The 1-D Schrodinger's equation is ;

$$\boxed{-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\,\psi(x) = E\,\psi(x)} \qquad -(1)$$

This equation determines the solution of a 1-D problem of a particle of mass $\underline{m}$ moving in potential $V(x)$.

The different constants in the equation are :

$\hbar$ = Planck constant.

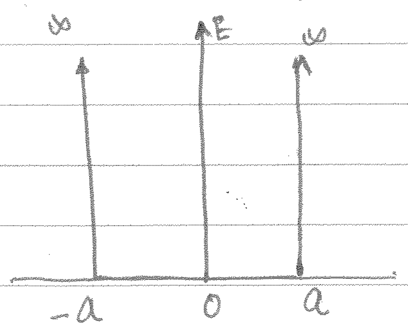
E = Total energy.

ψ = wavefunction describing the system.

= solution for which we are looking for,

$\psi^*\psi\,dx$ = Probability of finding the particle between x and $x+dx$.

For an infinite square well.



$$V(x) = \begin{cases} 0 & , -a \leq x \leq a \\ \infty & ; \text{otherwise} \end{cases}$$

S.E solved in the interval $-a \leq x \leq a$ gives the solution

$$\psi(x) = A\sin kx + B\cos kx.$$

$$\text{where } k = \sqrt{\frac{2mE}{\hbar^2}}.$$

For $-a \leq x \leq a$, $V = \infty$, so the particle should not be found there.

What are the Boundary conditions?

When I said, the particle is not found outside the well, then it means that the wavefunction is zero in that region.

Also, Probability of finding a particle is a continuous function

$\Rightarrow$ ψ must vanish at $x = \pm a$.

Working on with boundary conditions :-

For $x = -a$

$$\psi = -A\sin ka + B\cos ka = 0 \quad \text{---} (2)$$

For $x = +a$

$$\psi = A\sin ka + B\cos ka = 0 \quad \text{---} (3)$$

So, adding / substracting gives us :-

$$B\cos ka = 0 \quad \text{---} (4)$$
$$A\sin ka = 0 \quad \text{---} (5)$$

Analysis,

$A\sin ka = 0$

so, $A = 0$ or $\sin ka = 0$

let's suppose $A = 0$ is taken

$\therefore B\cos ka = 0$, & if $B \neq 0$ for soln to exist

$\Rightarrow \cos ka = 0$

$= (2n+1)\frac{\pi}{2}$ $\quad n = 0, 1 \ldots$

So, $ka = \sqrt{\frac{2mE}{\hbar^2}}\, a = \left(n+\frac{1}{2}\right)\pi \qquad n = 0, 1 \ldots$

$$\text{or} \quad \boxed{E_n = \frac{\left(n+\frac{1}{2}\right)^2 \hbar^2 \pi^2}{2ma^2}} \qquad n = 0, 1 \ldots$$

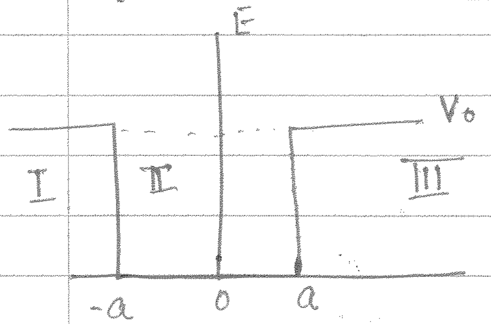
But if $A \neq 0$, then $\sin ka = 0$

$\Rightarrow ka = 0, \pi, 2\pi \ldots$

$$\Rightarrow \boxed{E_n = \frac{n^2 \pi^2 \hbar^2}{2ma^2}} \qquad n = 1, 2 \ldots$$

· Infinite = NO.

Let's solve for a finite square well potential.



In region I

Schrodinger equation is :-

$$\frac{d^2\psi}{dx^2} - \frac{2m}{\hbar^2}(V_0 - E)\psi = 0$$

soln

$$\psi_I(x) = C e^{\beta x} + D e^{-\beta x}$$

$$\beta = \sqrt{\frac{2m}{\hbar^2}(V_0 - E)}$$

Region II

$$\psi_{II} = A\sin \alpha x + B\cos \alpha x$$

$$\alpha = \sqrt{\frac{2m}{\hbar^2}E}$$

Region III

$$\psi_{III} = F e^{-\beta x}$$

Since $\psi(x)$, $\psi'(x)$ are continuous

At $x = -a$.

$$-A\sin\alpha a + B\cos\alpha a = Ce^{-\beta a}$$

and $$-\alpha A\cos\alpha a + \alpha B\sin\alpha a = \beta C e^{-\beta a}$$

At $x = a$

$$A\sin\alpha a + B\cos\alpha a = Fe^{-\beta a}$$

and $$\alpha A\cos\alpha a - \alpha B\sin\alpha a = -\beta F e^{-\beta a}$$

⋮

Let's do this algebra in Matlab

Even states $A = 0$, $B \neq 0$, $C = F$, $\alpha\tan\alpha a = \beta$

Odd states $A \neq 0$, $B = 0$, $C = -F$, $\alpha\cot\alpha a = -\beta$

⇓ Problem of finding energies and ψ of the finite square well has evolved into finding roots of a transcedental equation.