Homework 3

Discrete Structures 2

due: 28 February 2023, 8:00am

Your task for this homework will be to answer the following questions without using any calculating resources. Your responses should be submitted via blackboard by the due date above as a PDF (submissions in any other format will be returned to the user and a resubmissions will be requested). You are free to use whatever tools you would like to generate the response document: scanned hand-written paper, tablet generated hand-written, microsoft word (with this option, please use the equation editor to correctly format your responses), LaTeX, etc. Your TA, IA, and Instructor are available to help during their designated office hours or via email (note that emails sent during non-business hours may not be responded to until the next working day).

> **Note:** all of these questions are on topics from chapters 5; thus you will only be proving by induction in this homework assignment.

1. Prove the following claim holds $\forall n \in \mathbb{Z}^{\geq 0}$ by induction on n:

$$\sum_{i=0}^{n} i^3 = \frac{n^4 + 2n^3 + n^2}{4}$$

2. Prove Bernoulli's inequality by induction: For an arbitrary $x \in \mathbb{R}$, such that $x \geq -1$, prove by induction on n that

$$(1 + x)^n \geq 1 + nx$$

   for any positive integer n.

3. Prove by induction on n that $8^n - 3^n$ is divisible by 5 for any nonnegative integer.

> This seems like it may use modulus somewhere in the proof, it does not.

4. Prove Cassini's identity: $f_{n-1} \cdot f_{n+1} - (f_n)^2 = (-1)^n$ for any $n \geq 2$, there f_n is the n-th Fibonacci number.

> remember, the n-th Fibonacci number $f_n = f_{n-1} + f_{n-2}$ for all $n \geq 3$ and $f_1 = f_2 = 1$.

5. Consider a sport in which teams can score two types of goals, worth either 3 points or 7 points. For example, Team Miners might (theoretically speaking) score 32 points by accumulating, in succession, 3, 7, 3, 7, 3, 3, 3, and 3 points. Find the smallest possible n_0 such that, for any $n \geq n_0$, a team can score exactly n points in a game. Prove your answer correct by strong induction.

   > This means you need to first figure out n_0, then prove that the claim is true for all larger n.

   > You may need to employ more than one base case here