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PHYS265

Lab 1 Report

I. Introduction

In the next few years, the main goal of the National Aeronautics and Space Administration or NASA is to send people to the Moon. To do so we will have to perform numerous calculations regarding the potentials and the forces of the Earth-Moon system. Potential refers to the work needed to move an object from infinity to the point specified. In the Earth-Moon system, we must take into account the potential within Earth's orbit and within the Moon's orbit. As we get closer to a mass, the gravitational potential from that object increases. Force is the push or pull an object feels. Again we must take into account the force of both the Earth's gravity as well as the force for the Moon's gravity. Using these values we can find out what forces must be overcome on our journey to the Moon. We can also do calculations to find the change in the rocket's velocity, the height of the rocket at burnout when all of the fuel will be used up, and the total burn time of the rocket.

II. The Gravitational Potential of the Earth-Moon System

When calculating the gravitational potential in the Earth-Moon system, we can use Equation 1 below to calculate the gravitational potential of both systems and then add them together as seen in Equation 2.

$$\Phi(r) = -\frac{Gm}{r}$$

Equation 1. Where Φ in the gravitational potential, G is the gravitational constant, m is the mass of the Earth or Moon, and r is the magnitude of the distance a certain radius from the Earth or Moon.

$$\Phi_{tot}(r) = (-\frac{Gm_e}{r_e}) + (-\frac{Gm_m}{r_m})$$

Equation 2. Where Φ in the gravitational potential, G is the gravitational constant, m_e is the mass of the Earth, m_m is the mass of the Moon, r_e is the magnitude of the distance a certain radius from the Earth, and r_m is the magnitude of the distance a certain radius from the Moon.

Using Equations 1 and 2, we were able to find the gravitational potential at varying distances within the Earth-Moon system. We made the following graph. Figure 1 is a 2D color-mesh plot of the combined Earth-Moon system. As we can see the closer we are to the Earth, the stronger the gravitational potential. We can also see a small dot above and to the right of the Earth. This is the Moon and its gravitational potential. As we can see by comparing the two, the gravitational potential of Earth is much larger than that of the Moon.

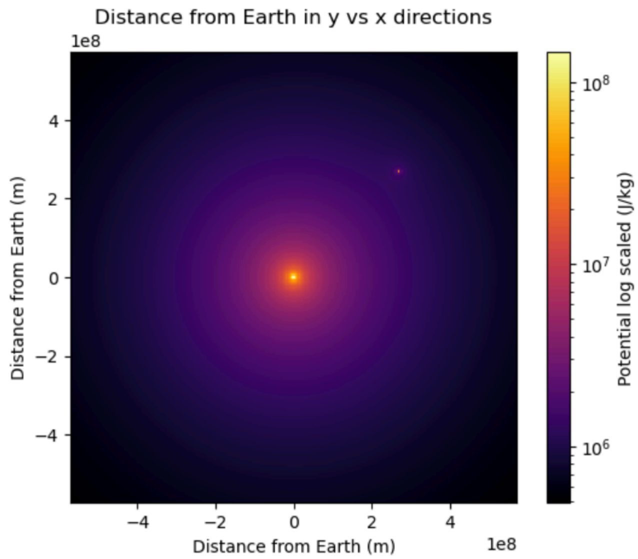

Figure 1. Where the gravitational potential of the Earth-Moon system is shown at varying distances from the Earth and Moon.

III. The Gravitational Force of the Earth-Moon System

Our next course of action was finding out what forces, and how much of them, we would have to counteract while in space. We used Equation 3 to find the force at each place in the Earth-Moon system.

$$\vec{F}_{21} = -G\frac{M_1 m_2}{|\vec{r}_{21}|^2}\hat{r}_{21}$$

Equation 3. Where $\vec{F}_{21}$ is the force of they system, G is the gravitational constant, M_1 of object 1, m_2 is the mass of object 2, $|\vec{r}_{21}|$ is the magnitude of the distance between the objects, and $\hat{r}_{21}$ is the difference in the x and y direction divided by the radius.

Something important about this equation to keep in mind is that there were four different forces found. The forces found were between the Earth and the Apollo 11 command modulus in both the x and y directions, and between the Moon and the Apollo 11 command modulus in both the x and y directions. Using this equation we were able to create Figure 2 which is a stream plot showing the direction and magnitude of the forces in the Earth-Moon system.

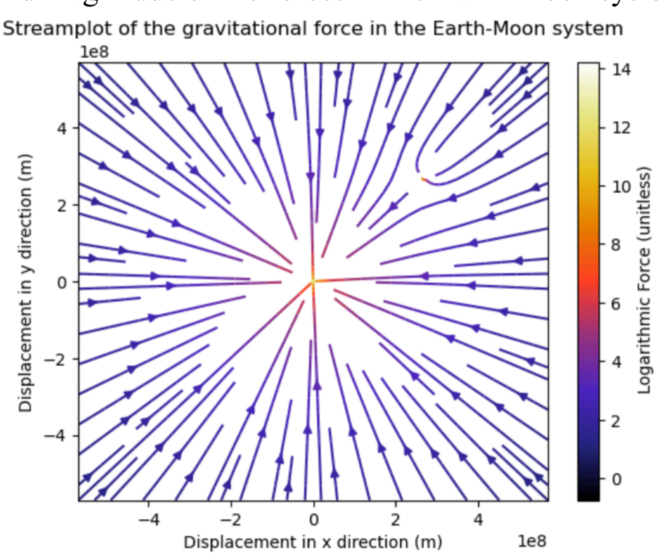

Figure 2. Where the direction and magnitude of the gravitational force is shown at different places in the Earth-Moon system.

IV. Projected Performance of the Saturn V Stage 1

We know that propelling the rocket forward is done by gas leaving the bottom end of the rocket with a high force. The change in the rocket's velocity as a function of time can be seen in Equation 4.

$$\Delta v(t) = v_e ln(\frac{m_0}{m_0 - \frac{dm}{dt}t}) - gt$$

Equation 4. Where Δv is the change in rocket's velocity, v_e is the exhaust velocity of Saturn V Stage 1, m_0 is the wet mass (which is the fuel + rocket parts + payload), $\frac{dm}{dt}$ is the fuel burn rate, t is the time since launch, and g is the gravitational acceleration.

We can also find the time at which all the fuel will be used up, or the burnout time, using Equation 5.

$$T = \frac{m_0 - m_f}{\frac{dm}{dt}}$$

Equation 5. Where T is total burn time of the rocket, m_0 is the wet mass (which is the fuel + rocket parts + payload), $\frac{dm}{dt}$ is the fuel burn rate, and m_f is the dry mass.

We can use Equations 4 and 5 plugged into Equation 6 to find the altitude of the rocket at the time when all the fuel is burned up by integrating using Equation 6.

$$h = \int_0^T \Delta v(t)dt$$

Equation 6. Where h is the height of the rocket at "burnout" which is the time at which all the fuel will be used up, 0 is time 0, T is the time at which ll the fuel will be used up, $\Delta v(t)$ is the change is rocket's velocity, and dt is an infinitesimally small time increment.

Using Equations 4, 5, and 6 we were able to find the burnout time to be 157.7 seconds, and the height of the rocket at burnout to be $7.41x10^4 \pm 5.85x10^{-8}$ m.

V. Discussion and Future Work

In the future, we hope to apply these concepts in the engineering of Apollo 11. We can use the information about the forces to find out how much propelling force will need to be created by the gas, so that we can escape Earth's gravity. We can use the information about gravitational potential to figure out again how much propelling force will need to be created to reach the moon. Through these experiments and findings, we assumed that the Earth and Moon are point sources, and that the burn rate of the fuel was constant. Our other assumption was that the Earth and Moon are on the same plane when they aren't in reality. Last week we received test results from a Saturn V prototype and our burnout time was slightly lower than the test results, and the height at burnout in the results was slightly lower than our results. Both of these discrepancies are probably due to us neglecting the drag force in our calculations.