

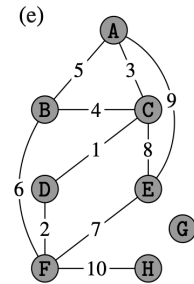
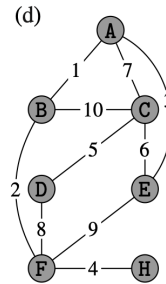
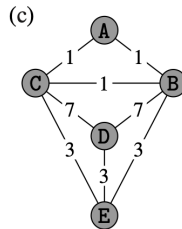
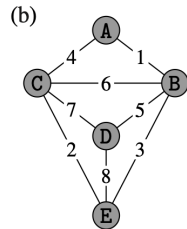
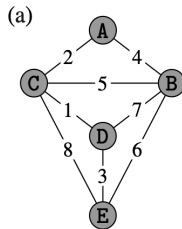
Homework 7

Discrete Structures 2

due: 27 April 2023, 8:00am

Your task for this homework will be to answer the following questions without using any calculating resources. Your responses should be submitted via blackboard by the due date above as a PDF (submissions in any other format will be returned to the user and a resubmissions will be requested). You are free to use whatever tools you would like to generate the response document: scanned hand-written paper, tablet generated hand-written, microsoft word (with this option, please use the equation editor to correctly format your responses), L^AT_EX, etc. Your TA, IA, and Instructor are available to help during their designated office hours or via email (note that emails sent during non-business hours may not be responded to until the next working day).

1. Prove or disprove: in any tree with 3 or more nodes, there is a node of degree equal to 2.
2. Prove or disprove: in any rooted binary tree, there are an even number of leaves. (In a binary tree, all nodes have 0,1, or 2 children.)
3. Prove or disprove: if an undirected graph $G = \langle V, E \rangle$ has $|V| - 1$ edges, then G must be a forest.
4. Prove by induction that a complete binary tree of height h contains precisely $2^{h+1} - 1$ nodes.
5. Identify a minimum spanning trees in each of the following graphs. If a minimum spanning tree does not exist explain why under the graph.



6. Prove or disprove: Let $G = \langle L \cup R, E \rangle$ be an undirected bipartite graph with $|L| = |R|$. Suppose every node in the graph (that is, all nodes in L and R) has at least one neighbor. Then the graph is connected.