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Quiz-5

(1)

~~a = 10,000 km — H =~~

$$T = \frac{2\pi}{\sqrt{H}} a^{3/2} = 100 \times 60 \text{ s}$$

$$r_p = 6700 \text{ km}$$

Also for Earth; $H = 398600.4 \text{ km}^3/\text{s}^2$

$$\Rightarrow 100 \times 60 = \frac{2\pi}{\sqrt{398600.4}} \times a^{3/2}$$

$$\Rightarrow \left(\frac{6000 \times \sqrt{398600.4}}{2\pi} \right)^{2/3} = a$$

$$\Rightarrow a = 7136.63 \text{ km}$$ ✓

(a) orbital speed at minimum radius -

$$V_p = \sqrt{\frac{2H}{r_p} - \frac{H}{a}} \text{ km/sec}$$

$$= \sqrt{\frac{2 \times 398600.4}{6700} - \frac{398600.4}{7139.05}} = \text{km/s} \quad ?$$

where has this value come from? ✗

$$= 7.95 \text{ km/s}$$ ✓

(b) orbital speed at maximum radius -

maximum rad - $r_a = a(1+e)$

$r_p = a(1-e) = 6700$ km

$$6700\left(1+1-\frac{6700}{7139.05}\right) \text{ km} = r_a$$

~~$r_a = 7112.05$ km~~ $r_a = 7109.9$ km

$$V_a = \sqrt{\frac{2\times 398600.4}{7112.05} - \frac{398600.4}{7139.05}} \text{ km/s}$$

$= 7.50$ km/s

(c) $e = 1 - \frac{6700}{7139.05} = 0.06$

$$r_p = \frac{p}{1+e} = 6700 \text{ km}$$

$$\Rightarrow p = 6700\,(1+0.06) \text{ km}$$

$= 7102$ km

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$$r(\theta) = \frac{p}{1+e\cos\theta}$$

$$r = \frac{7102 \text{ km}}{1+(0.06)(\cos 270^\circ)} = \text{~~7102 km~~} \; 7120 \text{ km}$$

Radius when true anamoly radius is 270° is 7102 km

2) we have

$V = 8.5 \text{ km sec}^{-1}$

$\phi = -25^\circ$

$r = 11500 \text{ km}$

first ; using energy eq we have

$$\frac{V^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$$

$$\Rightarrow 36.125 - 34.66 = \frac{-3.98600.4}{2a}$$

$$\Rightarrow a \overset{(-)}{=} 136134.01 \text{ km}$$

Now $h = rV\cos\phi$

$\Rightarrow h = 8859.6 \text{ km}^2 \text{ s}^{-1}$

hence : $p = \frac{h^2}{\mu} = 19690.06 \text{ km}$

$$e = \sqrt{1 - p/a}$$

$$e = \sqrt{1 - (-0.049)} = 1.024$$

Now $\tan\phi = \frac{e\sin\theta}{1 + e\cos\theta}$

$\Rightarrow \tan(-25^\circ) = \frac{1.024\sin\theta}{1 + 1.024\cos\theta}$

$\Rightarrow -0.466 = \frac{1.024\sin\theta}{1 + 1.024\cos\theta}$

$$\Rightarrow \quad -0.466 - 0.477\cos\theta = 1.024\sin\theta$$

Squaring

$$0.217 + 0.227\cos^2\theta + 0.445\cos\theta$$

$$= 1.048 - 1.048\cos^2\theta$$

$$\Rightarrow \quad 1.275\cos^2\theta + 0.445\cos\theta - 0.831 = 0$$

$$\Rightarrow \quad \cos\theta = \frac{-0.445 \pm \sqrt{0.198 + 4.238}}{2.55}$$

$$\Rightarrow \cos\theta = \frac{-0.445 \pm 2.10}{2.55}$$

$$\cos\theta = 1.0004 \quad , \quad \cos\theta = 0.649$$

$$\cos\theta = -49.53^\circ \quad = 310.46^\circ$$

(3) $a = 10,000$ km , $e = 0.5$ $t - t_0 = 30$ min = 1800 sec

r.o.u.p 2. Earth orbit $\Rightarrow \mu = 398600.4$

Soln $0 \le e < 1 \rightarrow$ eliptic orbit

mean frequency $n = \sqrt{\mu/a^3} = 0.000631348$ rad/sec

mean anomaly $M = n/(t - t_0) = 1.13642654$ rad

Applying Newton's method :- $\Delta = 10^{-7}$ rad

let $E = M = 1.13642654$ rad

E	$F(E)$	$F'(E)$	ΔE
1.13642654	-0.45535677	0.7895806	0.57447413
1.171086786	0.079338102	1.0698697	-0.0741613
1.6367065	0.001365295	1.0329324	0.00132209 14
1.6353844	$4.3604 5 \times 10^{-7}$	1.0322716	$-4.22432 99 \times 10^{-7}$
1.635384016	$4.4408920 \times 10^{-14}$	1.0322713 96	$-4.30205866 \times 10^{-14}$

$E = 1.63538401$ rad

True anomaly $\theta = 2 \tan^{-1} \sqrt{\frac{1+e}{1-e}} \times \tan\left(\frac{E}{2}\right)$

$\theta = 2.1494457$ rad $= 123.154172°$

rad $r = a(1 - e \cos E)$

$= 10322.7139$ km

Speed $v = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}} = 6.1129199$ km/s

flight path angle $\phi = \tan^{-1}\left(\frac{e\sin\theta}{1+e\cos\theta}\right) = 0.522695$ rad
$= 29.94824°$

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~~$h = r v \cos$~~ $h = r v \cos\phi = 54676.34772$ km²/sec

$h = \sqrt{\mu a(1-e^2)} = 54676.34772$ km²/sec

all calculation and right
