AE-777A (Optimal Space Flight Control)

Quiz No. 1

Quiz Procedure

1. Clearly write out your solution to the quiz problems within the specified time on blank sheets of paper. (Marks will be given only for complete calculation/derivation steps.)

2. Take *low-resolution* pictures of your solution, convert them into a single PDF file (about 1MB), and send it to me by email (ashtew@iitk.ac.in) from your *registered* email account.

3. Submit your solution only *once*. In case of multiple submissions, only the *earliest* one will be accepted.

4. The time limit will be *strictly enforced*, and late submissions will *not* be accepted.

Quiz No. 1 (Time 60 min)

(Marks for each part are indicated in parentheses.)

1. Consider the planar orbital dynamics around a spherical body of gravitational constant, μ, governed by the following differential equations:

$$\ddot{r} - \frac{h^2}{r^3} + \frac{\mu}{r^2} = a_r$$

$$\dot{h} = r a_\theta$$

Here r gives the radial location of the spacecraft from the centre of the spherical body, h is its angular momentum, and a_r and a_θ are the acceleration inputs applied to the spacecraft in the radial and circumferential directions, respectively.

   (a) Derive the state equations when only the radial acceleration input, $u = a_r$, is applied, and $a_\theta = 0$. (*Hint*: Use r and $\dot{r}$ as state variables). (10)

   (b) Linearize the system about a circular orbit, $r = c =$ const., and determine the state-space coefficient matrices, A and B. (10)

2. Derive the state-transition matrix of a system whose state equations are the following:

$$\begin{aligned} \dot{x}_1 &= 2x_2 \\ \dot{x}_2 &= -2x_1 + u \end{aligned}$$

(15)

3. A system has the following state equations:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u \end{aligned}$$

   (a) Investigate the stability of the system. (5)

   (b) Investigate the controllability of the system. (5)

   (c) Is the system observable with $y = x_2$ being the only output? (5)

   (d) If possible, design a state-feedback regulator for the system such that the closed-loop characteristic polynomial is the following:

$$s^2 + s + 1 = 0$$

(10)

Please send your solution to me (ashtew@iitk.ac.in) before 1:00 p.m. today.