Homework 5

Discrete Structures 2

due: 13 April 2023, 8:00am

Your task for this homework will be to answer the following questions without using any calculating resources. Your responses should be submitted via blackboard by the due date above as a PDF (submissions in any other format will be returned to the user and a resubmissions will be requested). You are free to use whatever tools you would like to generate the response document: scanned hand-written paper, tablet generated hand-written, microsoft word (with this option, please use the equation editor to correctly format your responses), LaTeX, etc. Your TA, IA, and Instructor are available to help during their designated office hours or via email (note that emails sent during non-business hours may not be responded to until the next working day).

1. For two strings x and y, let's call a shuffle of x and y any interleaving of the letters of the two strings (that maintains the order of the letters within each string, but may repeatedly alternate between blocks of x letters and blocks of y letters). For example, the words `ALE` and `LID` can be shuffled into

   `A L L I E D` *or* `A L L I D E` *or* `A L L I D E` *or* `L I D A L E`.

   How many different strings can be produced as shuffles of the following pairs of words?

   (a) `BACK` and `FORTH`

   (b) `DAY` and `NIGHT`

   (c) `SUPPLY` and `DEMAND`

   (d) `LIFE` and `DEATH`

   (e) `ON` and `ON`

   (f) `OUT` and `OUT`

2. What is the smallest even integer n for which the following statement is true? If we flip an unbiased coin n times, as in the example from class, the probability that we get exactly $\frac{n}{2}$ heads is less than 10%.

3. How many ways are there to choose 32 out of 202 options if

   (a) repetition is allowed and order matters?

   (b) repetition is forbidden and order matters?

   (c) repetition is allowed and order doesn't matter?

   (d) repetition is forbidden and order doesn't matter?

4. Consider the equation $a + b + c = 202$. How many solutions are there where a, b, and c are all nonnegative integers?

5. How many solutions are there to the equation $a + b + c + d + e = 8$, where all of $\{a, b, c, d, e\}$ must be nonnegative integers?

6. What about for $a+b+c+d+e = 88$, again where all variables must be nonnegative integers?

7. What about for $a + 2b + c = 128$, again where a, b, and c must be nonnegative integers? (Hint: sum over the possible values of b and use the slide "Choosing with repetition when order doesn't matter" from the slides/Theorem 9.16 in the book.)