

# Sliding Mode Speed Control for Brushless DC Motor Based on Sliding Mode Torque Observer

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**Abstract** - Brushless DC (BLDC) motor control system is a multi-parameter nonlinear systems, and the ultra-low inductance of it makes more stringent requirements on the performance of the control system, which make it difficult using the traditional PI control method and the intelligent control methods, such as fuzzy control method and neural network control method, to effectively control a low-inductance BLDC motor with high performance demands. Considering this, we propose a new dual-loop control strategy in which the current hysteresis control is adopted for the current loop, and the speed loop control is realized based on the exponential approach law sliding mode speed controller and sliding mode observer to construct the control system for a low-inductance BLDC motor. The simulation and experimental verification show that compared to the traditional PI control strategy, the proposed control strategy shows obvious advantages in overshoot control, response speed and adaptability to load variations.

**Keywords** - BLDC motor, Variable Structure Control, Sliding Mode Control, Sliding Mode Observer

## I. INTRODUCTION

Brushless DC motor (BLDC motor) is a kind of permanent magnet synchronous motor with a trapezoidal back electromotive force (EMF) wave. Due to its large power density, high output torque, good dynamic characteristics, simple structure and control process and high reliability, BLDC motor has been widely used in machine tools, electric automotive, aerospace and other fields, and has been gradually replacing traditional induction motor and brush motor in many application fields.

Usually, dual-loop control strategy consisting the current loop control and speed loop control or direct torque strategy is used to construct the control system of BLDC motor, and the former is much more frequently used. In the dual-loop control strategy, the current hysteresis control method (CHCM), voltage pulse width modulation (PWM) method, and current predictive control method are three commonly used methods for the current loop control, while among them, owing to its simple control process, rapid current response, and inherent over-current protection capability [1], CHCM has become the preferred method for high-performance control system of BLDC motor, especially that with ultra-small induction. As regards the speed loop control, a large amount of researches on it using different control methods [2], such as PI control method, model reference adaptive control method, fuzzy control method, neural network control method, variable

structure control method and so on, have been conducted. However, the simple control process of the PI control method cannot meet the demand of multi-variable nonlinear control system for high performance, moreover, the performance of the PI control method in the adaptability to load variations is very poor. Turn to the intelligent control methods, such as the neural network method and fuzzy control method, they need continuous learning, optimization and adjustment in the control process, and consequently, high-performance underlying controller, mostly, beyond reality, which make it hard for the control system based on them to realize both in hardware and software. In a word, it is hard to use either of these two kinds of control strategies to construct the control system with high performance for a BLDC motor. On the other hand, the sliding mode control method is of great robustness, easy to realize, and especially applicable to the motor control process with strong nonlinearity and frequently-varying load, and it has been used in the position loop control, speed loop control, current loop control in the BLDC motor control system [3] [4] [5].

In this paper, the dual loop control structure is used to construct the control system for the BLDC motor with an ultra-small inductor. In this control system, the current hysteresis control is adopted for current loop, and the speed loop control is realized based on the exponential approach law sliding mode speed controller and sliding mode observer. The simulative and experimental results show that compared with that using the traditional PI control method, the control system of BLDC motor using the control method proposed in this paper has better speed performance and adaptability to load variations. Meanwhile, because of its simplicity to realize, this control system is easy to use and promote, and hence with great engineering value.

## II. MATHEMATICAL MODEL OF A BLDC MOTOR

For the sake of simplicity of the design and simulation, assumptions about the physical structure and working state of the BLDC motor are made as follows,

1. Star winding with three completely symmetrical phases and full pitch is used.
2. The back EMF waveform is trapezoidal with a flat top width of  $120^\circ$ .
3. The motor works in the two-phase-in-conduction state, and the conducted electrical angle is  $120^\circ$ .

4. The motor core is not saturated, and the eddy current loss, hysteresis loss and cogging are all ignored.

5. System power inverters, diodes all work in perfect condition.

The governing voltage equations for a BLDC motor is as follows,

$$\begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} = \begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} + \begin{bmatrix} l & 0 & 0 \\ 0 & l & 0 \\ 0 & 0 & l \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}, \quad (1)$$

$$i_a + i_b + i_c = 0, \quad (2)$$

where  $u$ ,  $i$ ,  $e$  are the phase voltage, phase current, and winding back EMF respectively, and the subscripts a, b and c correspond to the three phases respectively,  $r$  is the stator resistance,  $l$  is the equivalent inductance considering both of the self and mutual inductances of the stator.

Regardless of the dynamic process of reversing, (1) and (2) can be written as

$$u = Ri + L \frac{di}{dt} + k_e \omega, \quad (3)$$

where  $u$  is the line voltage,  $R$  is the line resistance,  $L$  is the line inductance,  $k_e$  is the back EMF coefficient, and  $\omega$  is the motor speed.

Motor torque equations are given by

$$T_e = J \frac{d\omega}{dt} + B\omega + T_l, \quad (4)$$

$$T_e = k_t i, \quad (5)$$

where  $T_e$  is the electromagnetic torque,  $T_l$  is the load torque,  $J$  is the moment of inertia,  $B$  is the damping coefficient, and  $k_t$  is the torque coefficient.

### III. EQUIVALENT CONTROL BASED ON EXPONENTIAL APPROACH LAW

The sliding motion is composed of the reaching motion and sliding mode motion, according to the principle of sliding mode control, while the reaching condition can only guarantee that system state can hit the sliding surface from anywhere of the phase plane, however, the reaching trajectory is not described explicitly [5]. In order to improve the reaching trajectory and reduce the chattering, the exponential approach law is selected for system designing.

Exponential approach law is given by

$$\dot{s} = -\varepsilon \operatorname{sgn}(s) - ks \quad \varepsilon > 0, k > 0, \quad (6)$$

where  $\dot{s} = -ks$  is the exponential approach term, and its solution is  $s = s(0)e^{-kt}$ . The function of the exponential term is guaranteeing that system state can approach the sliding surface rapidly when the value of  $s$  is larger, and the exponential term will decrease gradually and to zero eventually with the system state approaching the sliding surface. In order to reach the sliding surface within finite time, the constant reaching law  $\dot{s} = -\varepsilon \operatorname{sgn}(s)$  is added. In addition, by adjusting parameters  $\varepsilon$  and  $k$ , a faster approach speed and alleviated chattering can be achieved.

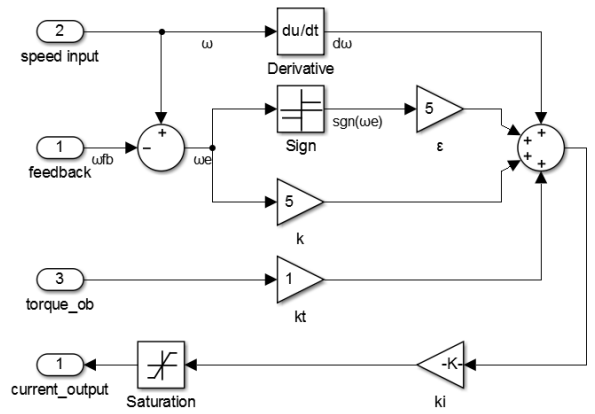


Fig. 1 Structure of sliding mode controller based on exponential approach law

Assume the input of current loop in the controller is  $u$ . Since frequency response of the current loop is much faster than the speed loop, it is reasonable to render that the current loop input is equal to the actual one within the limited range of the current loop, i.e.  $u = i$ . The motion equation of motor is described by

$$J\dot{\omega} = k_t u - B\omega - T_l. \quad (7)$$

The tracking error is

$$e = \omega_d - \omega. \quad (8)$$

The sliding surface is designed as

$$s(\omega) = e. \quad (9)$$

Let  $\dot{s}(\omega) = 0$ , then

$$\begin{aligned} \dot{s}(\omega) &= \dot{e} \\ &= \dot{\omega}_d - \dot{\omega} \\ &= \dot{\omega}_d - \frac{k_t}{J} u + \frac{B}{J} \omega + \frac{1}{J} T_l = 0 \end{aligned} \quad (10)$$

The equivalent control  $u_{eq}$  is

$$u_{eq} = \frac{1}{k_t} (J\dot{\omega}_d + B\omega + T_l). \quad (11)$$

The switching control  $u_{sw}$  is

$$u_{sw} = \frac{1}{k_t} (\varepsilon \operatorname{sgn}(s) + ks). \quad (12)$$

The actual control is

$$u = u_{eq} + u_{sw} = \frac{1}{k_t} (J\dot{\omega}_d + B\omega + T_l + \varepsilon \operatorname{sgn}(s) + ks). \quad (13)$$

Substituting Eq. (13) into Eq. (10), the following equation can be obtained

$$\dot{s}(\omega) = \frac{1}{J} (-\varepsilon \operatorname{sgn}(s) - ks). \quad (14)$$

Obviously, the stability condition is satisfied

$$\dot{s}(\omega)s(\omega) = \frac{1}{J} (-\varepsilon \operatorname{sgn}(s) - ks)s \leq 0. \quad (15)$$

However, considering that the load torque  $T_l$  is unknown,  $T_l$  can be seen as disturbance and eliminated from the control law, then the reaching condition can be represented by

$$\dot{s}(\omega)s(\omega) = \frac{1}{J} (-\varepsilon \operatorname{sgn}(s) - ks - T_l)s \leq 0. \quad (16)$$

In order to meet stability condition, the values of  $\varepsilon, k$  should be large enough. Nevertheless, in practice, too-large values of them may bring about serious chattering phenomenon owing to some factors such as the backlash, and the control period, which will ruin the control performance. In order to solve this problem, an observed value of  $T_l$  from observer is compensated into  $u$ . The sliding control schematic is shown by Fig. 1.

#### IV. LOAD TORQUE OBSERVER DESIGN BASED ON SLIDING MODE CONTROL

From the motion equation of motor, the following equation is obtained

$$\dot{\omega} = \frac{k_t}{J} u - \frac{B}{J} \omega - \frac{T_l}{J}. \quad (17)$$

The extended load torque observer is designed by

$$\dot{\hat{\omega}} = \frac{k_t}{J} u - \frac{B}{J} \hat{\omega} - \frac{\hat{T}_l}{J} + \eta \operatorname{sgn}(s). \quad (18)$$

$$\dot{\hat{T}_l} = g \eta \operatorname{sgn}(s). \quad (19)$$

where  $\hat{\omega}$  is the estimation value of  $\omega$ ,  $g$  is the sliding mode coefficient,  $\eta$  is the switching coefficient,  $s$  is the sliding surface, and  $s = e_\omega = \hat{\omega} - \omega$ .

The observed error dynamic equation can be achieved by

$$\dot{\hat{\omega}} - \dot{\omega} = -\frac{B}{J}(\hat{\omega} - \omega) - \frac{\hat{T}_l - T_l}{J} + \eta \operatorname{sgn}(s). \quad (20)$$

The stability condition of the sliding mode is

$$\begin{aligned} s\dot{s} &= (\hat{\omega} - \omega)(\dot{\hat{\omega}} - \dot{\omega}) \\ &= (\hat{\omega} - \omega)\left(-\frac{B}{J}(\hat{\omega} - \omega) - \frac{\hat{T}_l - T_l}{J} + \eta \operatorname{sgn}(s)\right) < 0. \end{aligned} \quad (21)$$

From the above stability condition,  $\eta$  should be selected by

$$\eta < \left| \frac{B}{J}(\hat{\omega} - \omega) + \frac{\hat{T}_l - T_l}{J} \right|. \quad (22)$$

When the system reaches the sliding surface, we get

$$\dot{\hat{\omega}} - \dot{\omega} = \hat{\omega} - \omega = 0. \quad (23)$$

Substituting Eq. (23) into Eq. (20), we can obtain

$$e = J \eta \operatorname{sgn}(s), \quad (24)$$

$$\dot{e} = g \eta \operatorname{sgn}(s). \quad (25)$$

The observed error of load torque is the solution of

$$\dot{e} - \frac{g}{J} e = 0. \quad (26)$$

Obviously,  $g < 0$  can guarantee the error converges to zero. In addition, the rate of convergence of observer should much faster than that of the controller.

#### V. SIMULATION AND EXPERIMENT

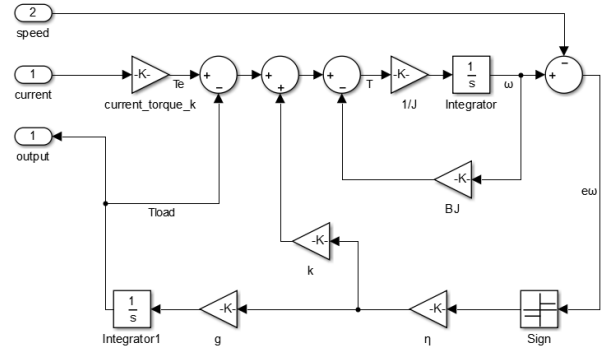


Fig. 2 Structure of the load torque observer designed based on sliding mode control

To validate the proposed algorithm, a system model is built using the Matlab/Simulink software. Parameters of the BLDC motor model are as follows: the rated voltage  $u$  is 48V, the rated power  $P$  is 1kW, the rated speed  $n$  is 500rpm, the number of pole pairs  $p$  is 23, the stator phase resistance  $R$  is 0.1743Ω, the stator phase inductance  $L = 0.139\text{mH}$ , the maximum allowable stator current  $I$  is 50A, the motor inertia  $J$  is 1.36kg·m<sup>2</sup>. Powergui is used to build inverter power module parts of the model, and the discrete system interval is  $T_s = 5e - 6s$ .

As a comparison, another control system using the traditional PI controller is also built. Back-calculation is applied as Anti-windup algorithm. The maximum output of PI controller is set to 50A, namely, the maximum allowed current of the motor. The proportional coefficient and integral coefficients are 5 and 1 respectively. System model is shown in Fig. 3. A 300rpm step input is given at  $T = 1$ , and the step response is shown in Fig. 4. A 30N·M disturbance load is given at  $T = 10$ , and the disturbance response is represented in Fig. 5.

From the simulation results, we can see that when using the PI control method, as a result of the maximum current limitation, a uniform acceleration process occurs at the initial stage of the step response. When added an integrator to, the system has no static error, a clear overshoot of 125rpm arises. When anti-windup algorithm is applied, the overshoot is reduced to about 21rpm, but cannot be eliminated either. After a disturbance load is conducted, the speed fluctuates significantly, and the maximum reduction is about 33rpm, while the rate of speed change is over 11%, and the recovery time is over 3.0s. Moreover, if increasing the integral coefficient, the output would fluctuate, and it is hard to get a good control performance.

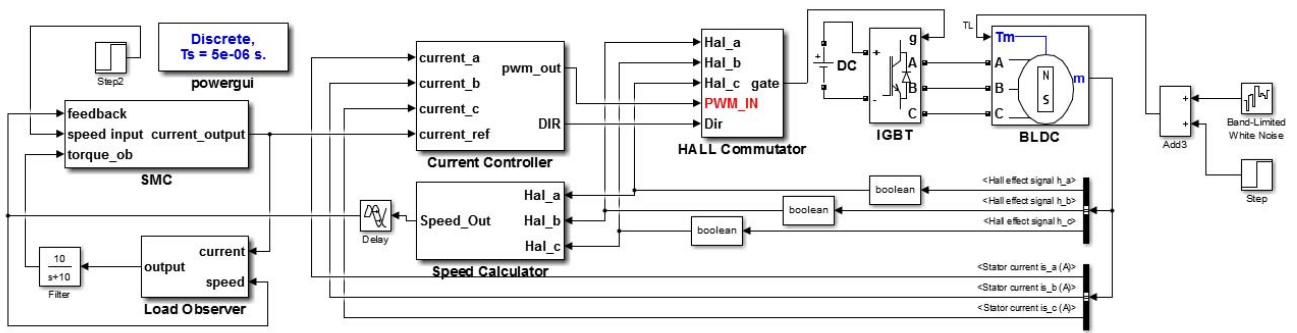


Fig. 3 Matlab model based on sliding mode control

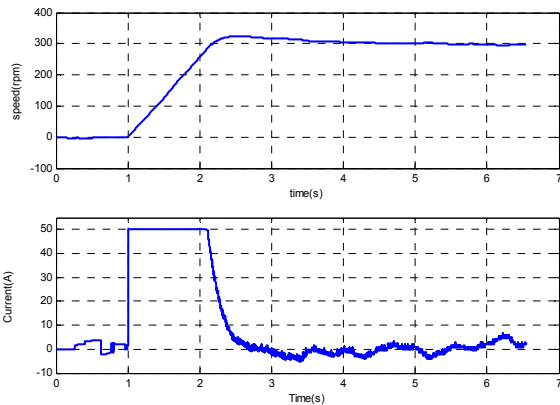


Fig. 4 Step response and temporal evolution of the current of the PI controller

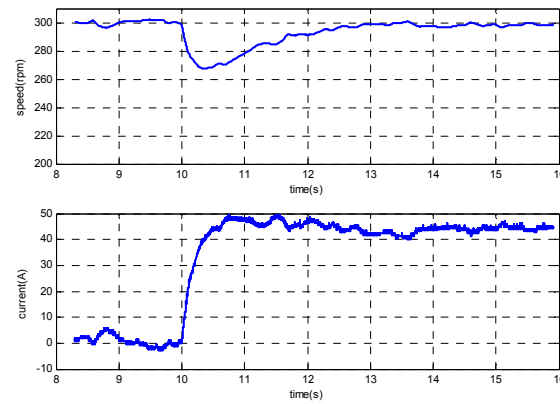


Fig. 5 Disturbance response and temporal evolution of the current of the PI controller

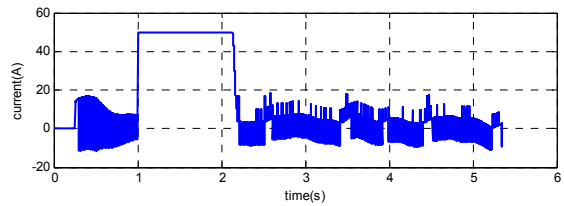
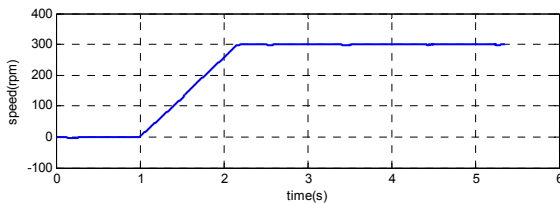


Fig. 6 Step response and temporal evolution of the current of the sliding mode controller

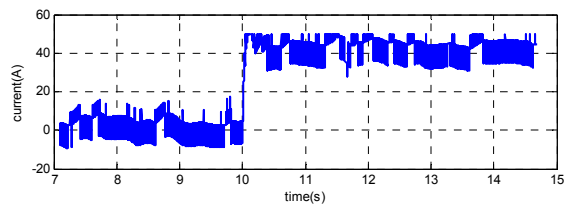
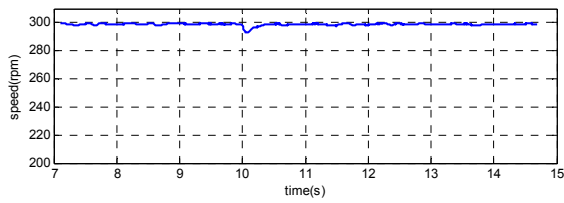


Fig. 7 Disturbance response and temporal evolution of the current of the sliding mode controller

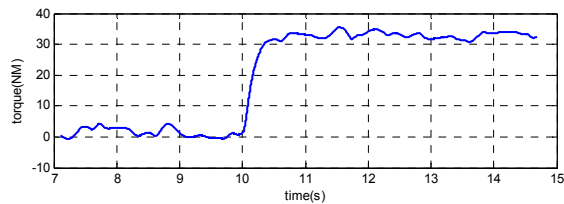


Fig. 8 Output of the sliding mode torque observer with filter

Response of the controller using sliding mode torque observer and sliding mode controller in the same condition as that using in the PI controller is shown in Fig. 6 and 7. The estimated value of disturbance load is shown in Fig. 8. The overshoot is below 3rpm, much reduced compared to that in the condition using PI controller. The speed change after a disturbance is applied is also greatly reduced, and the maximum reduction is 8rpm, with a rate of speed change within 2.7%. It can be seen that the performance is much better than that using PI controller. Though, a bad influence that the current chatters slightly chatter is brought about. This results from some inherent reasons such as the digital control

interval, delay in the speed calculation with hall sensor, which lead to the chattering of the system in a tight neighbourhood of the sliding surface. This chatter amplitude is positive correlated with the sliding coefficient  $\varepsilon$ . On the premise of satisfying the system reaching condition, the chattering can be reduced by reducing  $\varepsilon$ .

In order to verify the effectiveness of the proposed control strategy, an experimental platform for BLDC motor control is built using DSP (TI TMS320F28335) and CPLD (ALTERA EPM1270), both the proposed control strategy and the PI control strategy are applied. In this platform, digital current hysteresis controller is implemented by CPLD and high-speed ADC, while phase current collection, hysteresis judgment and power module control signal output are completed by CPLD. Output frequency of the current loop is 20 kHz. The speed loop is implemented on DSP, and the output frequency is 2 kHz. System structure is shown in Fig. 9.

PI control experiment and sliding mode experiment are both carried out on this platform, and results are shown in Fig. 10 and 11 respectively. When the PI controller is used, there is a clear overshoot of 36rpm and if increasing the integral coefficient, the output will fluctuate. It is can be seen that the control performance becomes better after using the sliding mode controller.

## VI. CONCLUSIONS

Taking into account the non-linear characteristics of BLDC motor control system and the demand of low-inductance motor control system for high-performance current control, this paper proposes a control strategy combining the current hysteresis control and the speed loop control based on the exponential approach law sliding mode speed controller and sliding mode torque observer to construct the high-performance control system for an ultra-low-inductance BLDC motor. The simulation and experimental verification show that compared to the traditional PI control strategy, the proposed control strategy represents better performance in overshoot control, response speed and adaptability to load variations. In addition, due to its simplicity, the proposed control strategy has obvious project value.

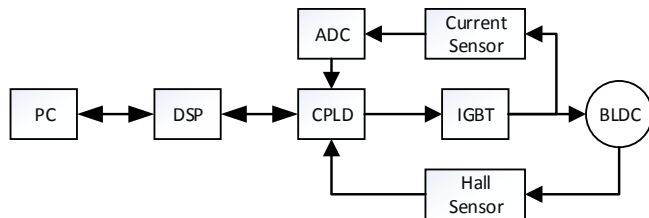


Fig. 9 Structure of the experimental platform

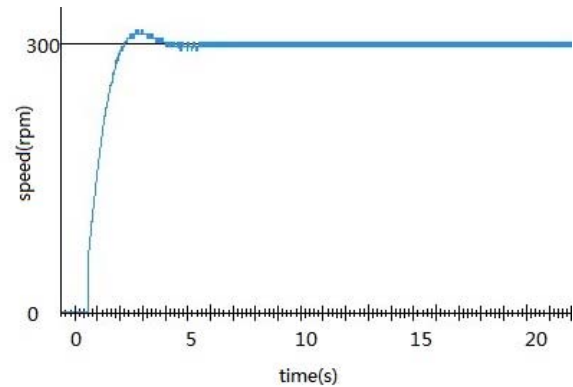


Fig. 10 Experimental step response of the PI controller

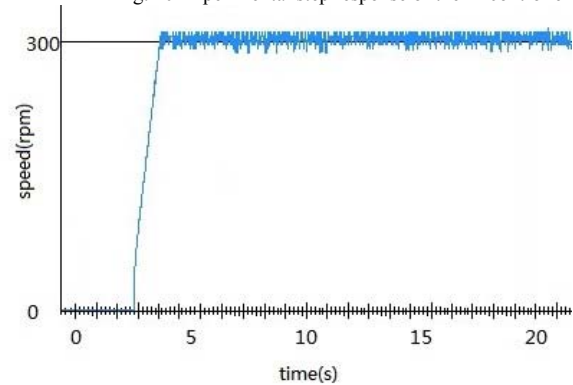


Fig. 11 Experimental step response of the sliding mode controller

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