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Quiz 2

1.) Given function:

$$L(U) = \frac{1}{3}U^3 + \frac{1}{2}U^2 + \frac{1}{4}U$$

on differentiating it:

$$L'(U) = \frac{d\left(\frac{1}{3}U^3 + \frac{1}{2}U^2 + \frac{1}{4}U\right)}{dU}$$

$$= U^2 + U + \frac{1}{4}$$

$$L''(U) = 2U + 1$$

Both are ~~also~~ continuous $\forall\ U \in R$.

for stationary point:

$$L'(U) = 0$$

$$U^2 + U + \frac{1}{4} = 0$$

$$\left(U + \frac{1}{2}\right)^2 = 0$$

$U = -\frac{1}{2}$, which is real.

$\therefore\ U = -\frac{1}{2}$ is stationary point.

Now, $L''(-\frac{1}{2}) = 2 \times (-\frac{1}{2}) + 1 = 0$

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$U = -\frac{1}{2}$, is a singular point of $L(U)$ it doesn't satisfy necessary condition for minima.

2.) Given

$$L(U) = U_1^2 - 3U_1U_2 + 2U_2$$

where

$$U = (u_1, u_2)^T \in R^2$$

Now,

$$L_U = \frac{dL}{dU} = \left[\frac{\partial L}{\partial U_1}, \frac{\partial L}{\partial u_2}\right] = 0$$

$$= \left[2u_1 - 3u_2, -3u_1 + 2\right] = [0, 0]$$

For stationary points : $L_U = (0,0) \Rightarrow U_1 = \frac{2}{3} \therefore 2-3u_1=0$

$U_2 = \frac{2U_1}{3} \therefore 2u_1 - 3u_2 = 0$

$u_2 = \frac{4}{9}$

$$\boxed{\begin{aligned} u_1 &= \frac{2}{3} \\ u_2 &= \frac{4}{9} \end{aligned}}$$

'u' should be a column vector!

checking $U = \left(\frac{2}{3}, \frac{4}{9}\right)$ is minimum

$\boxed{\text{eigen Values } (L_{UU}) > 0}$ $= \frac{\delta^2 L}{\delta U^2} = \begin{bmatrix} \frac{d}{dU_1}(2U_1 - 3u_2) & , \frac{d}{dU_2}(-3U_1+2) \\ \frac{d}{dU_2}(2U_1 - 3U_2) & , \frac{d}{dU_2}(-3u_1+2) \end{bmatrix}$

$$= \begin{bmatrix} 2 & -3 \\ -3 & 0 \end{bmatrix}$$

& $\det(s\bar{I} . L_{UU}) = 0$ - for eigen values

$\det\begin{pmatrix} s-2 & -3 \\ -3 & s \end{pmatrix} = 0 \Rightarrow s^2 - 2s - 9 = 0$

$$s = \frac{2 \pm \sqrt{40}}{2}$$

$$s = 1 \pm \sqrt{10}$$

$S_1 = 1 - \sqrt{10}$ $\quad S_2 = 1 + \sqrt{10}$ 10

Since eigen value $S_1 < 0$ why not? Explain.

$U = \left(\frac{2}{3}, \frac{4}{9}\right)$ is stationary point, not minimum.

(3) $L(x, U) = \frac{1}{2}x^2 + \frac{1}{2}U^T \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} u$; $u \in R^2, x \in R$

$f(x, U) = x + [2, -1]u + 2 = 0$

Hamiltonian Function →

$H = L(x, U) + \lambda + f(x; U)$ ($x \in R$ is a Scalar lagrange multiplier)

$H_x = x + \lambda \cdot 1 = 0$

$H_u = u^T \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} + \lambda (2 \; -1) = (0, 0)$ } necessary condition

Equality Constraint →

$x + [2 \; -1]U + 2 = 0$

Equations to solve become → (assuming $u = (u_1, u_2)^T$)

$x + \lambda = 0$ — (1)

$(2u_1 + 2\lambda, u_2 - \lambda) = (0, 0)$ — (2)

$x + 2u_1 - u_2 + 2 = 0$ → (3)

From (1) → $\boxed{\lambda = -x}$

Substituting λ in Eqn (2) →

$(2u_1 - 2x, u_2 + x) = (0, 0)$

$\Rightarrow u_1 = x$; $u_2 = -x$

Substituting u_1 & u_2 in (3) →

$x + 2x + x + 2 = 0 \Rightarrow \boxed{x = -\frac{1}{2}}$

& $(u_1, u_2)^T = (-1/2, 1/2)^T = u$

$\therefore$ stationary points (x^*, u^*) become $\rightarrow$

$$\boxed{(x^*, u^*) = \left(-\tfrac{1}{2}, \begin{pmatrix} -1/2 \\ 1/2 \end{pmatrix}\right)}$$

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