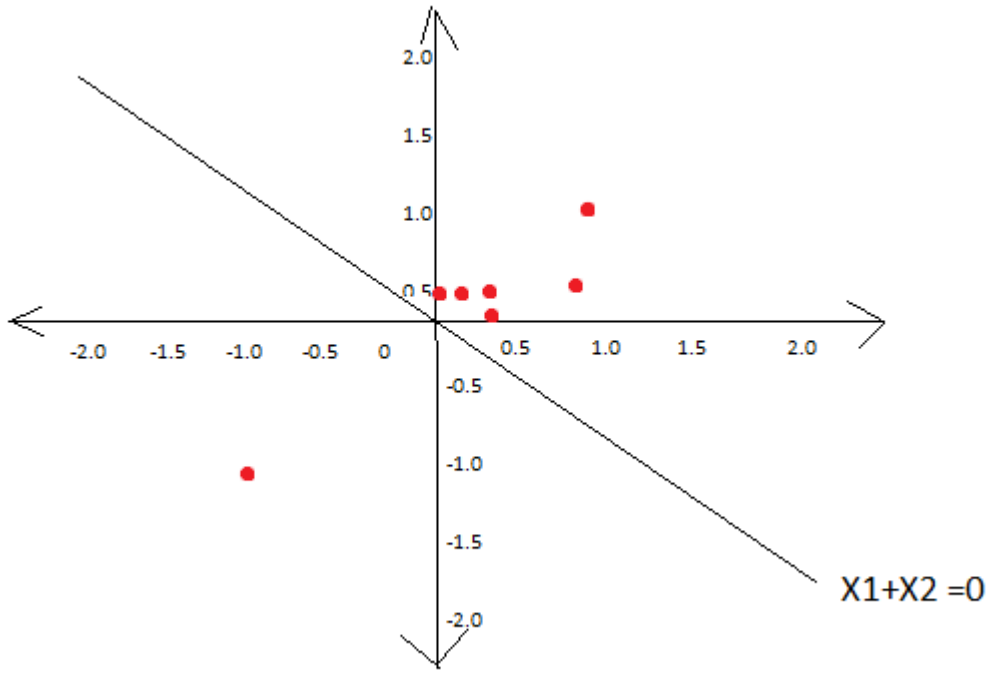
Assume weight vector of initial decision boundary $w^T x = 0$ as w = [1,1]

$y_{in} = w_1^T \dot{x}_i + b = w_i x_1 + w_i x_2 + b$




Assume learning rate as 1

$y = \{$ 1 if $y_{in} > 0$

0 if $y_{in} = 0$

-1 if $y_{in} < 0$

$\Delta\omega_1 = \alpha t x_1$

$\Delta\omega_2 = \alpha t x_2$

$\Delta b = \alpha t$

| $x_1$ | $x_2$ | Class(t) | $y_{in}$ | $y$ | $\Delta\omega_1$ | $\Delta\omega_2$ | $\Delta b$ | $w_1$ | $w_2$ | $b$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | +1 | 2 | +1 | 0 | 0 | 0 | 1 | 1 | 0 |
| -1 | -1 | -1 | -2 | -1 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 0.5 | -1 | 0.5 | +1 | 0 | -0.5 | -1 | 1 | 0.5 | -1 |
| 0.1 | 0.5 | -1 | -0.65 | -1 | 0 | 0 | 0 | 1 | 0.5 | -1 |
| 0.2 | 0.2 | +1 | -0.1 | -1 | 0.2 | 0.2 | 1 | 1.2 | 0.7 | 0 |
| 0.9 | 0.5 | +1 | 1.43 | +1 | 0 | 0 | 0 | 1.2 | 0.7 | 0 |

| $x_1$ | $x_2$ | Class(t) | $y_{in}$ | $y$ | $\Delta\omega_1$ | $\Delta\omega_2$ | $\Delta b$ | $w_1$ | $w_2$ | $b$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | +1 | 1.9 | +1 | 0 | 0 | 0 | 1.2 | 0.7 | 0 |
| -1 | -1 | -1 | -1.9 | -1 | 0 | 0 | 0 | 1.2 | 0.7 | 0 |
| 0 | 0.5 | -1 | 0.35 | +1 | 0 | -0.5 | -1 | 1.2 | 0.2 | -1 |
| 0.1 | 0.5 | -1 | -0.78 | -1 | 0 | 0 | 0 | 1.2 | 0.2 | -1 |
| 0.2 | 0.2 | +1 | -0.72 | -1 | 0.2 | 0.2 | 1 | 1.4 | 0.4 | 0 |
| 0.9 | 0.5 | +1 | 1.46 | +1 | 0 | 0 | 0 | 1.4 | 0.4 | 0 |

| $x_1$ | $x_2$ | Class(t) | $y_{in}$ | $y$ | $\Delta\omega_1$ | $\Delta\omega_2$ | $\Delta b$ | $w_1$ | $w_2$ | $b$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | +1 | 1.8 | +1 | 0 | 0 | 0 | 1.4 | 0.4 | 0 |
| -1 | -1 | -1 | -1.8 | -1 | 0 | 0 | 0 | 1.4 | 0.4 | 0 |
| 0 | 0.5 | -1 | 0.2 | +1 | 0 | -0.5 | -1 | 1.4 | -0.1 | -1 |
| 0.1 | 0.5 | -1 | -0.81 | -1 | 0 | 0 | 0 | 1.4 | -0.1 | -1 |
| 0.2 | 0.2 | +1 | -0.74 | -1 | 0.2 | 0.2 | 1 | 1.6 | 0.1 | 0 |
| 0.9 | 0.5 | +1 | 1.49 | +1 | 0 | 0 | 0 | 1.6 | 0.1 | 0 |

| $x_1$ | $x_2$ | Class(t) | $y_{in}$ | $y$ | $\Delta\omega_1$ | $\Delta\omega_2$ | $\Delta b$ | $w_1$ | $w_2$ | $b$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | +1 | 1.7 | +1 | 0 | 0 | 0 | 1.6 | 0.1 | 0 |
| -1 | -1 | -1 | -1.7 | -1 | 0 | 0 | 0 | 1.6 | 0.1 | 0 |
| 0 | 0.5 | -1 | 0.05 | +1 | 0 | -0.5 | -1 | 1.6 | -0.4 | -1 |
| 0.1 | 0.5 | -1 | -1.04 | -1 | 0 | 0 | 0 | 1.6 | -0.4 | -1 |
| 0.2 | 0.2 | +1 | -0.76 | -1 | 0.2 | 0.2 | 1 | 1.8 | -0.2 | 0 |
| 0.9 | 0.5 | +1 | 1.52 | +1 | 0 | 0 | 0 | 1.8 | -0.2 | 0 |

| $x_1$ | $x_2$ | Class(t) | $y_{in}$ | $y$ | $\Delta\omega_1$ | $\Delta\omega_2$ | $\Delta b$ | $w_1$ | $w_2$ | $b$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | +1 | 1.6 | +1 | 0 | 0 | 0 | 1.8 | -0.2 | 0 |
| -1 | -1 | -1 | -1.6 | -1 | 0 | 0 | 0 | 1.8 | -0.2 | 0 |
| 0 | 0.5 | -1 | -0.1 | +1 | 0 | 0 | 0 | 1.8 | -0.2 | 0 |
| 0.1 | 0.5 | -1 | -0.08 | -1 | -0.1 | -0.5 | -1 | 1.7 | -0.7 | -1 |
| 0.2 | 0.2 | +1 | -0.8 | -1 | 0.2 | 0.2 | 1 | 1.9 | -0.5 | 0 |
| 0.9 | 0.5 | +1 | 1.46 | +1 | 0 | 0 | 0 | 1.9 | -0.5 | 0 |

| $x_1$ | $x_2$ | Class(t) | $y_{in}$ | $y$ | $\Delta\omega_1$ | $\Delta\omega_2$ | $\Delta b$ | $w_1$ | $w_2$ | $b$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | +1 | 1.4 | +1 | 0 | 0 | 0 | 1.9 | -0.5 | 0 |
| -1 | -1 | -1 | -1.4 | -1 | 0 | 0 | 0 | 1.9 | -0.5 | 0 |
| 0 | 0.5 | -1 | -0.25 | -1 | 0 | 0 | 0 | 1.9 | -0.5 | 0 |
| 0.1 | 0.5 | -1 | -0.06 | -1 | 0 | 0 | 0 | 1.9 | -0.5 | 0 |
| 0.2 | 0.2 | +1 | -0.28 | +1 | 0 | 0 | 0 | 1.9 | -0.5 | 0 |
| 0.9 | 0.5 | +1 | 1.46 | +1 | 0 | 0 | 0 | 1.9 | -0.5 | 0 |

The perceptron learning algorithm converged in 6 steps:

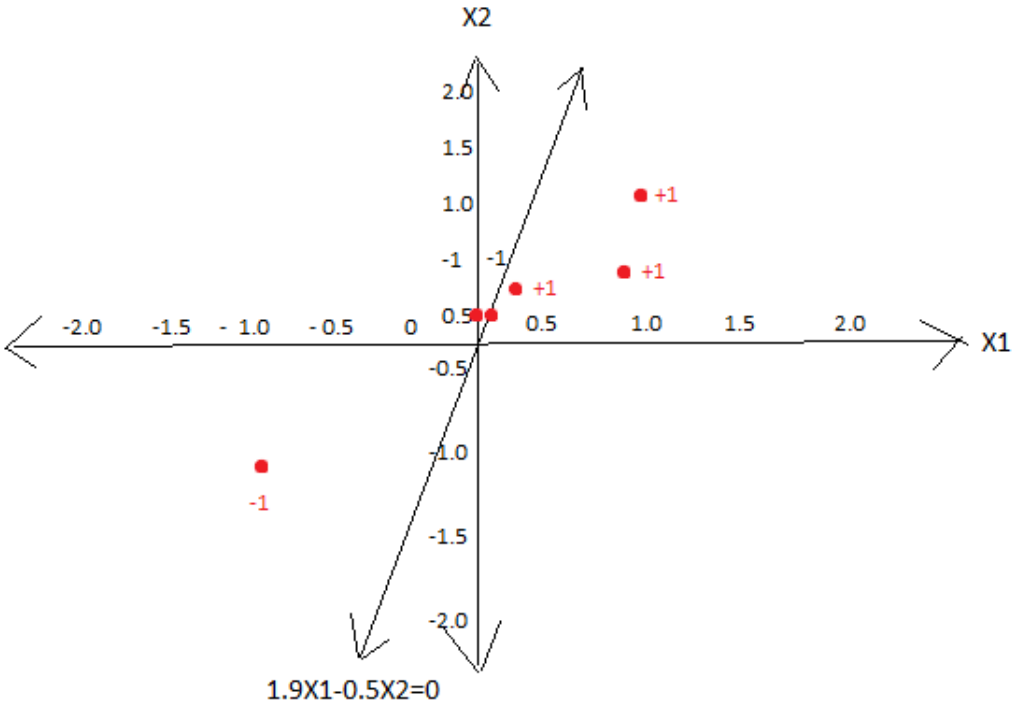
The final weight vector of the decision boundary is w = [1.9,-0.5]

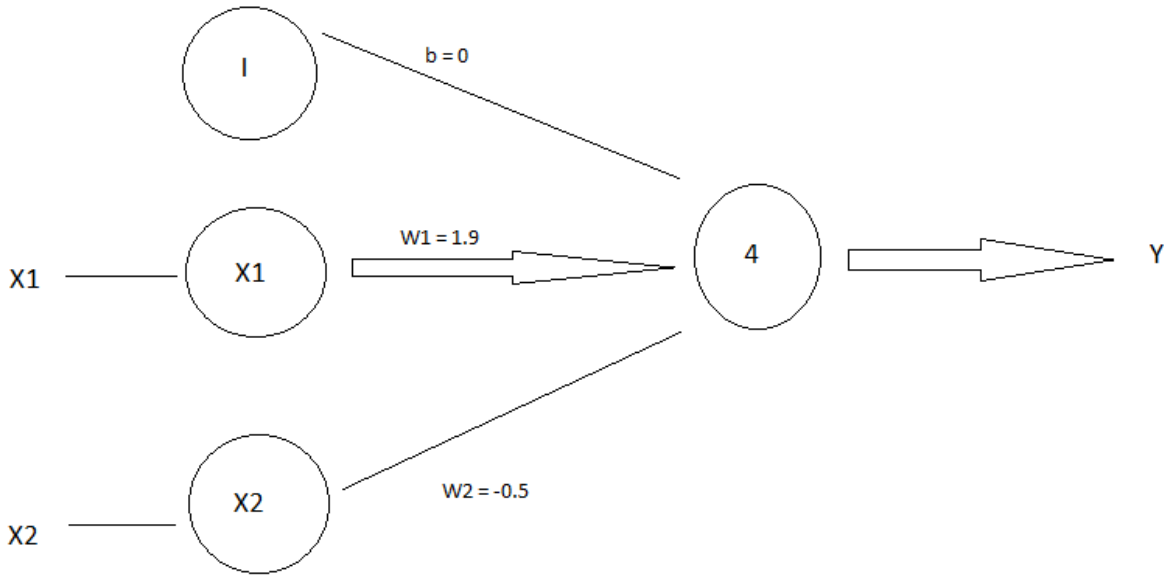
1.9$x_1$ + (-0.5)$x_2$ = 0

=> 1.9$x_1$ -0.5$x_2$ = 0

Let's plot the final decision boundary

We can see that 1.9$x_1$ -0.5$x_2$ = 0 line separates the two classes correctly






Neural Network Corresponding to the perceptron.