# Homework 7

Discrete Structures 1

due: 27 April 2023, 8:00am

Your task for this homework will be to answer the following questions without using any calculating resources. Your responses should be submitted via blackboard by the due date above as a PDF (submissions in any other format will be returned to the user and a resubmissions will be requested). You are free to use whatever tools you would like to generate the response document: scanned hand-written paper, tablet generated hand-written, microsoft word (with this option, please use the equation editor to correctly format your responses), LaTeX, etc. Your TA, IA, and Instructor are available to help during their designated office hours or via email (note that emails sent during non-business hours may not be responded to until the next working day).

1. Prove or disprove: *For any number $x \in \mathbb{Z}$, $x^2$ is even.*

2. Prove that the area of a right triangle with legs $x$ and $y$ is $xy/2$.

3. Prove the following by **contrapositive**: For $n \in \mathbb{Z}^{\geq 0}$. If $2n^4 + n + 5$ is odd, then $n$ is even.

4. Identify whether the following arguments are valid or fallacious. Justify your answers.

   (a) Premise A: Every programming language that uses garbage collection is slow.
   Premise B: C does not use garbage collection.
   Conclusion: Therefore, C is not slow.

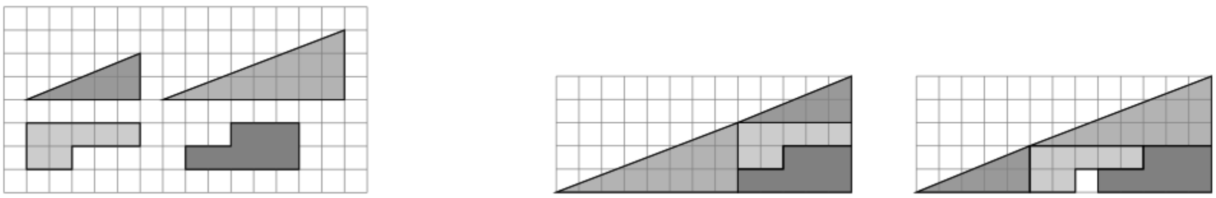
   (b) Premise A: Every data structure is either slow at insertions or lookups.
   Premise B: The data structure called the Hackmatack tree is slow at insertions.
   Conclusion: Therefore, the Hackmatack tree is slow at lookups.

5. Here is a (nonobviously) bogus proof of the (obviously) bogus claim that $0 = 1$. Identify precisely the flaw in the argument.

   **Proof that $\mathbf{0 = 1}$.** Consider the four shapes in Figure 1a, and the two arrangements thereof in Figure 1b. The area of the triangle in the first configuration is $\frac{13 \cdot 5}{2} = \frac{65}{2}$, as it forms a right triangle with height 5 and base 13. But the second configuration also forms a right triangle with height 5 and base 13 as well, and therefore it too has area $\frac{65}{2}$. But the second configuration has one unfilled square in the triangle, and thus we have

$$
\begin{aligned}
0 = & \frac{65}{2} - \frac{65}{2} \\
= & \text{area of the second bounding triangle} - \text{area of the first bounding triangle} \\
= & (1 + \text{area of four constituent shapes}) - (\text{area of four constituent shapes}) \\
= & 1.
\end{aligned}
$$

hint: you may want to look at where exactly the 3 objects meet along the "hypotenuse"



(a) The shapes. (b) Two configurations.

Figure 1: Figures for Question 4