Midterm Exam

Discrete Structures 1

9 March 2023, **12noon**

Name: _____

(please write legibly)

Question	**Topic**	**Value**
1	(modulus)	/20
2	(modulus)	/20
3	(sums/products)	/90
4	(sets)	/90
5	(sets)	/40
6	(vectors)	/90
7	(logic)	/120
	bonus	
Total		/470

There are a few rules:

(1) You are not allowed to use outside online resources. No outside help (e.g., from a classmate, a friend, online search, documents on your own laptop, etc.) will be tolerated. Any attempt to obtain help or information about the exam will be reported.

(2) **You are not allowed to use headphones during the exam.** Your phone should be in your backpack.

(3) You should only have a pencil, an eraser, and a pencil sharpener with you on the table at the time of the exam. Everything else should be safely packed in your backpack and not to be used at any time during the exam.

(4) You were told to attend to the bathroom before the exam starts: you will not be allowed to leave the room during the first hour of the exam (unless you have a doctor's note to indicate otherwise).

A few pieces of advice:

(1) Read the questions carefully and try the tracing exercises on draft paper before you answer on your exam copy.

(2) Pay careful attention to the instructions written in the exam.

Please write legibly and in a structured manner: keep in mind that what you write needs to be read. Answers that are unreadable or hard to follow will not receive full credit.

1. For a given $x \in \mathbb{Z}^{\geq 1}$, what is minimum value of $x \bmod 19$? What is the maximum?

2. For a given $b \in \mathbb{Z}^{\geq 1}$ and $a \in \mathbb{Z}^{\geq b}$, what is the relationship between the values of $a \bmod b$ and $(a - b) \bmod b$?

3. Write the equation represented by the following (that is, do not give a single number, write out all of the terms):

   (a) $\prod_{i=1}^{6} i^3$

   (b) $\sum_{i=1}^{4} i^{-2}$

   (c) $\sum_{i=1}^{4} \sum_{j=i}^{3} (i \cdot j)$

4. Let $A = \{0, 1, 3, 4, 5, 7, 8, 9\}$, $B = \{0, 4, 5, 9\}$, and $C = \{0, 3, 6, 9\}$.

   (a) What is $A \cap B$?

   (b) What is $B \cup C$?

   (c) What is $|B \cup C|$?

   (d) What is $C - B$?

   (e) Is $A \subseteq B$?

   (f) Is $A \supseteq B$?

5. True or false: $6 \in \left\{k \in \mathbb{Z}^{\geq 1} : k^2 \leq 10\right\}$? Explain your response.

6. For the vectors $a = \langle 2, -2 \rangle$ and $b = \langle 1, 3 \rangle$:

   (a) What is $a \cdot b$?

   (b) What is $a \cdot a$?

   (c) What is $b - a$?

7. Fill in the following truth table on two atomic propositions p and q, then answer the question below:

p	q	$\neg p$	$\neg q$	$q \wedge p$	$\neg(p \wedge q)$	$\neg p \vee \neg q$

   Are $\neg(p \wedge q)$ and $\neg p \vee \neg q$ equivalent? Explain your answer.

8. BONUS:

   (a) $|\mathbb{Z}| =$

   (b) $|\mathbb{Q}| =$

   (c) $|\mathbb{R}| =$

   (d) True or False: $|\mathbb{Z}| = |\mathbb{R}|$

   (e) True or False: $|\mathbb{Q}| \leq |\mathbb{R}|$