Homework 1

Discrete Structures 2

due: 7 February 2023, 8:00am

Your task for this homework will be to answer the following questions without using any calculating resources. Your responses should be submitted via blackboard by the due date above as a PDF (submissions in any other format will be returned to the user and a resubmissions will be requested). You are free to use whatever tools you would like to generate the response document: scanned hand-written paper, tablet generated hand-written, microsoft word (with this option, please use the equation editor to correctly format your responses), LaTeX, etc. Your TA, IA, and Instructor are available to help during their designated office hours or via email (note that emails sent during non-business hours may not be responded to until the next working day).

1. Are $\lfloor |x| \rfloor$ and $|\lfloor x \rfloor|$ always equal? Why or why not?
2. Without using a computer or calculator decide which is bigger: $\log_{10} 17$ or $\log_{17} 10$? (show each step used to make the decision)
3. What is the smallest positive integer n, where $n \bmod 2 = 0$, $n \bmod 3 = 0$, and $n \bmod 5 = 0$?
4. Compute the following nested sum (and show each step): $\sum_{i=1}^{4} \sum_{j=i}^{4} (j^i)$
5. In general, $A - B$ and $B - A$ do *not* denote the same set. But your friends Evan and Yasmin wander by and tell you the following. Let E denote the set of CS homework questions that Evan has not yet solved. Let Y denote the set of CS homework questions that Yasmin has not yet solved. Evan and Yasmin claim that $E - Y = Y - E$. Is this possible? If so, under what circumstances? If not, why not? Justify your answer.
6. Define $S = \{x \in \mathbb{Z} : x \in [1, 100]\}$. Let $W = \{x \in S : x \bmod 2 = 0\}$, $H = \{x \in S : x \mod 3 = 0\}$, and $O = S - H - W$. Is $\{W, H, O\}$ a partition of S?
7. What is the power set of$\{1, a\}$?
8. Let $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$, and let T be an unknown set. suppose that $S \times T = \varnothing$. What can you conclude about T?
9. Let $f(x) = 3x + 1$ and let $g(x) = 2x$. Identify a function h such that $g \circ h$ and f are identical. (For example, the function $f \circ g$ is given by the definition $(f \circ g)(x) = f(g(x)) = 6x + 1$.)
10. Consider $f : \mathbb{R} \to \mathbb{R}$, where $f(x) = 3x + 1$. What is the inverse of f?