Problem - 3 (HW #1 - ML) (CS 273A)

(a)

Maximum Likelihood Estimate.

- $P(y) \ \forall y \in \{-1, 1\}$

$$P(y = 1) = \frac{\sum_{i : y^{(i)}=1} y_i}{\sum_i 1} = 0.4$$

$$P(y = -1) = \frac{6}{10} = 0.6$$

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	$y = -1$				$y = 1$			
Feature	0's	1's	$P(x_i=1/y)$	$P(x_i=0/y)$	0's	1's	$P(x_i=1/y)$	$P(x_i=0/y)$
x_1	3	3	0.5	0.5	1	3	0.75	0.25
x_2	1	5	0.83	0.17	4	0	0	1
x_3	2	4	0.67	0.33	1	3	0.75	0.25
x_4	1	5	0.83	0.17	2	2	0.5	0.5
x_5	4	2	0.33	0.67	3	1	0.25	0.75

(b) Naive Bayes

(i) $\therefore$

$$P(y = 1 / x = \langle 0,0,0,0,0 \rangle)$$

$$= P(y=1) \times P(x_1=0/y=1) \times P(x_2=0/y=1) \ldots P(x_5=0/y=1).$$

$$= 0.4 \times 0.25 \times 1 \times 0.25 \times 0.5 \times 0.75$$

$$= 0.009$$

$$P(y = -1 / x = \langle 0,0,0,0,0 \rangle)$$

$$= 0.6 \times 0.5 \times 0.17 \times 0.33 \times 0.17 \times 0.67$$

$$= 0.002.$$

$\therefore$ The class chosen is $y = 1$.

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(ii) $P(Y=1 \mid x = (1\,1\,0\,1\,0))$

$= 0.4 \times 0.75 \times 0 \times 0.25 \times 0.5 \times 0.75 = 0$.

$P(Y=-1 \mid x = (1\,1\,0\,1\,0))$

$= 0.6 \times 0.5 \times 0.83 \times 0.33 \times 0.83 \times 0.67$

$= 0.046$.

∴ ~~the~~ chosen class is $y = -1$.

(c) $P(Y=1 \mid \underline{x} = \langle 1,1,0,1,0 \rangle)$

$= 0$

(d) 5 features (binary) ∴ we have 2^{32} parameters but 10 data points

∴ Bayes Classifier would overfit.