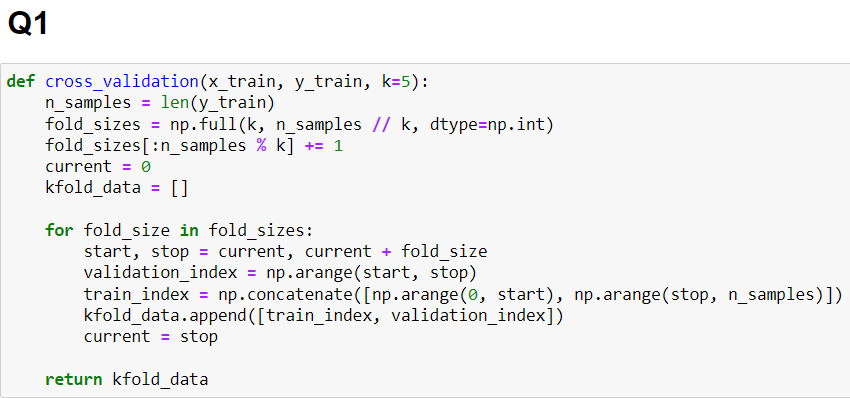
NYCU Pattern Recognition, Homework 4

**311553010, 陳姿羽**

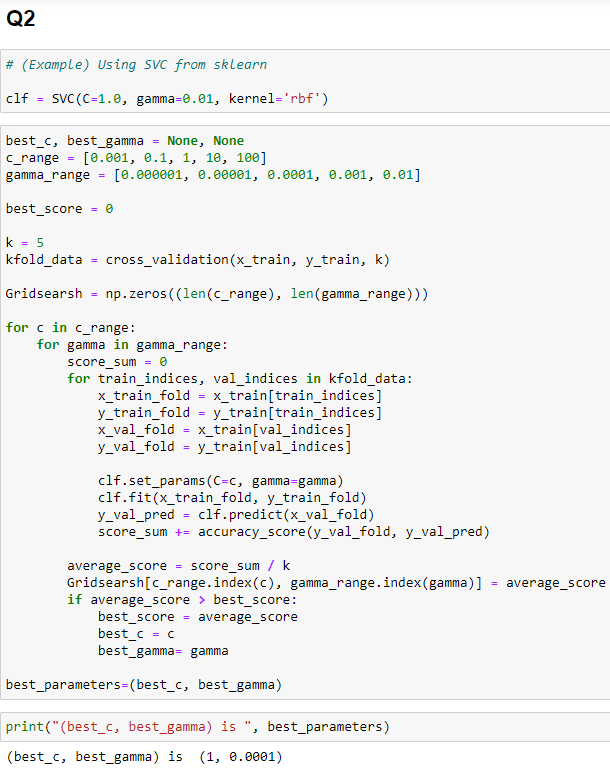
**Part. 1, Coding (70%)**:

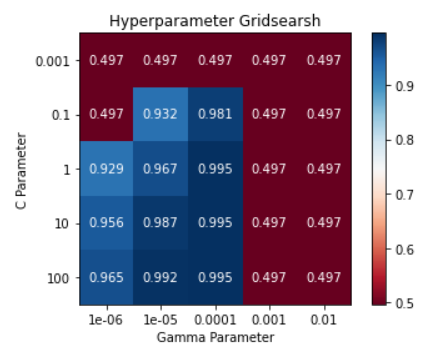
1. (5%) Implement K-fold data partitioning.

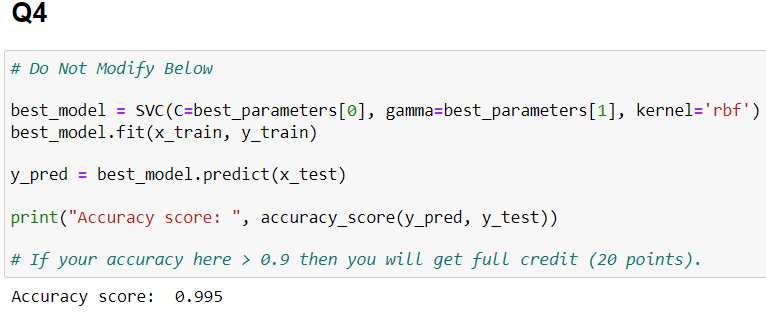


1. (10%) Set the kernel parameter to 'rbf' and do grid search on the hyperparameters **C** and **gamma** to find the best values through cross-validation. Print the best hyperparameters you found. Note that we suggest using K=5 for the cross-validation.

(best\_c, best\_gamma) is (1, 0.0001)



1. (10%) Plot the results of your SVM's grid search. Use "gamma" and "C" as the x and y axes, respectively, and represent the average validation score with color.  
   
2. (5%) Train your SVM model using the best hyperparameters found in Q2 on the entire training dataset, then evaluate its performance on the test set. Print your testing accuracy.  
   Accuracy score: 0.995



**Part. 2, Questions (30%):**

1. Show that the kernel matrix should be positive semidefinite is the necessary and sufficient condition for to be a valid kernel.

According to Mercer's theorem, for a valid kernel matrix , the dot product of any vector with must be greater than or equal to zero. Additionally, the kernel matrix must be positive semidefinite, which means that all of its eigenvalues are greater than or equal to zero. If the kernel matrix is not positive semidefinite, there may exist a vector such that , which means that is not valid as a kernel function.

Therefore, the kernel matrix being positive semidefinite is the necessary and sufficient condition for to be a valid kernel.

1. Given a valid kernel , explain that is also a valid kernel. (Hint: Your answer may mention some terms like \_\_\_\_ series or \_\_\_\_ expansion.)

is a valid kernel, so it is also positive semidefinite. According to the eigenvalue decomposition theorem, any positive semidefinite can be represented by eigenvectors and eigenvalues. Thus, can be expressed as , where is the eigenvalue and is the corresponding eigenvector.

After expanding using Taylor expansion, we substitute into the definition of the kernel matrix to obtain a new matrix *K*.

Since is a real number, this Taylor expansion converges. The matrix *K* can be expressed as an infinite series sum.

Since is positive semidefinite, raising it to the power of *m*, , also results in positive semidefinite. Consequently, *K* is positive semidefinite as well. Therefore, is a valid kernel.

1. Given a valid kernel , prove that the following proposed functions are or are not valid kernels. If one is not a valid kernel, give an example of that the corresponding is not positive semidefinite and show its eigenvalues.
2. =
3. =
4. Consider the optimization problem

State the dual problem. (Full points by completing the following equations)

*=*

=

when ,

*=*

= =