Homework 8

Discrete Structures 1

due: 4 May 2023, 8:00am

Your task for this homework will be to answer the following questions without using any calculating resources. Your responses should be submitted via blackboard by the due date above as a PDF (submissions in any other format will be returned to the user and a resubmissions will be requested). You are free to use whatever tools you would like to generate the response document: scanned hand-written paper, tablet generated hand-written, microsoft word (with this option, please use the equation editor to correctly format your responses), LaTeX, etc. Your TA, IA, and Instructor are available to help during their designated office hours or via email (note that emails sent during non-business hours may not be responded to until the next working day).

1. What is the smallest positive integer n, where $n \bmod 2 = 0$, $n \bmod 3 = 0$, and $n \bmod 5 = 0$?

2. Compute the following nested sum (and show each step): $\sum_{i=1}^{4} \sum_{j=i}^{4} (j^i)$

3. What is the power set of$\{1, a\}$?

4. Given the vectors $v = \langle \sqrt{2}, 5 \rangle$ and $w = \langle \sqrt{2}, 12 \rangle$, is $v \cdot w$ rational or irrational? (note this is the dot product not multiplication since v and w are vectors.)

5. Let $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$, and let T be an unknown set. suppose that $S \times T = \varnothing$. What can you conclude about T?

6. Let $f(x) = 3x + 1$ and let $g(x) = 2x$. Identify a function h such that $g \circ h$ and f are identical. (For example, the function $f \circ g$ is given by the definition $(f \circ g)(x) = f(g(x)) = 6x + 1$.)

7. Consider $f : \mathbb{R} \to \mathbb{R}$, where $f(x) = 3x + 1$. What is the inverse of f?

8. For the following false claim and a bogus proof of that false claim (a) identify the precise error in the proof, and (b) give a counterexample to the claim. (Note that saying why the claim is false does not address (a) in the slightest—it would be possible to give a bogus proof of a true claim!)

   False Claim 1. Let n be a positive integer and let $p, q \in \mathbb{Z}^{\geq 2}$, where p and q are prime. If n is evenly divisible by both p and q, then n is also evenly divisible by pq.

   Bogus proof. Because $p|n$, there exists a positive integer k such that $n = pk$. Thus, by assumption, we know that $q|pk$. Because p and q are both prime, we know that p does not evenly divide q, and thus the only way that $q|pk$ can hold is if $q|k$. Hence $k = q\ell$ for some positive integer ℓ, and thus $n = pk = pq\ell$. Therefore $pq|n$.