LAB 4 - Principal axes of a given cross-section in a thin-walled beam
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AE351
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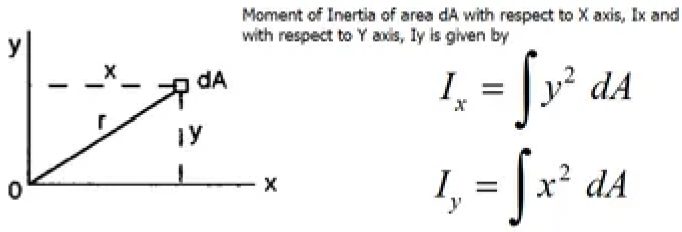
- **OBJECTIVE:**

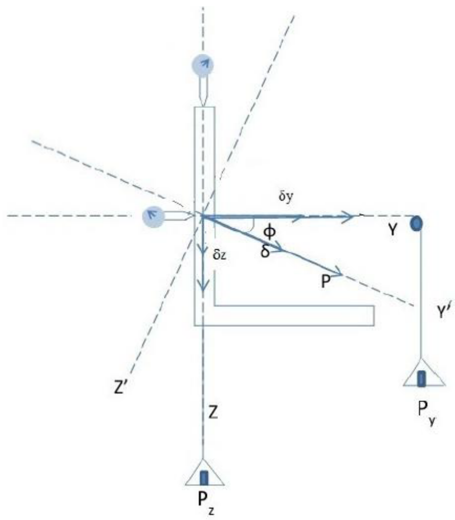
  To determine the principal axes and the orientation of principal planes of an L section beam. Determine the orientation of principal axes from the four sets of data included in the attached file. Compare your result with theoretical calculations

- **INTRODUCTION AND THEORY:**



Our target is to find the principal centroidal axis in an unsymmetrical beam bending case by doing experiments.

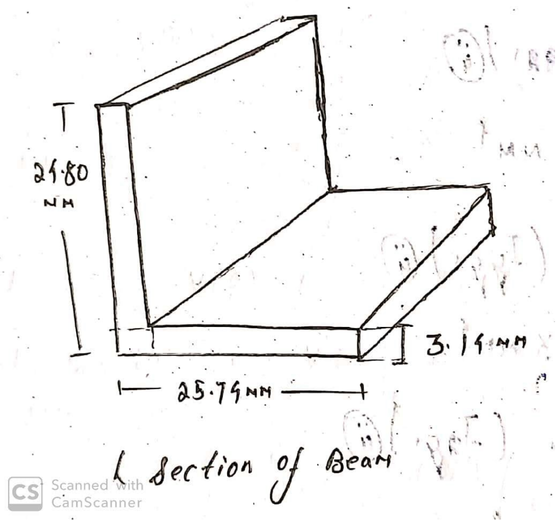
- ***The principal centroidal axis*** is the axis about which the Area Moment of Inertia is either maximum or minimum. For a symmetrical CrossSection body the Horizontal and the vertical axis is the principal centroidal axis. Area moment of inertia decides how stiff/compliant will be your structure for a specific geometry. The moment of inertia of a section depends upon the reference axes as well as the orientation of the axes about the origin. The variation of MOI wrt the axis location is governed by the parallel Axis theorem.



- If the origin of the axis is located at the centroid of the section then the axis of symmetry/neutral axis coincides with the Principal axes.

- Asymmetric Bending: If the loading doesn't act on the plane of symmetry then the deflection also occurs perpendicular to the loading direction besides in the loading direction. The coupling between deflection in loading and its perpendicular direction can be avoided if the loading direction coincides with principal axes

- Z sections and L sections are widely used in Aerospace engineering because we want to reduce the weight but at the same time, we don't want to compromise with the stiffness.



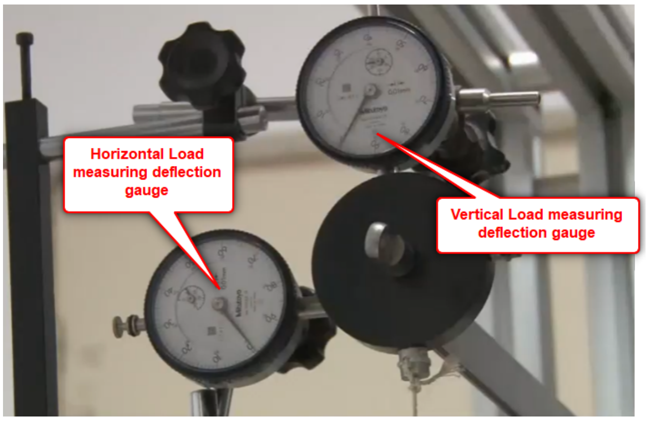
Theoretical formula: $tan(2\phi) = \frac{-2Iyz}{Iyy-Izz}$

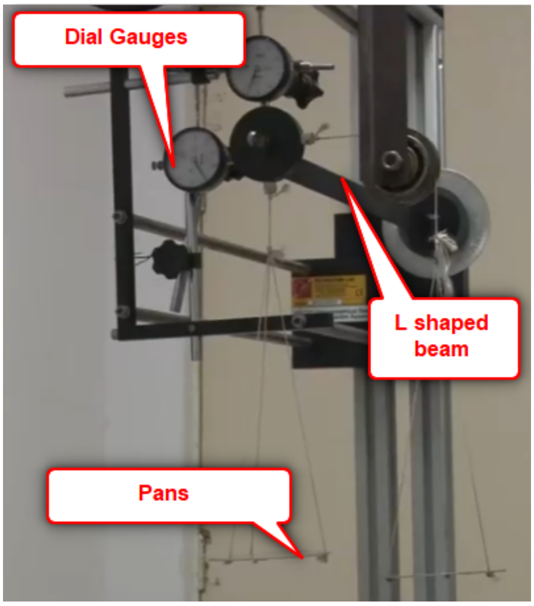
Experimental formula: $tan(\phi) = \frac{Pz}{Py} = \frac{\delta z}{\delta y}$

- **EQUIPMENT AND OPERATING CONDITIONS:**

The experimental setup includes:

1. L -Cross-section beam subjected to the two loads - one vertical and another one horizontal. The horizontal load is provided with the help of the frictionless pulley. The combined load provides a moment
2. Two dial gauges- to capture the horizontal as well as the vertical deflection. The dial gauge is to be kept at zero.
3. Two pans to hold the weights (that will act as a force)





- Some Important Dimensions:

Dial Gauge	
Least Count of the Dial Gauge	0.01mm
No. of divisions in one round	100div
L section of Beam	
Thickness	3.14mm
Web	24.80mm
Flange	25.74mm

- **PROCEDURE :**

i) Measure the thickness of the web and flange of the L section. Also measure the length of the flange and the height of the web to determine the values of Izz, Iyy, Iyz.
ii) Adjust the dial gauges to remove any zero error while supporting the pans with your hands to have the no-load initial setup.

iii) Fix the y-direction load Py, and for some random z-direction load Pz, note the beam deflections δy and δz
iv) Increase the loads in each of the pans and calculate the ratio of loads and the ratio of deflections produced. They should be almost equal i.e., the difference between these two ratios should be very small.
v) Repeat the steps above for different values of Py and Pz.

Collected Data: [we are using the 4dataset provide to us]

S. No.	Py(N)	Pz(N)	δ_y(mm)	δ_z(mm)	Pz/Py	δ_z/δ_y
1.1	5	0.98	0.25	0.06	0.196	0.240
1.2	5	1.96	0.23	0.04	0.392	0.174
1.3	5	3.92	0.18	0.08	0.784	0.444
1.4	5	5	0.15	0.15	1	1
1.5	5	6.96	0.1	0.30	1.392	3.00
1.6	6.96	5	0.23	0.14	0.718	0.609
1.7	8.92	5	0.31	0.11	0.561	0.355
1.8	8.92	6.96	0.3	0.20	0.780	0.667

S. No.	Py(N)	Pz(N)	δ_y(mm)	δ_z(mm)	Pz/Py	δ_z/δ_y
2.1	15	5	0.68	0.06	0.333	0.088
2.2	15	10	0.54	0.25	0.667	0.463
2.3	15	13	0.48	0.42	0.867	0.875
2.4	15	14	0.45	0.47	0.933	1.044
2.5	15	15	0.41	0.57	1	1.390
2.6	15	20	0.27	0.88	1.333	3.259

S. No.	Py(N)	Pz(N)	δ_y(mm)	δ_z(mm)	Pz/Py	δ_z/δ_y
3.1	5	10	0.01	0.48	2	48
3.2	10	10	0.28	0.33	1	1.179
3.3	15	10	0.55	0.22	0.667	0.4
3.4	20	10	0.81	0.10	0.5	0.123
3.5	25	10	1.08	0.02	0.4	0.019

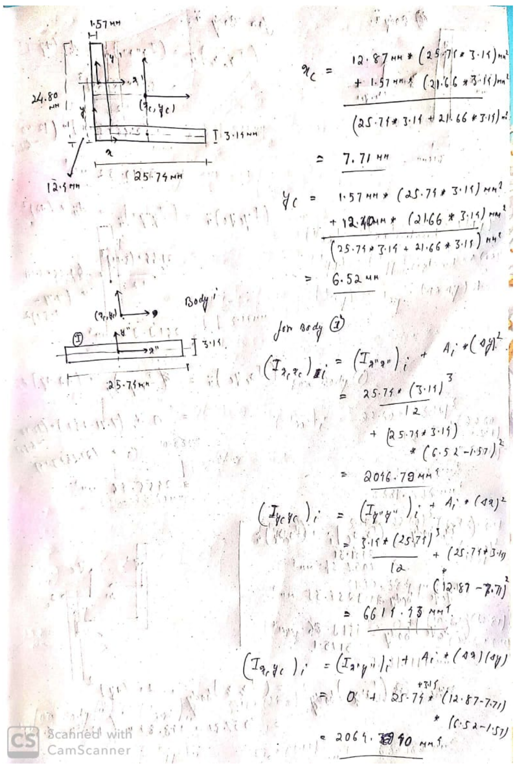
3.6	5	20	0.28	1.09	4	3.893
3.7	10	20	0.05	0.98	2	19.6
3.8	15	20	0.24	0.88	1.333	3.667
3.9	20	20	0.50	0.78	1	1.56

S. No.	Py(N)	Pz(N)	δ_y(mm)	δ_z(mm)	Pz/Py	δ_z/δ_y
4.1	5	15	0.15	0.80	3	5.33
4.2	10	15	0.09	0.72	1.50	8
4.3	15	15	0.38	0.56	1	1.47
4.4	20	15	0.64	0.46	0.75	0.72
4.5	25	15	0.93	0.34	0.60	0.37
4.6	10	5	0.38	0.4	0.5	0.11
4.7	10	10	0.26	0.34	1	1.31
4.8	10	15	0.10	0.68	1.5	6.8

- **RESULTS AND DISCUSSION :**

(a) Calculations and Plots

- Theoretical calculation of Φ:



$x_c = \dfrac{12.87\text{ mm} * (25.75 * 3.15)\text{ mm}^2 + 1.57\text{ mm} * (21.66 * 3.15)\text{ mm}^2}{(25.75 * 3.15 + 21.66 * 3.15)\text{ mm}^2} = 7.71\text{ mm}$

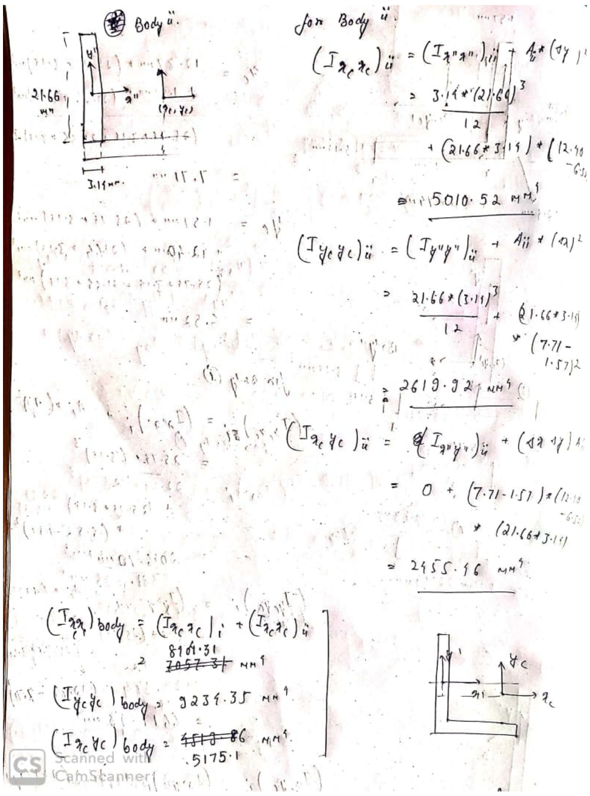
$y_c = \dfrac{1.57\text{ mm} * (25.75 * 3.15)\text{ mm}^2 + 12.40\text{ mm} * (21.66 * 3.15)\text{ mm}^2}{(25.75 * 3.15 + 21.66 * 3.15)\text{ mm}^2} = 6.52\text{ mm}$

for Body (i)

$(I_{x_c x_c})_i = (I_{x''x''})_i + A_i * (\Delta y)^2 = \dfrac{25.75 * (3.15)^3}{12} + (25.75 * 3.15) * (6.52 - 1.57)^2 = 2016.79\text{ mm}^4$

$(I_{y_c y_c})_i = (I_{y''y''})_i + A_i * (\Delta x)^2 = \dfrac{3.15 * (25.75)^3}{12} + (25.75 * 3.15) * (12.87 - 7.71)^2 = 6611.13\text{ mm}^4$

$(I_{x_c y_c})_i = (I_{x''y''})_i + A_i * (\Delta x)(\Delta y) = 0 + 25.75 * 3.15 * (12.87 - 7.71) * (6.52 - 1.57) = 2064.40\text{ mm}^4$



for Body ii

$(I_{x_c x_c})_{ii} = (I_{x''x''})_{ii} + A_{ii} * (\Delta y)^2 = \dfrac{3.15 * (21.66)^3}{12} + (21.66 * 3.15) * (12.40 - 6.52)^2 = 5010.52\text{ mm}^4$

$(I_{y_c y_c})_{ii} = (I_{y''y''})_{ii} + A_{ii} * (\Delta x)^2 = \dfrac{21.66 * (3.15)^3}{12} + (21.66 * 3.15) * (7.71 - 1.57)^2 = 2619.92\text{ mm}^4$

$(I_{x_c y_c})_{ii} = (I_{x''y''})_{ii} + (\Delta x \Delta y) A_{ii} = 0 + (7.71 - 1.57) * (12.40 - 6.52) * (21.66 * 3.15) = 2455.96\text{ mm}^4$

$(I_{x_c x_c})_{body} = (I_{x_c x_c})_i + (I_{x_c x_c})_{ii} = \cancel{7057.31}\ 8161.31\text{ mm}^4$

$(I_{y_c y_c})_{body} = 9231.35\text{ mm}^4$

$(I_{x_c y_c})_{body} = \cancel{4513.86}\ 5175.1\text{ mm}^4$

wrt the figure

$I_{zz} = 9234.35\ mm^4$

$I_{yy} = 8904.31\ mm^4$

$I_{yz} = 5175.1\ mm^4$

$$\tan 2\phi = \frac{-2I_{yz}}{I_{yy} - I_{zz}} = \frac{-2*(5175.1)}{8904.31 - 9234.35} \approx 12.967$$

$\phi = 0.5 \arctan(12.967)$

$\boxed{\phi = 42.69^\circ}$ ans.

$\Phi_{Theoretical}$ = 42.69°

- Calculation from Experimental data:

For each dataset I have considered the data were abs(Pz/Py - δ_z/δ_y) is minimum -
[*the green highlight* in the collected data table]

Dataset	Φ'=arctan(Pz/Py)	Φ''=arctan(δ_z/δ_y)	$\Phi_{experimental}$=(Φ'+Φ'')/2
1	arctan(1)=45°	arctan(1)=45°	45°
2	arctan(0.867)=40.97°	arctan(0.875)=41.19°	41.08°
3**	arctan(1)=45°	arctan(1.179)=49.70°	47.35°
4	arctan(0.75)=36.87°	arctan(0.72)=35.75°	36.31°

Note: ** for dataset 3 I had considered the ***yellow highlighted*** *data* because it was the second closest data in this case. The green marked data here would have given a large error.

$\Phi_{experimental\_average}=\frac{45°+41.08°47.35°+36.31°}{4}$=42.44°

Datasets	error% =|($\Phi_{experimental}$-$\Phi_{Theoretical}$)|/|$\Phi_{Theoretical}$ | * 100
1	5.41
2	3.77
3**	10.92

4	14.94

%Error(of the experimental averaged **Φ** wrt to the Theoretically calculated **Φ**)
= $|(\Phi_{experimenta\_averagel} - \Phi_{Theoretical})|/|\Phi_{Theoretical}| * 100$
= 0.58%

b. Sources of Error :

1. The dial gauge and reading should be set to 0 prior to placing the weight on the pan
2. Before taking the reading you should make sure that there is no oscillation in the pan after the loading as all our theoretical calculation assumes static stability.

- **Conclusion :**

    1. **$\Phi_{Theoretical}$ = 41.45°**
    2. **$\Phi_{experimental\_average}$ = 42.44°**
    3. **% Error**(of the $\Phi_{experimental\_average}$ wrt to $\Phi_{theoretical}$) **= 0.58%**

- **Reference:**
    - Pictures from Google, Lectures and Lecture Notes of AE351 (Lab4)
- **Appendix :**

The Matlab code used:

```
Py1=[5,5,5,5,5,6.96,8.92,8.92];
Py2=[15,15,15,15,15,15];
Py3=[5,10,15,20,25,5,10,15,20];
Py4=[5,10,15,20,25,10,10,10];

Pz1=[0.98,1.96,3.92,5,6.96,5,5,6.96];
Pz2=[5,10,13,14,15,20];
Pz3=[10,10,10,10,10,20,20,20,20];
Pz4=[15,15,15,15,15,5,10,15];

del_y1=[0.25,0.23,0.18,0.15,0.1,0.23,0.31,0.3];
del_y2=[-68,-54,-48,-45,-41,-27];
del_y3=[1,28,55,81,108,28,5,24,50];
del_y4=[15,-9,-38,-64,-93,-38,-26,-10];

del_z1=[0.06,0.04,0.08,0.15,0.30,0.14,0.11,0.20];
del_z2=[6,-25,-42,-47,-57,-88];
del_z3=[48,33,22,10,2,109,98,88,78];
del_z4=[-80,-72,-56,-46,-34,-4,-34,-68];

plot(Pz1,Py1,'-b');
hold;
plot(del_z1,del_y1,'-r');
hold;

index = func1(Pz4,Py4,del_z4,del_y4);
```

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