

Pole Projet P15 - Sketching

subtitle

Team Name

January 22, 2026

Problem Formulation: Mixed Poisson Recovery

Goal: Recover latent parameters of a Mixed Poisson distribution from count data.

- **Model:** $X \sim \sum_{k=1}^r \pi_k \text{Pois}(\lambda_k)$
- **Unknowns:** Rate matrix $\mathbf{A} = [\lambda_1, \dots, \lambda_r]$ and weights $\boldsymbol{\pi}$.

Challenge: High-dimensional estimation from unlabeled samples.

Sketching Approach:

- ① Compute **Empirical PGF** at specific points: $\mathbf{t}(\mathbf{u}, n) = \mathbf{1} + j\Delta n \mathbf{u}$.
- ② Map PGF samples to **Harmonic Retrieval** model.
- ③ Apply **JESPRIT** to recover parameters.

Sampling Strategy: The Empirical PGF

Why Empirical PGF?

- We lack the true distribution, but we have samples $\mathbf{x}^{(j)}$.
- PGF form: $\hat{G}_{\mathbf{x}}(\mathbf{t}) = \frac{1}{N_s} \sum_{j=1}^{N_s} e^{\langle \mathbf{x}^{(j)}, \ln(\mathbf{t}) \rangle}$.

Line Sampling for Shift Invariance

- Single points are not enough. We need a **sequence** to capture "rotations".
- For each direction vector \mathbf{u}_I , we sample along a line in the complex domain:

$$\mathbf{t}(\mathbf{u}_I, n) = \mathbf{1} + j\Delta n \mathbf{u}_I, \quad n = 0, 1, \dots, 2N - 1$$

Resulting Signal Sequence:

$$y_I[n] = \hat{G}_{\mathbf{x}}(\mathbf{t}(\mathbf{u}_I, n)) \approx \sum_{k=1}^r \pi_k (e^{j\Delta \langle \lambda_k, \mathbf{u}_I \rangle})^n$$

This maps the Poisson recovery problem to Harmonic Retrieval.

The JESPRIT Algorithm

Key Innovation: Global Subspace Estimation

- Instead of processing each direction independently, we **stack** Hankel matrices from all directions into \mathbf{X}_{glob} .
- **Global SVD:** $\mathbf{X}_{\text{glob}} \approx \mathbf{U}_{\text{glob}} \boldsymbol{\Sigma} \mathbf{V}^H$.
- Ensures all directional subspaces $\hat{\mathbf{U}}_I$ share a **coherent basis**.

Recovery Steps:

- ① **RIMs:** Solve $\hat{\mathbf{U}}_{I,\uparrow} \Psi_I \approx \hat{\mathbf{U}}_{I,\downarrow}$ for shifts.
- ② **Joint Diagonalization:** Find \mathbf{T} to diagonalize all Ψ_I simultaneously.
- ③ **Extraction:** Eigenvalues \rightarrow Rates λ_k . Least Squares \rightarrow Probs π .

Results: Parameter Analysis & Phase Unwrapping

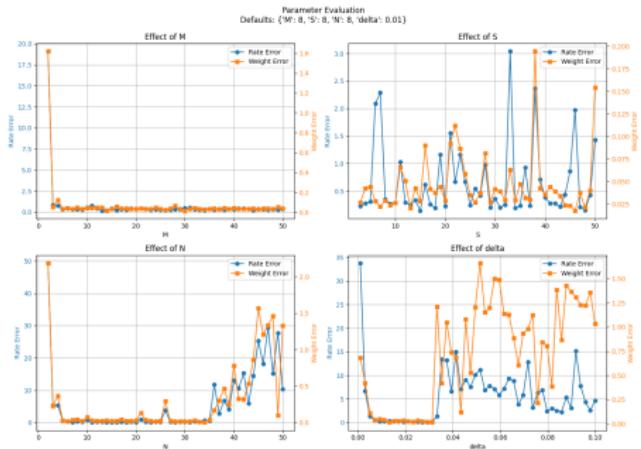


Figure: Error without Unwrapping

1. Phase Unwrapping:

- Unwrapping *increases* error here.
- Optimal scale: $\Delta \approx 1 / \max(A)$.

2. Parameter Sensitivity:

- Directions (M) & Snapshots (S):** Robust to over-sampling.
- Samples (N):** Sensitive. Large N causes aliasing/wrapping issues.

Results: Sample Complexity

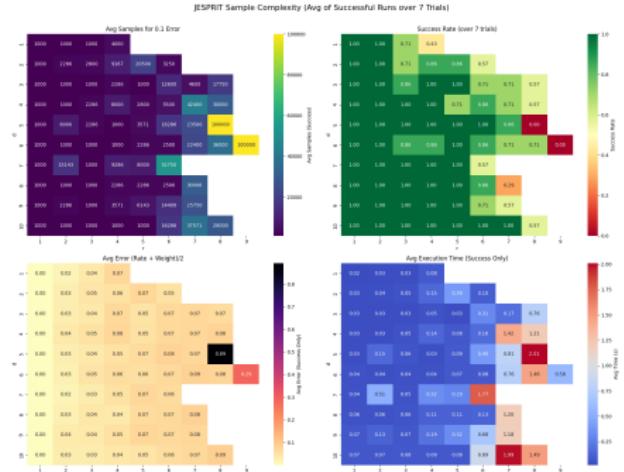


Figure: Success Rate (Large Range)

Finding 2: Dynamic Range

- Larger rate range $[0, 10^4]$ improves recovery.
- Distinct "directions" in count space act as higher SNR.

Scalability:

- Sample complexity depends on Rank r , not Dimension d .