AE-641A (Space Dynamics-I)

Quiz No. 1 (Solution)

1. Write either "True" or "False" against each of the following statements:

   (a) *Orbital mechanics* is the study of the rotational motion of the spacecraft about its centre of mass. (*False.*)

   (b) The motion of a system of n-bodies in mutual gravitational attraction always takes place in a constant plane containing the barycentre. (*True.*)

   (c) The barycentre of a two-body system moves in a circle. (*False.*)

   (d) The net angular momentum of a rigid body about its centre of mass is independent of the velocity of the centre of mass. (*True.*)

   (e) The gravitational potential of a spherical body depends upon the co-latitude angle. (*False.*)

2. Prove that the total energy, $T + V$, of an n-body problem is a constant.

   Solution:
   This problem was given as Exercise 2 of Lecture 2, and was solved in Lecture 3. It must be noted (see Lecture 2) that whatever holds for a system of n-particles, is also true for an n-body problem, because a large number of particles can be arbitrarily grouped into any number of bodies.

3. Show that the straight-line (*rectilinear*) relative motion is a possible solution to the two-body problem. (*Hint*: Apply the chain rule of vector differentiation to the radius vector, and substitute into the governing equation of relative motion).

   Solution:
   Recall from Lecture 3 that the relative motion between two spherical bodies is governed by

$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} = -\mu\frac{\mathbf{r}}{r^3} \tag{1}$$

   where $\mu = G(m_1 + m_2)$, $\mathbf{r}(t)$ is the radius vector measured from the centre of m_1, whose Euclidean norm is the radius, $r = \mid \mathbf{r} \mid$. The radius vector and its time derivatives are expressed as follows for rectilinear motion in a fixed direction given by the unit vector $\mathbf{i}$ (see Lecture 2 for the chain rule of vector differentiation):

$$
\begin{aligned}
\mathbf{r} &= r\mathbf{i} \\
\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} &= \dot{r}\mathbf{i} + r\frac{\mathrm{d}\mathbf{i}}{\mathrm{d}t} = \dot{r}\mathbf{i} \\
\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} &= \ddot{r}\mathbf{i} + \dot{r}\frac{\mathrm{d}\mathbf{i}}{\mathrm{d}t} = \ddot{r}\mathbf{i}
\end{aligned}
\tag{2}
$$

where the overdot represents the time derivative. Note that the time derivative of the unit vector $\mathbf{i}$ vanishes, because it is fixed in both magnitude and direction. Substituting the result obtained in Eq.(2) into the governing differential equation, Eq.(1), we have

$$\ddot{r}\mathbf{i} = -\mu\frac{\mathbf{i}}{r^2} \tag{3}$$

or,

$$\ddot{r} + \frac{\mu}{r^2} = 0 \tag{4}$$

which is the scalar differential equation governing rectilinear motion along the radius vector between the two bodies.