Yash Srivastava 180889 52/60

AE 777A

Quiz-1

Given eqn:-

$\dot{\varepsilon}_{1} = \varepsilon_{2}$

$\dot{\varepsilon}_{2} = \eta - \varepsilon_{2}^{2}$

a) $f = \begin{Bmatrix} \varepsilon_2 \\ \eta - \varepsilon_2^2 \end{Bmatrix}$

$A = \frac{\partial f}{\partial \varepsilon_1}(\varepsilon_r, \eta_r)$ $\quad B = \frac{\partial f}{\partial \eta}(\varepsilon_r, \eta_r)$

$A = \begin{bmatrix} 0 & 1 \\ 0 & -\varepsilon_2 \end{bmatrix}$ $\quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Reference Sol.

$\varepsilon_r = \begin{bmatrix} c \\ 0 \end{bmatrix}$

$A\Big|_{\varepsilon_1 = c,\ \varepsilon_2 = 0} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

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$B\Big|_{\varepsilon_1 = c,\ \varepsilon_2 = 0} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

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B.) Here, $\bar{x} = \begin{Bmatrix} \varepsilon_1(t) - c \\ \varepsilon_2(t) \end{Bmatrix}$

& $\bar{u}(t) = \eta(t) = S(t)$

$\bar{x}_i(t) = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$

then, $\Phi(t_f, t_i) = \mathcal{L}^{-1}(sI - A)^{-1}$

$$sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}$$

$$(sI - A)^{-1} = \begin{bmatrix} 1/s & 1/s^2 \\ 0 & 1/s \end{bmatrix}$$

then $\Phi(t, t_i) = \mathcal{L}^{-1}(sI - A) = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$

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~~the~~ state response

$$y(t) = e^{At} \int_0^t e^{-At} B(z) \bar{w}(z) dz$$

$$= \begin{bmatrix} 1 & e \\ 0 & 1 \end{bmatrix}^{\circ} \int_0^t \begin{bmatrix} 1 & \pi \\ 0 & \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} (S(z)) dz$$

$$= \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \int_0^t \begin{bmatrix} -t\,\delta(t) \\ \delta(t) \end{bmatrix} dz$$

$$= \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \left[\begin{bmatrix} -t\,S(t) \\ S(t) \end{bmatrix} - \begin{bmatrix} 0 \\ S(0) \end{bmatrix} \right]$$

$$= \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -t\,S(t) \\ S(t) - S(0) \end{bmatrix}$$

$$= \begin{bmatrix} -t\,S(t) + t\,S(t) - t\,S(0) \\ S(t) - S(0) \end{bmatrix} = \begin{bmatrix} -t\,S(0) \\ S(t) - S(0) \end{bmatrix}$$

c.) Stabality of Linearized system.

$$SI - A = \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}$$

$$\det(SI - A) = 0$$

$\Rightarrow s^2 = 0 \Rightarrow s_{1,2} = 0 \rightarrow$ Both eigenvalues are zero 7
$\Rightarrow$ unstable.

d.) controllability :-

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad AB = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

controllability test matrix

$$P = (A, AB) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow \text{Rank} = 2 = \text{order of system.}$$

$\Rightarrow$ system is controllable.

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e.) output $y = \xi_{1_2} \Rightarrow \vec{h} = \{\xi_{1_2}\}$

output coeffi matrix:

$$C = \frac{\partial h}{\partial \xi}(\xi_r, \eta_r, t) = [0, 1]$$

Now $\quad C^T = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad A^T C^T = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$(C^T, A^T C^T) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \Rightarrow \text{Rank} = 1 \neq \text{order of system}$$

$\Rightarrow$ system is not observable

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F) closed loop characteristic polynomial.

$$\det(s\hat{I} - A + BK) = s^2 + 2s + 2$$

$$\Rightarrow \hat{a} = (2, 2)$$

$$\det(s\hat{I} - A) = s^2$$

$$\Rightarrow a = (0, 0)$$

$$\Rightarrow (\hat{a} - a) = (2, 2)$$

$$P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad (PW)^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$W = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \text{Now } K = (\hat{a} - a)\,(PW)^{-1}$$

$$PW = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad K = [2, 2]\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = [2 \;\; 2]$$

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