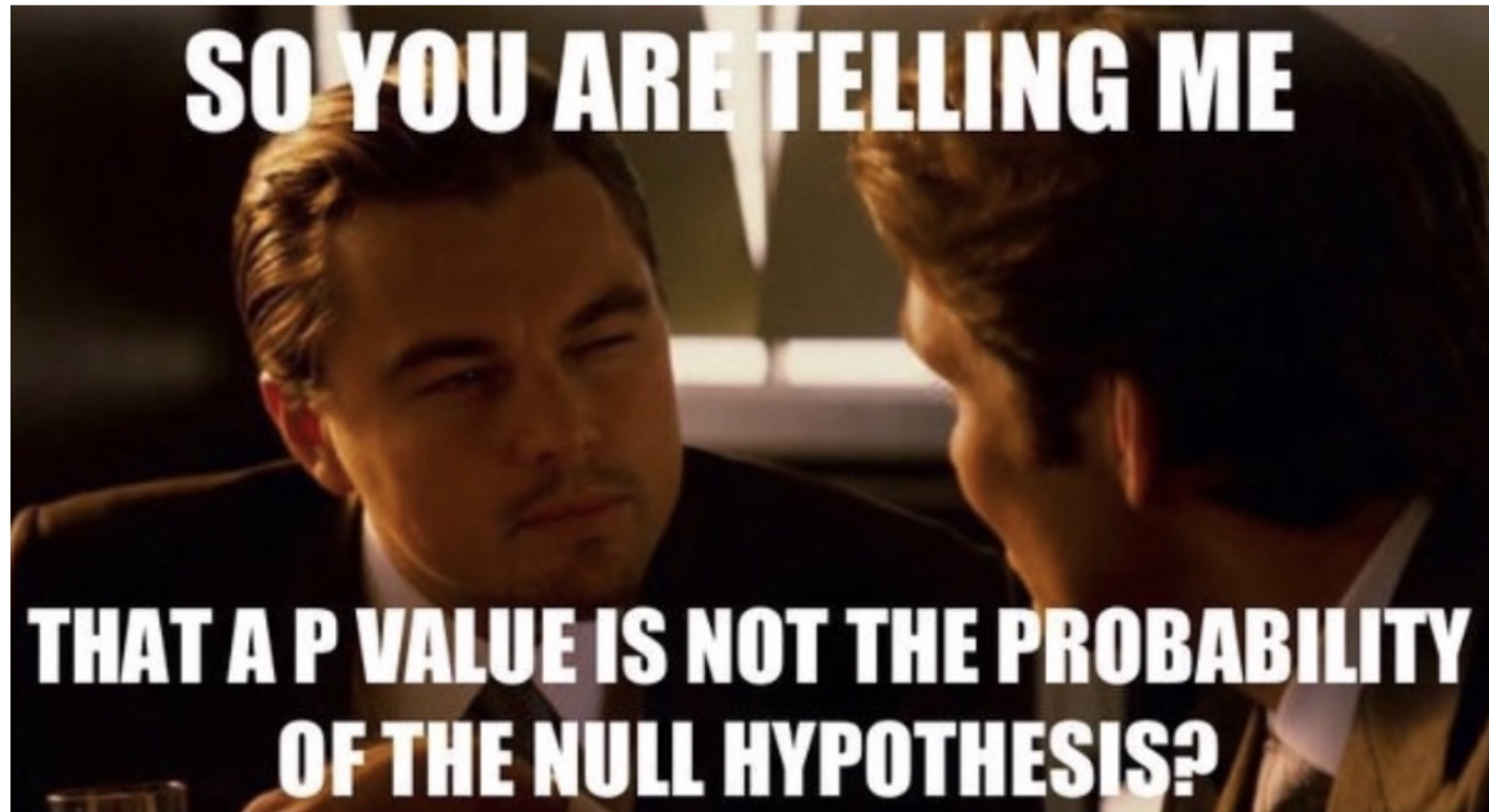


24th June '23



Pvalue — Prob. of H_0 being True
✓ X

- Recap framework

- Recap Z test

left Two tailed

- 1 sample z test

- 1 sample T-test

- 2 sample T-test.

Right, Left, Two tailed *

Framework

(i) H_0 & H_A

(ii) Test stat & db^n

(iii) which tail test.

(iv) compute p value

(v) compare p value
with α

↓
Sig. level.

If $p < \alpha \rightarrow \text{Reject } H_0$

* Premature childrens : 

Avg IQ = 100
std dev = 15

population
 σ

Medical Researches
wants to check.

↓
lower IQ.

@ 5% Significance
level

$\alpha = 0.05$

50 premature child.
↓
finding. → Avg. IQ = 95

$n = 50$

(i) $H_0: \mu = 100$
 $H_A: \mu < 100$

(ii) Test stat.

'm' → Sample mean
↓
R.V. → Normal

(iii) which Tail Test \rightarrow left tail.

$$\underline{E(m)} = 100$$

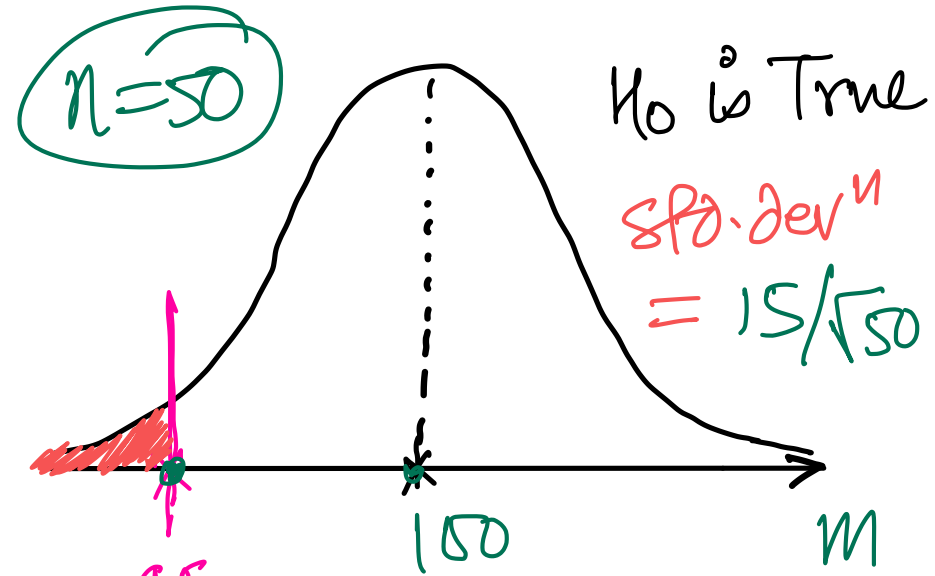
(iv) p-value.

\downarrow

$$P[m \leq 95 \mid H_0 \text{ is True}]$$

$$= \text{norm.cdf}\left(\frac{95 - 100}{15/\sqrt{50}}\right)$$

$$= 0.009$$



\downarrow
 $Z_{95} ? \rightarrow \text{norm.cdf}(Z_{95}).$

p-value $< \alpha$ (0.05) \rightarrow reject H_0

\swarrow
Premature babies have lower IQ.

(IInd)

lower IQ \longleftrightarrow higher IQ.

$\mu = 100$
 $\sigma = 15$ } Population

99% Confidence.
1% Significance.

$\alpha = 0.01$

50 premature child.

94

obs. mean.

$m \rightarrow$ Sample mean

frame work:

$H_0: \mu = 100$
 $H_A: \mu \neq 100$

$H_A: < 100 \rightarrow$ left.

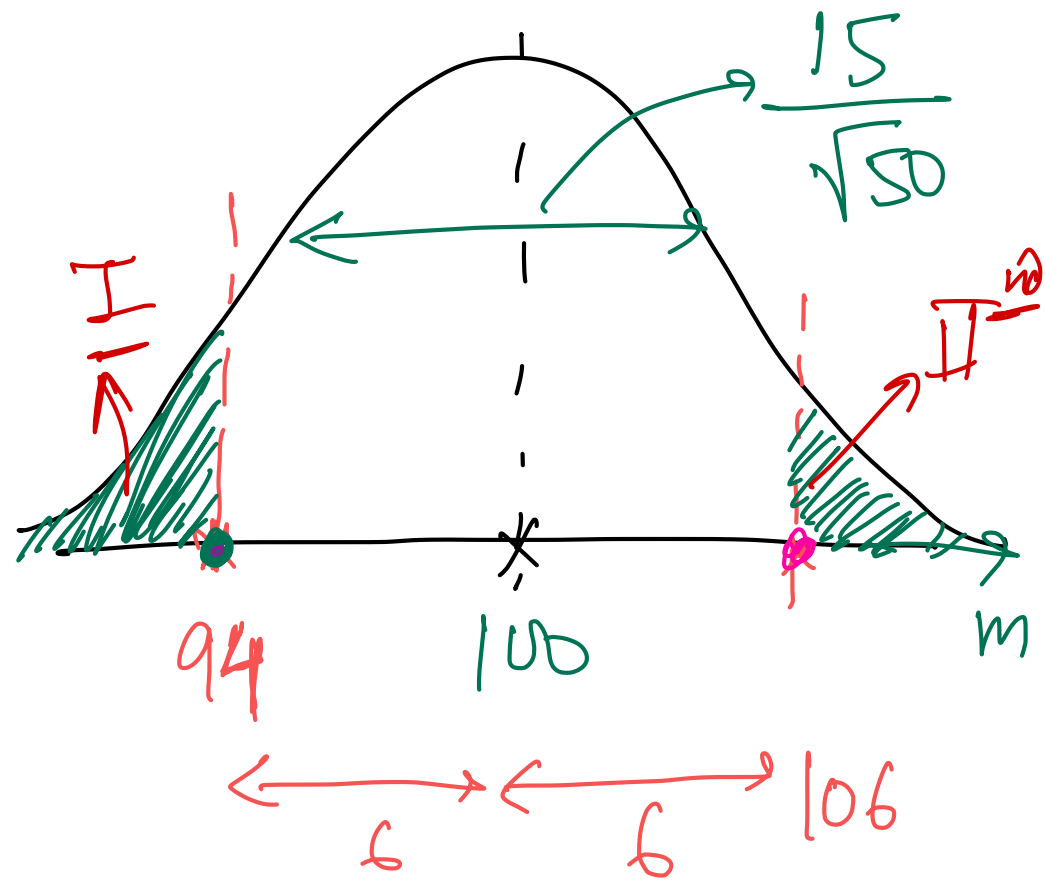
$H_A: > 100 \rightarrow$ Right.

$H_A: \neq 100 \rightarrow$ Two tailed Test.

②

sample mean $\rightarrow \underline{\bar{m}}$
 $m \sim \text{Normal}(\sigma^2)$

$m_{\text{obs}} = 94$



③ Two tailed Test

④

p value

I^{cf} : $\text{norm. cdf} \left[\frac{94 - 100}{15/\sqrt{50}} \right] = 0.0023$

I^{no} : $1 - \text{norm. cdf} \left[\frac{106 - 100}{15/\sqrt{50}} \right] = 0.0023$

$$\begin{aligned} p \text{ value} &= I + II^{nd} \\ &= 2 \times 0.0023 \end{aligned}$$

$$p \text{ value} = 0.0046$$

$$\alpha = 0.01$$

Reject H_0

$$p < \alpha$$

premature child have
significantly diff IQ than normal babies
at 99% conf. or 1% sig.

8:15

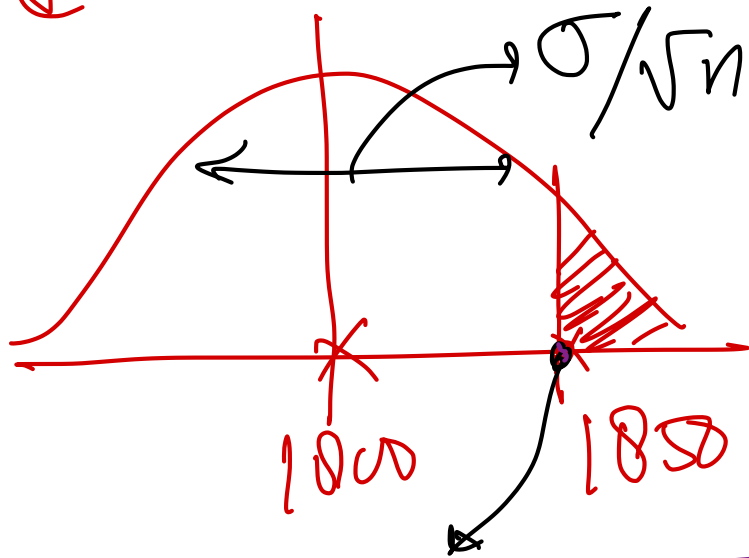
break!

σ : known

n : Sample size

Poplⁿ mean \rightarrow claimed
 \downarrow

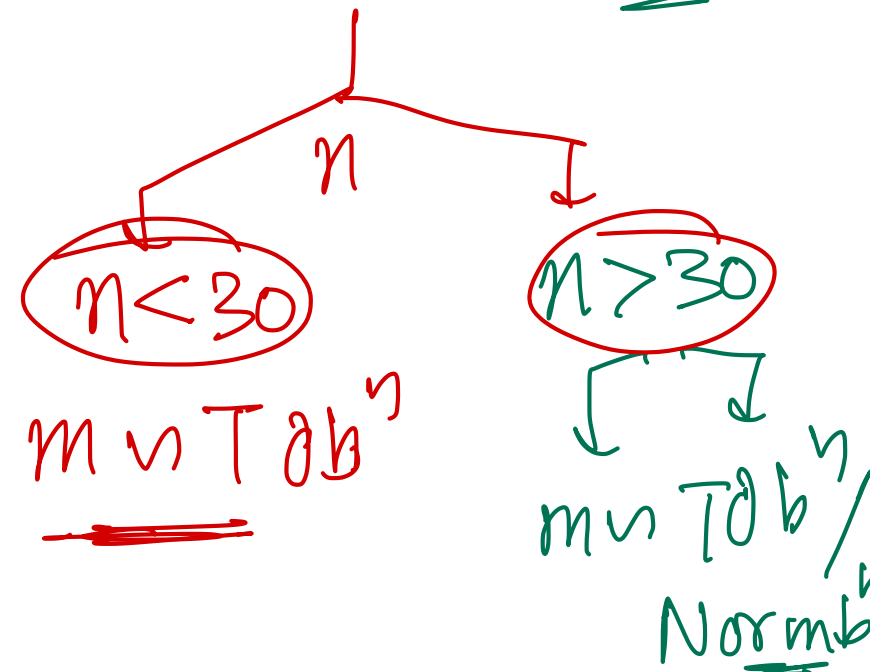
$m \rightarrow$ Sample mean $\sim N(\mu, \sigma/\sqrt{n})$



$$Z_{\text{stat}} = \left(\frac{1850 - 1800}{\sigma/\sqrt{n}} \right)$$

σ : is unknown

$m \sim T$ distribution

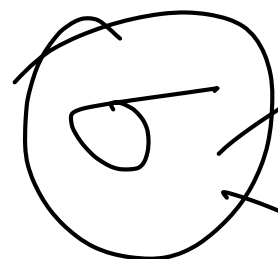


Pop lⁿ std - devⁿ

'n' doesn't matter.

Z test

Normal distⁿ



known

un known. \rightarrow n

$n < 30 \rightarrow$ T distⁿ
T test

$n > 30$

T test / Z test
almost same

IQ Pill

$$\mu = 100$$

σ is known.

Sample of people = $\{ \dots \}$

→ test statistic

$Z_{\text{stat}} = \left(\frac{\bar{x} - 100}{\sigma / \sqrt{n}} \right)$

↓
Normal

6 is Unknown

$$t_{\text{stat}} = \frac{x - \mu_0}{(s/\sqrt{n})}$$

↓

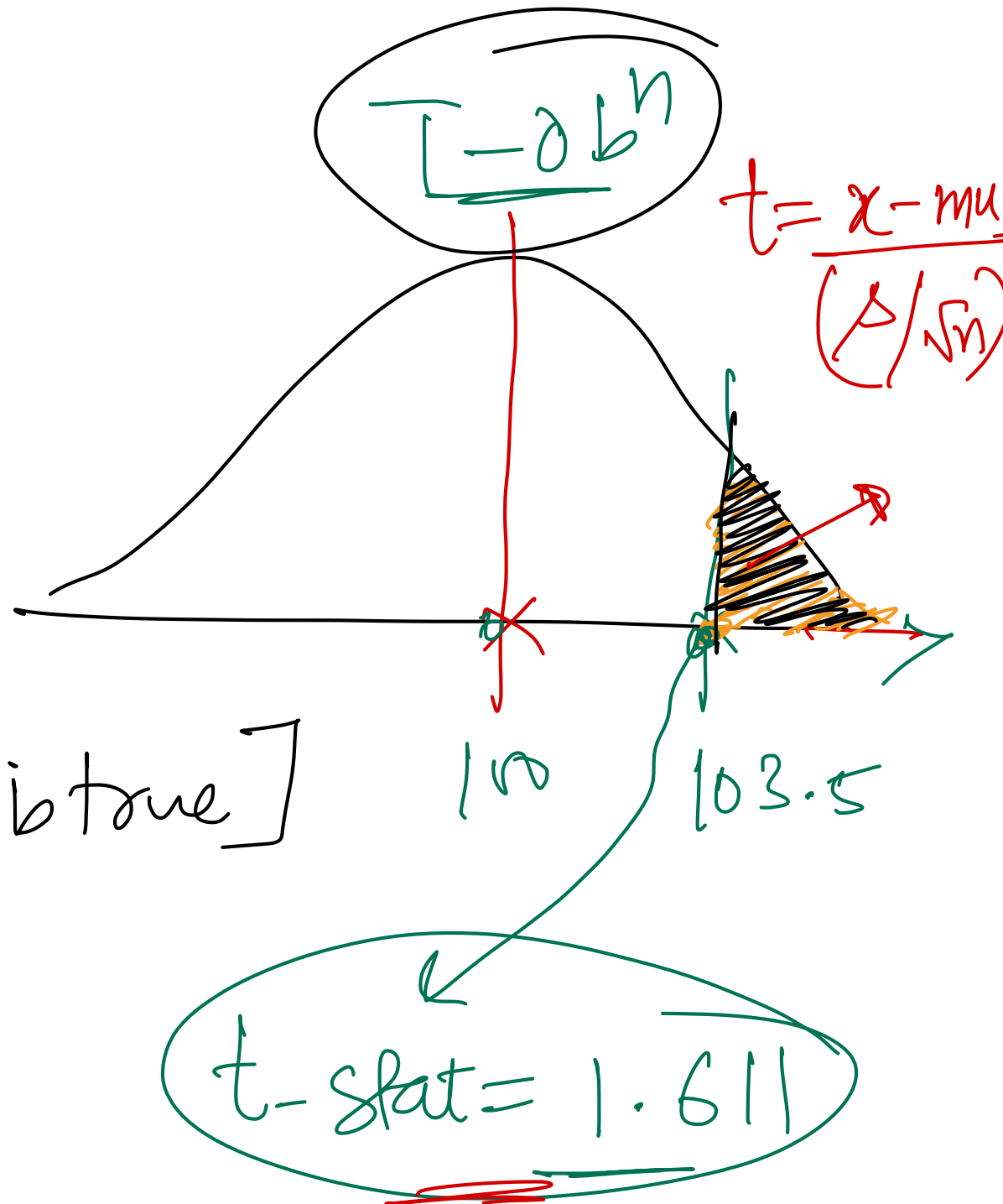
t distribution

\Rightarrow Avg IQ = 100

$$H_0: \mu = 100$$

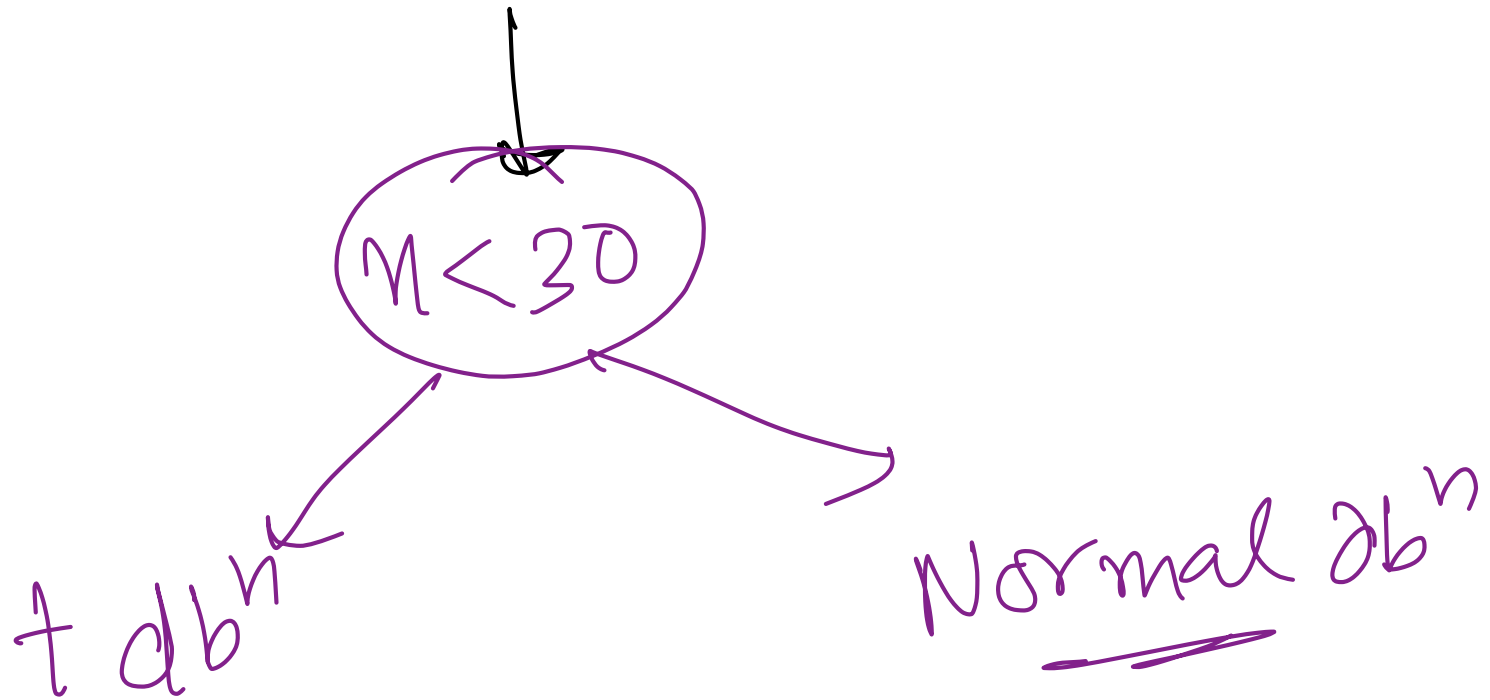
$$H_A: \mu > 100$$

$$P[t\text{-stat} \geq 1.611 \mid H_0 \text{ is true}]$$



CLT

$$\bar{x} \rightarrow \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right).$$



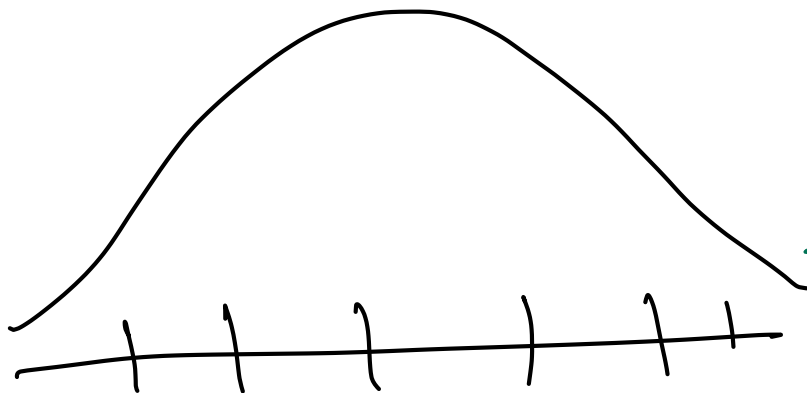
σ is known

↗

Normal



n doesn't matter



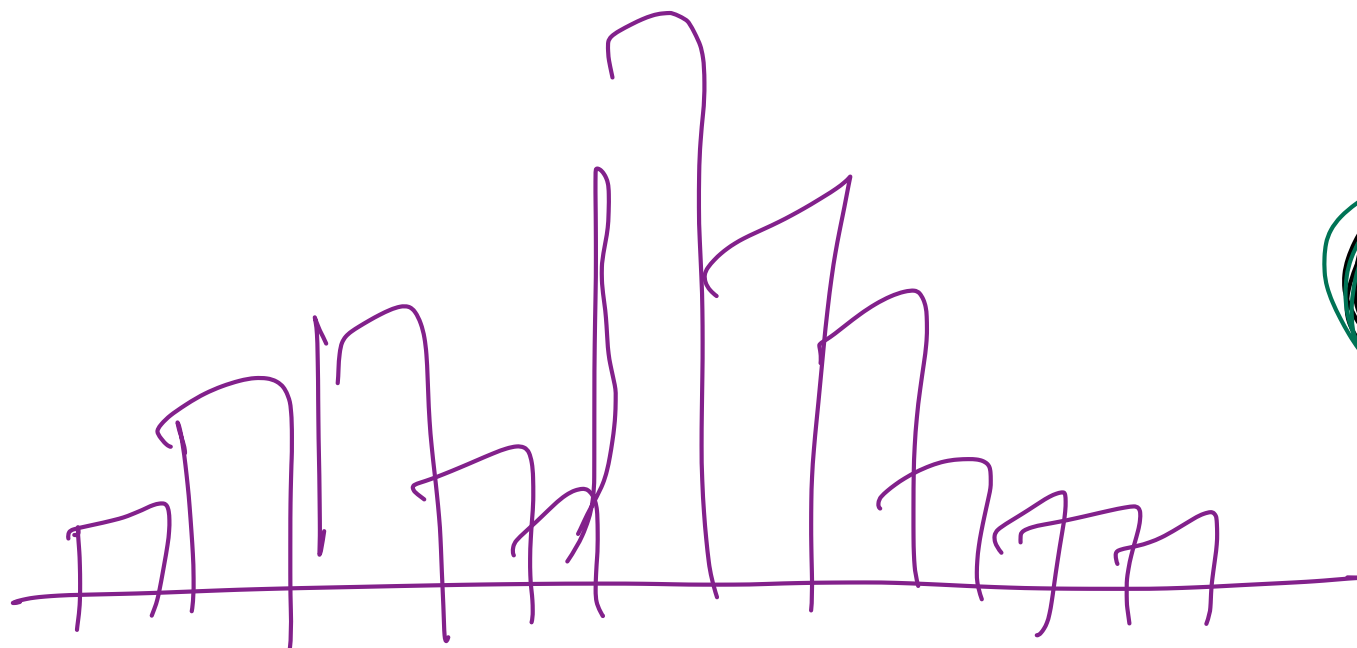
not normal



m_1

m_2

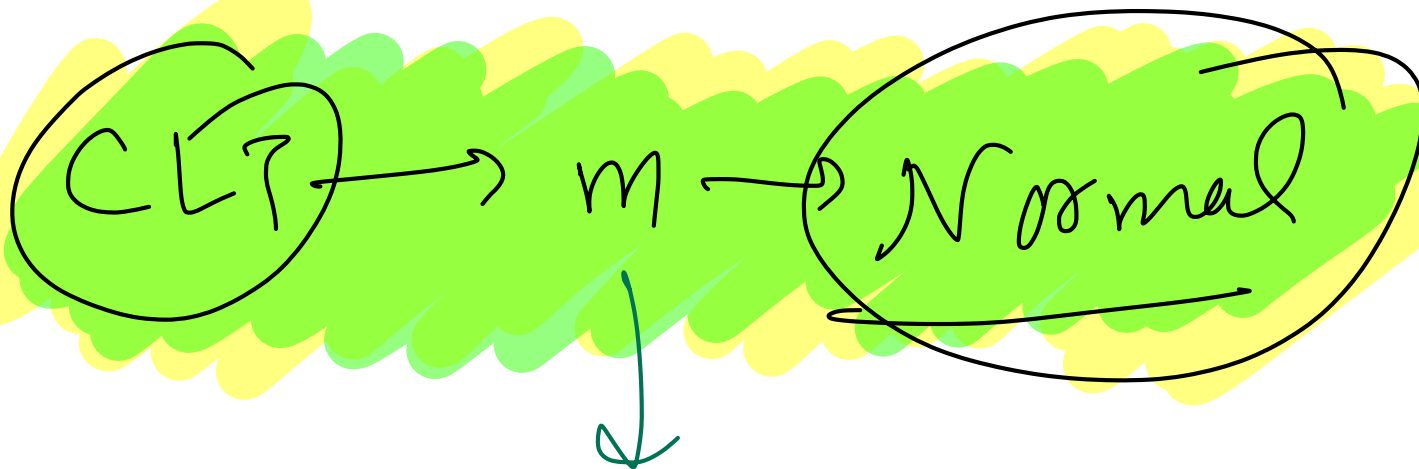
normal



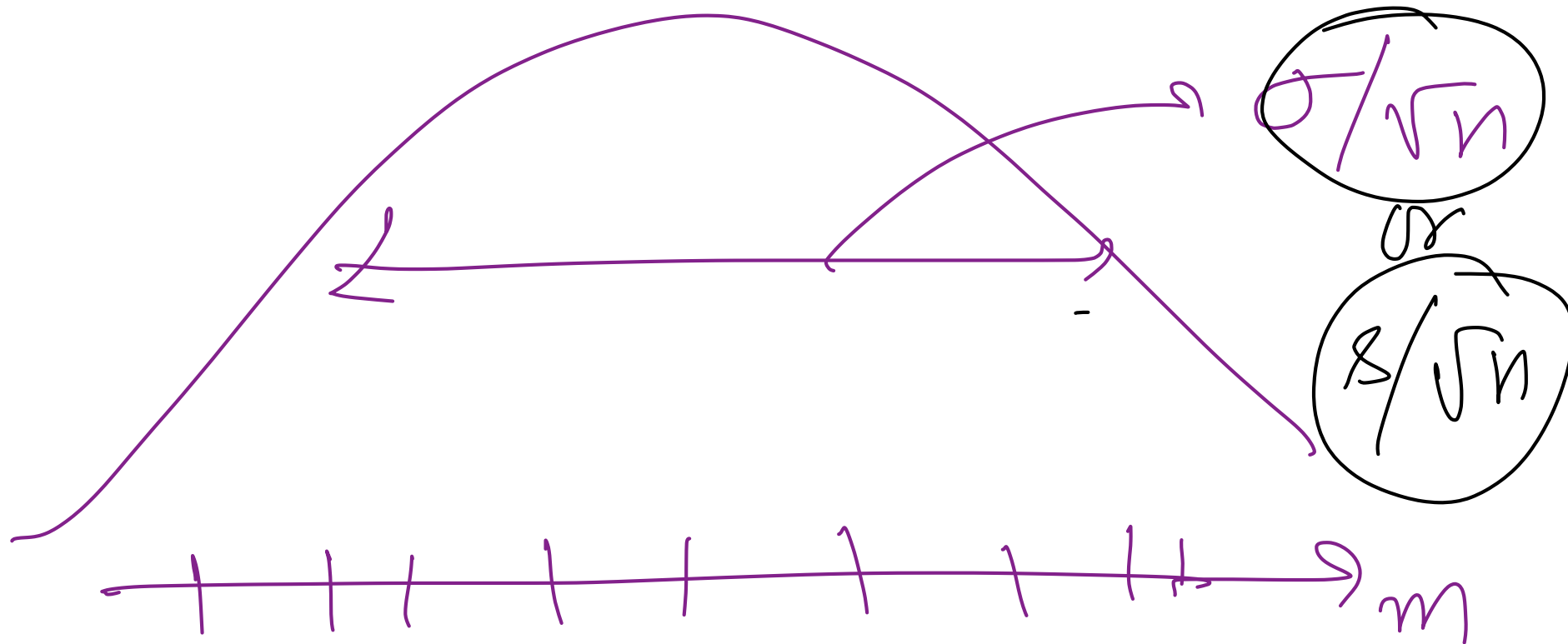
$n < 30$

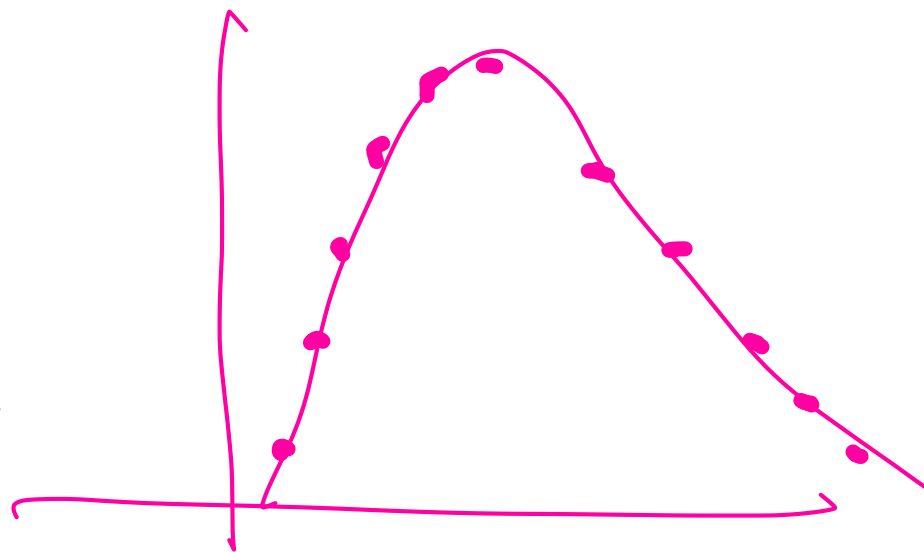
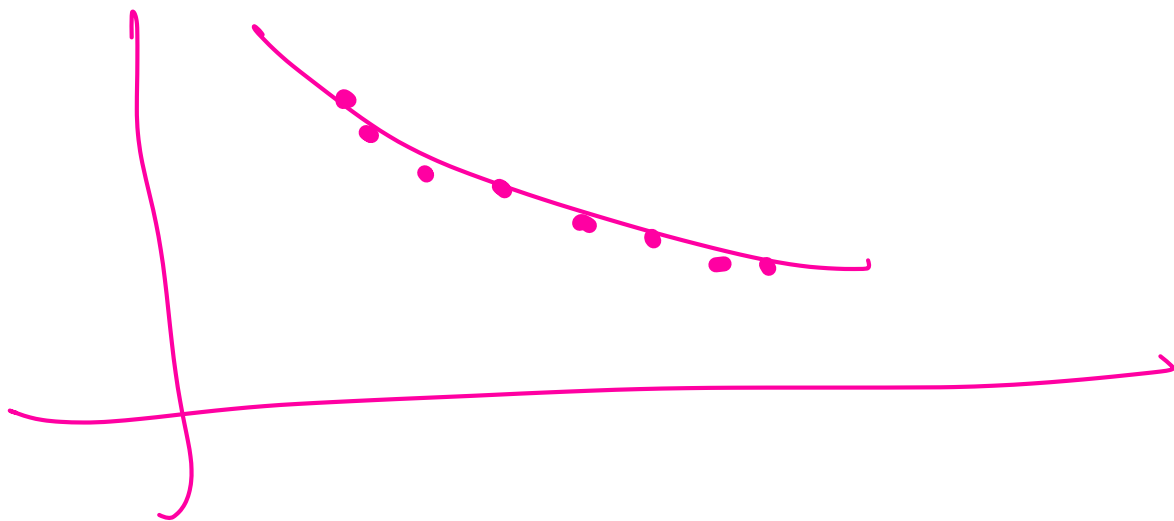
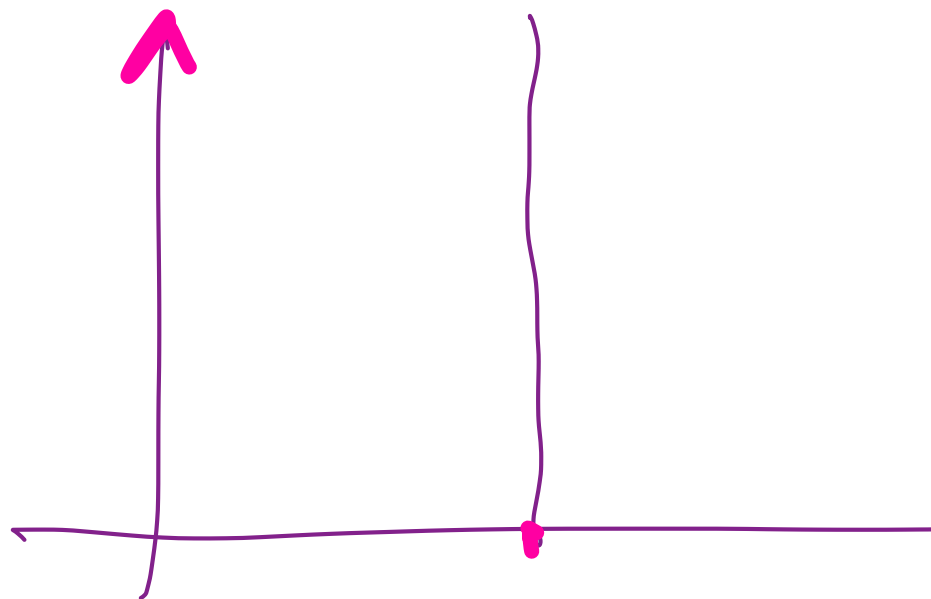
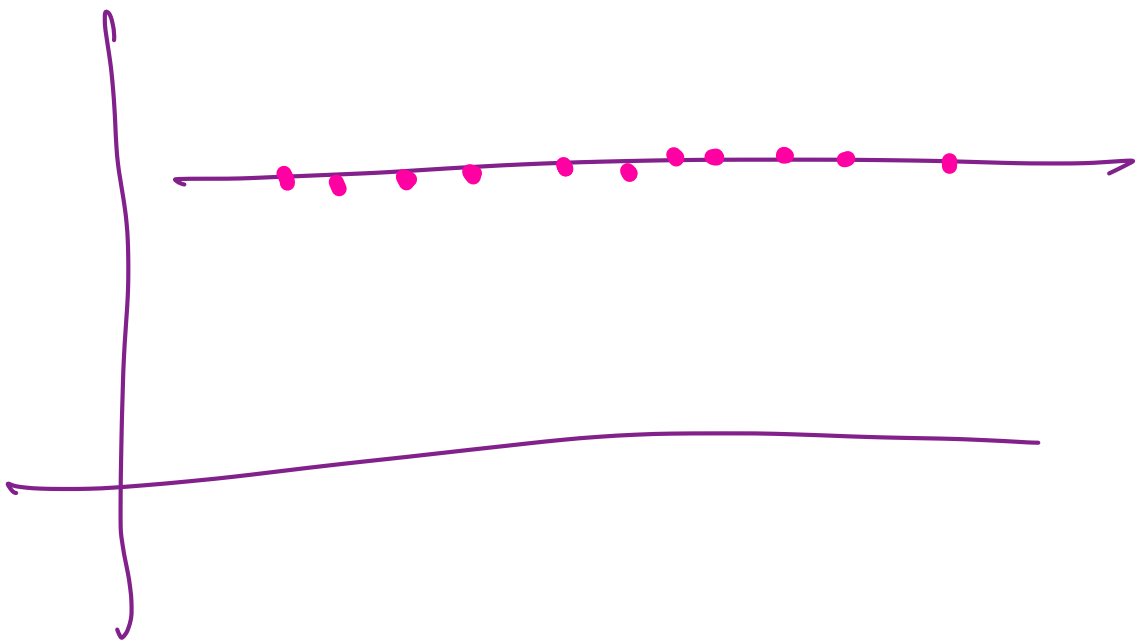
$\sigma = \text{pop}^2 \text{ std. dev.}$

population
variability

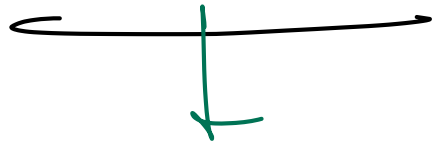


$s \rightarrow \text{Sample std. dev.}$





Theoretical



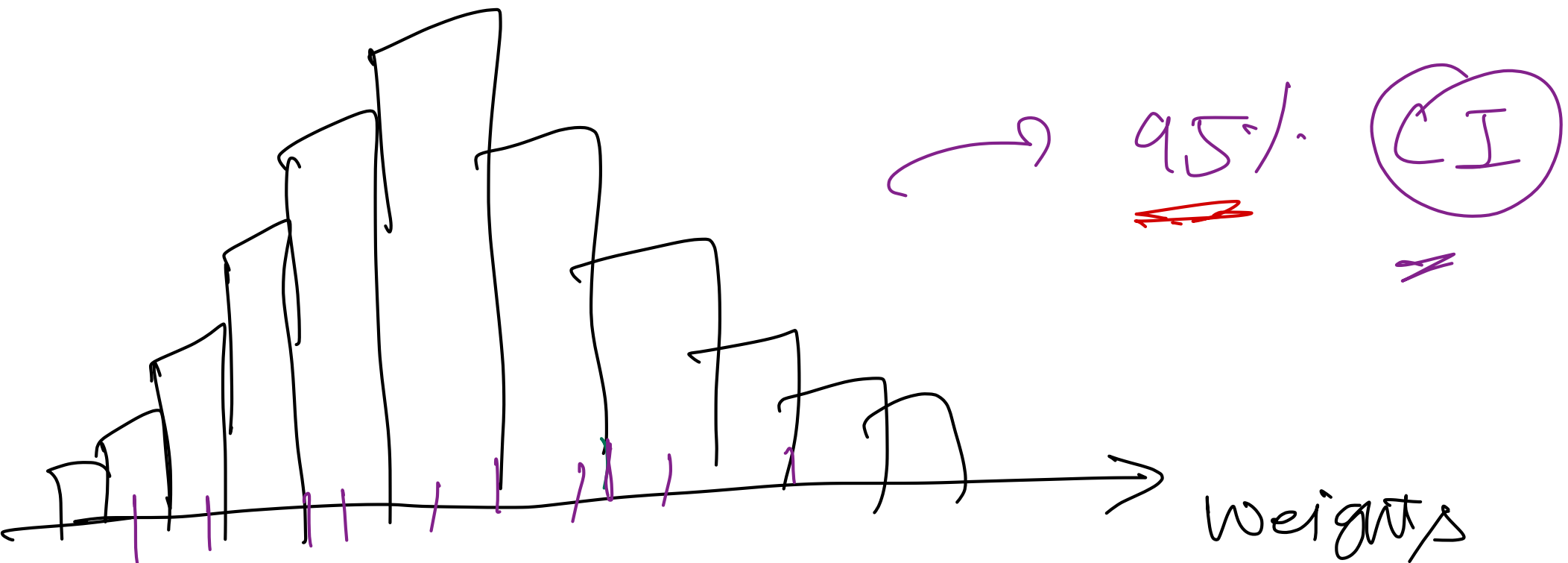
utilized the
prop. of normal
distn



Empirical



Bootstrapping.

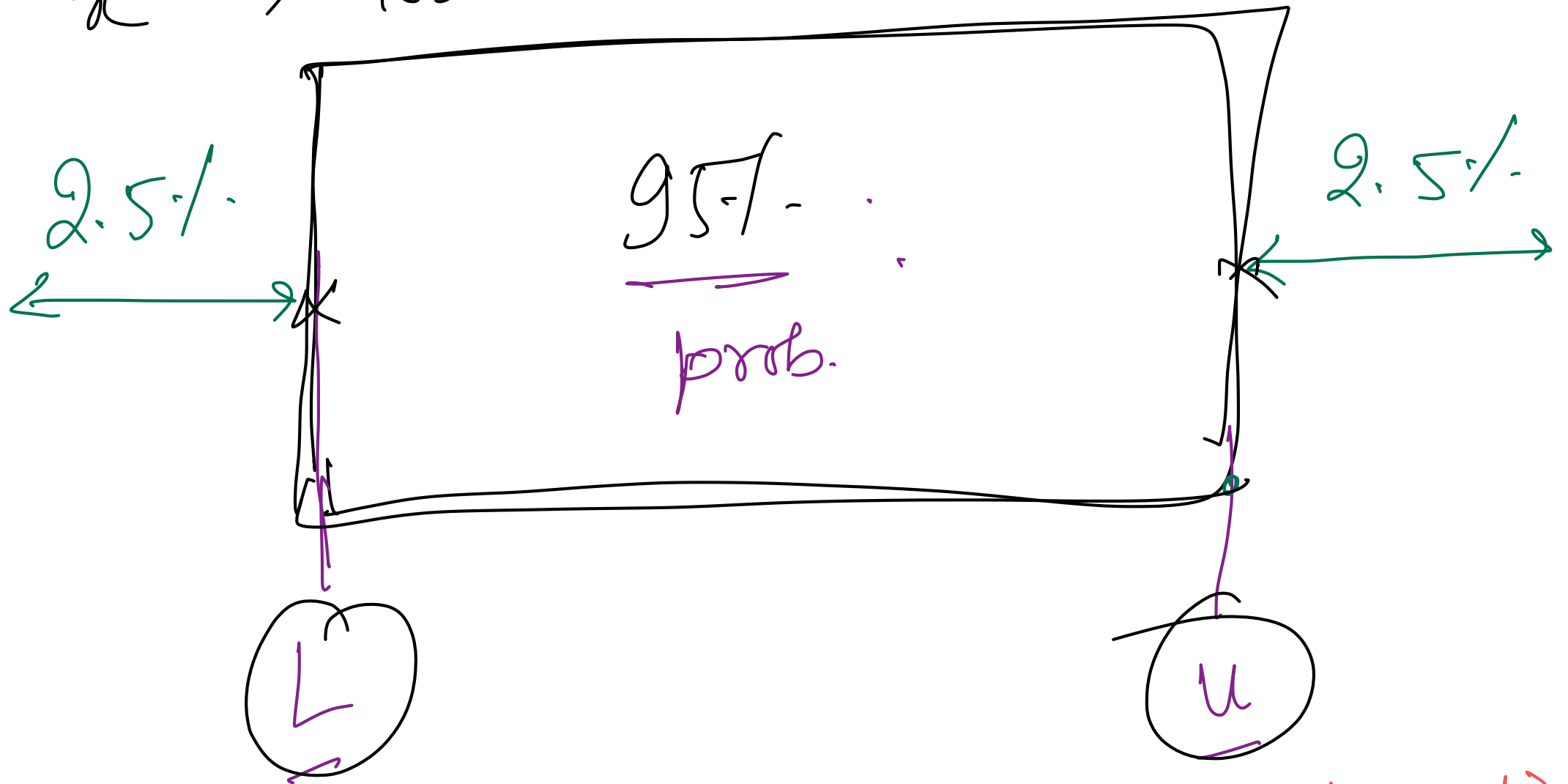


$w_1, \dots, w_i, \dots, w_n$

↓
Individual w_i

Range in which 95% weight lies.

$\mu \rightarrow 1000 \text{ wts.}$



np. percentile($\mu, 2.5$)

np. percentile
= ($\mu, 97.5$)

weights = poplⁿ mean is unknown.

$$n = 200$$

$$\bar{x}$$

$$= 65 \text{ kg}$$

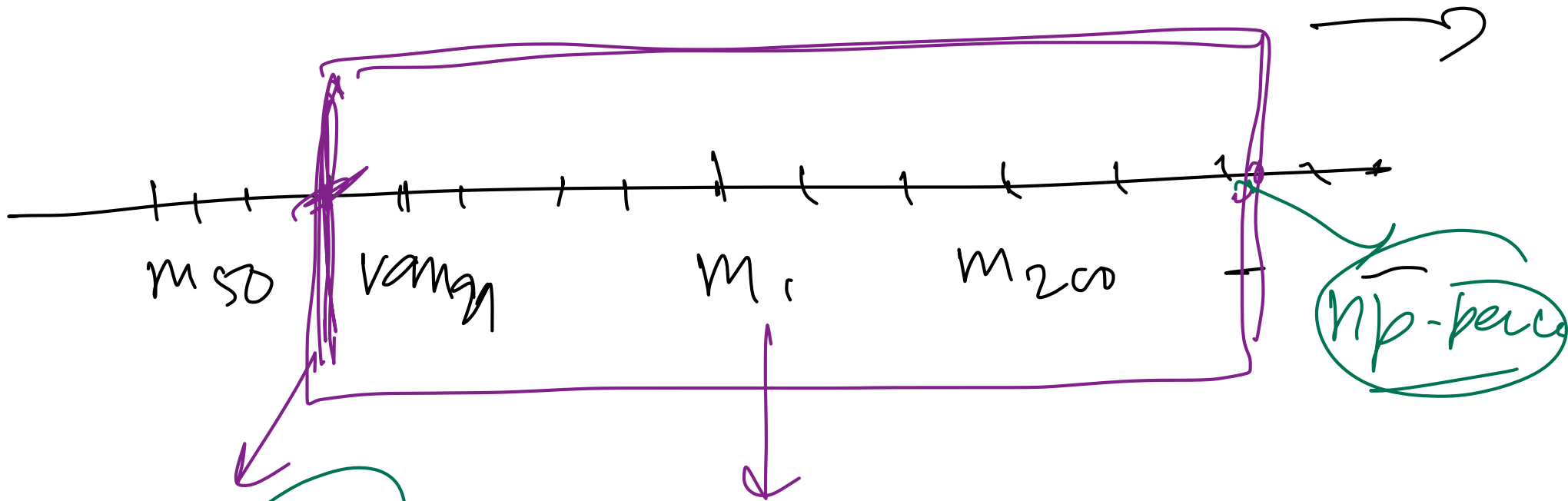
1 time activity

$$n = 200 \Rightarrow S_1 \Rightarrow m_1$$

$$= 200 \Rightarrow S_2 \Rightarrow m_2$$

$$\vdots$$
$$S_{1000} \Rightarrow m_{1000}$$

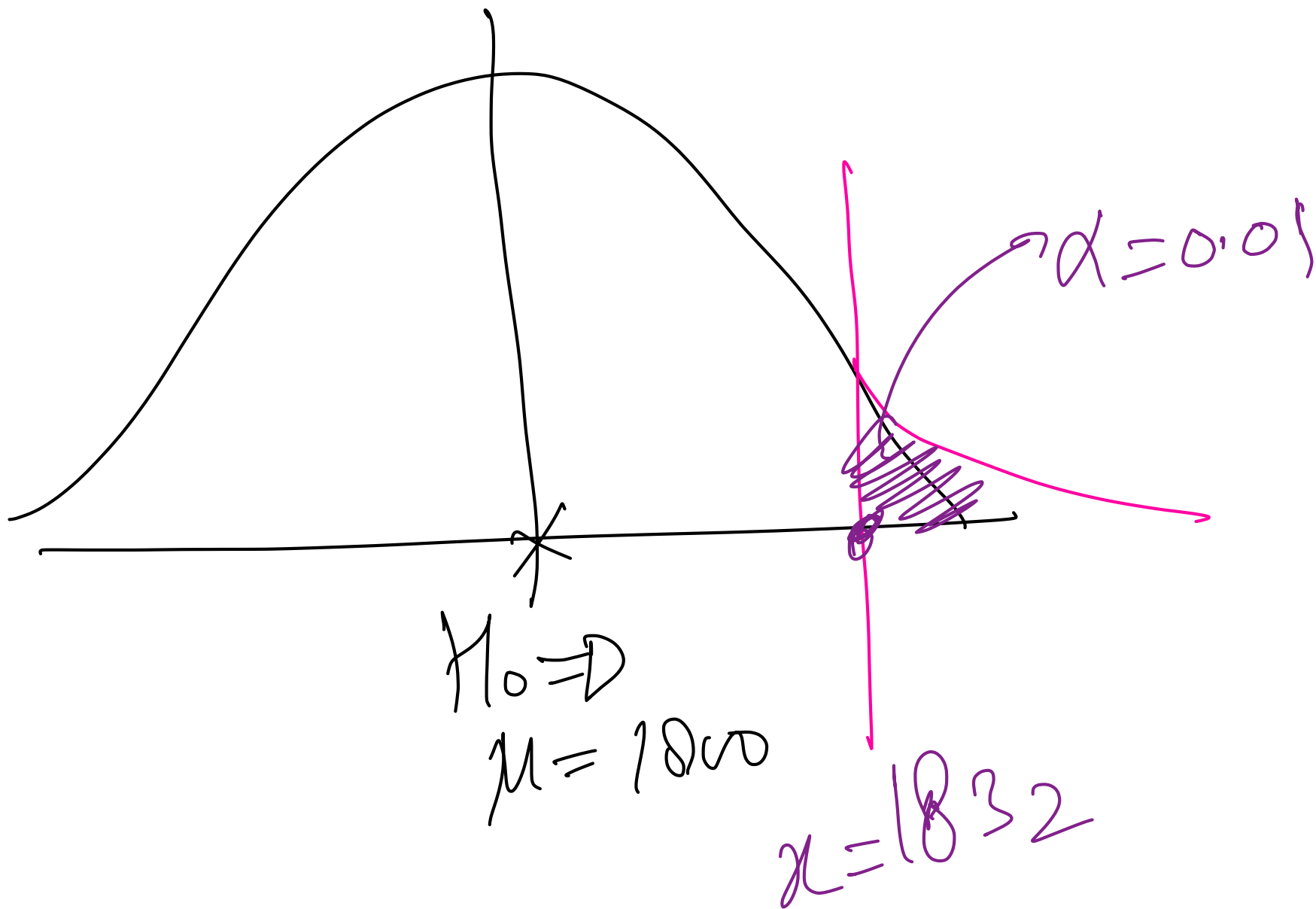
Sort the 1000 means.



np-perc Interval contain 95% of the means

[63.2, 67.5]

95% confidence interval



$$\alpha = 0.05$$

$$\alpha/2 = 0.025$$

$$\alpha/2 = 0.025$$

