AE-777A (Optimal Space Flight Control)

Quiz No. 3

Quiz Procedure

1. Clearly write out your solution to the quiz problems within the specified time on blank sheets of paper. (Marks will be given only for complete calculation/derivation steps.)

2. Take low-resolution pictures of your solution, convert them into a single PDF file (about 1MB), and send it to me by email (ashtew@iitk.ac.in) from your registered email account.

3. The time limit will be strictly enforced, and late submissions will not be accepted.

Quiz No. 3 (Time 60 min)

(Marks for each problem are indicated in parentheses.)

1. For the minimization of

$$L(u) = u_1^2 - 2u_1u_2 + 4u_2^3$$

with respect to $u = (u_1, u_2)^T \in \mathbb{R}^2$, and subject to

$$u_1 \geq 0\,; \quad u_2 \geq 0$$

find the minimum points, $\hat{u}$, (if any).

(20)

2. For the following scalar system [$x(t) \in \mathbb{R}$, $u(t) \in \mathbb{R}$]:

$$\dot{x} = u + 2x$$

determine the extremal trajectory and the corresponding control history, if $x(0) = 0$, and the following objective function must be minimized w.r.t. $u(t)$ in the control interval $0 \leq t \leq 1$:

$$J = \frac{1}{2}[x_1(1) - 1]^2 + \frac{1}{2}\int_0^1 u^2(t)\mathrm{d}t$$

(20)

3. Consider a system with the following state equations [$(x_1, x_2)^T \in \mathbb{R}^2$, $u(t) \in \mathbb{R}$]:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_1 + u \end{aligned}$$

Given the initial co-state vector to be $\lambda(0) = (c_1, c_2)^T$, derive the expressions for the extremal control, $u^*(t)$, and the boundary conditions to be satisfied by the state variables in terms of c_1, c_2, if $x(0) = (0, 0)^T$ and the following objective function must be minimized w.r.t. $u(t)$ in the control interval $0 \leq t \leq 5$:

$$J = \frac{1}{2}[x_1(5) - 1]^2 + \frac{1}{2}x_2^2(5) + \frac{1}{2}\int_0^5 u^2(t)\mathrm{d}t$$

(You are *not* required to solve for the constants, c_1, c_2.)

(20)

Please send your solution to me (ashtew@iitk.ac.in) before 1:00 p.m. today.