

1.

```
import numpy as np
import matplotlib.pyplot as plt

# Parameters for the sine wave
sampling_rate = 1000 # Samples per second
frequency = 5        # Frequency of the sine wave in Hz
duration = 1         # Duration in seconds

# Time array
t = np.linspace(0, duration, int(sampling_rate * duration), endpoint=False)

# Create a sine wave signal
signal = np.sin(2 * np.pi * frequency * t)

# Perform Fast Fourier Transform (FFT)
fft_result = np.fft.fft(signal)
fft_magnitude = np.abs(fft_result) # Magnitude of the FFT
frequencies = np.fft.fftfreq(len(fft_magnitude), 1/sampling_rate)

# Plot the original sine wave signal
plt.figure(figsize=(12, 6))

plt.subplot(2, 1, 1)
plt.plot(t, signal, label='Sine Wave (5 Hz)')
plt.title('Original Sine Wave Signal')
plt.xlabel('Time (s)')
plt.ylabel('Amplitude')
plt.grid()
plt.legend()
```

```

# Plot the frequency spectrum

plt.subplot(2, 1, 2)

plt.plot(frequencies[:len(frequencies)//2], fft_magnitude[:len(frequencies)//2], color='orange',
label='Frequency Spectrum')

plt.title('Frequency Spectrum')

plt.xlabel('Frequency (Hz)')

plt.ylabel('Magnitude')

plt.xlim(0, 50) # Limit x-axis for better visualization

plt.grid()

plt.legend()

plt.tight_layout()

plt.show()

```

2.

```

import numpy as np

from scipy.integrate import quad

def f(x):
    return x**2

a = 0

b = 5

result, error = quad(f, a, b)

printf("The integral of  $f(x) = x^2$  from {a} to {b} is approximately {result:.4f}")

printf("Estimated error in the result: {error:.4e}")

```

output:

The integral of $f(x) = x^2$ from 0 to 5 is approximately 41.6667

Estimated error in the result: 4.6259e-13

3.

```

import numpy as np

from scipy.optimize import minimize

```

```
# Define the function to minimize
```

```
def f(x):
```

```
    return (x - 3)**2 + 2
```

```
x0 = 0
```

```
# Perform the minimization
```

```
result = minimize(f, x0)
```

```
# Output the result
```

```
printf("The minimum value occurs at x = {result.x[0]:.4f}")
```

```
printf("The minimum value of the function is f(x) = {result.fun:.4f}")
```

```
output:
```

```
The minimum value occurs at x = 3.0000
```

```
The minimum value of the function is f(x) = 2.0000
```

```
4.
```

```
import numpy as np
```

```
# Coefficient matrix
```

```
A = np.array([[2, 3],  
              [4, 1]])
```

```
# Right-hand side vector
```

```
b = np.array([5, 6])
```

```
# Solve the system of equations
```

```
solution = np.linalg.solve(A, b)
```

```
# Output the result
```

```
x, y = solution
```

```
printf("The solution is x = {x:.4f}, y = {y:.4f}")
```

output:

The solution is $x = 1.3000$, $y = 0.8000$