AE-777A (Optimal Space Flight Control)

Quiz No. 3

Quiz Procedure

(i) Clearly write out your solution to the quiz problems within the specified time on blank sheets of paper. (Marks will be given only for complete calculation/derivation steps.)

(ii) Take *low-resolution* pictures of your solution, convert them into a single PDF file (about 1MB), and send it to me by email (ashtew@iitk.ac.in) from your *registered* email account.

(iii) Submit your solution only *once*. In case of multiple submissions, only the *earliest* one will be accepted.

(iv) The time limit will be *strictly enforced*, and late submissions will *not* be accepted.

Quiz No. 3 (Time 70 min)

(Marks for each problem are indicated in parentheses.)

1. For the minimization of

$$L(u) = u_1^2 - u_1 u_2 + \frac{1}{3}u_2^3$$

with respect to $u = (u_1, u_2)^T \in \mathbb{R}^2$, and subject to

$$u_1 \geq 0 \; ; \quad u_2 \geq 0$$

find the minimum points, $\hat{u}$, (if any).

(15)

2. Consider a system with the following state equations $[(x_1, x_2)^T \in \mathbb{R}^2$, $u(t) \in \mathbb{R}]$:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 + u \end{aligned}$$

Given the initial state vector to be $x(0) = (0, 0)^T$, determine the extremal control history, $u^*(t)$, such that the following objective function is minimized w.r.t. $u(t)$ in the control interval $0 \leq t \leq 1$:

$$J = \frac{1}{2}[x_1(1) - 1]^2 + \frac{1}{2}\int_0^1 u^2(t)\mathrm{d}t$$

(25)

3. For the system of Problem 2, find a state-feedback regulator (if any) such that the following objective function is minimized w.r.t. $u(t)$:

$$J = \frac{1}{2}\int_0^{\infty} \left\{x_1^2(t) + u^2(t)\right\} \mathrm{d}t$$

(20)

Please send your solution to me (ashtew@iitk.ac.in) before 1:10 p.m. today.