RCC INSTITUTE OF INFORMATION TECHNOLOGY

श्रमम् बिना न किमपि साध्यम्

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1. Show that for any two sets A and B,
$P(A) \cup P(B) \subseteq P(A \cup B)$
$P(A) \cap P(B) \subseteq P(A \cap B)$ where $P(X)$ is the power set of X.

SOLUTION:

$P(A) \cup P(B) \subseteq P(A \cup B)$

Let,

$d \in P(A) \cup P(B)$
$\Rightarrow d \in P(A)$ or $d \in P(B)$
$\Rightarrow d \subseteq A$ or $d \subseteq B$
$\Rightarrow d \subseteq A \cup B$
$\Rightarrow d \in P(A \cup B)$

Therefore,

$P(A) \cup P(B) \subseteq P(A \cup B)$

Hence, shown.

$P(A) \cap P(B) \subseteq P(A \cap B)$

Let,

$d \in P(A) \cap P(B)$
$\Rightarrow d \in P(A)$ and $d \in P(B)$
$\Rightarrow d \subseteq A$ and $d \subseteq B$
$\Rightarrow d \subseteq A \cap B$
$\Rightarrow d \in P(A \cap B)$

Therefore,

$P(A) \cap P(B) \subseteq P(A \cap B)$

Hence, shown.

2. Prove that for sets A,B,C if $A \cup C = B \cup C$ and $A \cap C = B \cap C$ then $A=B$.

SOLUTION:

Indeed,

$A = A \cup (A \cap C)$ [Law of absorption]

$= A \cup (B \cap C)$ [since, A=B, given]

$= (A \cup B) \cap (A \cup C)$ [by distributivity)

$= (A \cup B) \cap (B \cup C)$ [since A=B, given]

$= (A \cup B) \cap (C \cup B)$ [by commutivity]

$= (A \cap C) \cup B$ [by distributivity]

$= (B \cap C) \cup B$ [since, A=B, given]

$= B$ [by law of Absorption]

Therefore, A=B

Hence, shown.

3. Prove that (A-B)∪(B-A)=(A∪B) - (A∩B)

SOLUTION: We will prove this by showing that each side is a subset of the other.

Part 1: $(A-B)\cup(B-A)\subseteq(A\cup B)-(A\cap B)$

Let $x\in(A-B)\cup(B-A)$. This means $x\in(A-B)$ or $x\in(B-A)$.

- **Case 1:** $x\in(A-B)$. This means $x\in A$ and $x\notin B$. Since $x\in A$, we know $x\in(A\cup B)$. Since $x\notin B$, x cannot be in $(A\cap B)$. Therefore, $x\in(A\cup B)-(A\cap B)$.
- **Case 2:** $x\in(B-A)$. This means $x\in B$ and $x\notin A$. Since $x\in B$, we know $x\in(A\cup B)$. Since $x\notin A$, x cannot be in $(A\cap B)$. Therefore, $x\in(A\cup B)-(A\cap B)$.

In either case, $x\in(A\cup B)-(A\cap B)$. Thus, $(A-B)\cup(B-A)\subseteq(A\cup B)-(A\cap B)$.

Part 2: $(A\cup B)-(A\cap B)\subseteq(A-B)\cup(B-A)$

Let $x\in(A\cup B)-(A\cap B)$. This means $x\in(A\cup B)$ and $x\notin(A\cap B)$. Since $x\in(A\cup B)$, we know $x\in A$ or $x\in B$ (or both). Since $x\notin(A\cap B)$, we know that x is NOT in both A and B.

- **Case 1:** $x\in A$. Since x is not in both A and B, it must be that $x\notin B$. Therefore, $x\in(A-B)$. Thus, $x\in(A-B)\cup(B-A)$.
- **Case 2:** $x\in B$. Since x is not in both A and B, it must be that $x\notin A$. Therefore, $x\in(B-A)$. Thus, $x\in(A-B)\cup(B-A)$.

In either case, $x\in(A-B)\cup(B-A)$. Thus, $(A\cup B)-(A\cap B)\subseteq(A-B)\cup(B-A)$.

Conclusion

Since $(A-B)\cup(B-A)\subseteq(A\cup B)-(A\cap B)$ and $(A\cup B)-(A\cap B)\subseteq(A-B)\cup(B-A)$, we can conclude that:
$(A-B)\cup(B-A)=(A\cup B)-(A\cap B)$

4. Define: a. Equivalence relation and compatibility relation b. Equivalence class c. Partition d. Covering of a set e. Partial order relation

SOLUTION:

a. Equivalence Relation and Compatibility Relation

Equivalence Relation: An equivalence relation on a set is a relation that is reflexive, symmetric, and transitive.

Reflexivity: For every element (a) in set (A), (a \sim a).
Symmetry: For any elements (a, b) in set (A), if (a \sim b), then (b \sim a).
Transitivity: For any elements (a, b, c) in set (A), if (a \sim b) and (b \sim c), then (a \sim c).

Example: The relation (a \equiv b \mod 3) on integers, where (a) is equivalent to (b) if (a - b) is divisible by 3.

Compatibility Relation: A compatibility relation allows certain structures to be preserved under operations.

Example: In a group, two elements (a) and (b) are compatible if (a \cdot b) (the group operation) is also in the group.

b. Equivalence Class
An equivalence class is a subset of a set formed by grouping all elements that are equivalent under a given equivalence relation.

Example: For (a \equiv b \mod 3), the equivalence classes are:

([0] = {..., -6, -3, 0, 3, 6, ...})
([1] = {..., -5, -2, 1, 4, 7, ...})
([2] = {..., -4, -1, 2, 5, 8, ...})

c. Partition
A partition of a set is a grouping of its elements into non-empty subsets, such that every element is included in exactly one subset.

Example: For the set (S = {1, 2, 3, 4}), a partition could be ({{1, 2}, {3}, {4}}).

d. Covering of a Set
A covering of a set is a collection of subsets whose union contains the entire set.

Example: For (S = {1, 2, 3, 4}), a covering could be ({ {1, 2}, {2, 3}, {3, 4} }), since their union is ({1, 2, 3, 4}).

e. Partial Order Relation
A partial order relation is a binary relation that is reflexive, antisymmetric, and transitive.

Example: The relation (\leq) on the set of integers (\mathbb{Z}) is a partial order. For instance, (2 \leq 3) and if (2 \leq 3) and (3 \leq 4), then (2 \leq 4) (transitivity).

5. If relations R and S are reflexive,symmetric and transitive,show that R∩S is also reflexive,symmetric and transitive.

SOLUTION:

Let R and S be reflexive, symmetric, and transitive relations on a set A. We want to show that R ∩ S is also reflexive, symmetric, and transitive.

Reflexive:

Since R is reflexive, (a, a) ∈ R for all a ∈ A.
Since S is reflexive, (a, a) ∈ S for all a ∈ A.
Therefore, (a, a) ∈ R ∩ S for all a ∈ A.
Thus, R ∩ S is reflexive.

Symmetric:

Assume (a, b) ∈ R ∩ S.
This means (a, b) ∈ R and (a, b) ∈ S.
Since R is symmetric, (a, b) ∈ R implies (b, a) ∈ R.
Since S is symmetric, (a, b) ∈ S implies (b, a) ∈ S.
Therefore, (b, a) ∈ R and (b, a) ∈ S.
This means (b, a) ∈ R ∩ S.
Thus, R ∩ S is symmetric.

Transitive:

Assume (a, b) ∈ R ∩ S and (b, c) ∈ R ∩ S.

This means (a, b) ∈ R, (a, b) ∈ S, (b, c) ∈ R, and (b, c) ∈ S.
Since R is transitive, (a, b) ∈ R and (b, c) ∈ R implies (a, c) ∈ R.
Since S is transitive, (a, b) ∈ S and (b, c) ∈ S implies (a, c) ∈ S.
Therefore, (a, c) ∈ R and (a, c) ∈ S.
This means (a, c) ∈ R ∩ S.
Thus, R ∩ S is transitive.

In conclusion we can say that,

Since R ∩ S is reflexive, symmetric, and transitive, the intersection of two reflexive, symmetric, and transitive relations is also reflexive, symmetric, and transitive.

6. Let X= {ball, bed,dog,let,egg} and R is a relation defined on X as R={(x,y) | x and y contain some common letters}. Show that R is a compatibility relation and also find maximum compatibility blocks for R.

SOLUTION:

1. Compatibility Relation:

A relation R on a set A is called a compatibility relation if it satisfies the following properties:

- Reflexivity: For every $x \in A$, $(x, x) \in R$.
- Symmetry: For every $x, y \in A$, if $(x, y) \in R$, then $(y, x) \in R$.

2. Reflexivity:

- For each word $x \in A$, x contains itself so $(x, x) \in R$.
- For example, $(\text{ball}, \text{ball}) \in R$, $(\text{bed}, \text{bed}) \in R$, etc.
- Therefore, R is reflexive.

- Since sharing a common letter is a symmetric property, if x shares a letter with y, then y also shares a letter with x.
- For example, $(\text{ball}, \text{bed}) \in R$ because both contain the letter 'b', and $(\text{bed}, \text{ball}) \in R$ for the same reason.
- Therefore, R is symmetric.

4. Transitivity:

- To show that R is not transitive, we need to find elements $x, y, z \in A$ such that $(x, y) \in R$ and $(y, z) \in R$, but $(x, z) \notin R$.
- Consider the words ball, bed, and dog:
    - $(\text{ball}, \text{bed}) \in R$ because both contain the letter 'b'.
    - $(\text{bed}, \text{dog}) \in R$ because both contain the letter 'd'.
    - However, $(\text{ball}, \text{dog}) \notin R$ because they do not share any common letters.
- Therefore, R is not transitive.

Conclusion:

The relation R is a compatibility relation because it is reflexive and symmetric. However, R is not transitive, as demonstrated by the counterexample involving the words "ball", "bed", and "dog".

Now,
ball: shares 'b' with bed.
bed: shares 'b' with ball and 'e' with let and egg.
dog: shares 'd' with bed.
let: shares 'e' with bed and egg.
egg: shares 'e' with bed and let.
Now, we can form the following compatibility blocks:

{ball, bed}: Both share 'b'.
{bed, let, egg}: All share 'e' and 'b'.
{bed, dog}: Both share 'd'.
{let, egg}: Both share 'e'.
The largest compatibility blocks are:

{bed, let, egg}: This block contains three elements where every pair shares at least one common letter.
Thus, the maximum compatibility block for (R) is {bed, let, egg}.

Given S={1,2,3,4} and relation R on S defined by R={(1,2),(4,3),(2,2),(2,1),(3,1)}. Show that R is not transitive. Find a relation R1 ≥ R such that R1 is transitive. Can you find another relation R2 ≥ R which is also transitive.

SOLUTION:

We are given the set S={1,2,3,4} and the relation R on S, defined by the set of ordered pairs R={(1,2),(4,3),(2,2),(2,1),(3,1)}.

Step 1: Check if R is transitive.

A relation R is transitive if whenever (a,b)∈R and (b,c)∈R, it follows that (a,c)∈R.

Let's check if R is transitive by looking for pairs (a,b) and (b,c) in R, and seeing if (a,c) is also in R:
For (1,2) and (2,2):

We have (1,2) and (2,2) in R, so we need to check if (1,2) is in R.
Since (1,2)∈R, the c

We have (2,1) and (1,2) in R, so we need to check if (2,2) is in R.
Since (2,2)∈R, this condition is satisfied.
For (4,3) and (3,1):

We have,
(4,3) and (3,1) in R, so we need to check if (4,1) is in R.

However,
(4,1)∉R, so R is not transitive.
Since we found a counterexample where (4,3) and (3,1) imply (4,1) is not in R, we conclude that R is not transitive.

Step 2: Find a relation R1≥R such that R1 is transitive

From the counterexample above, we need to add the pair

Thus, we can define the new relation R1 by adding (4,1) to R: R1 $=R\cup\{(4,1)\}=\{(1,2),(4,3),(2,2),(2,1),(3,1),(4,1)\}$

Now, let's check if
R1 is transitive:

We have already verified that all the existing pairs in R were consistent with transitivity.
With the addition of (4,1), we ensure that the relation is transitive by closing the gaps.

Thus, $R1=\{(1,2),(4,3),(2,2),(2,1),(3,1),(4,1)\}$ is transitive.

Step 3: Find another relation

To ensure transitivity, we could add (1,1), (2,3), and any other pairs that might be necessary for closure.

Let's try:
$R2=R\cup\{(4,1),(1,1),(2,3)\}=\{(1,2),(4,3),(2,2),(2,1),(3,1),(4,1),(1,1),(2,3)\}$

Now, let's check if R2 is transitive:

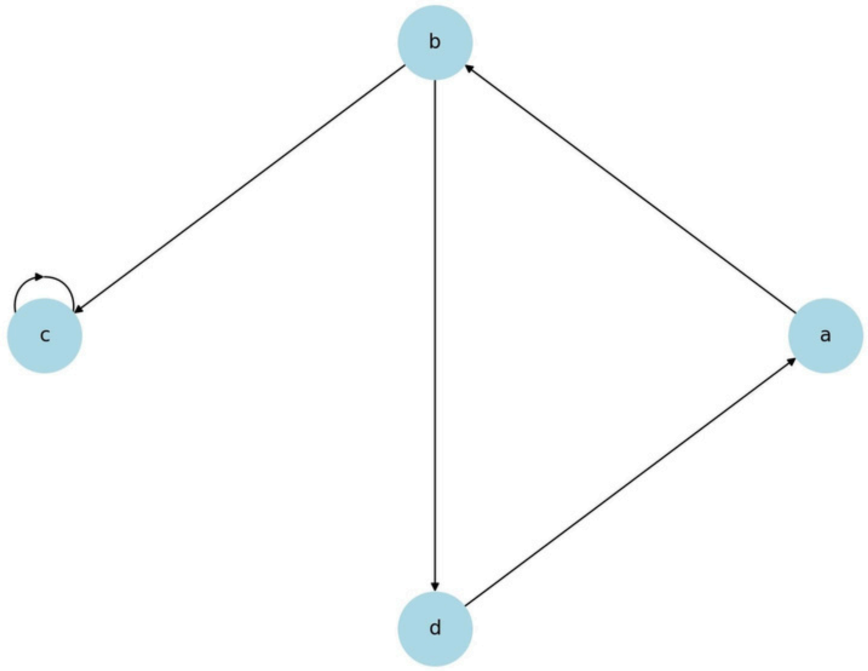
By adding these extra pairs, we ensure that any pair that would be a composition of two existing pairs now has a corresponding pair in the relation.

Thus,
$R=\{(1,2),(4,3),(2,2),(2,1),(3,1),(4,1),(1,1),(2,3)\}$ is also transitive.

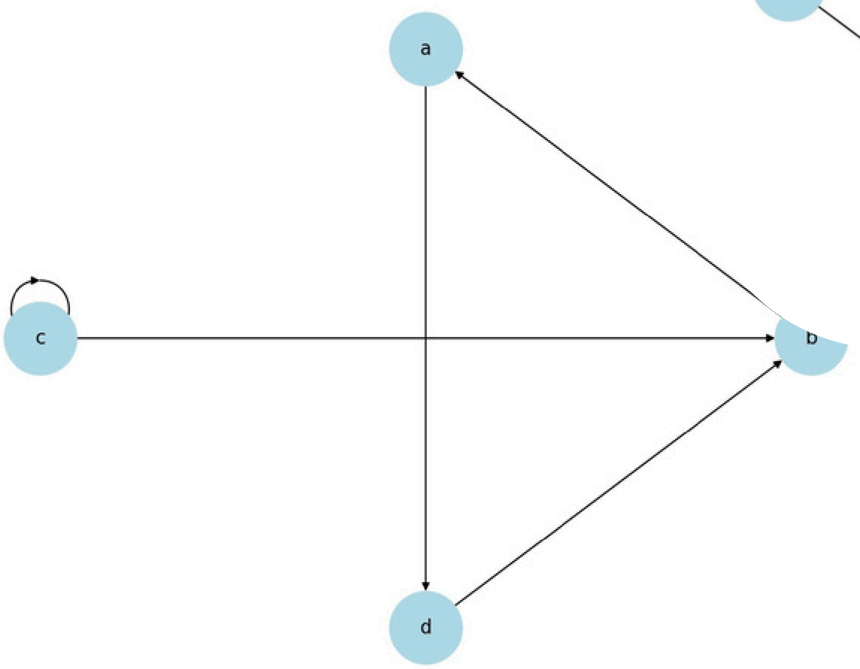
7. Consider the relation R ={(a,b),(b,c),(b,d),(d,a),(c,c)}.
a. Draw a digraph for the relation R.
b. Draw a digraph for the relation inverse of R that is R-1 .
c. Draw a digraph for the relation complement of R, that is RC.
d. Draw a digraph for the relation intersection of R and inverse of R, R∩R-1.

SOLUTION:

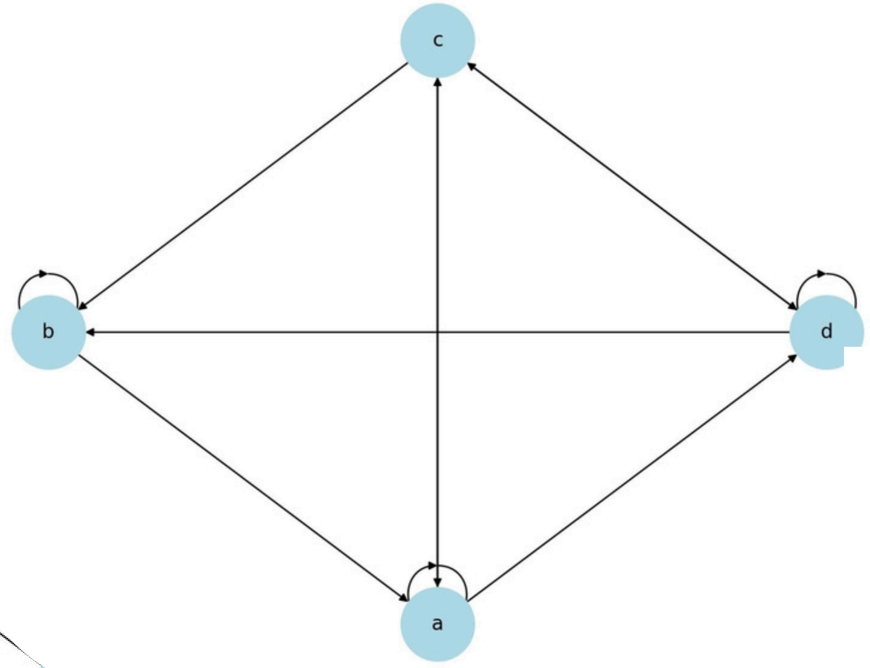
(a) Digraph for the relation R:



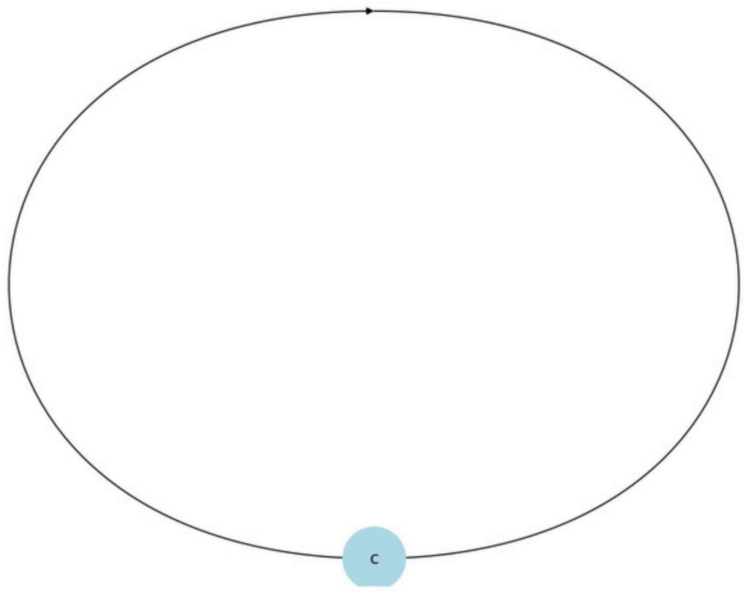
(b) Digraph for the relation R-1:



(c) Digraph for the relation R':



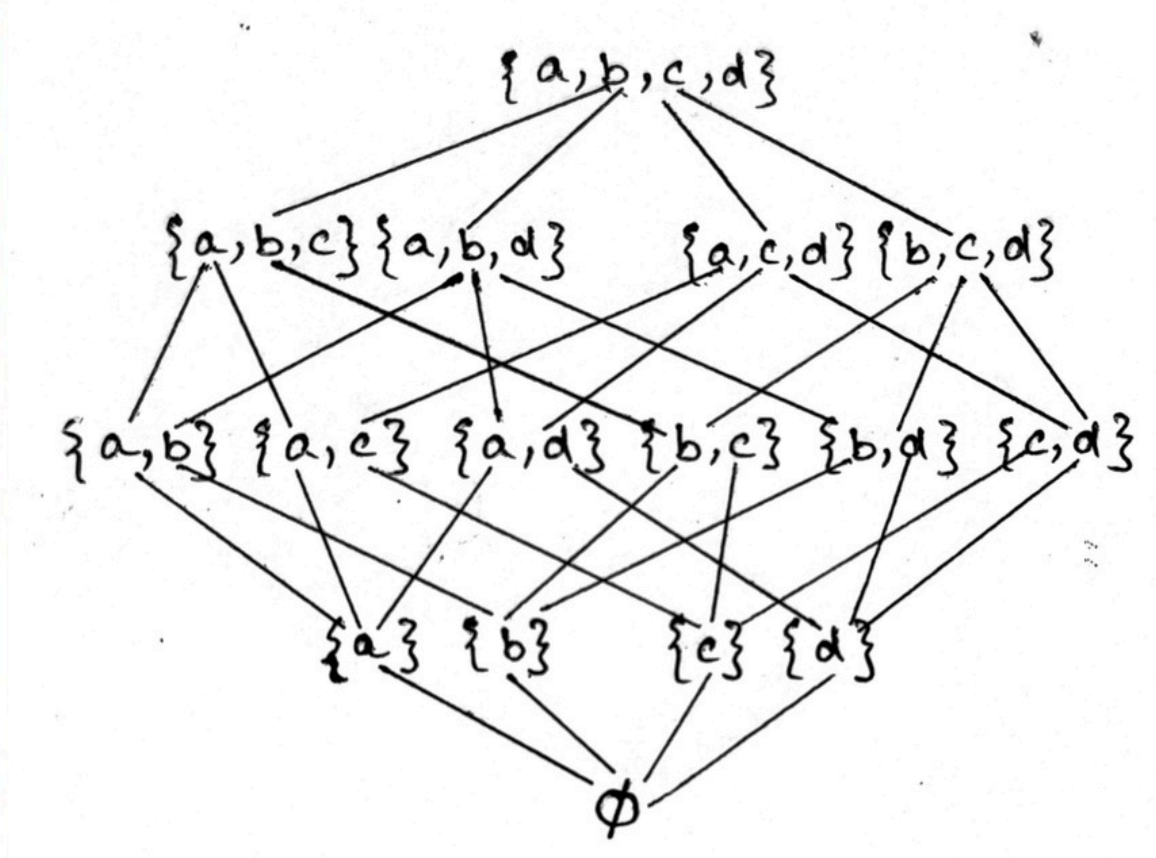
(d) Digraph for the relation R∩R-1:



9. Draw the Hasse diagram of (P(S),⊆) where the P(S) is the power set of the set S={a,b,c,d}.

SOLUTION:

The Set S = {a, b, c, d}
Since S has 4 elements
|S| = 4
Number of elements in Power Set P(S) = 2^4 = 16
P(S) = {∅, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, {a, b, c, d}}



Hasse diagram of (P(S),⊆)

10. Let A={1,2,3,4,5,6,12} on A , define a relation R by aRb if and only if a divides b. Prove that
R is partial ordering on R. Draw the Hasse diagram for this relation.

SOLUTION:

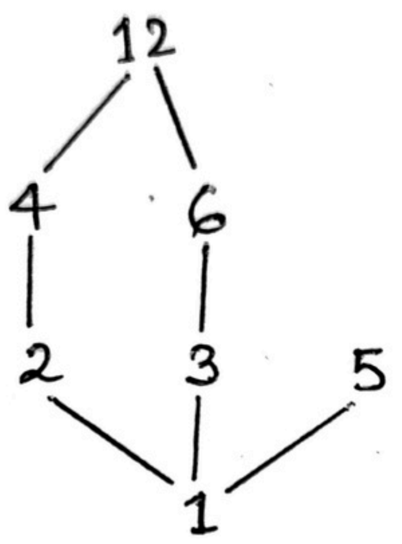
To prove that the relation R on A={1,2,3,4,5,6,12}, defined by aRb $\Leftrightarrow$ a/b (i.e., a divides b), is a partial ordering, we need to verify the following properties:

1. Reflexivity:
For all $a \in A$, a/a, as every number divides itself.Thus, R is reflexive.

2. Antisymmetry:
If aRb and bRa, then a=b.
If a/b and b/a, then a and b must be the same number since the only divisors a number shares with itself are its multiples.
Thus, R is antisymmetric.

3. Transitivity: If aRb and bRc, then aRc.
If a/b and b/c, then a/c because the divisibility relation is transitive.Thus, R is transitive.
Since R satisfies reflexivity, antisymmetry, and transitivity, R is a partial order on A.

Hence, proved.



Here is the Hasse diagram for the relation R on the set A= {1,2,3,4,5,6, 12}, where aRb <=> a/b

11. Let A={1,2,3,4,6,8,12,24} on A , define a relation R by aRb if and only if a divides b. Prove that R is partial ordering on R. Draw the Hasse diagram for this relation.

SOLUTION:

To prove that the relation R on A={1,2,3,4,6,8,12,24}, defined by aRb ⇔ a/b (i.e., a divides b), is a partial ordering, we need to verify the following properties:

1. Reflexivity

For all a∈A, a/a, as every number divides itself.Thus, R is reflexive.

2. Antisymmetry

If aRb and bRa, then a=b.

If a/b and b/a, then a and b must be the same number since the only divisors a number shares with itself are its multiples.

Thus, R is antisymmetric.

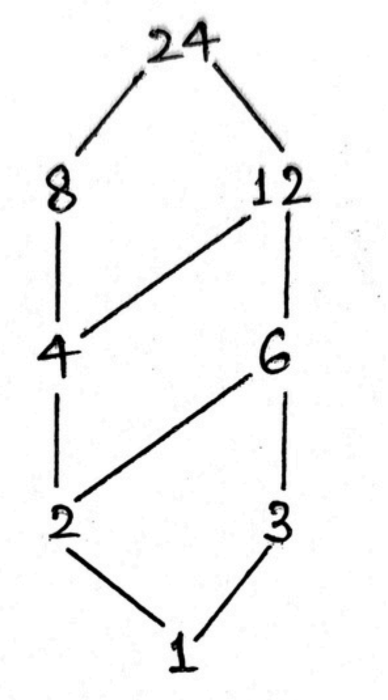
3. Transitivity

If aRb and bRc, then aRc.

If a/b and b/c, then a/c because the divisibility relation is transitive.Thus, R is transitive.

Since R satisfies reflexivity, antisymmetry, and transitivity, R is a partial order on A.

Hence, proved.



Here is the Hasse diagram for the relation R on the set A= {1,2,3,4,5,6,12,24}, where aRb <=> a/b

12. **Let Z be the set of integers and let R be the relation called congruence modulo 3 defined by R ={(x,y)| x∈z∧y∈z∧(x-y) is divisible by 3}. Determine the equivalence classes generated by the elements of Z.**

SOLUTION:

The relation R is defined as $R = \{(x, y) \mid x \in \mathbb{Z} \wedge y \in \mathbb{Z} \wedge (x - y) \text{ is divisible by } 3\}$. This is the congruence modulo 3 relation, denoted as $x \equiv y \pmod 3$.

To determine the equivalence classes generated by the elements of $\mathbb{Z}$, we need to find the sets of integers that are related to each other under this relation. Two integers x and y are related if their difference $(x - y)$ is divisible by 3. In other words, x and y have the same remainder when divided by 3.

There are three possible remainders when an integer is divided by 3: 0, 1, and 2. Therefore, there are three equivalence classes:

1. **Equivalence class of 0:** $[0] = \{x \in \mathbb{Z} \mid x \equiv 0 \pmod 3\} = \{\ldots, -6, -3, 0, 3, 6, \ldots\} = \{3k \mid k \in \mathbb{Z}\}$
2. **Equivalence class of 1:** $[1] = \{x \in \mathbb{Z} \mid x \equiv 1 \pmod 3\} = \{\ldots, -5, -2, 1, 4, 7, \ldots\} = \{3k + 1 \mid k \in \mathbb{Z}\}$
3. **Equivalence class of 2:** $[2] = \{x \in \mathbb{Z} \mid x \equiv 2 \pmod 3\} = \{\ldots, -4, -1, 2, 5, 8, \ldots\} = \{3k + 2 \mid k \in \mathbb{Z}\}$

These three equivalence classes partition the set of integers $\mathbb{Z}$. That is, every integer belongs to exactly one of these classes.

Equivalence Classes:

$$[0] = \{3k \mid k \in \mathbb{Z}\}$$
$$[1] = \{3k + 1 \mid k \in \mathbb{Z}\}$$
$$[2] = \{3k + 2 \mid k \in \mathbb{Z}\}$$

13. Let R = {(b,c),(b,e),(c,e),(d,a),(c,b),(e,c)}. Find the transitive closure of the relation R.

SOLUTION:

The transitive closure of

$$R = \{(b,c),\ (b,e),\ (c,e),\ (d,a),\ (c,b),\ (e,c)\}$$

is

$$R^{+} = \{(b,b),\ (b,c),\ (b,e),\ (c,b),\ (c,c),\ (c,e),\ (e,b),\ (e,c),\ (e,e),\ (d,a)\}.$$

Verification:

Within the subset $\{b, c, e\}$, the relation forms a cycle (since $b \to c$, $c \to b$, $b \to e$, $c \to e$, and $e \to c$). This cyclicity implies that every element of $\{b, c, e\}$ is reachable from every other element, including itself (as shown by $b \to c \to b$, etc.). The pair (d, a) stands alone with no further connections from a to any element. Hence, the transitive closure includes all ordered pairs among b, c, and e, plus the isolated pair (d, a).

14. A number of computer users are surveyed to find out if they have a printer, modem or scanner. Draw separate Venn diagrams and shade the areas, which represent the following configurations.

a. modem and printer but no scanner b. scanner but no printer and no modem

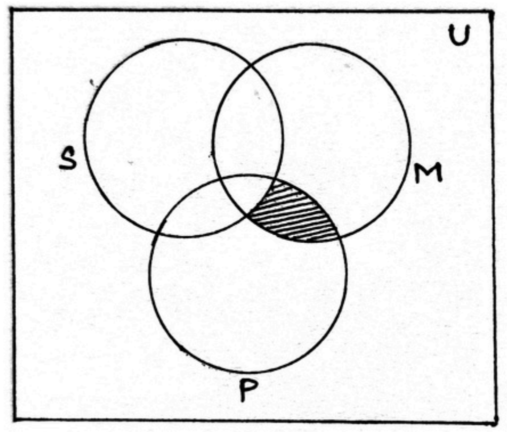
c. scanner or printer but no modem d. no modem and no printer.

SOLUTION: To solve this, let's define the sets:

P= users who have a printer M= users who have a modem S= users who have a scanner.

(a) Modem and Printer but no Scanner:

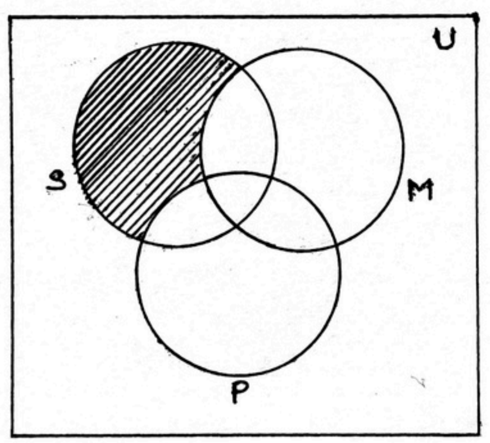
We need to find users who have both printer and a modem, but no Scanner. This means the intersection of M and P, but excluding S. Mathematically, (M∪P)-S



(b) Scanner but no Printer and no Modem:

User only uses Scanner, meaning exclusively S and not P or M.

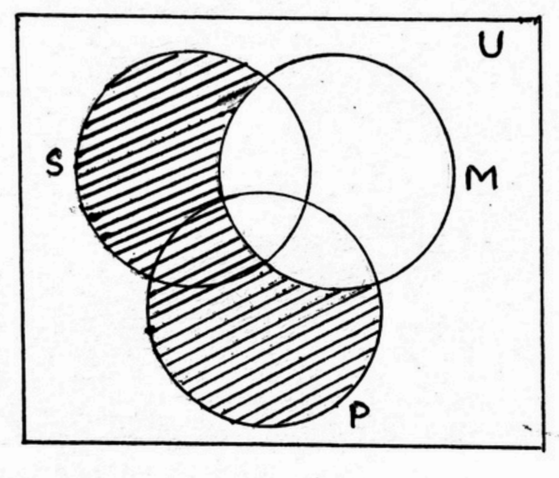
Mathematically, S-(M∪P)



(c) Scanner or Printer but no Modem:

User uses either Scanner or Printer but must not have a Modem.

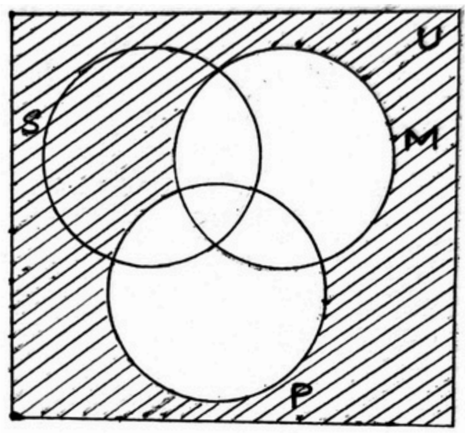
Mathematically, (S∪P)-M



(d) No Modem and no Printer

User do not have a Modem and do not have a Printer, that means user only have a Scanner.

Mathematically, Universal Set - (M∪P)



15. In a school, 100 students have access to three software packages, A, B and C
28 did not use any software
8 used only packages A
26 used only packages B
7 used only packages C
10 used all three packages
13 used both A and B
a. Draw a Venn diagram with all sets enumerated as far as possible. Label the two subsets which cannot be enumerated as x and y, in any order.
b. If twice as many students used package B as package A, write down a pair of simultaneous equations in x and y.
c. Solve these equations to find x and y.
d. How many students used package C?

SOLUTION:

A only = 8
B only = 26
C only = 7
All three = 10
Both A and B = 13
Both A and B but not C = 13-10 = 3
Both A and C but not B = x
Both B and C but not A = y
None = 28
Total = 100

Now, 8+26+7+10+3+x+y+28 = 100
=>82+x+y = 100
=>x+y = 18

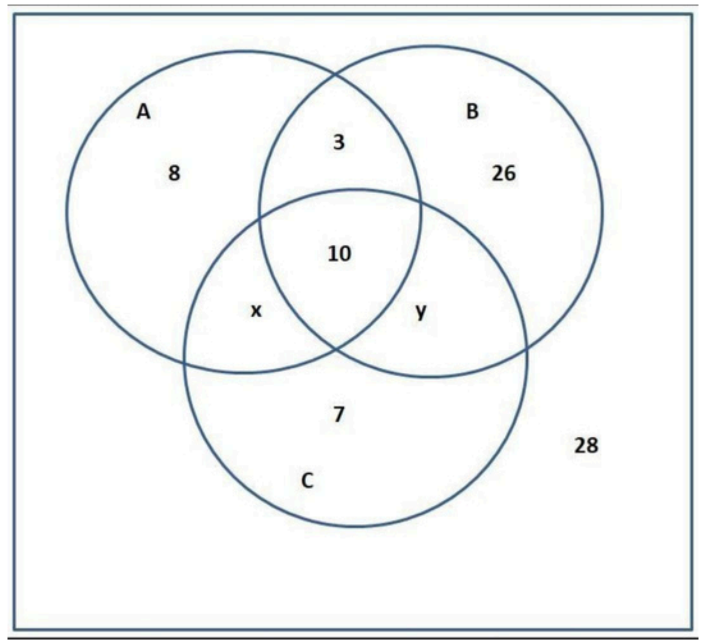
Twice as many students used package B as package A

B used = 26+13+y= 39+y
A used = 8+13+x= 21+x

By the sum,
39+y = 42+2x
=>y = 2x+3
Again, x+y = 18
=>x+2x+3 = 18
=>3x = 15
=> x = 5
Hence, y = 13

C used = 7+10+x+y = 17+18 = 35

The required Venn Diagram:



16. Find $P(P(P(\varnothing)))$ where $P(X)$ is the power set of X.

SOLUTION:

To find $P(P(P(\varnothing)))$, we need to carefully break down the problem by applying the concept of a power set at each step.

Step 1: Find $P(\varnothing)$

The power set of a set
X is the set of all subsets of
X.
The power set of the empty set, $\varnothing$, is: $P(\varnothing)=\{\varnothing\}$

This is because the only subset of the empty set is the empty set itself.

Step 2: Find $P(P(\varnothing))$

Now, we need to find the power set of $P(\varnothing)$, which is $P(\{\varnothing\})$.

The power set of $\{\varnothing\}$ is the set of all subsets of $\{\varnothing\}$, which are:
$P(\{\varnothing\})=\{\varnothing,\{\varnothing\}\}$
This is because the subsets of $\{\varnothing\}$ are the empty set and the set containing the empty set.

Step 3: Find $P(P(P(\varnothing)))$

Finally, we need to find the power set of $P(P(\varnothing))$, which is
$P(\{\varnothing,\{\varnothing\}\})$.
The power set of $\{\varnothing,\{\varnothing\}\}$ is the set of all subsets of $\{\varnothing,\{\varnothing\}\}$, which are:
$P(\{\varnothing,\{\varnothing\}\})=\{\varnothing,\{\varnothing\},\{\{\varnothing\}\},\{\varnothing,\{\varnothing\}\}\}$

Thus,
$P(P(P(\varnothing)))=\{\varnothing,\{\varnothing\},\{\{\varnothing\}\},\{\varnothing,\{\varnothing\}\}\}$.

THANK YOU