Pole Projet P15 - Sketching

subtitle

Team Name

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Section 1

JESPRIT for Parameters Estimation of Mixed Poisson distribution

Problem Formulation: Mixed Poisson Recovery

Goal: Recover latent parameters of a Mixed Poisson distribution from count data.

- **Model:** $X \sim \sum_{k=1}^{r} \pi_k \mathrm{Pois}(\boldsymbol{\lambda}_k)$
- **Unknowns:** Rate matrix $\mathbf{A} = [\boldsymbol{\lambda}_1, \ldots, \boldsymbol{\lambda}_r]$ and weights $\boldsymbol{\pi}$.

Sketching Approach:

1. Compute **Empirical PGF** at specific points: $\mathbf{t}(\mathbf{u}, n) = \mathbf{1} + j\Delta n\mathbf{u}$.
2. Map samples to theoretical expression for the PGF of the Mixed Poisson.
3. Apply **JESPRIT** to recover parameters.

Sampling Strategy: The Empirical PGF

Why Empirical PGF?

- We lack the true distribution, but we have samples $\mathbf{x}^{(j)}$.
- PGF form: $\hat{G}_{\mathbf{X}}(\mathbf{t}) = \frac{1}{N_s}\sum_{j=1}^{N_s} e^{\langle \mathbf{x}^{(j)}, \ln(\mathbf{t})\rangle}$.

Line Sampling for Shift Invariance

- Single points are not enough. We need a **sequence** to capture the rotation invariance.
- For each direction vector $\mathbf{u}_l$, we sample along a line in the complex domain:

$$\mathbf{t}(\mathbf{u}_l, n) = \mathbf{1} + j\Delta n \mathbf{u}_l, \quad n = 0, 1, \ldots, 2N-1$$

The JESPRIT Algorithm

Key Innovation: Global Subspace Estimation

- Instead of processing each direction independently, we **stack** directional samples matrices Z_l from all directions into $\mathbf{X}_{\text{glob}}$.
- **Global SVD: $\mathbf{X}_{\text{glob}} \approx \mathbf{U}_{\text{glob}} \boldsymbol{\Sigma} \mathbf{V}^H$.**
- Ensures all directional subspaces $\hat{\mathbf{U}}_l$ share a **coherent basis**.

Recovery Steps:

1. **RIMs:** Solve $\hat{\mathbf{U}}_{l,\uparrow} \Psi_l \approx \hat{\mathbf{U}}_{l,\downarrow}$ for shifts.
2. **Joint Diagonalization:** Find $\mathbf{T}$ such that for all l:
$$\Psi_l \approx \mathbf{T} \Phi_l \mathbf{T}^{-1}, \quad \Phi_l = \text{diag}(\phi_{1,l}, \dots, \phi_{r,l})$$
3. **Extraction:** Eigenvalues $\phi_{k,l}$ give Rates $\boldsymbol{\lambda}_k$ via:
$$\phi_{k,l} = e^{j\Delta\langle \boldsymbol{\lambda}_k, \mathbf{u}_l \rangle}$$
4. **Probabilities ($\boldsymbol{\pi}$):** Solve linear system $\mathbf{y} \approx \mathbf{E}\boldsymbol{\pi}$ via Least Squares:
$$\min_{\boldsymbol{\pi}} \|\mathbf{y} - \mathbf{E}\boldsymbol{\pi}\|_2^2 \quad \text{where } E_{l,k} = e^{j\Delta\langle \hat{\boldsymbol{\lambda}}_k, \mathbf{u}_l \rangle}$$

Results: Parameter Analysis & Phase Unwrapping

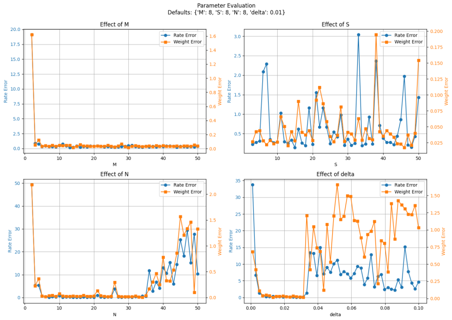

Figure: Error without Unwrapping

1. **Phase Unwrapping:**
   - Unwrapping *increases* error here.
   - Optimal scale: $\Delta \approx 1/\max(A)$.
2. **Parameter Sensitivity:**
   - **Directions (M) & Snapshots (S):** Robust to over-sampling.
   - **Samples (N):** Sensitive. Large N causes aliasing/wrapping issues.

Results: Sample Complexity

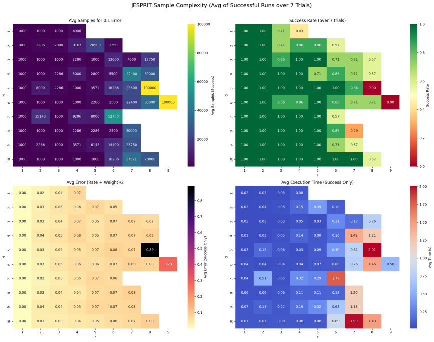

Figure: Success Rate (Large Range)

Experiment Setup:

- A = random(range, d, r).
- Increase number of samples until MRE $< 10\%$.
- Increase dimension r while success rate $> 75\%$.

Finding 1: Dynamic Range

- A larger rate range in A improves recovery.

Finding 2: Scalability

- Algorithm performance depends on Rank r, not Dimension d.