Homework 4

Discrete Structures 1

due: 30 March 2023, 8:00am

Your task for this homework will be to answer the following questions without using any calculating resources. Your responses should be submitted via blackboard by the due date above as a PDF (submissions in any other format will be returned to the user and a resubmissions will be requested). You are free to use whatever tools you would like to generate the response document: scanned hand-written paper, tablet generated hand-written, microsoft word (with this option, please use the equation editor to correctly format your responses), LaTeX, etc. Your TA, IA, and Instructor are available to help during their designated office hours or via email (note that emails sent during non-business hours may not be responded to until the next working day).

1. True or False: it is always the case that

$$a \mod 10 = (a + 100) \mod 10 = (a - 10) \mod 100$$

for some $a \geq 20$. Justify your answer.

2. What is

$$\left|\{x \mod y : x \in \mathbb{Z}^{\geq 0}\}\right|$$

for a fixed, but arbitrary, number $y \in \mathbb{Z}^{\geq 1}$? (note this is the set size, not the set itself.)

3. Reduce the following to a polynomial on x (that is an equation that involves only x):

$$\sum_{c=3}^{5} \sum_{d=1}^{c} x^d$$

4. Supposed $A \times B = \{\langle 1, 1\rangle, \langle 2, 1\rangle\}$. What are the sets A and B?
(note that $\times$ is not multiplication, its cartesian product)

5. For the vector $c = \langle 4, 0\rangle$, what is the value of $c \cdot c$?
(note that $\cdot$ is not multiplication, its dot product)

6. State the **domain** and **range** of the following functions, be as precise as possible (that means if you can place a restriction on one of our known sets, do so; i.e. $\mathbb{Z}^{\geq 1}$ rather than just $\mathbb{Z}$).
   (a) $f(x) = |x|$
   (b) $f(x) = \lfloor x \rfloor$
   (c) $f(x) = x \mod 2$
   (d) $f(x) = 2 \mod x$

7. Show (using a truth table) that the following equivalences hold:

(a) $p \vee (q \vee r) \equiv (p \vee q) \vee r$

(b) $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

(c) $p \oplus (q \oplus r) \equiv (p \oplus q) \oplus r$