

$$(a) \quad p(y=1) = \frac{4}{10} \quad p(y=-1) = \frac{6}{10}$$

~~$$H(x) = H(y) = H(x=y) = p(x=1) \log \left(\frac{1}{p(y=1)} \right)$$~~

$$\begin{aligned} H(y) &= \frac{4}{10} \log \frac{10}{4} + \frac{6}{10} \log \frac{10}{6} \\ &= 0.4 \log \frac{5}{2} + 0.6 \log \frac{5}{3} \\ &= \log 5 - 0.4 - 0.6 \log 3 \end{aligned}$$

$$= 0.9710$$

$$(b) \quad IG_c(x_1) = H(y) - [p(x_1=1) \cdot H(y|x_1=1) + p(x_1=0) \cdot H(y|x_1=0)]$$

~~$$= 0.9710 - [(0.6) \left(\frac{1}{2} \right) + (0.4) \left(\frac{1}{4} \log_2 4 \right)]$$~~

$$= 0.9710 - [(0.6) \left(\frac{1}{2} \right) + (0.4) \left(\frac{1}{4} \log_2 4 \right)]$$

$$= 0.4710$$

$$\begin{aligned} IG(x_2) &= 0.9710 - \left[\left(\frac{1}{2} \right) (0) + \left(\frac{1}{2} \right) \frac{4}{5} \log \frac{5}{4} \right] \\ &= 0.8422 \end{aligned}$$

$$IG(x_3) = 0.9710 - \left[\frac{4}{10} \cdot \frac{3}{7} \log \left(\frac{7}{3} \right) + \frac{6}{10} \cdot \frac{2}{3} \log \frac{3}{2} \right] = 0.4872$$

$$IG(x_4) = 0.9710 - \left[\frac{1}{10} \cdot \frac{2}{3} \log \frac{7}{2} + \frac{8}{10} \cdot \frac{2}{3} \log \frac{3}{2} \right] = 0.2 \left[\log 7 + \log 3 \right]$$

$$IG(x_5) = 0.9710 - \left[\frac{3}{10} \cdot \frac{2}{3} \log \frac{3}{2} + \frac{7}{10} \cdot \frac{3}{7} \log \frac{7}{3} \right]$$

$$= 0.9710 - \left(0.2 \log \frac{3}{2} + 0.3 \log \frac{7}{3} \right) = 0.4925$$

$$= 0.9710 - 0.4925$$

