Homework 4

Discrete Structures 2

due: 30 March 2023, 8:00am

Your task for this homework will be to answer the following questions without using any calculating resources. Your responses should be submitted via blackboard by the due date above as a PDF (submissions in any other format will be returned to the user and a resubmissions will be requested). You are free to use whatever tools you would like to generate the response document: scanned hand-written paper, tablet generated hand-written, microsoft word (with this option, please use the equation editor to correctly format your responses), LaTeX, etc. Your TA, IA, and Instructor are available to help during their designated office hours or via email (note that emails sent during non-business hours may not be responded to until the next working day).

1. Complete the proof below: We will prove by *weak* induction on n that

$$\sum_{i=1}^{n} f_i = f_{n+2} - 1$$

(where f_k is the k-th fibonacci number).

Base cases ($n = 1$ and $n = 2$):

$$\sum_{i=1}^{1} f_i = f_1 = 1 = 2 - 1 = f_3 - 1$$

and

$$\sum_{i=1}^{2} f_i = f_1 + f_2 = 1 + 1 = 2 = 3 - 1 = f_4 - 1.$$

Inductive case ($n \geq 3$):

We will assume the inductive hypothesis holds for $n - 1$, that is $\sum_{i=1}^{n-1} f_i = f_{n+1} - 1$.
(note you will likely use direct prove to complete the rest of the inductive case, remember to start with the original proposition)

2. A string over an alphabet Σ is a sequence of elements of a set — that is, a string x over Σ satisfies $x \in \Sigma^n$ for some length $n \geq 0$. How many strings of length *at most* n over the alphabet $\Sigma = \{\mathtt{A}, \mathtt{B}, ..., \mathtt{Z}, \text{␣}\}$ are there? How many contain exactly 2 “words” (that is they contain exactly one space and its not in the first or last position).

3. Define a palindrome bit string as one that satisfies one of the following conditions:

   - the empty string (containing no bits);
   - the string `0`;
   - the string `1`;
   - the string `0x0` where `x` is a palindrome bit string; or
   - the string `1x1` where `x` is a palindrome bit string.

   Further define $\#0(s)$ and $\#1(s)$ to be the number of `0`s and `1`s respectively in a string s.

   Prove by **structural induction**, that is induction not over the string s, that the proposition P holds for all palindrome bit strings s:

   $$P(s) : \text{ the value of } [\#0(s)] \cdot [\#1(s)] \text{ is even}$$

   (note that the two numbers are multiplied not summed).

   > The structure of your proof will likely be on the cases in the definition of palindrome bit strings, notice some cases rely on others