AE-641A (Space Dynamics-I)

Quiz No. 2 (Solution)

1. Write either "True" or "False" against each of the following statements:

   (a) The eccentricity vector points towards the *maximum* radius point of any two-body orbit. (Ans. *False*).

   (b) The orbital angular momentum of a two-body orbit is constant in *magnitude*, but varies in *direction*. (Ans. *False*).

   (c) The time period of an elliptic orbit is *independent* of its eccentricity. (Ans. *True*).

   (d) The speed in any two-body orbit *increases* as the radius *increases*. (Ans. *False*).

   (e) The *maximum* value of the flight-path angle in a hyperbolic orbit is $90°$. (Ans. *True*).

2. A spacecraft is detected by radar to be moving at a speed of 10 km/s around the Earth ($\mu = 398600.4$ km^3/s^2) with a flight-path angle of $-20°$ when its radius is 8000 km. Calculate each of the following :

   (a) Semi-major axis of the orbit.

   (b) Orbital eccentricity.

   (c) Minimum orbital radius.

   (d) True anomaly when the radar observation is taken.

   Solution:

   (a) Given $v = 10$ km/s and $r = 8000$ km, the semi-major axis is calculated via the orbital energy as follows:

$$\begin{aligned}\epsilon &= \frac{v^2}{2} - \frac{\mu}{r} = 0.17495 \text{ km}^2/\text{s}^2 \\ &= -\frac{\mu}{2a}\end{aligned}$$

   or

$$a = -\frac{\mu}{2\epsilon} = -1139183.767 \text{ km}$$

(b) The orbital eccentricity is calculated next from the given data ($r =$ 8000 km, $v = 10$ km/s, $\phi = -20°$):

$$h = rv\cos\phi = 75175.4097 \text{ km}^2/\text{s}$$

$$p = h^2/\mu = 14177.964 \text{ km}$$

$$e = \sqrt{1 - \frac{p}{a}} = 1.0062$$

(c) The minimum (perigee) radius is calculated from the semi-major axis and eccentricity as follows:

$$r_p = a(1-e) = 7067.0615 \text{ km}$$

(d) The true anomaly at the point of observation is calculated from the orbit equation,

$$r = \frac{p}{1 + e\cos\theta}$$

to be one of the following:

$$\theta = \cos^{-1}\left(\frac{p}{er} - \frac{1}{e}\right) = 39.8715° \text{ or } 320.1285°$$

Since $\phi < 0$ at the given point, the correct value of the true anomaly is

$$\theta = 320.1285°$$

3. A spacecraft is in an orbit around the Earth ($\mu = 398600.4$ km^3/s^2). When its true anomaly is 90°, the radius is observed to be 15,000 km, and when the true anomaly is 30°, the radius is 10,000 km. Calculate each of the following for the spacecraft's orbit:

   (a) Orbital parameter.
   (b) Orbital eccentricity.
   (c) Largest possible orbital radius.
   (d) Orbital period.

   Solution:

   (a) Given the radius is $r_1 = 15,000$ km when the true anomaly is $\theta_1 =$ 90°, we have from the orbit equation

$$r_1 = \frac{p}{1 + e\cos\theta_1} = \frac{p}{1 + e\cos(90°)} = p$$

which implies that the orbital parameter is the following:

$$p = r_1 = 15,000 \text{ km}$$

(b) Next, from the orbit equation we calculate the orbital eccentricity as follows, given that the radius is $r_2 = 10,000$ km when the true anomaly is $\theta_2 = 30°$:

$$r_2 = \frac{p}{1 + e\cos\theta_2}$$

or,

$$e = \frac{p/r_2 - 1}{\cos\theta_2} = \frac{15000/10000 - 1}{\cos(30°)} = \frac{1}{\sqrt{3}} = 0.57735$$

(c) Since the orbit is elliptical ($e < 1$), the largest possible radius is the apogee radius given by

$$r_a = a(1 + e) = \frac{p}{1 - e} = 35490.381 \text{ km}$$

(d) Calculating the semi-major axis first,

$$a = \frac{p}{1 - e^2} = 22500 \text{ km}$$

we use the expression derived in Lecture 5 for the orbital period of an elliptic orbit:

$$T = \frac{2\pi}{\sqrt{\mu}} a^{3/2} = 33588.049 \text{ s}$$