Homework 1

Discrete Structures 1

due: 7 February 2023, 8:00am

Your task for this homework will be to answer the following questions without using any calculating resources. Your responses should be submitted via blackboard by the due date above as a PDF (submissions in any other format will be returned to the user and a resubmissions will be requested). You are free to use whatever tools you would like to generate the response document: scanned hand-written paper, tablet generated hand-written, microsoft word (with this option, please use the equation editor to correctly format your responses), LaTeX, etc. Your TA, IA, and Instructor are available to help during their designated office hours or via email (note that emails sent during non-business hours may not be responded to until the next working day).

1. If x and y are integers, is $x + y$ necessarily an integer? Why or why not?

2. If x and y are rationals, is $x + y$ necessarily an rational? Why or why not?

3. What is the value of $(\lfloor 3.14159 \rfloor)^{\lceil 3.14159 \rceil}$?

4. Are $\lfloor |x| \rfloor$ and $||x||$ always equal? Why or why not?

5. Without using a computer or calculator decide which is bigger: $\log_{10} 17$ or $\log_{17} 10$? (show each step used to make the decision)

6. What is the smallest positive integer n, where $n \bmod 2 = 0$, $n \bmod 3 = 0$, and $n \bmod 5 = 0$?

7. What is the smallest positive integer n where $n \bmod 3 = 2$, $n \bmod 5 = 3$, and $n \bmod 7 = 5$?

8. Compute the following sum (and show each step): $\displaystyle\sum_{i=1}^{6} 2^{2i}$

9. Compute the following product (and show each step): $\displaystyle\prod_{i=1}^{6} 2^{2i}$

10. Compute the following nested sum (and show each step): $\displaystyle\sum_{i=1}^{4}\sum_{j=i}^{4} \left(j^i\right)$