Modeling a Space Elevator

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Abstract

A space elevator has been proposed by engineers, physicists, and entrepreneurs as a viable and relatively low-cost way to transport people and cargo into orbit. In this lab, we examined the various complexities and constraints of a space elevator from a mechanical perspective. The problems we endeavored to solve included the desired acceleration for ascension and braking, the mass of the counterweight, the work done by the climber, and the energy required to power the climber. In addition, we explored the effects of non-inertial/fictitious forces on the system, including the Coriolis force on the entire elevator as well as an analytic consideration of the centrifugal force on the climber itself.

1. Introduction

The concept of a space elevator, at its most fundamental level, is relatively simple. The primary components include: (1) a base station, (2) the cable, (3) the climber, and (4) the counterweight. We will now examine each component in detail:

First, the base station should be located at the equator, because of the Earth's rotation. Also, for practical reasons the equator has pleasant weather and is proximate to a large percentage of the population.

Second, we propose the cable should be constructed out of carbon nanotube for its high tensile strength, low mass, and resilience. We would begin by launching a satellite into orbit which contains an onboard carbon nanotube production factory, and spooling out the created fiber towards Earth.

Third, the climber would consist of a cabin for freight or people adjoining a rail. Wheels on the rail would propel the climber upwards along the cable, and a braking mechanism would decelerate the cable once it reaches the peak velocity of 90 m/s.

Finally, the counterweight will be placed past geosynchronous orbit and form the terminus of the cable. However, most cargo would be jettisoned before the climber reaches the counterweight because geosynchronous orbit is the preferred destination.

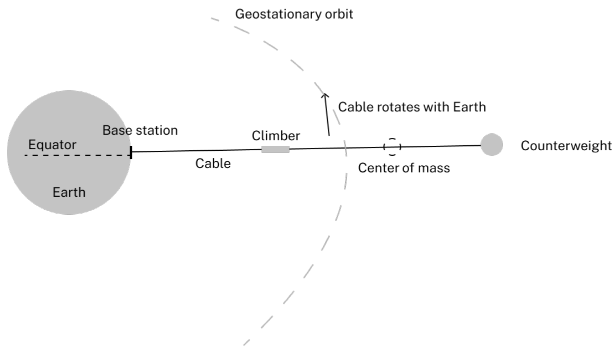

Figure 1: Diagram of space elevator system

2. Theory

Calculating the mass of the system with the Coriolis force C_p

We know that the Coriolis force operates in the x-direction and will flip based on the climber's direction (ascent or descent). Furthermore, the Coriolis force acts on the system as a whole: the climber-cable-counterweight system. Here is the net force in the x-direction:

$$F_{net,x} = 2m\dot{\vec{r}} \times \vec{\Omega} \tag{1}$$

The mass m here is the center of mass of the system, which is approaches the counterweight as the climber ascends. The elevator is designed such that the center of mass never drops below geosynchronous orbit. The Coriolis force is dependent on the velocity of the mass, and we will find this velocity using our computational simulation. It is critical that we limited the velocity in our simulation, because a velocity which exceeds 90 m/s will invoke dangerous oscillations due to the high Coriolis force.

Centrifugal force on the climber C_L

As noted in the introduction, we can analytically model the interaction of the centrifugal force on the climber itself, as displayed below:

$$F_{net,y} = +m(\Omega \times r) \times \Omega - mg \tag{2}$$

where r is the position of the climber relative to the center of the Earth and g is changing marginally as the climber ascends. To find g, we use the equation for universal gravitation:

$$g = \frac{Gm_1 m_2}{r^2} \tag{3}$$

where G is the gravitational constant and the two masses are the Earth and the elevator system respectively.

3. Method

We wrote our application in Python on Jupyter Notebook. The program runs on real-time (1 second) linspace, meaning that the computer simulation the full extent of the elevator's 5-day journey. This enabled us to produce precise and meaningful graphs.

3.1. Constraints

Constraints are noted in Method, not Theory, because many of these constrains were generated "on the fly" during the process of our computational simulation through in-built functions (rather than through a simple analytic formulation).

3.1.1. Coriolis force

As mentioned in the Introduction, we will be using a cable constructed from carbon nanotubes in this study, which offer a tensile strength of 11 - 63 GPa. At the highest end of this range (63 GPa), the cables should be able to withstand 131040000 N of tension force.

$$6.3 \times 10^1 9 \frac{N}{m^2} \tag{4}$$

. We will assume the Coriolis force is acting on the contact surface between the cable and the climber. Because the force acts laterally, the formula would assume this form:

$$F_{cor} <= T_C \frac{m}{s^2} \tag{5}$$

It is critical that we limit our velocity properly, because a climber with excessive speed would incur tremendous Coriolis forces. However, the complexity of the system makes the velocity difficult to produce, so we settled on 83 m/s as the desired cruising speed for the climber, with a max acceleration range of 80-90 m/s. This is in line with previously modeled engineering constraints on climber speed, based on an array of factors, including (1) friction on the climber, (2) power, and (3) trip length.

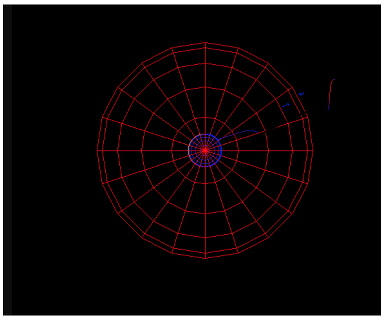

Figure 2: Simulation of climber with excessive Coriolis forces and loss of counterweight; notice the frayed cable and un-tethered counterweight on the right-hand side of the graph

3.1.2. Max acceleration on climber

The max comfortable acceleration on a human being is

$$a_{max} = 3g \tag{6}$$

so that is a critical constraint on the climbing force. Of course, cargo could be hypothetically transported at a greater acceleration, but we will keep the for the purposes of this simulation.

3.2. Power and Energy

We also wanted to model the energy usage of the climber, which is important when building a system of energy transfer from the cable to the climber. Since the trip is five days long, we imagine the primary power source would come from a conductive connection between the cable and the climber itself. Energy would be transferred from the base station upwards, and the base station itself would be covered in solar panels to provide a constant energy input to the elevator. The elevator would also keep an onboard battery for braking in case of sudden power shortages. If the power stopped, the elevator would naturally drop, and the onboard battery would provide emergency braking to avoid dangerous acceleration.

First, we looked at the power required for climbing:

$$P_{climb} = F_{climb} \times v \tag{7}$$

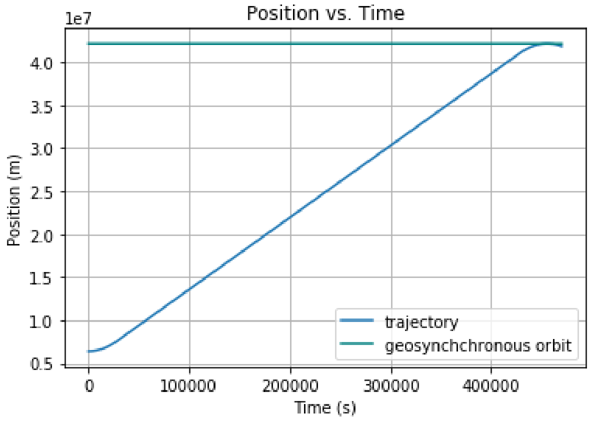

Figure 3: Total energy vs. time for the climber and constraint

$$P_{coast} = 0 \tag{8}$$

$$P_{brake} = F_{brake} \times v \tag{9}$$

4. Results

With regard to constraints, our computational model provided values for:

$$r_{coast} = 7571502\ m \tag{10}$$

$$r_{brake} = 40970698\ m \tag{11}$$

We also returned motion graphs for the climber's trajectory, subject to the constraints we applied in the computational model. Constraints are superimposed onto the graph for reference.

In the graph above, the geosynchronous orbit is marked by the green line. Notice that braking is activated to stop the within a few kilometers of geosynchronous orbit to allow for passenger/freight injection. The system allows for a small margin of error here, since the total length of the cable extends far beyond GSO to the counterweight.

In the v-t graph, we see that the climber never reaches the maximum operational velocity at the outer range of the accepted max velocity range (80 - 90 m/s).

Finally, we produced energy graphs for the climber, which enabled us to find the power needed to run the system.

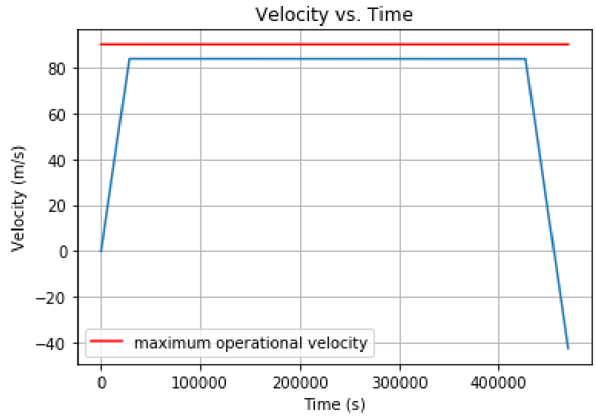

Figure 4: Velocity vs. time for the climber and constraint

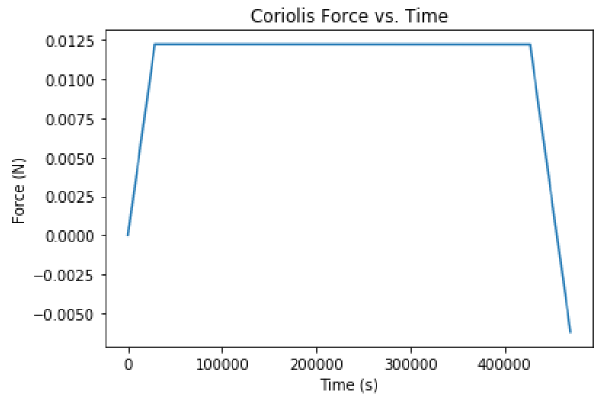

Figure 5: Coriolis force vs. time for the climber

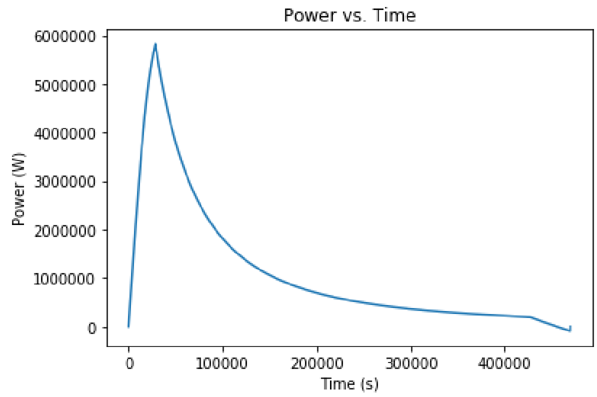

Figure 6: Power vs. time for the climber

$$P_{max} = 5.8 \times 10^6 \ W \tag{12}$$
$$P_{avg} = 1.1 \times 10^6 \ W \tag{13}$$

Thus, at peak the climber would require enough electricity to power about 4500 homes at once.

5. Discussion

We are satisfied with the results we achieved in this lab, which combined analytic solutions for various constraints with a computational model of the space elevator. We often used computational methods to find our solution, and thus the exact trajectories are unique to the specific runtime and cannot be replicated. However, the constants we put in place ensure that passengers are transported safely, maximum operational velocities are kept in check, acceleration and deceleration are kept at nominal rates, power consumption is viable, and the Coriolis force has minimal effects on cable rigidity and climber integrity.

Naturally, there are numerous complexities with a project of this scale and technological complexity. In this model, we were only able to explore a small slice of this system, and hopefully provided some useful insights on its operational design.

There are a number of interesting opportunities for additional exploration. These include:

- Modeling the forces of "microscopic interaction" between the wheels of the climber and the cable itself, taking into account the forces of friction, cable tensile strength/tension force, and centrifugal force
- Finding constraints for the mass of the counterweight and climber
- Computationally modeling scenarios of cable breakage due to excessive Coriolis forces

References

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