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AE 777A
Quiz 4

1.) State equations for orbital system

$$\ddot{r} = \frac{h^2}{r^3} + \frac{\mu}{r^2} = a_r$$

$$\dot{h} = r a_\theta$$

$$\dot{\alpha} = \frac{r}{h} a_n$$

We have Control Input

$$u(t) = a_r(t)$$

$$a_\theta = a_n = 0$$

$$\ddot{r} - \frac{h^2}{r^3} + \frac{\mu^2}{r^2} = a_r = \ldots$$

$$\left(\begin{array}{c} \dot{h} = 0 \\ \dot{\alpha} = 0 \end{array} \right)$$

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Both the orbital angular momentum, h & plane orientation, α are constants. So they are dropped as state equations.

We choose state variables

$$\xi_1 = r \qquad \xi_2 = \dot{r}$$

$$\xi = \begin{Bmatrix} \xi_1 \\ \xi_2 \end{Bmatrix} = \begin{Bmatrix} r \\ \dot{r} \end{Bmatrix}$$

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with control input, $\eta(t) = u(t) = a_r(t)$

$$f(\xi, \eta) = \dot{\xi} = \begin{Bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{Bmatrix} = \begin{Bmatrix} \dot{r} \\ \ddot{r} \end{Bmatrix}$$

$$f = \dot{\xi} = \begin{Bmatrix} \xi_2 \\ \eta + \frac{h^2}{\xi_1^3} - \frac{\mu}{\xi_1^2} \end{Bmatrix}$$

- Reference sol.^{n} when $\eta = 0$ is a circular orbit of radius $r = C$

0 $$\xi_r = \begin{Bmatrix} \xi_{1r}(t) \\ \xi_{2r}(t) \end{Bmatrix} = \begin{Bmatrix} C \\ 0 \end{Bmatrix}$$?

Q2.

$$\dot{x} = (t_f - t)u \quad x(t) \in \mathbb{R} \quad u(t) \in \mathbb{R}$$

&

$$J = k^2 [x(t_f)]^2 + \int_t^{t_f} u^2(\tau)\, d\tau$$

k is a real constant

Cost function, $\phi = k^2 [x(t_f)]^2$

Lagrangian, $L = u^2(\tau)$

Hamiltonian, $H = L + \lambda^T f$

$$H = u^2 + \lambda^T (t_f - t) u$$

$$H_u = 2u + \lambda^T (t_f - t)$$

$$H_{uu} = 2$$

$$H_u = 0$$

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$$2u^* + \lambda^{*T} (t_f - t) = 0$$

$$u^* = \frac{-\lambda^{*T}}{2} (t_f - t)$$

$\lambda = ?$

(3) $\dot{x} = f(x, u)$ state eqn

Langrangian $\rightarrow L(x, u)$

Hamiltonian $\rightarrow H = L(x, u) + \lambda^T f(x, u)$

For the Hamiltonian to be constant of a time invariant system along an external $x^*(t), u^*(t)$

The neccessary conditions:

$$\dot{\lambda}^* = -\left(\frac{\partial H}{\partial x}\right)^{*T}$$

$$\dot{x}^* = f(x^*, u^*) = \left(\frac{\partial H}{\partial \lambda}\right)^{*T}$$

$$\& \left(\frac{\partial H}{\partial u}\right)^* = 0$$

The Hamiltonian derivative

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$$\dot{H} = \left(\frac{\partial H}{\partial x}\right)^* \dot{x}^* + \left(\frac{\partial H}{\partial \lambda}\right)^* \dot{\lambda}^* + \left(\frac{\partial H}{\partial u}\right)^* \dot{u}^*$$

$$\Rightarrow \dot{H} = \left(\frac{\partial H}{\partial x}\right)^* \left(\frac{\partial H}{\partial \lambda}\right)^{*T} - \left(\frac{\partial H}{\partial \lambda}\right)^* \left(\frac{\partial H}{\partial \lambda}\right)^{*T} = 0$$

$\Rightarrow H =$ constant or external trajectory.

4.) $f = \dot{x} = u \qquad u \in R$

$|u(t)| \leq 1 \qquad x \in R$

$J = t_f$

$x(0) = 1$ to $x(t_f) = 0$

Langrangian ~~for~~

$L = 1$

$H = 1 + \lambda^T f$

$H = 1 + \lambda^T u = = 1 + \lambda u$

from Pontryajin's minimum principle.

Derivation?

$$\hat{u}(t) = \begin{cases} -1 & \lambda(t) > 0 \\ 1 & \lambda(t) < 0 \end{cases}$$

$$\dot{\lambda}^* = -\left(\frac{\partial H}{\partial x}\right)^{*T} = 0$$

$\lambda^* =$ constant $= c$

$\because x_0(0) = 1$ & $x(t_f) = 0$

$\therefore \hat{u} < 0$

~~Also~~ λ^* is const. $\Rightarrow$ no switching & $\hat{u}(t)$ will remain const. throughout.

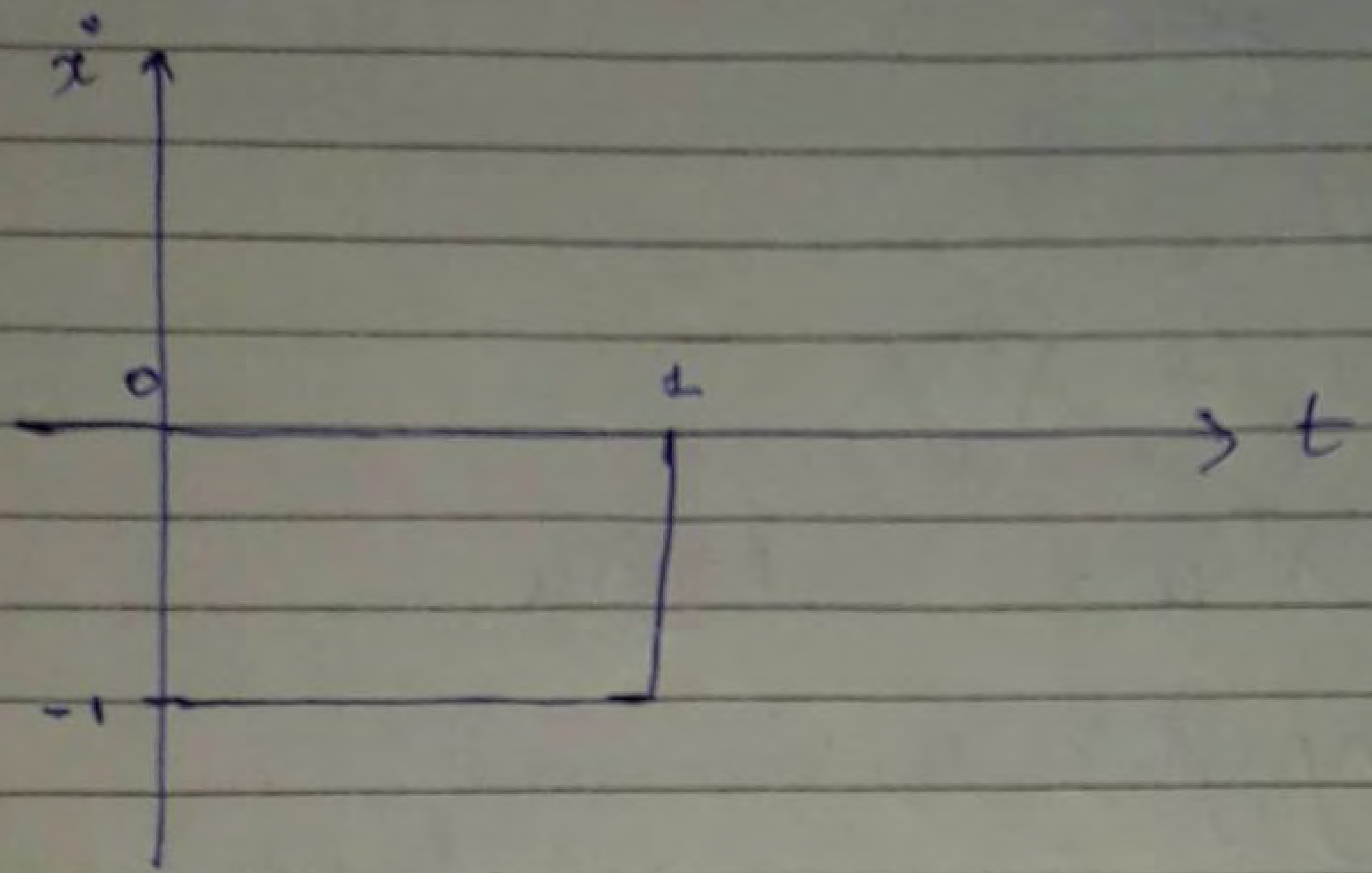
$\hat{u}(t) = -1$ if $\lambda(t) > 0$ if $c > 0$

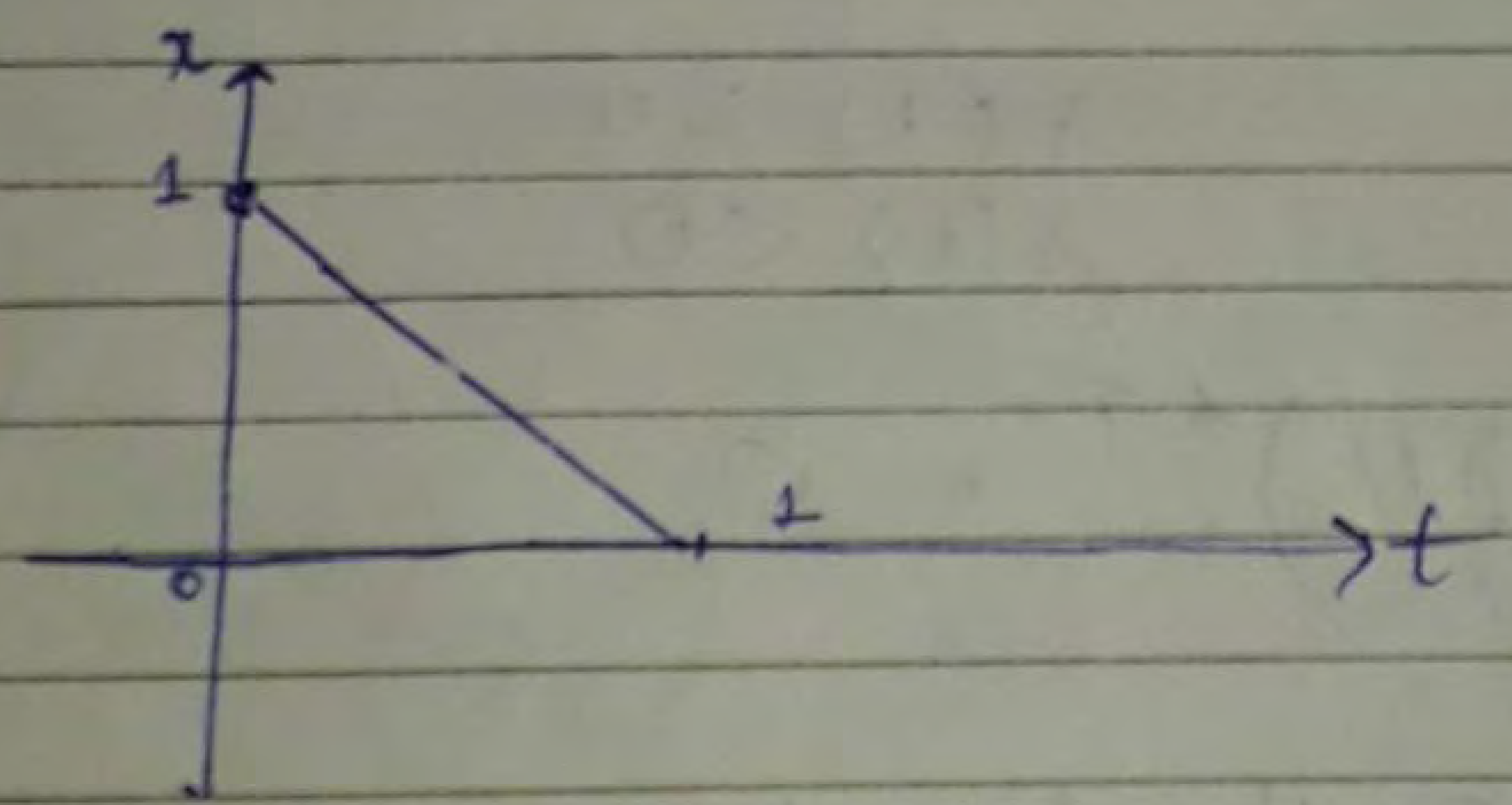
$$\dot{x} = -1$$

$$x = -t + R$$

$$x(t) = 1 - t \quad \{x(0) = 1\}$$

$$x(t_f) = 0 \Rightarrow 1 - t_f = 0 \Rightarrow \boxed{t_f = -1}$$





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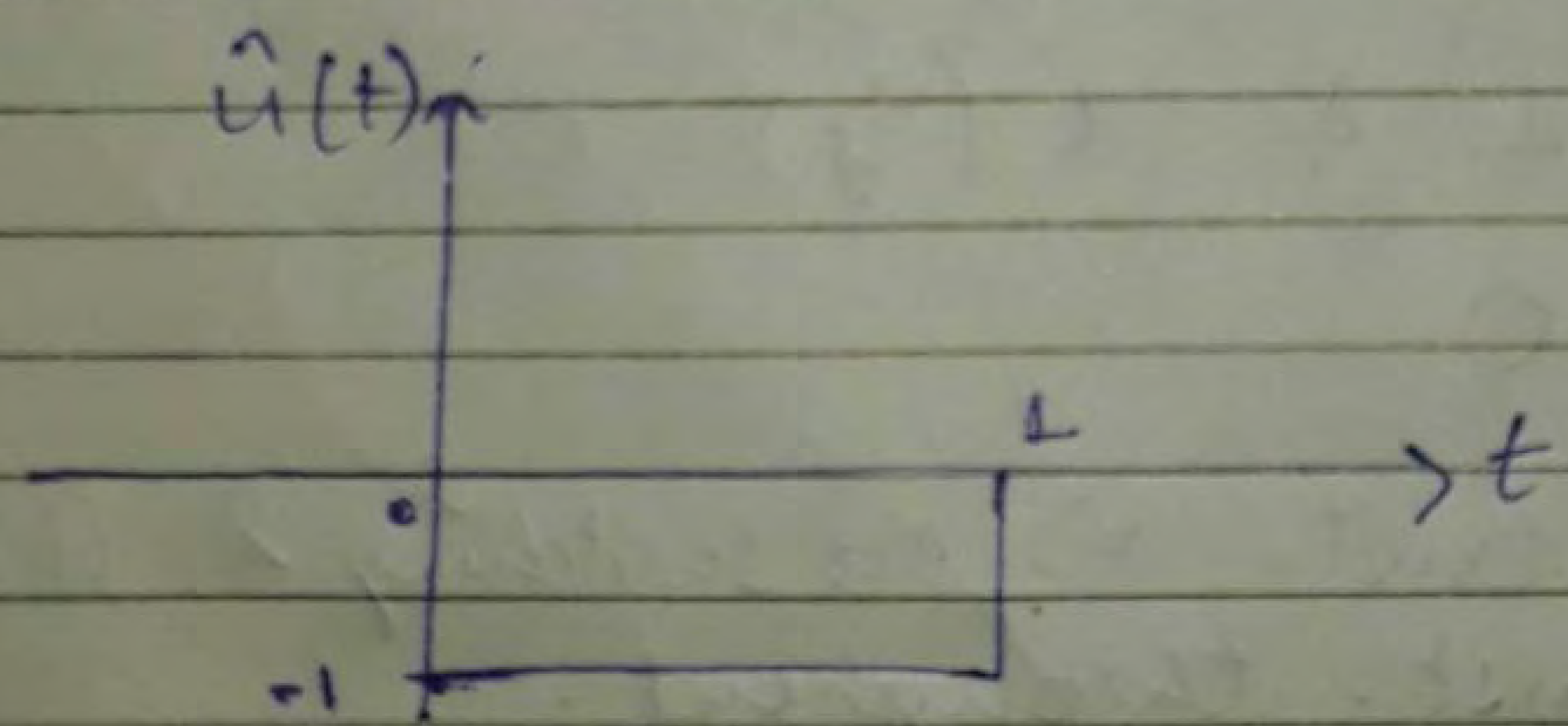

5.) a.) False
b.) True
c.) True
d.) False
e.) False

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