LAB 3 - Beam Deflection and Strains Debanjan Manna (190255) AE351 28th Jan 2022

OBJECTIVE:

Experimentally measure the strain and deflection in a beam subjected to transverse loading. And determine the strain and the deflection variation along the beam using Euler- Bernoulli beam theory and compare the results with experimental measurements.

INTRODUCTION AND THEORY:

- Euler-Bernoulli Beam Bending:

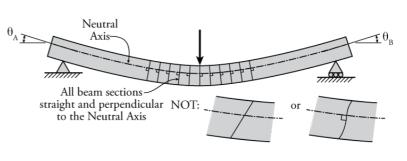
For a linear elastic material with Elastic modulus E and Polar Moment of Inertia I, We have

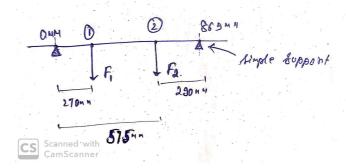
$$M(x) = EI \frac{d^2 v(x)}{dx^2}$$
$$\varepsilon xx = y \frac{M(x)}{EI}$$

Where v(x) -- deflection, M(x) -- moment

Strain Gauge -- to measure the strain (at a particular location)

Dial Gauge -- to measure the deflection (at a particular location)

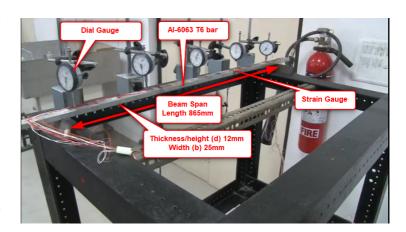




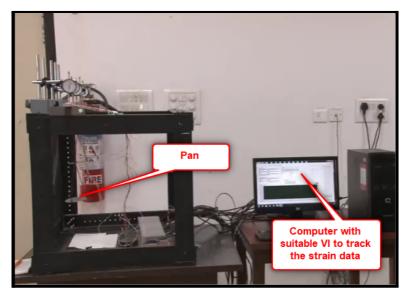
EQUIPMENT AND OPERATING CONDITIONS:

The experimental setup includes:

- a. A beam of the rectangular cross-section (made up of Al-6063 alloy T6 grade)
 - The beam is simply supported.
- b. Fifteen strain gauges were carefully mounted on the top surface of the beam.
- c. Strain indicator (with Wheatstone bridge circuits) to record strain gauge data
- d. A9-9273 C-DAC modules, which are specially designed for strain measurement.
- d. Five deflection dial gauges to measure beam deflection
 - The location of the dial gauges is mentioned later.
- e. 2 pans to hold the weights (that will act as a force)







- Some Important Dimensions:

Least Count of the Dial Gauge	0.01mm
No. of divisions in one round	100div
Beam Span Length (L)	865mm
Beam Cross Section height (d)	12mm
Beam Cross Section width (b)	25mm
Young's Modulus of the beam (E)	70GPa

Pan 1 position: (from x=0 position)	270mm
Pan 2 position: (from x=0 position)	575mm

PROCEDURE:

- 1. Mount the beam with simply supported boundary conditions. Measure beam dimensions and the location of strain gages with respect to the supports.
- 2. Apply a concentrated load of 1kg on each pan. Record all dial gauge readings and the strain values using strain indicator equipment and tabulate your data.
- 3. **Theoretically calculate strains** at each of the strain gage locations using Euler-Bernoulli beam theory and compare your results with experimentally measured strain values.
- 4. Generate graphs that show both your experimental measurements (as data points) and theoretical predictions (as solid lines/curves).
- 5. Calculate the percent errors and **discuss possible reasons** for the discrepancies.
- 6. Perform the analysis steps mentioned above in 3,4,5 for the beam deflection as well.
- 7. Remove all dial gages. Simulate symmetric four-point bend conditions by applying two concentrated loads of the same magnitude symmetrically with respect to the supports. Tabulate strain data recorded using strain indicator equipment and repeat the analysis steps mentioned in 3,4,5.

Collected Data : [from the five Dial gauges]

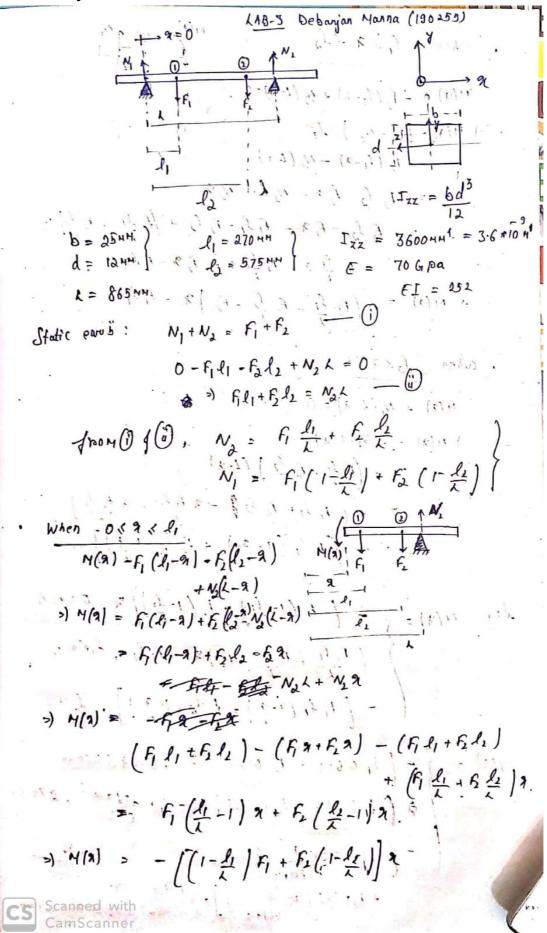
Dial Gauge No.	Position (mm)	Readings (of the Dial gauge)		
		Case1 Pan1 1Kg Pan2 0Kg	Case2 Pan1 1Kg Pan2 0Kg	Case3 Pan1 0Kg Pan2 1Kg
1	35	-0.08	-0.13	-0.03
2	225	-0.38	-0.79	-0.35
3	415	-0.44	-0.96	-0.56
4	605	-0.33	-0.78	-0.5
5	795	-0.09	-0.2	-0.19

- Collected Data : [from the fifteen Strain gauges]				
Strain Gauge No.	Position (mm)	Readings (Strain data in 10 ⁻⁶ mm/mm)		
		Case1 Pan1 1Kg Pan2 0Kg	Case2 Pan1 1Kg Pan2 0Kg	Case3 Pan1 0Kg Pan2 1Kg
1	26	-6.2404842	-8.134273	-5.2581246
2	85	-17.132253	-24.3465482	-11.1861586
3	145	-27.88941	-40.56093755	-16.93612955
4	205	-39.9428823	-57.7849215	-23.6619249
5	265	-50.2287377	-74.2941824	-29.9108772
6	325	-45.9132942	-75.41916395	-36.1662525
7	385	-40.5957683	-75.7341069	-42.24140595
8	445	-35.4617	-76.97573335	-48.8528695
9	505	-31.3017337	-80.11006035	-56.3217372
10	565	-25.8556194	-80.37432	-61.62379185
11	625	-20.9079855	-66.19042	-52.1282333
12	685	-15.6737955	-48.6537329	-38.6457681
13	745	-10.4002196	-32.02986065	-25.6696015
14	805	-5.7131479	-16.2006845	-13.32244285
15	865	-1.9110928	-2.2529271	2.64988115

RESULTS AND DISCUSSION:

(a) Calculations and Plots

Theoretically calculated distribution of Strain:



When
$$l_1 \leqslant 2 \leqslant l_2$$
 $M(a) \circ -i f_1 (l_1 - a) + N_1 (l_1 - a) = 0$
 $M(a) \circ -i f_2 (l_1 - a) + N_2 (l_1 - a) = 0$
 $M(a) \circ -i f_2 (l_1 - a) + N_2 (l_1 - a) = 0$
 $= f_1 l_2 - f_2 a - N_2 l_1 + N_2 a$
 $= f_2 l_3 - f_2 a - f_2 l_1 - f_2 l_2 + f_3 l_4 + f_4 l_4 l_4$
 $= f_1 l_2 + f_2 l_2 - f_2 l_3 - f_3 l_4$
 $M(a) \circ -i f_1 (l_1 - a) = 0$
 $M(a) \circ -i f_1 (l_2 - a) = 0$
 $M(a) \circ -i f_1 (l_3 - a) = 0$
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Albuming linean elastic Material,

we have, $M(x) = \frac{1}{EF} \frac{\partial^2 V(x)}{\partial x^2} \in I$ $\frac{\partial^2 V(x)}{\partial x^2} = \frac{\partial^2 V(x)}{\partial x^2} = \frac{\partial^2$ also Eq = - M(n) 4 then home 14 oblistance $\delta 0, \quad \xi_{q} = \begin{cases} \frac{1}{EI} \left[\frac{F_{h} \left(1 - I_{h} \right) + F_{h} \left(1 - I_{h} \right) \right]}{2} \right] \frac{qd}{2}, \quad 0 < q < I_{h} \end{cases}$ EI [(51 + 51 - 61) a - 51) d , l, (24) EI [Fl, (1-2) + Fl, (1-2)] d , l, 5956 [F, (1-2) + F, (1-2)] 2 (ad), OSASA, Eg = { [F, [1-2]] ... L.] (-d), lisass, [F, l, (1-2)+ F21, (1-2)] (d) 1 12 (25) on betting numerical values we have, [F, * 0.270(1- 2) 270 1 9 x 525 47 + F2 # 0.575 (1-2 (2.381 +10-5) i.e. En = { (1.638 F1 + 0.798 F1 + 10 - 2 ,059 5270444 -[(0.713 F1 - 0.798 F2) 2 - 0.88 F1] *10 - 5 (0.613 (1-2 1.369 (1-2 1.369 (1-2 1.369) F2 1 10-5

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\xi_{9} = \begin{cases}
  \left(1.638 F_{1} + 0.798 F_{2}\right) 9 * 10^{-5}, 0.44.2704- \\
  \left(0.643 F_{1} - 0.713 F_{1} - 0.798 F_{2}\right) * 10^{-5}, 20.52.575...
  \left(0.643 F_{1} + 1.363 F_{2}\right) \left(1-\frac{2}{865*10}\right) * 10^{-5}, 57553.866
\end{cases}

  Case J: Fi = 1 mg (-9.8) No 2 Fi = 0.
\xi_{n} = \begin{cases} 16.0520 + 10^{-5} & 0 < 9 < 270 \text{ MM.} \\ \frac{2.646}{6.3010} - 7.281 & 1*10^{-5} & 270 < 9 < 575 \text{ MM.} \\ \frac{6.3010}{865 + 10^{-5}} & 575 \text{ MM.} < 9 < 865 \text{ MM.} \end{cases}
         = (-16.052 * 10-5 2 , 0 < 2 < 270 MM
             . [ 6-301 -7-2819 ) x10-5, 270 < 2 < 865 MM
 Cabe II : F = 10-17 ( Fax = 9.8 M)
 \begin{cases} 7.820 \times 10^{-5} & 2.0 < 2.270 \text{ AM} \\ 7.820 \times 10^{-5} & 2.70 < 2.575 \text{ ASSSS} \\ 13.116 & (1-\frac{2}{865 \times 10^{-5}}) \times 10^{-5} & 575 < 2.585 \end{cases}
 - 13.416 (1-2 10-5 1575 575 8865 MM
       Case II: F. =-9.8N F, =-9.8N
         Ex = (-25.87.2, + 10-5 2, 0 < 2 < 2.75 HH

-(6.301+0.5392) +10-5, 270 < 2 < 5.75 HH
                                   (-19.718 + (1-2 865 x 10-3) + 10-5 , 575 < 25 865 MM
```

Data Used: 1. Strain Gauge Data

Total DataPoints -- $15 \times 3 = 45$

NOTE: - Strain Gauge data is used to measure the strain

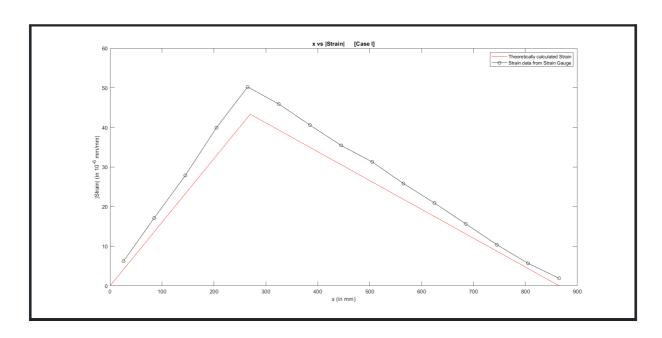
Note: Percent Difference = |(Measured-Predicted)|/|Predicted| * 100

Case 1: Pan1 -- 1Kg , Pan2 -- 0Kg

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Strain Gauge No.	x (in mm)	Mx (in N.m)	ε _x (x 10 ⁻⁶) [Measured]	ε _x (x 10 ⁻⁶) [Predicted]	Percent Difference
1	26	0.175	-6.240	-4.172	49.53
2	85	0.573	-17.132	-13.643	25.56
3	145	0.978	-27.889	-23.274	19.82
4	205	1.382	-39.942	-32.905	21.38
5	265	1.787	-50.229	-42.536	18.08
6	325	1.652	-45.913	-39.347	16.69
7	385	1.469	-40.596	-34.979	16.06
8	445	1.285	-35.462	-30.610	15.85

9	505	1.102	-31.302	-26.242	19.29
10	565	0.918	-25.856	-21.873	18.21
11	625	0.735	-20.908	-17.504	19.45
12	685	0.551	-15.674	-13.136	19.33
13	745	0.368	-10.400	-8.767	18.64
14	805	0.184	-5.713	-4.399	29.90
15	865	0.001	-1.911	0	large_value

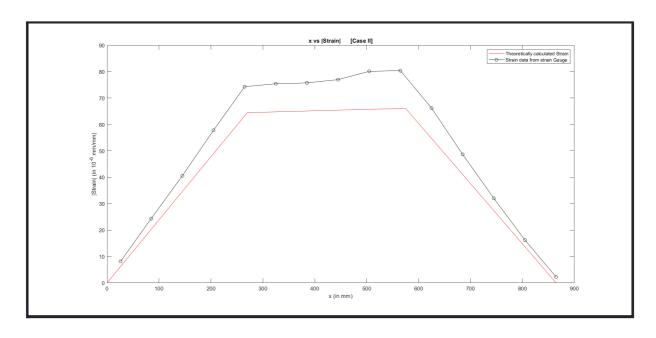
%average Error (of the above table) = $\frac{49.53+25.56+19.82+21.38+18.08+16.69+16.06+15.85+19.29+18.21+19.45+19.33+18.64+29.90}{14} = 21.99\%$



Case 2: Pan1 -- 1Kg , Pan2 -- 1Kg

	Out of the state o				
Strain Gauge No.	x (in mm)	Mx (in N.m)	ε _x (x 10 ⁻⁶) [Measured]	ε _x (x 10 ⁻⁶) [Predicted]	Percent Difference
1	26	0.261	-8.13	-6.21	31.06
2	85	0.852	-24.35	-20.29	19.99
3	145	1.454	-40.56	-24.61	17.18
4	205	2.055	-57.78	-48.94	18.08
5	265	2.657	-74.29	-63.26	17.44
6	325	2.719	-75.42	-64.76	16.46
7	385	2.733	-75.73	-65.09	16.36
8	445	2.746	-76.98	-65.41	17.68
9	505	2.760	-80.11	-65.73	21.87

10	565	2.773	-80.37	-66.06	21.68
11	625	2.298	-66.19	-54.71	20.99
12	685	1.724	-48.65	-41.03	18.58
13	745	1.149	-32.03	-27.35	17.09
14	805	0.575	-16.20	-13.68	18.45
15	865	0.001	-2.25	0	large_value

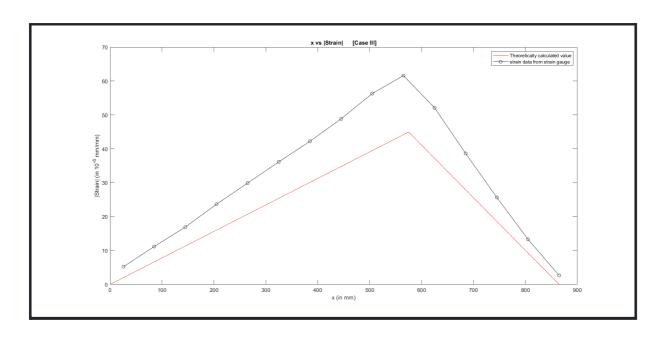


%average Error (of the above table) = $\frac{31.06+19.99+17.18+18.08+17.44+16.46+16.36+17.68+21.87+21.68+20.99+18.58+17.09+18.45}{14} = 19.49\%$

Case 3: Pan1 -- 0Kg , Pan2 -- 1Kg

Strain Gauge No.	x (in mm)	Mx (in N.m)	ε _x (x 10 ⁻⁶) [Measured]	ε _x (x 10 ⁻⁶) [Predicted]	Percent Difference
1	26	0.175	-5.26	-2.03	158.61
2	85	0.573	-11.19	-6.65	68.29
3	145	0.978	-16.94	-11.34	49.36
4	205	1.382	-23.66	-16.03	47.6
5	265	1.787	-29.91	-20.72	44.34
6	325	1.652	-36.17	-25.41	42.30
7	385	1.469	-42.24	-30.11	40.30
8	445	1.285	-48.85	-34.80	40.39
9	505	1.102	-56.32	-39.49	42.62
10	565	0.918	-61.62	-44.18	39.47
11	625	0.735	-52.13	-37.22	40.04

12	685	0.551	-38.64	-27.92	38.43
13	745	0.368	-25.67	-18.61	37.92
14	805	0.184	-13.32	-9.31	43.16
15	865	0.001	-2.65	0	Large_value



%average Error (of the above table) =

 $\frac{158.61 + 68.29 + 49.36 + 47.60 + 44.34 + 42.30 + 40.39 + 42.62 + 39.47 + 40.04 + 38.43 + 37.92 + 43.16}{14} = 52.35\%$

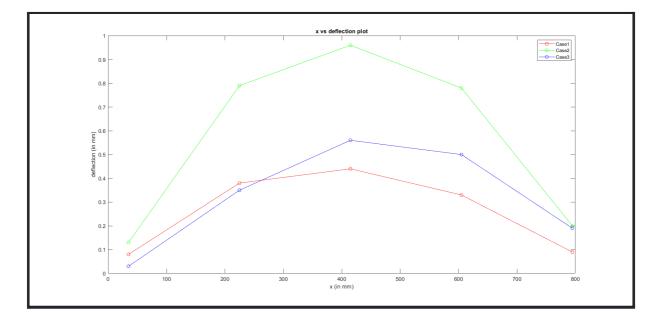
Data Used: 2. Dial Gauge Data

Total DataPoints -- $5 \times 3 = 15$

NOTE: - Dial Gauge data is used to measured the deflection

- The -ve sign in the deflection indicates that the bar is deflecting downwards

Dial Gauge No.	x (in mm)	Reading of Dial Gauge (in mm)		
		Case 1: (d1) Pan1 1Kg Pan2 0Kg	Case 2: (d2) Pan1 1Kg Pan2 1Kg	Case 3: (d3) Pan1 0Kg Pan2 1Kg
1	35	-0.08	-0.13	-0.03
2	225	-0.38	-0.79	-0.35
3	415	-0.44	-0.96	-0.56
4	605	-0.33	-0.78	-0.5
5	795	-0.09	-0.20	-0.19



Deflection in case1, d1+ Deflection in case3, d3 (in mm)	Deflection in case2 d2 (in mm)	% error = (d1+d3)-d2 /d2 * 100
-0.1100	-0.1300	15.38
-0.7300	-0.7900	7.59
-1.0000	-0.9600	4.17
-0.8300	-0.7800	6.41
-0.2800	-0.2000	40

%average Error (of the above table) =
$$\frac{15.38+7.59+4.17+6.41+40}{5}$$
 = 14.71%

b. Sources of Error:

- 1. The dial gauge and strain gauge reading should be set to 0 prior to placing the weight on the pan
- 2. Before taking the strain reading you should make sure that there is no oscillation in the pan after the loading as all our theoretical calculation assumes static stability.
- 3. Insert the connections of the strain gauge into the electrical strain measuring device properly.

Conclusion :

- Experimental Strain data vs theoretically calculated Strain data:

	%average error
Case1	21.99
Case2	19.49
Case3	52.35

- Validity of Superposition principle:

%average Error (of the deflection data) = 14.71%

This deviation arises mainly because the material cant be approximated as a linear elastic material.

However the Superposition principle can be used in this case (specially in the middle sections of the bar) because the error is fairly low.

Reference:

- http://asm.matweb.com/search/SpecificMaterial.asp?bassnum=MA6063T6 (for Al6063-T6 data)
- Pictures from Google and Lectures of AE351 (Lab3)

Appendix :

The Matlab code used:

```
% data recorded by five Dial Gauges
DG = I
    -0.08,-0.13,-0.03;
    -0.38,-0.79,-0.35;
    -0.44, -0.96, -0.56;
    -0.33,-0.78,-0.50;
    -0.09, -0.20, -0.19;
DG_position = [35;225;415;605;795];
% data recorded by fifteen strain gauges
SG = [
  -6.2404842,-8.134273,-5.2581246;
  -17.132253,-24.3465482,-11.1861586;
  -27.88941,-40.56093755,-16.93612955;
  -39.9428823,-57.7849215,-23.6619249;
  -50.2287377,-74.2941824,-29.9108772;
  -45.9132942,-75.41916395,-36.1662525;
  -40.5957683,-75.7341069,-42.24140595;
  -35.4617,-76.97573335,-48.8528695;
  -31.3017337,-80.11006035,-56.3217372;
  -25.8556194,-80.37432,-61.62379185;
  -20.9079855,-66.19042,-52.1282333;
  -15.6737955,-48.6537329,-38.6457681;
  -10.4002196,-32.02986065,-25.6696015;
  -5.7131479,-16.2006845,-13.32244285;
  -1.9110928,-2.2529271,2.64988115;
SG_position = [26;85;145;205;265;325;385;445;505;565;625;685;745;805;865];
%% deflection data plot
plot(DG position,-DG(:,1),'-or');
hold on:
plot(DG_position,-DG(:,2),'-og');
hold on;
plot(DG_position,-DG(:,3),'-ob');
hold on;
xlabel('x (in mm)');
ylabel('deflection (in mm)');
title('x vs deflection plot');
%% Theoretically calculated value of Strain
%{
x = [0:0.01:865];
y1 = [];% strain in case1
y2 = [];% strain in case2
y3 = [];% strain in case3
% Case I
for t=0:0.01:269.99
  y1=[y1;(16.052*10^-5)*t*10^-3];
end
for t=270:0.01:865
  y1=[y1;(6.301-7.281*t*10^-3)*10^-5];
% Case II
for t=0:0.01:269.99
  y2=[y2;(23.872*10^-5)*t*10^-3];
end
```

```
for t=270:0.01:574.99
 y2=[y2;(6.301+0.539*t*10^-3)*10^-5];
for t=575:0.01:865
 y2=[y2;19.718*(1-t/865)*10^-5];
end
% Case III
for t=0:0.01:574.99
  y3=[y3;7.820*10^-5*t*10^-3];
for t=575:0.01:865
  y3=[y3;13.416*(1-t/865)*10^-5];
%}
%{
plot(x,y1*10^6,'-r');
hold on;
plot(SG_position,abs(SG(:,1)),'-ok');
hold on;
title("x vs |Strain|
                    [Case I]");
xlabel("x (in mm)");
ylabel("|Strain| (in 10^{-6} mm/mm)");
%{
plot(x,y2*10^6,'-r');
hold on;
plot(SG_position,abs(SG(:,2)),'-ok');
hold on;
title("x vs |Strain|
xlabel("x (in mm)");
                     [Case II]");
ylabel("|Strain| (in 10^{-6} mm/mm)");
%{
plot(x,y3*10^6,'-r');
hold on:
plot(SG\_position,abs(SG(:,3)),'-ok');
hold on;
title("x vs |Strain| [Case III]");
xlabel("x (in mm)");
ylabel("|Strain| (in 10^{-6} mm/mm)");
plot(x,y1*10^6,'-r');
hold on;
plot(x,y2*10^6,'-b');
hold on;
plot(x,y3*10^6,'-g');
hold on;
xlabel("x (in mm)");
ylabel("Strain (in 10^{-6} mm/mm)");
title("x vs |\epsilon_x(x)| --- Theoretical value");
\%\% Theoretically calculated values of Moment
%{
m1=[];
m2=[];
m3=[];
% Case I
for t=0:0.01:269.99
 m1=[m1;6.742*t*10^-3];
for t=270:0.01:865
 m1=[m1;(2.646-3.058*t*10^-3)];
end
% Case II
for t=0:0.01:269.99
  m2=[m2;10.025*t*10^-3];
for t=270:0.01:574.99
  m2=[m2;(2.646+0.225*t*10^-3)];
```

```
for t=575:0.01:865
    m2=[m2;8.281*(1-1.156*t*10^-3)];
end

% Case III
for t=0:0.01:574.99
    m3=[m3;3.283*t*10^-3];
end

for t=575:0.01:865
    m3=[m3;5.635*(1-1.156*t*10^-3)];
end

plot(x,m1);
hold on;
plot(x,m2);
hold on;
plot(x,m3);
hold on;
title("x vs M(x)");
xlabel('x (in mm)');
ylabel('Moment(X) (in N.m)');
%}
```