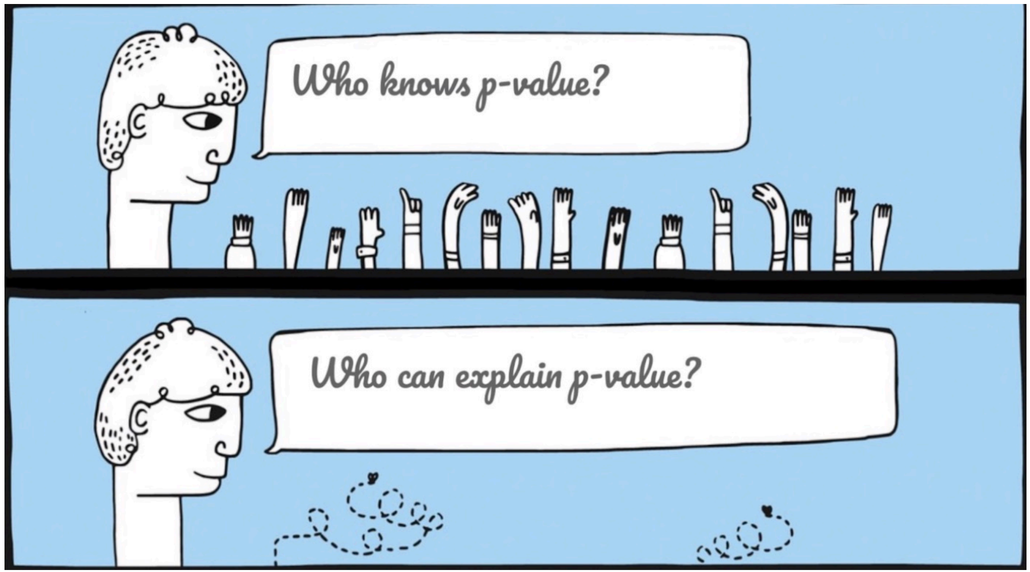
22nd June '23

Hypothesis Testing - 2



Agenda:

- Quick recap of Imp terminologies
- Hypothesis testing framework Implementation
- diff. approaches to test hypothesis
- Examples of Left, Right & Two tailed Tests

Recap of CLT :

Avg. Indian height $= 65$ in $\rightarrow (\mu)$.

std. devⁿ $= \sigma = 2.5$ inches.

$n=50$

$\underbrace{x_1, x_2, \ldots, x_{50}}$

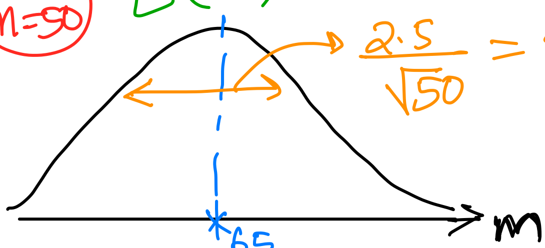
$m \rightarrow$ Sample mean

Random Variable.

· dbⁿ of 'm' $\sim$ Normal dbⁿ.

$n=50$ $\quad E(m) = 65$

$\dfrac{2.5}{\sqrt{50}} = s.d.$



$n=5$

$\{x_1, x_2, \ldots, x_5\} \rightarrow m$

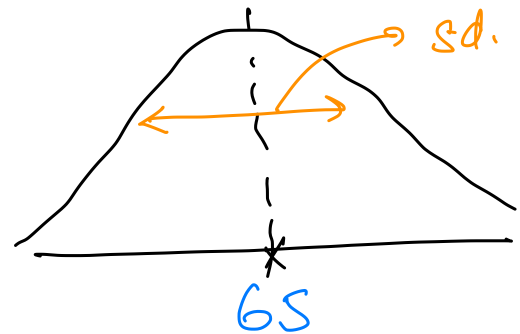
Sample mean

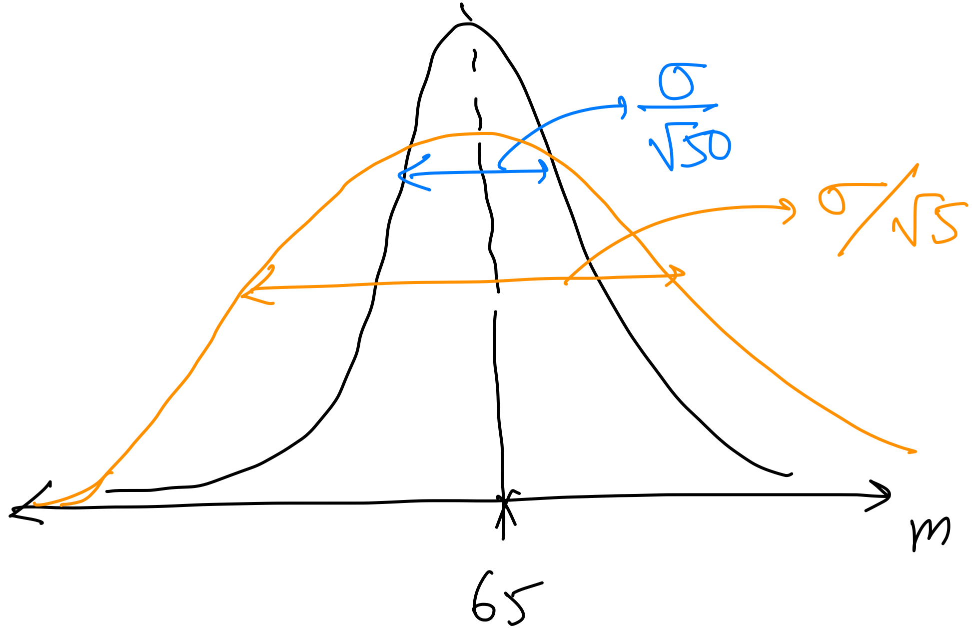
$n=5$

$E(m) = 65$

$m \sim$ Normal

$sd. = \left(\dfrac{2.5}{\sqrt{5}}\right)$





$\frac{\sigma}{\sqrt{50}}$
$\sigma/\sqrt{5}$
m
65

Terminologies of H.T. / Framework of H.T

10T → 7H

which tailed test.

① H_0, H_A

$T_{obs} = 7$

Acc. = 85%

② Test statistic → observed value fm data.

(a single pt. value fm the observed data)

③ Left, Right, Two tailed Test.

④ Compute the "p_value" → P [data | H_0 is True]

⑤ Compare p value with 'α' → significance level (0.05) 5% or 95%.

Supply chain example:

A retailer has 2100 stores.

On avg. the weekly sale avg. = 1800 → μ

std. dev^n = σ = 100 → σ.

own team of Analysts

* 99% confidence

= significance 1%

⇒ $\alpha = 0.01$

Sales team hires marketing Team.

team A

deploying their strategy on 50 stores.

$(x_1 + x_2 - - - x_{50}) / 50$

• avg. weekly sales = 1850

team B

5 stores

$x_1, x_2, ----, x_5$

avg. sales = 1900

Team-A : $\mu = 1800$, $\sigma = 100$, $\alpha = 0.01$

50 stores , $m_a = 1850 = \left(\dfrac{x_1 + x_2 - - - - - + x_{50}}{50}\right)$

$x_i \rightarrow$ denotes weekly sales.

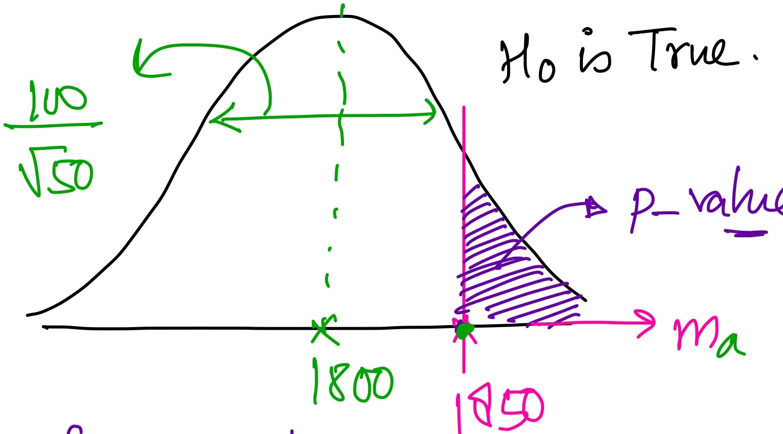
$m_a \rightarrow$ Sample mean

- $H_0 : \mu_a = 1800$ (no effect)

  $H_A : \mu_a > 1800$

$sd = \dfrac{100}{\sqrt{50}}$

H_0 is True.



- $m_a \sim$ Normal → R.V

  $E(m_a) = 1800$

- std dev. $(m_a) = \dfrac{\sigma}{\sqrt{n}} = \dfrac{100}{\sqrt{50}}$

$\rightarrow P[m_a \geq 1850 \mid H_0 \text{ is True}] = p\_value$

$= 1 - \text{norm.cdf}(Z_{1850})$

$\Rightarrow$ observed value of $m_a = 1850$

$\Rightarrow$ Right Tailed test.

$\Rightarrow Z = \dfrac{x - \text{mean}}{sd.} \Rightarrow \dfrac{1850 - 1800}{(100/\sqrt{50})} \approx 3.353$

$z = 3.353$

$p\_value = 1 - norm.cdf(3.353)$.

$p\_value = 0.0002$

$\alpha = 0.01$

$p\_value < \alpha$.

reject H_0 ⟹ Null hypothesis.

⟹ The Team A has true effect on Sales wrt. 99% confidence or 1% sig.

Team-B

$\mu = 1800$, $\sigma = 100$, $\alpha = 0.01$

$n=5$, $m_b = 1900 = \left(\dfrac{x_1 + x_2 - - - - - + x_5}{5}\right)$

Sample mean

$x_i \to$ denotes weekly sales.

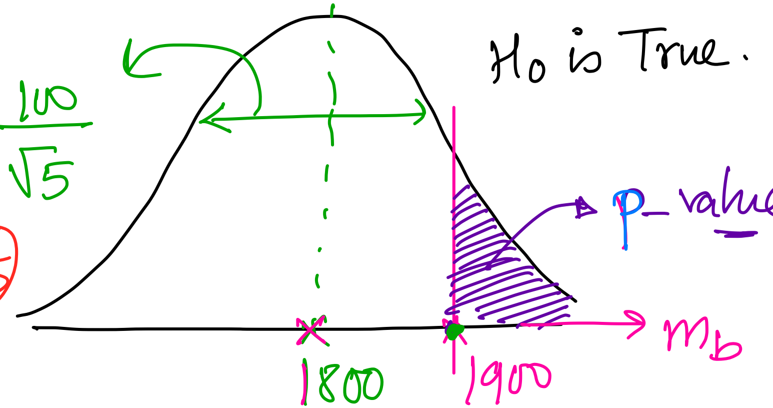
- H_0: $\mu_b = 1800$ (no effect)

  H_A: $\mu_b > 1800$

H₀ is True.

$sd = \dfrac{100}{\sqrt{5}}$

$n=5$



- $m_b \sim$ Normal

  R.V

  $E(m_b) = 1800$

- std dev. $(m_b) = \dfrac{\sigma}{\sqrt{n}} = \dfrac{100}{\sqrt{5}}$

$\Rightarrow$ Observed value of $m_a = 1900$

$\Rightarrow$ Right Tailed test.

$\to$ $\underbrace{P[m_b \geq 1900 \mid H_0 \text{ is True}] = p\_value}$

$= 1 - \text{norm.cdf}(Z_{1900})$

$\Rightarrow Z = \dfrac{x - \text{mean}}{sd.} \Rightarrow \dfrac{1900 - 1800}{(100/\sqrt{5})} = 2.236$

$Z = 2.236$

$p\_value = 1 - norm.cdf(2.236)$

$p\_value = 0.012$

$\alpha = 0.01$

$p\_value > \alpha$

not able to reject H_0

· Team B has no statistical significat Effect on sales improvement wrt 99% confidence or 1% significance

P-VALUE APPROACH

Supply chain example

$\alpha = 0.01$

$\mu = 1800$

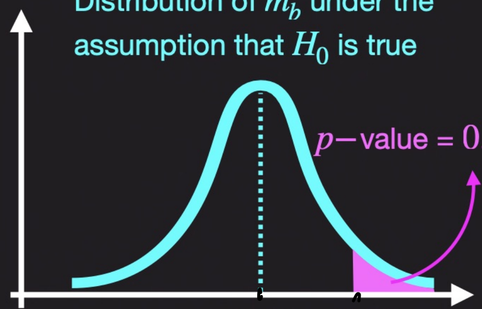
$\sigma = 100$

50 stores with average of 1850

$H_0 : \mu_b = 1800$

$H_a : \mu_b > 1800$

Distribution of m_b under the assumption that H_0 is true



$$\text{std dev} = \frac{100}{\sqrt{50}}$$

sd_1

$$z = \frac{1850 - 1800}{100/\sqrt{50}} = 3.53$$

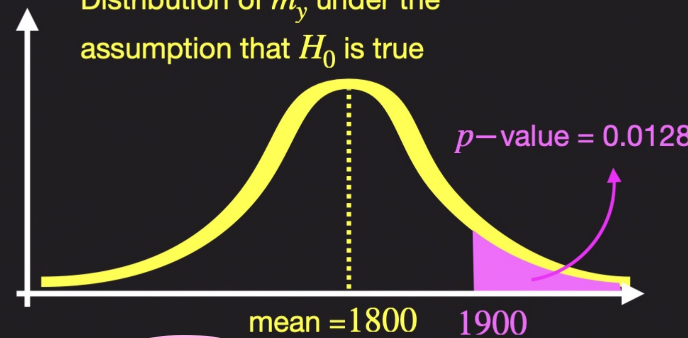
Reject H_0

5 stores with average of 1900

$H_0 : \mu_y = 1800$

$H_a : \mu_y > 1800$

Distribution of m_y under the assumption that H_0 is true



$$\text{std dev} = \frac{100}{\sqrt{5}}$$

sd_2

$$z = \frac{1900 - 1800}{100/\sqrt{5}} = 2.23$$

Fail to reject H_0

1850 is 3.53 (sd_1) away f^m mean

1900 $(2.23) sd_2$ away f^m mean

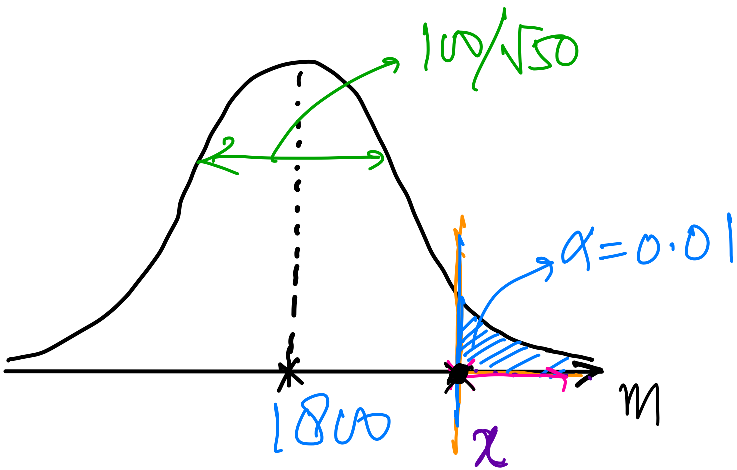
• $\mu = 1800, \quad \sigma = 100, \quad \alpha = 0.01$

n = # of stores. → 1, 2, - - - - . , 2000

$n \in [1, 2000]$.

$\boxed{n=50}$ → m → Sample mean (weekly sales from 'n' stores).

R.V $\sim$ Normal db^n



$E(m) = 1800$

$S.d(m) = 100/\sqrt{n} = 100/\sqrt{50}$

• Critical value of 'm' → mean sales above which I reject my H_0

$p < \alpha$

x : critical value

$x \rightarrow ?$

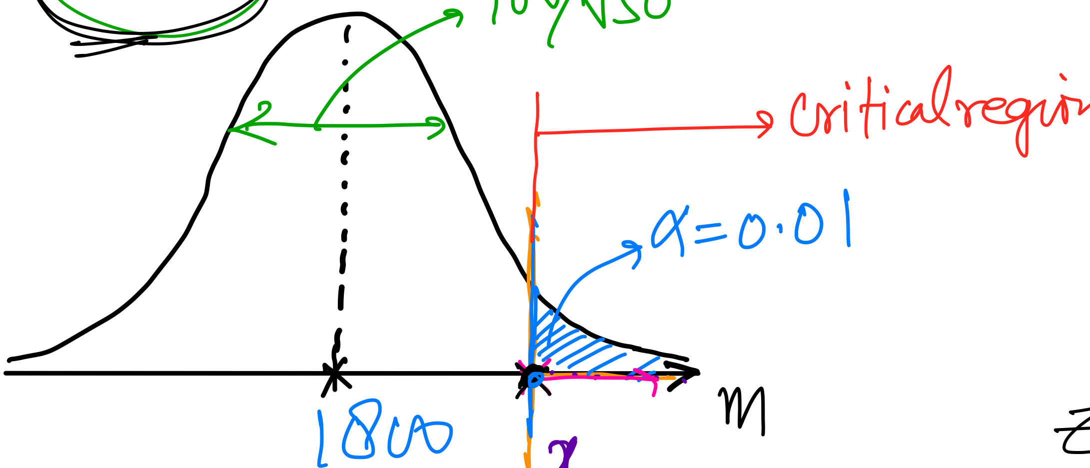
(i) Z score of 'x'.

$n = 50$

$100/\sqrt{50} = sd$

norm.ppf(0.99)

$$Z_x = 2.33$$



$$Z = \frac{x - \text{mean}}{s.d.}$$

critical value

$$x = \text{mean} + Z_x(sd)$$

$$\boxed{x = 1800 + (2.33)(100/\sqrt{50})}$$

$x = 1832.9$

$x = 1832.9 \longrightarrow$ Critical sales value.

$1833 > x$

$1834 > x$

⋮

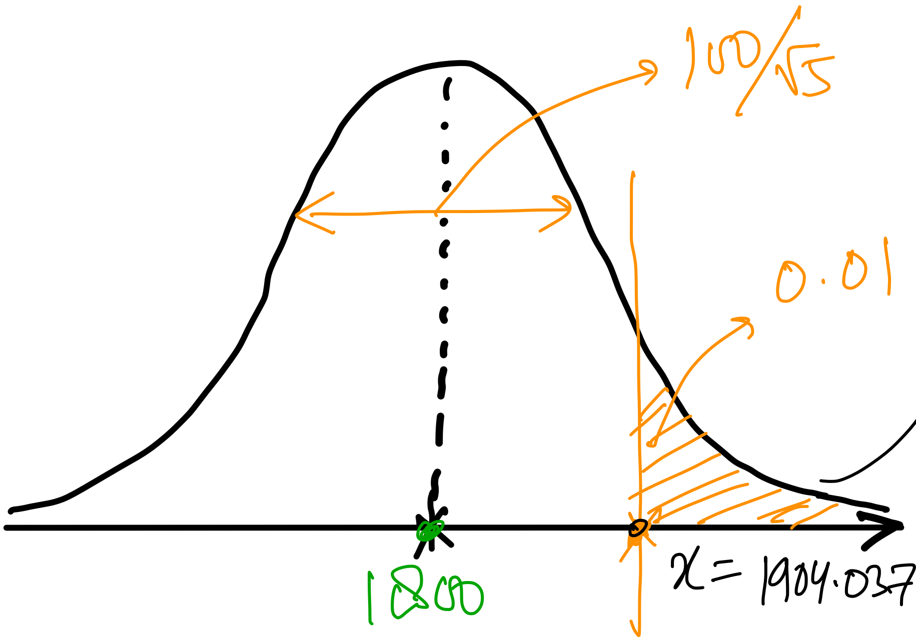
Sales value (m) $> x$

$\boxed{\text{Critical value Approach}}$ *

Critical value for $n=5$.

$\mu = 1800$, $\sigma = 100$, $\alpha = 0.01$

$M \sim RV \sim N\left(1800, \frac{100}{\sqrt{5}}\right)$



$z_\alpha = \text{norm.ppf}(0.99)$
$= 2.33$

$z_\alpha = \dfrac{x - \text{mean}}{\text{sd.}}$

$\Rightarrow x = \text{mean} + z_\alpha \cdot \text{sd.}$

$x = 1800 + (2.33) \cdot \left(\frac{100}{\sqrt{5}}\right)$

$x = 1904.037$

If sale < x → no impact.

n=5

If sale > x → true impact.

* Team B → n=5, 1900

Supply chain example

$\alpha = 0.01$

$\mu = 1800$

$\sigma = 100$

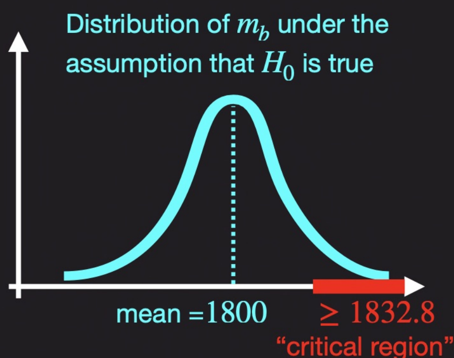
1900

50 stores

$H_0 : \mu_b = 1800$

$H_a : \mu_b > 1800$

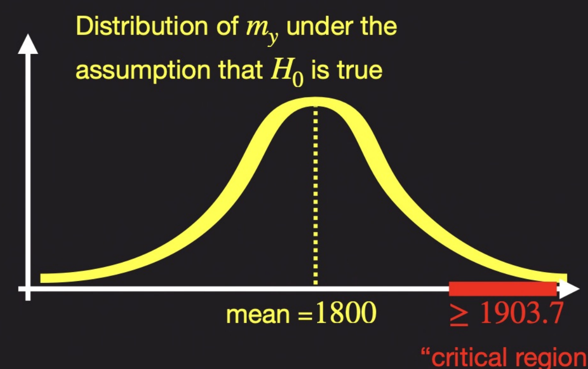
1850



5 stores

$H_0 : \mu_y = 1800$

$H_a : \mu_y > 1800$



Note: For right-tailed test, the critical region is on the right

The probability associated with critical region is α

The rule to reject is very simple: If the observed test statistic is in the critical region, then reject the null hypothesis

Hypothesis Testing Framework

1) Setup the Null and Alternate Hypothesis

2) Choose the right test statistic

3) Left-tailed Vs Right-tailed Vs Two-tailed

4) Compute p-value   (Or compute the critical region)

5) If p-value is less than alpha, then reject the null hypothesis

    (Or check if observed test statistic is in the critical region. If so, reject the null hypothesis)

$n = 5, \; 50 \rightarrow \dfrac{\sigma}{\sqrt{n}}$

$1800 \rightarrow \mu$

$100 \rightarrow \sigma$

$\left(\dfrac{x_1 + \dots + x_{50}}{50}\right)$

$n = 50$. $\dfrac{100}{\sqrt{50}}$

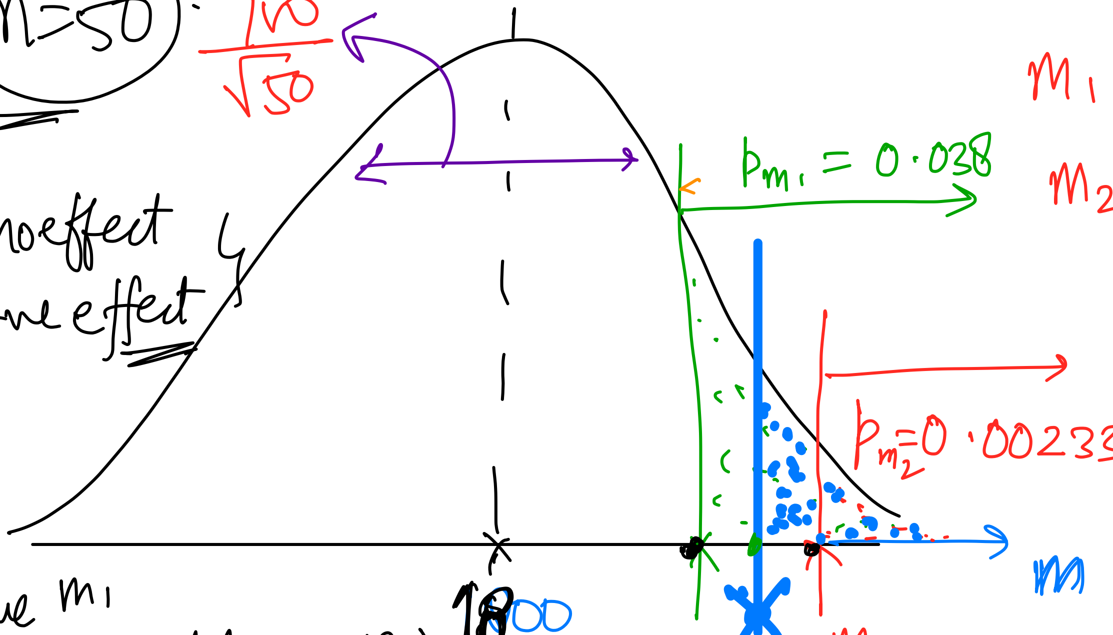
H_0: no effect

H_A: true effect

$m_1 \rightarrow 1825$ ✓

$m_2 \rightarrow 1840$ ✓

p values.



$m_1 = 1825$

$m_2 = 1840$

α.

$= \alpha = 0.01$

→ pvalue m_1

$= 1 - \text{norm.cdf}\left(\dfrac{1825 - 1800}{100/\sqrt{50}}\right)$

$m_2 = 1 - \text{norm.cdf}\left(\dfrac{1840 - 1800}{100/\sqrt{50}}\right)$

at X → Right Area after X

= 0.01

left hand side Area = 0.99

Z score of x → norm.ppf(0.99)

= 2.33

$$z = \frac{x - \text{mean}}{\text{sd}} \Rightarrow x = \text{mean} + z(\text{sd})$$

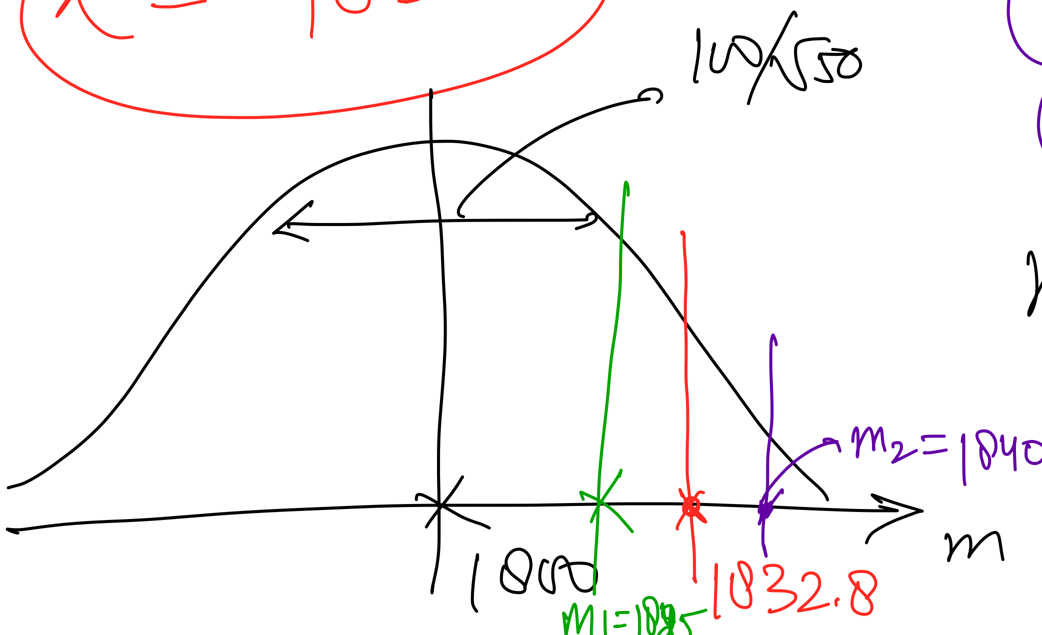
$$x = 1800 + 2.33\left(\frac{100}{\sqrt{50}}\right)$$

$n=50$

$$x = 1832.8$$

$m_1 = 1825$

$m_2 \rightarrow 1840$

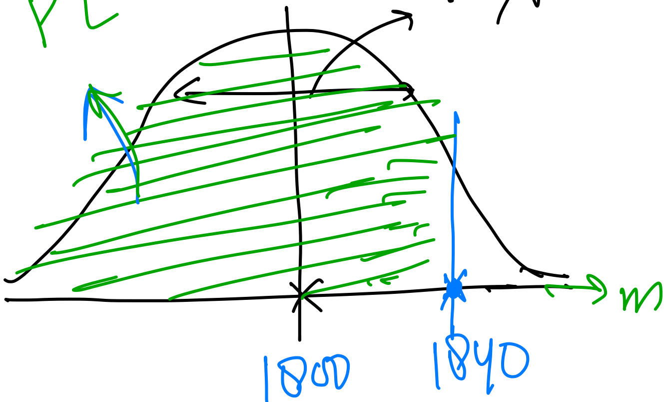


H_0: no effect

H_A: true effect.

ppf v/s cdf → norm

$P[M \le 1840]$



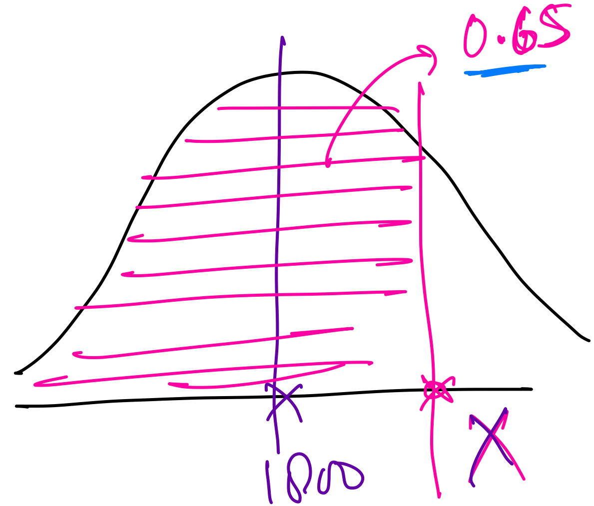
Z_{1840} ✓ (known)

↓

Area = ?

norm.cdf(Z_{1840})

= area



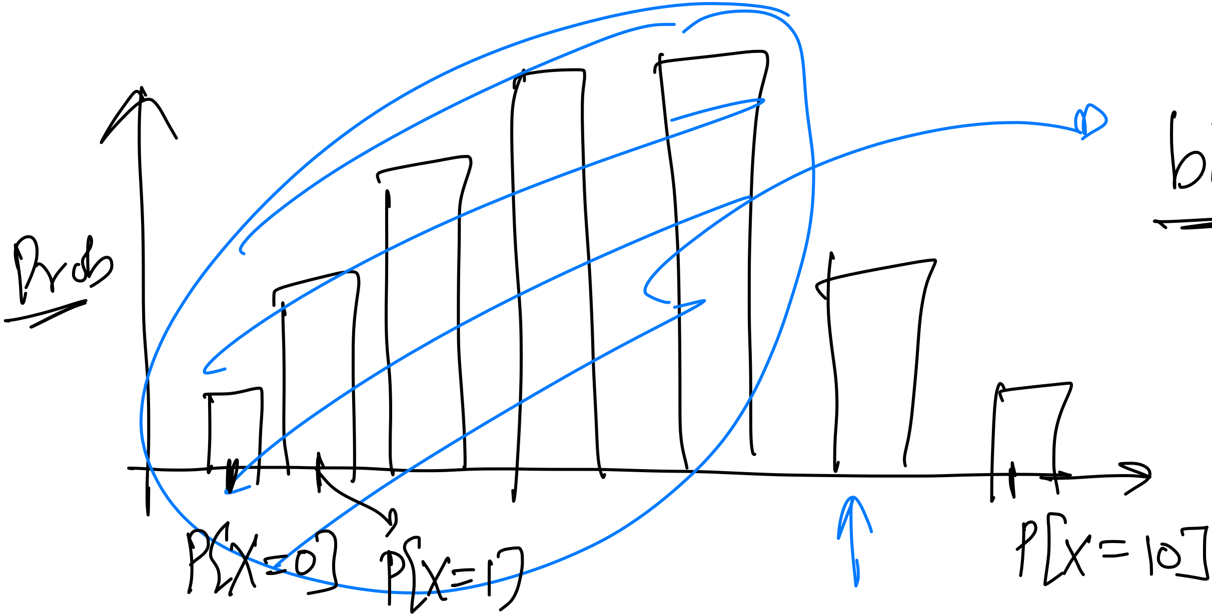
norm.ppf(0.65)

↓

Z_x

↓

$Z_x = \dfrac{x - \text{mean}}{\text{sd}}$ ✓



binom.cdf ( )

n=10 times

#of heads.