Homework 6

Discrete Structures 1

due: 20 April 2023, 8:00am

Your task for this homework will be to answer the following questions without using any calculating resources. Your responses should be submitted via blackboard by the due date above as a PDF (submissions in any other format will be returned to the user and a resubmissions will be requested). You are free to use whatever tools you would like to generate the response document: scanned hand-written paper, tablet generated hand-written, microsoft word (with this option, please use the equation editor to correctly format your responses), LaTeX, etc. Your TA, IA, and Instructor are available to help during their designated office hours or via email (note that emails sent during non-business hours may not be responded to until the next working day).

1. If you wanted to prove the following claim *by cases*, what would the cases be that you're going to divide the problem into:

   claim: *For any number $x \in \mathbb{Z}$, x^2 is even.*

2. Prove that the binary representation of any odd integer ends with a 1.

3. We want a *proof by contrapositive* for the following claim, what is the contrapositive of the statement:

   claim: *Let $n \in \mathbb{Z}^{\geq 0}$. If $n \mod 4 \in \{2, 3\}$, then n is not a perfect square.*

4. If you were to prove the following claim using *proof by contradiction*, what would be the initial assumption:

   claim: *Let $x, y \in \mathbb{R}$. If $x^2 - y^2 = 1$, then x or y (or both) is not an integer.*

5. Disprove the following *by counterexample*: *If $x - y$ is rational, then x and y are rational.*

6. Prove that the area of a right triangle with legs x and y is $xy/2$.