```python
import matplotlib.pyplot as plt
import pulp
import math
import random
import numpy as np
```

Hints for students: The utility section contains code you need to use *unchanged* to generate the test data required. You may use this code for your final solution, beware not to overwrite function definitions in this section. Otherwise you can ignore the code in this section.

Utilities

Note: Section 1.1-1.4 is identical to the MST example (Week 3). 1.5-1.10 are adjustments of the data structures for the current problem.

Points and Distances

```python
resolution = 10000  # city of about 10km diameter

def dist(p1, p2):
    (x1, y1) = p1
    (x2, y2) = p2
    return int(math.sqrt((x1-x2)**2+(y1-y2)**2))

def nearest(X, P):
    minD = math.inf
    minP = None
    for p in P:
        d=dist(X, p)
        if d<minD:
            minP, minD = p, d
    return minP

def generatePoints(n):

    def gen():
        x = int(random.gauss(mu=resolution/2, sigma=resolution/8))
        return min(resolution-50, max(50, x))

    mindist = resolution/(2*math.sqrt(n))
    # avoid points to neart to each other
    P = []
    while len(P)<n:
        i=len(P)

        x0=gen()
        y0=gen()
```

```
        # don't place new points near existing points
        P.append((x0,y0))
        for j in range(0,i):
            if dist(P[i],P[j])<mindist:
                P=P[:-1]
                break
    return P
```

Lines

Lines are representes as pairs of Points, but there is no preferred order of points.

```
def equalLines(l1, l2):
    if l1==l2:
        return True
    else:
        return l1[0]==l2[1] and l1[1]==l2[0]

def rev(L):
    return L[1],L[0]

def solve(x11, x12, y1, x21, x22, y2):

    def Det(x11, x12, x21, x22):
        return x11*x22-x21*x12

    D = Det(x11, x12, x21, x22)
    Ds = Det(y1, x12, y2, x22)
    Dt = Det(x11, y1, x21, y2)
    if D==0:
        return False
    s=Ds/D
    t=Dt/D
    return 0 <= s and s <= 1 and 0 <= t and t <= 1

def intersecting(l1, l2):
    p1, p2 = l1
    q1, q2 = l2
    if p1==q1 or p1==q2 or p2==q1 or p2==q2:
        return False
    xp1, yp1 = p1
    xp2, yp2 = p2
    xq1, yq1 = q1
    xq2, yq2 = q2
    return solve(xp2-xp1, xq1-xq2, xq1-xp1,
                 yp2-yp1, yq1-yq2, yq1-yp1)
```

Triangles

```python
def equalTriangles(t1, t2):
    P1, P2, P3 = t1
    Q1, Q2, Q3 = t2
    if P1==Q1:
        if P2==Q2:
            return P3==Q3
        elif P2==Q3:
            return P3==Q2
        else:
            return False
    elif P1==Q2:
        if P2==Q1:
            return P3==Q3
        elif P2==Q3:
            return P3==Q1
        else:
            return False
    elif P1==Q3:
        if P2==Q1:
            return P3==Q2
        elif P2==Q2:
            return P3==Q1
        else:
            return False
    else:
        return False

def removeTriangle(t, T):
    for tt in T:
        if equalTriangles(t, tt):
            T.remove(tt)
            return True
    return False

def addTriangle(t, T):
    for tt in T:
        if equalTriangles(t, tt):
            return
    T.append(t)

def area(p1, p2, p3):
    x1, y1 = p1
    x2, y2 = p2
    x3, y3 = p3
    return abs(x1*(y2-y3)+x2*(y3-y1)+x3*(y1-y2))/2

def insideTriangle(x, t):
    p, q, r = t
```

```
    if x==p or x==q or x==r:
        return False
    return abs(area(p,q,x) + area(p,r,x)+ area(q,r,x) -
area(p,q,r))<0.00001

def sides(t):
    A, B, C = t
    return [(B, C), (C, A), (A, B)]

def longestSide(t):
    return sorted(sides(t), key=lambda s: dist(s[0], s[1]),
reverse=True)[0]

def commonSide(t1, t2):
    S1 = sides(t1)
    S2 = sides(t2)
    for s1 in S1:
        for s2 in S2:
            if equalLines(s1, s2):
                return s1
    return None

def oppositePoint(t, l):
    A, B, C = t
    P, Q = l
    if A==P:
        return C if B==Q else B
    elif A==Q:
        return C if B==P else B
    else:
        return A

def intersectingTriangles(c1, c2):
    c= commonSide(c1, c2)
    if c is None:
        return False
    A, B = c
    C1 = oppositePoint(c1, c)
    C2 = oppositePoint(c2, c)
    return intersecting((A, C1), (B,C2)) or \
            intersecting((A, C2), (B, C1))

def intersectingLines(c1, c2):
    c= commonSide(c1, c2)
    if c is None:
        return None
    A, B = c
    C1 = oppositePoint(c1, c)
    C2 = oppositePoint(c2, c)
    if intersecting((A, C1), (B,C2)):
```

```
        return (A, C1), (B,C2)
    elif intersecting((A, C2), (B, C1)):
        return (A, C2), (B, C1)
    else:
        return None

def defuse(t1, t2):
    # is only called if intersectingTriangles(t1, t2)
    l1, l2 = intersectingLines(t1, t2)
    c = commonSide(t1, t2)
    if c is None:
        return None
    A, B = c
    C1 = oppositePoint(t1, c)
    C2 = oppositePoint(t2, c)
    if intersecting((A, C1), (B,C2)):
        return [C1, C2, A]
    elif intersecting((A, C2), (B, C1)):
        return [C1, C2, B]

def slimTriangle(t):
    A, B, C = t
    a = dist(B,C)
    b = dist(A,C)
    c = dist(A,B)
    [a, b, c] = sorted([a, b, c])
    return a+b<1.07*c

def plotTriangle(t, style='r-o', lw=1, ms=3):
    p1, p2, p3 = t
    plt.plot( [ p1[0], p2[0], p3[0], p1[0] ],
              [ p1[1], p2[1], p3[1], p1[1] ],
              style, lw=lw, ms=ms)

def plotTriangles(T, style='r-o', lw=1, ms=3):
    plt.xlim(0,resolution)
    plt.ylim(0,resolution)
    plt.axis('off')
    for t in T:
        plotTriangle(t, style, lw=lw, ms=ms)
    plt.show()
```

Triangulation

The triangulation algorithm is an only slightly modified version as documented by:

Agryzkov T., Oliver J.L., Tortosa L., Vicent J.F. (2014) A Method to Triangulate a Set of Points in the Plane. In: Murgante B. et al. (eds) Computational Science and Its Applications – ICCSA 2014. ICCSA 2014. Lecture Notes in Computer Science, vol 8580. Springer, Cham DOI 10.1007/978-3-319-09129-7_25

```
def triangulation(P, show=False):
    x = [ p[0] for p in P ]
    y = [ p[1] for p in P ]
    minx = min(x)
    maxx = max(x)
    miny = min(y)
    maxy = max(y)
    nmaxx = 1.1 * maxx
    nmaxy = 1.1 * maxy
    nminx = max(0.5 * minx, 0)
    nminy = max(0.5 * miny, 0)
    distx = nmaxx-nminx
    disty = nmaxy-nminy
    d = math.sqrt(distx*disty/(2*len(P)))
    dx = math.ceil(distx/d)
    dy = math.ceil(disty/d)


    def kx(i):
        return nminx+i*d

    def ky(j):
        return nminy+j*d

    def k(i, j):
        return (kx(i), ky(j))

    # def n(i, j):
    #     return nearest( (kx(i), ky(j)), P)
    def n(i, j):
        X = (kx(i), ky(j))
        minD = math.inf
        minP = None
        for p in P:
            d=dist(X, p)
            if d<minD:
                minP, minD = p, d
        return minP

    if show:

        plt.figure(0)
        plt.xlim(0,resolution)
        plt.ylim(0,resolution)
        plt.axis('off')

        plt.plot([ p[0] for p in P ],
                 [ p[1] for p in P ], 'ro', ms=10)

        for i in range(0, dx+1):
```

```
        for j in range(0, dy):
            plt.plot( [ kx(i), kx(i) ],
                      [ ky(j), ky(j+1) ], 'b:.')

    for j in range(0, dy+1):
        for i in range(0, dx):
            plt.plot( [ kx(i), kx(i+1) ],
                      [ ky(j), ky(j) ], 'b:.')

    for i in range(0, dx):
        for j in range(0, dy):
            plt.plot( [ kx(i), kx(i+1) ],
                      [ ky(j), ky(j+1) ], 'b:.')

    for i in range(0, dx+1):
        for j in range(0, dy+1):
            nx, ny = n(i,j)
            plt.plot( [ kx(i), nx ],
                      [ ky(j), ny ], 'g:')



# set of triangles
T = []
for i in range(0, dx):
    for j in range(0, dy):
        p1=n(i,j)
        p2=n(i+1, j)
        p3=n(i, j+1)
        p4=n(i+1, j+1)
        if p1!=p2 and p2!=p4 and p4!=p1:
            addTriangle([p1, p2, p4], T)
        if p1!=p3 and p3!=p4 and p4!=p1:
            addTriangle([p1, p3, p4], T)

# Replace a triangle containing an inner point
# with three triangles formed from this inner point
C = T.copy()
for p in P:
    for t in T:
        if insideTriangle(p, t):
            p1, p2, p3 = t
            C.remove(t)
            C.append( (p1, p2, p) )
            C.append( (p2, p3, p) )
            C.append( (p3, p1, p) )

PLOT = 0

def showTriangulation(T, style='r-o', check=False):
```

```python
        nonlocal PLOT
        PLOT += 1
        plt.figure(PLOT)
        plt.xlim(0,resolution)
        plt.ylim(0,resolution)
        plt.axis('off')
        for t in T:
            plotTriangle(t, 'r-o', lw=0.5)

        if check:
            found=False
            for t1 in T:
                for t2 in T:
                    if t1!=t2 and intersectingTriangles(t1, t2):
                        plotTriangle(t1, 'b-o')
                        plotTriangle(t2, 'g-o')
                        found=True
                        break
                if found:
                    break

    found = True
    while found:

        if show:
            showTriangulation(C)

        D=C.copy()
        found = False
        for c1 in C:
            for c2 in C:
                if c1!=c2 and intersectingTriangles(c1, c2):
                    # print(f"replacing")
                    # print(triangle_String(c1))
                    # print(triangle_String(c2))
                    c3=defuse(c1, c2)
                    removeTriangle(c2, D)
                    # print(triangle_String(c3))
                    addTriangle(c3, D)
                    found=True
                    break
            if found:
                break
        C=D

    # Check for isolated points
    # This can now probably not happen anymore,
    # but just leave it in for now

    def isolated(p, C):
```

```
        for c in C:
            p1, p2, p3 = c
            if p==p1 or p==p2 or p==p3:
                return False
        return True

    for p in P:
        if isolated(p, D):
            plt.plot( [ p[0] ], [ p[1] ], 'ko', ms=15)

    return C
```

Graphs

```
def element(p, S):
    for s in S:
        if s==p: # s[0]==p[0] and s[1]==p[1]:
            return True
    return False

def vertices(T):
    S = []
    for t in T:
        A, B, C = t
        if not element(A, S):
            S.append(A)
        if not element(B, S):
            S.append(B)
        if not element(C, S):
            S.append(C)
    return sorted(S, key=lambda p: p[0])

def edges(T):
    S = []
    for t in T:
        A, B, C = sides(t)
        if not element(A, S) and not element(rev(A), S):
            S.append(A)
        if not element(B, S) and not element(rev(B), S):
            S.append(B)
        if not element(C, S) and not element(rev(C), S):
            S.append(C)
    return sorted(S, key=lambda p: p[0][0])

def graph(T):
    return vertices(T), edges(T)
```

Lists and Paths

```python
def pathLength(P):
    return 0 if len(P)<=1 else \
            dist(P[0], P[1])+pathLength(P[1:])

def reverse(P):
    return [ P[-i] for i in range(1,len(P)+1) ]

def index(x, L):
    for i in range(len(L)):
        if x==L[i]:
            return i
    return None

def addWithoutDuplicates(L, X):
    for i in range(len(X)):
        if X[i] not in L:
            L.append(X[i])
    return L
```

Maps

```python
def createMap(P, p=0.2):
    T = triangulation(P)
    V, E = graph(T)

    for t in T:
        if slimTriangle(t):
            s = longestSide(t)
            if s in E:
                E.remove(s)
            elif (s[1], s[0]) in E:
                E.remove((s[1], s[0]))
        else:
            for tt in T:
                if not equalTriangles(t, tt):
                    s = commonSide(t, tt)
                    if s is None:
                        continue
                    if random.random()<p:
                        if s in E:
                            E.remove(s)
                            break
                        elif (s[1], s[0]) in E:
                            E.remove((s[1], s[0]))
                            break

    return V, E
```

Place Warehouse Somewhere Inside

```
def placeWarehouse(M):
    V, _ = M
    dmin = math.inf
    pos = (random.randint(resolution/4, resolution*3/4),
            random.randint(resolution/4, resolution*3/4))
    for p in V:
        dp = dist(p, pos)
        if dp<dmin:
            w = p
            dmin = dp
    return w
```

Generate Delivery Points

```
def splitEdge(V, E, s):
    A, B = s
    p = random.uniform(0.2,0.8)
    x = int(A[0]+p*(B[0]-A[0]))
    y = int(A[1]+p*(B[1]-A[1]))
    t = (x,y)
    E.remove(s)
    E.append((A, t))
    E.append((t, B))
    V.append(t)
    return (V, E), t

def addTargets(M, n=5):
    V, E = M
    V, E = V.copy(), E.copy()
    T = []
    # we want to ensure that the beginning of the
    # sequence of points generated randomly stays
    # the same
    mindist = 0.05
    while len(T)<n:
        S = random.sample(E, 1)
        s = S[0]
        A, B = s
        if dist(A,B)>resolution/20: # avoid targets placed narrowly
            (V, E), t = splitEdge(V, E, s)
            T.append(t)
    return (V, E), T
```

Plot Map with Delivery Route

```
def plotMap(G, T=[], P=[], w=None,
            style='r-o', lw=1, ms=3,
```

```
            styleT='go', msT=5,
            styleP='b-o', lwP=3, msP=1,
            stylePT='go', msPT=7,
            styleW='bo', msW=7,
            text=None, grid=False):
    fig = plt.gcf()
    fig.set_size_inches(6, 6)
    plt.xlim(0,resolution)
    plt.ylim(0,resolution)
    if not grid:
        plt.axis('off')
    V, E = G
    for e in E:
        p1, p2 = e
        plt.plot( [ p1[0], p2[0] ],
                  [ p1[1], p2[1] ],
                  style, lw=lw, ms=ms)
    for t in T:
        plt.plot( [ t[0] ], [ t[1] ],
                  styleT, ms=msT)
    plt.plot( [ p[0] for p in P ],
              [ p[1] for p in P ],
              styleP, lw=lwP, ms=msP)
    for p in P:
        if p in T:
            plt.plot( [ p[0] ], [ p[1] ],
                      stylePT, ms=msPT)
    if w is not None:
        plt.plot( [ w[0] ], [ w[1] ],
                      styleW, ms=msW)
    if text is not None:
        plt.text(0.8*resolution, 0, text)
    if grid:
        plt.grid()
    plt.show()
```

Generate Data

```
def generateData(seed=None, nodes=50, customers=100,
                 plot=False, log=False):

    if seed is None:

        print("Usage:  M, W, C = generateData(seed=None, plot=False, 
log=False)")
        print("")
        print("  seed  the seed value to be used for data generation. 
")
        print("        To test the application use seed=0, it will
```

```
create")
        print("        a small map, with a very few customer locations
and")
        print("        a small set of delivery data.")
        print("        For the generating proper simulation data, use
the last")
        print("        four digits of your student ID as seed value.")
        print("")
        print("  log   Controls print output during data generation.")
        print("")
        print("  plot  Controls graphical output during data
generation.")
        print("")
        print("Returns:")
        print("")
        print("  M = (V, E) is the generated map given as a graph")
        print("    where V is a list of vertices, with each vertice ")
        print("    given as a pair (x, y) of integer coordinates, ")
        print("    and E is a list of edges, with each edge given")
        print("    as a pair (A, B) of vertices, with each vertex
again")
        print("    given as a pair (x, y) of integer coordinates")
        print("")
        print("  W ∈ V  is the location of the distribution
warehouse")
        print("    given as a pair (x, y) of integer coordinates")
        print("")
        print("  C ⊆ V  is a list of customer locations")
        print("    given as pairs (x, y) of integer coordinates")
        print("    len(C) gives the number of customers generated")
        print("")

        seed = 0

    if seed==0:           # generate very simple test data
        nodes = 11        # number of points in map
        customers = 6     # number of  customers
        grid = True

    else:
        grid = False

    random.seed(seed)

    P = generatePoints(nodes)

    M = createMap(P)
    W = placeWarehouse(M)
    MT, C = addTargets(M, customers)
```

```
    if log:
        print(f"Generated map with {nodes:d} nodes and "
              f"{customers:d} customer locations")
    if plot:
        label="" if seed==0 else f"seed={seed:4d}"
        plotMap(MT, T=C, w=W, text=label, grid=grid)

    return MT, W, C
```

Data Generation is reproducible

```
D1 = generateData(2101)
D2 = generateData(2101)
D1 == D2
```

```
True
```

Generate Data

This section demonstrates how you can generate the test data for the problem.

General Help Message

If you use `generateData()` without any parameters you will get a general help message.

```
M, W, C = generateData()
```

```
Usage:  M, W, C = generateData(seed=None, plot=False, log=False)

  seed  the seed value to be used for data generation.
        To test the application use seed=0, it will create
        a small map, with a very few customer locations and
        a small set of delivery data.
        For the generating proper simulation data, use the last
        four digits of your student ID as seed value.

  log   Controls print output during data generation.

  plot  Controls graphical output during data generation.

Returns:

  M = (V, E) is the generated map given as a graph
    where V is a list of vertices, with each vertice
    given as a pair (x, y) of integer coordinates,
    and E is a list of edges, with each edge given
    as a pair (A, B) of vertices, with each vertex again
```

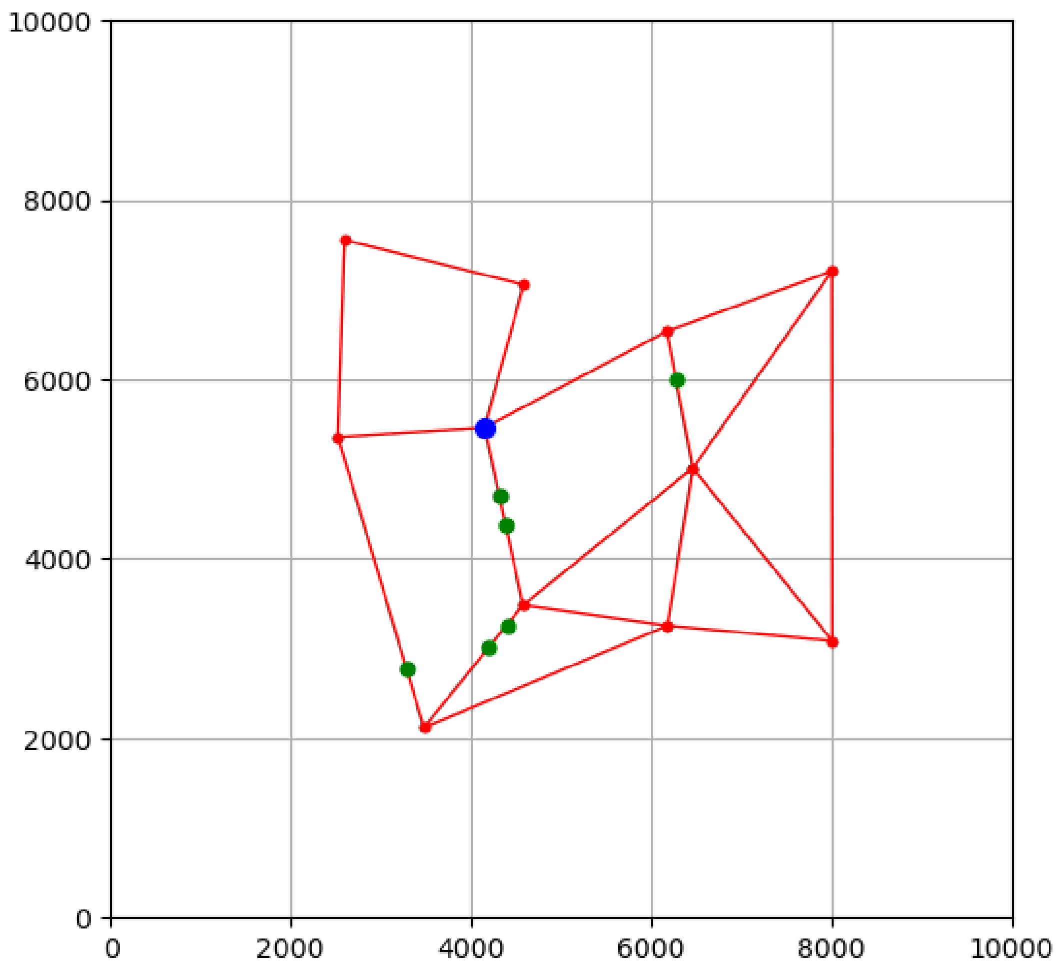
```
    given as a pair (x, y) of integer coordinates

  W ∈ V  is the location of the distribution warehouse
    given as a pair (x, y) of integer coordinates

  C ⊆ V  is a list of customer locations
    given as pairs (x, y) of integer coordinates
    len(C) gives the number of customers generated
```

Analysing Simple Test Data

This section illustrates the data structure generated.

```
M, W, C = generateData(seed=0, log=True, plot=True)
```

```
Generated map with 11 nodes and 6 customer locations
```



The Graph

You can identify the points in the grid above. The vertices of the graph are:

```
V, E = M
V

[(2523, 5360),
 (2596, 7555),
 (3477, 2118),
 (4150, 5463),
 (4575, 3484),
 (4582, 7057),
 (6171, 6537),
 (6177, 3254),
 (6458, 5010),
 (8001, 7207),
 (8005, 3086),
 (6271, 6001),
 (3285, 2769),
 (4311, 4709),
 (4197, 3014),
 (4397, 3263),
 (4383, 4371)]
```

The edges of the graph are:

```
E

[((2523, 5360), (2596, 7555)),
 ((2596, 7555), (4582, 7057)),
 ((4150, 5463), (2523, 5360)),
 ((4150, 5463), (4582, 7057)),
 ((4150, 5463), (6171, 6537)),
 ((4575, 3484), (6177, 3254)),
 ((6177, 3254), (3477, 2118)),
 ((6177, 3254), (6458, 5010)),
 ((6458, 5010), (4575, 3484)),
 ((6458, 5010), (8005, 3086)),
 ((6458, 5010), (8001, 7207)),
 ((8001, 7207), (6171, 6537)),
 ((8001, 7207), (8005, 3086)),
 ((8005, 3086), (6177, 3254)),
 ((6458, 5010), (6271, 6001)),
 ((6271, 6001), (6171, 6537)),
 ((3477, 2118), (3285, 2769)),
 ((3285, 2769), (2523, 5360)),
 ((4311, 4709), (4150, 5463)),
 ((4197, 3014), (3477, 2118)),
 ((4575, 3484), (4397, 3263)),
```

```
 ((4397, 3263), (4197, 3014)),
 ((4575, 3484), (4383, 4371)),
 ((4383, 4371), (4311, 4709))]
```

Customer Addresses

The customer addresses (green dots in the map) are:

```
C
```

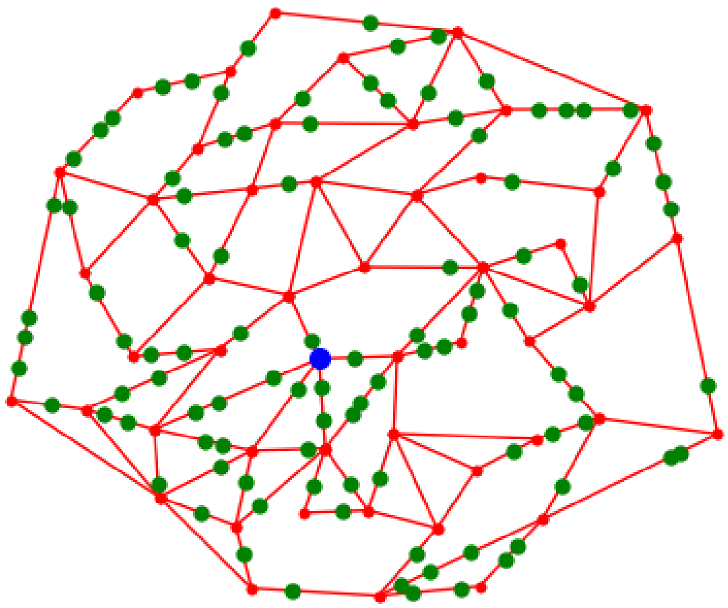
```
[(6271, 6001),
 (3285, 2769),
 (4311, 4709),
 (4197, 3014),
 (4397, 3263),
 (4383, 4371)]
```

Real Sample Data

This section shows sample data as you you may get them for your required simulation.

```
_ = generateData(1234, plot=True, log=True)
```

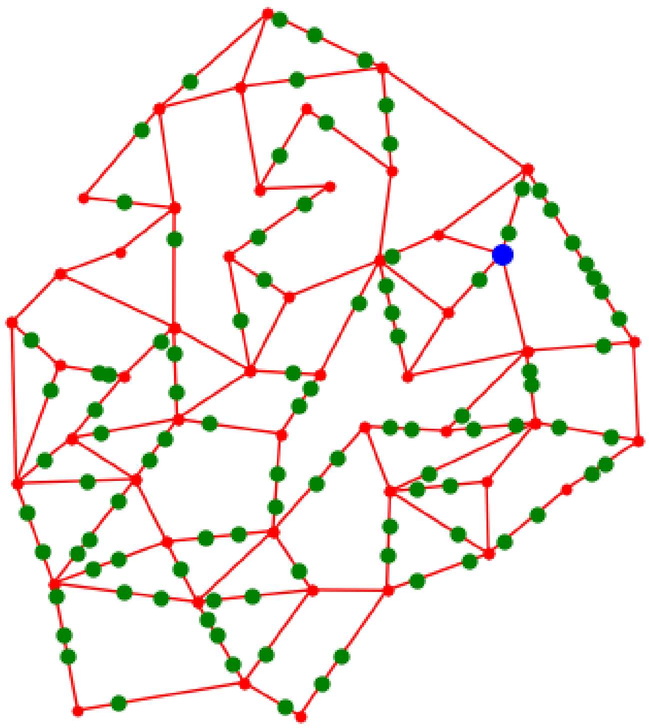
```
Generated map with 50 nodes and 100 customer locations
```



seed=1234

```
data = generateData(2101, plot=True, log=True)
```

```
Generated map with 50 nodes and 100 customer locations
```



seed=2101

Save sample data as pickle file:

```
import pickle
with open('/Users/debmalyadeb/Documents/NCI_Learning/Semester
2/Modelling, Simulation and Optimisation /CA 60%/Final CA1 Submission
code/2 Data Files/data.pickled', 'wb') as f:
    pickle.dump(data, f)

sampleData = generateData(seed=0)

import pickle
with open('/Users/debmalyadeb/Documents/NCI_Learning/Semester
2/Modelling, Simulation and Optimisation /CA 60%/Final CA1 Submission
code/2 Data Files/sampleData.pickled', 'wb') as f:
    pickle.dump(sampleData, f)
```