Final Exam

Discrete Structures 1

8 May 2023

Name: ____
(please write legibly)

Question	**Topic**	**Value**
1	(functions)	/36
2	(predicate logic)	/30
3	(propositional logic)	/35
4	(propositional logic)	/30
5	(proofs)	/15
6	(proofs)	/30
7	(proof by contradiction)	/30
8	(proof by contrapositive)	/40
9	(proof by cases)	/40
10	(proofs)	/15
	bonus	
Total		/301

There are a few rules:

(1) You are not allowed to use outside online resources. No outside help (e.g., from a classmate, a friend, online search, documents on your own laptop, etc.) will be tolerated. Any attempt to obtain help or information about the exam will be reported.

(2) **You are not allowed to use headphones during the exam.** Your phone should be in your backpack.

(3) You should only have a pencil (or pen), an eraser, and a pencil sharpener with you on the table at the time of the exam. Everything else should be safely packed in your backpack and not to be used at any time during the exam.

(4) You were told to attend to the bathroom before the exam starts: you will not be allowed to leave the room during the first hour of the exam (unless you have a doctor's note to indicate otherwise).

A few pieces of advice:

(1) Read the questions carefully and try the tracing exercises on draft paper before you answer on your exam copy.

(2) Pay careful attention to the instructions written in the exam.

Please write legibly and in a structured manner: keep in mind that what you write needs to be read. Answers that are unreadable or hard to follow will not receive full credit.

1. Select the smallest size set that represents the domain and range of each of the following functions:

Function	Domain	Range
$f(x) = x \bmod 3$	__________	__________
$g(x) = 3 \bmod ((x \bmod 2) + 4)$	__________	__________
$h(x) = 3x^2 + \pi x + \sqrt{2}$	__________	__________
$\ell(x) = \lvert\lfloor x \rfloor\rvert$	__________	__________
$e(x) = \lvert x \rvert$	__________	__________
$r(x) = x \bmod 4 + \frac{3}{2}$	__________	__________

(A) $\mathbb{Z}$	(H) $\{0\}$
(B) $\mathbb{Z}^{\geq 1}$	(K) $\{1\}$
(C) $\mathbb{Z}^{\geq 0}$	(M) $\{2\}$
(D) $\mathbb{R}$	(N) $\{3\}$
(E) $\mathbb{R}^{\geq 1}$	(O) $\{0, 1\}$
(F) $\mathbb{R}^{\geq 0}$	(P) $\{0, 1, 2\}$
(G) $\varnothing$	(Q) $\{0, 1, 2, 3\}$

2. For each of the following statements, write a qualified predicate logic statement. The variables will be given in parenthesis.

   (a) Some student ($s \in S$) in the class has the highest grade ($g(s)$) in the class. (you will likely need to define a second student $t \in S$).

   (b) For all topics topics ($t \in T$) there is at least one questions ($q \in Q$) that at least one student ($s \in S$) will get correct ($C(t, q, s) = true$).

3. Construct a truth table to evaluate the following expression:

$$(p \land \neg q) \lor (\neg q \land \neg r)$$

4. Using the expression and truth table from Question 3, answer the following:
   (a) Is the expression a tautology? why or why not?

   (b) Is the expression satisfiable? Why or why not?

   (c) Is the expression in Disjunctive Normal Form, Conjunctive Normal Form, or neither? Explain your response.

5. Disprove the following: *A coin system with coins of value 4 and 7 cents can make any amount of change above 11 cents.*

6. Prove the following claim:

   There is a course taught in the CS department, that is one credit hour.

7. We want to prove the following using contradiction, **what is the initial assumption**?

   Any product xy of two odd numbers x and y is odd.

8. If instead we wanted to prove the claim in the previous question by *contrapositive*,
   (a) assuming we represent the initial implication as $p \implies q$, what is p?

   (b) what is q?

   (c) what is $\neg q$?

   (d) what is $\neg p$?

9. We want to prove the claim below:

$$\textit{For all numbers } x \in \mathbb{Z}^{\geq 1}\textit{, if } x \bmod 8 \in \{2,3,6,7\}$$
$$\textit{then the second to last bit in the binary representation is a 1.}$$

   (a) what would the different cases be in a proof by cases?

   (b) once we have proven each of the cases (no need to do it here, just assume we did), what is the final step in a proof by cases? (you should provide the final statement/argument.)

10. Identify and describe the flaw in the following direct proof.

    False Claim: 1=0

    Proof. Let $x, y \in \mathbb{Z}^{\geq 1}$ where $x = y$.

$x = y$	by assumption
$x^2 = x \cdot y$	multiply both sides by x
$x^2 - y^2 = x \cdot y - y^2$	subtract y^2 from both sides
$(x-y)(x+y) = y(x-y)$	algebra
$x + y = y$	divide both sides by $(x - y)$
$2y = y$	because $x = y$
$2 = 1$	divide by y, which is non-zero
$1 = 0$	subtract 1 from both sides

$\square$