AE-777A (Optimal Space Flight Control)

Quiz No. 4

Quiz Procedure

1. Clearly write out your solution to the quiz problems within the specified time on blank sheets of paper. (Marks will be given only for complete calculation/derivation steps.)
2. Take low-resolution pictures of your solution, convert them into a single PDF file (about 1MB), and send it to me by email (ashtew@iitk.ac.in) from your registered email account.
3. The time limit will be strictly enforced, and late submissions will not be accepted.

Quiz No. 4 (Time: 60 min; Total Marks: 60)

(There are a total of five (5) problems, 3 on this page, and 2 on the next page.)

1. For the orbital system of a satellite in a circular orbit of radius, C, and frequency, n, around a spherical planet, use the linearized state equations with a radial acceleration control input, $u(t) = a_r(t)$, to find a state-feedback regulator (if any) such that the following objective function is minimized w.r.t. $u(t)$:

$$J = \frac{1}{2}\int_0^\infty \left\{[r(t) - C]^2 + k^2 u^2(t)\right\} \mathrm{d}t$$

with $k > 0$ being a constant.

(15)

2. A system is governed by the differential equation

$$\dot{x} = (t_f - t)u$$

where $x(t) \in \mathbb{R}$ and $u(t) \in \mathbb{R}$. Determine the state-feedback control law for minimizing the following objective function:

$$J = k^2[x(t_f)]^2 + \int_t^{t_f} u^2(\tau)\mathrm{d}\tau$$

where k is a real constant.

(10)

3. For a time-invariant system with the state equation

$$\dot{x} = f(x, u)$$

and the Lagrangian, $L(x, u)$, where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$, and $L(x, u)$ and $f(x, u)$ being continuously differentiable with their arguments, prove that the Hamiltonian is constant on an extremal trajectory.

(10)

(PTO)

4. The system with the state equation, $\dot{x} = u \in \mathbb{R}$, has its input bounded by

$$|u(t)| \leq 1$$

Derive the control input $u(t)$ for the time-optimal transfer ($J = t_f$) from $x(0) = 1$ to $x(t_f) = 0$, and solve for the optimal trajectory.

(15)

5. Write either "True" or "False" against each of the following statements:

   (a) *Orbital dynamics* refers to the rotational motion of the spacecraft about its centre of mass.

   (b) Space navigation is the control of the translational motion of the spacecraft's centre of mass.

   (c) The navigational *feedforward controller* compares the actual trajectory with the specific waypoints, and generates corrective inputs.

   (d) The navigational system acts as a slave to the attitude control system.

   (e) The idealized navigational control system neglects the time scale of the orbital dynamics system.

(10)

Please send your solution to me (ashtew@iitk.ac.in) before 1:00 p.m. today.