

CD estimator

Debkrishna Manna

10/19/2020

```
N=1000;n=100
#quantile_calculation
e=rnorm(10000,0,sqrt(0.1))
X=rgamma(10000,shape=1,scale = 2)
Y=X+e
p=ecdf(Y)
t1=Y[which(p(Y)==0.25)];t2=Y[which(p(Y)==0.5)]
t3=Y[which(p(Y)==0.75)];t4=Y[which(p(Y)==1)]
#Iteration process
ITR=function(r,t){
e=rnorm(N,0,sqrt(0.1))
X=rgamma(N,shape=1,scale = 2)
Y=X+e
#Finite population cdf
Q=ecdf(Y)
F_Nt=Q(t)
#CD_estimator & Naive estimator
s=sample(c(1:1000),n,replace=FALSE)
y=Y[s];x=X[s]
x_ns=X[-s]
beta_est=as.numeric((solve(t(x)%*%x))%*%t(x)%*%y)
x_ij=matrix(NA,nrow = length(x),ncol=length(x_ns))
cnt=0;
for (i in 1:length(x)) {
  for(j in 1:length(x_ns)){
    x_ij[i,j]=x_ns[j]-x[i]
    if(y[i]<=t-(beta_est*x_ij[i,j])){
      cnt=cnt+1
    }
  }
}
CD=((length(y[y<=t]))/N)+((cnt)/(N*n))
F_est=((length(y[y<=t]))/n)
list(F_Nt=F_Nt,CD=CD,F_est=F_est)
}
R=500
k=c(1:R)
H=lapply((k),t=t1,ITR)
F_Nt=CD=F_est=array(NA,R)
for(i in 1:R){
  F_Nt[i]=H[[i]]$F_Nt
  CD[i]=H[[i]]$CD
}
```

```
F_est[i]=H[[i]]$F_est
}
```

The CD estimator of distribution function at Quartile 1 is

```
H[[1]]$CD
```

```
## [1] 0.26082
```

The Naive estimator of distribution function at Quartile 1 is

```
H[[1]]$F_est
```

```
## [1] 0.15
```

The Relative Bias of CD estimator and Naive estimator respectively

```
#Relative Bias
mean(CD-F_Nt)/mean(F_Nt);mean(F_est-F_Nt)/mean(F_Nt)
```

```
## [1] -0.0005234272
```

```
## [1] -0.01799205
```

The Relative MSE of CD estimator and Naive estimator respectively

```
# Relative MSE
((sum((CD-F_Nt)^2))/R)/mean(F_Nt);((sum((F_est-F_Nt)^2))/R)/mean(F_Nt)
```

```
## [1] 0.0004604046
```

```
## [1] 0.006228866
```