Correctness Quiz 1

Discrete Structures 2

21 February 2023, 9am

Name: _____
(please write legibly)

1. $\forall x, y \in \mathbb{Z}^{>1}$ such that $x < y$, what is the value of x mod y?

2. $\forall z, q \in \mathbb{Z}^{>1}$ (with no other restriction), what is the maximum possible value of any number q mod z?

3. Let
   - $U = \{x \in \mathbb{Z} : x \in [1, 20)\}$,
   - $A = \{y \in U : (y \bmod 4 = 2) \text{ or } (y \bmod 4 = 0)\}$, and
   - $B = \{z \in U : (z \bmod 6 = 4) \text{ or } (z \bmod 6 = 2) \text{ or } (z \bmod 6 = 0)\}$.

   (a) Write A by enumerating the items:

   (b) Write B by enumerating the items:

   (c) What is the value of $|A \cup B|$?

   (d) What is the value of $|A \cap B|$?

   (e) What is the set $A - B$?

   (f) What is the set $B - A$?

   (g) Is the statement $A \subseteq B$ true? why or why not?

   (h) Is the statement $A \subset B$ true? why or why not?

4. Prove by **strong induction** on n that, for every integer $n \geq 4$, it is possible to make n dollars using only two- and five-dollar bills. (That is, prove that any integer $n \geq 4$ can be written as $n = 2a + 5b$ for some integer $a \geq 0$ and some integer $b \geq 0$.)