Homework 6

Discrete Structures 2

due: 20 April 2023, 8:00am

Your task for this homework will be to answer the following questions without using any calculating resources. Your responses should be submitted via blackboard by the due date above as a PDF (submissions in any other format will be returned to the user and a resubmissions will be requested). You are free to use whatever tools you would like to generate the response document: scanned hand-written paper, tablet generated hand-written, microsoft word (with this option, please use the equation editor to correctly format your responses), LaTeX, etc. Your TA, IA, and Instructor are available to help during their designated office hours or via email (note that emails sent during non-business hours may not be responded to until the next working day).

1. Draw a graph with the following nodes and edges? (make sure to ask yourself the following questions: Does it make sense for the graph to be directed or undirected? Is the graph going to be simple?)
   (a) The nodes $V = \{1, 2, ..., 10\}$; and edge connects x and y if the greatest common denominator of the two numbers is 1.
   (b) The nodes $V = \{1, 2, ..., 10\}$; and edge connects x and y if y is evenly divisible by x.
   (c) The nodes $V = \{1, 2, ..., 10\}$; and edge connects x and y if $x < y$.
2. If $G = \langle V, E \rangle$ is an *undirected*, simple graph with n nodes, what is the largest $|E|$ can be? smallest? explain your answers.
3. If $G = \langle V, E \rangle$ is an *directed*, simple graph with n nodes, what is the largest $|E|$ can be? smallest? explain your answers.
4. If $G = \langle V, E \rangle$ is an *directed* graph with self-loops allowed and n nodes, what is the largest $|E|$ can be? smallest? explain your answers.
5. Determine if the pairs of graphs in Figure 1 are isomorphic? Justify your answers. (note there are two pairs of graphs in the figure and thus you will be providing two answers.)

Figure 1: Graph pairs for Question 5.

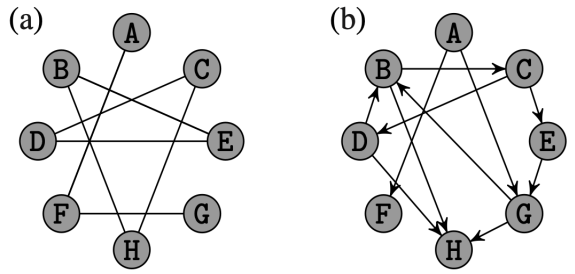

Figure 2: Graph pairs for Question 7.

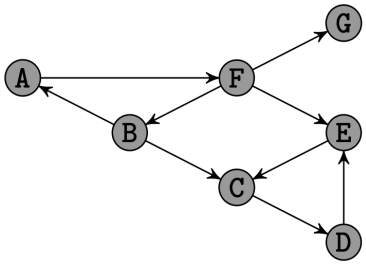

Figure 3: Graph pairs for Question 8.

6. Consider a bipartite graph with a set L of nodes in the left column and a set of nodes R on the right column, where $|L| = |R|$. Prove or disprove the following claims:

   (a) The sum of the degrees of the nodes in L must equal the sum of the degrees of the nodes in R.

   (b) The sum of the degrees of the nodes in L must be even.

7. Use the graphs in Figure 2 to determine the following:

   (a) Enumerated the connected components of Figure 2a. Is the graph connected?

   (b) Enumerated the strongly connected components of Figure 2b. Is the graph strongly connected?

8. If we run the BFS algorithm on the graph in Figure 3 thating at each of the following nodes, what is the *last* node BFS discovers. (if there is a tie, list all of the nodes.)

   (a) `A`

   (b) `B`

   (c) `E`