Response to Referee #2

We are very pleased that the referee recommends publication. We also greatly appreciate the referee's constructive comments, which have proved very helpful in guiding our thinking on this topic. We address each of the referee's comments below. All of the differences between the original and new versions of the manuscript are indicated by change bars in the margin.

1. I dont understand why the 2π factor is included in the definition of the Froude number. I know the authors tend to do this, but does anyone else? If not, why bother - it just makes comparisons with other studies more difficult. On page 2, it is suggested that the 2π is included because it is part of the buoyancy period, and the Froude number is a ratio of time scales. But one could equally argue that there should be a 2π in the eddy time scale L/U (e.g. in a circular eddy with azimuthal velocity U at radius L). It just seems arbitrary. Furthermore, it seems to be omitted in some cases (Fr_h in the middle of page 2) and it makes several resulting equations cumbersome, e.g. for E_p and buoyancy flux on page 15, potential energy dissipation rate on page 17 (why is it not $(2\pi)^2$?), relationship between Gn and R, etc.

We appreciate that we have been unable to persuade the community to adopt the factor of 2π in the Froude number. Nevertheless, we find our definition to be informative because $Fr \sim O(1)$ is an important theoretical threshold and an order of magnitude change in Fr significantly changes the flow because it is Fr^2 that appears in the dimensionless momentum equation. For example, scaling analysis suggest that $Fr \approx 1$ a buoyancy period after stratification is imposed, and we use this prediction to verify the current simulations. Observing that $Fr \approx 0.16$ after 6.3 buoyancy times, which is the equivalent statement when the factors of 2π are omitted, gives the impression of an emprical result, not the exhibiting of a result predicted by theory.

In our opinion, the main difficulty in comparing Fr in the literature is not the factor of 2π but the different length scales that are used. The literature shows that the horizontal longitidunal integral length L_h is the dynamically relevant length scale rather than the turbulence length scale $L_t = u^3/\epsilon$, and that L_h/L_t is a function of Fr. So the common practice of defining a Froude number defined in terms of L_t makes it a function of itself and also makes the threshold for strong stratifcation effects $Fr \ll O(1)$.

So we think that our inclusion of 2π in the the definition of Fr has important advantages and presents a very minor problem in comparing Froude numbers between papers relative to the other challenges in doing so.

Jim On page 2 and the other pages of the introduction, many definitions of the Froude number are used as review the published literature. On page 2 we have change modified the notation to show, e.g., that Fr_h scales with U_h/NL_h.

The definition of χ on page 17 was in error and has been corrected.

Gn is defined Gibson and is extensively used in the oceanography community without it being interpreted as $(L_o/L_k)^{4/3}$. If it is interpreted as this ratio, then it would be advantageous to include the factor of 2π because L_o is much smaller than the length that it is hypothesised to represent, that is, the largest length scale unaffected by buoyancy. Waite [2011] and Almalkie and de Bruyn Kops [2012] recongnise this problem and include 2π in the defintion of L_o so that L_o is a physically plausible length scale in simulations. However, since the use of Gn predates its interpretation by Gargett et al. [1984] as a ratio of length scales, and is routinely used outside this context, we do not see any inconsistency with our omitting the factor of 2π in its definition.

In our opinion, the factor of 2π is not a significant cause of the confusion related to $Gn/R \neq 1$ because different length scales and velocity scales are used in the defintions of Gn and R. Reconciling the two quantities requires accounting for L_h/L_t and u/u_h, both of which vary with, at a minimum, with Froude and Reynolds number. In fact we emphasise that Gn and R are different quantities, as shown by figure 11, not the same quantity written in terms of different multiplicative constants.

Jim – you worked through the derivation and may still have your notes.

2. Some of the figure captions are difficult to understand. For example, in figure 3, there are reference lines for all the curves (this should be stated in caption) but only one is labelled in panels a and b. Also in figure 3, the caption can be misinterpreted as saying that symbols are for non-stratified cases in panel d only. It took some work for me to fully understand this figure. Other captions are lacking in any detail (e.g. figure 17).

We have reworked almost all of the figures and captions in an attempt to make them as clear as possible.

3. There doesnt seem to be any real reason for including Case IV in table 1, since it is not discussed or presented.

Jim – another review wonders why Case IV is introduced in the conclusions. I think we can just state the result (that it does not matter what Fr we start with) and omit Case IV

4. On page 12: how can stratification reduce the dissipation rate in forced simulations, where the dissipation rate must balance the forcing? Im not sure this is a meaningful generic statement, since it will depend on the type of forcing used.

In general, forcing can maintain constant energy or inject constant power. In the latter case, stratification cannot change the dissipation rate and instead changes the energy in the flow [c.f. Lindborg 2006]. Our forcing maintains constant energy in a range of wave numbers and so stratification can, and does, change the dissipation rate. The text has been changed to indicate that the simulations referenced used constant-energy forcing.

5. It would be interesting to include some energy spectra from the late time results. Does the horizontal spectrum look like kh5 at these times, as seen in forced studies with viscous layering (Waite & Bartello 2004, Brethouwer et al 2007)?

6. Page 4, third paragraph: consistent others should be consistent with others

Corrected.

References

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