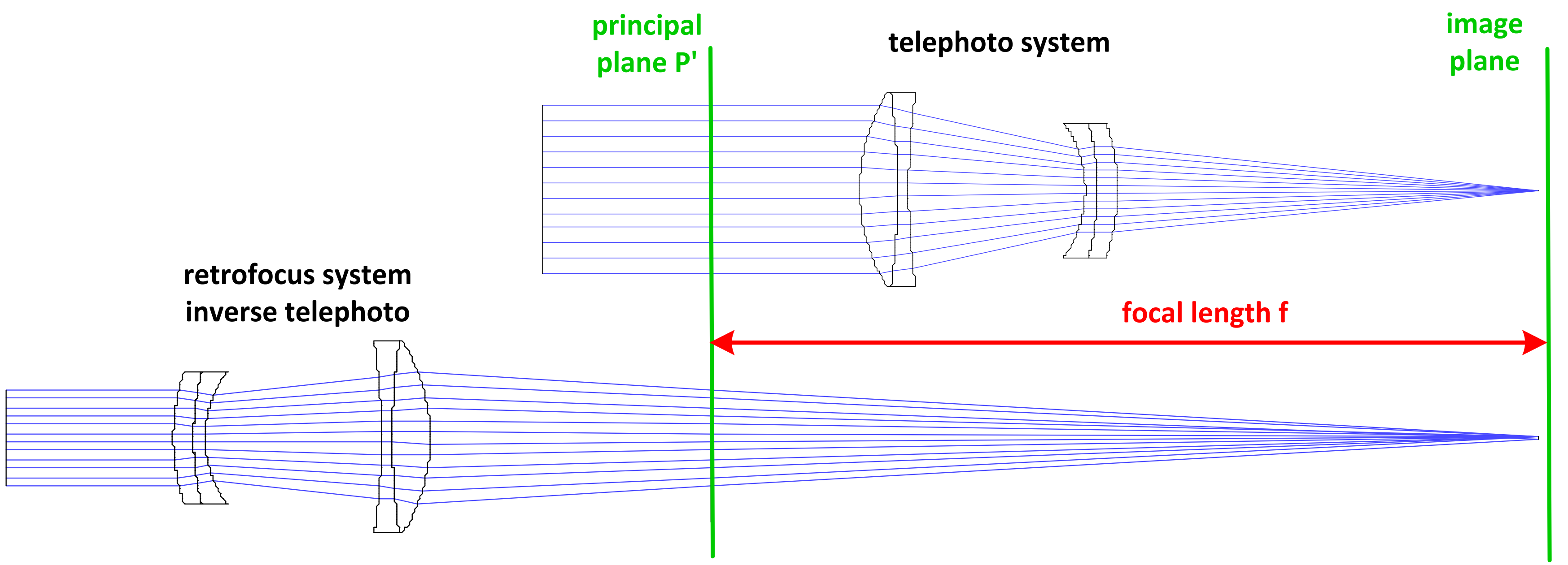
Retrofocal System

- Combination of a negative and a positive lens:
  Shift of the second principal plane behind the system
- The intersection length is larger than the focal length
- Application: systems for large free working distance
- Corresponds to an inverse telephoto system



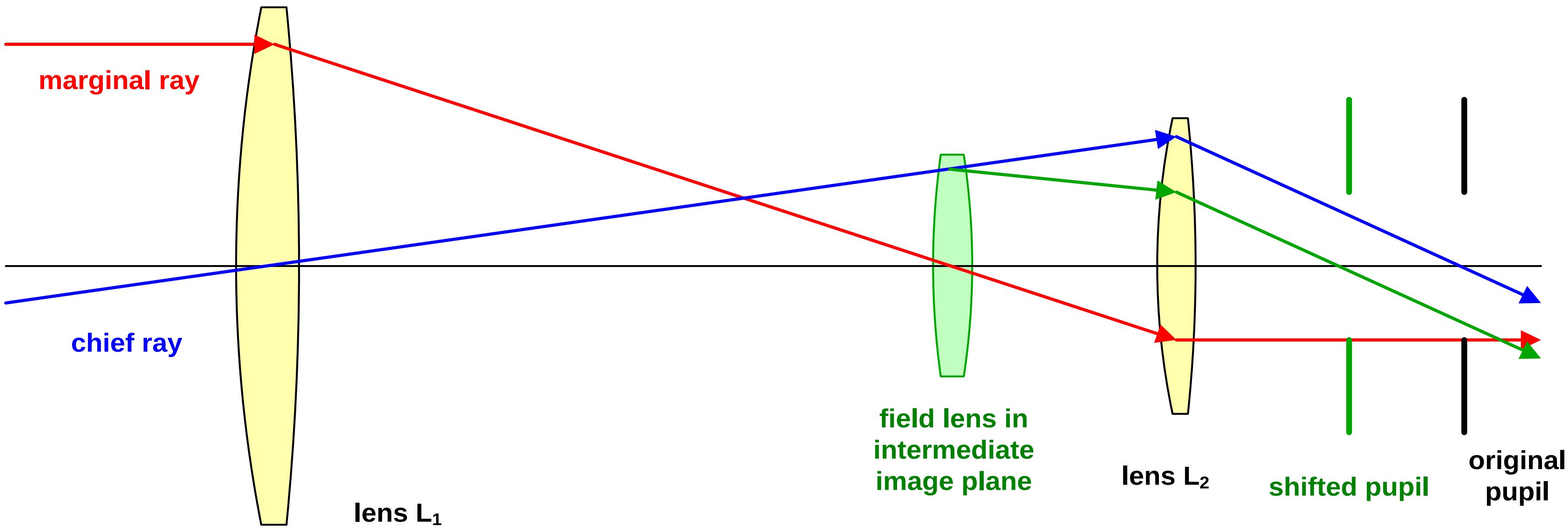
Telephoto and inverse Telephoto Principle

Retrofocus system results form a telephoto system by inversion

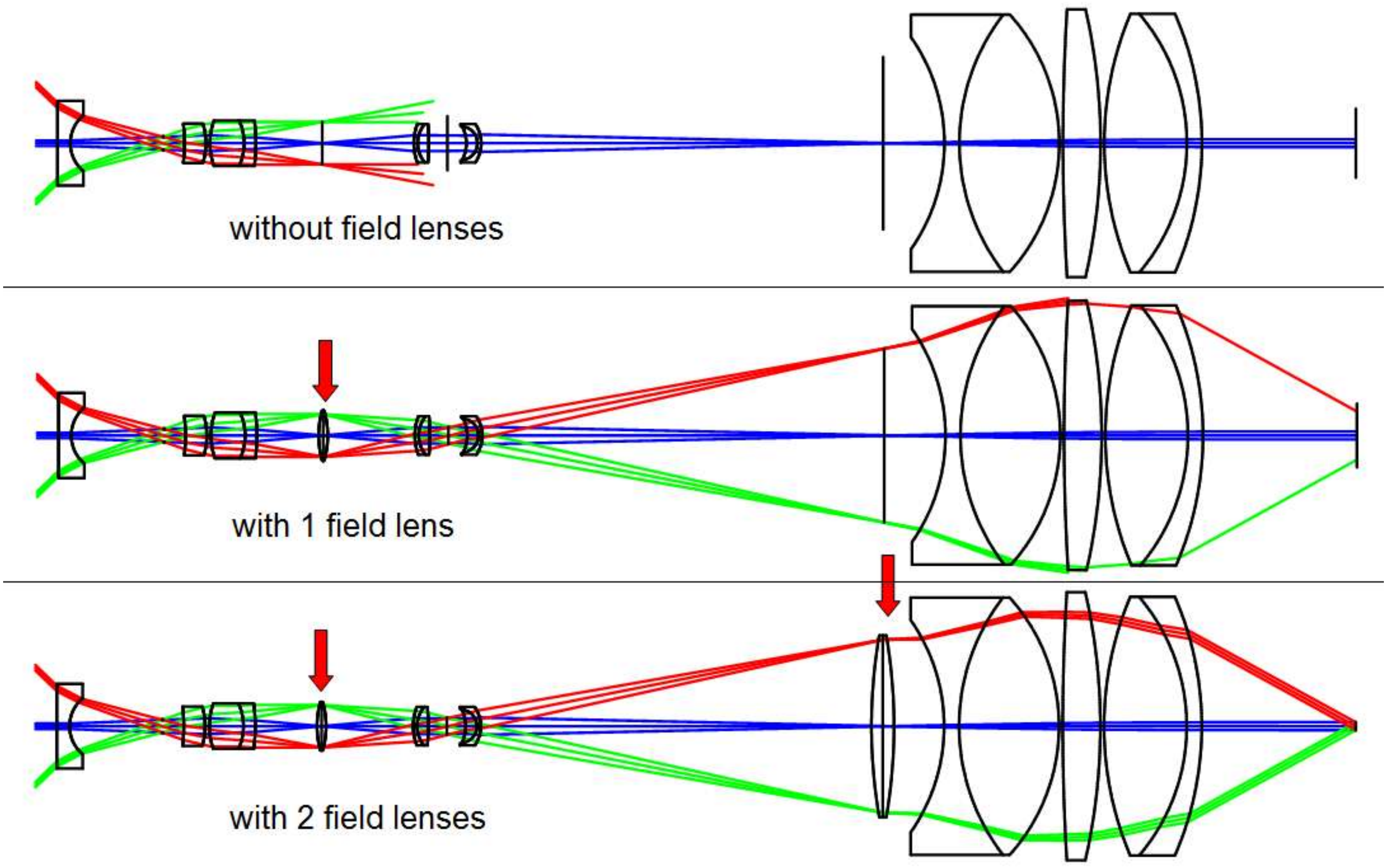


Field Lenses

- Field lens: in or near image planes
- Influences only the chief ray: pupil shifted
- Critical: conjugation to image plane, surface errors sharply seen



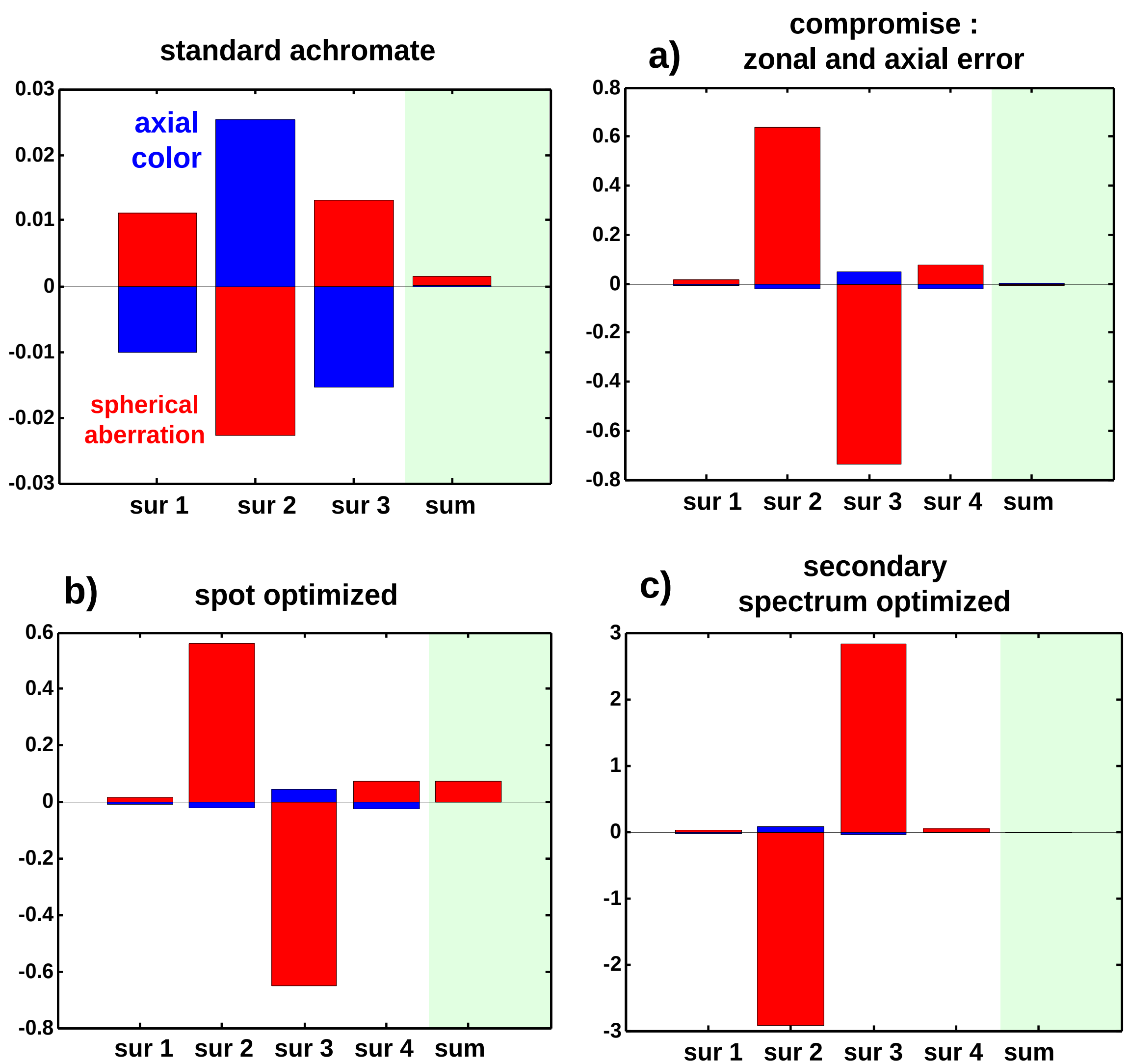
Field Lens im Endoscope



Ref : H. Zügge

Coexistence of Aberrations : Balance

- Example: Achromate
- Balance :
  1. zonal spherical
  2. Spot
  3. Secondary spectrum



Ref : H. Zügge

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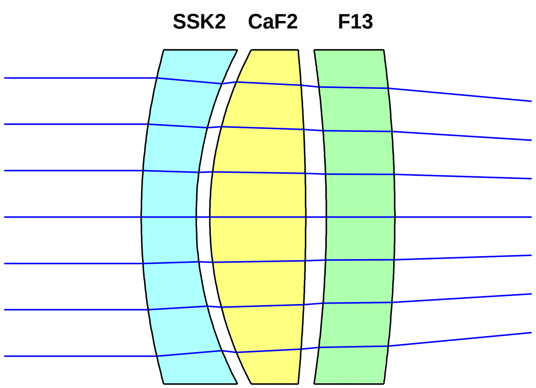

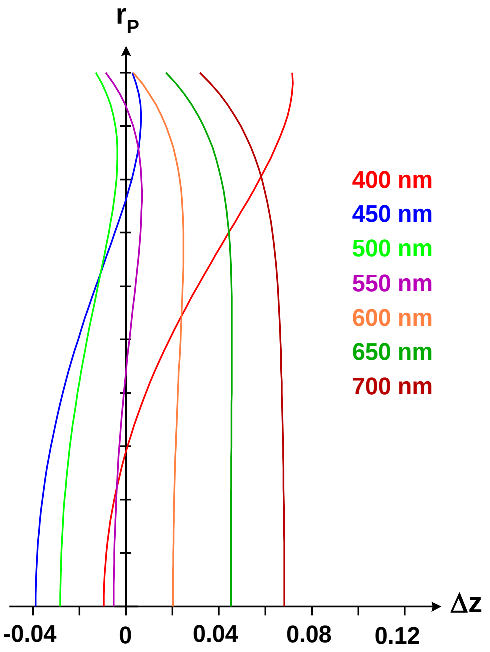

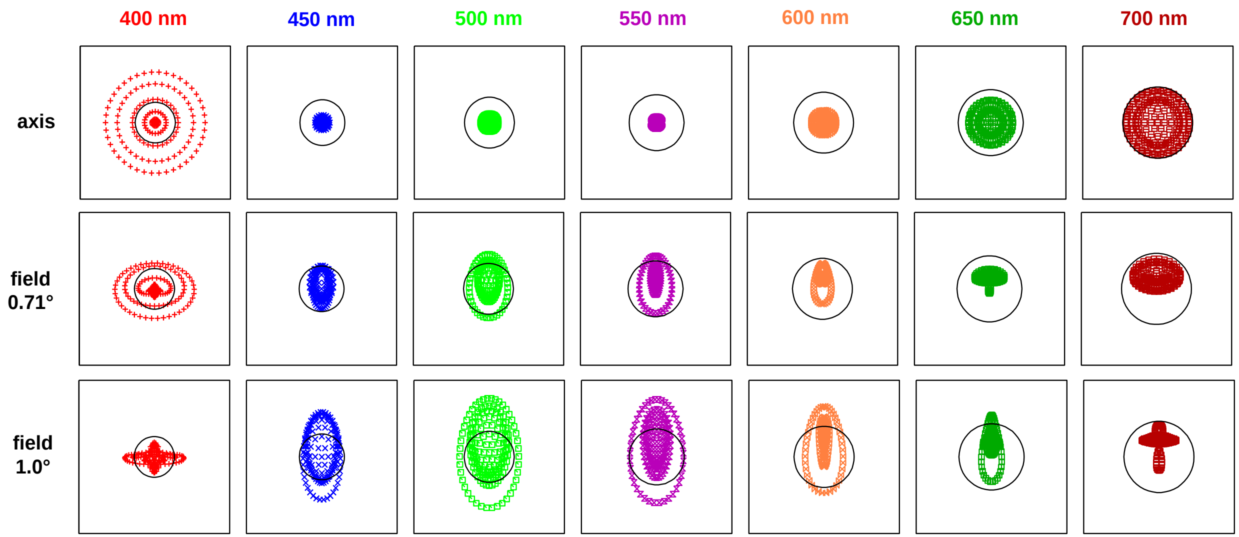

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Symmetrical Dublet

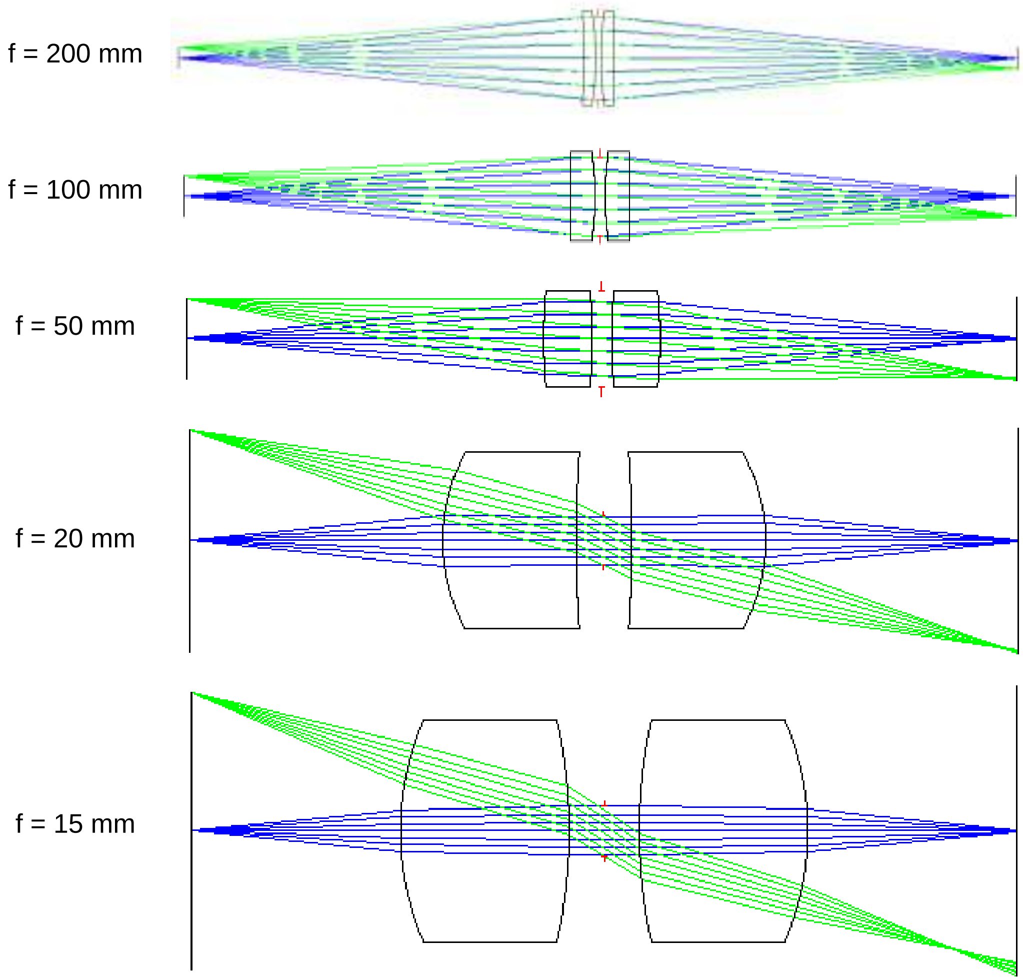
- Variable focal length
  f = 15 ...200 mm

- Invariant:
  object size y = 10 mm
  numerical aperture NA = 0.1

- Type of system changes:
  - dominant spherical for large f
  - dominant field for small f

- Data:

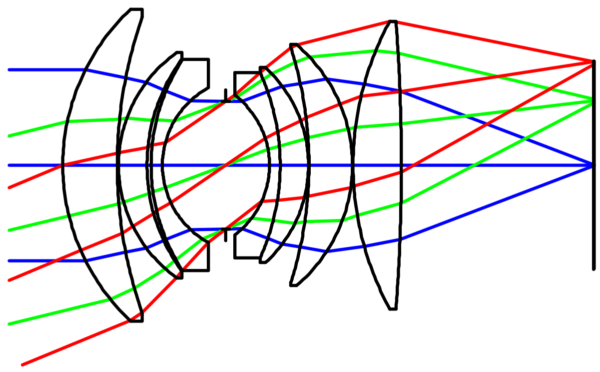
No	focal length [mm]	Length [mm]	spherical c_9	field curvature c_4	astigma-tism c_5
1	200	808	3.37	-2.01	-2.27
2	100	408	1.65	1.19	-4.50
3	50	206	1.74	3.45	-7.34
4	20	75	0.98	3.93	2.31
5	15	59	0.20	16.7	-5.33

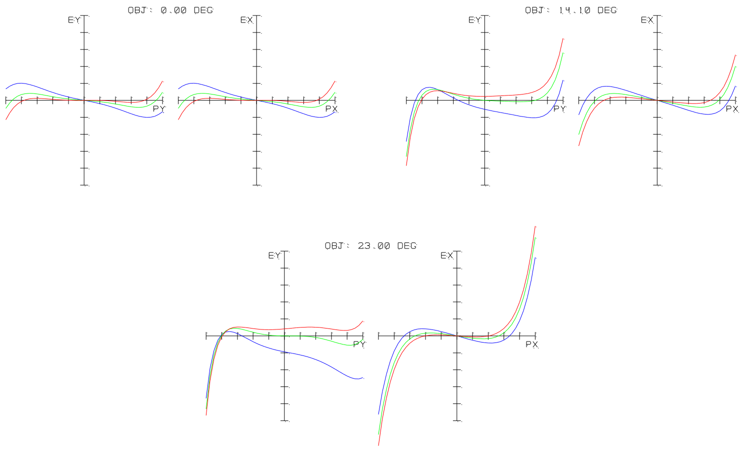


Aberrations Limited by Vignetting

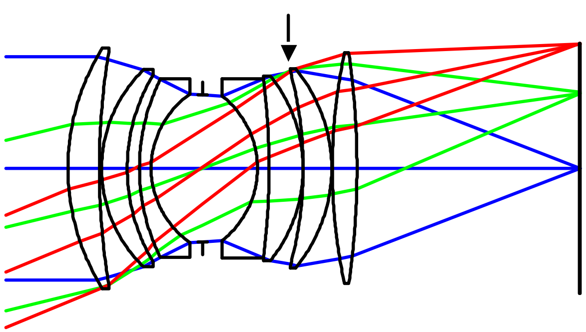
- Clipping of outer coma rays by vignetting
- Consequences:
  - reduced brightness
  - anisotropic resolution

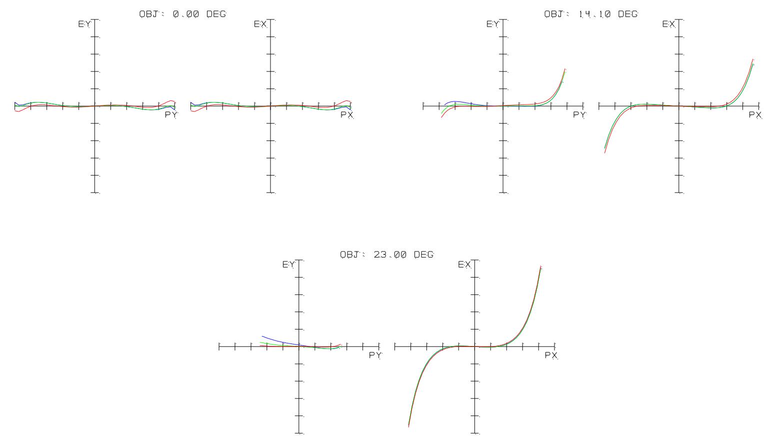
No Vignetting





With Vignetting





Ref: H.Zügge

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Anamorphotic Imaging Setup

- Anamorphotic imaging:
  different magnifications in x- and y-cross section,
  tangential and sagittal magnification

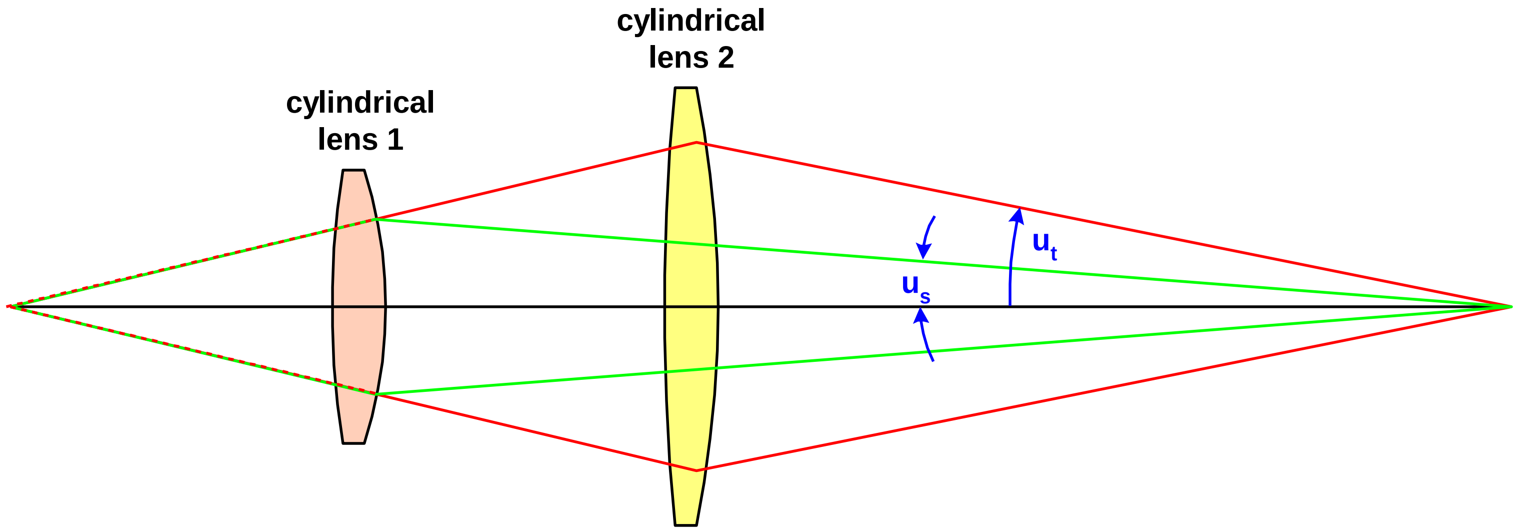
$$\beta_t = \frac{n_1 \cdot u_{t,1}}{n_k \cdot u_{t,k}}$$

$$\beta_s = \frac{n_1 \cdot u_{s,1}}{n_k \cdot u_{s,k}}$$

- Identical image location in both sections

- Anamorphotic factor

$$F_{anamoph} = \frac{\beta_s}{\beta_t}$$



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Anamorphotic Imaging Setup

Realization of an anamorphotic imaging with cylindrical lenses