Homework 3

Discrete Structures 1

due: 7 March 2023, 8:00am

Your task for this homework will be to answer the following questions without using any calculating resources. Your responses should be submitted via blackboard by the due date above as a PDF (submissions in any other format will be returned to the user and a resubmissions will be requested). You are free to use whatever tools you would like to generate the response document: scanned hand-written paper, tablet generated hand-written, microsoft word (with this option, please use the equation editor to correctly format your responses), LaTeX, etc. Your TA, IA, and Instructor are available to help during their designated office hours or via email (note that emails sent during non-business hours may not be responded to until the next working day).

1. What is the truth values of the following propositions: $22 + 32 = 42$
2. The identity of a binary operator $\diamond$ is a value i such that, for any x, the expressions $\{x, xi, ix\}$ are all equivalent. An example from arithmetic: the identity of $+$ is 0, because $x+0 = 0+x = x$ for any number x.
   (a) Identify the identity of $\vee$, and justify your answer. (Some operators do not have an identity; if there is no identity, explain why it doesn't exist.)
   (b) What is the identity of $\wedge$? (Or, if it doesn't exist, explain why not.)
   (c) What is the identity of $\leftrightarrow$? (Or, if it doesn't exist, explain why not.)
   (d) What is the identity of $\oplus$? (Or, if it doesn't exist, explain why not.)
3. In addition to purely logical operations, computer circuitry has to be built to do simple arithmetic very quickly. Consider a number $x \in 0, ..., 15$ represented as a 4-bit binary number, and denote by x_0 the least-significant bit of x, by x_1 the next bit, and so forth. (Think of 0 as false and 1 as true.) Here you'll explore some pieces of using propositional logic and binary representation of integers to express arithmetic operations. (It's straightforward to convert your answers into circuits.) See Figure 1.
   (a) Give a proposition over $\{x_0, x_1, x_2, x_3\}$ that expresses that x is greater than or equal to 8.
   (b) Give a proposition over $\{x_0, x_1, x_2, x_3\}$ that expresses that x is evenly divisible by 4.
   (c) Give a proposition over $\{x_0, x_1, x_2, x_3\}$ that expresses that x is evenly divisible by 9.

you may want to make a truth table

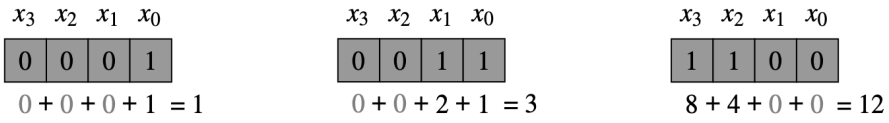

Figure 1: Some examples for Question 3