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PHYS265

Mine Crafting Report

I. Introduction

The goal of this project is to learn how long a test mass will take to hit the bottom of our mine shaft, and use this information to find out exactly how deep the mineshaft is. As discussed, we have prepared calculations that can tell us information about the depth of the mine we operate by dropping a test mass. To further understand this, we looked at many cases and took into account different forces. These forces and variables include: drag, a varying gravitational constant, and Coriolis force. We also looked at some additional cases like an infinitely deep mine, and Earth's non-uniform density. All of these cases will have varying effects on the test mass as we drop it through the mineshaft. We hope to learn more about whether or not this idea of dropping a mass to find the depth of the mineshaft is feasible.

II. Calculations of Fall Time

In this first case, we initially calculated the fall time of the mass using the kinematic equations of motion. This gave us a theoretical value to move forward with. This was found using Equation 1.

$$t = \sqrt{\frac{2*\Delta x}{a}}$$

Equation 1. Where t = fall time of the mass, Δx is the approximate depth of the mineshaft, and a is the acceleration due to gravity.

We then numerically integrated Equation 2 with α being zero to find the fall time in a different and slightly more accurate way. Using both of these methods we got that the fall time is 28.6 seconds. These two numbers agree.

$$\frac{dv}{dt} = -g - \alpha(v)^{\gamma}$$

Equation 2. Where $\frac{dv}{dt}$ is the change in velocity with respect to time, g is the acceleration due to gravity, α is the drag coefficient, v is the velocity, and γ is the speed dependence of the drag.

Our next step was to include the drag force in our calculations. This is a force that acts in the opposite direction of the motion of the mass. The drag force will slow the mass down so we expect a slower fall time using Equation 2, where we used a drag coefficient calibrated according to the terminal velocity. When taking into account drag, we got 83.5 seconds for the mass to hit the bottom. This is an exponential increase from the initial time value we got without taking into account drag force. Figure 1 shows the position and velocity of the mass as it falls through the mineshaft with drag and a varying gravitational acceleration accounted for.

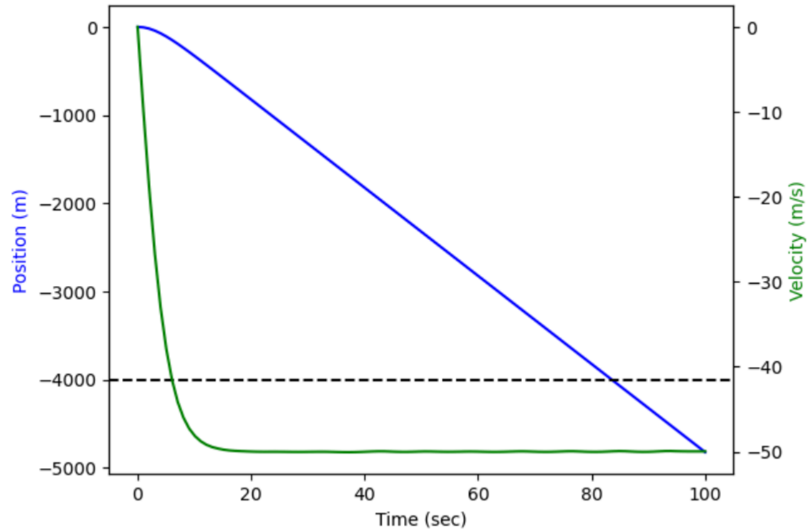

Figure 1. Position and Velocity of the mass as it falls through the mineshaft with a drag and a varying gravitational constant, and a dashed line showing the approximate depth of the mineshaft.

III. Feasibility of Depth Measurement Approach

Following taking into account drag, we looked at Coriolis force. This force is caused by Earth's rotation and causes objects to "move" transversely because Earth is moving. The main goal here is to see if the mass will hit the side of the mineshaft before hitting the bottom. If it does, we need to rethink this project.

Using Equation 3 we found the Coriolis force on the mass.

$$\vec{F}_c = -2m(\vec{\Omega} \times \vec{v})$$

Equation 3. Where $\vec{F}_c$ is the Coriolis force, m is the mass of the object, $\vec{\Omega}$ is the Earth's rotation rate for a vector along $\hat{z}$, and $\vec{v}$ as the velocity vector.

When calculating the Coriolis force, we must take into account the force in both the x and y directions. Using these calculations and having our mineshaft at 5 feet in diameter, we can calculate if the mass will hit the side of the shaft before hitting the bottom of it. We found that the mass would hit the side at 21.9 seconds at a depth of 2354.0 meters below the surface. This was just taking into account Coriolis force. The mass will hit the bottom at 28.6 seconds. This is a clear indication that this project is not possible. Due to the Coriolis force, the mass would hit the side before hitting the bottom so the time for the mass to reach the bottom would not be a clear indication of the depth of the mineshaft. Adding drag slows down the mass, so the mass would hit the side at a later time but at a higher depth. Figure 2 shows the mass's transverse position and depth position.

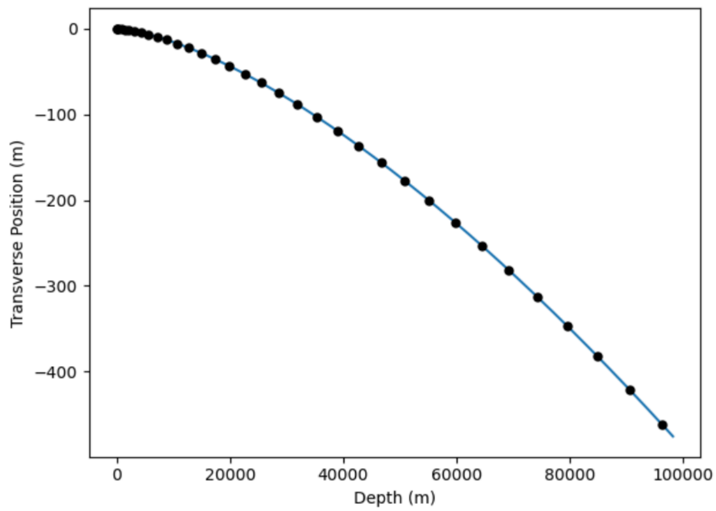

Figure 2. Transverse position vs Depth position with dots labelling 3 second intervals assuming we don't stop at 4000 meters.

IV. Calculation of Crossing Times

In addition to the other calculations, we calculated the crossing time for a homogeneous Earth, which is not practical. The time for the mass to cross to the other side of Earth in this case is 2497.1 seconds without drag. If we take into account the non-homogeneous density of Earth, at a density concentration of zero, the crossing time of the mass is 1267.3 seconds. At a density concentration of nine, the mass's crossing time is 943.8 seconds. Through these values, we can see a clear correlation in that with a higher density concentration, the mass moves slower and therefore takes longer to cross to the other side of Earth.

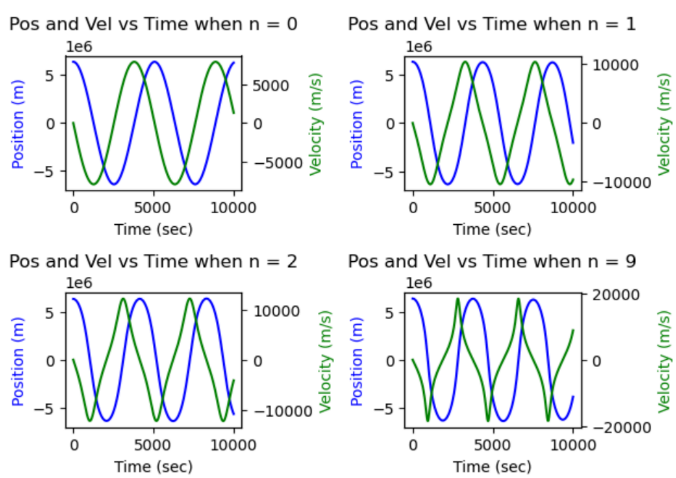

Figure 3. Position and Velocity versus Time of the test mass at density profiles of n = 0, 1, 2, and 9.

We also found the crossing time for the moon. It was found to be 3250.2 seconds. The ratio of the moon's density to the Earth's is 0.6. The lower density is probably the reason for the 2.5 times greater crossing time for the moon.

V. Discussion and Future work

Throughout these simulations, we approximated the values of γ and α. γ being the speed dependence of the drag and α being the drag coefficient. We also would, in reality, need a more precise terminal velocity because we assumed the value was the same as the terminal velocity of a skydiver, but this is wrong. Additionally we assumed a spherical Earth which Earth is not. Lastly, we assumed a continuous density of Earth, which is also not true. We learned that because of the Coriolis force this simulation can't be used to find the depth of the mineshaft.