Masters Project Presentation

Deep Galerkin Method for Solving European-Option Pricing Model

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- It has applications in a wide range of fields, from natural language processing and computer vision to robotics and self-driving cars.
- **Deep Learning** is a subfield of Machine Learning and AI, that is concerned with developing algorithms that can learn from large amounts of data. It involves building and training neural networks, which are computational models that are inspired by the structure and function of the human brain. And deep learning we used various fields, like natural language processing, speech recognition, image processing and many more.

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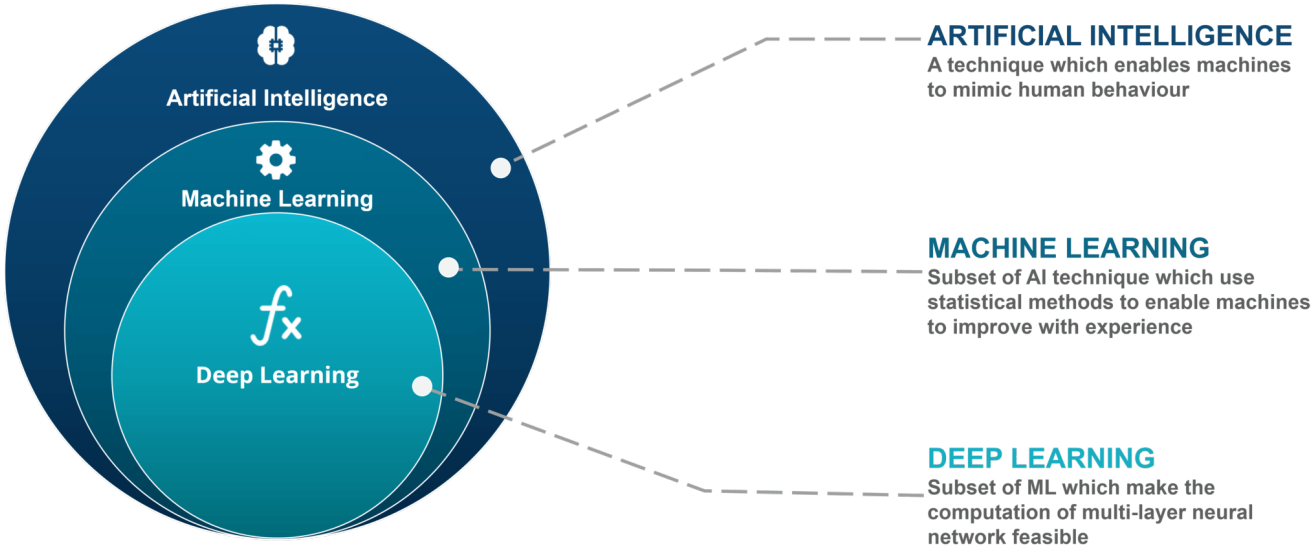

Figure: AI vs ML vs Deep-Learning

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- Fraud Detection: Machine learning algorithms can be used to identify fraudulent activities in real-time, such as credit card fraud or identity theft.
- Predictive Analytics: Machine learning algorithms can be used to analyze historical data and predict future outcomes, such as sales forecasting, customer churn, or demand forecasting.

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- In the DGM, a neural network is trained to approximate the solution to a differential equation on a set of discretized points. The neural network takes in the input variables (e.g., time and space) and returns an approximation of the solution.
- The **Black-Scholes model** is a mathematical model used to estimate the price of financial instruments, such as stock options. It was developed by Fischer Black and Myron Scholes in 1973 and this model is widely used by traders, investors, and financial institutions to price options and manage risk.

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What is a Neural Network

Neural Networks is a computational learning system that uses a network of functions to understand and translate a data input of one form into a desired output, usually in another form. The concept of the *Artificial Neural Network (ANN)* was inspired by human biology and the way neurons of the human brain function together to understand inputs from human senses. In simple words, Neural Networks are a set of algorithms that tries to recognize the patterns, relationships, and information from the data through a process that is inspired by and works like the human brain/biology.

Neural Network

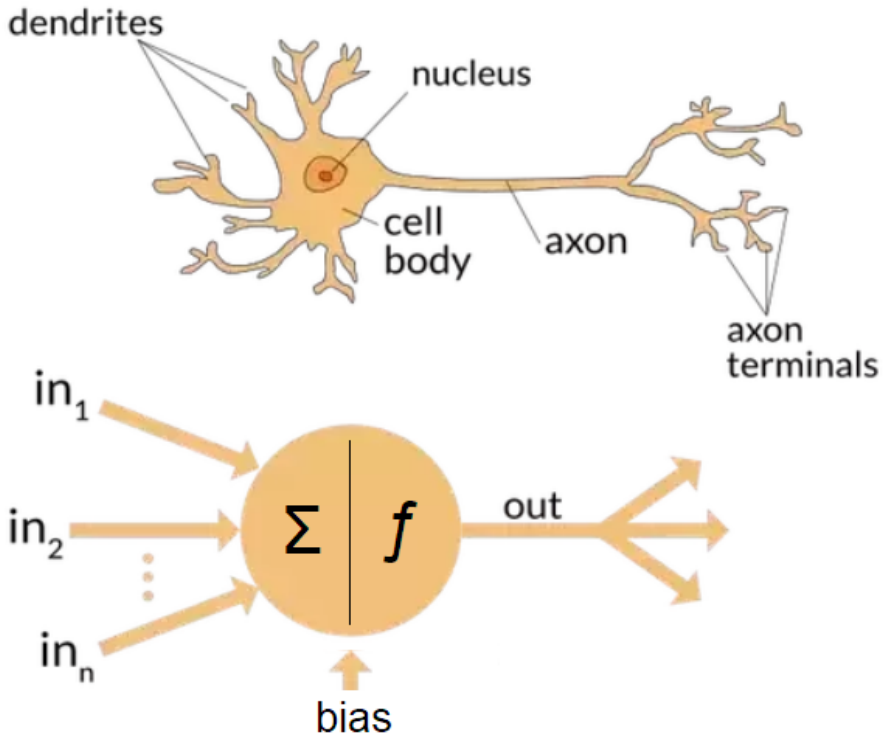

Figure: Human brain vs Neural Network

Components of a Neural Network

A simple Neural Network consists of three components :

1. Input Layer
2. Hidden Layer
3. Output Layer

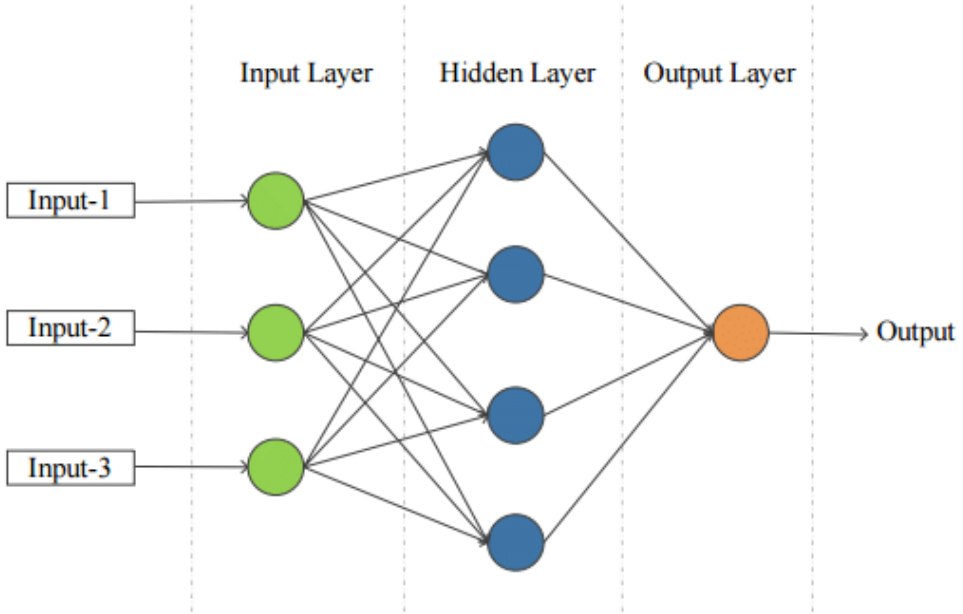

Figure: A Basic Neural Network

Components of a Neural Network

- **Input Layer:** Also known as Input nodes are the inputs/information from the outside world which is provided to the model to learn and derive conclusions from.
- **Hidden Layer:** Hidden layer is the set of neurons where all the computations are performed on the input data.
- **Output layer:** The output layer is the output/conclusions of the model derived from all the computations performed. There can be single or multiple nodes in the output layer.

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- **Bias** terms are additional constants attached to neurons and added to the weighted input before the activation function is applied.Bias terms help models represent patterns that do not necessarily pass through the origin.

Components of a Neural Network

- **Activation functions** modify the data they receive before passing it to the next layer. It give neural networks their power allowing them to model complex non-linear relationships. By modifying inputs with non-linear functions neural networks can model highly complex relationships between features. Popular activation functions include ReLU and Sigmoid.

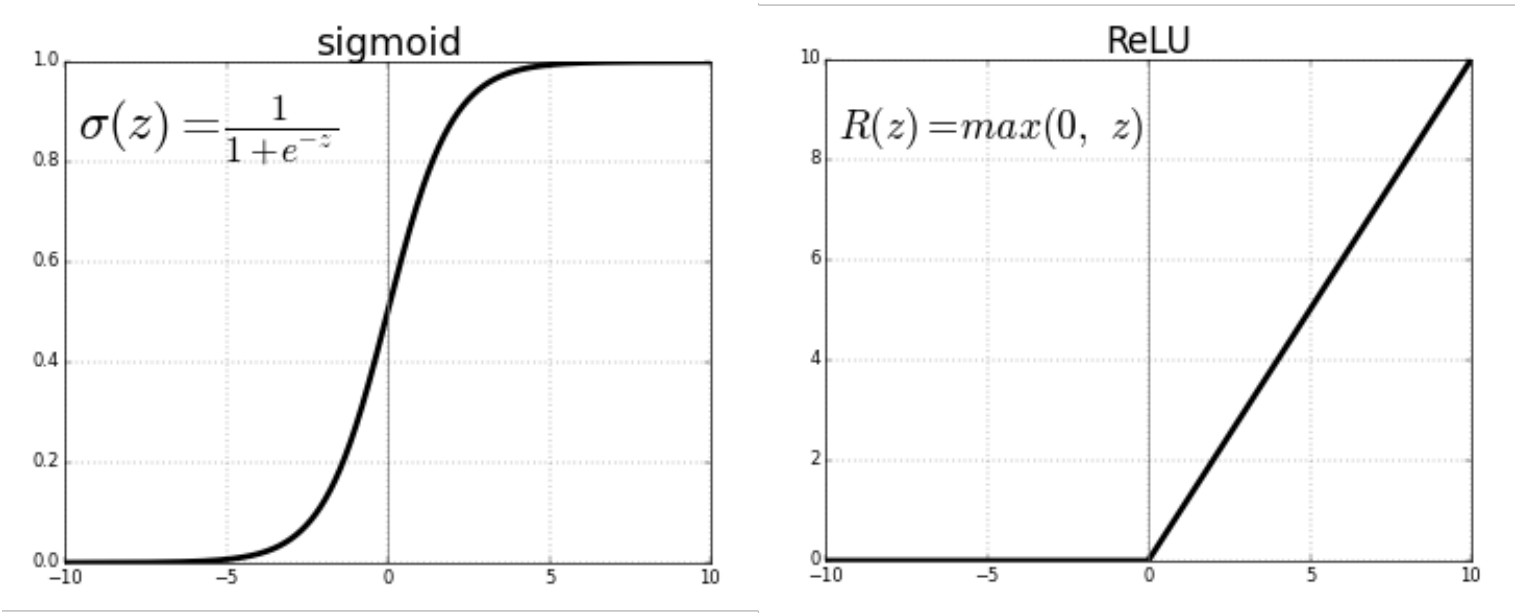

Figure: Sigmoid (left) and ReLU (right)

LSTM

- **LSTM (Long Short-Term Memory)** is a type of recurrent neural network (RNN) that is designed to handle the vanishing gradient problem in traditional RNNs. The vanishing gradient problem occurs when the gradients that are used to update the weights of the neural network during training become too small, making it difficult for the network to learn long-term dependencies.

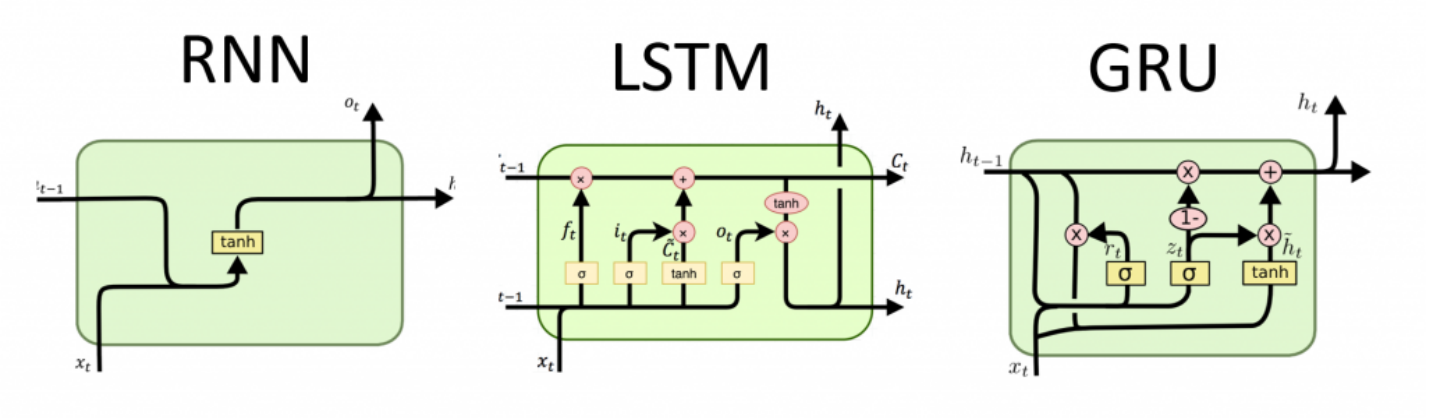

Figure: RNN vs LSTM vs GRU

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- Measure that how well the approximation satisfies the differential operator,

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An option is a financial instrument that allows its owner to buy or sell(but not the obligation) an underlying asset at a pre-specified fixed price within a specific period.

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- **Put Option**: An option that gives its holder the right to sell the underlying asset at an agreed price by the maturity data.

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- **American Option**: American-style option allows the holder to exercise the option at any time prior to the expiration date.

Definition(Pay-off)

The pay-off of an option is its value at the time of exercise T. It is basically gain or loss that can be described in the following figure.

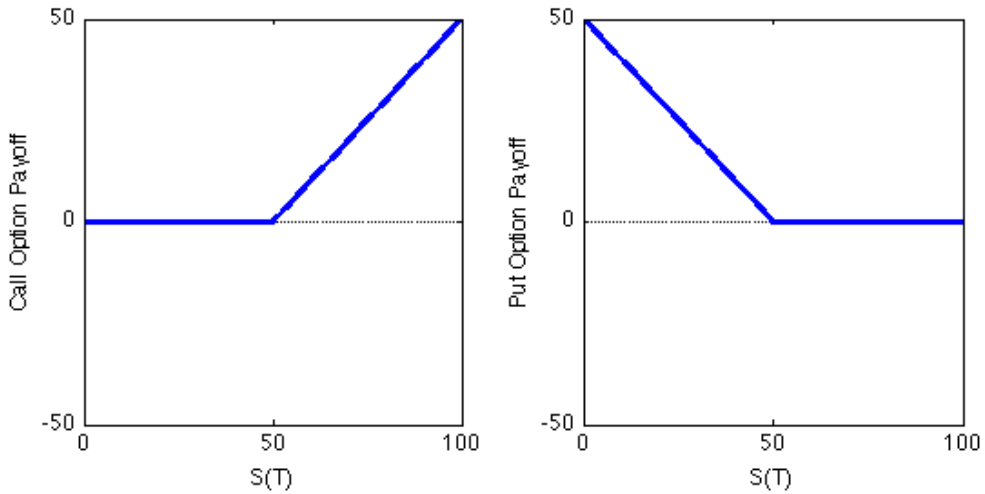

Figure: Payoff diagram of European Call and European Put.

$$V(S_T, T) = \max\{S_T - K, 0\}.$$

$$V(S_T, T) = \max(K - S_T, 0).$$

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Black-Scholes Formula

Now we have boundary conditions and final condition. So when the interest rate r and volatility σ are constant.The exact solution for European call is

$$C(S,t) = SN\left(d_1\right) - Ke^{-r(T-t)}N\left(d_2\right), \tag{1}$$

where $N(\cdot)$ is the cumulative distribution function for a standardised normal random variable, given by

$$N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}y^2}dy. \tag{2}$$

Here

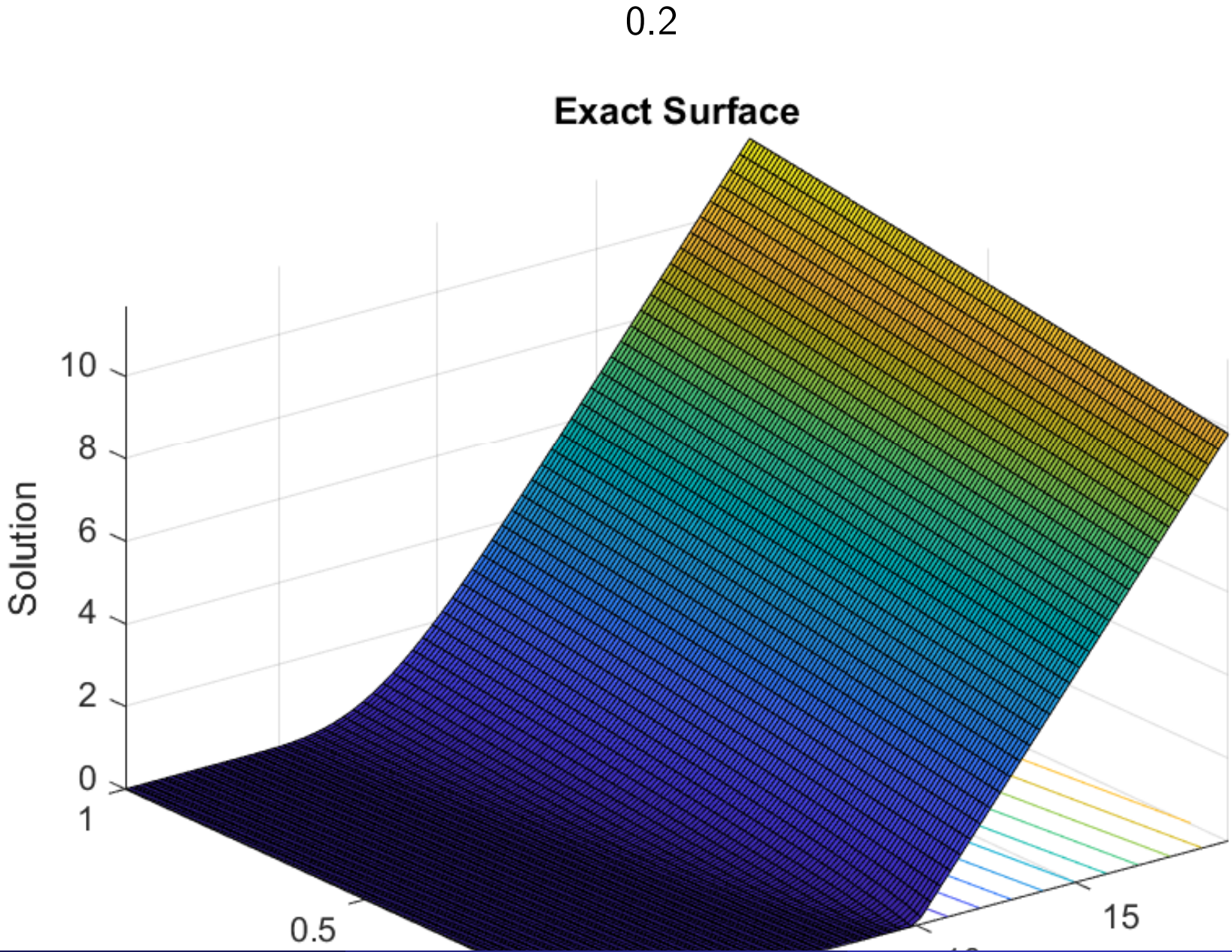
$$\begin{cases} d_1 = \dfrac{\log(\frac{S}{K}) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}, \\ d_2 = \dfrac{\log(\frac{S}{K}) + \left(r - \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}. \end{cases} \tag{3}$$

For a European put, the exact solution is

$$P(S,t) = Ke^{-r(T-t)}N\left(-d_2\right) - SN\left(-d_1\right). \tag{4}$$

Considering the values of parameters as $T = 1, K = 10, r = 0.2$, $\sigma = 0.25$ and $S_{max} = 20$. We get the following plots for $V(S, t)$

Plot of Exact Solution of Black-Scholes Equation



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4. Take a descent step at the random point s_n with Adam-based learning rates

$$\theta_{n+1} = \theta_n - \alpha_n . \nabla_\theta L(\theta_n; s_n)$$

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   - Compute $L_2(\theta_n; , \tau_n, z_n) = \left(f(\tau_n, z_n) - g(\tau_n, z_n)\right)^2$
   - Compute $L_3(\theta_n; w_n) = \left(f(0, w_n) - u_0(w_n)\right)^2$
4. Take a descent step at the random point s_n with Adam-based learning rates

$$\theta_{n+1} = \theta_n - \alpha_n . \nabla_\theta L(\theta_n; s_n)$$

5. Repeat the steps (2) and (4) until $\|(\theta_{n+1} - \theta_n)\|$ is small.

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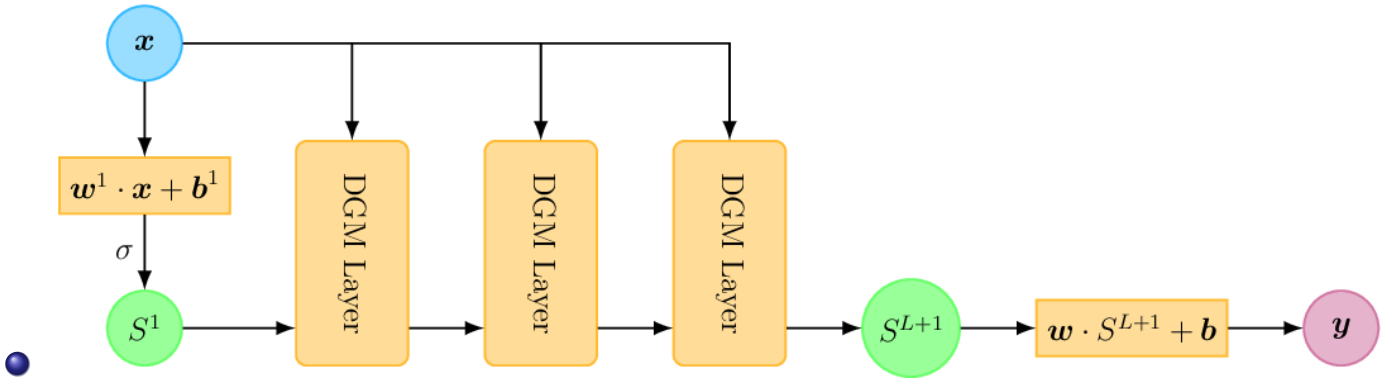

Figure: DGM Architecture

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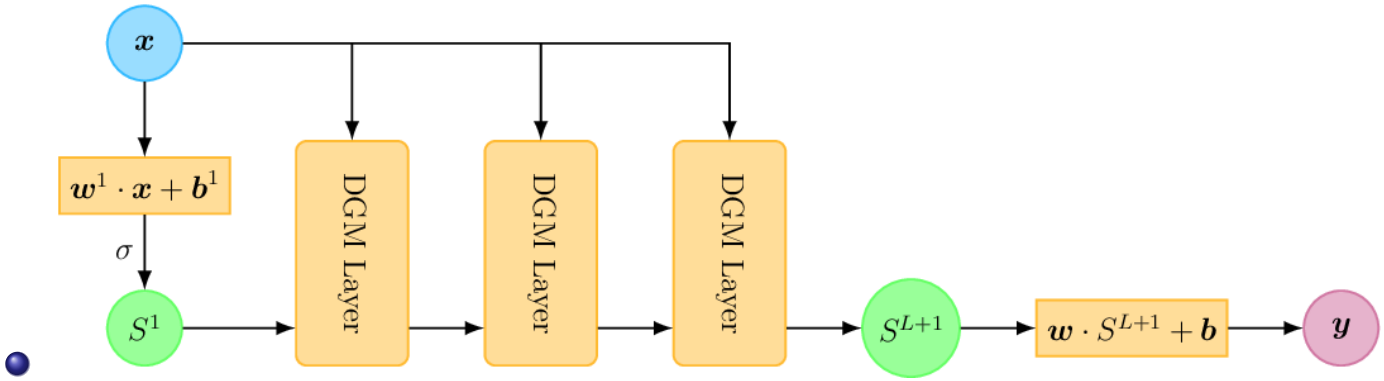

Figure: DGM Architecture

- So now we take the input k mini-batch point on the domain (i.e x in our case and also we chose random normal distribution to take the input on the domain) and also an output of the previous DGM layer. So in this process, we get a vector value output y which consists of the Neural Network and approximates the function of u.

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- $$R_l = \sigma(u_{r,l}.x + w_{r,l}.S_l + b_{r,l})$$
- $$H_l = \sigma(u_{h,l}.x + w_{h,l}.(S_l \odot R_l) + b_{h,l})$$

Implementation

- $$S_{l+1} = (1 - G_l) \odot H_l + Z_l \odot S_l$$

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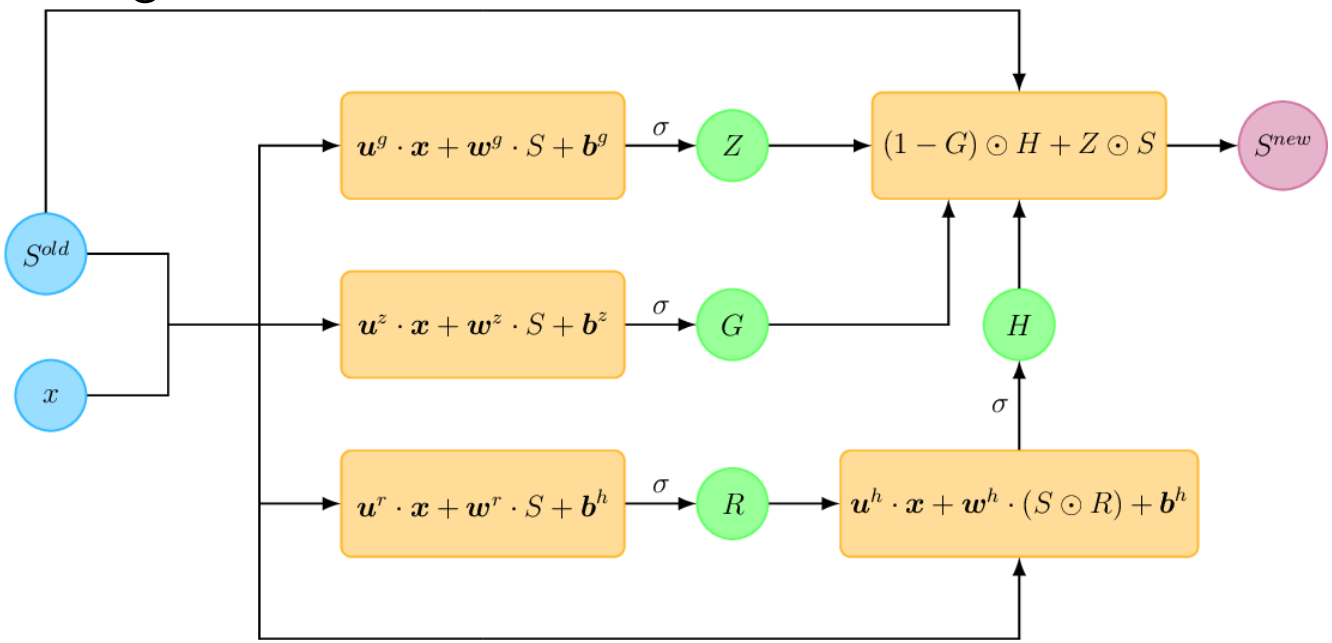

Figure: Notation each single DGM Layer

Experimentation and Results

European Call Option
One-Dimension Black-Scholes PDE

- $$\frac{\partial s(t,x)}{\partial t} + rs.\frac{\partial s(t,x)}{\partial x} + 1/2\sigma^2 x^2.\frac{\partial^2 s(t,x)}{dx^2} = r.s(t,x)$$

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- $$s(T,x) = F(x)$$
- **Solution**
- $$s(t,x) = x.N(v_+) - K.e^{-r(T-t)}N(v_-$$
- where

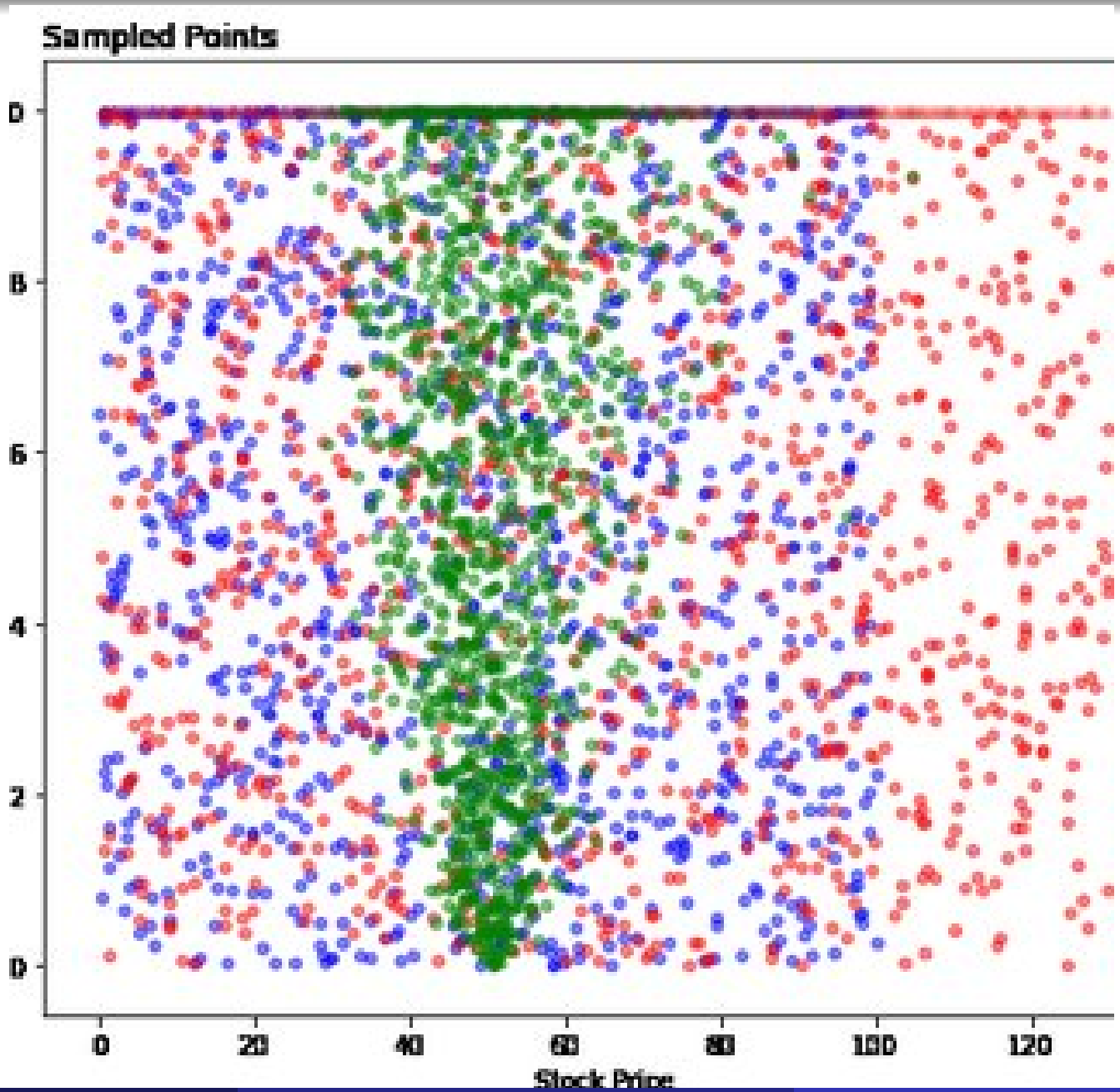
$$v_+ = \frac{\ln(x/K) + (r + 1/2\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$v_- = v_+ - \sigma\sqrt{T-t}$$

Experimentation and Results

Suppose you want to price a European call option on a non-dividend paying stock with a strike price of $K = 50$, a current stock price of $S = 50$, a time to maturity of $T = 1$, a continuously compounded risk-free interest rate of $r = 5\%$, and a volatility of $\sigma = 20\%$.

Experimentation and Results



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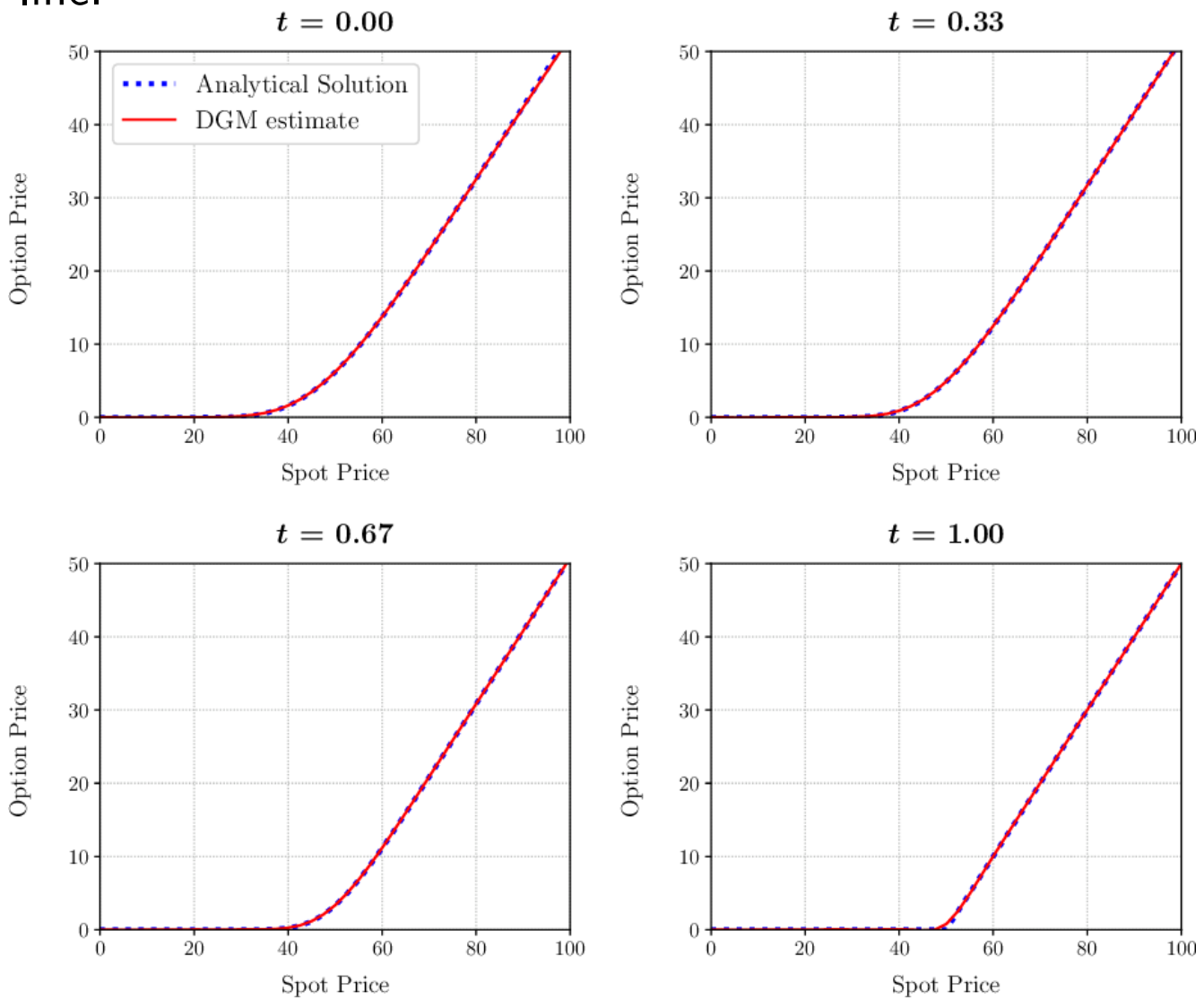

Conclusions

- We have shown that the Deep Galerkin Method(DGM) is a promising approach for solving the European-option pricing model.Our results demonstrate that the DGM can accurately and efficiently approximate the solution to the Black-Scholes equation, but in traditional numerical methods struggle to converge.

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