Macroeconomic effects of Central Bank Digital Currency[*†]

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Abstract

This paper examines the potential effects of Central Bank Digital Currency (CBDC) on the intermediation of banks and lending. Given the growing popularity of cryptocurrencies, central banks have shown a keen interest in exploring CBDC as a potential alternative. To investigate this, the paper presents a DSGE model that considers bank monopoly, default risk of banks and firms, and the inclusion of CBDC in the economy. The results indicate that the introduction of CBDC can lead to improvements in inflation dynamics and a reduction in interest rates on loans and deposits by banks. However, there appears to be no significant impact on bank deposits. Additionally, the study finds that the remuneration scheme for CBDC can have a different effect on the economy. This analysis offers crucial insights into the impact of CBDC on the performance of the economy, with a particular focus on price stability and financial intermediation.

Keywords: Central Bank Digital Currency, Monetary policy, DSGE, Market power, Disintermediation

1 Introduction

Central bank digital currency (CBDC) has taken a substantial part of the central bank research very quickly and many claim that these is due to central banks around the world having major

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fear of missing out. Presence of many cryptocurrencies has led to this belief and central banks do not want to leave any major controlling power in the hands of these private companies when it comes to payment and money. Gorton and Zhang [2022] goes to a great length discussing why the government should be cautious while considering the coexistence of privately issued digital money like stable coins and fiat currencies, as it could result in the relinquishment of their monopoly on issuing and circulating money.

Introduction of CBDC poses several concerns to the monetary authorities of the economies. For example, CBDC might substitute the conventional private money i.e deposit held at the bank leading to a substantial increase in funding cost of the banks, see Carapella and Flemming [2020]. Keister and Sanches [2022] have shown that if CBDCs become highly competitive with bank deposits, disintermediation is unavoidable. The impact of CBDCs on bank funding depends on whether it is cash-like or deposit-like. A cash-like CBDC would have no direct effect on bank funding, but a deposit-like CBDC would lead to a reduction in deposits and lending. Despite this, the introduction of a CBDC would increase the aggregate stock of liquid assets, leading to a more efficient exchange and improved social welfare. In their work, they have modeled the banking sector as a competitive market using the framework of Lagos and Wright [2005]. This outcome has a solid ground as households around the world consider the central bank's liability as the safest one as recently conveyed by the survey of OMFIF [2020]. Although the above claim can be contested as empirical studies seems to show that banks have market power. [1] In another work, Chiu et al. [2023] micro-founded general equilibrium model of payments and banking market and calibrated it to the US, finding an imperfectly competitive banking sector. An interest-bearing CBDC could increase bank intermediation by raising bank lending by 1.57%, deposit and loan rates but CBDC remuneration must be within a specific range. Garratt et al. [2022] created a simplified model where big and small banks compete in order to illustrate how the introduction of CBDC could have varying effects. They presumed that big banks offer nonpecuniary benefits to depositors, but this benefit could be mimicked by a bank-distributed CBDC, even if policymakers could independently set the convenience yield. In their scenario, introducing an interest-bearing CBDC reduces the convenience gap between small and large banks, thereby weakening the market power of big banks. Although bank might their profits being squeezed out by the CBDC, the degree of it will depend how easily they can switch to alternative market-based funding or increasing the lending rate (that could lead to a loss of potential borrowers) or deleveraging. Hence business strategy of the banks, how leveraged it is, the mere size of the bank and its reputation, all these questions are important to understand the effect of CBDC. Different hypothetical scenarios regarding these dynamics are discussed in detail in Adalid et al. [2022]

This paper contributes to this important and growing body of work regarding the intermediation of banks and lending in presence of CBDC by developing a Dynamic Stochastic General Equilibrium (DSGE) model with bank monopoly and adoption of liquid asset following Gerali et al. [2010]. The supply side of the credit market is primarily populated by private banks. Monopolistic competition along with the market power is one of the main features of these banks. Banks

[1] In their study, Drechsler et al. [2017] provided evidence of the market power that U.S. banks hold in deposit markets. Their findings indicated that when the federal funds rate increases, the policy rates and deposits exhibit a wider spread, ultimately leading to a decrease in the total volume of deposits

often take advantage of lock-in effects i.e. long term relationship with customers as the latter find it quite costly to change banks frequently. This long-term relationship often gives rise to asymmetric information in the contract between borrower and lender. (see Diamond [1984], Sharpe [1990]). Many empirical studies have confirmed that interest rate spreads and bank's profitability are highly affected by the degree of competition of the banks (for more detailed studies, see Berger et al. [2004], Degryse and Ongena [2008]). This monopoly power might attenuate the effect of crowding out of deposit as Andolfatto [2020] shows that bank might leverage this in order to fight CBDC. Moreover, monopolistic competition in the banking sector might lead to an increase the lending activity as banks might have to increase the deposit rate and subsequently decrease the lending rate amounting to increase in the volume of credit supplied to firms due to higher demand. Andolfatto [2020] developed an overlapping generation model to study these effects whereas in this paper a DSGE modeling approach has been considered allowing to study general equilibrium effects and policy rules. Hence contribution of the paper are manifolds and they are namely (i) incorporation of banking monopoly and market power in the bank intermediation and lending literature of CBDC, (ii) introducing cash-in-advance constraint and liquidity properties to model the adaptation of CBDC (iii) policy analysis incorporating several shocks to the lending conditions of the bank to understand the effect of CBDC. The paper follows Gerali et al. [2010] to develop the DSGE model and incorporate CBDC into the economy. Among the two types of households, patient households draw utility from holding cash and CBDC whereas impatient households are net borrowers in the economy and do not hold any liquid asset. Cash has a quadratic storage cost increasing in the level of cash holding following Burlon et al. [2022] where as CBDC is devoid of any such costs thus having potential advantage over cash. Central bank can adjust the degree to which CBDC behaves as cash or deposit by adjusting its elasticity of substitution and scaling parameter following Agur et al. [2019]. Impatient households smooth their consumption by borrowing from the banks and banks hold housing as collateral Iacoviello [2005]. Entrepreneurial firms produce wholesale goods by borrowing from the banks to buy new capital. They face a borrowing constrain with present value of capital as a collateral following Kiyotaki and Moore [1997]. Banking sector has two divisions namely, retail sector and wholesale sector. In wholesale sector gives the possibility to find the spread of deposit rate and policy rate. In the retail sector, banks decide the interest rate to charge to households and firms. Banks' market power are established by interest rate elasticity of deposit and loan demand. Banks also face quadratic adjustment cost for changing the interests.

The rest of the paper is structured as follows: Section 2 describes the model used to features of the economy, Section 3 explains the calibration and estimation of the model and Section 4 describes the results and finally the Section 5 draws conclusion on the paper.

2 Model

My model is constructed upon the closed economy DSGE framework model by Gerali et al. [2010] in which the economy consists of households, firms, capital-good producers, entrepreneurs, retailers, one representative bank and a central bank.

Households are of two types, namely impatient and patient. They are differentiated by how they

discount the future or how impatient they are. Hence by construction, discount factor of patient households (β^P) is greater than that of impatient households (β^I). They work, accumulate housing services and use financial instruments for consumption smoothing. Entrepreneurs on the other hand, hire labor from both types of households, buy capital-goods, produce intermediate goods and sell those to retailers.
Workers supply labor in a competitive framework without any friction to entrepreneurs and their wages are set to match the demand of the firms. Capital goods producers are responsible for transforming retail goods and depreciated capital to new capital that is to be used in the production in the intermediate goods. Retailers buy intermediate goods from entrepreneurs in a competitive market and bundle them in a differentiated monopolistic market subjected to nominal price rigidity following Rotemberg [1982].
In the financial sector, there is a representative bank that provide loans to impatient households against housing service as collateral. They also provide loans to entrepreneurs against inflation adjusted capital as collateral. Banks face regulation in the form of costly deviation of optimal bank-capital and asset ratio. Patient households hold deposits at the bank and gain interest rate set in monopolistic competition by banks. Central Bank has to make two policy decisions: set policy interest rate or supply of cash and set either supply or interest rate of the CBDC. Patient households hold cash and CBDC supplied by the Central Bank.

2.1 Patient Household

The infinitely lived patient household maximize their following utility function by choosing level of consumption, how much to work and a financial portfolio for saving.

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_t^P \left[(1-a^P)\epsilon_t^Z \log(c_t^P(i) - a^P c_{t-1}^P) + \epsilon_t^h \log h_t^P(i) + \psi_m \frac{m_t^{1-\sigma_m}}{1-\sigma_m} + \psi_{cbdc} \frac{cbdc_t^{1-\sigma_{cbdc}}}{1-\sigma_{cbdc}} - \frac{l_t^P(i)^{1+\phi}}{1+\phi} \right]$$

Households consumption c_t^P is affected by the external and group-specific consumption habit of the last period c_{t-1}^P. The steady state impact of habits are offset by the factor of $1-a^P$. They supply l_t^P units of labor and are paid an hourly wage of w_t^P. The labor dis-utility is parameterized by the inverse Frisch elasticity parameter ϕ. The patient household opts for h_t^P units of housing services which is subjected to housing demand shock ϵ_t^h.

$$\begin{aligned} & c_t^P(i) + q_t^P \Delta h_t^P(i) + d_t^P(i) + m_t^P(i) + cbdc_t^P(i) + f(m_t^P(i)) \leq \\ & w_t^P l_t^P(i) + (1+r_{t-1}^d)\frac{d_{t-1}^P(i)}{\pi_t} + (1+r_{t-1}^{cbdc})\frac{cbdc_{t-1}^P(i)}{\pi_t} + \frac{m_{t-1}^P(i)}{\pi_t} + t_t^P(i) \end{aligned} \tag{1}$$

The cash is subjected to storage cost in the form of $f(m_t) = \frac{\Psi_m}{2} m_t^2$ with $\Psi_m > 0$ following Burlon et al. [2022]. The preference to hold cash is set by the parameter ψ_m and relative risk aversion of holding cash is denoted by parameter σ_m. CBDC parameters are set relative to the cash as many central banks around the world have communicated that CBDC will have some cash like features. We set the parameters of CBDC following Ferrari et al. [2022] as $\psi_{cbdc} = \theta\psi_m$ and $\sigma_{cbdc} = \sigma_m + (1-\theta)\sigma_m$. Here θ captures the level of similarities between cash and CBDC. The patient household is also constrained by cash-in-advance requirement as it needs to hold liquid asset for transaction in the goods market. We consider that household can use only cash and CBDC to buy consumption goods. Cash and CBDC are differentiated by the acceptance of them. Cash is generally considered widely accepted mode of payment but CBDC may not be accepted everywhere due to lack of adoption, infrastructure etc. Thus household faces a following constraint,

$$c_t^P \leq \frac{m_{t-1}}{\pi_t} + \gamma_t \left(1 + r_{t-1}^{CBDC}\right) \frac{cbdc_{t-1}}{\pi_t} \tag{2}$$

where γ_t captures fraction of CBDC that the household can bring to market. Deposit holding is remunerated by r_t^d from bank and CBDC holding is remunerated by r_t^{CBDC} from the central bank. π_t is the gross inflation rate and t_P is the lump-sum transfer that includes dividends from firms and banks.

2.2 Impatient Household

Impatient households are net borrowers of the economy and maximizes the following utility function by choosing the level of consumption c_t^I, number of hours to work l_t^I and housing services h_t^I.

$$\max_{c_t^I, h_t^I, l_t^I, b_t^I} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_I^t \left[(1 - a^I)\epsilon_t^Z \log(c_t^I(i) - a^I c_{t-1}^I) + \epsilon_t^h \log h_t^I(i) - \frac{l_t^I(i)^{1+\phi}}{1+\phi} \right]$$

subjected to the budget constrain

$$c_t^I(i) + q_t^h \Delta h_t^I(i) + (1 + r_{t-1}^{bH}) \frac{b_{t-1}^I(i)}{\pi_t} \leq w_t^I l_t^I(i) + b_t^I(i) + t_t^I(i) \tag{3}$$

In order to smooth their consumption, impatient households borrow b_t^I units of loan against the housing service as collateral thus subjected to eq 4

$$(1 + r_t^{bH}) b_t^I(i) \leq m_t^I \mathbb{E}_t[q_{t+1}^h h_t^I(i) \pi_{t+1}] \tag{4}$$

where m^I stands for the Loan-to-Value ratio for the credit that bank lends to the impatient household. The m^I has been considered exogenous and subjected to shocks representing the fact that during the bad or good times bank might faces different level of risks and thus changing the percentage of capital required as collateral.

2.3 Entrepreneurs

The entrepreneurs maximizes following utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_E^t \log(c_t^E(i) - a^E c_{t-1}^E)$$

by choosing their consumption c_t^E, level of utilization of physical capital u_t , level of physical capital k_{t+1}. Their optimization problem is subjected to two constrains, namely budget constraint and borrowing constraint. Budget constrain for the entrepreneurs is as following

$$c_t^E(i) + w_t^P l_t^{E,P}(i) \; + \; w_t^I l_t^{E,I}(i) \; + \; (1 + r_{t-1}^{bE}) \frac{b_{t-1}^E(i)}{\pi_t} \; + \; q_t^k k_t^E(i) \; + \; \psi(u_t(i)) k_{t-1}^E(i) =$$
$$\frac{y_t^E(i)}{x_t} \; + \; b_t^E(i) \; + \; q_t^k (1-\delta) k_{t-1}^E(i) \tag{5}$$

Entrepreneurs produce intermediate goods according the eq 6. They hire labors from patient and impatient households and pay them hourly wage w_t^P and w_t^I respectively. Total labor employed is l_t^E and it is aggregated using cob-douglus function as $l_t^E = [l_t^{E,P}]^{\mu}[l_t^{E,I}]^{1-\mu}$ where μ is the labor income share of the patient household (Iacoviello and Neri [2010]). Physical capital is the priced at q_t^k and its utilization comes at a cost of $\psi(u_t(i))$. x_t is the relative price of the wholesale goods and retail goods and defined as $\frac{P_t^W}{P_t} = \frac{1}{x_t}$

$$y_t^E(i) = a_t^E [k_{t-1}^E(i) u_t(i)]^{\alpha} l_t^E(i)^{1-\alpha} \tag{6}$$

In order to meet their funding gap, entrepreneurs borrow b_t^E units of loans from bank at the interest of r_t^{bE} per unit. Banks ask for physical capital as collateral. Therefore, the borrowing constrain becomes discounted value of next period stock of physical capital. Like in the case of impatient households, borrowing constraint of the entrepreneurs is also subjected to Loan-to-Value ratio m_t^E.

$$(1 + r_t^{bE}) b_t^E(i) \leq m_t^E \mathbb{E}_t [q_{t+1}^k (1-\delta) k_t^E(i) \pi_{t+1}] \tag{7}$$

2.4 Loan and Deposit Demand

Banks are subjected to monopolistic competition in deposit and loan market. Each financial product is differentiated and their demand is inversely proportional to the return/cost of the product. Therefore demand of two types of loans to bank j is given by

$$b_t^I(j) = \left(\frac{r_t^{bH}(j)}{r_t^{bH}} \right)^{-\epsilon_t^{bH}} b_t^I \qquad\qquad b_t^E(j) = \left(\frac{r_t^{bE}(j)}{r_t^{bE}} \right)^{-\epsilon_t^{bE}} b_t^E \tag{8}$$

with

$$r_t^{bH} = \left[\int_0^1 r_t^{bH}(j)^{\epsilon^{bH}-1} dj \right]^{\frac{1}{\epsilon^{bH}-1}}$$

and

$$r_t^{bE} = \left[\int_0^1 r_t^{bH}(j)^{\epsilon^{bE}-1} dj \right]^{\frac{1}{\epsilon^{bE}-1}}$$

Similarly aggregate deposit demand for patient household j is given by

$$d_t^P(j) = \left(\frac{r_t^d(j)}{r_t^d} \right)^{-\epsilon_t^d} d_t^P \tag{9}$$

and the interest on the deposit is given by

$$r_t^d = \left[\int_0^1 r_t^{bH}(j)^{1-\epsilon^d} dj\right]^{\frac{1}{1-\epsilon^d}}$$

Here elasticities of the financial products have values $\epsilon^{bH}, \epsilon^{bE} > 1$ for the loans and $\epsilon^d < -1$ for the deposit. These elasticities are considered to be stochastic to allow for the unexpected changes in different bank rate spreads. For more details, see Gerali et al. [2010]

2.5 Banking sector

Banking sector model in Gerali et al. [2010] gives a good starting point for analyzing the effect of disintermediation if central bank will introduce CBDC. Market power of a bank is arising due to differentiated financial products and slow accumulation of bank capital through retained earnings, thus making the model a natural candidate for understanding the dynamics in these markets while facing disintermediation challenges and economic shocks.

Each representative bank of type j have two branches, namely *Wholesale branch* and *Retail branch*. The *Wholesale branch* is responsible for managing bank capital, wholesale loans and wholesale deposits. This branch of the bank can be typically thought to be with the other financial institutions, taking part in the open market operation of Federal Reserve in US etc. This branch accumulates bank capital K_t^B according to the following rule,

$$\pi_t K_t^b = (1-\delta^b)K_{t-1}^b + j_{t-1}^b \tag{10}$$

where j_{t-1}^b is the retained bank profit from the last period. Banks target for an exogenous leverage ratio and any deviation from that target would be costly for the bank implying poor portfolio management, bad economic conditions etc. We also assume that bank is able to maintain the exogenous leverage ratio in steady-state thus removing cost of deviation from this ratio in the steady-state. Bank capital is also subjected to exogenous shocks i.e. *Bank capital shock* to simulate phenomenon of exogenous changes in banks valuation or equity in the face of financial turmoil.

Wholesale branch of the bank maximize their profit by choosing wholesale loans B_t and wholesale deposit D_t, thus maximizing the following,

$$\max_{B_t, D_t} \mathbb{E}_o \sum_{t=0}^{\infty} \Lambda_{0,t}^P \left[\left(1+R_t^b\right) B_t - B_{t+1} + D_{t+1} - (1+R_t^d)D_t + \Delta K_{t+1}^b - \frac{\kappa_{KB}}{2}\left(\frac{K_t^b}{B_t} - \nu^b\right)^2\right]$$

subjected to the balance sheet constrain,

$$B_t = D_t + K_t^b \tag{11}$$

First order condition of the above optimization problem gives us the wholesale loan rate as,

$$R_t^b = R_t^d - \kappa_{KB}\left(\frac{K_t^b}{B_t} - \nu^b\right)\left(\frac{K_t^b}{B_t}\right)^2 \tag{12}$$

If the wholesale deposit rate R_t^d is the same as the policy rate of the central bank R_t^{CB} [2] then the spread between the wholesale loan rate and policy rate is driven by costs that bank obtains when they deviate from the optimal leverage ratio. Decrease in bank's leverage ratio decreases, that implies that bank face a more suitable financial conditions in the market. Therefore, more the cost of deviation, lower is the spread between wholesale interest rate, and policy rate and banks cannot bid interest rate higher than what central bank asks for.

The *Retail branch* of the bank borrows wholesale loans from the wholesale branch of the bank at the rate R_t^b and differentiate the loans to offer these loans to the impatient households and entrepreneurs. They apply markup over the wholesale rate R_t^b and face quadratic costs while changing the interest rate that they charge for the retail loans. These effects of these costs are captured by parameters κ_{bE} and κ_{bH}. Therefore, bank optimizes the profit by maximizing over the following expression by choosing the retail interest rate namely, r_t^{bH}, r_t^{bE} for loans to impatient households and entrepreneurs respectively-

$$\max_{r_t^{bH}(j), r_t^{bE}(j)} \mathbb{E}_o \sum_{t=0}^{\infty} \Lambda_{0,t}^P \Bigg[r_t^{bH}(j) b_t^I(j) + r_t^{bE}(j) b_t^E(j) - R_t^b B_t(j)$$
$$- \frac{\kappa_{bH}}{2} \left(\frac{r_t^{bH}(j)}{r_{t-1}^{bH}(j)} - 1 \right)^2 r_t^{bH} b_t^I - \frac{\kappa_{bE}}{2} \left(\frac{r_t^{bE}(j)}{r_{t-1}^{bE}(j)} - 1 \right)^2 r_t^{bE} b_t^E \Bigg]$$

subjected to the loan specific demands obtained in section 2.4 as

$$b_t^I(j) = \left(\frac{r_t^{bH}(j)}{r_t^{bH}} \right)^{-\epsilon_t^{bH}} b_t^I \quad ; \quad b_t^E(j) = \left(\frac{r_t^{bE}(j)}{r_t^{bE}} \right)^{-\epsilon_t^{bE}} b_t^E \tag{13}$$

and the supply of wholesale loan constrain,

$$B_t(j) = b_t^I(j) + b_t^E(j) \tag{14}$$

The *Retail branch* collects deposits from the patient households, pay them r_t^d for their deposit holding, and pass the collection on to the *Wholesale branch*, gaining the remuneration at the central bank's policy rate R_t^{CB}. Similar to the case of retail loans, bank faces adjustment cost parameterized by κ_d , while changing the deposit interest rate r_t^d. The optimization problem faced by the deposit collection is to maximize the profits of the deposit funding by choosing r_t^d as,

$$\max_{r_t^d(j)} \mathbb{E}_o \sum_{t=0}^{\infty} \Lambda_{0,t}^P \left[r_t D_t(j) - r_t^d(j) d_t^P(j) - \frac{\kappa_d}{2} \left(\frac{r_t^d(j)}{r_{t-1}^d(j)} - 1 \right)^2 r_t^d d_t \right]$$

[2] In open market operations, many central banks e.g. Federal reserve, use the window on the overnight repurchase agreement creating a ceiling and floor for the borrowing and lending. Banks in the interbank markets might use one of the rates, making this assumption viable

subjected to demand for deposits from the patient households as described in 2.4

$$d_t^P(j) = \left(\frac{r_t^d(j)}{r_t^d}\right)^{-\epsilon_t^d} d_t \tag{15}$$

and the wholesale deposit supply constrain,

$$D_t(j) = d_t^P(j) \tag{16}$$

The overall bank profit coming from these two branches of the bank is given by,

$$j_t^b = r_t^{bH} b_t^H + r_t^{bE} b_t^E - r_t^d d_t - \frac{\kappa_{KB}}{2}\left(\frac{K_t^b}{B_t} - \nu^b\right)^2 K_t^b - Adj_t^B \tag{17}$$

where Adj_t^B is the sum of all adjustment costs incurred during the adjustment of the retail loan and deposit rates.

2.6 Capital Producers

Capital producers buy depreciated physical capital $(1-\delta)k_{t-1}$ from the entrepreneurs at price Q_t^k and final goods from the retailers at price P_t in a perfectly competitive. Using these as input, they increase the stock of new physical capital and sell it to the entrepreneurs at price Q_t^k. Hence their optimization problem is given by,

$$\max_{\overline{x}_t, i_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^E \left[q_t^k \Delta \overline{x}_t - i_t\right]$$

subjected to

$$\overline{x}_t = \overline{x}_{t-1} + \left[1 - \frac{\kappa_i}{2}\left(\frac{i_t \epsilon_t^{qk}}{i_{t-1}} - 1\right)^2\right] i_t \tag{18}$$

where $\Delta\overline{x}_t = k_t - (1-\delta)k_{t-1}$ and $q_t^k = \frac{Q_t^k}{P_t}$ is the real price of capital.

2.7 Retailers

A representative retailer produces in a monopolistically competitive market where prices are sticky and it follows a pricing using Rotemberg [1982]. They maximize the following by choosing an optimal $P_t(j)$ as

$$\max_{P_t(j)} \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^P \left[P_t(j) y_t(j) - P_t^W y_t(j) - \frac{\kappa_p}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - \pi_{t-1}^{\iota_p} \pi^{1-\iota_p}\right)^2 P_t y_t\right]$$

subject to the demand of the specific good j obtained from utilization maximization of a household,

$$y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t^y} y_t$$

2.8 Central bank

The central bank has two monetary policy tools in this economy. It sets the policy interest rate following

$$R_t^{CB} = (R^{CB})^{1-\phi_R}(R_{t-1}^{CB})^{\phi_R}\left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi}\left(\frac{y_t}{y_{t-1}}\right)^{\phi_y}\right]^{1-\phi_R}(1+\epsilon_t^r) \tag{19}$$

It also set the remuneration rate on the CBDC as,

$$R_t^{cbdc} = R_t^{CB} - \Delta^{CBDC} \tag{20}$$

following Schiller and Gross [2021]. This implies that the interest rate on the CBDC is allowed to go negative making it less preferred saving instrument than cash for the patient households.

2.9 Aggregation

The total consumption for the economy is the weighted sum of the group consumption of patient and impatient households and entrepreneurs.

$$C_t = \gamma_P c_t^P + \gamma_I c_t^I + \gamma_E c_t^E \tag{21}$$

The supply of the housing services in the economy is considered fixed in the economy to exogenous level of $\overline{h}$. Thus market clearing condition in the housing market is

$$\overline{h} = \gamma_P h_t^P + \gamma_I h_t^I \tag{22}$$

The aggregate output of the economy is the following,

$$\begin{aligned} Y_t = {} & C_t + q_t^k\left(K_t - (1-\delta_k)K_{t-1}\right) + \delta_{Kb}K_{t-1}^b + f(m_t) + \psi(u_t) + Y_t\frac{\kappa_p}{2}\left(\pi_t - \pi_{t-1}^{\iota_p}\pi^{1-\iota_p}\right)^2 \\ & + d_t^b r_t^b\frac{\kappa_d}{2}\left(\frac{r_t^d}{r_{t-1}^d} - 1\right)^2 + b_t^H r_t^{bH}\frac{\kappa_d}{2}\left(\frac{r_t^{bH}}{r_{t-1}^{bH}} - 1\right)^2 + b_t^E r_t^{bE}\frac{\kappa_{bE}}{2}\left(\frac{r_t^{bE}}{r_{t-1}^{bE}} - 1\right)^2 \end{aligned} \tag{23}$$

Please note that, in the first order approximation of the model, adjustment costs becomes irrelevant in deciding the dynamics of aggregate output. Goods and services in other markets also clear by having demand equal to the their supply.

3 Calibration

The model is calibrated to Euro area for the sample period 2000Q1-2018Q4. I have avoided the COVID period data to keep the analysis as simple as possible [3]. The patient households' discount factor (β^P) is set at 0.994, resulting in an annual deposit interest rate of 2.41%. The impatient

[3] as they show extreme changes to the growth rates for macro-variables

households' discount factor (β^I) and entrepreneur's discount factor (β^E) is set to 0.975 as in Iacoviello [2005]. Inverse Frisch-elasticity of labor supply is considered to be 1.
Cash substitution (ψ_m) and elasticity σ_m are set at 0.44 and 10.67 respectively. For the baseline simulations, I kept the cash-similarity parameter θ at 0.5. Effect of varying θ is studied in section 4.6. Please note that, when $\theta = 1$, CBDC is exactly same as cash in terms of elasticity wheres as $\theta = 0$ implies that patient households do not get any utilities in holding CBDC. The steady-state value of share of CBDC that can be taken to the goods market i.e. γ_{ss} is set to 0.270 as in Benigno and Nisticò [2017]. Cash-to-GDP ratio and CBDC-to-GDP ratio in steady state are calibrated at 0.3443. [4]. The physical capital depreciates by 2.5% each quarter. The patient household's labor income share μ is set to 0.8 and capital share of production α is set at 0.25.
Banking parameters are calibrated according to Gerali et al. [2010] if not mentioned otherwise. Parameters related to the cost of utilizing physical capital is set following Schmitt-Grohé and Uribe [2005]. Elasticity parameters of retail loans and deposits are in line with the demand of them.

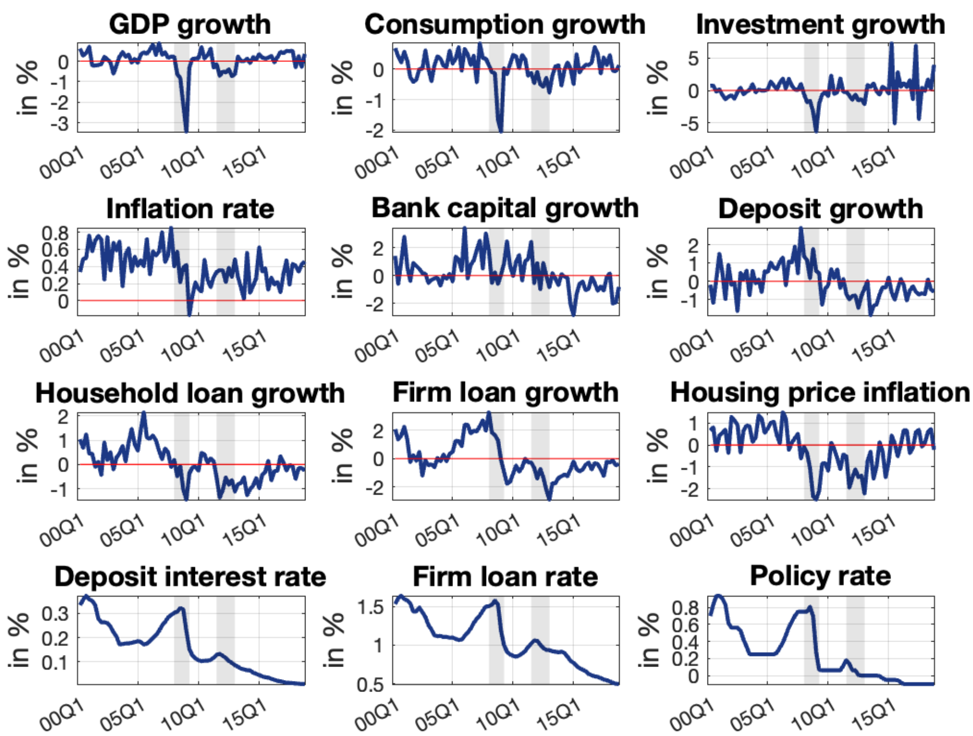


Figure 1: Time series data used for estimating the model to Euro area.

Monetary policy parameters are estimated in quite standard fashion, commonly found in Smets and Wouters [2007], Christiano et al. [2005] etc. In Taylor rule for the central bank policy rate, inflation weight ϕ_π is estimated with posterior mean 0.866, output weight ϕ_Y with posterior mean 0.746 and interest rate weight ϕ_R is at 0.753. These figures are reported in appendix A. The adjustment costs for the firm loans are found to be higher than that on mortgage loans which is in

[4] In one publication, ECB said the level of CBDC in the economy will be at the level of cash in the economy

contrast with Gerali et al. [2010]. Adjustment costs for changing bank capital is estimated to be 3.701 reflecting higher flexibility of changing the leverage ratio than reported in Gerali et al. [2010]. Among the shocks, liquidity shock is estimated to have high auto-correlation where as TFP shocks and monetary policy shocks are estimated to have low auto-correlation. Standard deviation of the shocks are estimated to be low except for TFP and preference shock for which it has value close to 1. I have discussed the calibration for the CBDC supply or interest rate rule in the section 4.5 and 4.4.

4 Results

4.1 TFP Shock

Here I have studied two scenarios of the economy. In baseline scenario, as a liquidity instrument, patient households have money and bank deposits without deriving utility from the bank deposit. In the alternative scenario, patient households have access to CBDC as well. Inflation goes up on impact for a positive technology shock for both cases but it comes back to steady state quicker than the baseline case. Dynamics of the inflation can also be explained with relative price of wholesale goods, where introduction of the CBDC leads to quicker stabilization. Higher housing holding of the impatient households is reflected in the borrowing loans from banks. As households' collateral

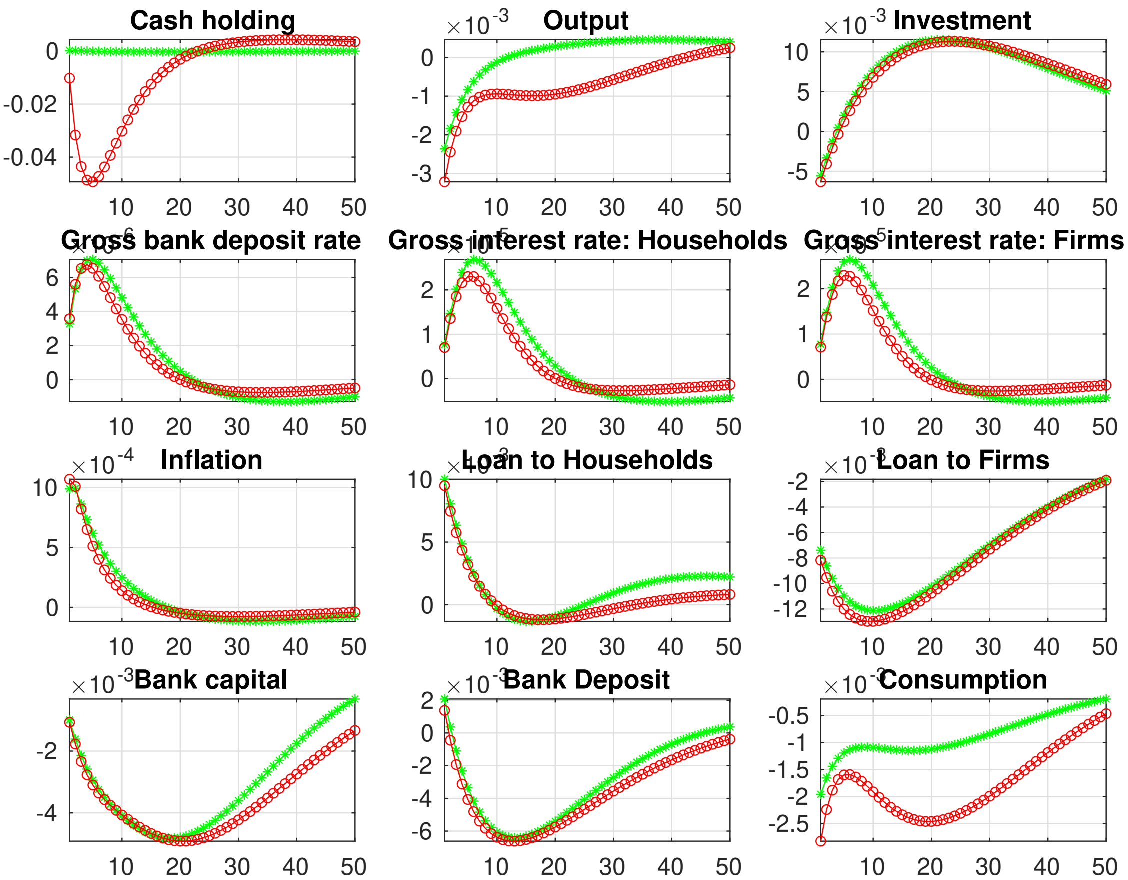


Figure 2: Response of variables to a positive technology shock for two models, (1) without CBDC (in green) (2) with CBDC (in red)

increases with high inflation and high holding of housing, borrowing is increased. Although interest rates for retail loans and deposit increases as bank can ask for more interest rate due to higher demand of credit, presence of CBDC makes the level of increase lower than in the absence of CBDC, clearly reducing profit margin of the banks. We can see adverse impact on the bank capital and deposit holdings at the bank, both having lower values when patient households have extra-liquidity instrument. Bank deposits decrease when CBDC is introduced but the impact is not very different from the case when there is no CBDC. Therefore we see bank disintermediation effect though it is not very significant. In fact we see that bank increases the interest rate on deposit holding to attract more funding but when CBDC is available, bank raises interest rate on deposits less than in the absence of CBDC. Bank capital decreases on impact and attains lowest level around 20 quarters from the time of impact. In presence of CBDC, bank capital takes longer to return to steady state which is in line with empirical results of Burlon et al. [2022] [5]

4.2 Monetary Policy Shock

Figure 3 shows the response of the model to a contractionary monetary policy shock. The unanticipated increase of 18 basis points in the central bank's policy rate slows down the economy as the output decrease on impact. If the patient household has access to CBDC then it takes longer time for the output to return to the steady state. Price puzzle is present in the case when the economy does not have CBDC and price puzzle is absent in the model with CBDC. In crease in the policy rate leads to increase in the remuneration of the CBDCs as well. This is reflected in the cash holdings of the patient households as CBDC becomes more attractive. Cash holdings drops on impact and attains lowest level at about 5 quarters with larger change in the holding compare to when the households do not have access to CBDC. The difference in the magnitude of response is clearly seen as green line is flatter compared to the red line for the cash holding. Due to the slowdown of the economic activities, overall borrowing decreases. Impatient household's demand for credit drops on impact but slowly recovers where as firms' demand for credit keeps decreasing for about 10 quarters reflecting the fact that their capital accumulation is sluggish. One interesting thing to note here is how CBDC improves the demand for credit in the case of firms and worsens the same for impatient households. We see similar effect on the bank capital as well i.e. in the absence of CBDC, bank capital decreases and keep decreasing for about 11 quarters after the shock has hit but recovers quickly compared to what happens in the presence of CBDC. Deposit holding at the bank also decreases on impact as holding CBDC has become more profitable. In presence of CBDC, bank deposits take longer time to return to the steady state though the magnitude of decrease is lower in the case when CBDC is not available. Results of the analysis of monetary policy transmission is line with different concerns discussed by Meaning et al. [2018]

[5] Burlon et al. [2022] shows that CBDC news has negative effect on the valuation of the bank which is reflected in the share price. Thus if equity can be considered as bank capital, this results underscores the empirical fact.

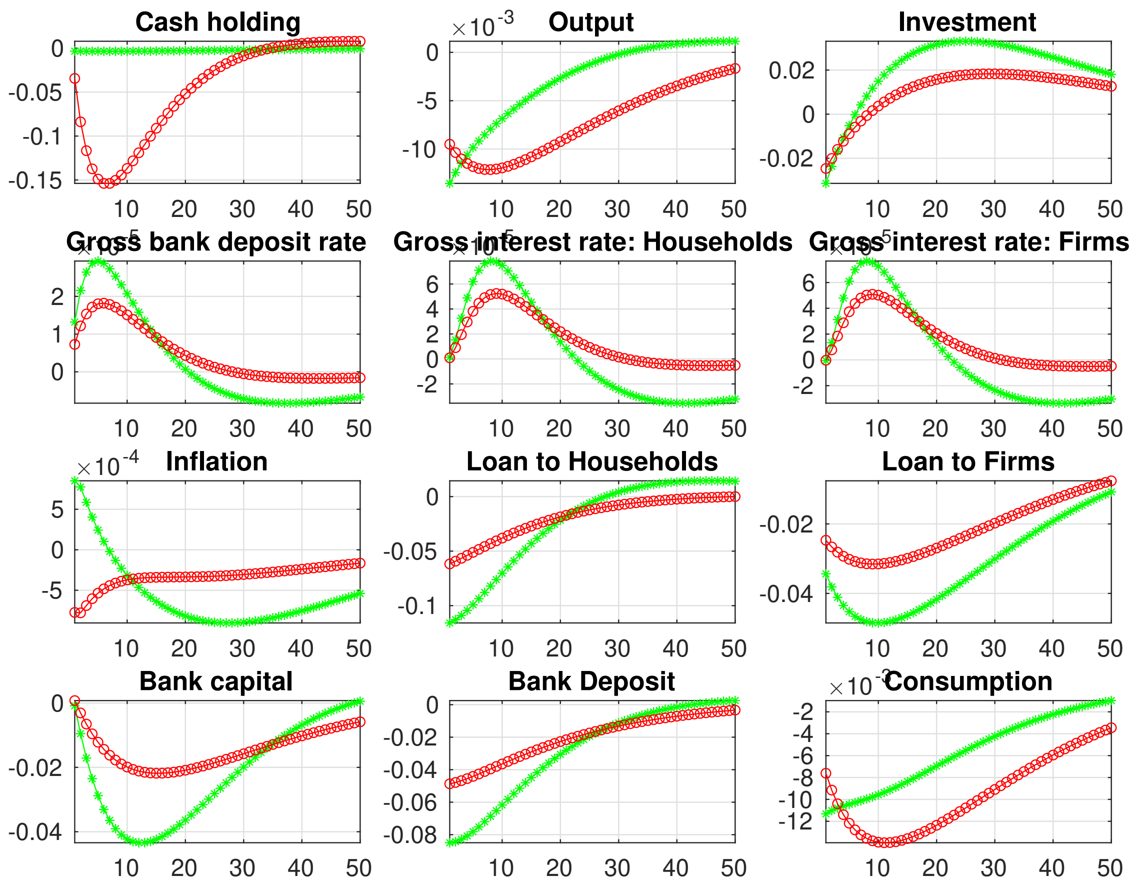


Figure 3: Response of variables to a contractionary monetary policy shock for two models, (1) without CBDC (in green) (2) with CBDC (in red)

4.3 Liquidity Shock and other shocks

Now we turn to the model with CBDC and cash-in-advance constrain. Here the liquidity has the interpretation of acceptance of CBDC or rather successful adoption of CBDC. When economy is hit with an unanticipated liquidity shock, the overall economy slows down as both output and inflation increase. Figure 41 and 42 report the impulse response of the key macroeconomic variables. Central bank decreases the policy interest rate by 15 basis points and this makes the CBDC holding less preferred for the household. Therefore although the overall the acceptance of CBDC is increased, households bring less CBDC to the goods market and therefore the consumption decreases. As economy slows down, the demand for credit also decreases. Banks therefore decreases the interest rates on retail loans and deposit holding. Capital accumulated by banks drops as well as they are targeting an optimal leverage ratio. Figure 33 and 33 show the effect of a positive shock to the deposit rates, leading to decrease in supply of deposits. As banks face funding issues, they have to decrease their loan issuance causing an economy wide slowdown and decrease in inflation. Central bank responds to this by decreasing the policy rate thus decreasing the interest rate on CBDC as well. Households quickly adjusts the level of CBDC holding with an increase about 5% on impact and slowly return to the steady state. When the economy faces an unanticipated shock to interest rate of mortgage loans, the banks are able to lower their losses by increasing interest rate on the firm loan and increase the deposit supply as shown in figure 35 and figure 36. Although CBDC holding increases, we do not see crowding in effect on deposit as banks do not pass through the central bank's policy interest rate. [6]

4.4 CBDC remuneration policies

In this policy exercise, I tried to understand the effect of different remuneration schemes. In recent times, many central banks are discussing various policy designs and there is no unanimity on the remuneration schemes, see Reserve [2022], Adalid et al. [2022]. Here I have considered two remuneration schemes i.e (1) CBDC is not remunerated and (2) CBDC is remunerated at a constant spread from the central bank's policy rate. The first might not seem a plausible scenario but it is quite feasible. Cash is not remunerated as well in the economy but storage cost of cash makes it less attractive than CBDC which as no storage cost [7]. Figure 4 shows the impulse response of the liquidity instruments available to the households when an unanticipated positive liquidity shock as hit the economy. Here we see that having different remuneration schemes only affects the

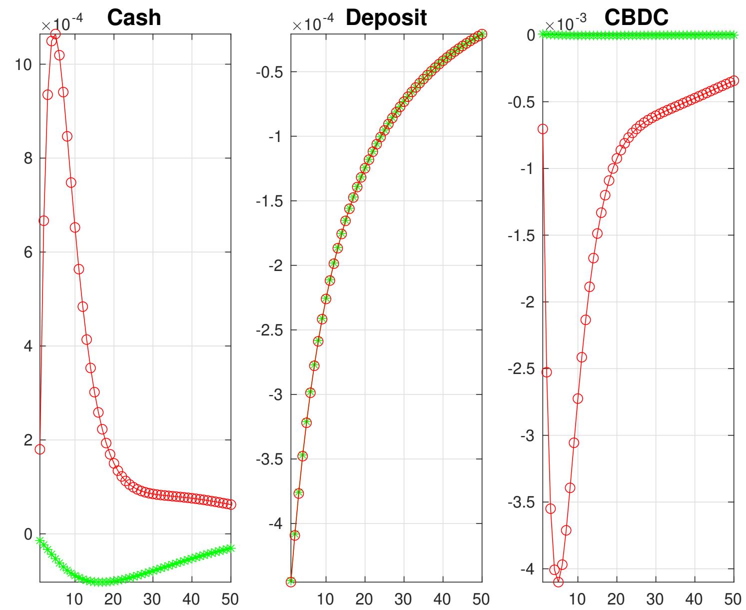


Figure 4: Response of various liquidity instrument to a liquidity shock under two different remuneration plans for CBDC i.e (1) no interest rate(green lines) (2) constant positive spread between policy rate and CBDC interest rate(red lines)

holding of cash and CBDC. There is potentially no impact on the deposit holding. CBDC holding decreases on impact and holding of cash increases to make up for the decrease in holding of CBDC. When CBDC is not remunerated, the magnitude of change in holding CBDC and cash is not so pronounced. Moreover in that case, household decides to hold less cash than CBDC as cash comes with a storage cost. Therefore, when CBDC is remunerated, variations in the level of holding of cash and CBDC is higher and becomes more sensitive to changes in policy rates.

[6] Notice the difference in order of the magnitude of response of R^{CB} and R^d

[7] Storage cost may arise due to several risks associated with holding cash. For more details, see Burlon et al. [2022]

4.5 CBDC supply policies

The figure 5 shows the impulse response function of the cash, CBDC and deposit holding by patient households. Here I have considered two different supply policies,

$$\text{Policy 1:} \quad cbdc_t = \phi_{CBDC} Y_t \tag{24}$$

$$\text{Policy 2:} \quad cbdc_t = \rho_{CBDC} cbdc_{t-1} + (1 - \rho_{CBDC})\phi_{CBDC}(Y_t - Y_{t-1}) \tag{25}$$

In case of the first policy, where CBDC supply simply a fraction of the output, under the con-

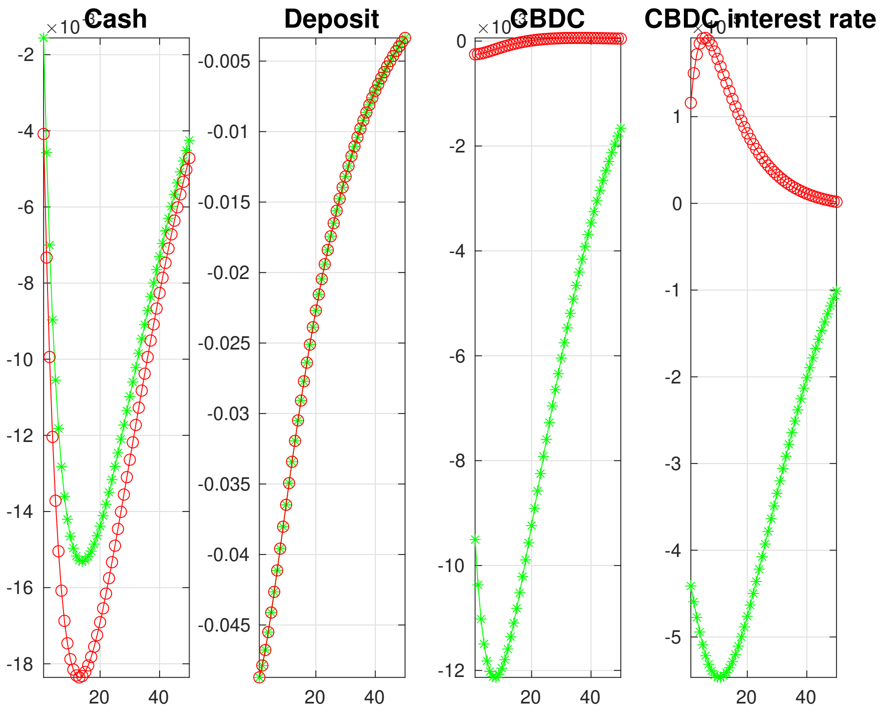


Figure 5: Response of various liquidity instrument to a monetary policy shock under two different supply plans for CBDC i.e (1) constant fraction ϕ_{CBDC} of Y (green lines) (2) Supply rule with feedback from economy(red lines)

tractionary monetary policy, supply of CBDC drops as well. Therefore, due to higher demand, remuneration of CBDC becomes quite low and even negative (green line). Cash holding and CBDC holding both decrease in the face of contractionary monetary policy, leading to decrease in the size of the central bank's balance sheet. When the central bank follows the second policy, changes in the supply of CBDC happens quite slowly, due to parameter ρ_{CBDC}. Higher remuneration of CBDC allows the patient households to hold more CBDC and therefore decreasing cash holding by a larger amount than in the case of first policy. Bank deposit shows no differential effect, it simply decrease on impact in the face of positive monetary policy shock. Therefore different supply policy rules do not matter for the deposit holdings, banks face same level of disintermediation in both cases.

4.6 Cash similarity

Another widely discussed feature of CBDC is that how closely it will replicate the features of cash. [8] To analyze different cash like features of CBDC, I have considered different values of $\theta \in [0, 1]$. This parameter controls the importance of CBDC in the utility of the household and the relative risk aversion of holding CBDC. When the θ is close to 1, CBDC replicates the features of cash,

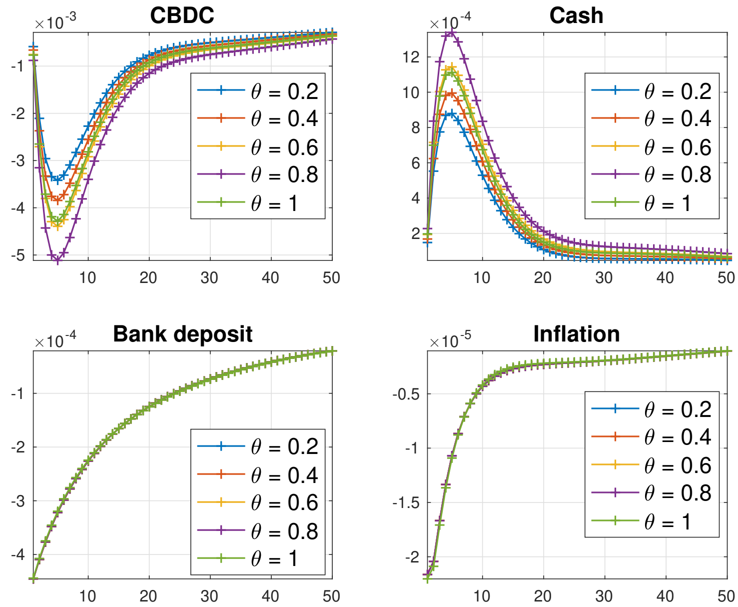


Figure 6: Response of various liquidity instrument to a liquidity shock under varying degree of similarity of CBDC to Cash captured by the parameter θ.

whereas when θ is close to zero, the household does not gain any utility from holding CBDC. When the economy is hit by unanticipated liquidity shock, the response of cash and CBDC are substituted with each other with having differential effects on the other macro variables as we can see in figure 6. As values of θ increases the magnitude of response of CBDC and cash increases attaining maximum value for $\theta = 0.8$ and then decreases when cash and CBDC are exactly similar. One might expect exact same dynamics of cash and CBDC for $\theta = 1$ but storage cost of cash leads to different dynamics for them. In figure 7, we see the impulse responses when the economy is hit by an unanticipated contractionary monetary policy. As policy rate increases, remuneration of CBDC also increases leading to higher holding of CBDC and decrease in the holding of cash. The magnitude of responses are increasing with θ and attain maximum at 0.8. Inflation decreases on impact, slightly less in case of $\theta = 1$. Thus having different levels of similarity of cash and CBDC make no difference in terms dynamics of other macro variables of the economy, only the substitution of CBDC and cash is pronounced.

[8] The main idea of CBDC is to allow households to have a direct claim to the central bank's balance sheet just like the case in cash. In today's payment system a new form of payment could only be useful from being different than the existing ones as the existing ones are quite efficient. Thus CBDC having features like cash can be useful and incentivise households to hold it. For more details, see Ferrari et al. [2022], Bordo and Levin [2017]

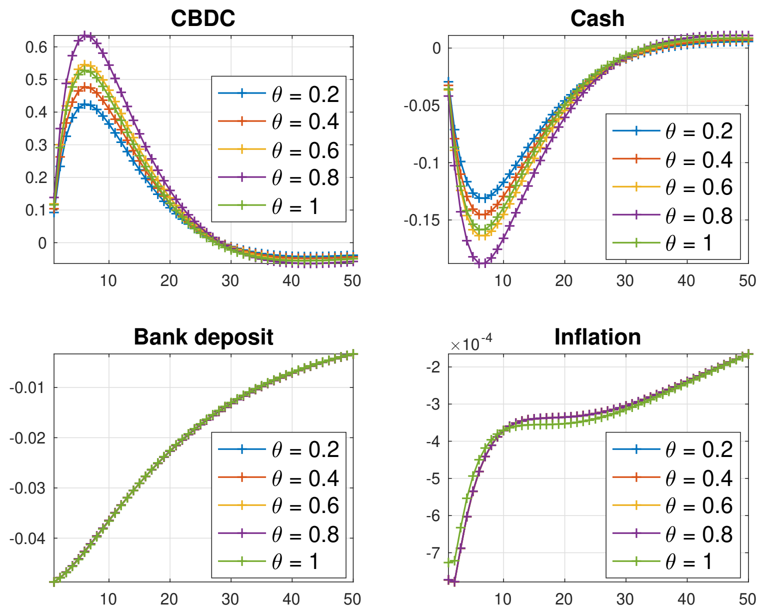


Figure 7: Response of various liquidity instrument to a monetary policy shock under varying degree of similarity of CBDC to Cash captured by the parameter θ.

5 Conclusion

The analysis of the model shows that introducing CBDC in the economy improves the dynamics of inflation, as the stabilization process is quicker than the baseline scenario without CBDC. Additionally, the presence of CBDC reduces the increase in interest rates on loans and deposits by the banks, which results in lesser profit margins for banks. However, there is no significant impact on the bank deposits due to the introduction of CBDC, and the bank disintermediation effect is not very pronounced.

In terms of monetary policy shocks, the study shows that the presence of CBDC can improve the response of the economy to such shocks. The price puzzle, which is present in the baseline scenario without CBDC, disappears when CBDC is introduced. Additionally, the study shows that different remuneration schemes for CBDC can have a significant impact on the holdings of cash and CBDC, with remunerated CBDC resulting in more pronounced changes in holdings.

The analysis also highlights some potential challenges associated with the introduction of CBDC, particularly with regards to the impact on bank capital and deposit holdings. The results indicate that the presence of CBDC can lead to a decrease in bank capital on impact, which takes a longer time to return to steady-state than in the absence of CBDC. Additionally, bank deposits decrease on impact when CBDC is introduced, which can have a potential impact on the banking system's stability.

Overall, the results of this study suggest that the introduction of CBDC can have significant effects on the economy and financial intermediation. While CBDC can improve price stability and reduce profit margins for banks, it can also have potential adverse effects on bank capital and deposit holdings. Therefore, policymakers must carefully weigh the potential benefits and risks of introducing CBDC before making any decisions on its implementation. Additionally, further research is needed to better understand the potential long-term effects of CBDC on the economy and financial intermediation.

References

Ramón Adalid, Álvaro Álvarez-Blázquez, Katrin Assenmacher, Lorenzo Burlon, Maria Dimou, Carolina López-Quiles, Natalia Martín Fuentes, Barbara Meller, Manuel Muñoz, Petya Radulova, et al. Central bank digital currency and bank intermediation. *ECB Occasional Paper*, (2022/293), 2022. URL https://www.ecb.europa.eu/pub/pdf/scpops/ecb.op293~652cf2b1aa.en.pdf?985167870ac2551e31097f06382d01d9.

Itai Agur, Anil Ari, and Giovanni Dell'Ariccia. Designing central bank digital currencies. *IMF Working Papers*, 2019, 2019. ISSN 1018-5941. doi: 10.5089/9781513519883.001.

David Andolfatto. Assessing the Impact of Central Bank Digital Currency on Private Banks. *The Economic Journal*, 131(634):525–540, 09 2020. ISSN 0013-0133. doi: 10.1093/ej/ueaa073. URL https://doi.org/10.1093/ej/ueaa073.

Pierpaolo Benigno and Salvatore Nisticò. Safe assets, liquidity, and monetary policy. *American Economic Journal: Macroeconomics*, 9(2):182–227, April 2017. doi: 10.1257/mac.20150073. URL https://www.aeaweb.org/articles?id=10.1257/mac.20150073.

Allen N. Berger, Asli Demirgüç-Kunt, Ross Levine, and Joseph G. Haubrich. Bank concentration and competition: An evolution in the making. *Journal of Money, Credit and Banking*, 36(3): 433–451, 2004. ISSN 00222879, 15384616. URL http://www.jstor.org/stable/3838945.

Michael D Bordo and Andrew T Levin. Central bank digital currency and the future of monetary policy. 2017. URL https://www.nber.org/papers/w23711.

Lorenzo Burlon, Carlos Montes-Galdon, Manuel A Muñoz, and Frank Smets. The optimal quantity of cbdc in a bank-based economy, 2022. URL www.cepr.org.

Francesca Carapella and Jean Flemming. Central bank digital currency: A literature review. *FEDS Notes. Washington: Board of Governors of the Federal Reserve System*, 2020. doi: https://doi.org/10.17016/2380-7172.2790.

Jonathan Chiu, Seyed Mohammadreza Davoodalhosseini, Janet Jiang, and Yu Zhu. Bank market power and central bank digital currency: Theory and quantitative assessment. *Journal of Political Economy*, 2023. doi: 10.1086/722517. URL https://doi.org/10.1086/722517.

Lawrence J. Christiano, Martin Eichenbaum, and Charles L. Evans. Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, 113(1):1–45, 2005. ISSN 00223808, 1537534X. URL http://www.jstor.org/stable/10.1086/426038.

Hans Degryse and Steven Ongena. Competition and regulation in the banking sector: A review of the empirical evidence on the sources of bank rents. *Handbook of financial intermediation and banking*, 2008:483–554, 2008.

Douglas W. Diamond. Financial Intermediation and Delegated Monitoring. *The Review of Economic Studies*, 51(3):393–414, 07 1984. ISSN 0034-6527. doi: 10.2307/2297430. URL https://doi.org/10.2307/2297430.

Itamar Drechsler, Alexi Savov, and Philipp Schnabl. The Deposits Channel of Monetary Policy*. *The Quarterly Journal of Economics*, 132(4):1819–1876, 05 2017. ISSN 0033-5533. doi: 10.1093/qje/qjx019. URL https://doi.org/10.1093/qje/qjx019.

Minesso Massimo Ferrari, Arnaud Mehl, and Livio Stracca. Central bank digital currency in an open economy. *Journal of Monetary Economics*, 127:54–68, 2022. ISSN 0304-3932. doi: https://doi.org/10.1016/j.jmoneco.2022.02.001. URL https://www.sciencedirect.com/science/article/pii/S0304393222000186.

Rodney Garratt, Jiaheng Yu, and Haoxiang Zhu. How central bank digital currency design choices impact monetary policy pass-through and market composition. *Available at SSRN 4004341*, 2022.

Andrea Gerali, Stefano Neri, Luca Sessa, and Federico M. signoretti. Credit and banking in a dsge model of the euro area. *Journal of Money, Credit and Banking*, 42(s1):107–141, 2010. doi: https://doi.org/10.1111/j.1538-4616.2010.00331.x. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1538-4616.2010.00331.x.

Gary B Gorton and Jeffery Zhang. Protecting the sovereign's money monopoly. *U of Michigan Law & Econ Research Paper*, (22-031), 2022. URL https://ssrn.com/abstract=4162884.

Matteo Iacoviello. House prices, borrowing constraints, and monetary policy in the business cycle. *American Economic Journal: Macroeconomics*, 2005.

Matteo Iacoviello and Stefano Neri. Housing market spillovers: Evidence from an estimated dsge model. *American Economic Journal: Macroeconomics*, 2:125–164, 4 2010. ISSN 19457715. doi: 10.1257/mac.2.2.125.

Todd Keister and Daniel Sanches. Should Central Banks Issue Digital Currency? *The Review of Economic Studies*, 90(1):404–431, 03 2022. ISSN 0034-6527. doi: 10.1093/restud/rdac017. URL https://doi.org/10.1093/restud/rdac017.

Nobuhiro Kiyotaki and John Moore. Credit cycles. *Journal of Political Economy*, 105(2):211–248, 1997. doi: 10.1086/262072. URL https://doi.org/10.1086/262072.

Ricardo Lagos and Randall Wright. A unified framework for monetary theory and policy analysis. *Journal of Political Economy*, 113(3):463–484, 2005. ISSN 00223808, 1537534X. URL http://www.jstor.org/stable/10.1086/429804.

Jack Meaning, Ben Dyson, James Barker, and Emily Clayton. Broadening narrow money: monetary policy with a central bank digital currency. 2018. URL https://www.ijcb.org/journal/ijcb21q2a1.pdf.

OMFIF. Digital currencies: A question of trust. *OMFIF report on global public confidence in monetary, financial and payment institutions*, 2020. URL https://www.omfif.org/wp-content/uploads/2020/02/Digital-currencies-A-question-of-trust-1.pdf.

Federal Reserve. Money and payments: The us dollar in the age of digital transformation. *January, https://www. federalreserve. gov/publications/moneyand-payments-discussion-paper. html*, 2022.

Julio J. Rotemberg. Monopolistic price adjustment and aggregate output. *The Review of Economic Studies*, 49(4):517–531, 1982. ISSN 00346527, 1467937X. URL http://www.jstor.org/stable/2297284.

Jonathan Schiller and Jonas Gross. A model for central bank digital currencies: Implications for bank funding and monetary policy. 2021. URL http://hdl.handle.net/10419/242350.

Stephanie Schmitt-Grohé and Martín Uribe. Optimal fiscal and monetary policy in a medium-scale macroeconomic model. *NBER Macroeconomics Annual*, 20:383–425, 2005. ISSN 08893365, 15372642. URL http://www.jstor.org/stable/3585431.

Steven A. Sharpe. Asymmetric information, bank lending, and implicit contracts: A stylized model of customer relationships. *The Journal of Finance*, 45(4):1069–1087, 1990. doi: https://doi.org/10.1111/j.1540-6261.1990.tb02427.x. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1540-6261.1990.tb02427.x.

Frank Smets and Rafael Wouters. Shocks and frictions in us business cycles: A bayesian dsge approach. *American Economic Review*, 97(3):586–606, June 2007. doi: 10.1257/aer.97.3.586. URL https://www.aeaweb.org/articles?id=10.1257/aer.97.3.586.

A Bayesian Estimation

Table 1: Calibrated Parameters for Households

Parameter	Value	Description
a_p	0.850	Degree of habit formation: Patient households
a_E	0.850	Degree of habit formation: Entrepreneurs
β^p	0.994	Discount factor patient households
β^I	0.975	Discount factor impatient households
β^E	0.975	Discount factor patient entrepreneurs
ϕ	1.000	Inverse Frisch elasticity of labor supply
γ_{ss}	0.270	Liquidity of CBDC in SS
ψ_m	0.002	Cash substitution
σ_m	10.670	Cash elasticity
Ψ_m	0.440	Cash storage cost

Table 1 – Continued

Parameter	Value	Description
$\frac{m}{Y}$	0.344	M/Y in SS
σ_{dc}	16.005	CBDC elastcity
θ	0.500	Cash similarity
ψ_{dc}	0.220	CDBC substitution
$\frac{cbdc}{Y}$	0.344	CBDC/Y in SS

Table 2: Calibrated Parameters of Production side

Parameter	Value	Description
δ^k	0.025	Capital depreciation
r^k_{ss}	0.047	Steady state int. rate capital
α	0.250	Capital share in the production function
μ	0.800	Wage share of patient households
ψ_1	0.048	Capital utilization cost parameter 1
ψ_2	0.005	Capital utilization cost parameter 2
x_{ss}	1.200	Price ratio in ss
ϵ^y_{ss}	6.000	Demand price elasticity RETAILERS
κ_p	28.650	Price adjustment cost parameter of retailers

Table 3: Banking Sector Parameters

Parameter	Value	Description
m^I_{ss}	0.700	Loan-to-value ratio impatient households
m^E_{ss}	0.350	Loan-to-value ratio entrepreneurs
ϵ_{bH}	2.790	Elasticity of substitution of loan household
ϵ_{bE}	3.120	Elasticity of substitution. of loan entrepreneur
ϵ_d	-1.460	Elasticity of substitution of deposits
ν_i	0.090	Banking Capital ratio over Loans (Basel II)
δ^b	0.105	Depreciation rate of bank capital

A.1 Model Estimation

Table 4: Prior and posterior distribution of the structural parameters

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
ϕ_R	Beta	0.800	0.1000	0.753	0.0308	0.7069	0.7991
ϕ_π	Gamma	2.000	0.5000	0.866	0.1137	0.7133	1.0390
ϕ_Y	Normal	0.200	0.1500	0.746	0.1025	0.6206	0.9343
κ_i	Gamma	2.500	1.0000	5.167	0.9989	3.4674	6.8073
κ_{Kb}	Gamma	10.000	5.0000	3.701	1.3907	1.4904	5.6144
κ_d	Gamma	10.000	2.5000	5.258	0.6583	4.0722	6.1712
κ_{bE}	Gamma	3.000	2.5000	9.959	2.6100	5.7154	14.0360
κ_{bH}	Gamma	6.000	2.5000	6.251	2.4834	2.9129	10.5563

Table 5: Prior and posterior distribution of structural parameters- exogenous process

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
AR coefficients							
ρ_z	Beta	0.800	0.1000	0.979	0.0102	0.9598	0.9931
ρ_a	Beta	0.500	0.2000	0.063	0.0286	0.0202	0.1063
ρ_r	Beta	0.800	0.1000	0.357	0.0393	0.2950	0.4112
ρ_h	Beta	0.500	0.2000	0.469	0.1788	0.1982	0.7514
ρ_{m^I}	Beta	0.800	0.1000	0.939	0.0644	0.8336	0.9939
ρ_{m^E}	Beta	0.800	0.1000	0.814	0.1086	0.6262	0.9645
ρ_d	Beta	0.800	0.1000	0.980	0.0069	0.9703	0.9920
ρ_{bH}	Beta	0.800	0.1000	0.769	0.0996	0.6268	0.9321
ρ_{bE}	Beta	0.800	0.1000	0.921	0.0401	0.8751	0.9782
ρ_γ	Beta	0.960	0.0300	0.967	0.0155	0.9416	0.9888
Standard deviations							
ε^z	Inv. Gamma	0.100	2.0000	0.947	0.1169	0.7673	1.1277
ε^a	Inv. Gamma	0.100	2.0000	0.735	0.0644	0.6255	0.8338
ε^r	Inv. Gamma	0.100	2.0000	0.057	0.0083	0.0438	0.0691
ε^h	Inv. Gamma	0.100	2.0000	0.115	0.0467	0.0614	0.1949
ε^{mI}	Inv. Gamma	0.100	2.0000	0.297	0.0667	0.1888	0.3862
ε^{mE}	Inv. Gamma	0.100	2.0000	0.076	0.0266	0.0367	0.1110

(Continued on next page)

Table 5: (continued)

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
ε^d	Inv. Gamma	0.100	2.0000	0.078	0.0062	0.0695	0.0874
ε^{bH}	Inv. Gamma	0.100	3.0000	0.049	0.0160	0.0238	0.0762
ε^{bE}	Inv. Gamma	0.100	2.0000	0.155	0.0250	0.1152	0.1855
ε^{γ}	Inv. Gamma	0.100	2.0000	0.104	0.0335	0.0497	0.1553

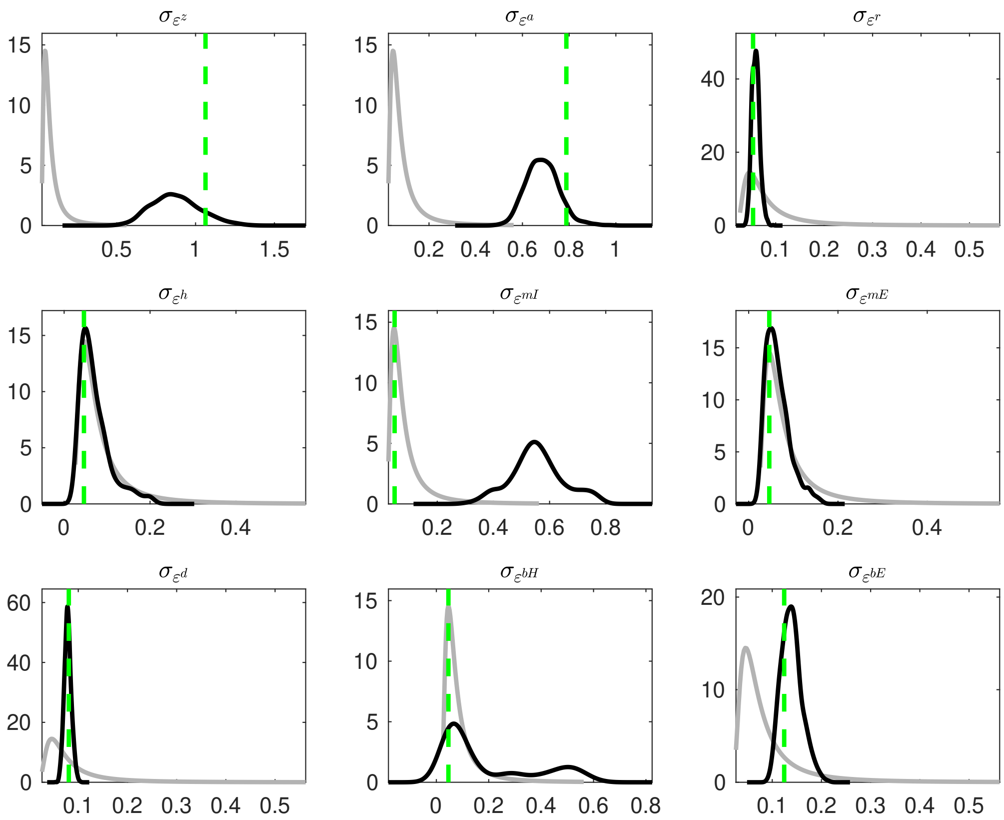


Figure 8: Priors and posteriors.

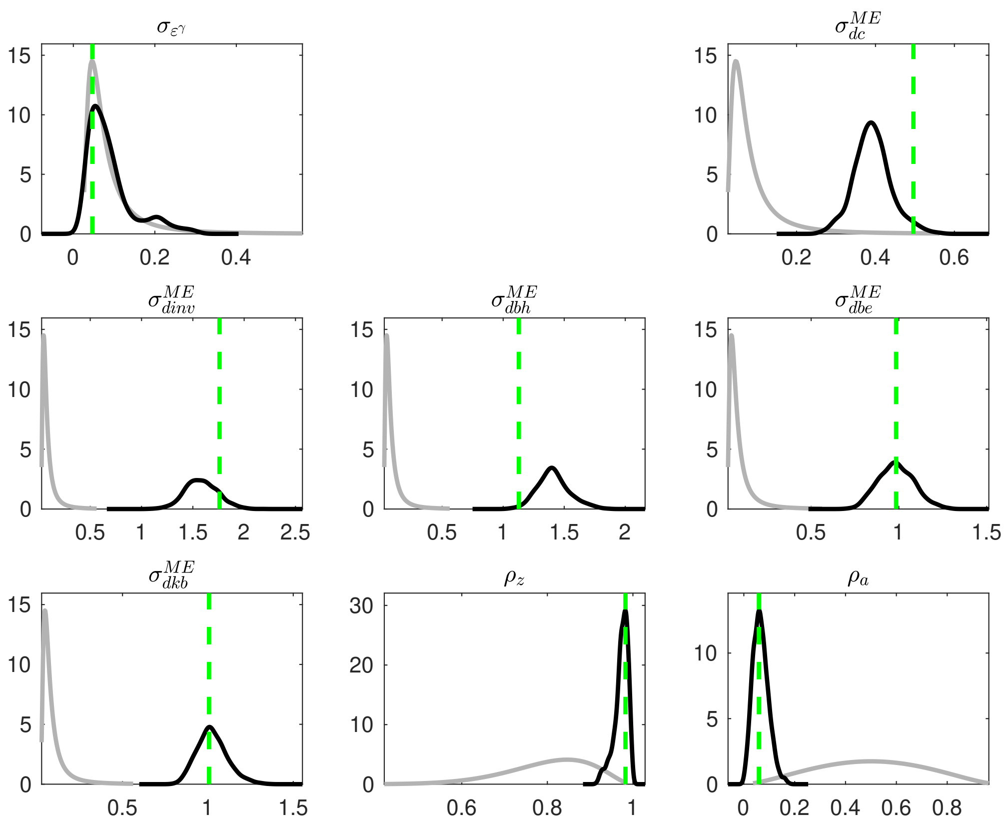


Figure 9: Priors and posteriors.

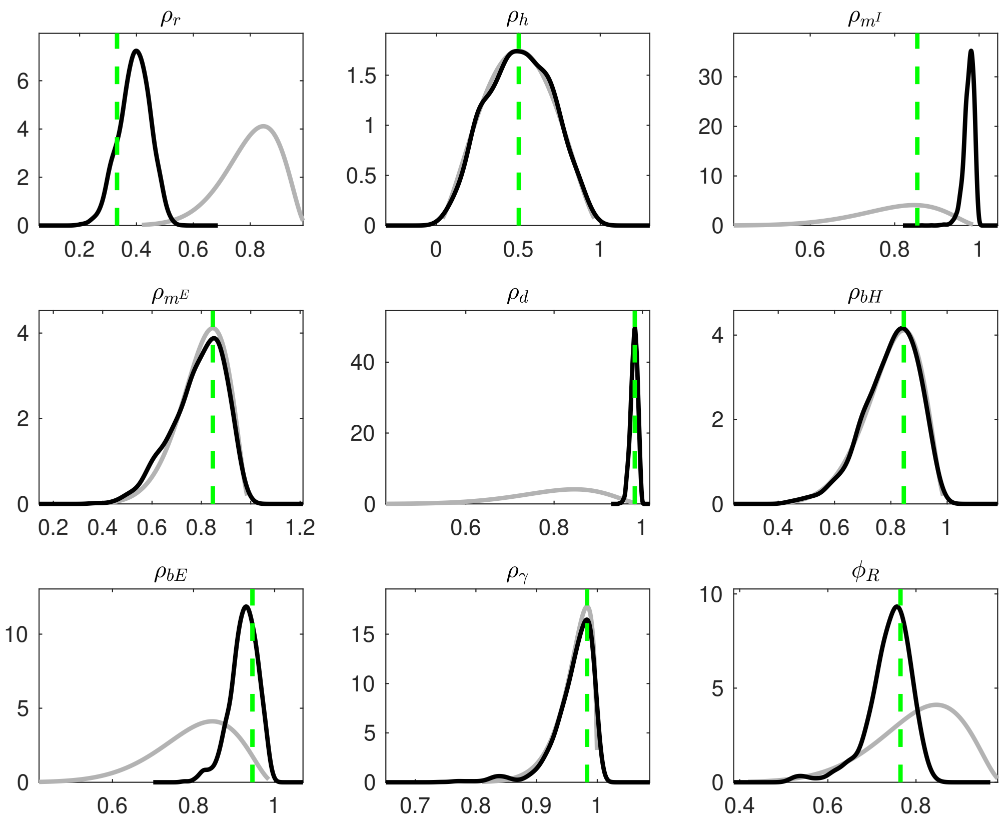


Figure 10: Priors and posteriors.

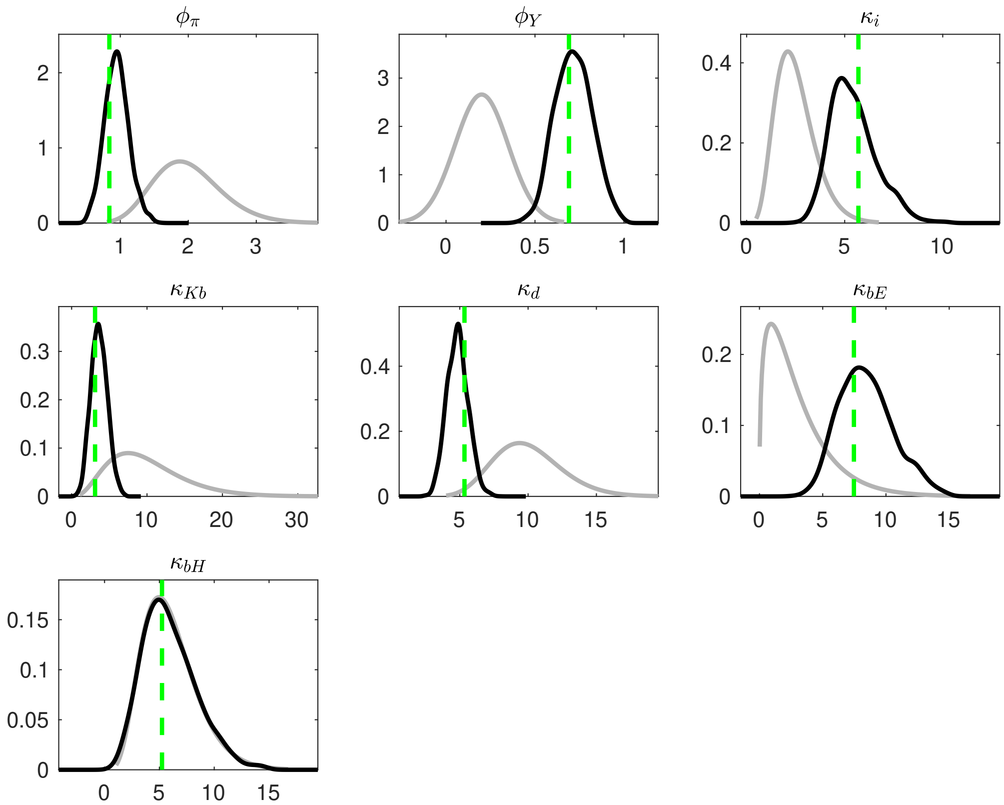


Figure 11: Priors and posteriors.

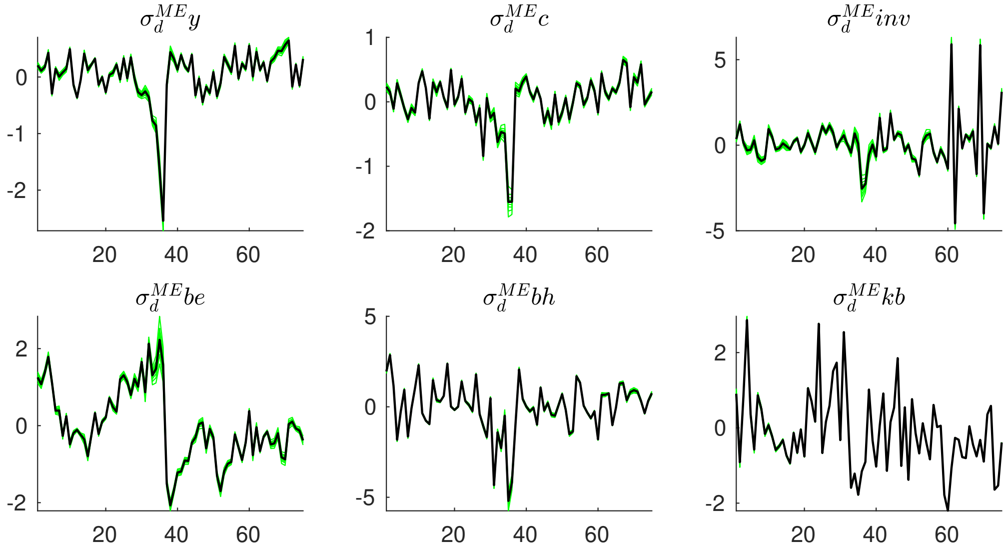


Figure 12: Smoothed measurement errors

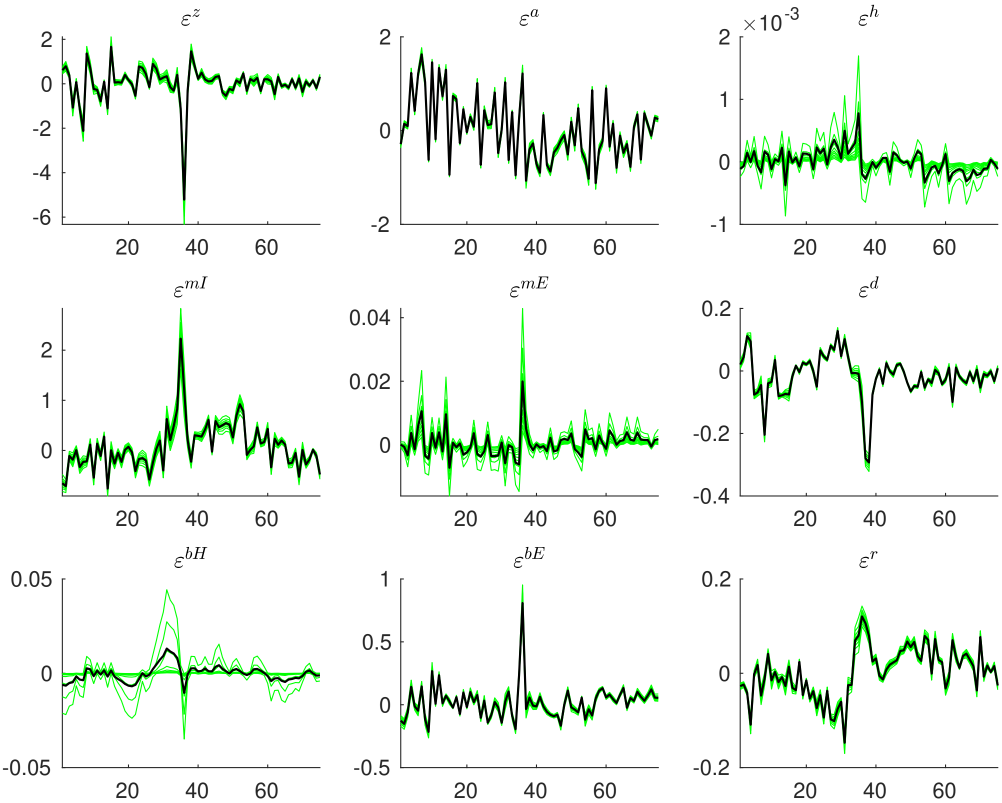


Figure 13: Smoothed shocks

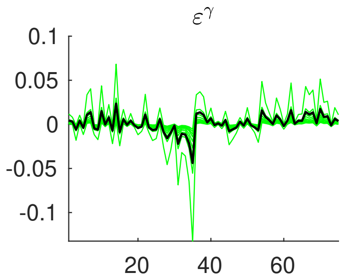


Figure 14: Smoothed shocks

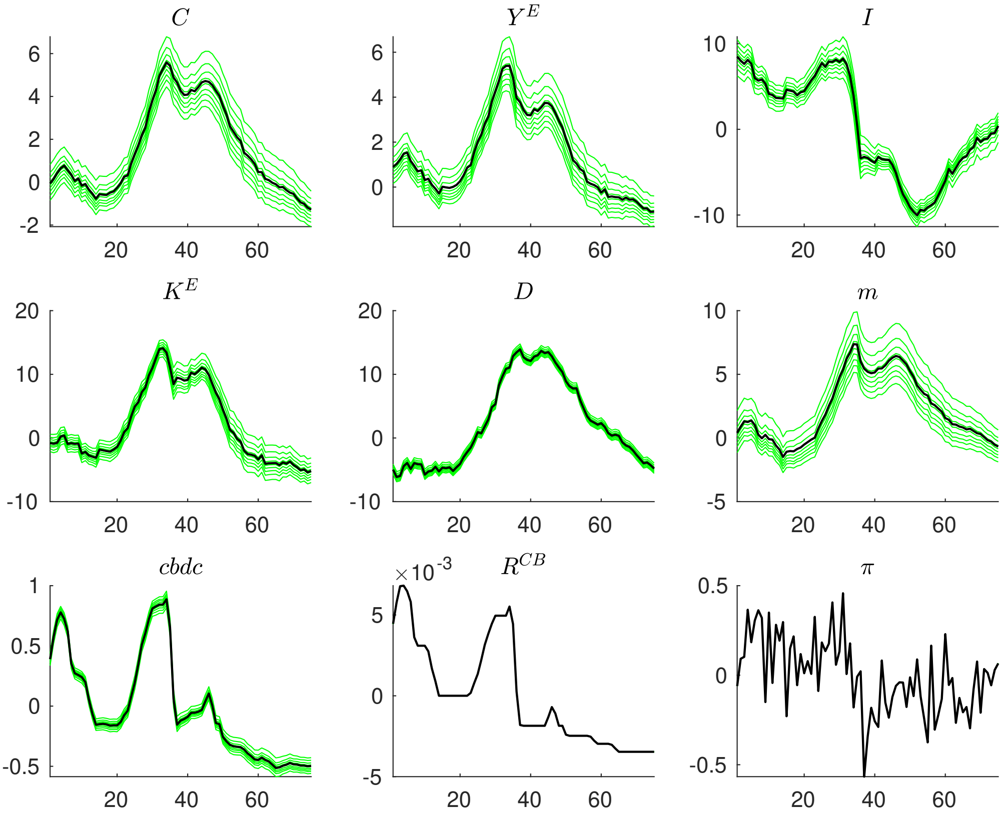


Figure 15: Smoothed variables

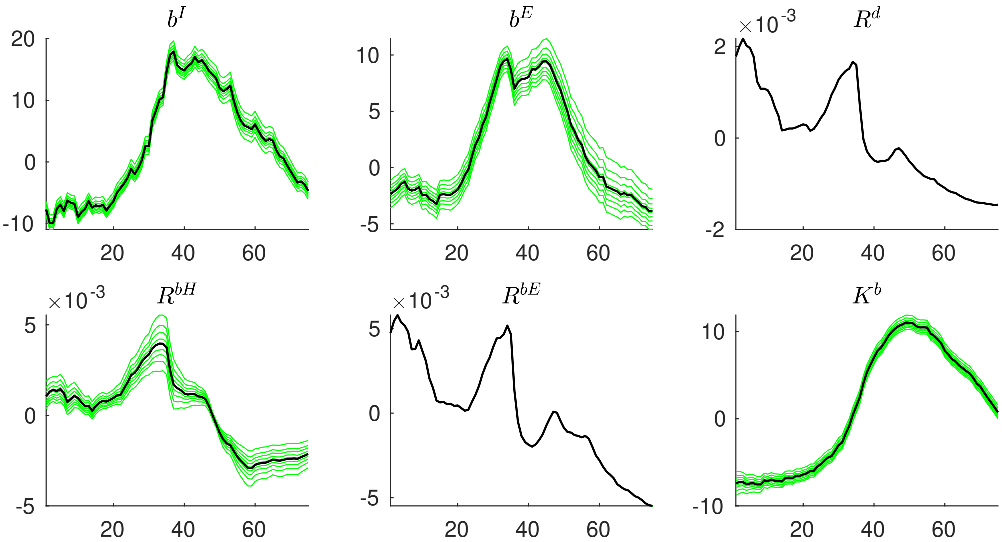


Figure 16: Smoothed variables

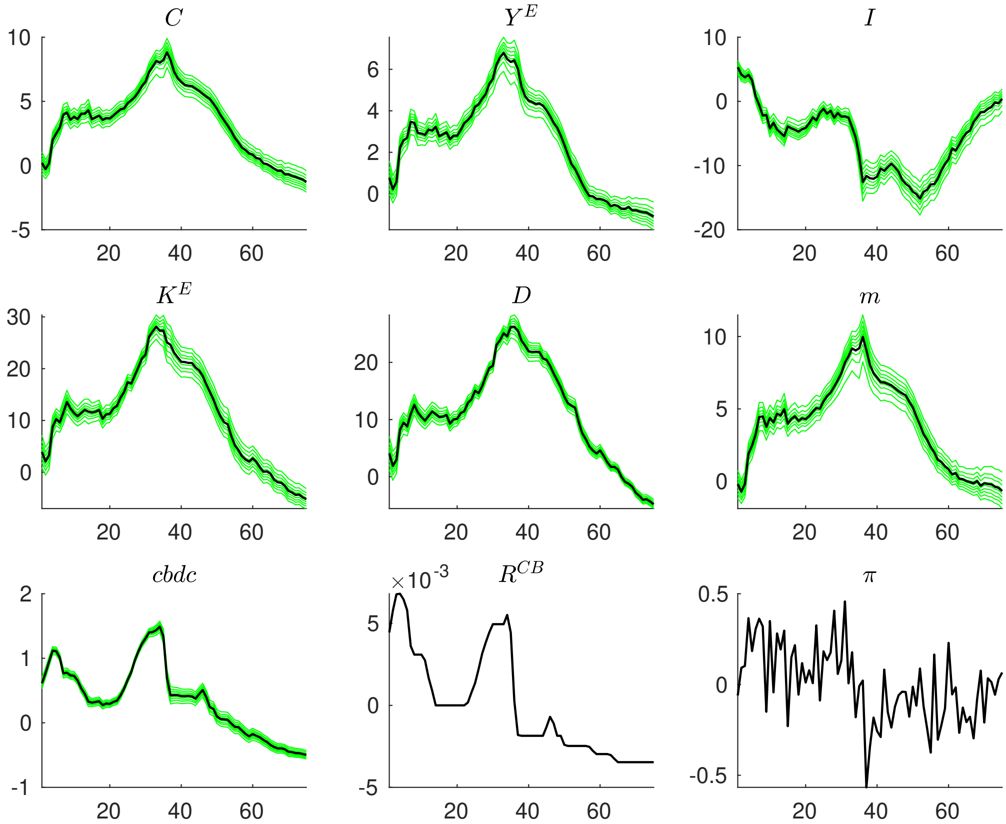


Figure 17: Updated Variables

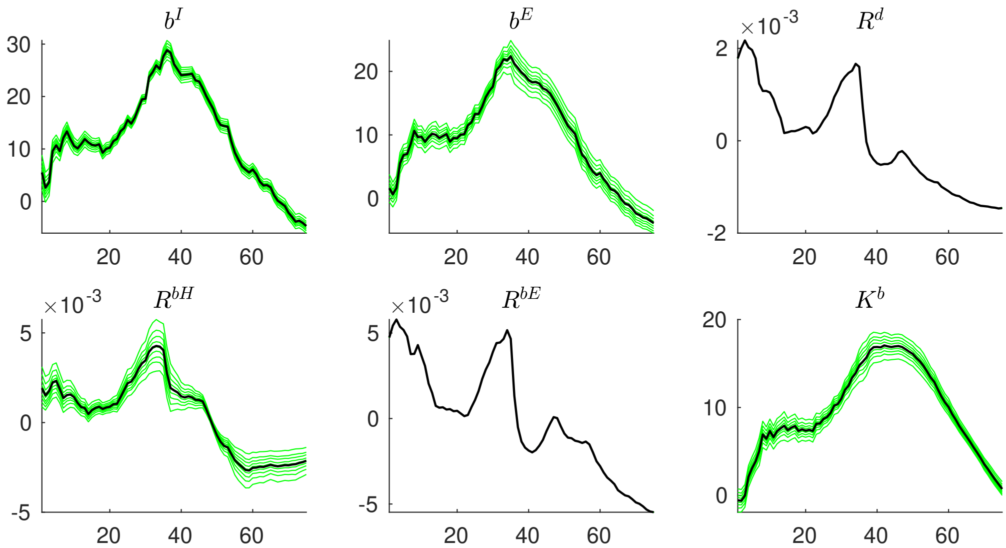


Figure 18: Updated Variables

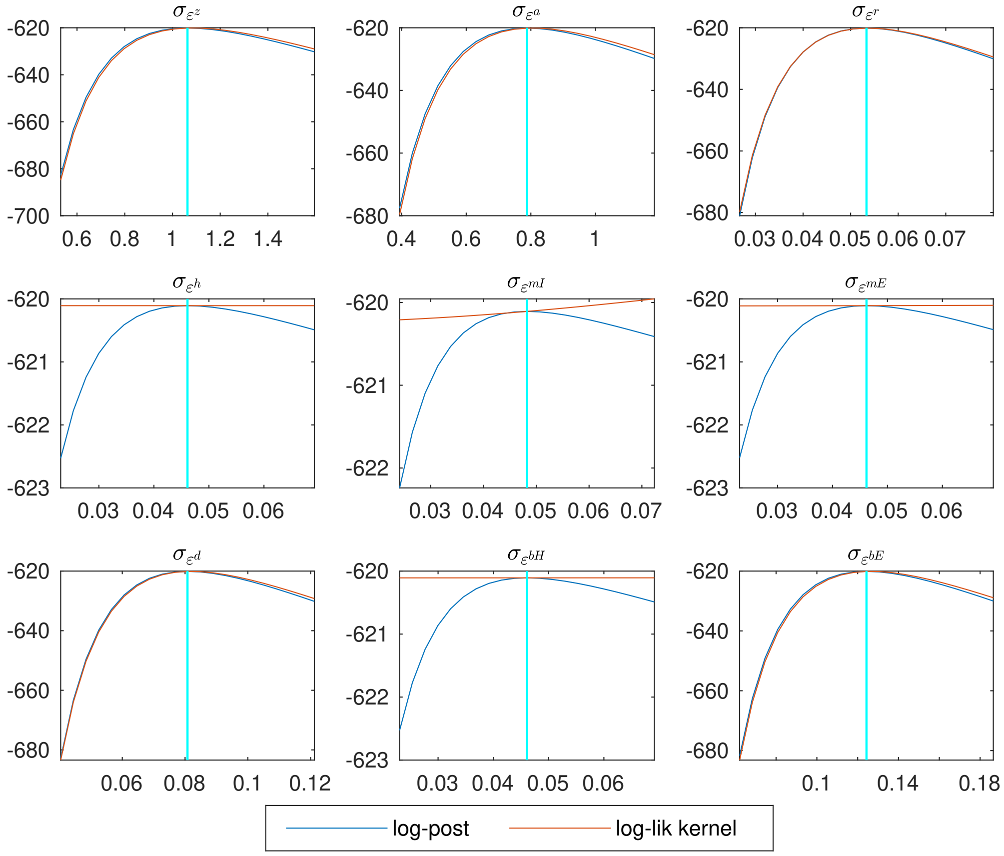


Figure 19: Check plots.

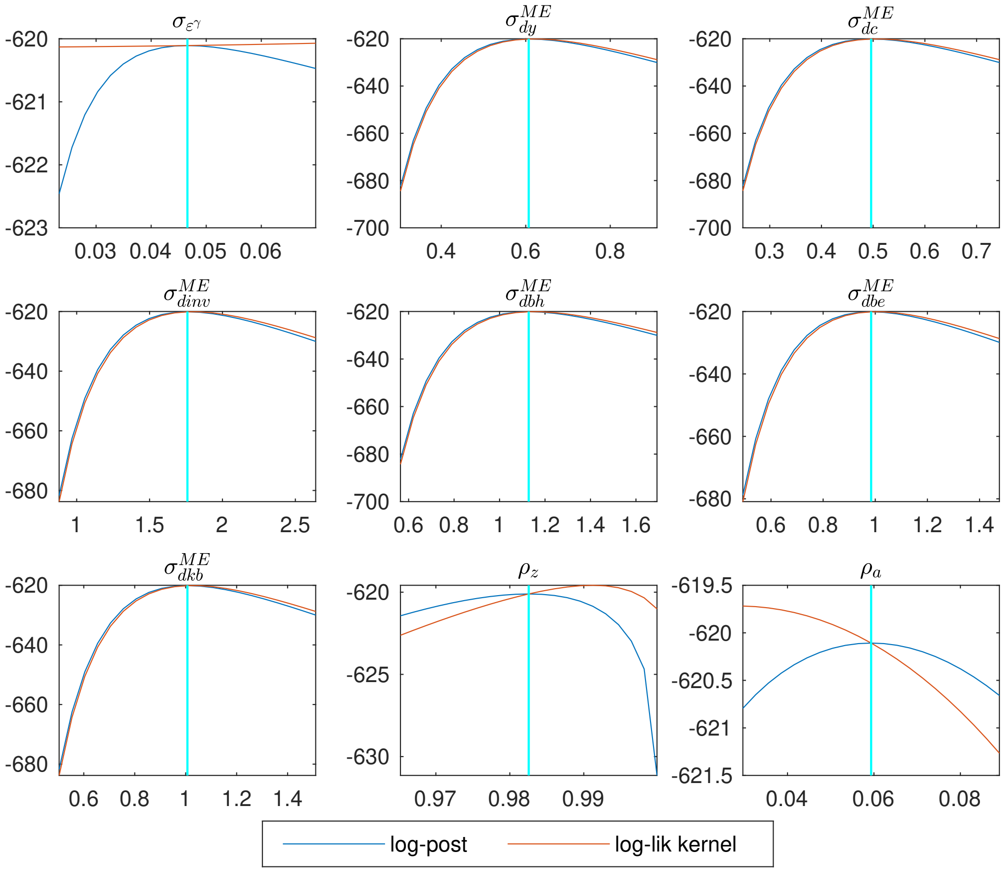


Figure 20: Check plots.

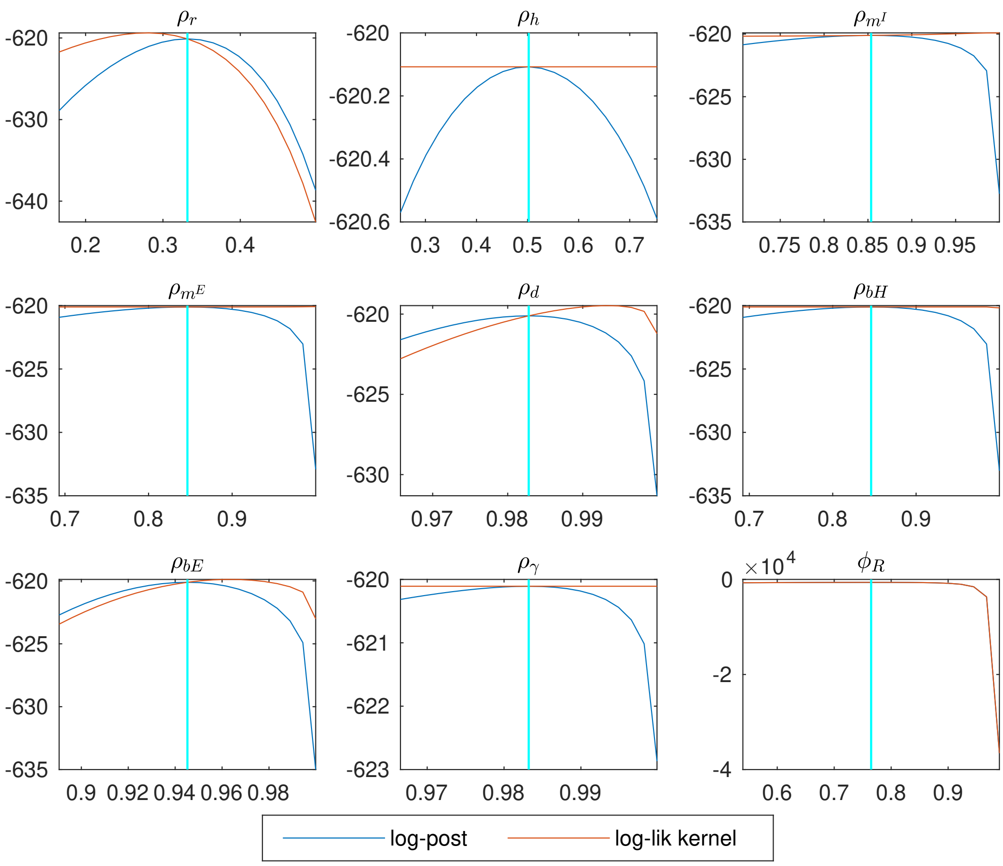


Figure 21: Check plots.

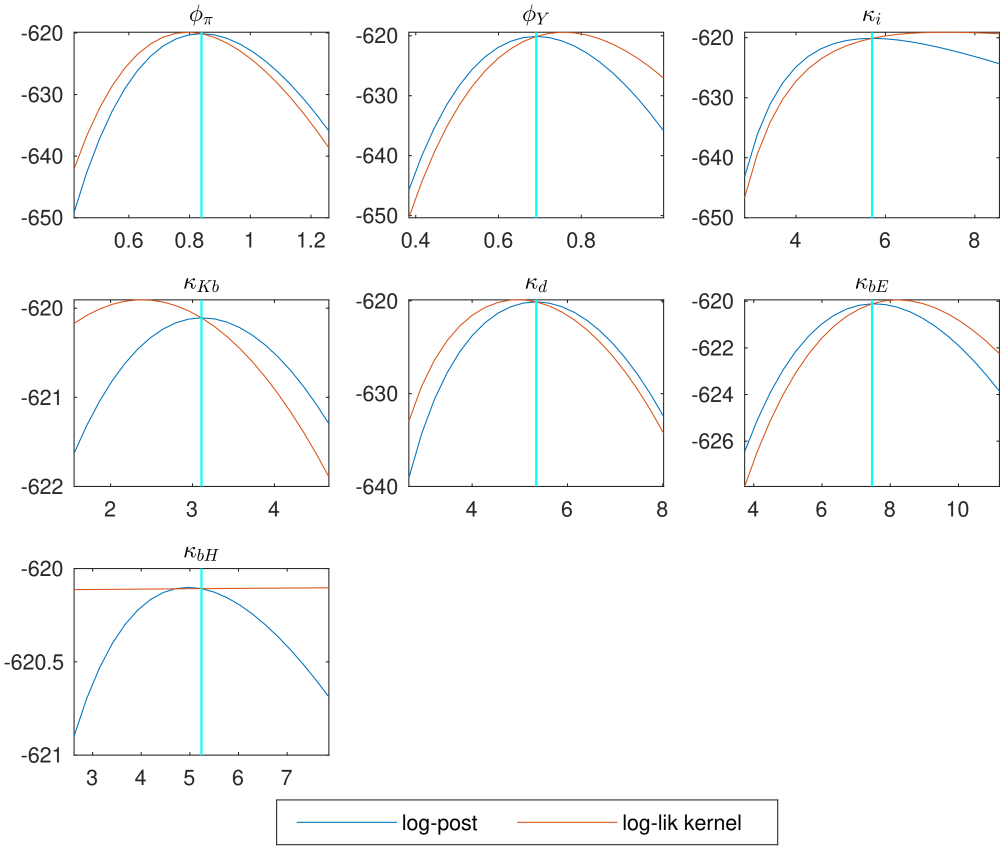


Figure 22: Check plots.

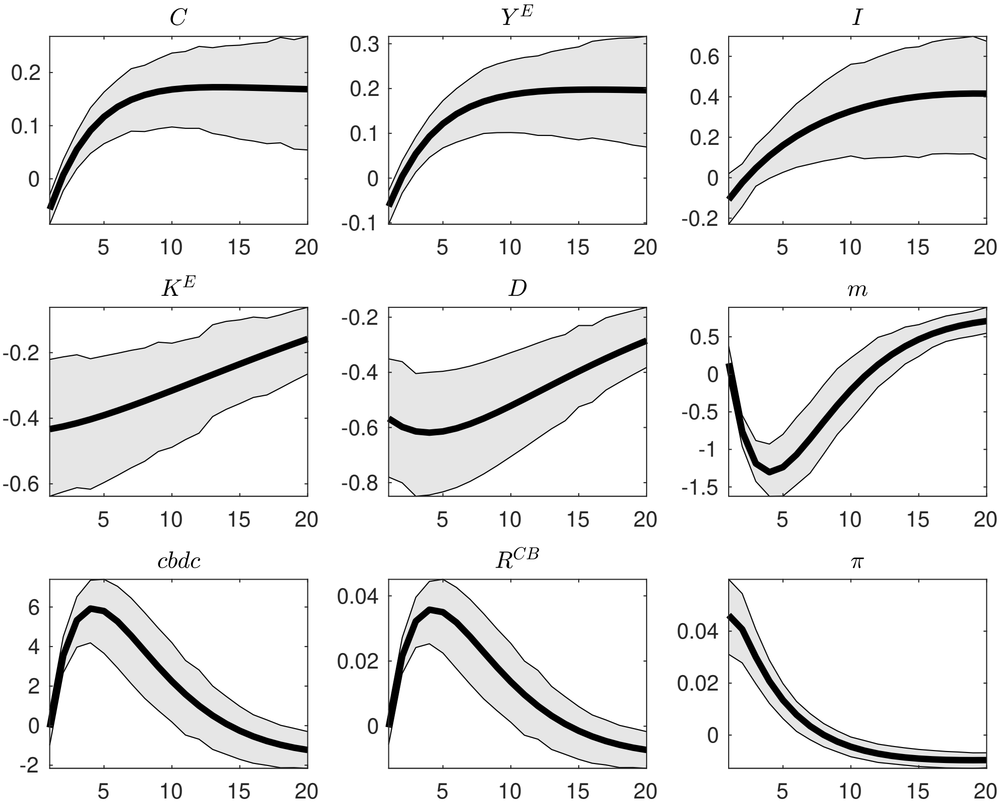


Figure 23: Bayesian IRF: Orthogonalized shock to ε^z.

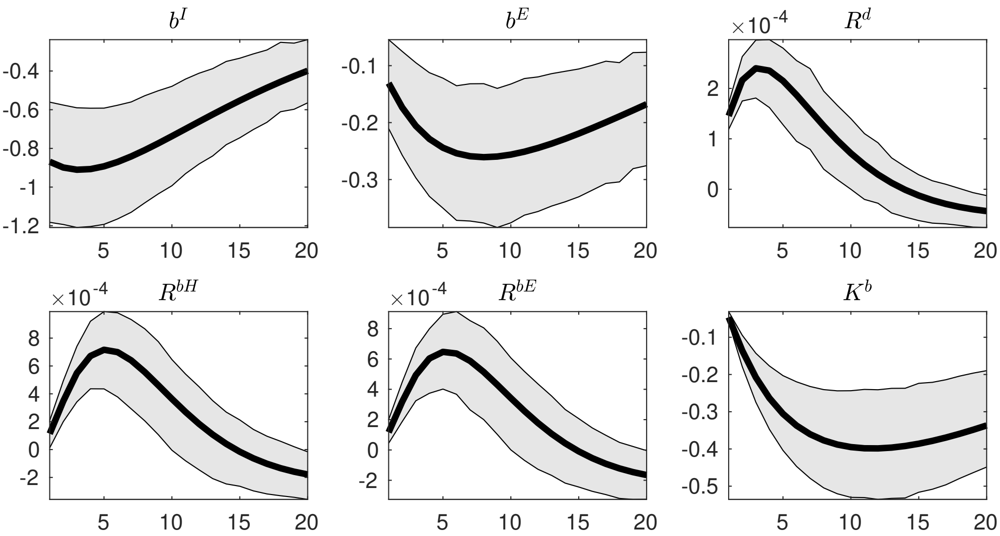


Figure 24: Bayesian IRF: Orthogonalized shock to ε^z.

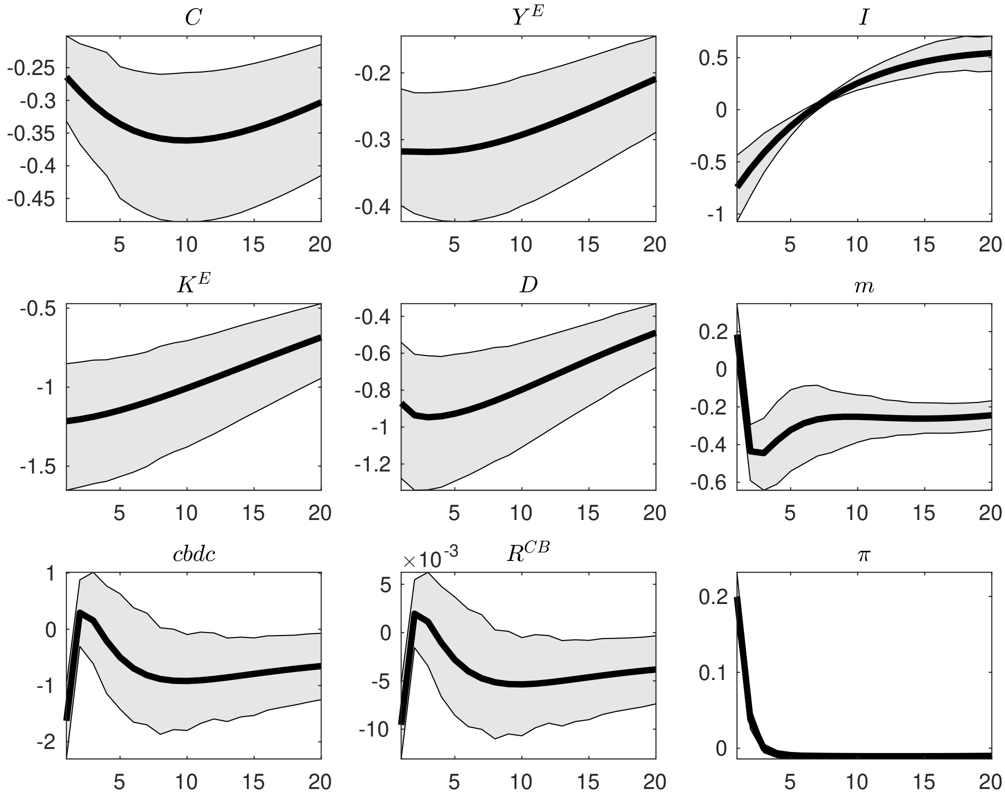


Figure 25: Bayesian IRF: Orthogonalized shock to ε^a.

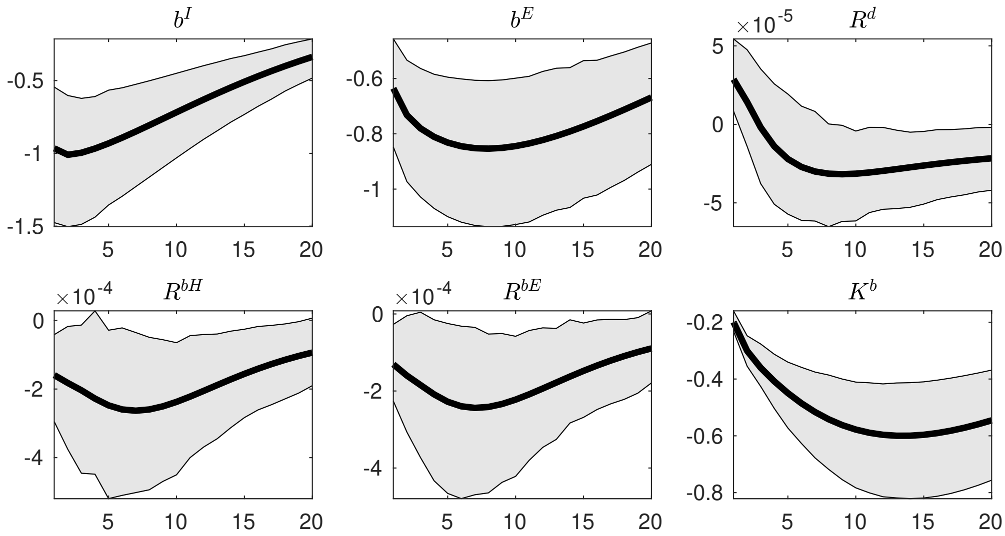


Figure 26: Bayesian IRF: Orthogonalized shock to ε^a.

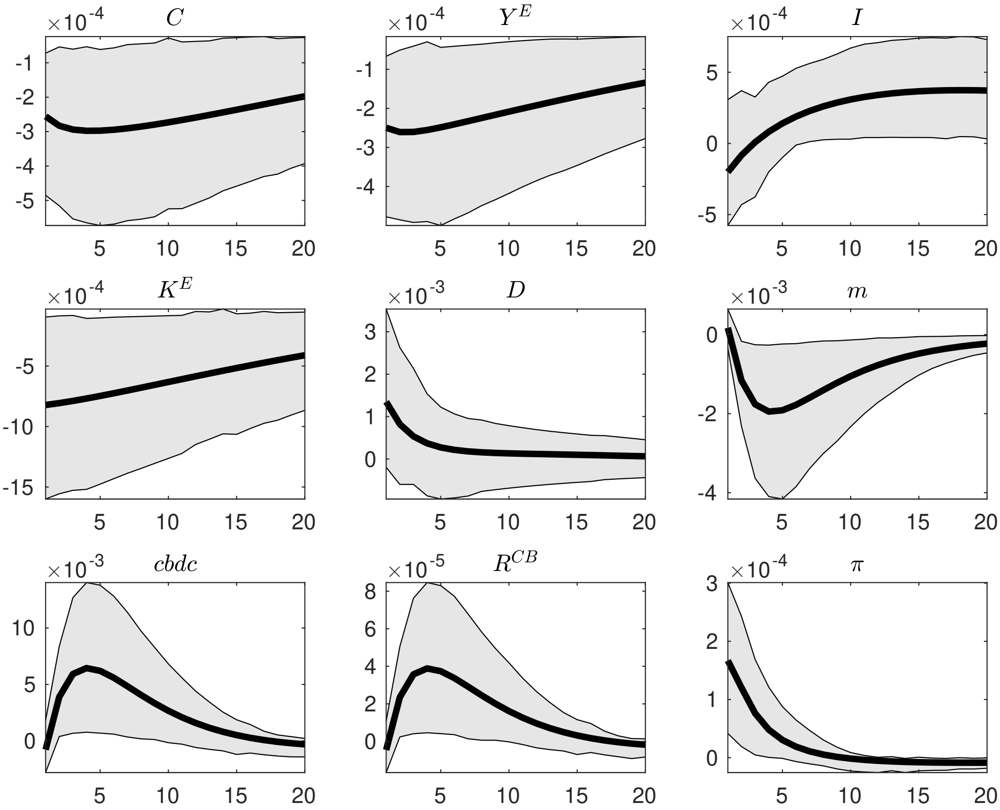


Figure 27: Bayesian IRF: Orthogonalized shock to ε^h.

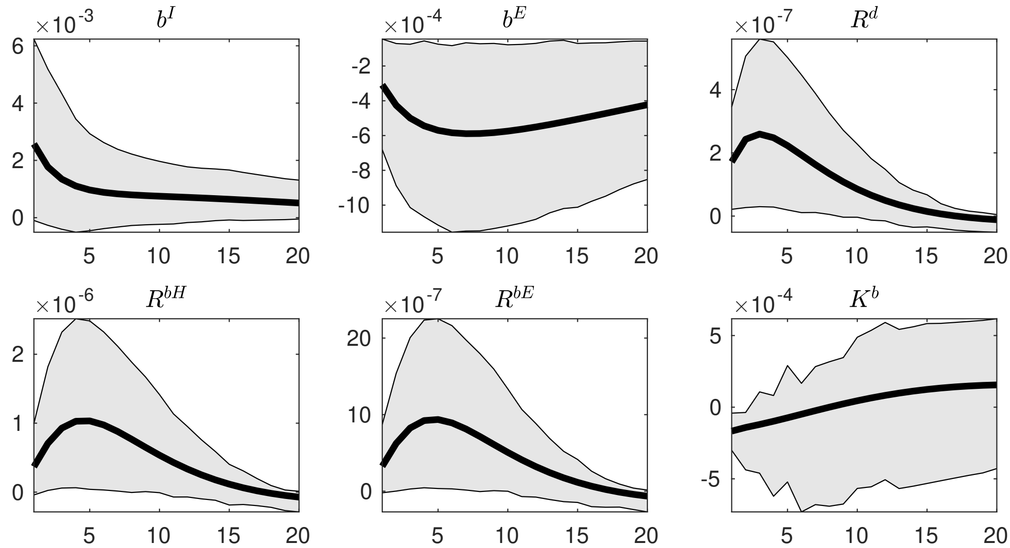


Figure 28: Bayesian IRF: Orthogonalized shock to ε^h.

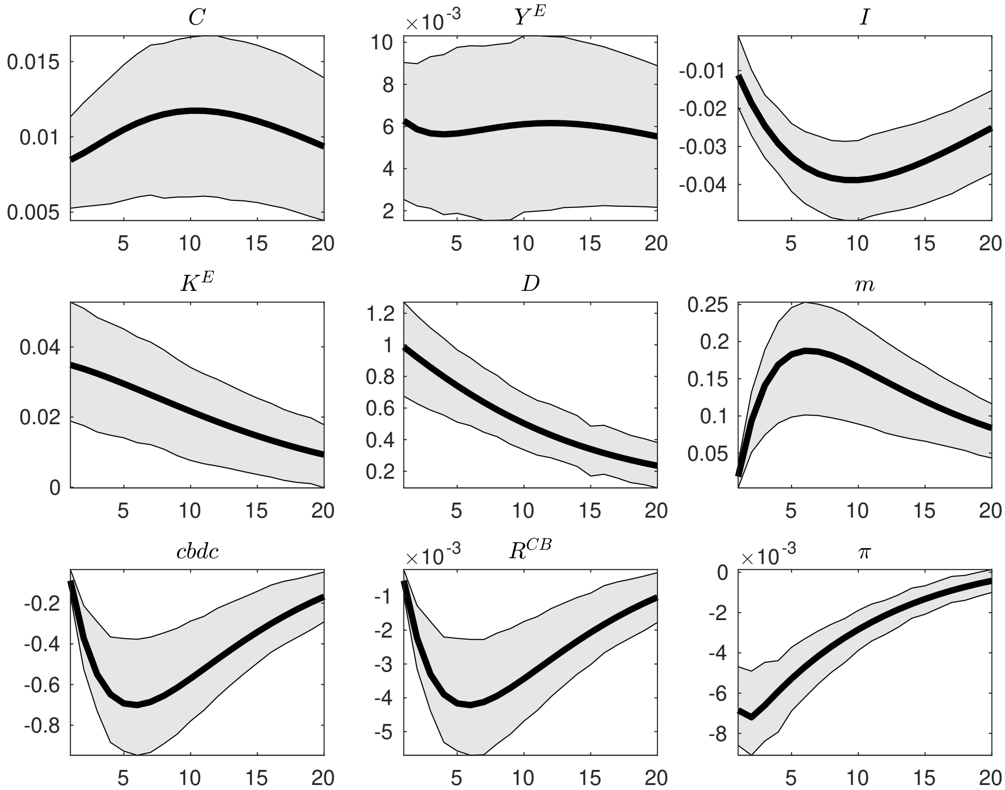


Figure 29: Bayesian IRF: Orthogonalized shock to ε^{mI}.

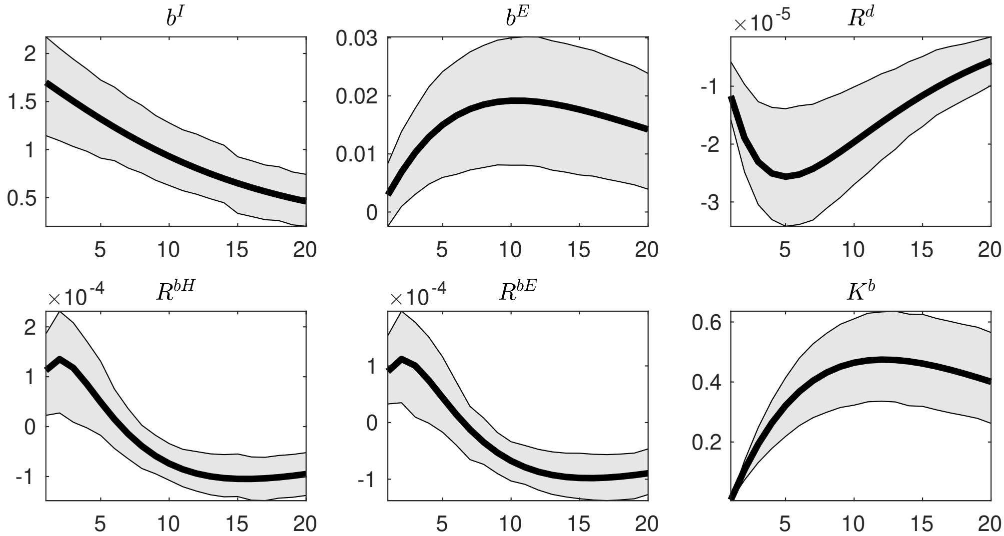


Figure 30: Bayesian IRF: Orthogonalized shock to ε^{mI}.

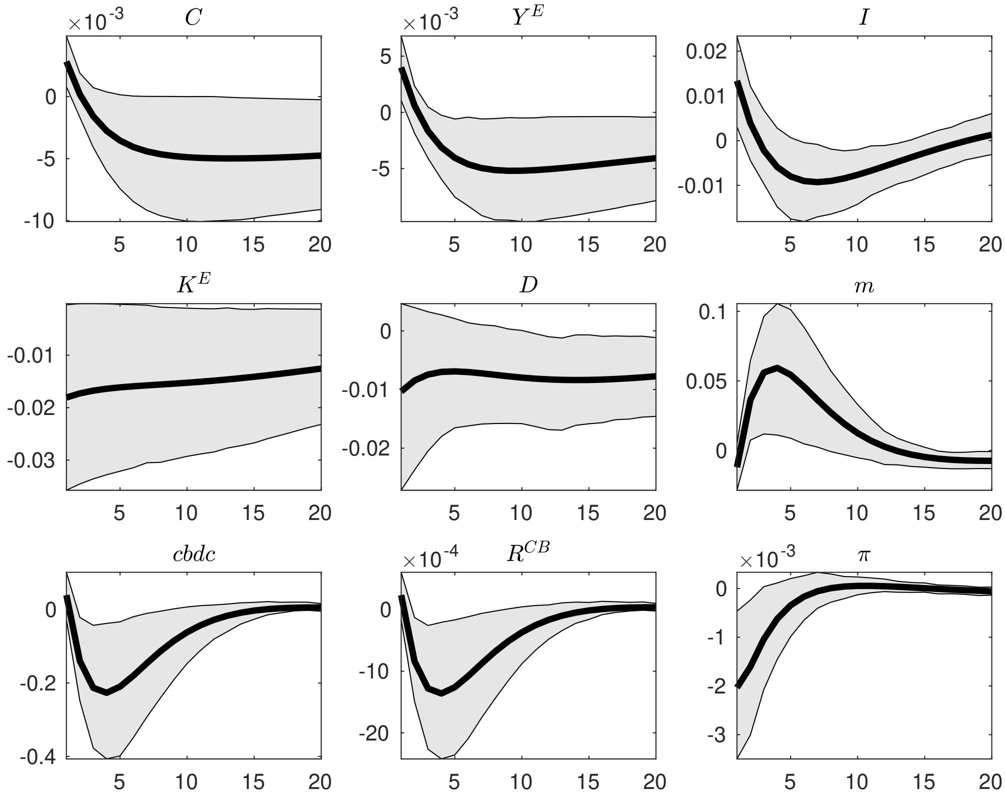


Figure 31: Bayesian IRF: Orthogonalized shock to ε^{mE}.

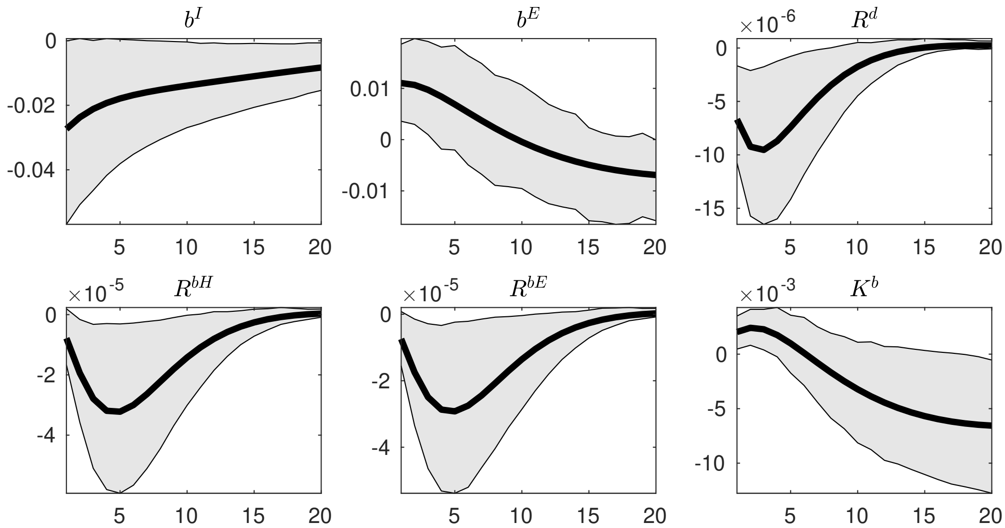


Figure 32: Bayesian IRF: Orthogonalized shock to ε^{mE}.

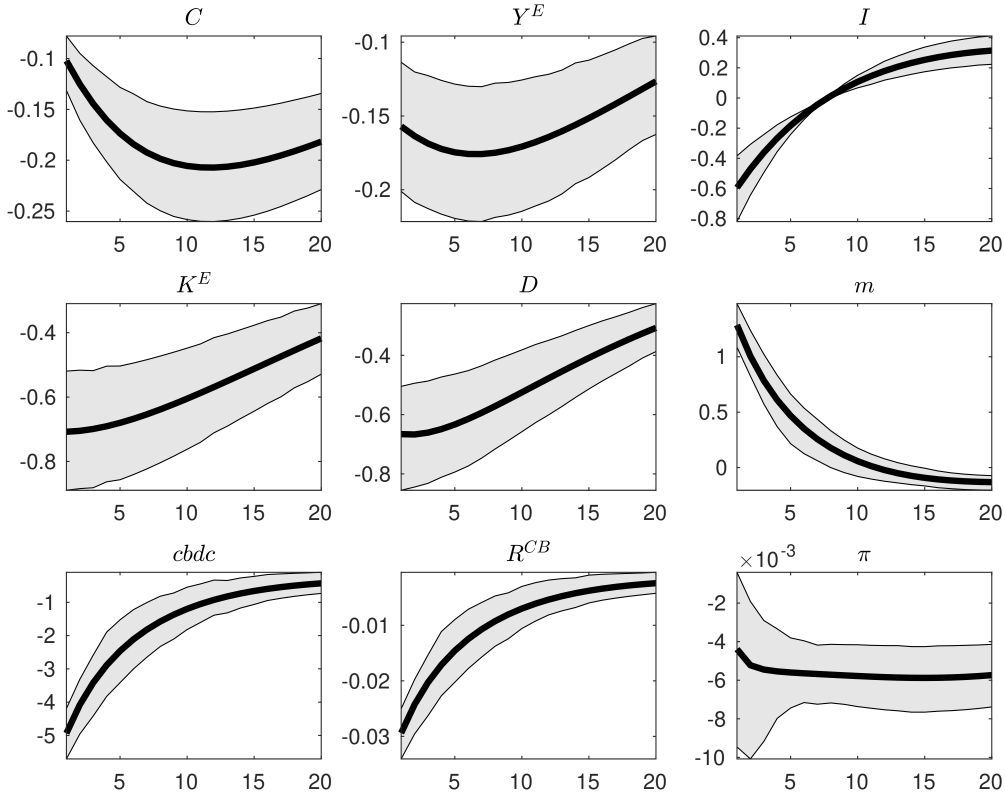


Figure 33: Bayesian IRF: Orthogonalized shock to ε^d.

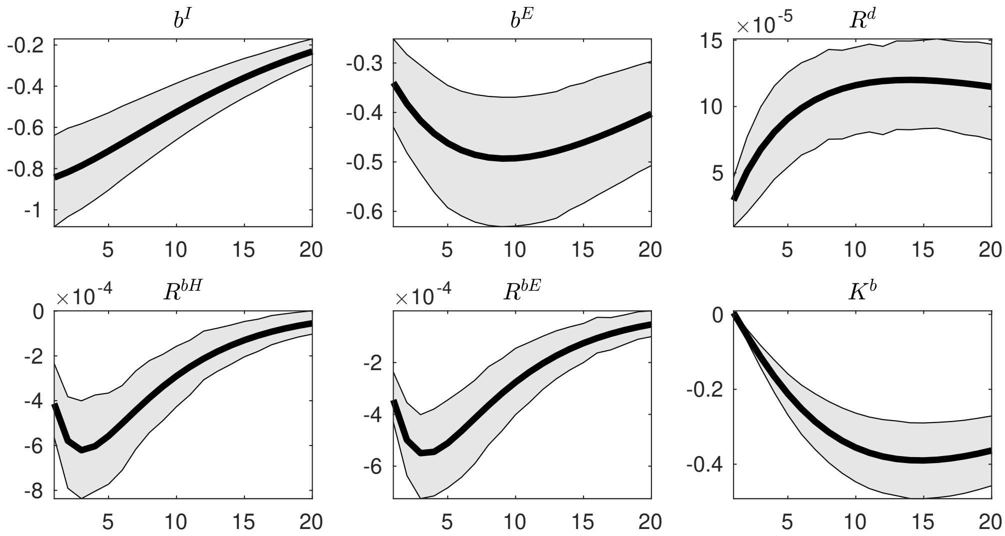


Figure 34: Bayesian IRF: Orthogonalized shock to ε^d.

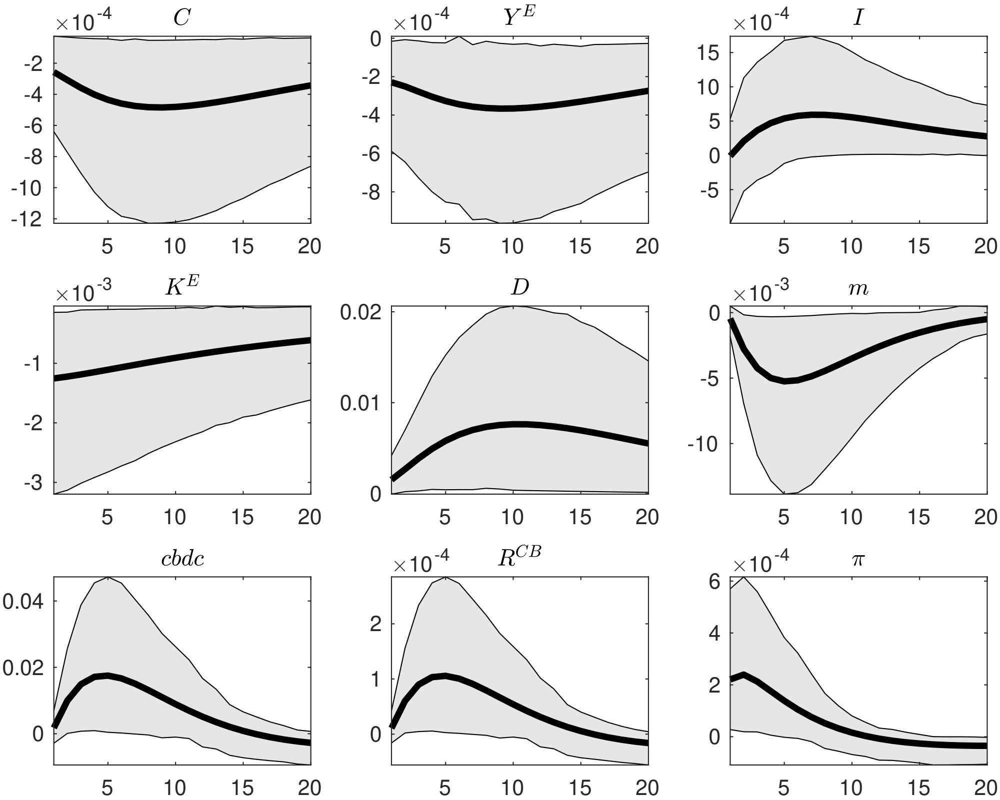


Figure 35: Bayesian IRF: Orthogonalized shock to ε^{bH}.

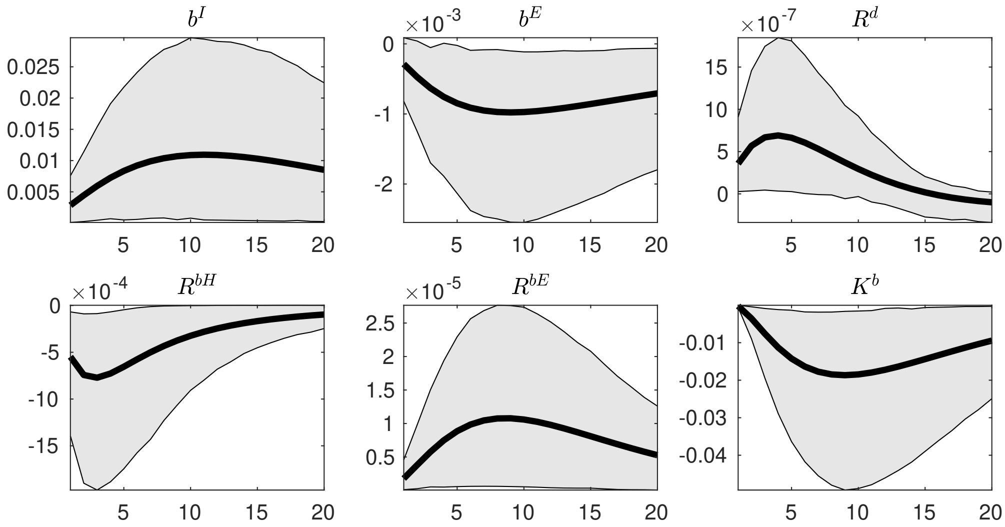


Figure 36: Bayesian IRF: Orthogonalized shock to ε^{bH}.

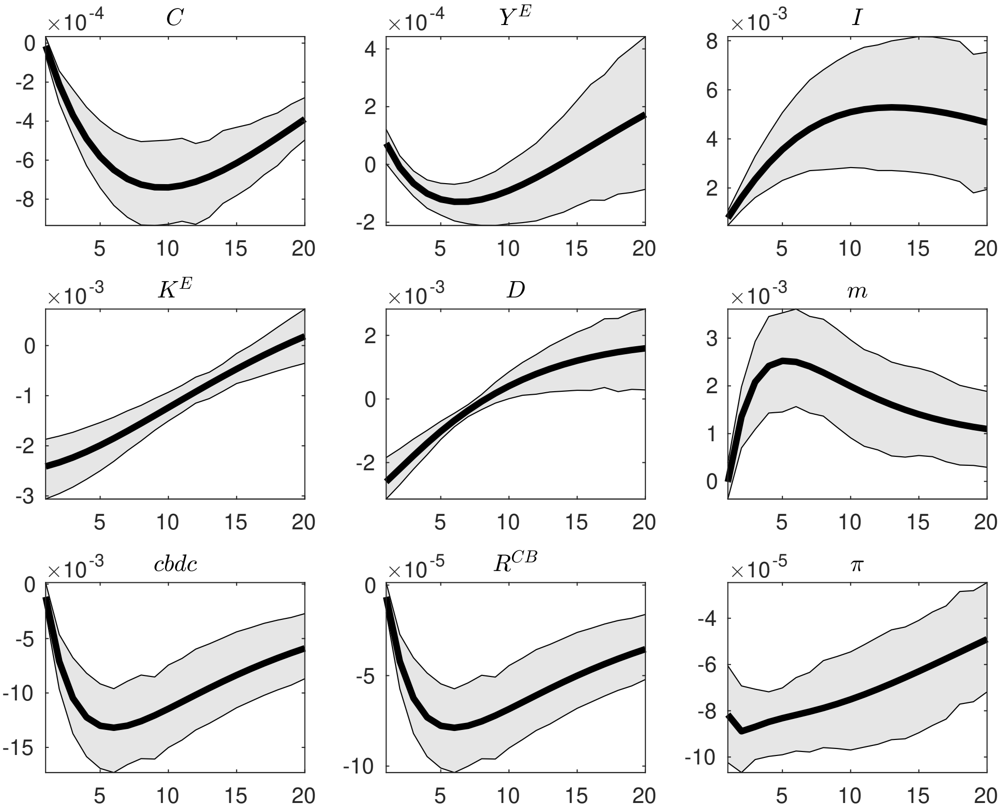


Figure 37: Bayesian IRF: Orthogonalized shock to ε^{bE}.

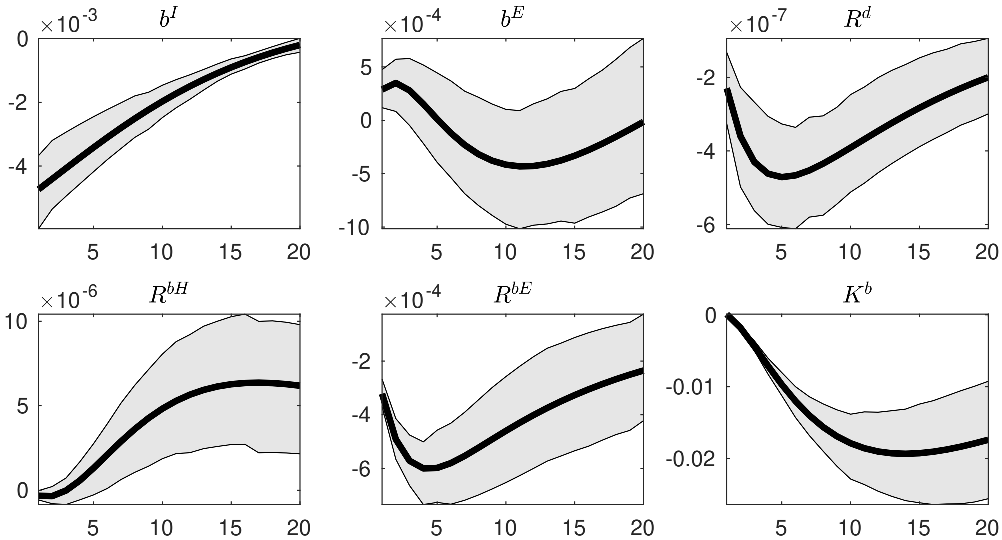


Figure 38: Bayesian IRF: Orthogonalized shock to ε^{bE}.

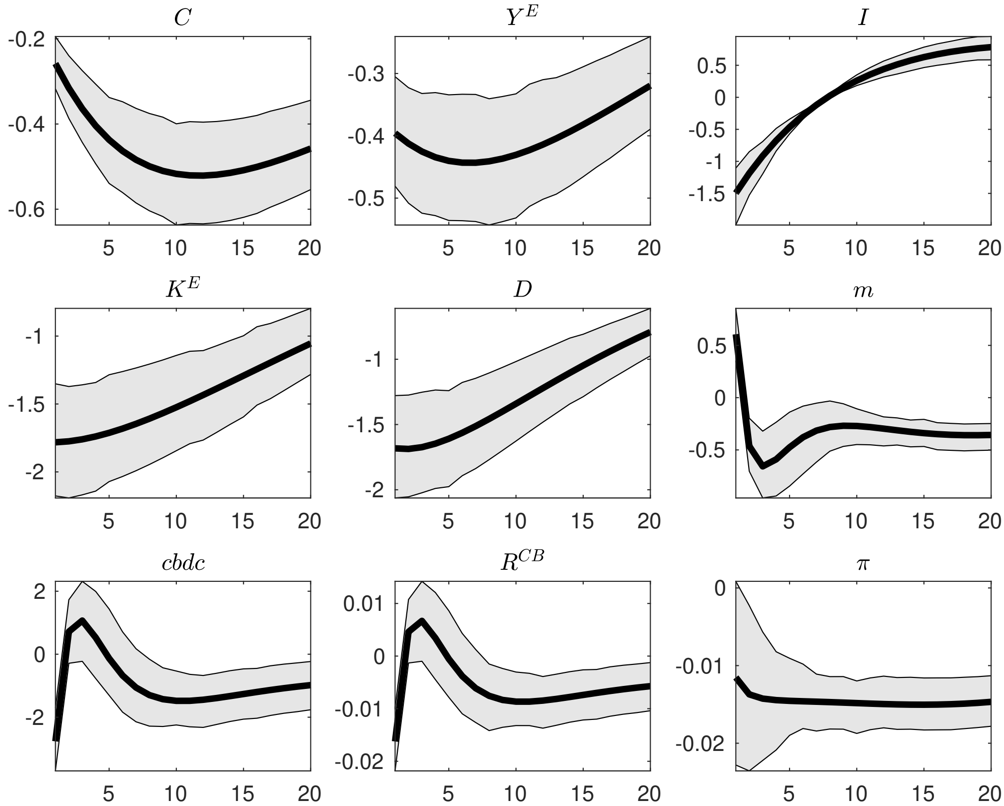


Figure 39: Bayesian IRF: Orthogonalized shock to ε^r.

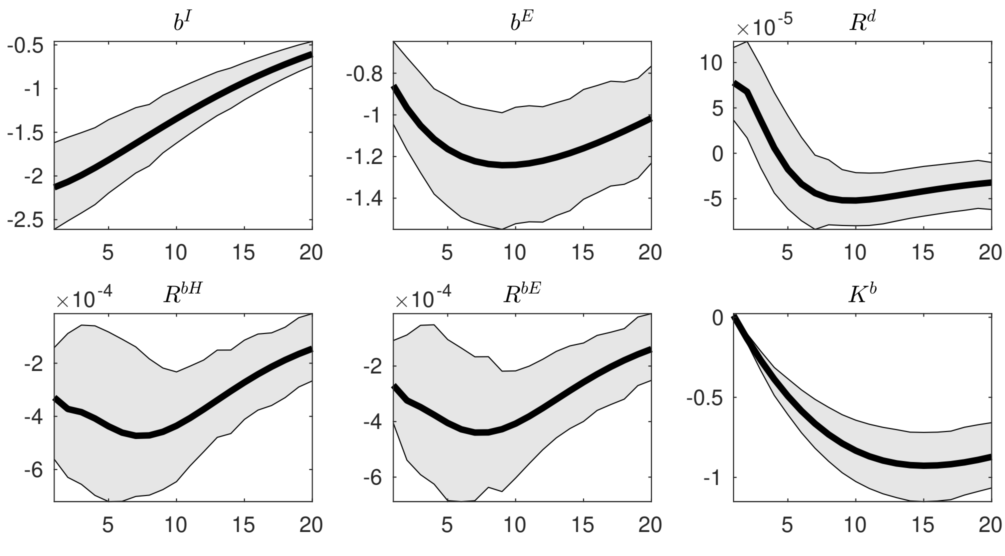


Figure 40: Bayesian IRF: Orthogonalized shock to ε^r.

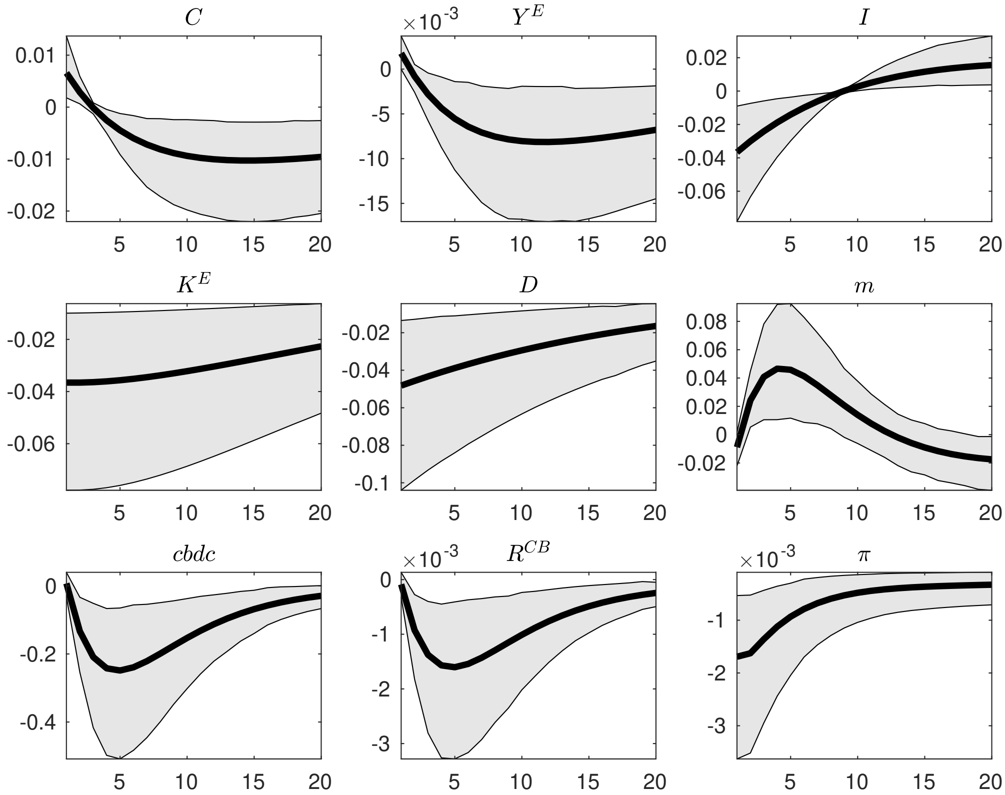


Figure 41: Bayesian IRF: Orthogonalized shock to ε^{γ}.

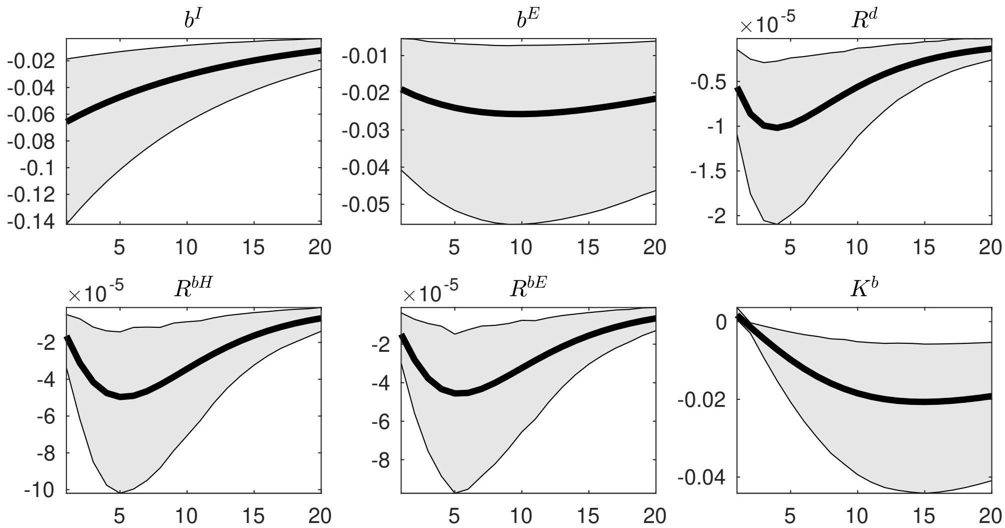


Figure 42: Bayesian IRF: Orthogonalized shock to ε^{γ}.

B Full model equations

Patient household:

$$\frac{(1-a^P)\epsilon_t^z}{c_t^P - a^P c_{t-1}^P} = \lambda_t^P + \Theta_t \tag{26}$$

$$\frac{j\epsilon_t^h}{h_t^P} + \beta^P \lambda_{t+1}^P q_{t+1}^h = \lambda_t^P q_t^h \tag{27}$$

$$\lambda_t^P = \beta^P \, \mathbb{E}_t \frac{\left(\lambda^P{}_{t+1}\right)\,\left(1+r_t^d\right)}{(\pi_{t+1})} \tag{28}$$

$$\lambda_t^P = \psi_{cbdc}\,\left(cbdc_t\right)^{(-\sigma_{cbdc})} + \beta^P \, \mathbb{E}_t \frac{\left(\lambda^P{}_{t+1} + \Theta_{t+1}\gamma_{t+1}\right)\,\left(1+r_t^{cbdc}\right)}{(\pi_{t+1})} \tag{29}$$

$$\lambda_t^P(1+\Psi_m m_t) = \psi_m\,\left(m_t\right)^{(-\sigma_m)} + \beta^P \, \mathbb{E}_t \frac{\left(\lambda^P{}_{t+1} + \Theta_{t+1}\right)}{(\pi_{t+1})} \tag{30}$$

$$\lambda_t^P w_t^P = (l_t^P)^\phi \tag{31}$$

$$c_t^P = \frac{m_{t-1}}{\pi_t} + \gamma_t \frac{1+r_{t-1}^{CBDC}}{\pi_t} cbdc_t \tag{32}$$

$$\begin{aligned} c_t^P + q_t^P \Delta h_t^P + d_t^P + m_t + cbdc_t + f(m_t) = \\ w_t^P l_t^P + (1+r_{t-1}^d)\frac{d_{t-1}^P}{\pi_t} + (1+r_{t-1}^{cbdc})\frac{cbdc_{t-1}}{\pi_t} + \frac{m_{t-1}}{\pi_t} + t_t^P \end{aligned} \tag{33}$$

Impatient household:

$$\frac{(1-a^I)\epsilon_t^z}{c_t^I - a^I c_{t-1}^I} = \lambda_t^I \tag{34}$$

$$\frac{j\epsilon_t^h}{h_t^I} + \beta^I \mathbb{E}_t \lambda_{t+1}^I q_{t+1}^h + s_t^I m_t^I \mathbb{E}_t \pi_{t+1} q_{t+1}^h = \lambda_t^I q_t^h \tag{35}$$

$$\lambda_t^I = \beta^I \, \mathbb{E}_t \frac{\left(\lambda^I{}_{t+1}\right)\,\left(1+r_t^{bH}\right)}{(\pi_{t+1})} + s_t^I(1+r_t^{bH}) \tag{36}$$

$$\lambda_t^I w_t^I = (l_t^I)^\phi \tag{37}$$

$$c_t^I + q_t^h \Delta h_t^I + (1+r_{t-1}^{bH})\frac{b_{t-1}^I(i)}{\pi_t} = w_t^I l_t^I + b_t^I + t_t^I \tag{38}$$

$$(1+r_t^{bH}) b_t^I = h_t^I m_t^I \mathbb{E}_t \pi_{t+1} q_{t+1}^h \tag{39}$$

Capital Producer:

$$K_t = (1-\delta_k)K_{t-1} + \left[1 - \frac{\kappa_i}{2}\left(\frac{i_t \epsilon_t^{qk}}{i_{t-1}} - 1\right)^2\right] i_t \tag{40}$$

$$1 = q_t^k \left[1 - \frac{\kappa_i}{2}\left(\frac{i_t \epsilon_t^{qk}}{i_{t-1}} - 1\right)^2 - \epsilon_t^{qk}\frac{i_t}{i_{t-1}}\kappa_i\left(\frac{i_t \epsilon_t^{qk}}{i_{t-1}} - 1\right)\right] + \kappa_i \beta^E \mathbb{E}_t \epsilon_{t+1}^{qk} \frac{\lambda_{t+1}^E}{\lambda_t^E} q_{t+1}^k \left(\frac{i_{t+1}\epsilon_t^{qk}}{i_t} - 1\right)\left(\frac{i_{t+1}}{i_t}\right)^2 \tag{41}$$

Entrepreneur:

$$\frac{(1-a^E)\epsilon_t^z}{c_t^E - a^E c_{t-1}^E} = \lambda_t^E \tag{42}$$

$$(1-\delta_k)\pi_{t+1} q_{t+1}^k s_t^E m_t^E + \beta^E \lambda_{t+1}^E \left[(1-\delta_k) q_{t+1}^k + r_{t+1}^k u_{t+1} - \psi(u_{t+1})\right] = q_t^k \lambda_t^E \tag{43}$$

$$w_t^P = \frac{\mu(1-\alpha)}{l_t^{E,P}} \frac{y_t^E}{x_t} \tag{44}$$

$$w_t^I = \frac{(1-\mu)(1-\alpha)}{l_t^{E,I}} \frac{y_t^E}{x_t} \tag{45}$$

$$\lambda_t^E = \beta^E \, \mathbb{E}_t \frac{\left(\lambda^I{}_{t+1}\right)\left(1 + r_t^{bE}\right)}{(\pi_{t+1})} + s_t^E (1 + r_t^{bE}) \tag{46}$$

$$r_t^k = \psi_1 + \psi_2(u_t - 1) \tag{47}$$

$$\begin{aligned} c_t^E + w_t^P l_t^{E,P} \ + \ w_t^I l_t^{E,I} \ + \ (1 + r_{t-1}^{bE})\frac{b_{t-1}^E}{\pi_t} \ + \ q_t^k k_t^E \ + \ \psi(u_t) k_{t-1}^E = \\ \frac{y_t^E}{x_t} \ + \ b_t^E \ + \ q_t^k (1-\delta) k_{t-1}^E \end{aligned}$$

$$y_t^E = a_t^E \left[k_{t-1}^E u_t\right]^\alpha \left(l_t^E\right)^{1-\alpha} \tag{48}$$

$$(1 + r_t^{bE}) b_t^E = (1-\delta_k) k_t^E m_t^E \mathbb{E}_t \pi_{t+1} q_{t+1}^k \tag{49}$$

$$r_t^k = \left(\left[l_t^{E,P}\right]^\mu \left[l_t^{E,I}\right]^{1-\mu}\right)^{1-\alpha} \alpha a_t^E u_t^{\alpha-1} (k_{t-1}^E)^{\alpha-1} / x_t \tag{50}$$

Bank:

$$R_t^b = r_t + -\kappa_{Kb}\left(\frac{K_t^b}{B_t} - \nu_i\right)\left(\frac{K_t^b}{B_t}\right)^2 \tag{51}$$

$$\pi_t K_t^b = (1-\delta_{Kb})K_{t-1}^b + j_{t-1}^b \tag{52}$$

$$b_t^E + b_t^H = K_t^b + d_t^b \tag{53}$$

$$1-\epsilon_t^{bH} + \epsilon_t^{bH}\frac{R_t^b}{r_t^{bH}} - \kappa_{bH}\left(\frac{r_t^{bH}}{r_{t-1}^{bH}}-1\right)\frac{r_t^{bH}}{r_{t-1}^{bH}} + \beta^P\mathbb{E}_t\left[\frac{\lambda_{t+1}^P}{\lambda_t^P}\kappa_{bH}\left(\frac{r_{t+1}^{bH}}{r_t^{bH}}-1\right)\left(\frac{r_{t+1}^{bH}}{r_t^{bH}}\right)^2\frac{b_{t+1}^H}{b_t^H}\right] = 0 \tag{54}$$

$$1-\epsilon_t^{bE} + \epsilon_t^{bE}\frac{R_t^b}{r_t^{bE}} - \kappa_{bE}\left(\frac{r_t^{bE}}{r_{t-1}^{bE}}-1\right)\frac{r_t^{bE}}{r_{t-1}^{bE}} + \beta^P\mathbb{E}_t\left[\frac{\lambda_{t+1}^P}{\lambda_t^P}\kappa_{bE}\left(\frac{r_{t+1}^{bE}}{r_t^{bE}}-1\right)\left(\frac{r_{t+1}^{bE}}{r_t^{bE}}\right)^2\frac{b_{t+1}^E}{b_t^E}\right] = 0 \tag{55}$$

$$-1+\epsilon_t^d - \epsilon_t^d\frac{r_t}{r_t^d} - \kappa_d\left(\frac{r_t^d}{r_{t-1}^d}-1\right)\frac{r_t^d}{r_{t-1}^d} + \beta^P\mathbb{E}_t\left[\frac{\lambda_{t+1}^P}{\lambda_t^P}\kappa_d\left(\frac{r_{t+1}^d}{r_t^d}-1\right)\left(\frac{r_{t+1}^d}{r_t^d}\right)^2\frac{d_{t+1}}{d_t}\right] = 0 \tag{56}$$

$$\begin{aligned} j_t^b = {} & r_t^{bH}b_t^I + r_t^{bE}b_t^E - r_t^d d_t^b - d_t^b r_t^b\frac{\kappa_d}{2}\left(\frac{r_t^d}{r_{t-1}^d}-1\right)^2 - b_t^H r_t^{bH}\frac{\kappa_d}{2}\left(\frac{r_t^{bH}}{r_{t-1}^{bH}}-1\right)^2 \\ & - b_t^E r_t^{bE}\frac{\kappa_{bE}}{2}\left(\frac{r_t^{bE}}{r_{t-1}^{bE}}-1\right)^2 - K_t^b\frac{\kappa_{Kb}}{2}\left(\frac{K_t^b}{K_{t-1}^b}-\nu_i\right)^2 \end{aligned} \tag{57}$$

Retailer:

$$J_t^r = Y_t\left[1-\frac{1}{x_t}-\frac{\kappa_p}{2}\Big(\pi_t - \pi_{t-1}^{\iota_p}\pi^{1-\iota_p}\Big)^2\right] \tag{58}$$

$$1-\epsilon_t^y + \frac{\epsilon_t^y}{x_t} - \pi_t\kappa_p\Big(\pi_t - \pi_{t-1}^{\iota_p}\pi^{1-\iota_p}\Big) + \beta^P\kappa_p\mathbb{E}_t\frac{\lambda_{t+1}^P}{\lambda_t^P}\Big(\pi_{t+1} - \pi_t^{\iota_p}\pi^{1-\iota_p}\Big)\pi_{t+1}^2\frac{Y_{t+1}}{Y_t} = 0 \tag{59}$$

Aggregate/ Market Clearance:

$$C_t = \gamma_P c_t^P + \gamma_I c_t^I + \gamma_E c_t^E \tag{60}$$

$$b_t^E + b_t^H = B_t \tag{61}$$

$$\overline{h} = \gamma_P h_t^P + \gamma_I h_t^I \tag{62}$$

$$Y_t = C_t + q_t^k\left(K_t - (1-\delta_k)K_{t-1}\right) + \delta_{Kb}K_{t-1}^b + f(m_t) + \psi(u_t) + Y_t\frac{\kappa_p}{2}\left(\pi_t - \pi_{t-1}^{\iota_p}\pi^{1-\iota_p}\right)^2$$
$$+ d_t^b r_t^b \frac{\kappa_d}{2}\left(\frac{r_t^d}{r_{t-1}^d} - 1\right)^2 + b_t^H r_t^{bH}\frac{\kappa_d}{2}\left(\frac{r_t^{bH}}{r_{t-1}^{bH}} - 1\right)^2 + b_t^E r_t^{bE}\frac{\kappa_{bE}}{2}\left(\frac{r_t^{bE}}{r_{t-1}^{bE}} - 1\right)^2 \tag{63}$$

Central bank :

$$r_t^{cbdc} = r_t - 0.0059 \tag{64}$$

$$1 + r_t = (1+r)^{1-\phi_R}(1+r_{t-1})^{\phi_R}\left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi}\left(\frac{y_t}{y_{t-1}}\right)^{\phi_y}\right]^{1-\phi_R}(1+\epsilon_t^r) \tag{65}$$

Shocks

$$\text{Consumption preference shock:} \quad \epsilon_t^z = \rho_z\epsilon_{t-1}^z + \varepsilon_t^z \tag{66}$$
$$\text{TFP shock:} \quad a_t^E = \rho_z a_{t-1}^E + \varepsilon_t^a \tag{67}$$
$$\text{Housing preference shock:} \quad \epsilon_t^h = \rho_z\epsilon_{t-1}^h + \varepsilon_t^h \tag{68}$$
$$\text{LTV ratio shock households:} \quad m_{t-1}^I = \rho_{m^I} m_{t-1}^I + \varepsilon_t^{m^I} \tag{69}$$
$$\text{LTV ratio shock entrepreneurs:} \quad m_t^E = \rho_{m^E} m_{t-1}^E + \varepsilon_t^{m^E} \tag{70}$$
$$\text{deposit markup shock:} \quad \epsilon_t^d = \rho_d\epsilon_{t-1}^d + \varepsilon_t^d \tag{71}$$
$$\text{loan markup (entrepreneurs):} \quad \epsilon_t^{bE} = \rho_z\epsilon_{t-1}^{bE} + \varepsilon_t^{bE} \tag{72}$$
$$\text{loan markup (households):} \quad \epsilon_t^{bH} = \rho_z\epsilon_{t-1}^{bH} + \varepsilon_t^{bH} \tag{73}$$
$$\text{Monetary policy shock:} \quad \epsilon_t^r = \rho_d\epsilon_{t-1}^r + \varepsilon_t^r \tag{74}$$
$$\text{Liquidity Shock:} \quad \gamma_t = \rho_\gamma\gamma_{t-1} + \epsilon_t^\gamma \tag{75}$$

B.1 Log-linearization

Patient Households:

$$\frac{(1-a^P)\epsilon_t^z}{c_t^P - a^P c_{t-1}^P} = \lambda_t^P + \Theta_t$$

Taking log of both sides we get,

$$\log\epsilon_t^z + \log(1-a^P) - \log(c_t^P - a^P c_{t-1}^P) = \log(\lambda_t^P + \Theta_t)$$

Doing Taylor expansion around the steady states we get,

$$\log\epsilon^z + \hat{\epsilon_t^z} + \log(1-a^P) - \log(c^P - a^P c^P) - \frac{1}{c^P(1-a^P)}\{c^P\hat{c}_t^P - a^P c^P\hat{c}_{t-1}^P\} = \log(\lambda^P + \Theta) + \frac{1}{\lambda^P + \Theta}(\lambda^P\hat{\lambda}_t^P + \Theta\hat{\Theta}_t)$$

Using steady state version of eq(B.1) and canceling appropriate terms, we get,

$$\boxed{\frac{1}{\lambda^P+\Theta}(\lambda^P\hat{\lambda}_t^P+\Theta\hat{\Theta}_t)=\frac{1}{(1-a^P)}(a^P\hat{c}_{t-1}^P-\hat{c}_t^P)+\hat{\epsilon}_t^z} \tag{76}$$

$$\frac{\epsilon_t^h}{h_t^P}+\beta^P\lambda_{t+1}^P q_{t+1}^h=\lambda_t^P q_t^h$$

Taking 2nd term on LHS to RHS and taking log, we get,

$$\log\epsilon_t^h-\log h_t^P=\log\left(\lambda_t^P q_t^h-\beta^P\lambda_{t+1}^P q_{t+1}^h\right)$$

Doing Taylor expansion around the steady states we get,

$$\begin{aligned}\log\left(\lambda^P q^h-\beta^P\lambda^P q^h\right)+\frac{1}{\lambda^P q^h-\beta^P\lambda^P q^h}\{\lambda^P q^h(\hat{\lambda}_t^P+\hat{q}_t^h)-\beta^P\lambda^P q^h(\hat{\lambda}_{t+1}^P+\hat{q}_{t+1}^h)\}\\=\log\epsilon^h-\log h^P+\hat{\epsilon}_t^h-\hat{h}_t^P\end{aligned}$$

Hence finally we get,

$$\boxed{(\hat{\lambda}_t^P+\hat{q}_t^h)-\beta^P(\hat{\lambda}_{t+1}^P+\hat{q}_{t+1}^h)=\hat{\epsilon}_t^h-\hat{h}_t^P} \tag{77}$$

For cash we have,

$$\lambda_t^P(1+\Psi_m m_t)=\psi_m\ (m_t)^{(-\sigma_m)}+\beta^P\,\mathbb{E}_t\frac{\left(\lambda^P{}_{t+1}+\Theta_{t+1}\right)}{(\pi_{t+1})}$$

Taking log on both sides we get,

$$\log\lambda_t^P+\log(1+\Psi_m m_t)=\log\Big(\psi_m\ (m_t)^{(-\sigma_m)}+\beta^P\,\mathbb{E}_t\frac{\left(\lambda^P{}_{t+1}+\Theta_{t+1}\right)}{(\pi_{t+1})}\Big)$$

Therefore, doing Taylor expansion around the steady state we get,

$$\boxed{\hat{\lambda}_t^P+\Big(\frac{m\Psi_m}{1+m\Psi_m}+\frac{\sigma_m m^{-\sigma_m}\psi_m}{\lambda^P}\Big)\hat{m}_t=\beta^P(\hat{\lambda}_{t+1}^P-\hat{\pi}_{t+1})+\beta^P\frac{\Theta}{\lambda^P}(\hat{\Theta}_{t+1}-\hat{\pi}_{t+1})} \tag{78}$$

For CBDC, we have the following FOC,

$$\lambda_t^P=\psi_{cbdc}\ (cbdc_t)^{(-\sigma_{cbdc})}+\beta^P\,\mathbb{E}_t\frac{\left(\lambda^P{}_{t+1}+\Theta_{t+1}\gamma_{t+1}\right)\ \left(1+r_t^{cbdc}\right)}{(\pi_{t+1})}$$

Taking log of the both sides we get,

$$\log\lambda_t^P=\log\Big(\psi_{cbdc}\ (cbdc_t)^{(-\sigma_{cbdc})}+\beta^P\,\mathbb{E}_t\frac{\left(\lambda^P{}_{t+1}+\Theta_{t+1}\gamma_{t+1}\right)\ \left(1+r_t^{cbdc}\right)}{(\pi_{t+1})}\Big)$$

Taking Taylor expansion around the steady state we get,

$$\hat{\lambda}_t^P=B1\hat{cbdc}_t+\beta^P R^{CBDC}(\hat{R}_t^{CBDC}+\hat{\lambda}_{t+1}^P-\hat{\pi}_{t+1})+B2(\hat{R}_t^{CBDC}+\hat{\Theta}_{t+1}+\hat{\gamma}_{t+1}-\hat{\pi}_{t+1}) \tag{79}$$

where $B1 = -\frac{\sigma_{CBDC}\psi_{CBDC}cbdc^{-\sigma_{CBDC}}}{\lambda^P}$ and $B2 = \beta^P \frac{\Theta}{\gamma}\overline{\gamma}R^{CBDC}$
For deposits, we have the following FOC

$$\lambda_t^P = \beta^P \, \mathbb{E}_t \frac{\left(\lambda^P{}_{t+1}\right)\, \left(1+r_t^d\right)}{\left(\pi_{t+1}\right)}$$

Taking log of both sides we get,

$$\log\lambda_t^P = \log\beta^P + \log\lambda_{t+1}^P + \log(1+r_t^d) - \log\pi_{t+1}$$

Doing Taylor expansion around the steady states we get,

$$\boxed{\hat{\lambda}_t^P - \hat{\lambda}_{t+1}^P = \hat{R}_t^d - \hat{\pi}_{t+1}} \tag{80}$$

where $R_t^d = 1 + r_t^d$

Labor supply of the patient household becomes,

$$\boxed{\hat{\lambda}_t^P + \hat{w}_t^P = \phi \hat{l}_t^P} \tag{81}$$

Impatient Households:

$$\boxed{(1-a^I)\hat{\lambda}_t^I = a^I \hat{c}_{t-1}^I - \hat{c}_t^I} \tag{82}$$

$$\frac{\epsilon_t^h}{h_t^I} + \beta^I \mathbb{E}_t(\lambda_{t+1}^I q_{t+1}^h + s_t^I m_t^I \pi_{t+1} q_{t+1}^h) = \lambda_t^I q_t^h$$

Taking log doing some side changes we obtain,

$$\log\epsilon_t^h - \log h_t^I = \log(\lambda_t^I q_t^h - \beta^I \mathbb{E}_t(\lambda_{t+1}^I q_{t+1}^h + s_t^I m_t^I \pi_{t+1} q_{t+1}^h))$$

Doing Taylor expansion we get,

$$\begin{aligned}
\frac{1}{\lambda^I q^h - \beta^I \lambda^I q^h - \beta^I s^I m^I \pi q^h} \Big\{ & \lambda^I q^h (\hat{\lambda}_t^I + \hat{q}_t^h) \\
& - \beta^I \lambda^I q^h (\hat{\lambda}_{t+1}^I + \hat{q}_{t+1}^h) - \beta^I s^I m^I q^h \pi (\hat{s}_t^I + \hat{m}_t^I + \hat{q}_{t+1}^h + \hat{\pi}_{t+1}) \Big\} \\
& = \hat{\epsilon}_t^h - \hat{h}_t^I
\end{aligned}$$

Finally,

$$\boxed{\lambda^I q^h \{(\hat{\lambda}_t^I + \hat{q}_t^h) - \beta^I(\hat{\lambda}_{t+1}^I + \hat{q}_{t+1}^h)\} - \beta^I s^I m^I q^h \pi (\hat{s}_t^I + \hat{m}_t^I + \hat{q}_{t+1}^h + \hat{\pi}_{t+1}) = \frac{\epsilon^h}{h^I}(\hat{\epsilon}_t^h - \hat{h}_t^I)} \tag{83}$$

$$\lambda_t^I = \beta^I \, \mathbb{E}_t \frac{\left(\lambda^I{}_{t+1}\right)\, \left(1+r_t^{bH}\right)}{\left(\pi_{t+1}\right)} + s_t^I(1+r_t^{bH}) \tag{84}$$

Taking log doing some side changes we obtain,

$$\log\left(\lambda_t^I - \beta^I\,\mathbb{E}_t\frac{(\lambda^I{}_{t+1})\ (1+r_t^{bH})}{(\pi_{t+1})}\right) = \log\Big(s_t^I(1+r_t^{bH})\Big)$$

Doing Taylor expansion we get,

$$\boxed{\lambda^I\hat{\lambda}_t^I - \frac{\beta^I\lambda^I R^{bH}}{\pi}(\hat{\lambda}_{t+1}^I + \hat{R}_t^{bH} - \hat{\pi}_{t+1}) = s^I R^{bH}(\hat{s}_t^I + \hat{R}_t^{bH})} \tag{85}$$

$$(1+r_t^{bH})b_t^I = h_t^I m_t^I \mathbb{E}_t\pi_{t+1}q_{t+1}^h \tag{86}$$

Taking log on both sides we get,

$$\log(1+r_t^{bH}) + \log b_t^I = \log h_t^I + \log m_t^I + \log\pi_{t+1} + \log q_{t+1}^h$$

Doing Taylor expansion we get,

$$\boxed{\hat{R}_t^{bH} + \hat{b}_t^I = \hat{h}_t^I + \hat{m}_t^I + \hat{\pi}_{t+1} + \hat{q}_{t+1}^h} \tag{87}$$

Labor supply of the impatient household becomes,

$$\boxed{\hat{\lambda}_t^I + \hat{w}_t^I = \phi\hat{l}_t^I} \tag{88}$$

Entrepreneurs:

$$\boxed{(1-a^E)\hat{\lambda}_t^E = a^E\hat{c}_{t-1}^E - \hat{c}_t^E} \tag{89}$$

$$\begin{aligned}\lambda^E q^k(\hat{\lambda}_t^E + \hat{q}_t^k) &- \beta^E\lambda^E\Big[(1-\delta^k)q^k + r^k\Big]\hat{\lambda}_{t+1}^E \\ &- \beta^E\lambda^E\Big[(1-\delta^k)q^k\Big]\hat{q}_{t+1}^k - \beta^E\lambda^E r^k(\hat{r}_{t+1}^k + \hat{u}_{t+1}) \\ &= s^E m^E q^k \pi(1-\delta^k)(\hat{s}_t^E + \hat{m}_t^E + \hat{q}_{t+1}^k + \hat{\pi}_{t+1})\end{aligned} \tag{90}$$

$$\boxed{\lambda^E\hat{\lambda}_t^E - \frac{\beta^E\lambda^E R^{bE}}{\pi}\Big(\hat{\lambda}_{t+1}^E + \hat{R}_t^{bE} - \hat{\pi}_{t+1}\Big) = s^E R^{bE}\Big(\hat{s}_t^E + \hat{R}_t^{bE}\Big)} \tag{91}$$

$$\boxed{\hat{R}_t^{bE} + \hat{b}_t^E = \hat{h}_t^E + \hat{m}_t^E + \hat{\pi}_{t+1} + \hat{q}_{t+1}^k} \tag{92}$$

$$\boxed{\hat{y}_t^E = \hat{a}_t^E + \alpha\Big(\hat{u}_t + \hat{k}_{t-1}^E\Big) + (1-\alpha)\Big\{\mu\hat{l}_t^P + (1-\mu)\hat{l}_t^I\Big\}} \tag{93}$$

$$\boxed{\hat{w}_t^P = \hat{y}_t^E - \hat{l}_t^P - \hat{x}_t} \tag{94}$$

$$\boxed{\hat{w}_t^E = \hat{y}_t^E - \hat{l}_t^I - \hat{x}_t} \tag{95}$$

$$\boxed{\hat{r}_t^k = \hat{a}_t^E + (\alpha - 1)\Big(\hat{u}_t + \hat{k}_{t-1}^E\Big) + (1-\alpha)\Big\{\mu \hat{l}_t^P + (1-\mu)\hat{l}_t^I\Big\} - \hat{x}_t} \tag{96}$$

$$\boxed{\hat{r}_t^k = \frac{\psi_2}{\psi_1}\hat{u}_t} \tag{97}$$

Capital producers

$$K_t = (1-\delta^k)K_{t-1} + \left[1 - \frac{\kappa_i}{2}\left(\frac{i_t \epsilon_t^{qk}}{i_{t-1}} - 1\right)^2\right] i_t$$

Taking log after bringing 1st term on RHS to LHS, we get

$$\log\Big(K_t - (1-\delta^k)K_{t-1}\Big) = \log\left[1 - \frac{\kappa_i}{2}\left(\frac{i_t \epsilon_t^{qk}}{i_{t-1}} - 1\right)^2\right] + \log\, i_t$$

Doing Taylor expansion we get,

$$\log\Big(K - (1-\delta^k)K\Big) + \frac{1}{K-(1-\delta^k)K}\Big\{K\hat{K}_t - (1-\delta^k)K\hat{K}_{t-1}\Big\} = \log\, i + \hat{i}_t$$

Finally,

$$\boxed{\hat{K}_t - (1-\delta^k)\hat{K}_{t-1} = \delta^k\ \hat{i}_t} \tag{98}$$

$$1 = q_t^k\left[1 - \frac{\kappa_i}{2}\left(\frac{i_t\epsilon_t^{qk}}{i_{t-1}} - 1\right)^2 - \epsilon_t^{qk}\frac{i_t}{i_{t-1}}\kappa_i\left(\frac{i_t\epsilon_t^{qk}}{i_{t-1}} - 1\right)\right] + \kappa_i\beta^E\mathbb{E}_t\epsilon_{t+1}^{qk}\frac{\lambda_{t+1}^E}{\lambda_t^E}q_{t+1}^k\left(\frac{i_{t+1}\epsilon_{t+1}^{qk}}{i_t} - 1\right)\left(\frac{i_{t+1}}{i_t}\right)^2$$

Taking log of both sides we get,

$$\begin{aligned} 0 = \log\Bigg(& q_t^k\left[1 - \frac{\kappa_i}{2}\left(\frac{i_t\epsilon_t^{qk}}{i_{t-1}} - 1\right)^2 - \epsilon_t^{qk}\frac{i_t}{i_{t-1}}\kappa_i\left(\frac{i_t\epsilon_t^{qk}}{i_{t-1}} - 1\right)\right] \\ & + \kappa_i\beta^E\mathbb{E}_t\epsilon_{t+1}^{qk}\frac{\lambda_{t+1}^E}{\lambda_t^E}q_{t+1}^k\left(\frac{i_{t+1}\epsilon_{t+1}^{qk}}{i_t} - 1\right)\left(\frac{i_{t+1}}{i_t}\right)^2\Bigg) \end{aligned}$$

Taylor expansion gives us,

$$0 = \hat{q}_t^k - \kappa_i(\hat{i}_t + \hat{\epsilon}_t^{qk} - \hat{i}_{t-1}) + \kappa_i\beta^E(\hat{i}_{t+1} + \hat{\epsilon}_{t+1}^{qk} - \hat{i}_t)$$

or more compact form,

$$\boxed{\hat{q}_t^k = \kappa_i(\hat{i}_t + \hat{\epsilon}_t^{qk} - \hat{i}_{t-1}) - \kappa_i\beta^E(\hat{i}_{t+1} + \hat{\epsilon}_{t+1}^{qk} - \hat{i}_t)} \tag{99}$$

Retailer:

$$\boxed{\hat{j}_t^R = \hat{y}_t + \frac{1}{x-1}\hat{x}_t} \tag{100}$$

$$1-\epsilon_t^y+\frac{\epsilon_t^y}{x_t}-\pi_t\kappa_p\Big(\pi_t-\pi_{t-1}^{\iota_p}\pi^{1-\iota_p}\Big)+\beta^P\kappa_p\mathbb{E}_t\frac{\lambda_{t+1}^P}{\lambda_t^P}\Big(\pi_{t+1}-\pi_t^{\iota_p}\pi^{1-\iota_p}\Big)\pi_{t+1}\frac{Y_{t+1}}{Y_t}=0$$

or,

$$1=\epsilon_t^y-\frac{\epsilon_t^y}{x_t}+\pi_t\kappa_p\Big(\pi_t-\pi_{t-1}^{\iota_p}\pi^{1-\iota_p}\Big)-\beta^P\kappa_p\mathbb{E}_t\frac{\lambda_{t+1}^P}{\lambda_t^P}\Big(\pi_{t+1}-\pi_t^{\iota_p}\pi^{1-\iota_p}\Big)\pi_{t+1}\frac{Y_{t+1}}{Y_t}$$

In steady state, we get,

$$1=\epsilon^y-\frac{\epsilon^y}{x} \qquad \Rightarrow x=\frac{\epsilon^y}{\epsilon^y-1}$$

Taking log of the actual equation we get,

$$0=\log\left(\epsilon^y-\frac{\epsilon^y}{x}\right)+\left\{\epsilon^y\hat{\epsilon}_t^y-\frac{\epsilon^y}{x}\hat{\epsilon}_t^y+\frac{\epsilon^y}{x}\hat{x}_t-\pi^2\kappa_p\Big(\hat{\pi}_t-\iota_p\hat{\pi}_{t-1}\Big)+\beta^P\pi^2\kappa_p\Big(\hat{\pi}_{t+1}-\iota_p\hat{\pi}_t\Big)\right\}$$

or more compact form,

$$\boxed{\hat{\epsilon}_t^y+(\epsilon^y-1)\hat{x}_t=\pi^2\kappa_p\Big(\hat{\pi}_t-\iota_p\hat{\pi}_{t-1}\Big)-\beta^P\pi^2\kappa_p\Big(\hat{\pi}_{t+1}-\iota_p\hat{\pi}_t\Big)} \tag{101}$$

Bank:

$$R_t^b=r_t+-\kappa_{Kb}\left(\frac{K_t^b}{B_t}-\nu_i\right)\left(\frac{K_t^b}{B_t}\right)^2$$

Taking log we get,

$$\log R_t^b=\log\left(r_t+-\kappa_{Kb}\left(\frac{K_t^b}{B_t}-\nu_i\right)\left(\frac{K_t^b}{B_t}\right)^2\right)$$

Considering that in steady state, $K^b/B=\nu_i$ (which implies $R^b=r^{CB}$ i.e. steady state policy rate is same as wholesale loan rate) we get,

$$\hat{R}_t^b=\frac{1}{r^{CB}}\left\{r^{CB}\hat{r}_t^{CB}-\kappa_{Kb}\left(\frac{K^b}{B}\right)^2(\hat{K}_t^b-\hat{B}_t)\right\}$$

Finally,

$$\boxed{\hat{R}_t^b=\hat{r}_t^{CB}-\frac{\kappa_{Kb}}{r^{CB}}\upsilon_i^2(\hat{K_t^b}-\hat{B}_t)}$$

$$\pi_tK_t^b=(1-\delta^b)K_{t-1}^b+j_{t-1}^b \tag{102}$$

Taking Taylor expansion, we get

$$\hat{\pi}_t+\hat{K}_t^b=\frac{1}{K^b}\left\{(1-\delta^b)K^b\hat{K}_{t-1}^b+j^b\hat{j}_{t-1}^b\right\}$$

Now from steady state, we obtain $j^b = \delta^b K^b$. Substituting this in the above,

$$\boxed{\hat{\pi}_t + \hat{K}_t^b = (1-\delta^b)\hat{K}_{t-1}^b + \delta^b \hat{j}_{t-1}^b} \tag{103}$$

$$B_t = K_t^b + D_t^b$$

In steady state, we get $B = K^b + d^b$. Dividing this by B, we have $1 = \frac{K^b}{B} + \frac{D}{B}$. Thus we get $\frac{D}{B} = 1 - \nu_i$. Hence the Taylor expansion of the balance sheet constrain gives us,

$$\boxed{\hat{B}_t = \nu_i \hat{K}_t^b + (1-\nu_i)\hat{D}_t} \tag{104}$$

Similarly for the total loan, we have

$$\boxed{\hat{B}_t = \frac{b^I}{B}\hat{b}_t^I + \frac{b^E}{B}\hat{b}_t^E} \tag{105}$$

$$1 = \epsilon_t^{bH} - \epsilon_t^{bH}\frac{R_t^b}{r_t^{bH}} + \kappa_{bH}\left(\frac{r_t^{bH}}{r_{t-1}^{bH}} - 1\right)\frac{r_t^{bH}}{r_{t-1}^{bH}} - \beta^P \mathbb{E}_t\left[\frac{\lambda_{t+1}^P}{\lambda_t^P}\kappa_{bH}\left(\frac{r_{t+1}^{bH}}{r_t^{bH}} - 1\right)\left(\frac{r_{t+1}^{bH}}{r_t^{bH}}\right)^2 \frac{b_{t+1}^I}{b_t^I}\right]$$

Taking log and doing Taylor expansion we get,

$$\epsilon^{bH}\hat{\epsilon}_t^{bH} - \epsilon^{bH}\frac{R^b}{r^{bH}}(\hat{\epsilon}_t^{bH} + \hat{R}_t^b - \hat{r}_t^{bH}) + \kappa_{bH}(\hat{r}_t^{bH} - \hat{r}_{t-1}^{bH}) - \beta^p\kappa_{bH}(\hat{r}_{t+1}^{bH} - \hat{r}_t^{bH}) = 0$$

or,

$$\hat{\epsilon}_t^{bH} - (\epsilon^{bH}-1)\hat{R}_t^b + (\epsilon^{bH}-1)\hat{r}_t^{bH} + \kappa_{bH}(1+\beta^p)\hat{r}_t^{bH} - \kappa_{bH}\hat{r}_{t-1}^{bH} - \beta^p\kappa_{bH}\hat{r}_{t+1}^{bH} = 0$$

Finally,

$$\begin{aligned}\hat{r}_t^{bH} = &\left(\frac{\epsilon^{bH}-1}{\epsilon^{bH}-1+\kappa_{bH}(1+\beta^p)}\right)\hat{R}_t^b + \left(\frac{\beta^p\kappa_{bH}}{\epsilon^{bH}-1+\kappa_{bH}(1+\beta^p)}\right)\hat{r}_{t+1}^{bH} \\ &+ \left(\frac{\kappa_{bH}}{\epsilon^{bH}-1+\kappa_{bH}(1+\beta^p)}\right)\hat{r}_{t-1}^{bH} - \left(\frac{1}{\epsilon^{bH}-1+\kappa_{bH}(1+\beta^p)}\right)\hat{\epsilon}_t^{bH}\end{aligned} \tag{106}$$

$$\begin{aligned}\hat{r}_t^{bE} = &\left(\frac{\epsilon^{bE}-1}{\epsilon^{bE}-1+\kappa_{bE}(1+\beta^p)}\right)\hat{R}_t^b + \left(\frac{\beta^p\kappa_{bE}}{\epsilon^{bE}-1+\kappa_{bE}(1+\beta^p)}\right)\hat{r}_{t+1}^{bE} \\ &+ \left(\frac{\kappa_{bE}}{\epsilon^{bE}-1+\kappa_{bE}(1+\beta^p)}\right)\hat{r}_{t-1}^{bE} - \left(\frac{1}{\epsilon^{bE}-1+\kappa_{bE}(1+\beta^p)}\right)\hat{\epsilon}_t^{bE}\end{aligned} \tag{107}$$

$$\begin{aligned}\hat{r}_t^{d} = &\left(\frac{\epsilon^{d}-1}{\epsilon^{d}-1-\kappa_{d}(1+\beta^p)}\right)\hat{r}_t^{CB} + \left(\frac{\beta^p\kappa_{d}}{\epsilon^{d}-1-\kappa_{d}(1+\beta^p)}\right)\hat{r}_{t+1}^{d} \\ &+ \left(\frac{\kappa_{d}}{\epsilon^{d}-1-\kappa_{d}(1+\beta^p)}\right)\hat{r}_{t-1}^{d} - \left(\frac{1}{\epsilon^{d}-1-\kappa_{d}(1+\beta^p)}\right)\hat{\epsilon}_t^{d}\end{aligned} \tag{108}$$

$$\boxed{j^b\hat{j}_t^b = r^{bH}b^I(\hat{r}_t^{bH} + \hat{b}_t^I) + r^{bE}b^E(\hat{r}_t^{bE} + \hat{b}_t^E) - r^d d^b(\hat{r}_t^d + \hat{d}_t^b)} \tag{109}$$

Central Bank:

$$\boxed{\hat{R}_t^{CB} = \phi_R \hat{R}_{t-1}^{CB} + \phi_\pi(1-\phi_R)\hat{\pi}_t + \phi_y(1-\phi_R)(\hat{Y}_t - \hat{Y}_{t-1}) + \hat{\epsilon}_t^r} \tag{110}$$

Aggregate:

$$\boxed{\hat{Y}_t = \left(1 - \delta^k \frac{K}{Y}\right)\hat{C}_t + \frac{K}{Y}\hat{K}_t - (1-\delta^k)\frac{K}{Y}\hat{K}_{t-1}} \tag{111}$$

B.1.1 Calibrating linearized equations

Patient Households:

$$\boxed{(1-a^P)\hat{\lambda}_t^P = a^P \hat{c}_{t-1}^P - \hat{c}_t^P} \tag{112}$$

$$\boxed{(\hat{\lambda}_t^P + \hat{q}_t^h) - \beta^P(\hat{\lambda}_{t+1}^P + \hat{q}_{t+1}^h) = \hat{\epsilon}_t^h - \hat{h}_t^P} \tag{113}$$

$$\boxed{\hat{\lambda}_t^P - \hat{\lambda}_{t+1}^P = \hat{R}_t^d - \hat{\pi}_{t+1}} \tag{114}$$

Impatient Households:

$$\boxed{(1-a^I)\hat{\lambda}_t^I = a^I \hat{c}_{t-1}^I - \hat{c}_t^I} \tag{115}$$

$$\boxed{\lambda^I q^h\{(\hat{\lambda}_t^I + \hat{q}_t^h) - \beta^I(\hat{\lambda}_{t+1}^I + \hat{q}_{t+1}^h)\} - \beta^I s^I m^I q^h \pi(\hat{s}_t^I + \hat{m}_t^I + \hat{q}_{t+1}^h + \hat{\pi}_{t+1}) = \frac{\epsilon^h}{h^I}(\hat{\epsilon}_t^h - \hat{h}_t^I)} \tag{116}$$

Dividing above equation by $\lambda^I q^h$, we get

$$\boxed{\{(\hat{\lambda}_t^I + \hat{q}_t^h) - \beta^I(\hat{\lambda}_{t+1}^I + \hat{q}_{t+1}^h)\} - \beta^I \frac{s^I}{\lambda^I} m^I(\hat{s}_t^I + \hat{m}_t^I + \hat{q}_{t+1}^h + \hat{\pi}_{t+1}) = \frac{1}{h^I \lambda^I q^h}(\hat{\epsilon}_t^h - \hat{h}_t^I)}$$

From FOC of loans of Impatient HHs in SS, we get

$$\frac{s^I}{\lambda^I} = \frac{1}{R^{bH}}\left(1 - \beta^I R^{bH}\right)$$

Substituiting above in the FOC of housing services of Impatient HHs,

$$\frac{1}{h^I \lambda^I q^h} = (1-\beta^I) - \frac{s^I}{\lambda^I} m^I$$

Hence the equation finally becomes,

$$\boxed{\{(\hat{\lambda}_t^I + \hat{q}_t^h) - \beta^I(\hat{\lambda}_{t+1}^I + \hat{q}_{t+1}^h)\} - \beta^I \frac{s^I}{\lambda^I} m^I(\hat{s}_t^I + \hat{m}_t^I + \hat{q}_{t+1}^h + \hat{\pi}_{t+1}) = \left[(1-\beta^I) - \frac{s^I}{\lambda^I} m^I\right](\hat{\epsilon}_t^h - \hat{h}_t^I)}$$

$$\begin{aligned}
\lambda^E q^k(\hat{\lambda}_t^E + \hat{q}_t^k) &- \beta^E \lambda^E \Big[(1-\delta^k)q^k + r^k\Big]\hat{\lambda}_{t+1}^E \\
&- \beta^E \lambda^E \Big[(1-\delta^k)q^k\Big]\hat{q}_{t+1}^k - \beta^E \lambda^E r^k(\hat{r}_{t+1}^k + \hat{u}_{t+1}) \\
&= s^E m^E q^k \pi(1-\delta^k)(\hat{s}_t^E + \hat{m}_t^E + \hat{q}_{t+1}^k + \hat{\pi}_{t+1})
\end{aligned}$$

Using $q^k = 1$ in SS and diving the whole equation by λ^E, we obtain

$$(\hat{\lambda}_t^E + \hat{q}_t^k) - \beta^E \Big[1 - \delta^k + r^k\Big] \hat{\lambda}_{t+1}^E - \beta^E \Big[(1-\delta^k)\Big] \hat{q}_{t+1}^k - \beta^E r^k (\hat{r}_{t+1}^k + \hat{u}_{t+1}) = \frac{s^E}{\lambda^E} m^E (1-\delta^k)(\hat{s}_t^E + \hat{m}_t^E + \hat{q}_{t+1}^k + \hat{\pi}_{t+1})$$

We get $\frac{s^E}{\lambda^E}$ from FOC of loans entrepreneurs inn S,

$$\frac{s^E}{\lambda^E} = \frac{1}{R^{bE}} \left(1 - \beta^{bE} R^{bE}\right)$$

For the profit of the bank, we obtain in steady state as,

$$\frac{j^b}{B} = r^{bH} \frac{b^I}{B} + r^{bE} \frac{b^E}{B} - r^d (1 - \nu_i)$$

where we have used the fact that $\frac{K^b}{B} = \nu_i$ in steady state. Using this in the steady state profit equation of banks we get

$$\frac{j^b}{B} = \delta^b \frac{K^b}{B} = \delta^b \nu_i$$

Substituting this back in j^b/B we get,

$$\begin{aligned} \delta^b \nu_i &= r^{bH} \frac{b^I}{B} + r^{bE} \frac{b^E}{B} - r^d (1 - \nu_i) \\ &= r^{bH} \frac{b^I}{B} + r^{bE} \left(1 - \frac{b^I}{B}\right) - r^d (1 - \nu_i) \\ &= (r^{bH} - r^{bE}) \frac{b^I}{B} + r^{bE} - r^d (1 - \nu_i) \end{aligned}$$

Finally,

$$\frac{b^I}{B} = \frac{(\delta^b - r^d)\nu_i + (r^d - r^{bE})}{r^{bH} - r^{bE}} \quad \text{and} \quad \frac{b^E}{B} = 1 - \frac{b^I}{B}$$

Therefore we have every coefficient identified in the profit equation of the bank i.e.

$$\boxed{\frac{j^b}{B} \hat{j}_t^b = r^{bH} \frac{b^I}{B} (\hat{r}_t^{bH} + \hat{b}_t^I) + r^{bE} \frac{b^E}{B} (\hat{r}_t^{bE} + \hat{b}_t^E) - r^d (1 - \nu_i)(\hat{r}_t^d + \hat{d}_t^b)}$$

Now lets look at the budget constrain of entrepreneurs in steady state,

$$\begin{aligned} c^E + w^P l^P + w^I l^I + R^{bE} b^E &= y^E/x + b^E - \delta^k k^E \\ \text{or, } c^E + (1-\alpha) y^E/x + (R^{bE} - 1) b^E &= y^E/x - \delta^k k^E \\ \text{or, } c^E + (R^{bE} - 1) b^E &= \alpha y^E/x - \delta^k k^E \\ \text{or, } \frac{c^E}{k^E} + (R^{bE} - 1) \frac{b^E}{k^E} &= \alpha \frac{y^E}{x k^E} - \delta^k \end{aligned}$$

Now we know that $\frac{b^E}{k^E} = \frac{m^E}{R^{bE}}(1-\delta)$ and $r^k = \alpha \frac{y^E}{x k^E}$, replacing these in the above equation we get,

$$
\begin{aligned}
&\frac{c^E}{k^E} + (R^{bE} - 1)\frac{m^E}{R^{bE}}(1-\delta) = r^k - \delta^k \\
\text{or,} \qquad &\frac{c^E}{k^E} = r^k - \delta^k - (R^{bE} - 1)\frac{m^E}{R^{bE}}(1-\delta)
\end{aligned}
$$

Thus dividing the budget constrain of entrepreneurs become,

$$
\begin{aligned}
&\left(\frac{c^E}{k^E}\right)\hat{c}_t^E + \left(\frac{\mu(1-\alpha)}{x}\frac{y^E}{k^E}\right)(\hat{w}_t^P + \hat{l}_t^E) + \left(\frac{(1-\mu)(1-\alpha)}{x}\frac{y^E}{k^E}\right)(\hat{w}_t^I + \hat{l}_t^I) \\
&\quad + \left(R^{bE}\frac{b^E}{k^E}\right)(\hat{b}_{t-1}^E + \hat{R}_{t-1}^{bE} - \hat{\pi}_t) = \left(\frac{1}{x}\frac{y^E}{k^E}\right)(\hat{y}_t^E - \hat{x}_t) + \left(\frac{b^E}{k^E}\right)\hat{b}_t^E + (1-\delta^k)(\hat{q}_t^k + \hat{k}_{t-1}^E)
\end{aligned}
$$

Now lets look at the budget constrain of the patient household. In steady state, the budget constrain becomes,

$$
\begin{aligned}
c^P + d^P &= w^P l^P + R_{ss}^d d^P \\
\text{or,} \quad c^P &= w^P l^P + (R_{ss}^d - 1)d^P \\
\text{or,} \quad \frac{c^P}{K^E} &= \frac{w^P l^P}{K^E} + (R_{ss}^d - 1)\frac{d^P}{K^E} \qquad \text{dividing the whole equation by } K^E \\
\text{Therefore,} \quad \frac{c^P}{K^E} &= \frac{(1-\alpha)\mu}{x}\frac{y^E}{K^E} + (R_{ss}^d - 1)(1-\nu_i)
\end{aligned}
$$

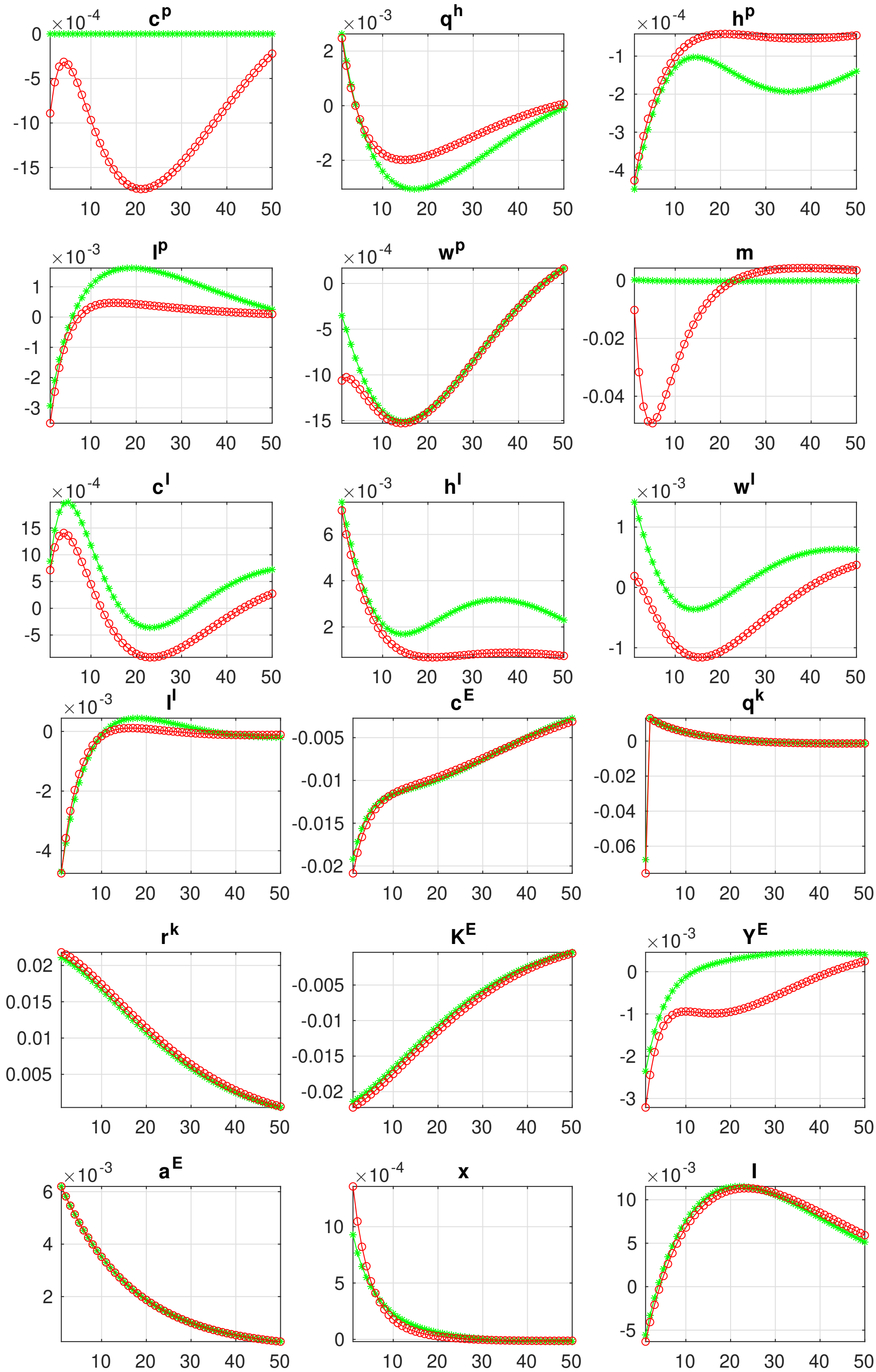
Now lets turn towards budget constrain of impatient household. In steady state we have,

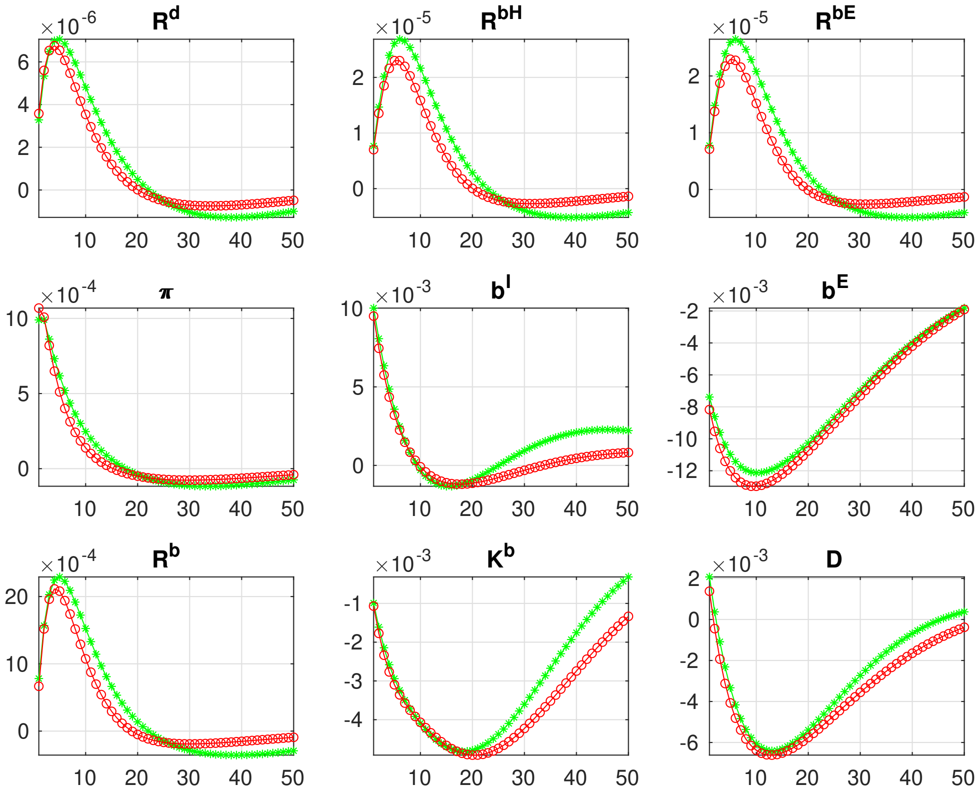
$$
\begin{aligned}
c^I + R_{ss}^{bH} b^I &= w^I l^I + b^I \\
\text{or,} \quad c^I + (R_{ss}^{bH} - 1)b^I &= w^I l^I \\
\text{or,} \quad \frac{c^I}{K^E} + (R_{ss}^{bH} - 1)\frac{b^I}{K^E} &= \frac{w^I l^I}{K^E} \qquad \text{dividing the whole equation by } K^E \\
\text{or,} \quad \frac{c^I}{K^E} + (R_{ss}^{bH} - 1)\frac{b^I}{B}\frac{B}{b^E}\frac{b^E}{K^E} &= \frac{(1-\alpha)(1-\mu)}{x}\frac{y^E}{K^E} \\
\text{therefore,} \quad \frac{c^I}{K^E} &= \frac{(1-\alpha)(1-\mu)}{x}\frac{y^E}{K^E} - (R_{ss}^{bH} - 1)\frac{b^I}{B}\frac{B}{b^E}\frac{b^E}{K^E}
\end{aligned}
$$

C All IRFs

C.1 Response of the models to an unanticipated TFP shock

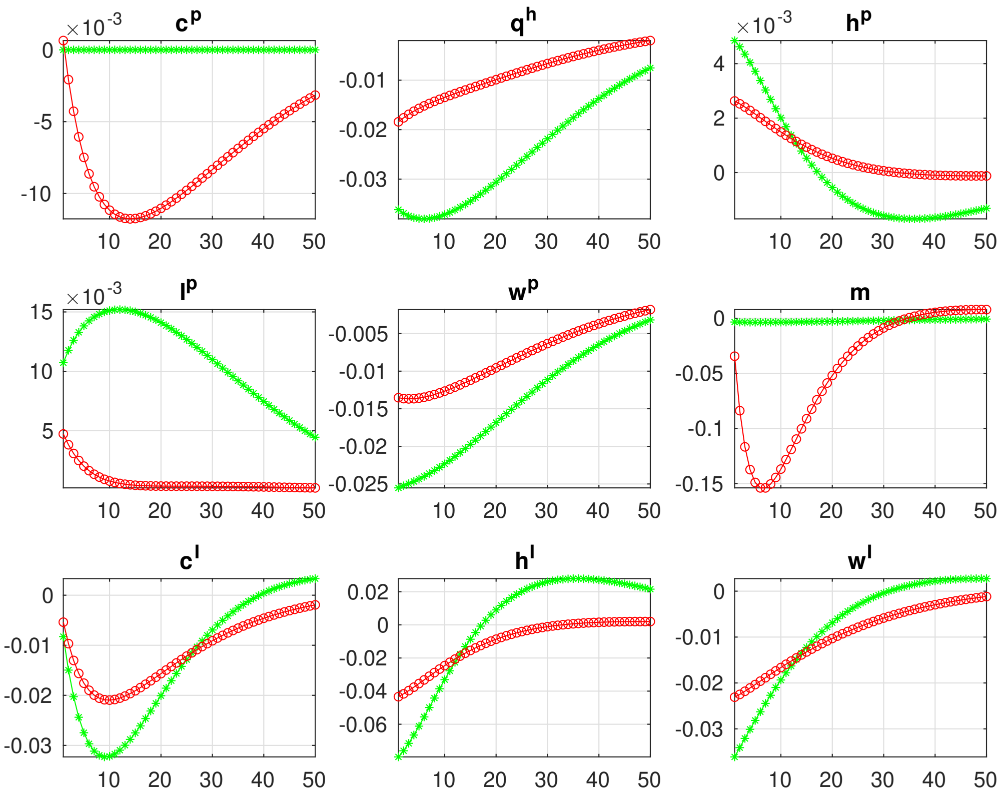
Here I report response of all variables to an unanticipated TFP shock. The green lines are for the model with only cash and the red lines are for the model with CDBC.

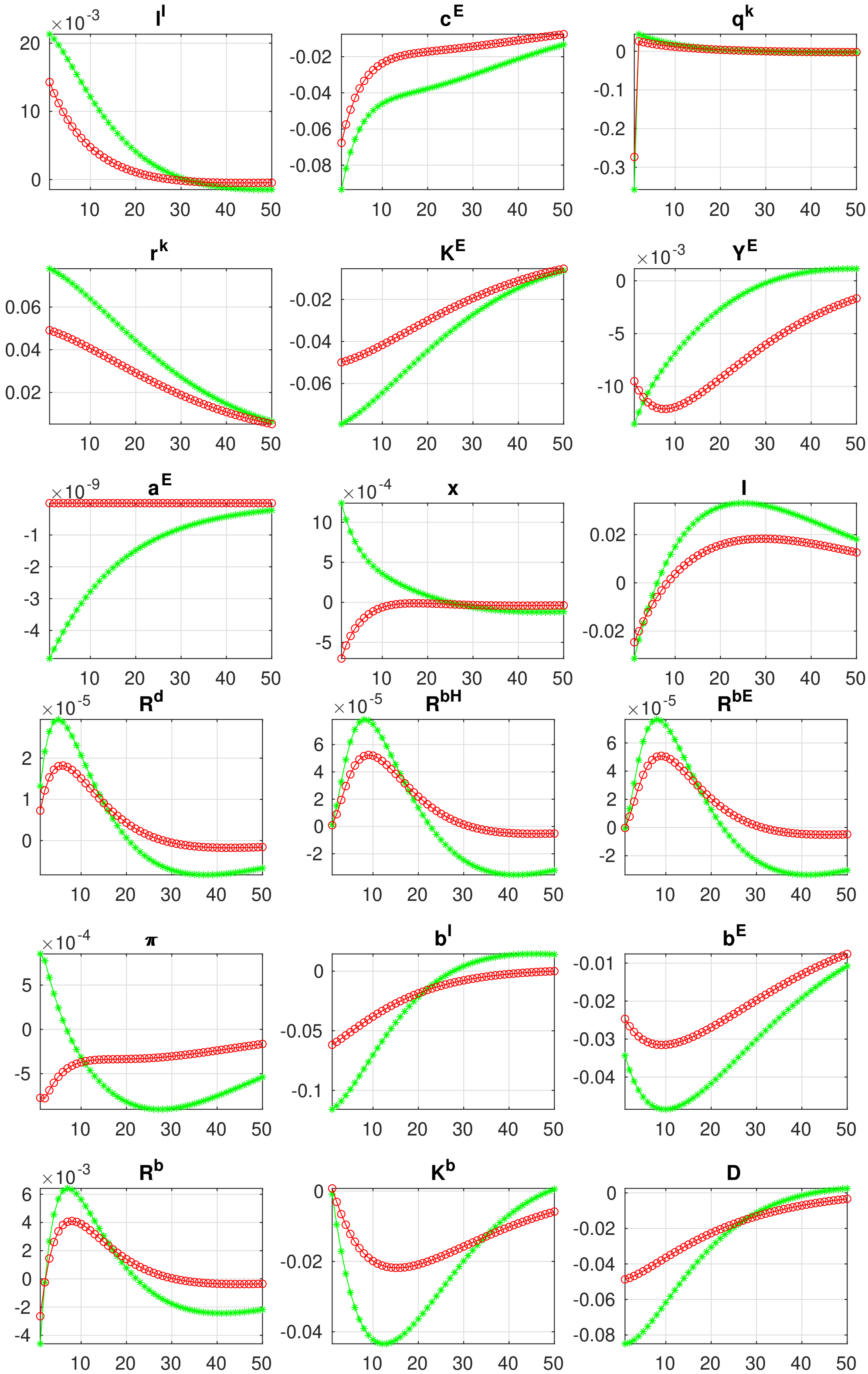




C.2 Response of the models to an unanticipated monetary policy shock

Here I report response of all variables to an unanticipated monetary policy shock. The green lines are for the model with only cash and the red lines are for the model with CDBC.




l^l
c^E
q^k
r^k
K^E
Y^E
a^E
x
l
R^d
R^bH
R^bE
π
b^l
b^E
R^b
K^b
D