Depositor-banker relationship and CBDC[*][†]

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Abstract

This paper explores the intricate dynamics of the banking sector and the macroeconomy, focusing on the introduction of CBDC and its impacts on monetary policy effectiveness, financial stability, and societal welfare. Using a medium scale DSGE model with financial friction and deep-habits in deposit market, it analyzes responses to shocks such as monetary policy changes, capital quality fluctuations, and shifts in the depositor-banker relationship. Additionally, the study evaluates the welfare implications of CBDC introduction, emphasizing potential benefits underimproved depositor-banker relationships and reduced transaction costs, while recognizing variations based on factors such as existing relationships and implementation costs. These insights offer crucial guidance for policymakers navigating decisions regarding CBDC adoption and digital currency framework design.

Keywords: Central Bank Digital Currency, Monetary policy, DSGE, Market power, Disintermediation

1 Introduction

The possibility of a Central Bank Digital Currency (CBDC) has swiftly captured significant attention within central bank research circles, with many attributing this phenomenon to the prevalent fear of missing out among central banks worldwide. The proliferation of various cryptocurrencies has amplified this apprehension, compelling central banks to assert control over payment systems and currency issuance, wary of ceding significant authority to private entities in monetary matters. Gorton and Zhang [2022] extensively elaborate on the importance of government vigilance concerning the concurrent existence of privately issued digital currencies such as stable-coins alongside fiat currencies. They

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caution against the potential ramifications, emphasizing the risk of central banks relinquishing their monopoly on money issuance and circulation.
Introduction of CBDC poses several concerns to the monetary authorities of the economies. For example, CBDC might substitute the conventional private money i.e deposit held at the bank leading to a substantial increase in funding cost of the banks, see Carapella and Flemming [2020]. Keister and Sanches [2022] have shown that if CBDCs become highly competitive with bank deposits, disintermediation is unavoidable. The impact of CBDCs on bank funding depends on whether it is cash-like or deposit-like. A cash-like CBDC would have no direct effect on bank funding, but a deposit-like CBDC would lead to a reduction in deposits and lending. Despite this, the introduction of a CBDC would increase the aggregate stock of liquid assets, leading to a more efficient exchange and improved social welfare. In their work, they have modeled the banking sector as a competitive market using the framework of Lagos and Wright [2005]. Their findings are bolstered by global sentiment, as evidenced by a recent survey conducted by OMFIF [2020], which indicates that households worldwide view central bank liabilities as the safest option. Bindseil [2020] shows how an tiered-remuneration system might mitigate this risk faced by banks. Brunnermeier and Niepelt [2019] established conditions equilibrium and welfare effects of funding thorough public and private money for banks. Disintermediation effect on the other hand might force bankers to retain more capital and alternate volatile funding options. Piazzesi and Schneider [2020] points to another feature of banking sector i.e. credit-line provision for depositors for payment purposes. They argue that if CBDC comes without the possibility of consumer loans, it might be welfare reducing.

The validity of the aforementioned claims could be subject to debate, as empirical studies have

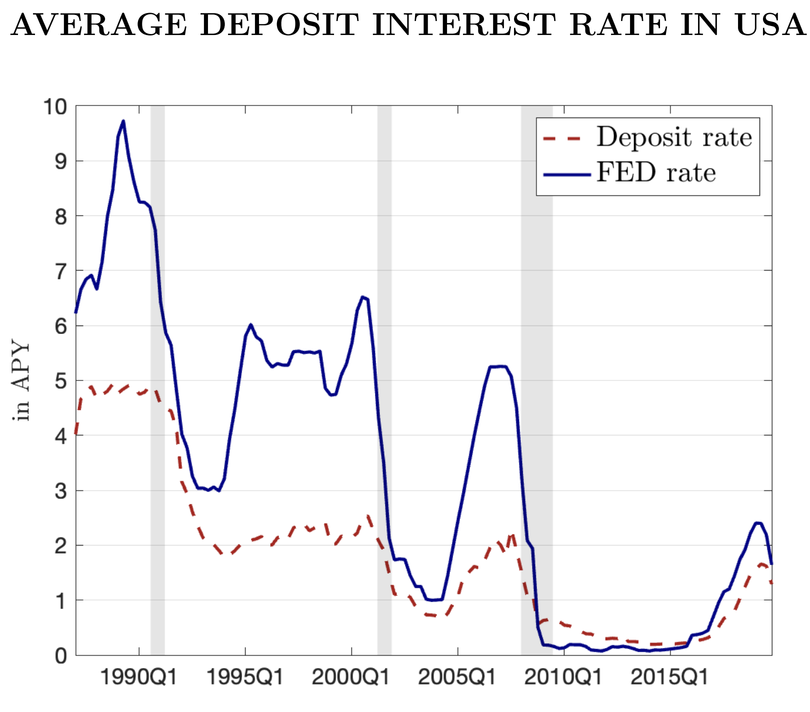


Figure 1: This figure plots average deposit interest rate of chartered banks in USA from 1987Q1-2019Q4. Deposit interest rate here refers to the transaction deposit and interest expense on transaction deposit reported by banks in the FDIC call report elements RCON3485 and RIAD4508. The policy rate is FEDFUNDS from St. Loius FED. Shaded areas are NBER recession dates. (data reference: Drechsler et al. [2017] and McCracken and Ng [2021])

indicated the existence of market power among banks and thus disintermediation effect could be significantly small due to a new payment method. In their study, Drechsler et al. [2017] provided evidence of the market power that U.S. banks hold in deposit markets. Their findings indicated that when the federal funds rate increases, the policy rates and deposits exhibit a wider spread, ultimately leading to a decrease in the total volume of deposits. This market power in deposit market allows bank to have low interest sensitivity of expenses, matching with the low sensitivity of income by from credit issuance. Thus maturity mismatch in their portfolio is offset by their ability to match interest sensitivities in both sides of their balance-sheets, as pointed out by DRECHSLER et al. [2021]. Figure 1 shows the average interest rate paid on transaction accounts and most liquid form of deposits in USA. Notice that, interest rate on these accounts does not follow one-for-one the policy rate and therefore subdue the transmission of the policy rate to real economy. When ever policy rate approaches zero lower bound (ZLB), deposit rate is above the policy rate as no interest on these account would cause depositors to switch to other risk-free forms of liquid assets like cash. Similar trends can be seen for Euro area banks and Canadian banks. A closer look to figures3 and 4 reveals that transmission of policy rate in case of Canadian banks have been extremely poor. Canadian banks even managed to stay at ZLB even when the policy rate had lift off from ZLB. Large deposit base and consumer's reluctance to change banks are often pointed out as a reasons for low sensitivity of deposit interest rates offered by Canadian banks. Chiu et al. [2023] micro-founded general equilibrium model of payments and banking market and calibrated it to the US, finding an imperfectly competitive banking sector. An interest-bearing CBDC could increase bank intermediation by raising bank lending by 1.57%, deposit and loan rates but CBDC remuneration must be within a specific range. Garratt et al. [2022] created a simplified model where big and small banks compete in order to illustrate how the introduction of CBDC could have varying effects. They presumed that big banks offer nonpecuniary benefits to depositors, but this benefit could be mimicked by a bank-distributed CBDC, even if policymakers could independently set the convenience yield. Hence business strategy of the banks, how leveraged it is, the mere size of the bank and its reputation, all these questions are important to understand the effect of CBDC. Different hypothetical scenarios regarding these dynamics are discussed in detail in Adalid et al. [2022]. Niepelt [2022] notes that CDBC could discipline 'too big to fail' banks when other measures of addressing market power are too costly for central bank. Chen et al. [2024] finds that CBDC rates directly impact liquidity costs for households and indirectly affect deposit rates through bank competition, while reserve rates affect deposit rates directly, with their effectiveness influenced by market concentration. Cirelli and Nyffenegger [2023] argues that cause of movement in spreads (i.e. is it from monopoly or leverage constraint) will determine the level of disintermediation. In similar line, Andolfatto [2020] shows that monopolistic competition in the banking sector might lead to an increase the lending activity as banks might have to increase the deposit rate and subsequently decrease the lending rate amounting to increase in the volume of credit supplied to firms due to higher demand.

This paper contributes to this important and growing body of work regarding the disintermediation effects on banks when CBDC will be introduced,in presence of bank's market power. A potential research gap exists as none of the existing literature of CBDC explores what happens there is hold-up effects due to long-term depositor banker relationship in the deposit market. In this paper, I have tried to answer this question by using a medium scale Dynamic Stochastic General Equilibrium (DSGE) model with financial friction of Gertler and Karadi [2011] and deep-habits banking of Airaudo and Olivero [2019], ALIAGA-DÍAZ and OLIVERO [2010]. Many empirical studies have confirmed that interest rate spreads and bank's profitability are highly affected by the degree of competition of the banks (for more detailed studies, see Berger et al. [2004], Degryse and Ongena [2008]). The main research questions of the paper are, (1) How bank intermediation of credit is effected in presence of CBDC? (2) Does depositor-banker relationship help banks when facing crowding out due to introduction of CBDC? (2) What are the implications of poor efficiency of central banks managing retail instruments as central bank faces cost of managing retail payments? The analysis unveils distinct responses to different shocks within the banking sector and the broader economy. Contractionary monetary policy shocks prompted adjustments in financial variables, with banks facing increased funding costs and consequent contractions in net worth and capital assets. However, the counter-cyclical nature of deposit and credit spreads allowed banks to mitigate profit shrinkage. Shocks to capital quality resulted in asset price declines and capital-triggered adjustments in bank behavior, with persistent counter-cyclicality of credit spreads but pro-cyclical deposit spreads. Analysis of shocks to the depositor-banker relationship revealed expansions in consumption and output due to reductions in the hold-up effect, indicating the significant influence of depositor-banker dynamics on economic activity. The study underscores the nuanced effects of introducing CBDC on societal welfare, emphasizing the

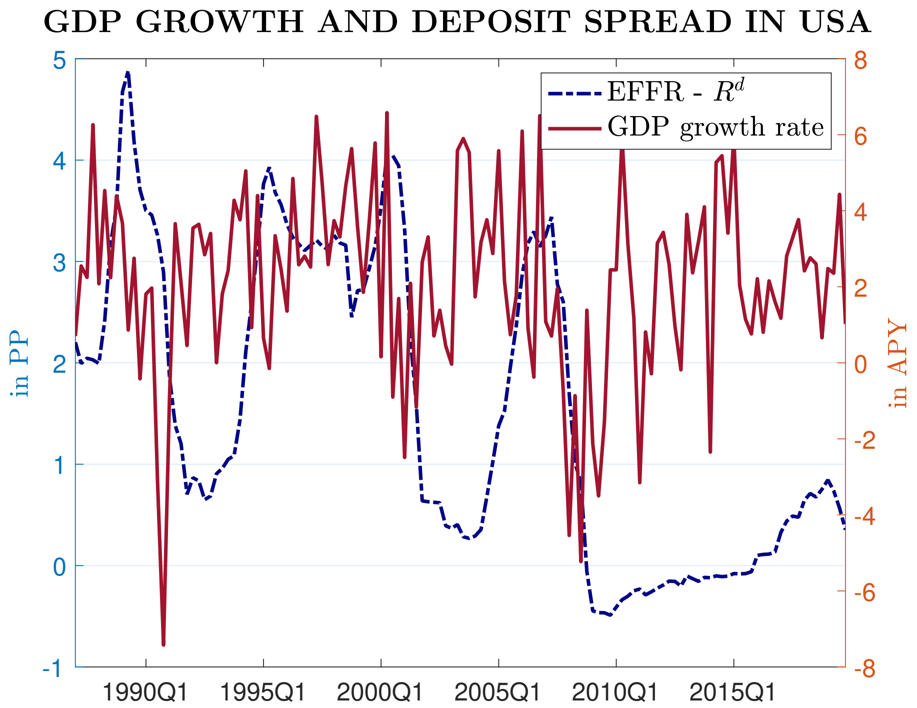


Figure 2: Deposit spread is defined as a difference of FEDFUNDS and average deposit rate as in figure1. GDP growth rate is calculated as a log difference of GDP series from St. Luis Fed (data reference: Drechsler et al. [2017] and McCracken and Ng [2021])

importance of considering existing relationships and implementation costs. These findings provide critical insights for policymakers navigating decisions regarding CBDC adoption and monetary policy effectiveness. The rest of the paper is structured as follows: Section 3 describes the model used to

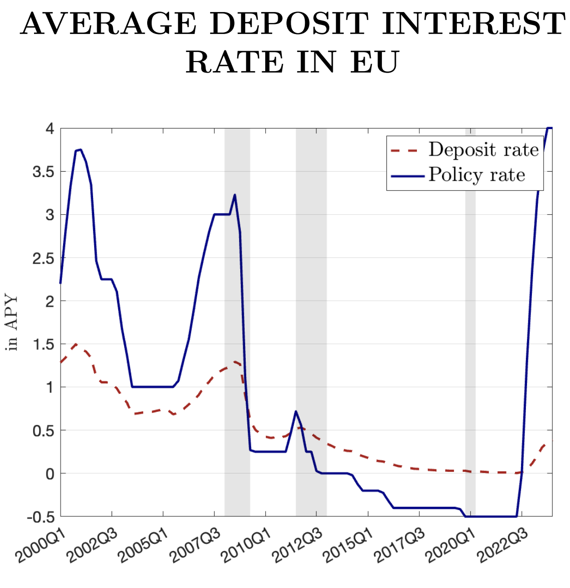


Figure 3: Deposit interest rate is Bank interest rates-overnight deposits from households-euro area (Series key: MIR.M.U2.B.L21.A.R.A.2250.EUR.N) and policy rate is deposit facility rate (ECBDFR). (Data source: ECB and FRED)

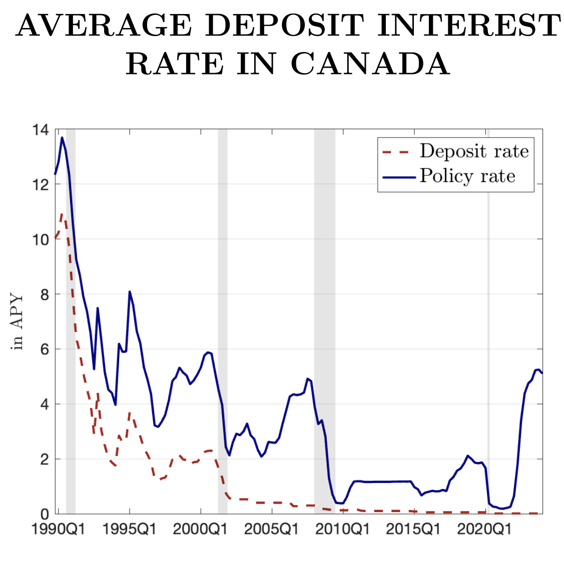


Figure 4: Deposit interest rate is the average deposit interest rate paid in transaction deposit account in six largest chartered banks in Canada. Policy rate is the 3-month interbank rate (Data source: Bank of Canada)

features of the economy, Section 4 explains the calibration and estimation of the model and Section 5 describes the results and finally the Section 6 draws conclusion on the paper.

2 Simple model to explore long-term relation

Lets consider a 2-period economy with households and banks. The household receives and endowment y in period 1 and decides consumption c and deposit d to hold at a representative bank in the economy. In each period households look for a bank to hold deposit and form a long-term relation. Importance of the long-term relationship is that household will not look for other banks in the next period and will hold their own deposit at their previous bank. This long-term relationship gets destructed exogenous with some probability λ and the household again have to look for a new bank. Therefore increase in the λ implies poor depositor-banker relationship in the economy. Whenever a household has to find a new bank, it has to bear a fixed cost ϕ representing search and matching friction. I denote the households with existing connection as c_O and newly formed connection as c_N. Therefore the budget constrain of the households in period 1 are,

$$C_O + d_O \leq y \qquad C_N + d_N + \phi \leq y \tag{1}$$

In the following period the households receives the interest on deposit from the bank and profit of the bank as well. The budget constrain of the household in the following period becomes,

$$C'_O \leq R^d d_O + \pi \qquad C'_N \leq R^d d_N + \pi \tag{2}$$

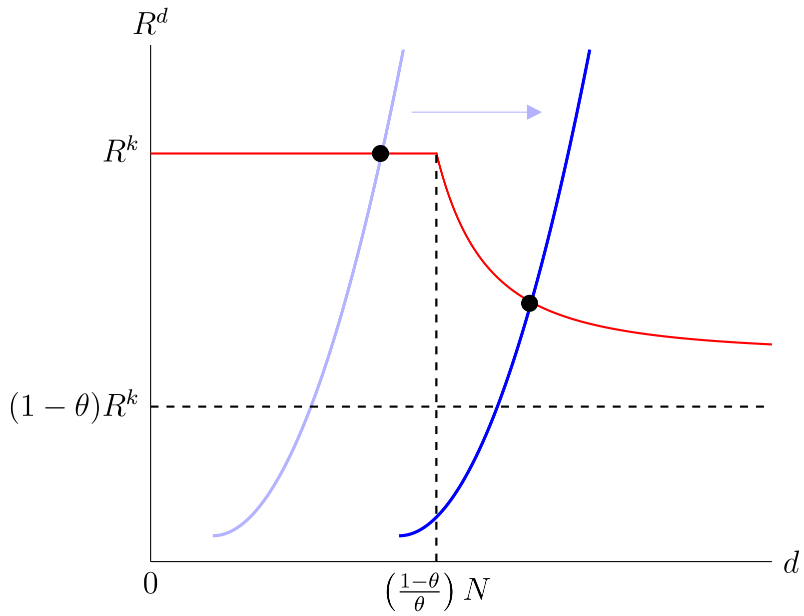


Figure 5: Equilibrium moves from first best to inefficient level when depositor-banker relation intensifies (i.e. when λ decreases)

I consider here that all households find a bank to hold deposit and each household is connected at most to one bank. Assuming the utility function of the households have the following form,

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma} \tag{3}$$

households of each respective types solves the following optimization,

$$\max_{C_i, C'_i, d'_i} U(C_i) + \beta U(C'_i) \qquad i \in \{O, N\} \tag{4}$$

subject to equations 1 and 2. Solution for the deposits demanded by each type of households is given by

$$d_O = \frac{y(\beta R^d)^{\frac{1}{\gamma}} - NR^k}{R^k + (\beta R^d)^{\frac{1}{\gamma}}} \qquad d_N = \frac{y(\beta R^d)^{\frac{1}{\gamma}} - NR^k - \phi(R^k + (\beta R^d)^{\frac{1}{\gamma}})}{R^k + (\beta R^d)^{\frac{1}{\gamma}}} \tag{5}$$

Therefore the total deposit demanded is

$$\begin{aligned} d &= \lambda d_N + (1-\lambda)d_O \\ &= d_O - \lambda\phi \end{aligned} \tag{6}$$

The banking sector is a 2-period version of the Gertler and Karadi [2011] model of financial friction where banks can run away with the fraction of assets and households do not get any remuneration on deposit. If banks can run away with θ fraction of the financial asset, then it will be indifferent between running away and still be in business if the returns are same in two cases. Thus banker's optimization problem is

$$\begin{aligned} &\max_d \quad R^k(N+d) - R^d d \\ \text{subject to} \quad &R^k(N+d) - R^d d \geq \theta R^k(N+d) \end{aligned}$$

Since the interest rate on deposit needs to clear the deposit market, it is determined from the Euler equation of the Household. Depending on what value R^d takes, the equilibrium occurs. The first-best in the economy occurs under $R^k = R^d$ which is obtained under absence of any friction in the financial market. In presence of market friction, we see that if $R^d = R^k$, the fist-best allocation is attained and the bank is indifferent on the level of d for $0 \leq d \leq \left(\frac{1-\theta}{\theta}\right)N$. In the case where $(1-\theta)R^k \leq R^d \leq R^k$ the equilibrium is no more first-best and bank demands largest possible d subject to the constrain. Level of deposit in this case is

$$d = \frac{R^k N(1-\theta)}{R^d - (1-\theta)R^k} \tag{7}$$

It is interesting to note that for sufficiently large decrease in λ i.e. when probability of new bank-depositor link decreases, deposit market may move to an inefficient equilibrium meaning. Therefore stronger depositor-banker relationship (implied by decrease in λ) leads to increase in bank's market power and an inefficient equilibrium of the economy.

3 Full Model

The model in this paper features a DSGE model with financial friction and *deep habits* in the deposit market. Figure 6 shows the interactions of different agents in the model. The representative households holds deposit at the bank, supply labor and hold a liquidity portfolio as it derives utility from it. (Burlon et al. [2022b], Ferrari et al. [2022], Schiller and Gross [2021]) Changing the nominal level of cash comes with a quadratic cost which can be interpreted as costs associated with visiting ATMs frequently or more than provisioned number by the bank. Banking sector is divided into retail banking and wholesale banking. The retail banking sector collects deposits from the households by remunerating these at bank specific deposit interest rate and bundle the deposits to produce a wholesale deposit and hold it at the wholesale bank and earn central bank's policy rate. I model long-term relationship of the bank and depositors following Airaudo and Olivero [2019] who use the *deep habits* model by Ravn et al. [2006] to model counter-cyclical credit spread in US economy. Wholesale branch of the bank is involve in purchasing equities from the firms and funds the credit line using wholesale deposit. These banks play a role of financial intermediary and not subjected to strong regulations from the central bank thus acts more like a shadow bank[1]. The banking sector features the financial friction a.k.a. agency problem as in Gertler and Karadi [2011]. Production side of the economy is standard New-Keynesian model as in many other literature.

[1] For more details about shadow banks, see Jiang et al. [2020], MOREIRA and SAVOV [2017]

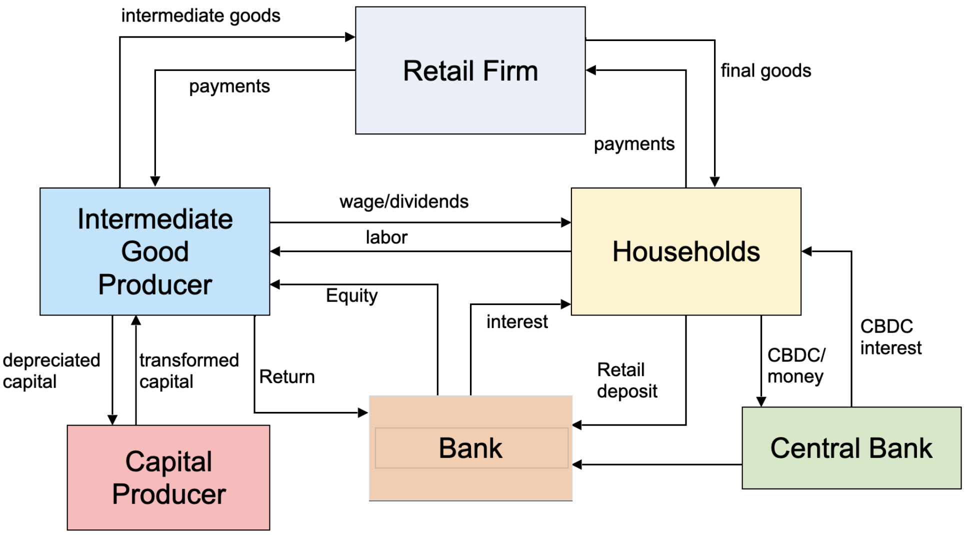


Figure 6: The above flow chart shows the cash flows in the economy.

3.1 Household

The household sector optimize the following maximization problem by choosing to consume, hold deposit at the bank and risk-free bond issued by the government. They also choose to hold cash and CBDC since it gives them liquidity services. In this economy, each representative household i holds a deposit composite $X_{i,t}$ which consists of stock of past deposit products of specific banks and current level of deposit held. The deposit composite is given by

$$X_{i,t}^d = \left[\int_0^1 (D_{i,j,t} - \theta^d S_{j,t-1})^{\frac{\epsilon^d - 1}{\epsilon^d}} \right]^{\frac{\epsilon^d}{\epsilon^d - 1}} \tag{8}$$

where $D_{i,j,t}$ is deposit holding of household i at bank j and $S_{j,t-1}$ is the stock of past deposit claims obtained from the bank j. Notice that it the cross-sectional average of all households holding deposit claims at the bank j. The parameter θ^d measures the intensity of relationship with bank[2]. The evolution of deposit stock is as following:

$$\tilde{S}_{j,t-1} = \rho_s \tilde{S}_{j,t-2} + (1 - \rho_s) D_{j,t-1} \tag{9}$$

$D_{j,t-1}$ is total deposit claims held at bank j by all households. Household considers this to be given and this makes the habit in this model as *external*. Notice that in the case $\rho_s = 0$ the deposit stock simply becomes $D_{j,t-1}$ i.e. average level of last period deposits held at jth bank. The optimization prpblem faced by household is the following:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ \log\left(C_t - hC_{t-1}\right) - \frac{\chi}{1+\varphi} L_t^{1+\varphi} + \frac{1}{1-\iota} \left[\omega_m (M_t/P_t)^{1-\iota} + \omega_d D_t^{1-\iota} + DC_t^{1-\iota} \right] \right\}$$

subject to the budget constraint,

$$C_t + X_t^d + m_t + DC_t + B_t = W_t L_t + \frac{R_{t-1}^{DC}}{\Pi_t} DC_{t-1} + \frac{m_{t-1}}{\Pi_t} + \int_0^1 \frac{R_{j,t-1}^d}{\Pi_t} D_{j,t-1} dj + \frac{R_{t-1}}{\Pi_t} B_{t-1}$$

where $X_t^d = \left[\int_0^1 (D_{j,t} - \theta^d S_{j,t-1})^{\frac{\epsilon^d - 1}{\epsilon^d}} \right]^{\frac{\epsilon^d}{\epsilon^d - 1}}$

Household first find the optimal relative demand for deposits issued by jth bank by maximizing the

[2] This is referred to *deep habits* as in Airaudo and Olivero [2019], Ravn et al. [2006]

total gain from deposit interest $\int_0^1 R^d_{j,t} D_{j,t}$ subject to X^d_t. The solution of the problem provides the expression for $D_{j,t}$ i.e. the representative household's optimal demand for deposits issued by jth bank as following

$$D_{j,t} = \left(\frac{R^d_{j,t}}{R^d_t}\right)^{-\epsilon^d} X^d_t + \theta^d S_{j,t-1} \tag{10}$$

Where,

$$R^d_t = \left[\int_0^1 (R^d_{j,t})^{1-\epsilon^d}\right]^{\frac{1}{1-\epsilon^d}} \tag{11}$$

is the aggregate deposit rate index. Notice that the deposit interest revenue can be simplified as $\int_0^1 R^d_{j,t-1} D_{j,t-1} dj = R^d_{t-1} X^d_{t-1} + \Delta^d_{t-1}$.

Therefore the budget constraint of the representative household can be written as,

$$C_t + X^d_t + m_t + DC_t + B_t = W_t L_t + \frac{R^{DC}_{t-1} DC_{t-1}}{\Pi_t} + \frac{m_{t-1}}{\Pi_t} + \frac{R^d_{t-1}}{\Pi_t} X^d_{t-1} + \Delta^d_{t-1} + \frac{R_{t-1}}{\Pi_t} B_{t-1}$$

where $\Delta^d_{t-1} = \theta^d \int_0^1 R^d_{j,t-1} S_{j,t-2} dj$
I assume that adjusting the cash is costly and the cost follows,

$$\begin{aligned} f(\cdot) &= \frac{\psi_m}{2}\left(\frac{M_t}{M_{t-1}} - 1\right)^2 m_t \\ &= \frac{\psi_m}{2}\left(\frac{m_t}{m_{t-1}}\Pi_t - 1\right)^2 m_t \end{aligned}$$

The expense can be understood as follows: whenever a household needs to adjust its nominal cash reserves, there are associated costs. For instance, the household may need to visit an ATM or bank counter, which consumes time, or encounter situations where cash is unavailable in the ATM. It's crucial to recognize that rising inflation increases the expenses linked with altering nominal cash balances because households may need to visit ATMs more frequently due to the diminished purchasing power of nominal cash. It's notable that this cost is zero in a steady state, thereby ensuring that our steady state calculation remains uninfluenced by the cash level. Given that households anticipate a fixed number of visits in the steady state, they do not bear any expense. However, deviations from the steady state incur costs, as these ATM visits may occur unexpectedly or exceed the permissible number of visits set by the bank.
The FOC of cash becomes

$$\begin{aligned} m_t^{-\iota} = \varrho_t \Bigg\{ 1 + \frac{\psi_m}{2}\left(\frac{m_t}{m_{t-1}}\Pi_t - 1\right)^2 + \psi_m \Pi_t \left(\frac{m_t}{m_{t-1}}\right)\left(\frac{m_t}{m_{t-1}}\Pi_t - 1\right)\Bigg\} \\ -\beta \mathbb{E}_t \varrho_{t+1} \Bigg\{ \frac{1}{\Pi_{t+1}} + \psi_m \Pi_{t+1}\left(\frac{m_{t+1}}{m_t}\Pi_{t+1} - 1\right)\left(\frac{m_{t+1}}{m_t}\right)^2 \Bigg\} \end{aligned}$$

3.2 Bank

From the household's demand for deposit optimization, we get, by summing all deposit demands from households $i \in [0,1]$

$$\begin{aligned} D_{j,t} &= \int_0^1 \left[\left(\frac{R_{j,t}}{R_t}\right)^{-\epsilon^d} X^d_{i,t} + \theta^d S_{j,t-1}\right] di \\ &= \left(\frac{R^d_{j,t}}{R^d_t}\right)^{-\epsilon^d} X^d_t + \theta^d S_{j,t-1} \end{aligned}$$

I consider each bank consists two operating arms, one retail and one wholesale. I assume that the wholesale branch is responsible for buying firm equities and managing whole sale deposits. Retail brunch is in charge of taking retail deposits $D_{j,t}$ from households by remunerating them at $R^d_{j,t}$ and bundle them into wholesale deposit D_t and earn risk-free government bond rate R_t. Thus retail branch of the bank solves the following problem by choosing its demand for wholesale deposit and interest rat paid on retail deposit to households as following,

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \lambda_{0,t} \Big[R_t D_t - R^d_{j,t} D_{j,t} \Big]$$

subject to,

$$D_t = D_{j,t} \tag{12}$$

$$D_{j,t} = \left(\frac{R^d_{j,t}}{R^d_t} \right)^{-\epsilon^d} X^d_t + \theta^d S_{j,t-1} \tag{13}$$

The Lagrangian corresponding to the profit maximization of the jth bank is formulated as:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \lambda_{0,t} \left[(R_t - R^d_{j,t}) D_{j,t} + \vartheta_{j,t} \left\{ \left(\frac{R^d_{j,t}}{R^d_t} \right)^{-\epsilon^d} X^d_t + \theta^d S_{j,t-1} - D_{j,t} \right\} \right] \tag{14}$$

Taking the FOC w.r.t $D_{j,t}$ and $R^d_{j,t}$, I get:

$$\vartheta_{j,t} = (R_t - R^d_{j,t}) + \theta^d (1-\rho_s) \beta \mathbb{E}_t \lambda_{t,t+1} \vartheta_{j,t+1} \tag{15}$$

$$D_{j,t} + \epsilon^d \vartheta_{j,t} \frac{X^d_t}{R^d_t} \left(\frac{R^d_{j,t}}{R^d_t} \right)^{-(1+\epsilon^d)} = 0 \tag{16}$$

Lets focus on equation 15. This variable represents the value of continuation of the business of retail banking. Its clear that in case of $\rho_s = 1$, the future value of retail bank becomes irrelevant for the present value and it becomes static. Also, the solution for the deposit stock S^d_{t-1} becomes zero and deposit-holdup effects become absent from the model. In contrast, the stronger hold-up effect implied by higher θ^d_t leads to higher accounting of value of future profits for retail banks on present value of profits.

From eq (13) and eq (16) I can get the expression for $\vartheta_{j,t}$. I use that in eq (15) to get,

$$R_t - R^d_{j,t} = \frac{R^d_{j,t}}{\epsilon^d} \left(\frac{\gamma_{j,t}}{\theta^d - \gamma_{j,t}} \right) - \theta^d (1-\rho_s) \beta \mathbb{E}_t \lambda_{t,t+1} \vartheta_{j,t+1} \tag{17}$$

Under symmetric equilibrium, I get the following expression for the deposit spread:

$$R_t - R^d_t = \frac{R^d_t}{\epsilon^d} \left(\frac{\gamma_t}{\theta^d - \gamma_t} \right) - \theta^d (1-\rho_s) \beta \mathbb{E}_t \lambda_{t,t+1} \frac{R^d_{t+1}}{\epsilon^d} \left(\frac{\gamma_{t+1}}{\theta^d - \gamma_{t+1}} \right) \tag{18}$$

where $\gamma_t = D_t / S_{t-1}$.

If I define the markdown on the policy rate as μ^d_t where $\mu^d_t = r^d_t / r_t$ the log-linerized equation is given by equation C.64 (See appendix C). This equation is central to the behaviour of the deposit spread in this model. Notice that an positive shock to the hold-up effect decreases the markup and thus increasing the deposit spread. Increasing the deposit demand reflected in the increase in the γ_t increases the markdown and decreases the spread consequently.

3.3 Wholesale bank

There are continuum of wholesale banks $j \in [0,1]$ which are modeled after Gertler and Karadi [2011]. They are engaged in buying equities from the non-financial firms and funding their invest through wholesale deposit obtained from the retail branch of the bank. The bank has to decide $S_{j,t}$ financial claims to buy which are priced at Q_t and $D^w_{j,t}$ wholesale deposits to hold at the retail branch. The return on the equity investment is R^k_t and wholesale deposit costs R_t per unit. If the net worth of the wholesale bank is $N_{j,t}$ then the balance sheet of the bank is given by

$$N_{j,t} + D^w_{j,t} = Q_t S_{j,t} \tag{19}$$

and the net worth evolves as,

$$N_{j,t+1} = R^k_{t+1} Q_t S_{j,t} - R_t D^w_{j,t} \tag{20}$$

The bank will not operate unless discounted return on equities are higher greater than discounted cost of maintaining deposit contract with the retail branch. Therefore at every period τ, the condition for the bank operation is

$$\beta^\tau \mathbb{E}_t \Lambda_{t,t+1}(R^k_{t+1+\tau} - R_{t+\tau}) \geq 0 \quad \text{for all } \tau > 0 \tag{21}$$

The stochastic discount factor $\Lambda_{t,t+1}$ is same as the one for household since all banks are owned by households. If the above condition holds, the bank operates in a period by maximizing expected wealth at exit as following

$$V_{j,t} = \max \mathbb{E}_t (1-\theta)\theta^\tau \beta^{\tau+1} \Lambda_{t,t+1+\tau} N_{j,t+1+\tau} \tag{22}$$

By defining the gross growth rate of assets $\mathcal{X}_{t,t+\tau} \equiv \frac{Q_{t+\tau} S_{j,t+\tau}}{Q_t S_{j,t}}$ and gross growth rate of net worth as $Z_{t,t+\tau} \equiv \frac{N_{j,t+\tau}}{N_{j,t}}$, it is possible to express the $V_{j,t}$ as

$$V_{j,t} = \nu_t Q_t S_{j,t} + \eta_t N_{j,t} \tag{23}$$

where,

$$\eta_t = \beta \mathbb{E}_t \Lambda_{t,t+1}[(1-\theta)R_t + \theta Z_{t+1}\eta_{t+1}] \tag{24}$$

$$\nu_t = (1-\theta)\beta \mathbb{E}_t \Lambda_{t,t+1}(R^k_{t+1} - R_t) + \theta\beta \mathbb{E}_t \lambda_{t,t+1} \mathcal{X}_{t+1}\nu_{t+1} \tag{25}$$

At each period, the wholesale banker can divert λ fraction of the funds to the owner household . This will lead to the bankruptcy of the bank since all depositors (retail bank in this case) will run and will recover the $1-\lambda$ fraction of the assets. Hence the incentive constraint for the wholesale banking operation is

$$V_{j,t} \geq \lambda Q_t S_{j,t} \tag{26}$$

From eq (13) and eq (26), I can write,

$$Q_t S_{j,t} = \phi_t N_{j,t} \tag{27}$$

where

$$\phi_t = \frac{\eta_t}{\lambda - \nu_t} \tag{28}$$

Therefore variation in the net worth of the wholesale bank will drive the demand for equities from the bank creating a channel for through which financial sector will be effecting the real sector of the economy.

At each period, $1-\theta$ fraction of the bankers leave with their net worth, the aggregate net worth of the remaining bankers is given by

$$N_{e,t} = \theta[(R^k_t - R_{t-1})\phi_{t-1} + R_{t-1}]N_{t-1} \tag{29}$$

Households transfer a $\frac{\omega}{1-\theta}$ fraction of aggregate net worth of the exiting bankers to the new bankers. Since the net worth of the exiting bankers is $(1-\theta)Q_tS_{t-1}$, the net worth of the new bankers are,

$$N_{n,t} = \omega Q_t S_{t-1} \tag{30}$$

Therefore, the aggregate net worth of the banking sector is,

$$N_t = \theta[(R_t^k - R_{t-1})\phi_{t-1} + R_{t-1}]N_{t-1} + \omega Q_t S_{t-1} \tag{31}$$

3.4 Capital producing firms

Capital producers purchase depreciated capital from the intermediate goods producers and create new capital. Both renovated capital and new capital to the intermediate goods producers. Market for capital is perfectly competitive. The new capital is priced at Q_t. The capital producers face investment adjustment cost associated with production of new capital. If depreciated capital is renovated at unit cost, the total investment is simply the sum of new investment and renovated capital. In mathematical terms,

$$I_t = I_t^n + \delta(U_t)\xi_t K_t \tag{32}$$

Notice that the depreciation of capital depends on the level of utilisation of capital i.e. wear and tear of capital. The functional form of the capital depreciation is given by,

$$\delta(U_t) = \bar{\delta} - \frac{\tilde{\delta}}{1+\zeta} + \frac{\tilde{\delta}}{1+\zeta}U_t^{1+\zeta} \tag{33}$$

where, $\bar{\delta}$ is the depreciation in the steady state. Note that the parameter $\tilde{\delta}$ will be endogenously determined in the model.
The capital producer solves the following profit maximizing problem,

$$\max_{I_t^n} \quad \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t,t+j} \Big[Q_{t+j} I_{t+j}^n - I_{t+j}^n - \frac{\eta^i}{2}\left(\frac{I_{t+j}^n + I^n}{I_{t+j-1}^n + I^n} - 1\right)^2 (I_{t+j}^n + I^n) \Big] \tag{34}$$

Taking derivative with respect to net investment gives the price equation of the capital as following

$$\begin{aligned} Q_t^k = 1 + \frac{\eta^i}{2}\left(\frac{I_t^n + I^n}{I_{t-1}^n + I^n} - 1\right)^2 + \eta^i \left(\frac{I_t^n + I^n}{I_{t-1}^n + I^n} - 1\right)\left(\frac{I_t^n + I^n}{I_{t-1}^n + I^n}\right) \\ - \beta \mathbb{E}_t \Lambda_{t,t+1} \eta^i \left(\frac{I_{t+1}^n + I^n}{I_t^n + I^n} - 1\right)\left(\frac{I_{t+1}^n + I^n}{I_t^n + I^n}\right)^2 \end{aligned} \tag{35}$$

3.5 Intermediate goods producers

In a perfectly competitive market, each m intermediate goods producer produce $Y_{m,t}$ using labor from the household and capital K_{t-1} which it buys using the funds raised by equity. It sells the produce to the retailers at price P_t^m. In other words, each intermediate goods producer issues a stock S_t priced at the price of capital and sell these stocks to wholesale banks to buy capital. Therefore value of the financial asset satisfies,

$$Q_t S_t = Q_t K_{t-1} \tag{36}$$

Please note that since the firm can buy the capital one period before the production and sell it to the capital producers as soon as the production is over, there is no capital accumulation on the firm's side. The production function of the firm is Cobb-Douglas with following specification,

$$Y_t^m = A_t \Big[U_t \xi_t K_{t-1}\Big]^{\alpha} L_t^{1-\alpha} \tag{37}$$

The firm solves the following profit maximizing problem by optimally choosing L_t and U_t,

$$\max \quad P_t^m Y_t^m + (Q_t - \delta(U_t))\xi_t K_{t-1} - W_t L_t \tag{38}$$

subject to the production function in eq (37). Taking first order with respect to L_t, U_t, I obtain the following optimality conditions,

$$(1-\alpha)P_t^m \frac{Y_t^m}{L_t} = W_t \tag{39}$$

$$\alpha P_t^m \frac{Y_t^m}{U_t} = \delta'(U_t)\xi_t K_{t-1} \tag{40}$$

The return on equity investment or capital can be interpreted in this way: since the intermediary earns zero profit in every period, the total return on capital is exactly same as the capital share of revenue and the depreciated capital itself. Thus the realised return on capital can be expressed as,

$$R_t^k = \frac{\left[\alpha P_t^m \frac{Y_t^m}{\xi_t K_{t-1}} + (Q_t^k - \delta_t)\right]\xi_t}{Q_{t-1}^k} \tag{41}$$

3.6 Retail producers

The retail producers a.k.a. final goods producers buy intermediate goods Y_t^m from the intermediate goods producers at price P_t^m and bundle them into single final goods Y_t in a monopolistically competitive market. So we have,

$$Y_t = \left(\int_0^1 (Y_t^f)^{\frac{\epsilon-1}{\epsilon}} df\right)^{\frac{\epsilon}{\epsilon-1}} \tag{42}$$

Each firm can re optimize their price occasionally[3] with probability $1-\gamma$. Firms that do not optimize the price, index their price using inflation and parameter μ. Therefore a retail firm optimizes the following problem,

$$\max_{P_t^f} \sum_{i=0}^{\infty} \gamma^i \beta^i \Lambda_{t,t+i} \left(\prod_{\tau=1}^{i} \Pi_{t+\tau-1}^{\mu} \frac{P_t^f}{P_{t+i}} - P_{t+i}^m\right) Y_{t+i}^f \tag{43}$$

subject to the demand

$$Y_{t+i}^f = \left(\prod_{\tau=1}^{i} \Pi_{t+\tau-1}^{\mu} \frac{P_t^f}{P_{t+i}}\right)^{-\epsilon} Y_{t+i} \tag{44}$$

where the aggregate price index is given by

$$P_t = \left(\int_0^1 (P_t^f)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \tag{45}$$

Also note that the price dispersion evolves as,

$$\varsigma_t = (1-\gamma)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon} + \gamma\left(\Pi_{t-1}^{\mu} \frac{P_{t-1}}{P_t}\right)^{-\epsilon} \varsigma_{t-1} \tag{46}$$

and the intermediate goods and final goods are related by price dispersion as

$$Y_t^m = \varsigma Y_t \tag{47}$$

[3] This the referred to Calvo pricing in the literature following Calvo [1983]

3.7 Government

I assume that government funds bond payment through taxes on the household and profit received from the central bank and issuing new bonds. The balance sheet of the government is following

$$G_t + R_{t-1}B_{t-1} = T_t + B_t + \Omega_t \tag{48}$$

where government bonds are purchased by both households and central banks i.e. $B_t = B_t^g + B_t^h$. I also assumes that government gives subsidy to the household, so that the flexible price equilibrium coincides with efficient equilibrium in steady state.

3.8 Central Bank

Central bank in this economy issue physical currency and digital currency and pays the remunerations if applicable. The profit of the central bank evolves as following:

$$\Omega_t = M_t + DC_t - \frac{R_{t-1}B_{t-1}^g}{\Pi_t} - \frac{R_{t-1}^{dc}DC_{t-1}}{\Pi_t} - \frac{M_{t-1}}{\Pi_t} - B_t^g \tag{49}$$

Due to presence of two public assets, central bank now has to set two policies i.e. (1) nominal interest rate on government bond and (2) either supply or remuneration of CBDC. The nominal interest rate is set according to a Taylor rule,

$$\frac{R_t^n}{R} = \left(\frac{R_{t-1}^n}{R}\right)^{\rho_R} \left[\Pi_t^{\phi_\pi} \left(\frac{Y_t}{Y_t^f}\right)^{\phi_Y}\right]^{1-\rho_R} \exp(\varepsilon_t^R) \tag{50}$$

where Y_t^f is the flexible price output or natural level of output and R_t^n is the nominal interest on bonds. Notice that nominal and real bond interest rates are connected using Fisher relation:

$$\frac{R_t^n}{\mathbb{E}_t\Pi_{t+1}} = R_{t+1} \tag{51}$$

For the baseline model, I consider that the household set the supply of CBDC as a fraction of output following Burlon et al. [2022b]. Thus the supply rule takes the following form:

$$R_t^{dc} = 1 \tag{52}$$

i.e. CBDC is not remunerated in the baseline model. Later on I will consider other policy specification for different scenario analyses.

3.9 Market clearing

Total available resource is given by

$$Y_t = C_t + G_t + I_t + \frac{\eta^i}{2}\left(\frac{I_t^n + I^n}{I_{t-1}^n + I^n} - 1\right)^2 (I_t^n + I^n) \tag{53}$$

where government expenditure G_t is $\bar{g}\bar{Y}$ for simplicity.

3.10 Aggregate shocks

There are in total 3 AR(1) processes that are subject to aggregate shocks in the economy. Namely A_t, the TFP, ξ_t capital quality and government spending μ_t :

$$A_t = \rho_A A_{t-1} + \varepsilon_t^A \tag{54}$$
$$\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_t^\xi \tag{55}$$
$$\mu_t = \rho_\mu \mu_{t-1} + \varepsilon_t^\mu \tag{56}$$

The monetary policy shock i.e. ε_t^R is considered to be purely exogenous.

4 Calibration

Table 1: Parameter Values

Parameter	Value	Description
β	0.990	Discount rate
σ	1.000	Consumption EIS
h	0.815	Habit formation
φ	0.276	Inverse Frisch elasticity
ζ	7.200	Elasticity of marginal depreciation wrt the utilization rate
θ	0.972	Survival prob. of bankers
α	0.330	Capital share
η^i	1.728	Elasticity of investment adjustment cost
ϵ^p	4.167	goods elasticity
γ	0.779	Calvo param
κ^Π	1.500	Taylor rule: Inflation coefficient
κ^Y	0.125	Taylor rule: Output gap
κ	10.000	Credit prolicy coefficient
δ^{ss}	0.025	SS depreciation
ψ_m	0.020	Cash storage cost Burlon et al. [2022a]
ϕ_{dc}	0.600	CBDC quantity param
θ^d	0.800	Intensity of bank-deposit relationship
ρ_s	0.750	deposit stock parameter
ω_{dc}	0.001	CBDC liquidity preference
ι	4.22	Elasticity of liquid assets

The standard parameters and bank specific parameters are taken from Gertler and Karadi [2011]. Model specific parameters are intensity of bank deposit θ^d, deposit stock parameter ρ_s. These parameters are suggestive and they can take values from 0 to 1. In the baseline model I considered 0.8 for the θ^d and 0.75 for ρ_s. In later section, I do sensitivity analysis on these parameters. I calibrate the elasticity parameter for liquid assets ι using the steady state ratio of cash and deposit composition in M1 measure of money for USA. I calibrate deposit markdown in USA to 0.85 following Jamilov and Monacelli [2023]. This markdown value is consistent with average deposit spread in USA of about 60 basis points.

5 Results

Here I document some model dynamics subject to various shocks.

5.1 Monetary policy shock

7 shows the response of the selected variables to 1 standard deviation contractionary monetary policy shock. Since the interest rate on the wholesale increases, the wholesale branches of the banks face costly funding option. This leads to contraction of the net worth of the banks. Since in this model, assets are fraction of the net worth of the wholesale branch, the capital asset drops. As a result the production decreases. On the retail branch of the bank, we see that the interest rate on the deposit does not increases as much as the risk-free interest rate in the economy. As a result, the deposit spread increases. It is important to note that both the deposit and credit spread are counter-cyclical in case of monetary policy shock. This indicates to the market power of the bank. Despite the expensive funding due to increase interest rate of the deposit rates, banks can now increase the return on the asset to make up for their profit shrinkage. Retail banks can also have their profit shrinkage delayed as they can afford to have lower deposit rate paid on the household's holding of deposit. Switching cost (as previously defined) increases by about 0.8% on impact due to increase in the deposit rate. It decreases gradually since both deposit holding and deposit interest rate decreases and it is cheaper for the households to opt for alternate deposit products in the market since the extra return they enjoy in presence of *deep-habits*. Both CBDC and cash drops about 5% on impact. In our baseline model, both cash and CBDC has not remuneration. Therefore although central bank's profit remains unchanged in this case, this implies about $234 billion reduction in the balance of the central bank considering CBDC and cash has the constitute the same level of currency in circulation as of today. Note that this would imply that central bank simply substituted the amount of cash for CBDC. This substituition will central bank to avoid and additional asset purchases(or sell-off) and potentially effecting the long-end of the yield curve. In Airaudo and Olivero [2019], credit-spread enters the NKPC through supply side of the economy. Working capital constraint on the firms lets the loan rate enter in the marginal cost. Through marginal cost loan interest rate enters the NKPC. In my model, deep habits in deposit market, will effect the demand side through aggregate demand equation.

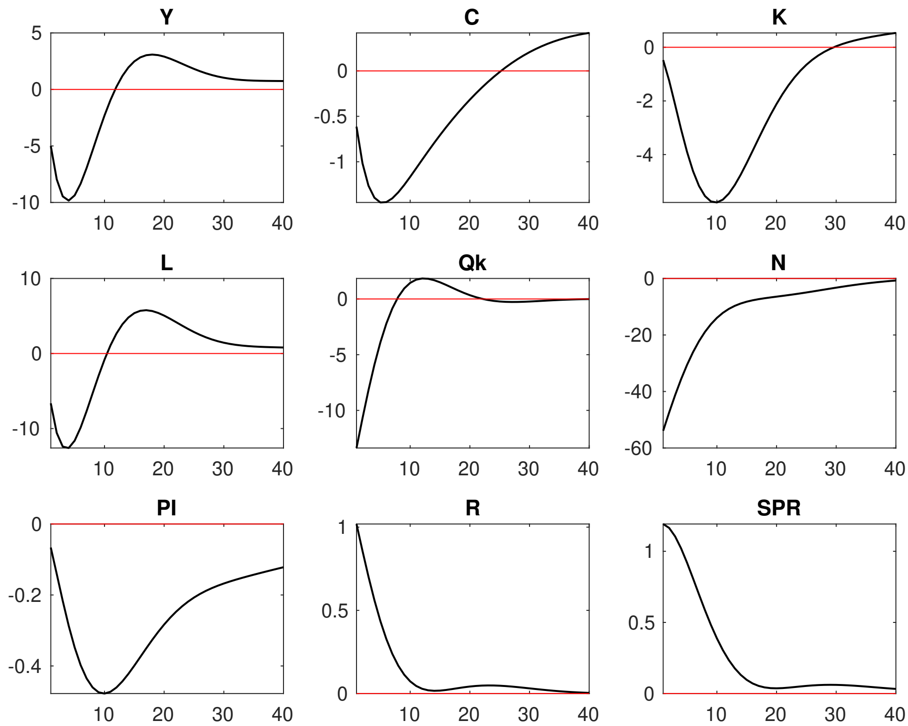


Figure 7: Impulse response function of some selected variables of the model to 1 s.d. contractionary monetary policy shock. Deviations are measured in percentage

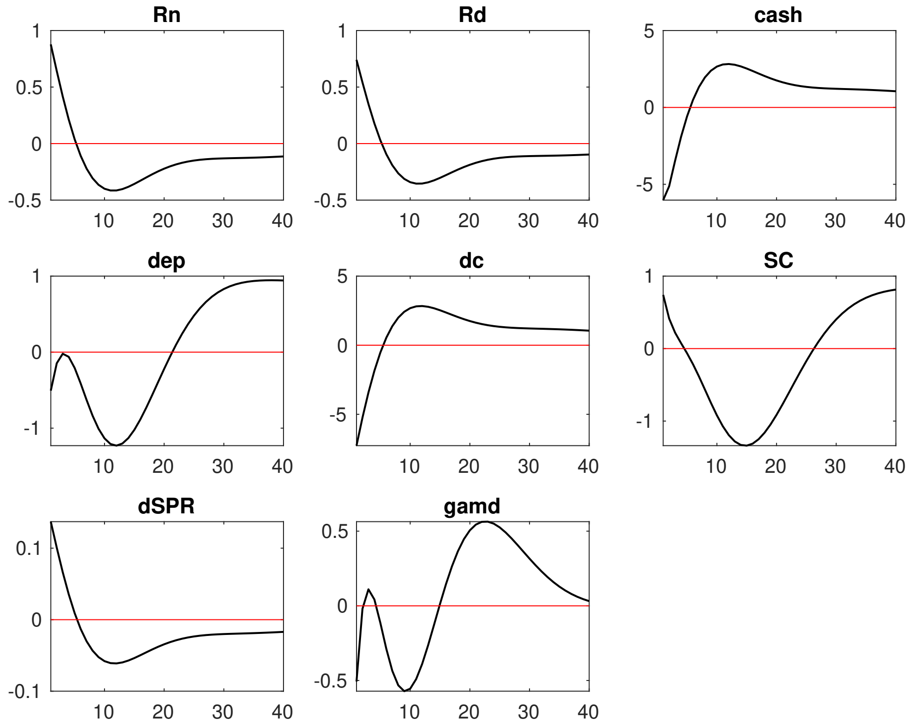


Figure 8: Impulse response function of some selected variables of the model to 1 s.d. contractionary monetary policy shock. Deviations are measured in percentage

5.2 Capital quality shock

When quality of the capital suddenly decreases it has an overall contractionary effect on the economy but the banking sector behaves differently in this case. As shown in the figure 9 and 10, the capital decreases by about 5% on impact due to the capital quality shock. The asset price drops about 15% initially but gradually recovers by reverting back to steady state in about 2.5 years whereas the capital attains the lowest in about the same time. The return on the asset increases due to the capital quality shock in the initial period. Gradual decrease of the capital in the economy and the increasing capital price, adds to the increase in the return on the capital. Notice that the counter-cyclicality of the credit spread holds in this case and confirms different empirical study (for example, Gertler and Karadi [2015]). Retail branch of the bank now behaves differently as oppose to monetary policy shock. Demand for deposit now decreases due to decrease in the net worth of the banks. Particularly, about 5% decrease of the total capital leads to decline of deposits by about 1% on impact. It continues to decrease for about 2.5 years and declining about 8% from its steady state value. As deposit declines, the deposit spread decreases as the demand for deposit leads to decrease in the spread according to equation 17. Therefore, although credit-spread remains counter-cyclical in presence of a monetary policy shock, the deposit-spread becomes pro-cyclical and decreases with output and consumption. As deposit interest decreases, the opportunity costs of holding cash and CBDC decreases. Therefore, the although cash and CBDC doesnot move on impact, they increase with a hump as deposit interest rate decreases. The subsequent decline in cash and CBDC, reflects the fact the deposit interest rate increases gradually after attaining lowest dip in about 2 years of the shock. It is interesting to note that although the deposit is declining, γ_t which reflects deposit per unit of deposit stock recovers earlier than deposit itself and starts increasing in about a year's time, thanks to the strong deposit-banker relationship.

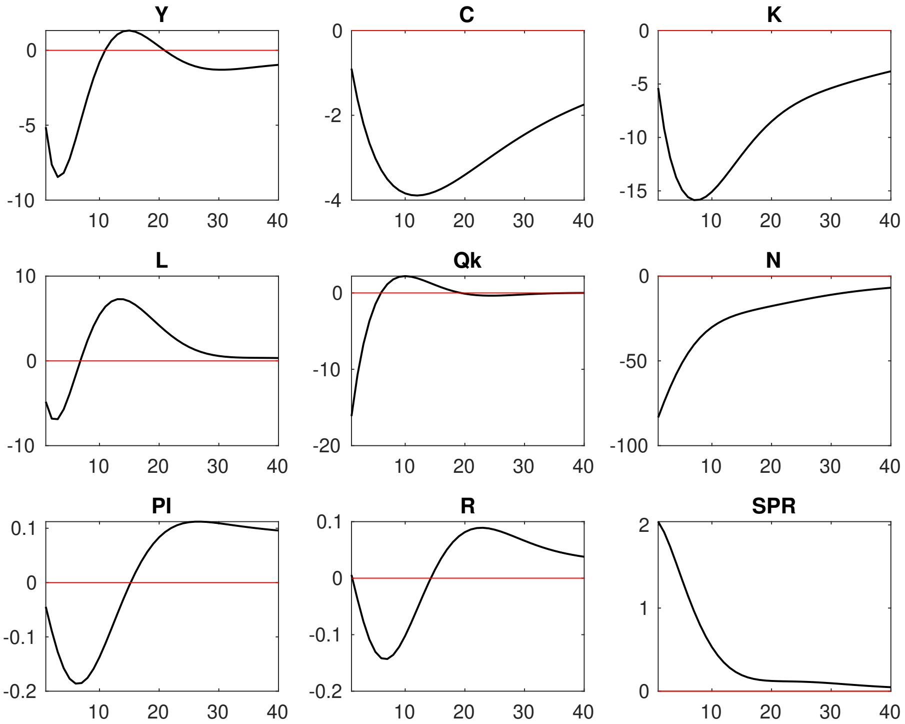


Figure 9: Impulse response function of some selected variables of the model to 1 s.d. negative capital quality shock. Deviations are measured in percentage

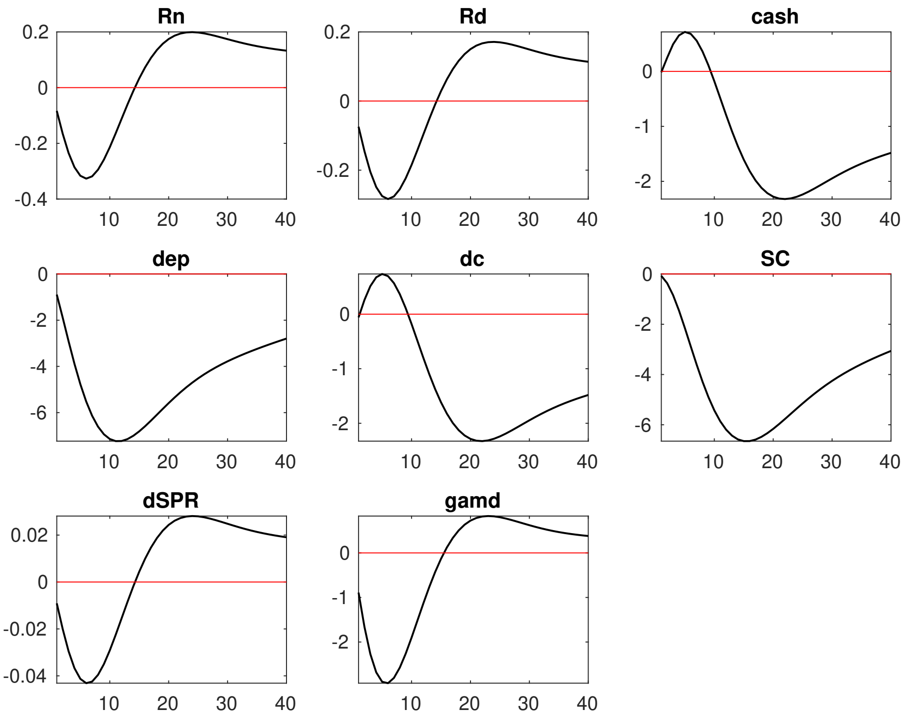


Figure 10: Impulse response function of some selected variables of the model to 1 s.d. negative capital quality shock. Deviations are measured in percentage

5.3 Shock to the depositor-banker relationship

Figures 11 and 12 show the response of the selected variables due to a shock to the depositor-banker relationship represented by the variable θ_t^d. The interpretation of the shock can be interpreted as decline in relationship with the banker due to sudden revelation of risk position of the banks but not amounting to a bank-run yet. 1 standard deviation decrease in the hold-up effect is expansionary in

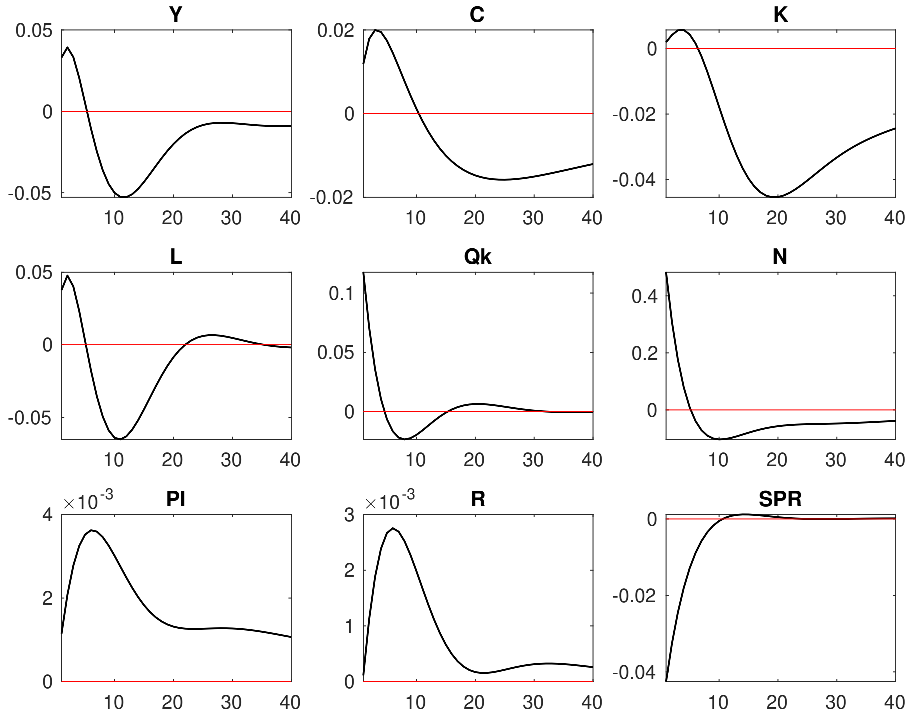


Figure 11: Impulse response function of some selected variables of the model to 1 s.d. negative shock to θ_t^d. Deviations are measured in percentage

the economy since the the effective monopoly power of the bank is now reduced. Thus we see an increase in consumption and output. θ_t^d takes gradual effect on the deposit level in the economy as we see that deposit barely moves on impact of the hold-up shock, the significantly decreases. The log-linearised version of the deposit spread (eq 17) gives us more insight in this case. Clearly both currently value and expected future value of θ^d leads to shrinkage of the spread. The retail firm faces a decline in the shadow value of per unit future profit due to decline in the hold-up effect. As hold-up effect reduces, the switching cost of consumers also decreases. To retain the deposit-holders, bank increases the interest rate paid on deposit. The asset prices increase by 0.1% and as a result the net worth of the bank increases. In this exercise, I have studied the sensitivity of the model to different long-run levels of θ_t^d i.e. the intensity of depositor-banker relationship or hold-up effect. In figure 13, we have response of selected variables under 1 s.d. contractionary shock with varying degree of long-run level of θ^d. With stronger depositor-banker relationship the contractionary effect of the monetary policy shock deepens. Stronger hold-up effect translate in to more market power of the bank, and

5.4 Welfare implications

In this section, I perform some welfare analysis based on the recursive welfare expressed in the following form

$$\begin{aligned} \text{WELFARE}_t = \log\left(C_t - hC_{t-1}\right) - \frac{\chi}{1+\varphi} L_t^{1+\varphi} \\ + \frac{1}{1-\iota}\left[\omega_m (M_t/P_t)^{1-\iota} + \omega_d D_t^{1-\iota} + DC_t^{1-\iota}\right] \\ + \beta \mathbb{E}_t \text{WELFARE}_{t+1} \end{aligned} \tag{57}$$

following Faia and Monacelli [2007]. In the first experiment, I tried to quantify the welfare improvement due to introduction of CBDC. Figure 15a shows the consumption equivalent welfare improvements in percentage where baseline model is the economy without any CBDC. It is interesting to note that any introduction of CBDC is welfare reducing but the most improvements comes in an economy where depositor-banker relationship is stronger. Thus stronger deep-habits in the depositor market would

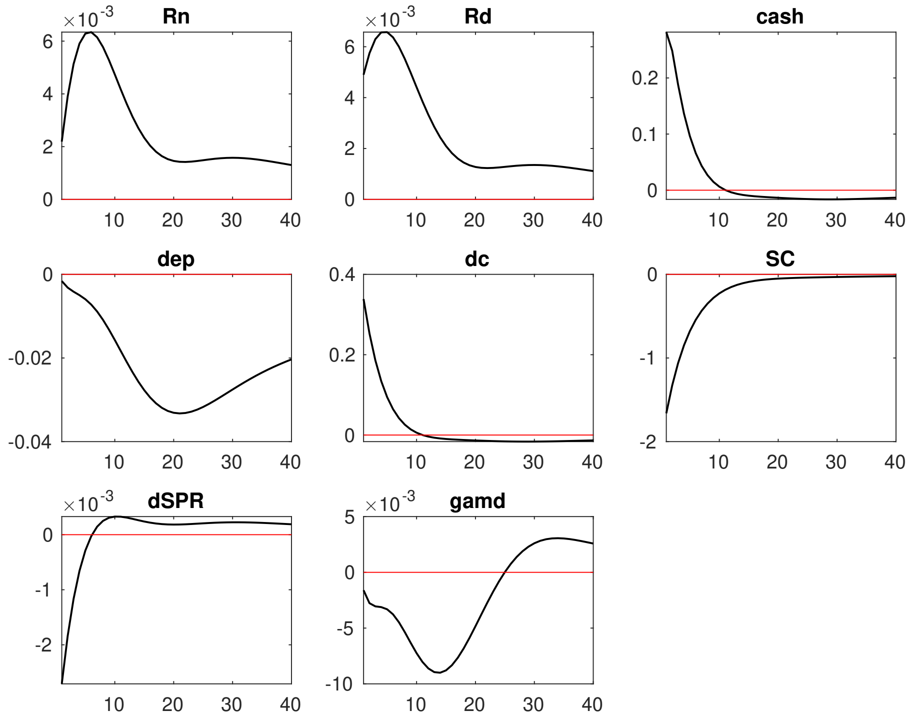


Figure 12: Impulse response function of some selected variables of the model to 1 s.d. negative shock to θ_t^d. Deviations are measured in percentage

lead to less welfare reductions in the economy after the introduction of the CBDC.
In the second experiment, I introduced quadratic cost introduction of CBDC for government in the form of $\frac{\psi_{dc}}{2}DC_t^2$. Retail payments are often associated with social costs and their presence can decide the implication of the introduction of the CBCDC. Schmiedel et al. [2012] documents that social cost associated with cash payments is about 1% of the GDP in European countries. Moreover, banks partner with payment intermediaries, to access their network of payments. This may allow them to be at a advantageous position as they may have more data on consumer behaviour about payments. Central bank moving into payment market may face large costs due to lack to information.
The modified cash-flow equation for central bank becomes:

$$\Omega_t = M_t + DC_t - \chi_{dc}\frac{DC_t^2}{2} + \frac{R_{t-1}B_{t-1}^g}{\Pi_t} - \frac{R_{t-1}^{dc}DC_{t-1}}{\Pi_t} - \frac{M_{t-1}}{\Pi_t} - B_t^g \tag{58}$$

and the total resource in the economy is given by

$$Y_t = C_t + G_t + I_t + \frac{\eta^i}{2}\left(\frac{I_t^n + I^n}{I_{t-1}^n + I^n} - 1\right)^2 (I_t^n + I^n) + \frac{\psi_{dc}}{2}DC_t^2 \tag{59}$$

Baseline model in this experiment is the economy where introducing CBDC is cost-free and there no depositor-holdup effect. In figure 15b, I show the consumption equivalent welfare heat map for different values of the CBDC introduction cost ψ_{dc} and depositor hold-up effect θ_t^d. Increasing the depositor hold-up effect in the economy increases the welfare in the economy but the improvements are the most when the introduction cost of CBDC would be very low.

6 Conclusion

The comprehensive analysis presented in this paper demonstrates the intricate dynamics of the banking sector and the broader economy in response to various shocks and policy changes, with a particular focus on the implications of CBDC introduction in banking economy with long-term depositor-banker relationship. Drawing on both theoretical modeling and empirical evidence, the study sheds light

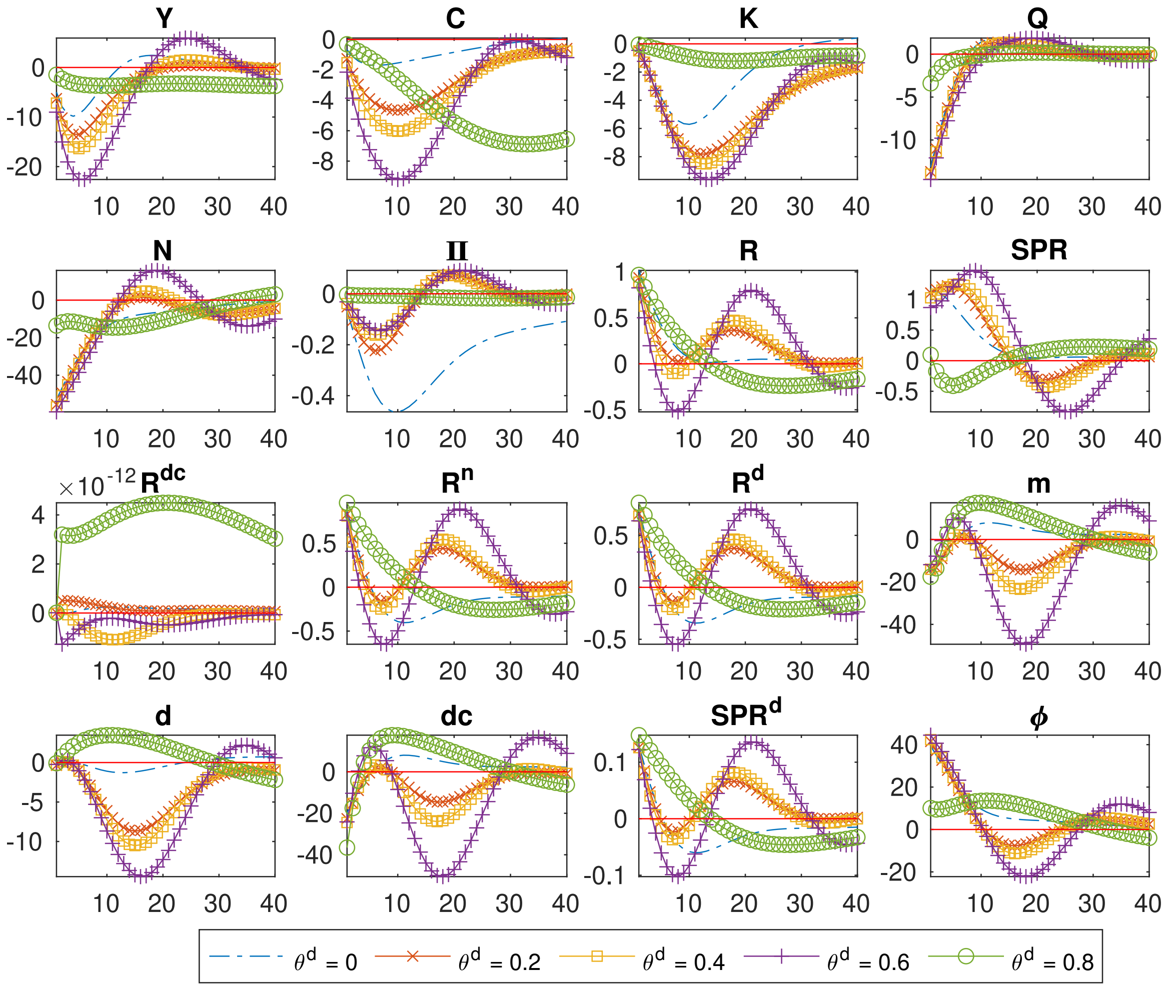


Figure 13: Impulse response function of some selected variables of the model to 1 s.d. contractionary monetary policy shock for different values of θ^d. Deviations are measured in percentage

on the multifaceted considerations surrounding CBDC implementation and its potential ramifications for monetary policy effectiveness, financial stability, and societal welfare.Notably, the response of the economy to monetary policy shocks, capital quality shocks, and shocks to the depositor-banker relationship each exhibited unique patterns, reflecting the underlying mechanisms at play within the banking sector and the broader economy.
The response to a contractionary monetary policy shock revealed significant adjustments in various financial variables. Banks faced increased funding costs, leading to a contraction in net worth and capital assets. However, the counter-cyclical nature of deposit and credit spreads underscored the market power of banks, allowing them to mitigate profit shrinkage through adjustments in asset returns and deposit rates. Additionally, the substitution of cash for CBDC highlighted potential implications for central bank balance sheets and monetary policy operations.
In contrast, a shock to capital quality resulted in a distinct response, with a decline in asset prices and capital triggering adjustments in bank behavior. The counter-cyclicality of credit spreads persisted, but the deposit spread exhibited pro-cyclical behavior, reflecting shifts in depositor behavior and bank strategies to maintain profitability. These findings emphasize the importance of capital quality in shaping financial stability and the transmission of shocks within the banking sector.
Furthermore, the analysis of shocks to the depositor-banker relationship elucidated the impact of changes in the effective monopoly power of banks. A reduction in the hold-up effect led to expansions in consumption and output, highlighting the role of depositor-banker dynamics in influencing economic activity. The subsequent adjustments in deposit levels, interest rates, and asset prices underscored the complex interplay between financial intermediaries and households in shaping macroeconomic outcomes.

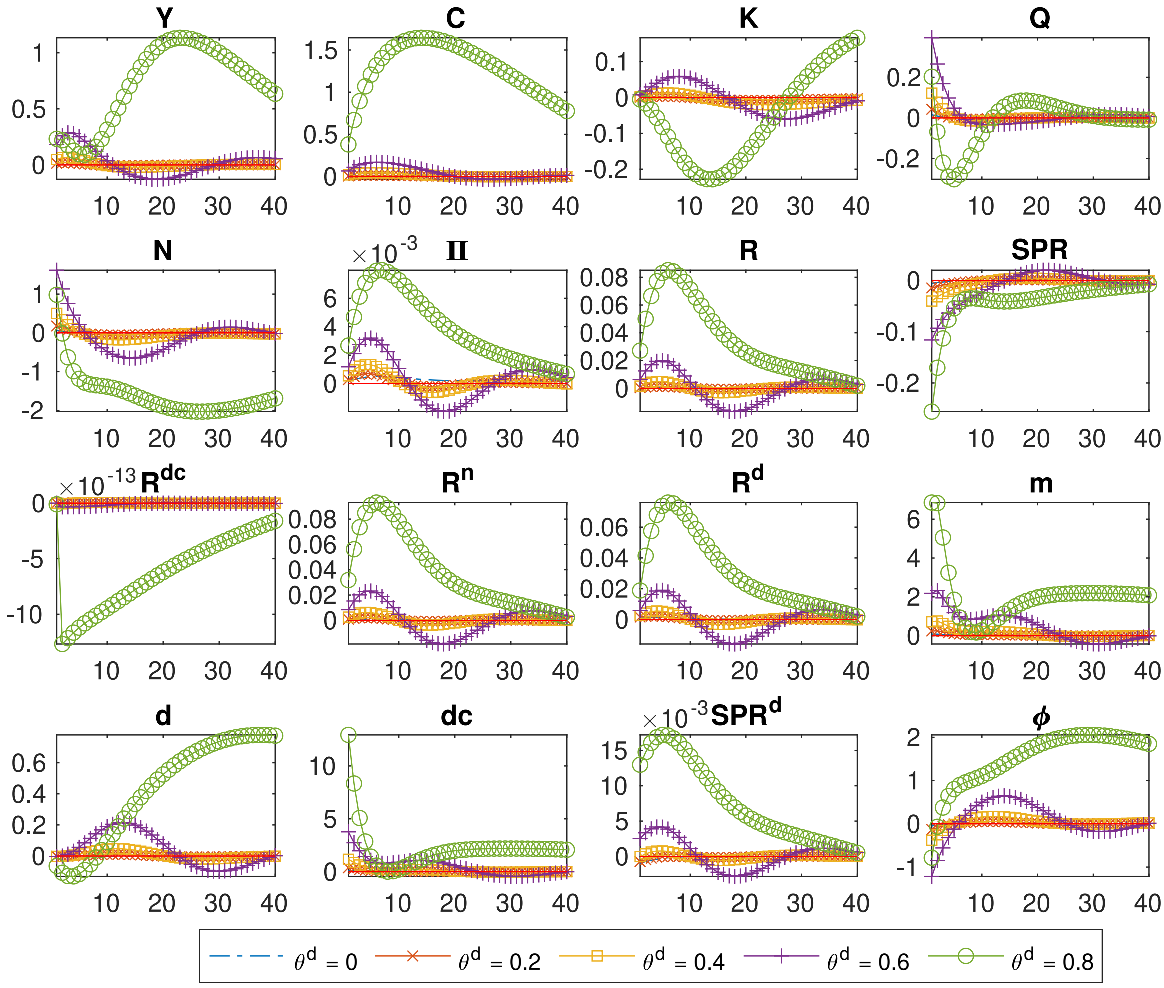
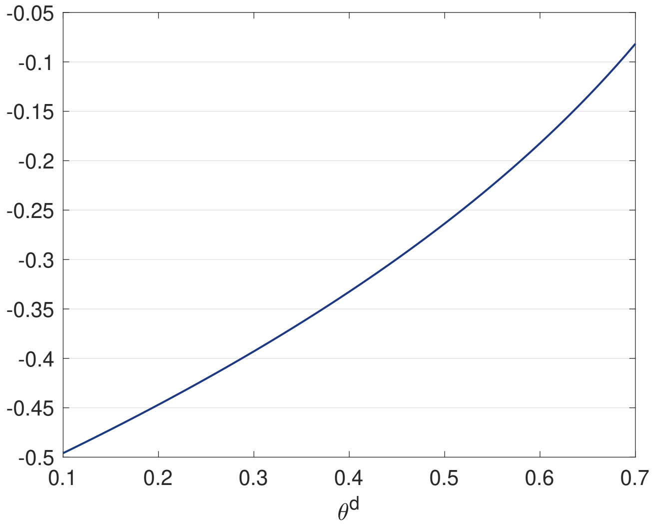


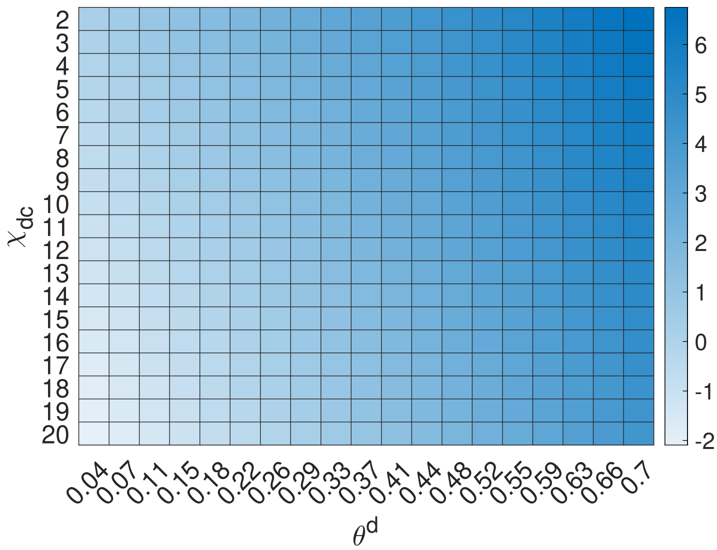
Figure 14: Impulse response function of some selected variables of the model to 1 s.d. negative shock to θ_t^d different values of θ^d. Deviations are measured in percentage

The welfare implications derived from the model underscored the nuanced effects of introducing CBDC and associated costs on societal welfare. While CBDC adoption offered potential benefits, such as improved depositor-banker relationships and reduced transaction costs, the magnitude of these benefits varied depending on factors such as the strength of existing relationships and the cost of CBDC implementation. These findings provide valuable insights for policymakers grappling with decisions regarding the adoption and design of CBDC initiatives.

Overall, the findings contribute to our understanding of the transmission mechanisms of monetary policy and the dynamics of financial intermediation in the presence of shocks. By incorporating detailed modeling of banking sector behavior and depositor-banker relationships, the study offers important implications for monetary policy effectiveness, financial stability, and the design of digital currency frameworks.



(a) Changes to welfare in comparison to the economy without CBDC and varying degree of hold-up effect



(b) Welfare differences in case in comparison to model with no hold-up effects and no cost of introducing CBDC

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A Full model equations

Household:

$$\varrho_t = (C_t - hC_{t-1})^{-\sigma} - \beta h \mathbb{E}_t (C_{t+1} - hC_t)^{-\sigma} \tag{A.1}$$

$$\lambda_{t,t+1} = \frac{\mathbb{E}_t \varrho_{t+1}}{\varrho_t} \tag{A.2}$$

$$D_t^{-\iota} = \varrho_t - \beta \mathbb{E}_t \frac{\varrho_{t+1} R_t^d}{\Pi_{t+1}} \tag{A.3}$$

$$\omega_{dc} DC_t^{-\iota} = \varrho_t - \beta \mathbb{E}_t \frac{\varrho_{t+1} R_t^{dc}}{\Pi_{t+1}} \tag{A.4}$$

$$\begin{aligned} \omega_m m_t^{-\iota} = {} & \varrho_t \left\{ 1 + \frac{\psi_m}{2} \left(\frac{m_t}{m_{t-1}} \Pi_t - 1 \right)^2 + \psi_m \Pi_t \left(\frac{m_t}{m_{t-1}} \right) \left(\frac{m_t}{m_{t-1}} \Pi_t - 1 \right) \right\} \\ & - \beta \mathbb{E}_t \varrho_{t+1} \left\{ \frac{1}{\Pi_{t+1}} + \psi_m \Pi_{t+1} \left(\frac{m_{t+1}}{m_t} \Pi_{t+1} - 1 \right) \left(\frac{m_{t+1}}{m_t} \right)^2 \right\} \end{aligned} \tag{A.5}$$

$$\chi L_t^{\varphi} = (1-\alpha) \varrho_t P_t^m \frac{Y_t}{L_t} \tag{A.6}$$

Retail bank:

$$R_t - R_t^d = \frac{R_t^d}{\epsilon^d} \left(\frac{\gamma_t}{\theta^d - \gamma_t} \right) - \theta^d (1-\rho_s) \beta \mathbb{E}_t \lambda_{t,t+1} \frac{R_{t+1}^d}{\epsilon^d} \left(\frac{\gamma_{t+1}}{\theta^d - \gamma_{t+1}} \right) \tag{A.7}$$

$$\gamma_t = \frac{D_t}{S_{t-1}} \tag{A.8}$$

$$S_{t-1} = \rho_s S_{t-2} + (1-\rho_s) D_{t-1} \tag{A.9}$$

Wholesale bank:

$$\eta_t = \beta \mathbb{E}_t \lambda_{t,t+1} [(1-\theta) R_t + \theta Z_{t+1} \eta_{t+1}] \tag{A.10}$$

$$\nu_t = (1-\theta) \beta \mathbb{E}_t \lambda_{t,t+1} (R_{t+1}^k - R_t) + \theta \beta \mathbb{E}_t \lambda_{t,t+1} X_{t+1} \nu_{t+1} \tag{A.11}$$

$$\phi_t = \frac{\eta_t}{\lambda - \nu_t} \tag{A.12}$$

$$Z_t = (R_t^k - R_{t-1}) \phi_{t-1} + R_{t-1} \tag{A.13}$$

$$X_t = \frac{\phi_t}{\phi_{t-1}} Z_t \tag{A.14}$$

$$Q_t^k K_t = \phi_t N_t \tag{A.15}$$

$$N_t = N_t^e + N_t^n \tag{A.16}$$

$$N_t^e = \theta Z_t N_{t-1} \epsilon_t^n \quad \text{where } \epsilon_t^n \text{ is exogenous shock to net worth} \tag{A.17}$$

$$N_t^n = \omega Q_t^k \xi_t K_{t-1} \quad \text{where } \xi_t \text{ is exogenous shock to capital quality} \tag{A.18}$$

Capital producer:

$$R_t^k = \frac{\left[\alpha P_t^m \frac{Y_t^m}{\xi_t K_{t-1}} + (Q_t^k - \delta_t)\right]\xi_t}{Q_{t-1}^k} \tag{A.19}$$

$$\begin{aligned} Q_t^k &= 1 + \frac{\eta^i}{2}\left(\frac{I_t^n + I^n}{I_{t-1}^n + I^n} - 1\right)^2 + \eta^i\left(\frac{I_t^n + I^n}{I_{t-1}^n + I^n} - 1\right)\left(\frac{I_t^n + I^n}{I_{t-1}^n + I^n}\right) \\ &\quad - \beta \mathbb{E}_t \lambda_{t,t+1}\eta^i\left(\frac{I_{t+1}^n + I^n}{I_t^n + I^n} - 1\right)\left(\frac{I_{t+1}^n + I^n}{I_t^n + I^n}\right)^2 \end{aligned} \tag{A.20}$$

$$\delta_t = \bar{\delta} - \frac{\tilde{\delta}}{1+\zeta} + \frac{\tilde{\delta}}{1+\zeta}U_t^{1+\zeta} \tag{A.21}$$

$$I_t^n = I_t - \delta_t \xi_t K_{t-1} \tag{A.22}$$

$$K_t = \xi_t K_{t-1} + I_t^n \tag{A.23}$$

$$\tag{A.24}$$

Production:

$$Y_t^m = A_t\Big(\xi_t U_t K_{t-1}\Big)^{\alpha} L_t^{1-\alpha} \tag{A.25}$$

$$\alpha P_t^m \frac{Y_t^m}{U_t} = \delta'(U_t)\xi_t K_{t-1} \tag{A.26}$$

$$\tag{A.27}$$

Pricing:

$$\varsigma_t = \gamma \varsigma_{t-1} P_{t-1}^{-\kappa\epsilon} P_t^{\epsilon} + (1-\gamma)\left[\frac{1}{1-\gamma}\left\{1 - \gamma P_{t-1}^{\kappa(1-\gamma)} P_t^{\gamma-1}\right\}\right]^{-\frac{\epsilon}{1-\gamma}} \tag{A.28}$$

$$\mu_t = \frac{1}{P_t^m} \tag{A.29}$$

$$F_t = Y_t P_t^m + \beta\gamma \mathbb{E}_t \lambda_{t,t+1} P_{t+1}^{\epsilon} P_t^{-\epsilon\kappa} F_{t+1} \tag{A.30}$$

$$H_t = Y_t + \beta\gamma \mathbb{E}_t \lambda_{t,t+1} P_{t+1}^{\epsilon-1} P_t^{(1-\epsilon)\kappa} H_{t+1} \tag{A.31}$$

$$P_t^* = \frac{\epsilon}{\epsilon - 1}\frac{F_t}{H_t}P_t \tag{A.32}$$

$$P_t^{1-\epsilon} = \gamma P_{t-1}^{(1-\epsilon)\kappa} + (1-\gamma)(P_t^*)^{(1-\epsilon)} \tag{A.33}$$

Government:

$$G_t = G\mu_t^g \quad \text{where } \mu_t^g \text{ is exogenous shock to government spending} \tag{A.34}$$

Aggregate resource:

$$Y_t = C_t + G_t + I_t + \frac{\eta^i}{2}\left(\frac{I_t^n + I^n}{I_{t-1}^n + I^n} - 1\right)^2 (I_t^n + I^n) \tag{A.35}$$

$$Y_t^m = \varsigma_t Y_t \tag{A.36}$$

$$\tag{A.37}$$

Central Bank:

$$\frac{R_t^n}{R} = \left(\frac{R_{t-1}^n}{R}\right)^{\rho_R}\left[\Pi_t^{\phi_\pi}\left(\frac{Y_t}{Y_t^f}\right)^{\phi_Y}\right]^{1-\rho_R}\exp(\varepsilon_t^R) \tag{A.38}$$

$$\frac{R_t^n}{\mathbb{E}_t\Pi_{t+1}} = R_{t+1} \tag{A.39}$$

$$DC_t = \phi_{DC}Y_t \tag{A.40}$$

B Derivation of total earning from deposit holding

$$\begin{aligned}
\int_0^1 R_{j,t-1}^d D_{j,t-1}dj &= \int_0^1 R_{j,t-1}^d\left[\left(\frac{R_{j,t-1}^d}{R_{t-1}^d}\right)^{-\epsilon^d} X_{t-1}^d + \theta^d S_{j,t-2}\right]dj \\
&= \int_0^1 R_{j,t-1}^d\left(\frac{R_{j,t-1}^d}{R_{t-1}^d}\right)^{-\epsilon^d} X_{t-1}^d dj + \theta^d\int_0^1 R_{j,t-1}^d S_{j,t-2}dj \\
&= \int_0^1 (R_{j,t-1}^d)^{1-\epsilon^d}(R_{t-1}^d)^{\epsilon^d} X_{t-1}^d dj + \Delta_{t-1}^d \\
&= (R_{t-1}^d)^{\epsilon^d} X_{t-1}^d\int_0^1 (R_{j,t-1}^d)^{1-\epsilon^d}dj + \Delta_{t-1}^d \\
&= (R_{t-1}^d)^{\epsilon^d} X_{t-1}^d (R_{t-1}^d)^{1-\epsilon^d} + \Delta_{t-1}^d \\
&= R_{t-1}^d X_{t-1}^d + \Delta_{t-1}^d
\end{aligned}$$

C Log-linearized version of the non-linear model

Household:

$$\hat{\varrho}_t = \frac{\beta h\sigma\mathbb{E}_t(\hat{C}_{t+1} - h\hat{C}_t)}{(1-\beta h)(1-h)} - \frac{\sigma\mathbb{E}_t(\hat{C}_t - h\hat{C}_{t-1})}{(1-\beta h)(1-h)} \tag{C.41}$$

$$\hat{\lambda}_{t,t+1} = \hat{\varrho}_{t+1} - \hat{\varrho}_t \tag{C.42}$$

$$-\iota(1-\beta R^d)\hat{X^d}_t = \hat{\varrho}_t - \beta R^d(\mathbb{E}_t\hat{\varrho}_{t+1} + \hat{R}_t^d - \hat{\pi}_{t+1}) \tag{C.43}$$

$$-\iota(1-\beta R^{dc})\hat{DC}_t = \hat{\varrho}_t - \beta R^{dc}(\mathbb{E}_t\hat{\varrho}_{t+1} + \hat{R}_t^{dc} - \hat{\pi}_{t+1}) \tag{C.44}$$

$$\begin{aligned}
\iota(1-\beta)\hat{m}_t &= \hat{\varrho}_t + \psi_m(\hat{m}_t + \hat{\pi}_t - \hat{m}_{t-1}) \\
&\quad - \beta\mathbb{E}_t\{\hat{\varrho}_{t+1} - \hat{\pi}_{t+1} + \psi_m(\hat{m}_{t+1} + \hat{\pi}_{t+1} - \hat{m}_t)\}
\end{aligned} \tag{C.45}$$

$$(1+\varphi)\hat{L}_t = \hat{\varrho}_t + \hat{P}_t^m + \hat{Y}_t \tag{C.46}$$

Retail bank:

$$\begin{aligned}
\left(\frac{1-(1-\rho_s)\beta\theta^d}{R-R^d}\right)(R\hat{R}_t - R^d\hat{R}_t^d) &= \hat{R}_t^d - \hat{\epsilon}_t^d + \left(\frac{\theta^d}{\theta^d-\gamma^d}\right)(\hat{\gamma}_t^d - \hat{\theta}_t^d) \\
&\quad - (1-\rho_s)\beta\theta^d\left[\hat{R}_{t+1}^d + \hat{\theta}_{t+1}^d + \hat{\lambda}_{t,t+1} - \hat{\epsilon}_{t+1}^d + \left(\frac{\theta^d}{\theta^d-\gamma^d}\right)(\hat{\gamma}_{t+1}^d - \hat{\theta}_{t+1}^d)\right]
\end{aligned} \tag{C.47}$$

$$\hat{\gamma}_t^d = \hat{D}_t - \hat{S}_{t-1}^d \tag{C.48}$$

$$\hat{S}_{t-1} = \rho_s\hat{S}_{t-2} + (1-\rho_s)\hat{D}_{t-1} \tag{C.49}$$

Wholesale bank:

$$\hat{\eta}_t = \hat{\lambda}_{t,t+1} + (1-\theta\beta Z)\hat{R}_t + \theta Z\beta(\hat{Z}_{t+1} + \hat{\eta}_{t+1}) \tag{C.50}$$

$$\hat{\nu}_t = \hat{\lambda}_{t,t+1} + \frac{(1-\theta\beta X)}{R^k - R}[R^k\hat{R}^k_{t+1} - R\hat{R}_t] + \theta\beta X[\hat{X}_{t+1} + \hat{\nu}_{t+1}] \tag{C.51}$$

$$\hat{\phi}_t = \hat{\eta}_t + \left(\frac{\nu}{\lambda-\nu}\right)\hat{\nu}_t \tag{C.52}$$

$$\hat{Z}_t = \left[\frac{\phi(R^k-R)}{Z}\right]\hat{\phi}_{t-1} + \left[\frac{\phi R^k}{Z}\right]\hat{R}^k_t + \left[\frac{(1-\phi)R}{Z}\right]\hat{R}_{t-1} \tag{C.53}$$

$$\hat{X}_t = \hat{\phi}_t - \hat{\phi}_{t-1} + \hat{Z}_{t-1} \tag{C.54}$$

$$\hat{Q}^k_t + \hat{K}_t = \hat{\phi}_t + \hat{N}_t \tag{C.55}$$

$$\hat{N}_t = \frac{N^e}{N}\hat{N}^e_t + \frac{N^n}{N}\hat{N}^n_t \tag{C.56}$$

$$\hat{N}^e_t = \hat{Z}_t + \hat{N}_{t-1} + \hat{\epsilon}^n_t \tag{C.57}$$

$$\hat{N}^n_t = \hat{Q}^k_t + \hat{K}_{t-1} + \hat{\xi}_t \tag{C.58}$$

$$\tag{C.59}$$

Capital producer:

$$\hat{R}^k_t = \left(\frac{\alpha P^m Y^m}{R^k K}\right)(\hat{P}^m_t + \hat{Y}^m_t - \hat{K}_{t-1}) + \frac{\hat{Q}^k_t}{R^k} - \hat{Q}^k_{t-1} - \left(\frac{\delta}{R^k}\right)\hat{\delta}_t + \frac{(1-\delta)}{R^k}\hat{\xi}_t \tag{C.60}$$

$$\tag{C.61}$$

Production:

$$\hat{Y}^m_t = \hat{A}_t + \alpha(\hat{U}_t + \hat{K}_{t-1} + \hat{\xi}_t) + (1-\alpha)\hat{L}_t \tag{C.62}$$

$$\hat{Y}^m_t + \hat{P}^m_t = (1+\zeta)\hat{U}_t + \hat{K}_{t-1} + \hat{\xi}_t \tag{C.63}$$

Expressing the deposit spread: Define $\mu^d_t = \frac{r^d_t}{r_t}$ as the deposit markdown on the risk-free interest rate. Then the deposit spread equation becomes

$$1 - \mu^d_t = \left(\frac{\mu^d_t}{\epsilon^d_t}\right)\left(\frac{\gamma_t}{\theta^d_t - \gamma_t}\right) - \theta^d_{t+1}(1-\rho_s)\beta\mathbb{E}_t\lambda_{t,t+1}\left(\frac{r_{t+1}}{r_t}\right)\left(\frac{\mu^d_{t+1}}{\epsilon^d_{t+1}}\right)\left(\frac{\gamma_{t+1}}{\theta^d_{t+1} - \gamma_{t+1}}\right)$$

Log-linearization around the deterministic steady state obtains

$$\begin{aligned}[1-\beta\theta^d(1-\rho_s)]\left(\frac{\mu_d}{\mu_d - 1}\right)\hat{\mu}^d_t &= (\hat{\mu}^d_t - \hat{\epsilon}^d_t) + \frac{1}{\theta^d - \gamma}(\hat{\gamma}_t - \hat{\theta}^d_t) \\ &\quad - \beta(1-\rho_s)\left(\frac{\theta^d}{\theta^d - 1}\right)\left\{\hat{\theta}^d_{t+1} + \hat{\lambda}_{t,t+1} + \hat{\mu}^d_{t+1} - \hat{\epsilon}^d_{t+1} + \hat{r}_{t+1} - \hat{r}_t + \theta^d(\hat{\gamma}_{t+1} - \hat{\theta}^d_{t+1})\right\}\end{aligned}$$

$$\begin{aligned}&[1-\beta\theta^d(1-\rho_s)]\left(\frac{\mu_d}{\mu_d - 1}\right)\hat{\mu}^d_t - \hat{\mu}^d_t = (-\hat{\epsilon}^d_t) + \frac{1}{\theta^d - 1}(\hat{\gamma}_t - \hat{\theta}^d_t) \\ &- \beta(1-\rho_s)\left(\frac{\theta^d}{\theta^d - 1}\right)\left\{(1-\theta^d)\hat{\theta}^d_{t+1} + \hat{\lambda}_{t,t+1} + \hat{\mu}^d_{t+1} - \hat{\epsilon}^d_{t+1} + \hat{r}_{t+1} - \hat{r}_t + \theta^d(\hat{\gamma}_{t+1})\right\}\end{aligned}$$

$$
\begin{aligned}
\mu^d\left[\frac{1}{(1-\theta^d)\epsilon^d}-1\right]\hat{\mu}_t^d = & \\
& + \frac{\mu^d\theta^d}{\epsilon^d(\theta^d-1)^2}\{\hat{\gamma}_t-\beta(1-\rho_s)\theta^d\hat{\gamma}_{t+1}\} \\
& + \frac{\mu^d\theta^d}{\epsilon^d(\theta^d-1)^2}\left[(1-\rho_s)\beta\hat{\theta}_{t+1}^d-\hat{\theta}_t^d\right] \\
& - \beta(1-\rho_s)\left(\frac{\mu^d}{\epsilon^d}\right)\left(\frac{\theta^d}{\theta^d-1}\right)\hat{\lambda}_{t,t+1} \\
& - \beta(1-\rho_s)\left(\frac{\mu^d}{\epsilon^d}\right)\left(\frac{\theta^d}{\theta^d-1}\right)(\hat{r}_{t+1}-\hat{r}_t) \\
& - \beta(1-\rho_s)\left(\frac{\mu^d}{\epsilon^d}\right)\left(\frac{\theta^d}{\theta^d-1}\right)\hat{\mu}_{t+1}^d \\
& + \left(\frac{\mu^d}{\epsilon^d(\theta^d-1)}\right)\left[(1-\rho_s)\beta\hat{\theta}^d\hat{\epsilon}_{t+1}^d-\hat{\epsilon}_t^d\right]
\end{aligned}
$$

or,

$$
\begin{aligned}
\left[1-(1-\theta^d)\epsilon^d\right]\hat{\mu}_t^d = & \\
& + \frac{\theta^d}{1-\theta^d}\{\hat{\gamma}_t-\beta(1-\rho_s)\theta^d\,\hat{\gamma}_{t+1}\} \\
& + \frac{\theta^d}{1-\theta^d}\left[(1-\rho_s)\beta\,\hat{\theta}_{t+1}^d-\hat{\theta}_t^d\right] \\
& + \beta(1-\rho_s)\theta^d\,\hat{\lambda}_{t,t+1} \\
& + \beta(1-\rho_s)\theta^d\,(\hat{r}_{t+1}-\hat{r}_t) \\
& + \beta(1-\rho_s)\theta^d\,\hat{\mu}_{t+1}^d \\
& + \left[\hat{\epsilon}_t^d-(1-\rho_s)\beta\theta^d\,\hat{\epsilon}_{t+1}^d\right]
\end{aligned}
\tag{C.64}
$$