

↳ section : Chat - window ]

→ Q&A : end

Today → Linear Algebra & CG → ML  
[fish - sorting]

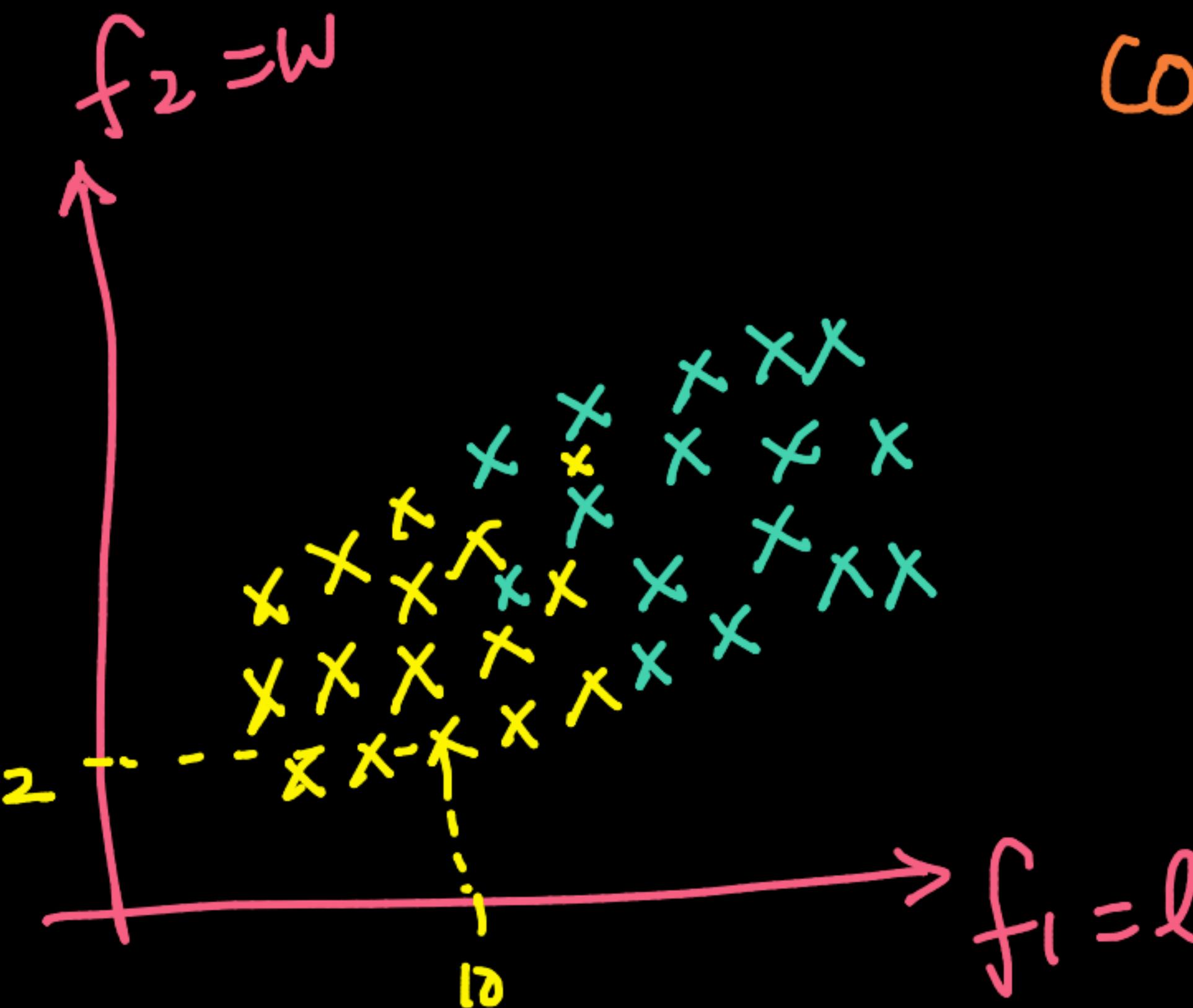
next → PCA ← eigen values & eigen vectors  
=====

Scatter plot:

$$f_2 = w$$

$D =$

	$l$	$w$	Type
10	2	1	
10	2	1	



200-fish

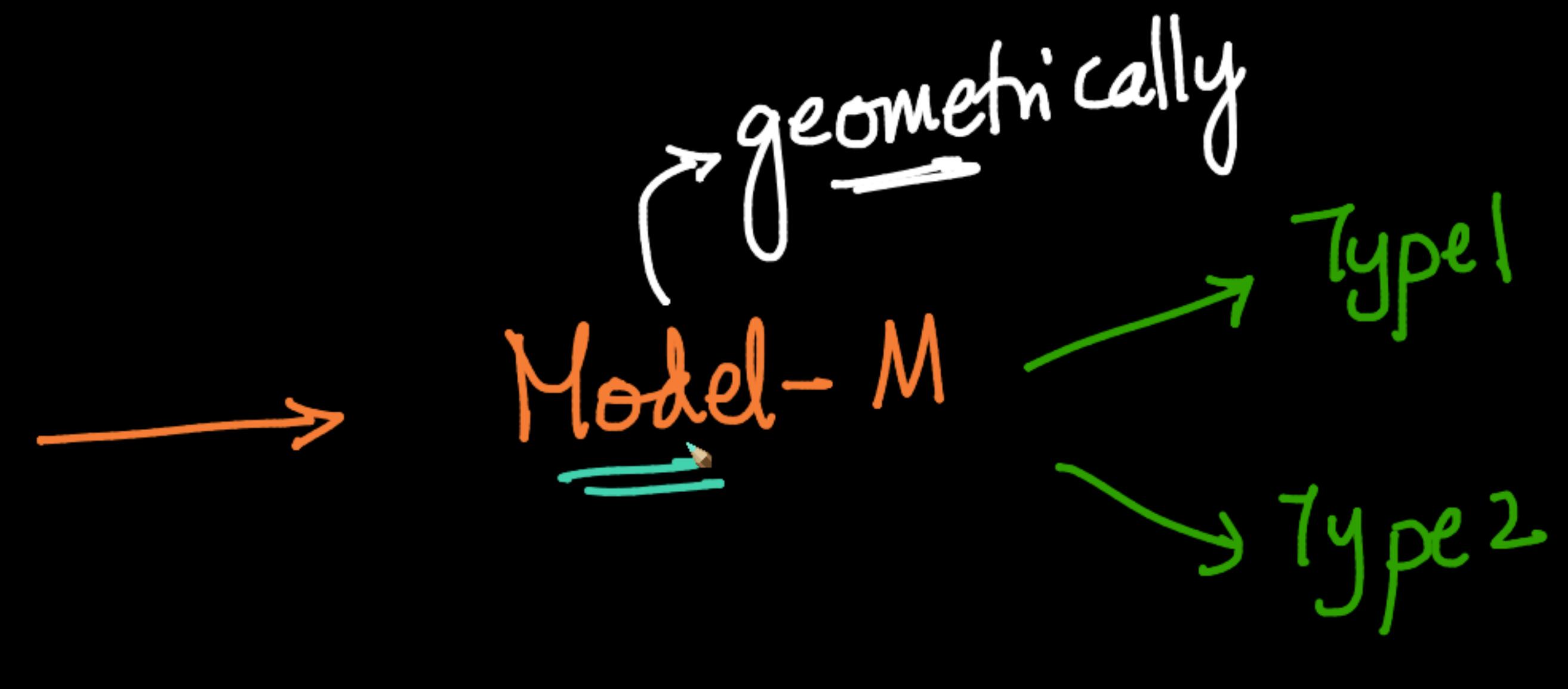
color-coding

Type 1:  $\times$  } legend  
Type 2:  $\times$  }

scanner

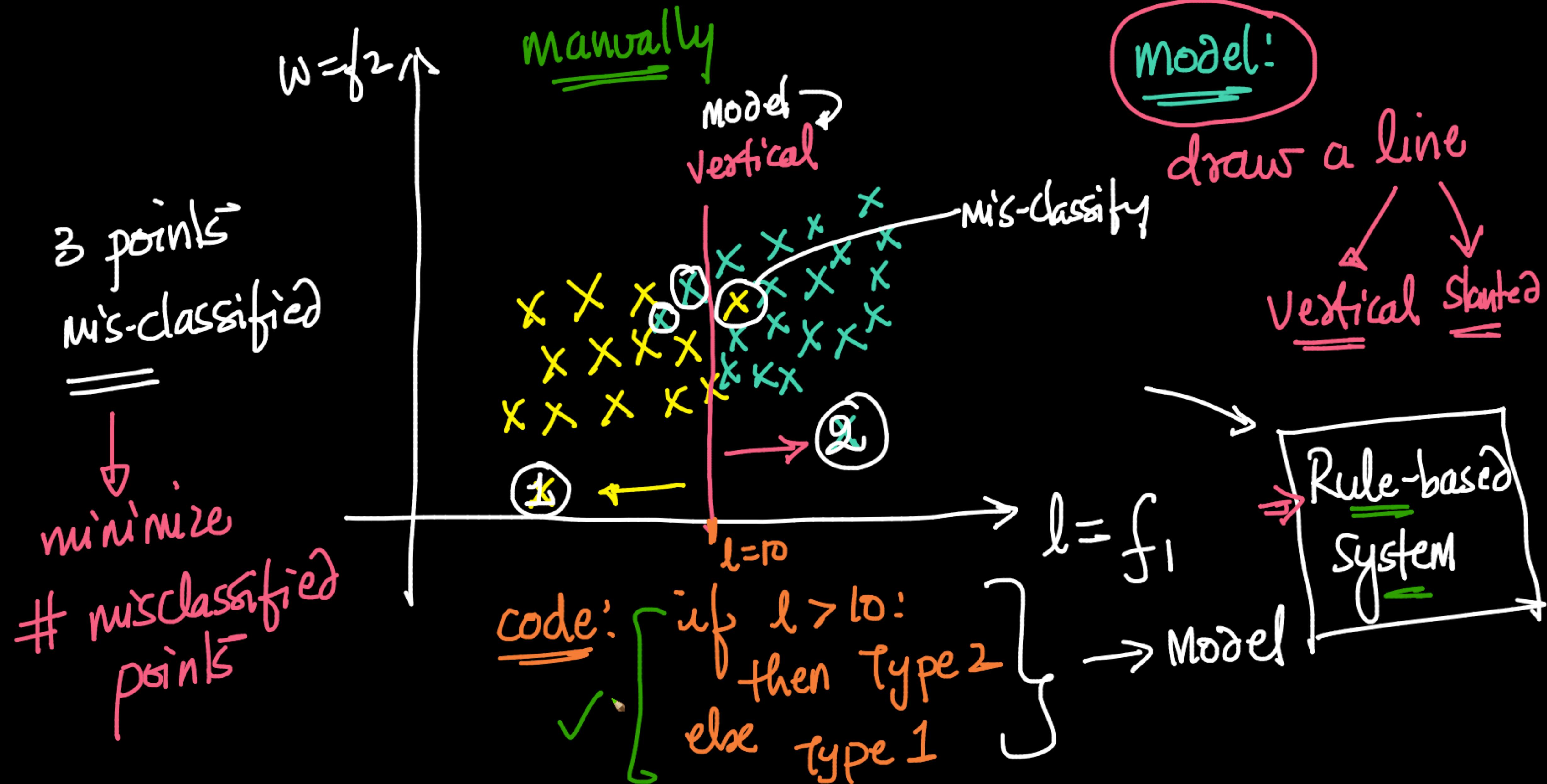
new fish

$$\begin{cases} l = 12 \text{ cm} \\ w = 3.4 \text{ cm} \end{cases}$$



train / build a model ?

```
graph LR; A1[ ] --- B1[ ]; A2[ ] --- B1; B1 --> C1[train]; B1 --> C2["build a model ?"];
```



Model → Vertical line → if ... else

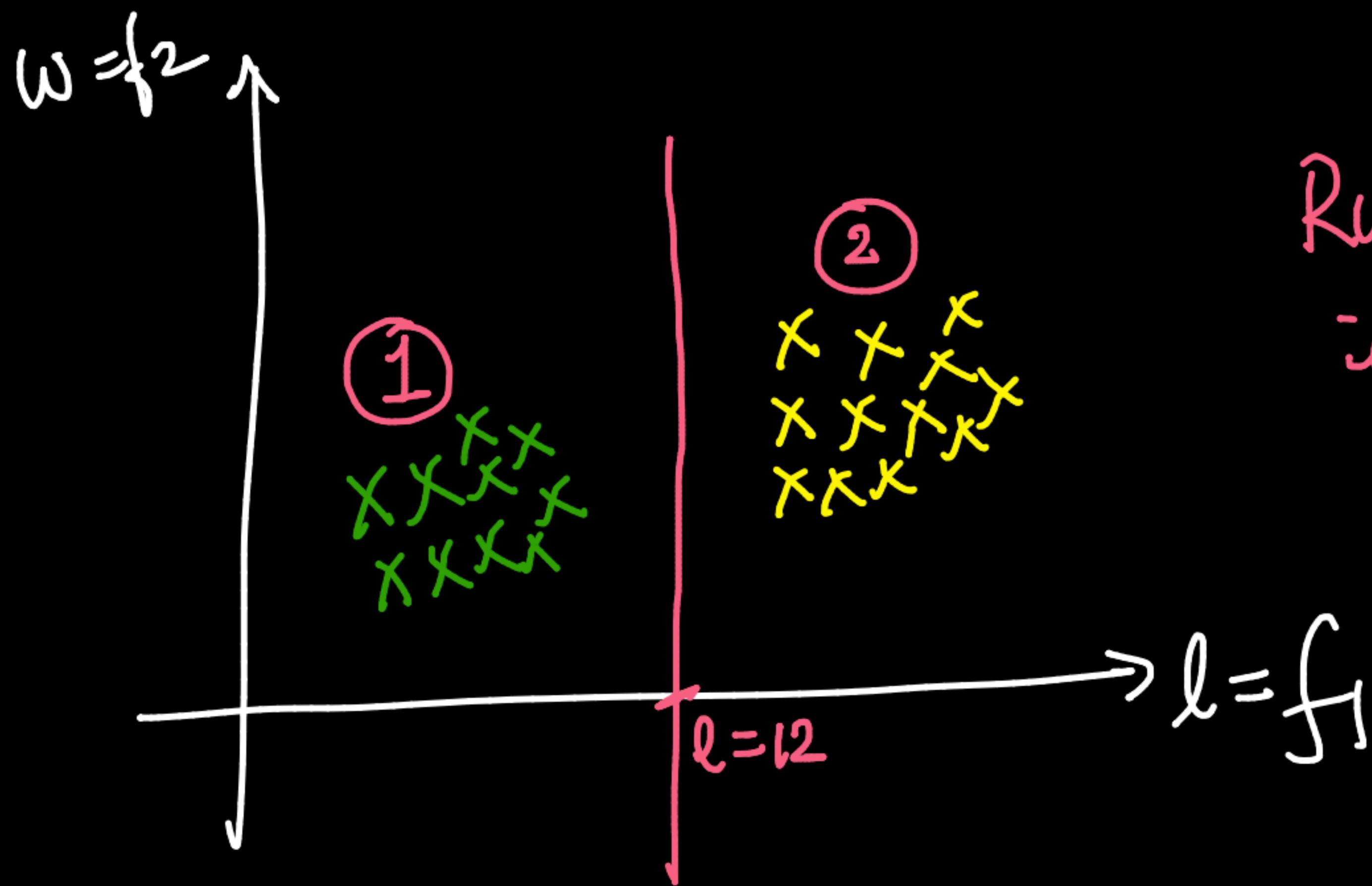


Decision Trees  
= RF  
= GBDT

: fancier

if ... else

Computationally  
= Discovered



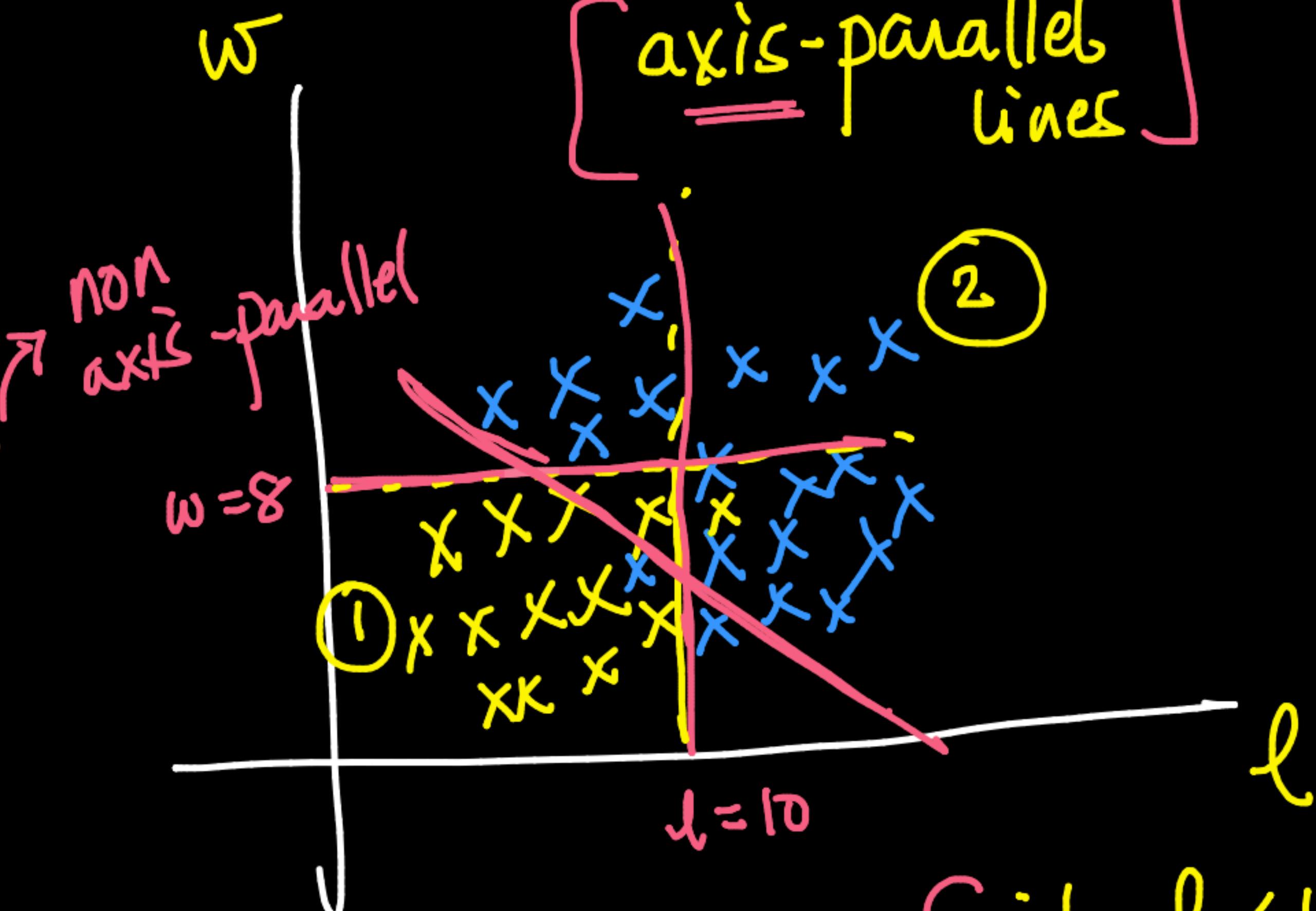
Rule-based  
if-else ↪

Q

slanted line



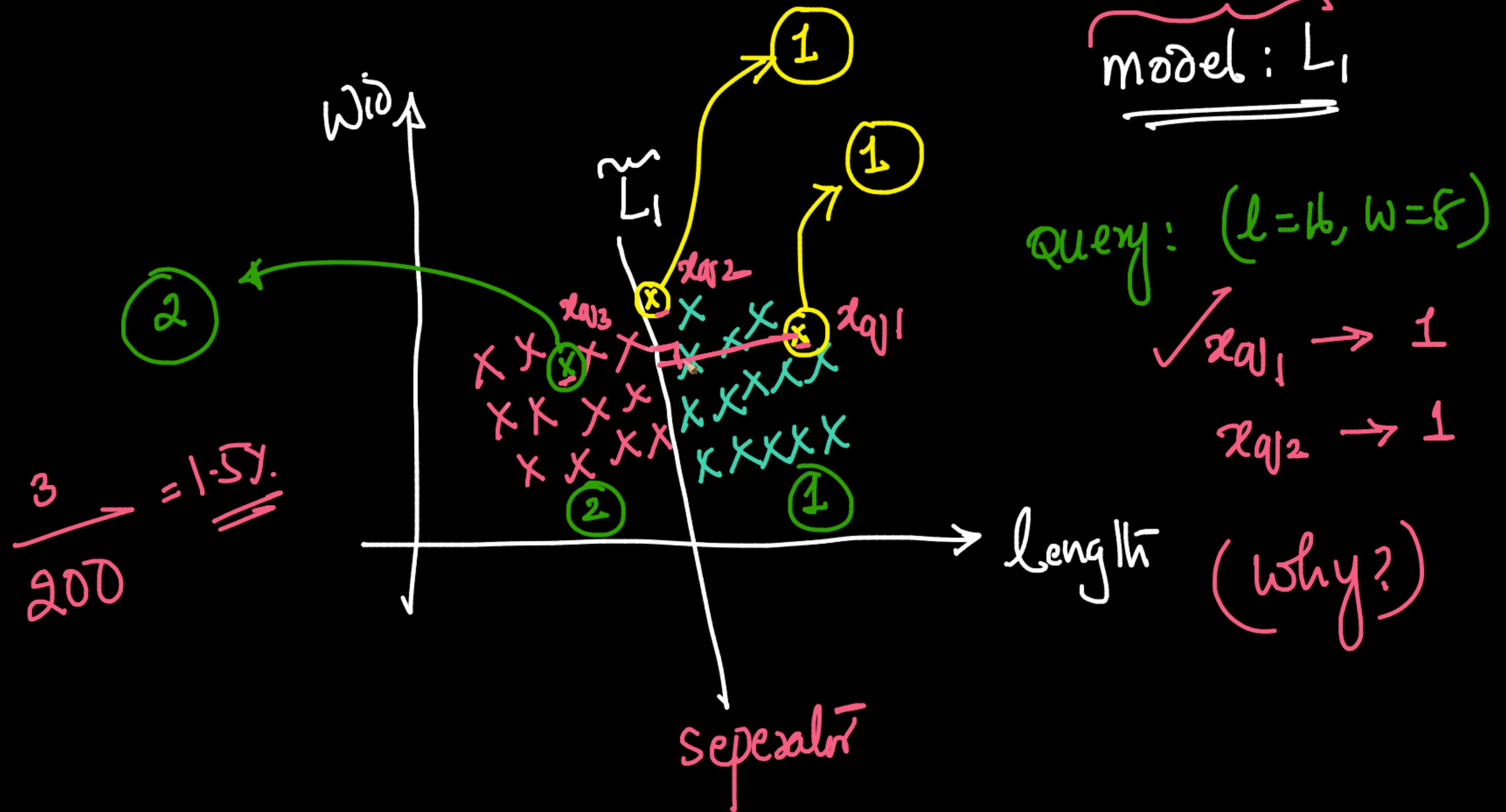
if ... else  
(NOT -easy)

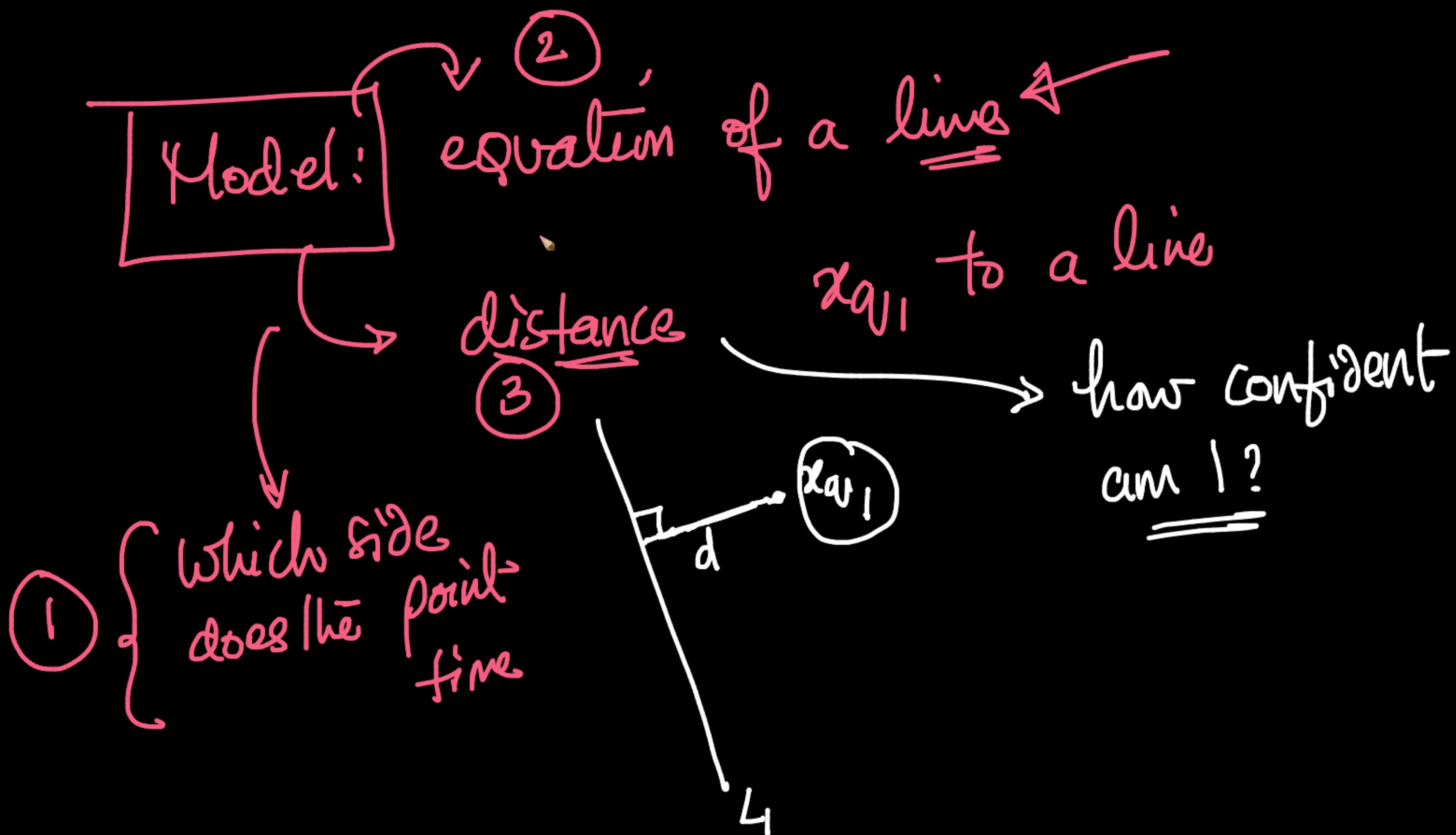


Set of axis-parallel

{ if  $l < 10 \& w < 8$   
else Type1  
Type2

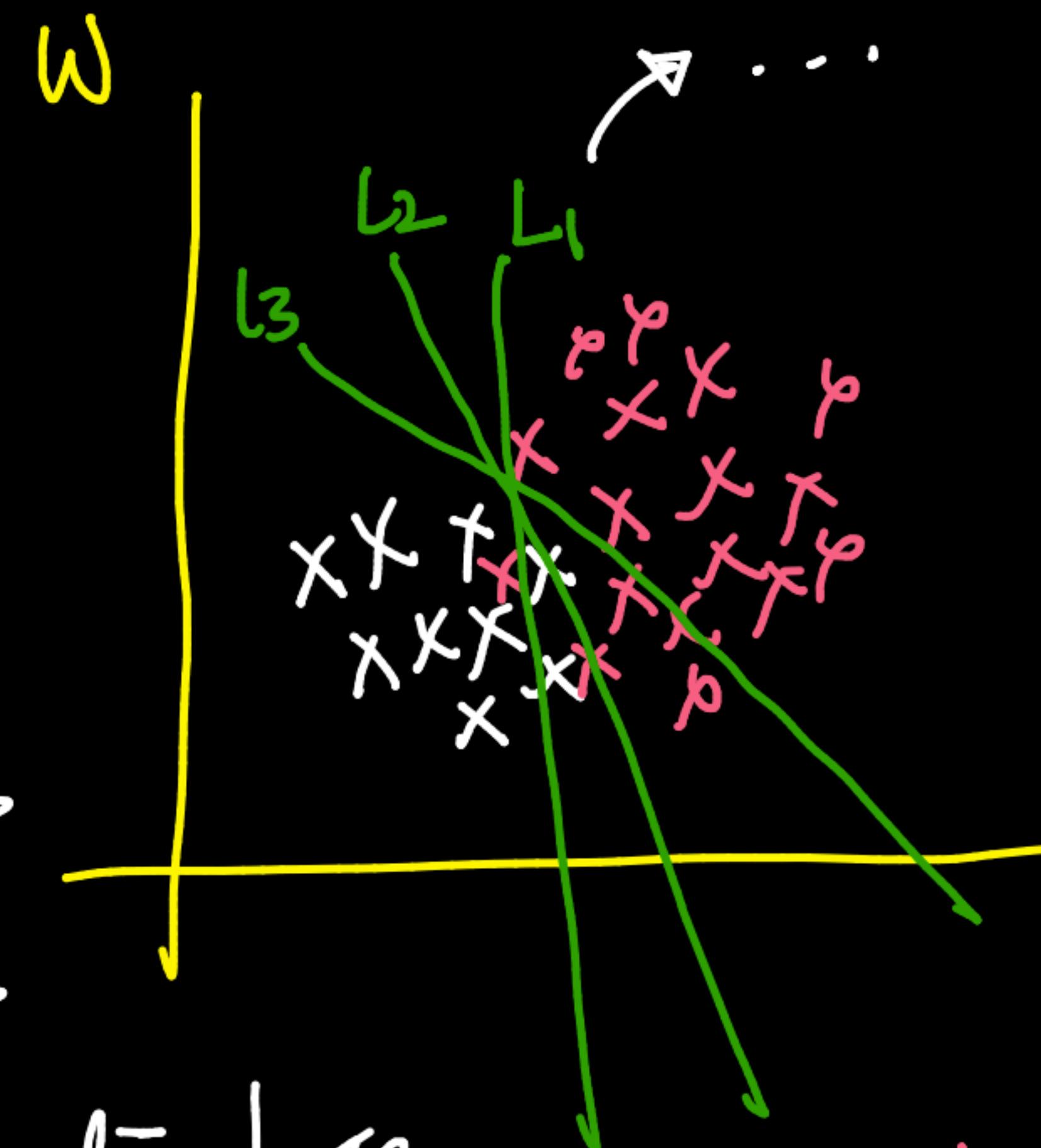
Rule-based:  
if ... else  
↓  
vertical or  
horizontal





how to find  
the line

try out multiple  
lines & pick the  
best



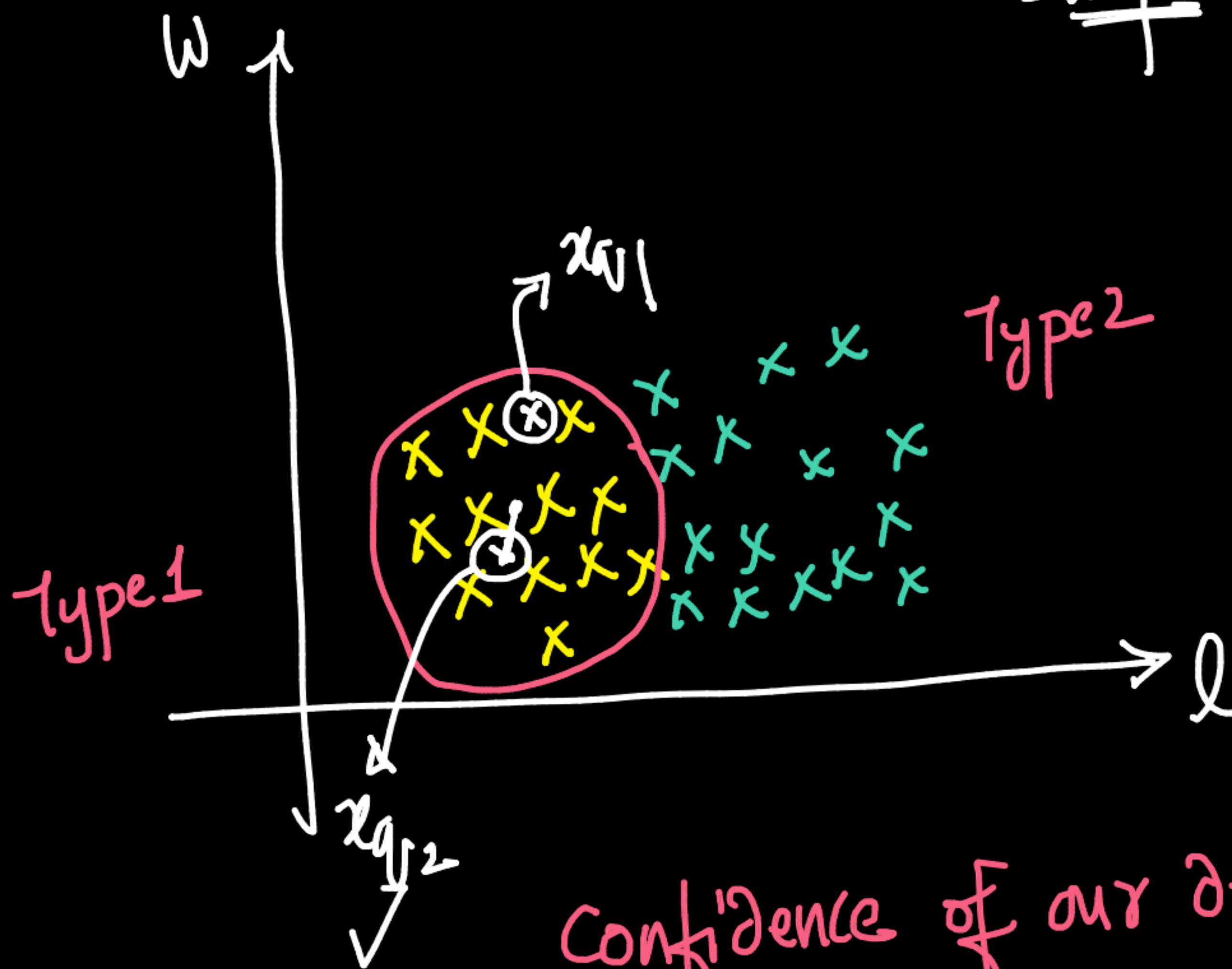
which line!

simplest

{ minimize the  
# mis-classification  
points

→ optimization  
maxima & minima  
→ calculus

Q



Shapes:

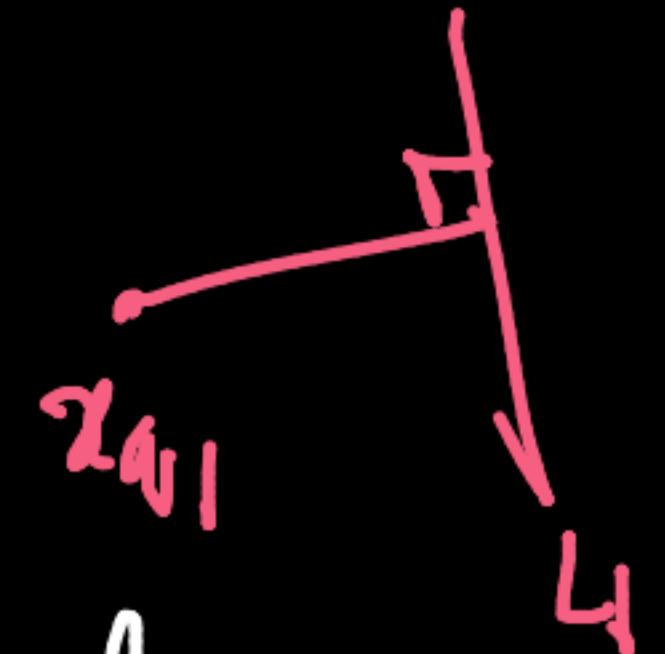
- ✓ - set of axis-parallel lines

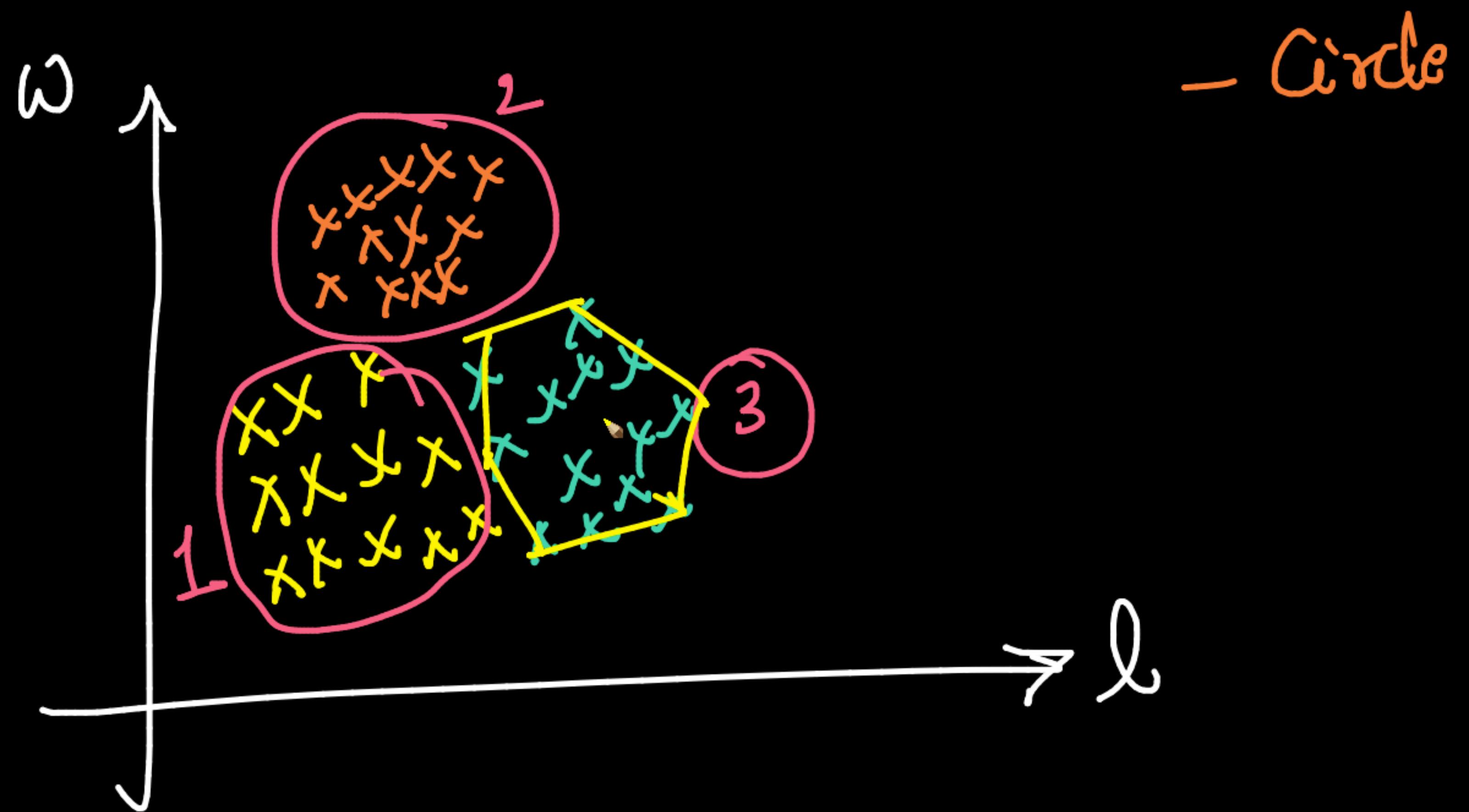
- ✓ - slanted line

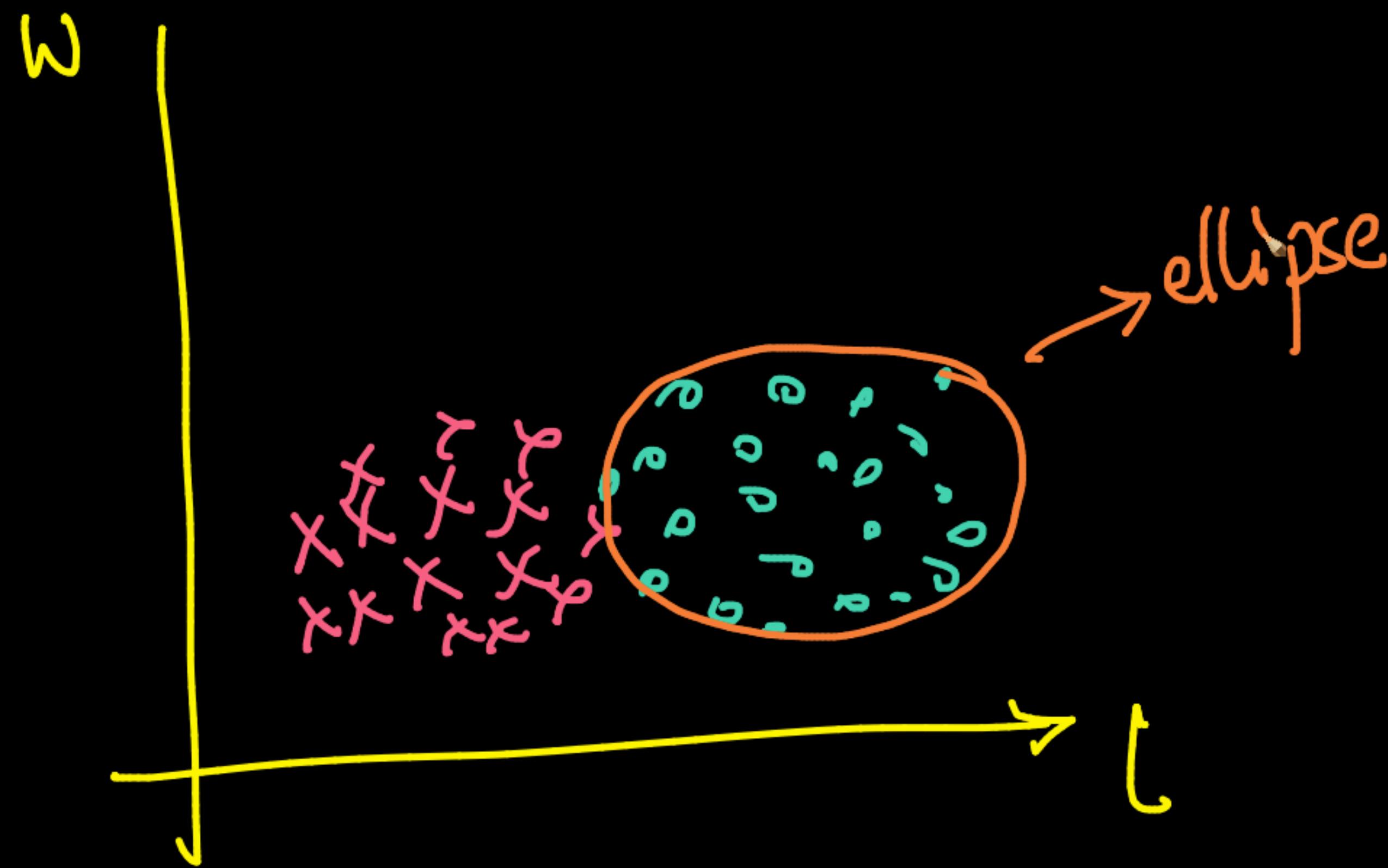
- ✓ - curved lines

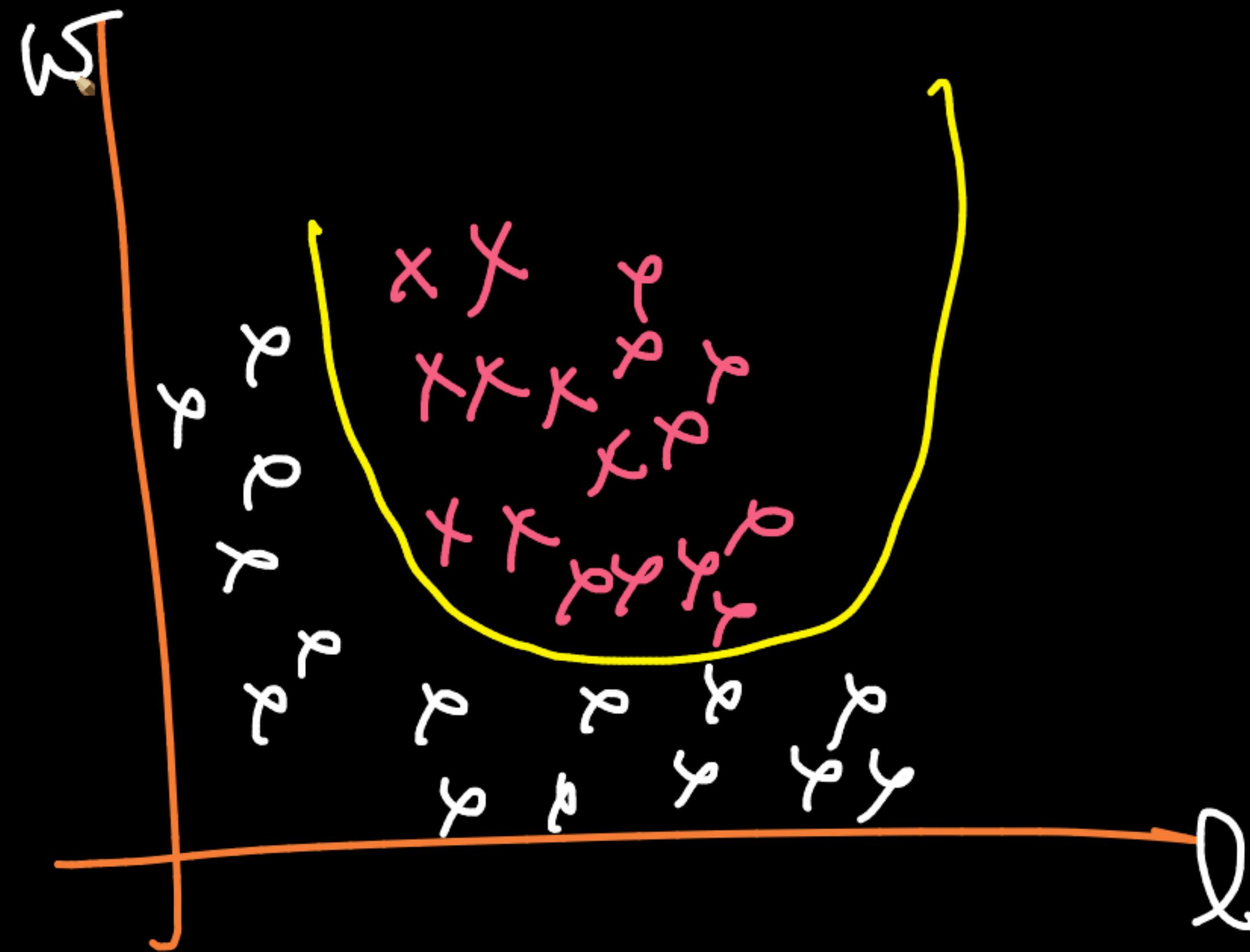
- ✓ - circles

confidence of our decision  
Close to the center of circle







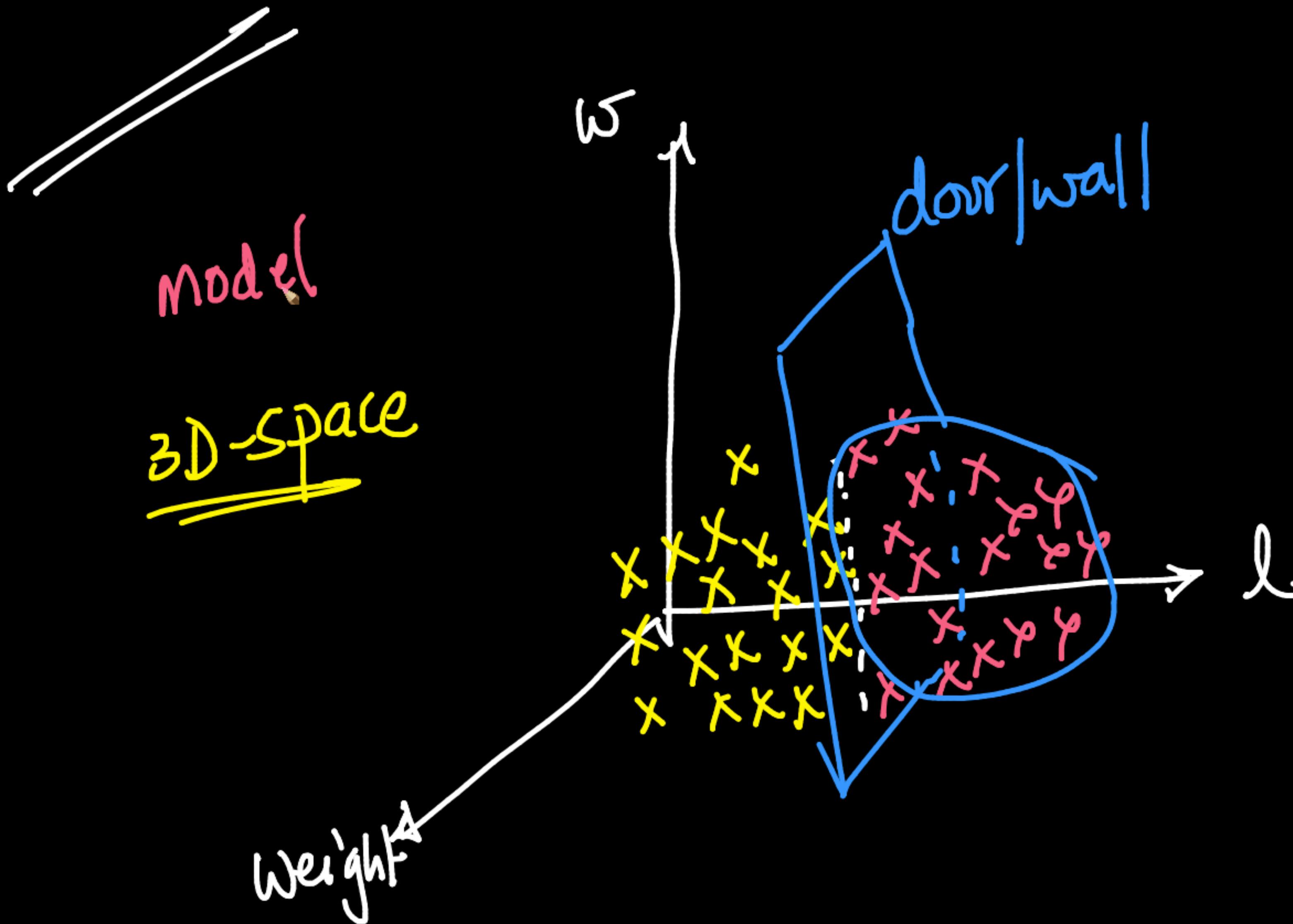


ML:

→ rectangles / squares  
Set of axis parallel - lines → DT; RF; GBDT  
Slanted lines → log-reg; SVM ; perceptron



↓  
plane in 3D

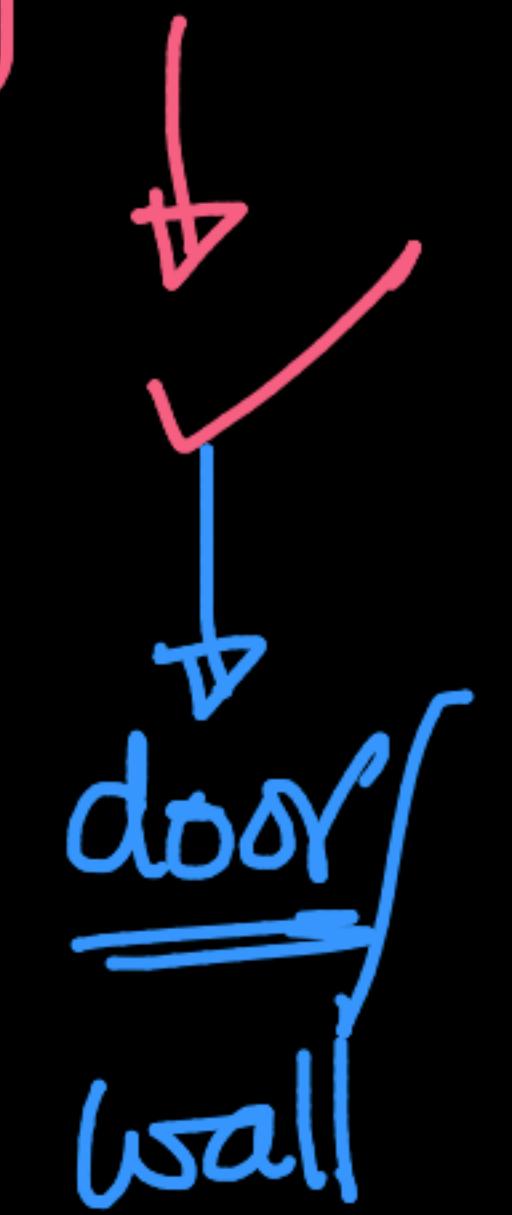


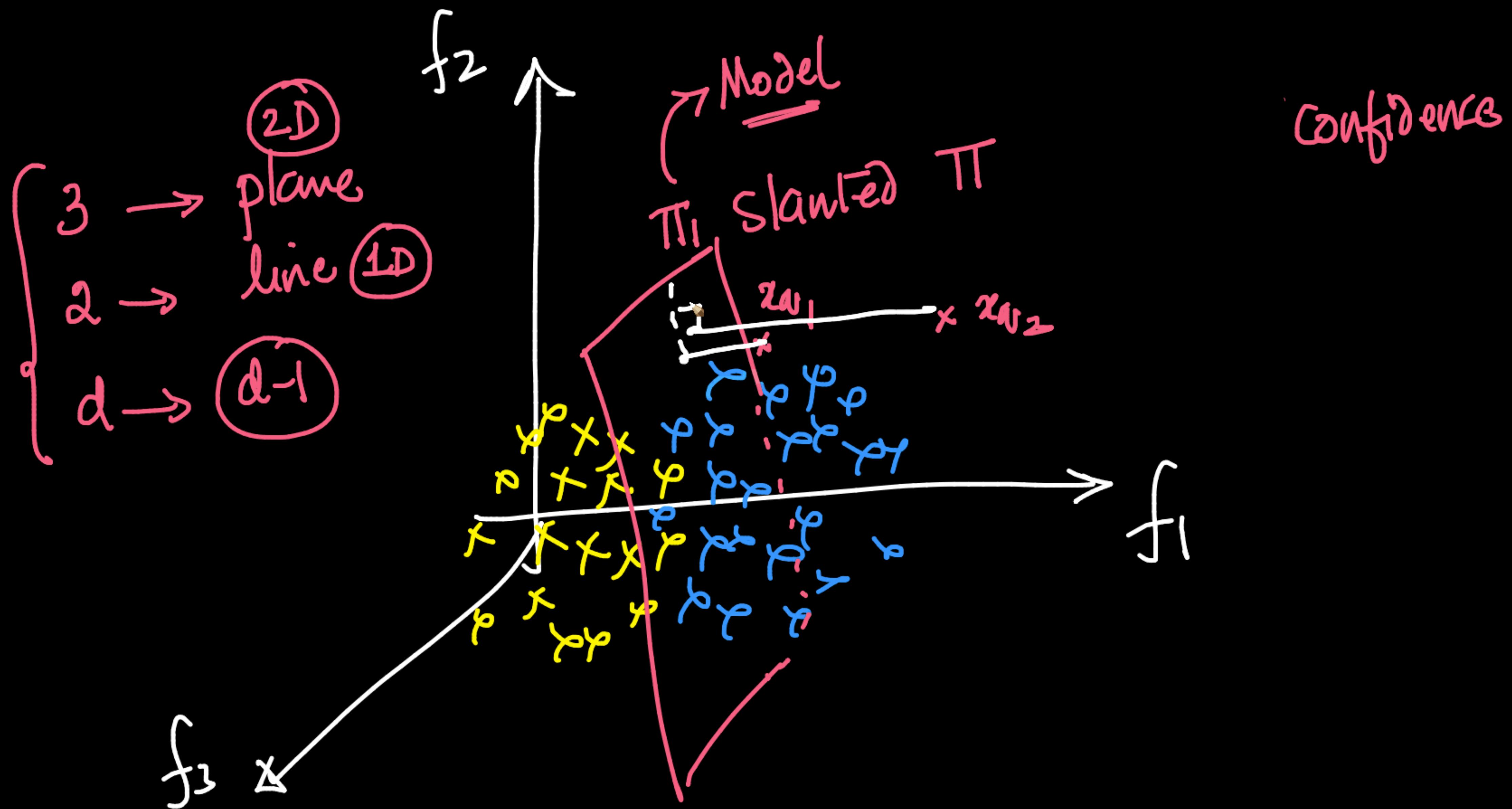
line vs plane

↓

Cannot separate in 3D space

pen





scanner

$l, w:$

2D  $\rightarrow$  line ;

circle; ellipse



$l, w, w_g:$

3D  $\rightarrow$  plane;

Sphere  
ball

ellipsoid

$l, w, w_g, f_L$

4D

5D

d-dim

✓  $L$  Line ; plane ;  $\Pi_3$   $\xrightarrow{\text{d-dim}}$   $\boxed{\text{hyperplane}}$   $\xleftarrow{\text{d-dim}} \Pi_d$

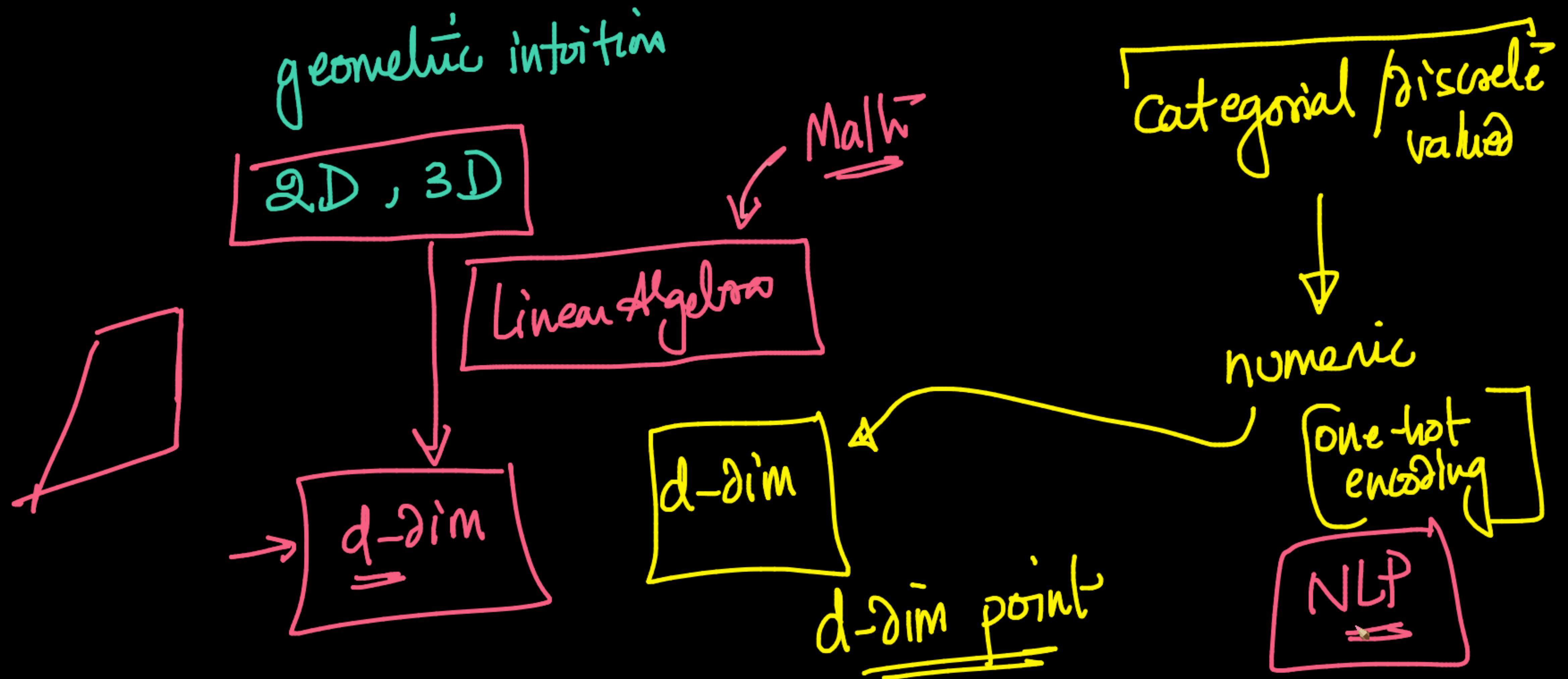
✓ Circle; sphere; hypersphere  
ellipse; ellipsoids; hyper-ellipsoids

✓ 2D — draw

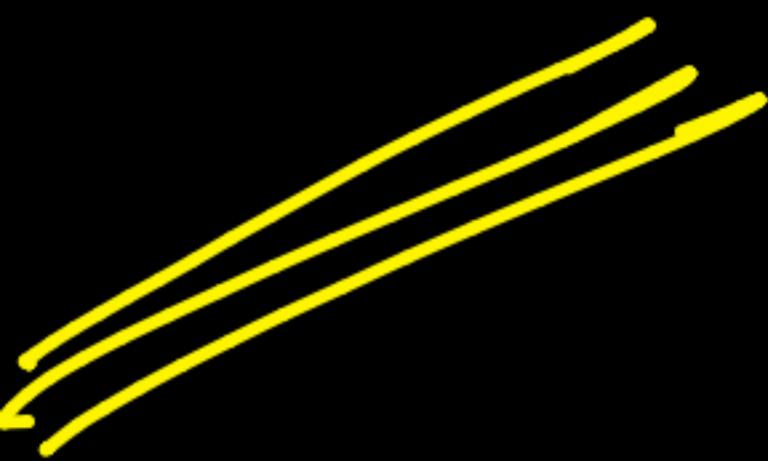
✓ 3D — exp

$\left\{ \begin{array}{l} \xrightarrow{\text{rule-based}} \\ \xrightarrow{\text{d-dim}} \\ d = 10 \text{ or } (n \approx 10,000) \\ \xrightarrow{\text{linear + Algebra}} (\text{eqns & notations}) \end{array} \right.$





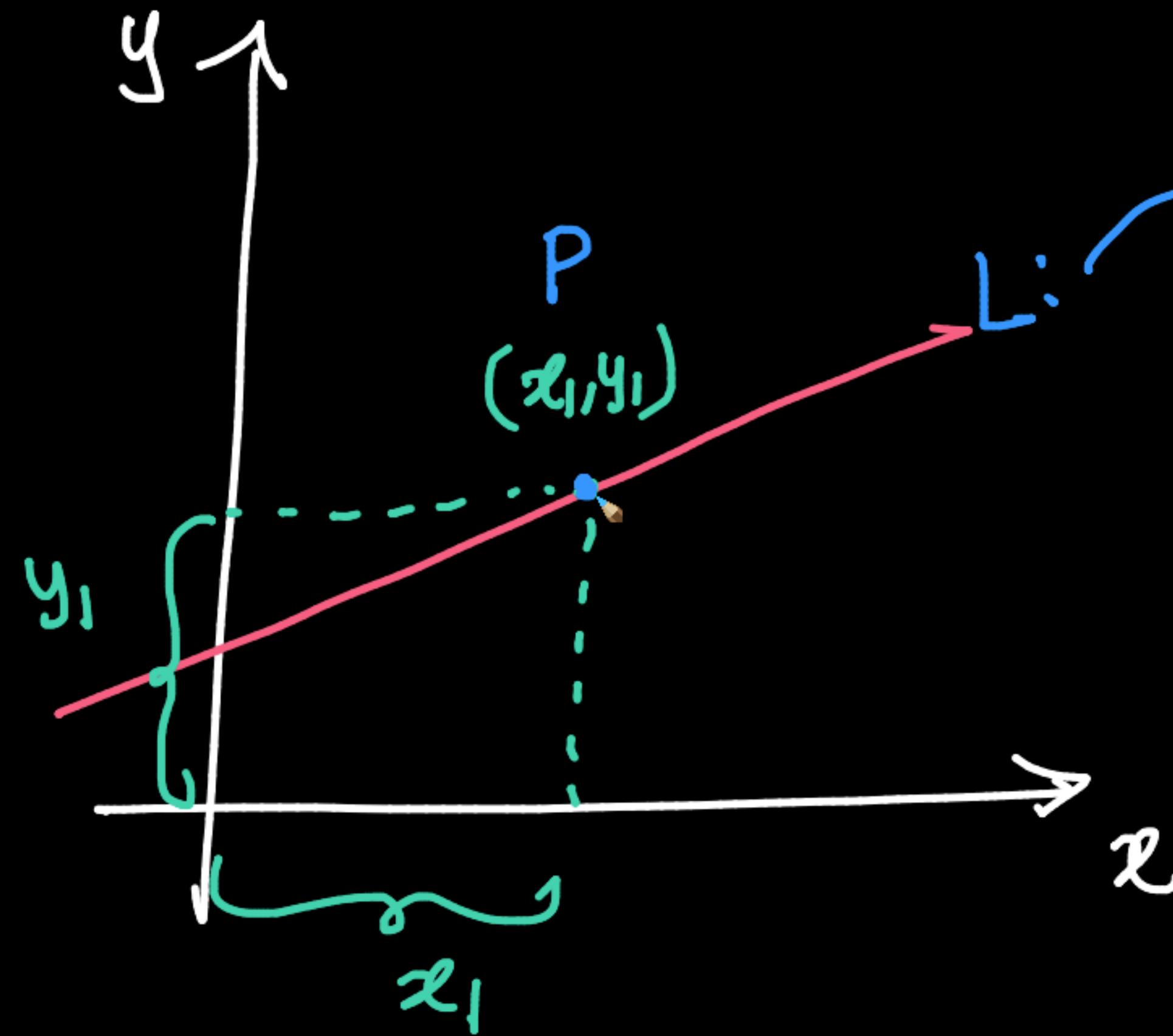
- Model: hyperplanes → Math-equation → code
- dist of  $x_0$  to model ( $\Pi_d$ )
- $x_q$  → side of  $\Pi_d$



↓

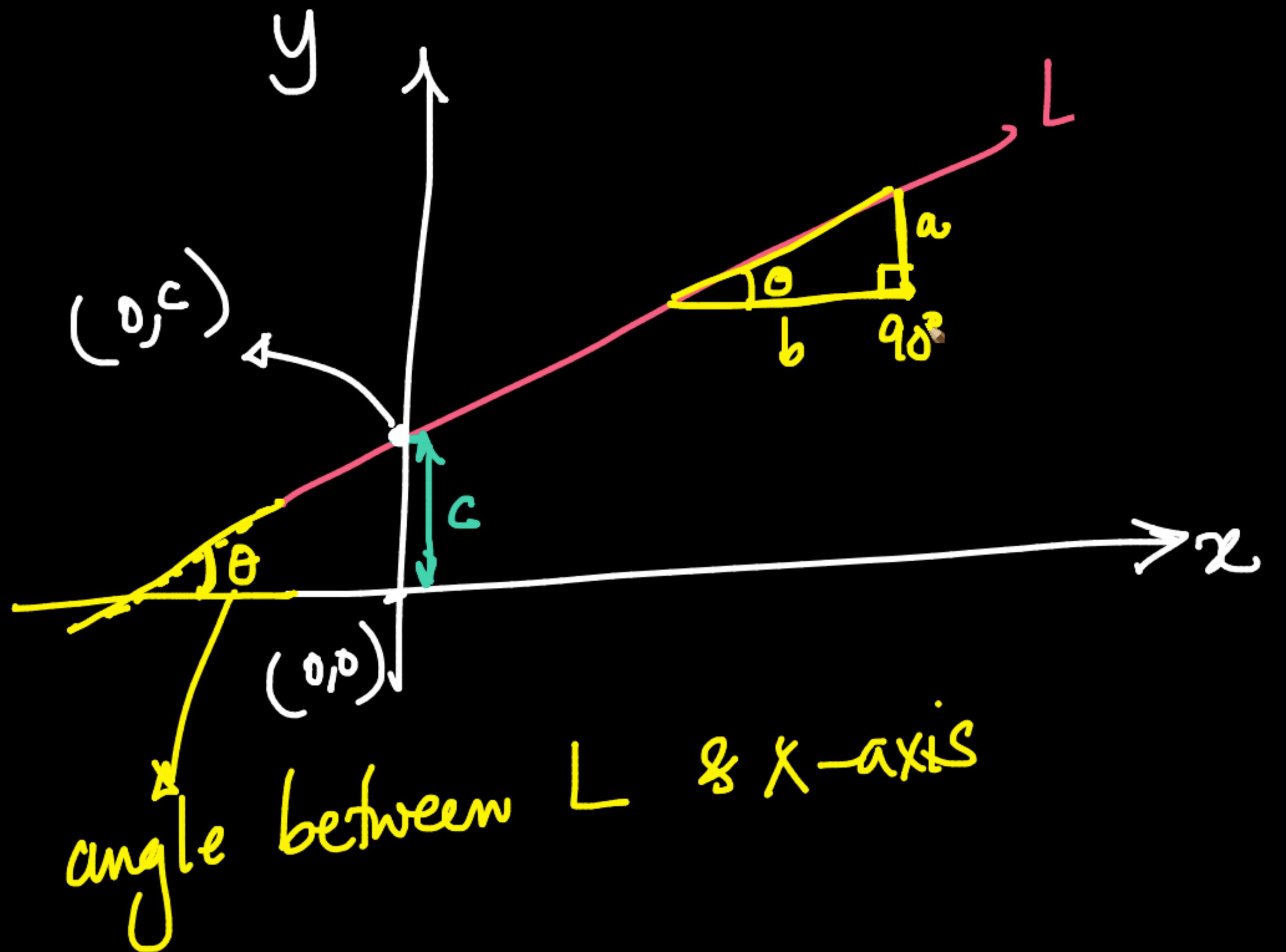
$$ax + by + c = 0$$

Lines:



$y = mx + c$   
P is on L if

$$y_1 = mx_1 + c$$

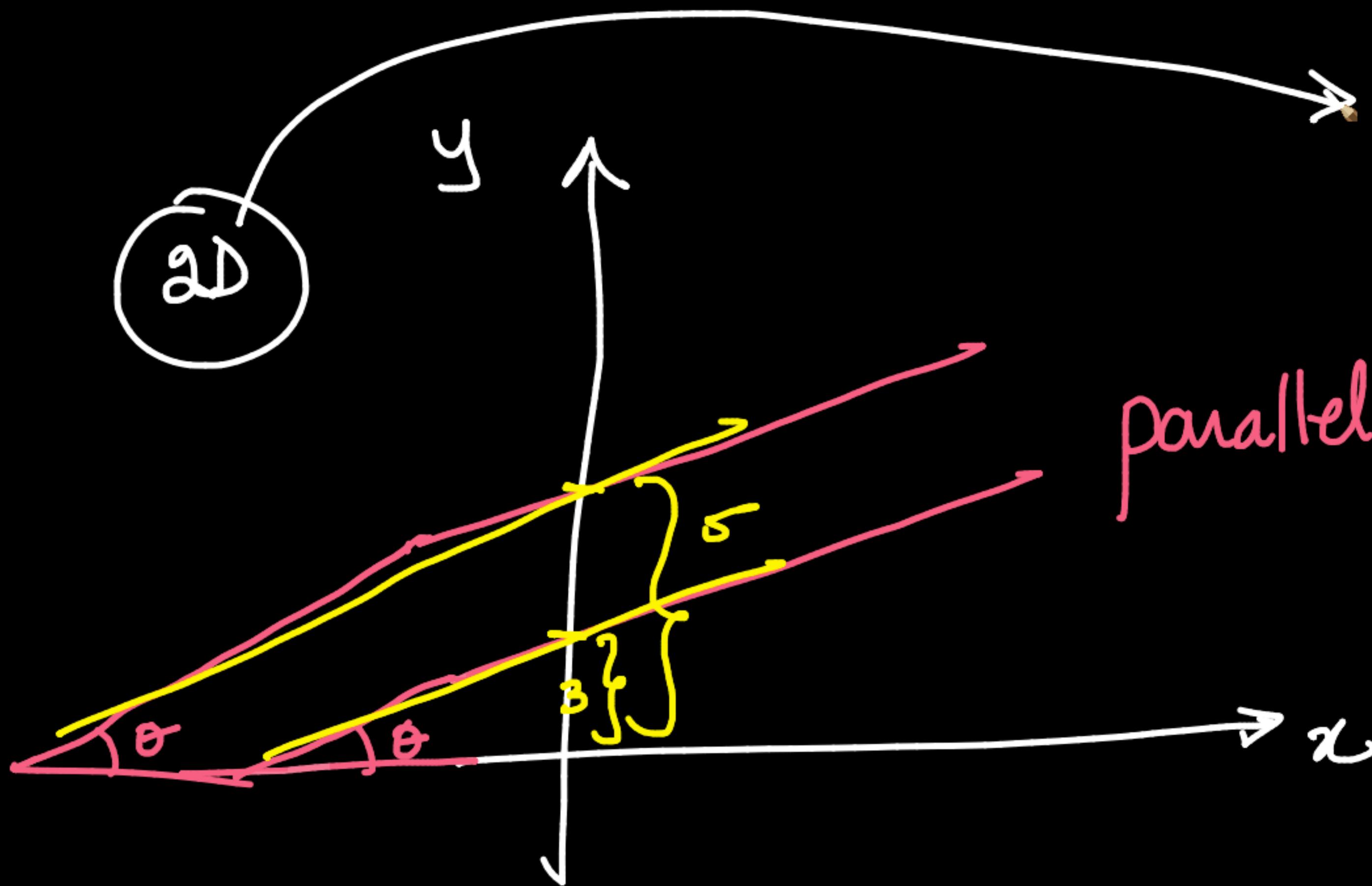


slope  $\downarrow$   $y$ -intercept  $\downarrow$

$L: y = mx + c$

$\uparrow$   $c$

$$m = \underline{\underline{\tan \theta}} = \frac{a}{b}$$

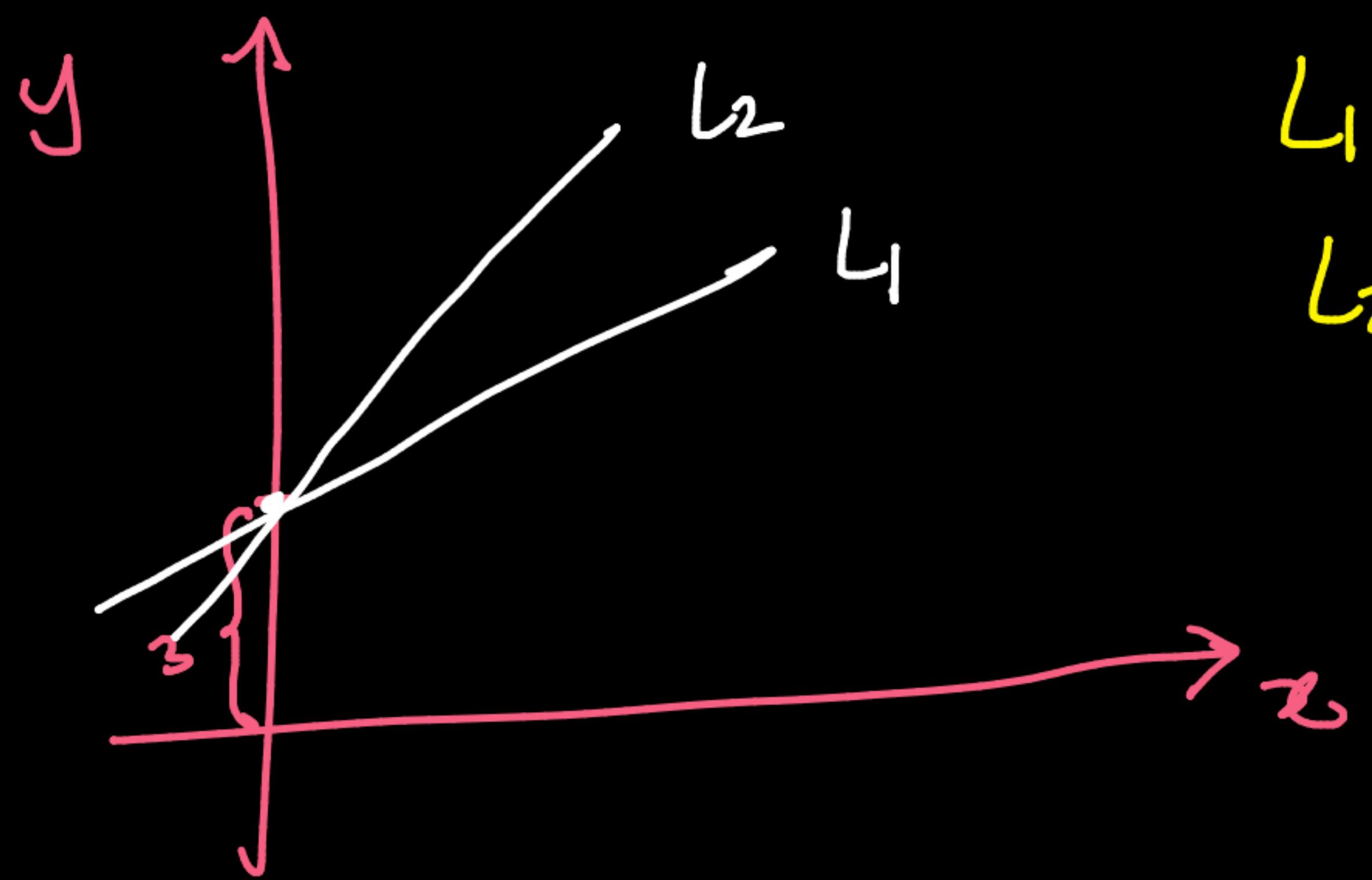


parallel lines

$$L: \underline{2x + 3} = y$$

$$L_1: \boxed{2x + 3} = y$$

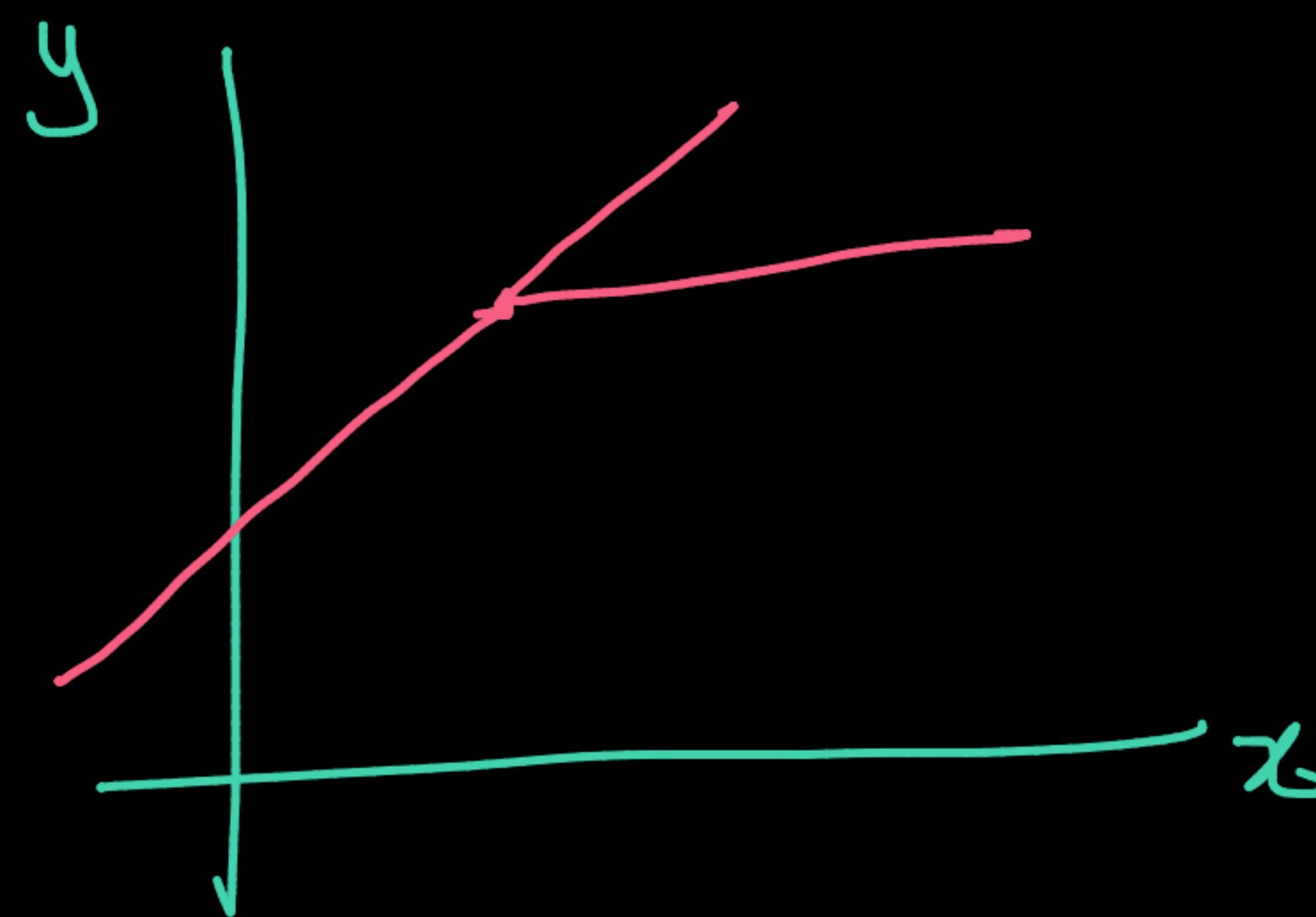
$$L_2: y = \boxed{2x + 5}$$



$L_1: (M, C)$   
2D

$$L_1: y = 2x + 3$$
$$L_2: y = 3x + 3$$

(Q) Can a lines have 2-slopes X



(Q)

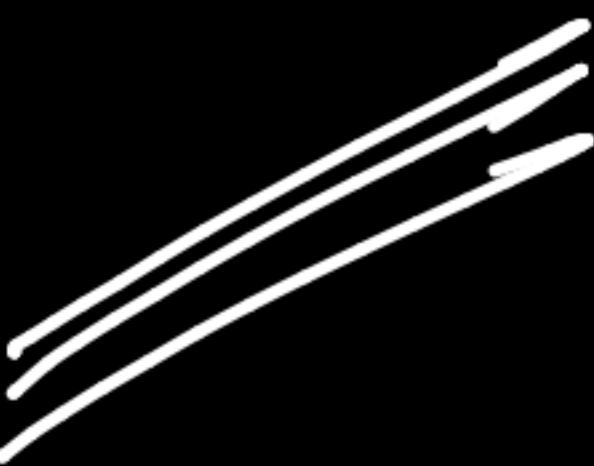
$$L_1: 3x + 2y - 1 = 0 \quad L_1: y = -\frac{3}{2}x + \frac{1}{2}$$

$$L_2: 6x + 4y = 2$$

✓ parallel  
same line / overlapping

$$L_2: y = -\frac{6}{4}x + \frac{2}{4}$$

General-form:



$$L: ax + by + c = 0$$

slope-intercept  
 $y = mx + c$

$$y = \frac{-ax}{b} + \frac{c}{b}$$

12<sup>th</sup>

slope-intercept

$$y = Mx + C$$



general-form:  $mx + (-1)y + c = 0$

$\uparrow \quad \uparrow \quad \uparrow$   
 $a \quad b \quad c$

$$y = MX + C$$

$$L: ax + by + c = 0$$

ML:

$$(w_1x_1 + w_2x_2 + w_0 = 0)$$

1st axis      2nd axis

Plane:  
3D  
ML:

$$ax + by + cz + d = 0$$
$$w_1x_1 + w_2x_2 + w_3x_3 + w_0 = 0$$

separation

2D

2D

L:

$$w_1 \overbrace{x_1} + w_2 \overbrace{x_2} + w_0 = 0 \quad \checkmark$$

3D

$\Pi_3$ :

$$w_1 \overbrace{x_1} + w_2 \overbrace{x_2} + w_3 \overbrace{x_3} + w_0 = 0$$

d-dim

$\Pi_d$ :

$$w_1 x_1 + w_2 x_2 + \dots + w_d x_d + w_0 = 0$$



$$\sum_{i=1}^d w_i x_i + w_0 = 0$$



algebra

d-dim  
hyper-plane

(Q)

$$\begin{array}{l} \curvearrowleft y = -\frac{1}{2}x - \frac{3}{2} \\ \curvearrowleft 3x + 2y + 1 = 0 \end{array} : L_1$$

$$\begin{array}{l} 4y + 6x + 1 = 20 \rightarrow \\ \curvearrowleft y = -\frac{6}{4}x + \frac{15}{4} \end{array} : L_2$$

parallel  
perpendicular  
 $\Downarrow$   
 $L_3 \perp L_4$

$$\checkmark L_1 \parallel L_2 \quad L_1 \neq L_2 \checkmark$$

(Q)

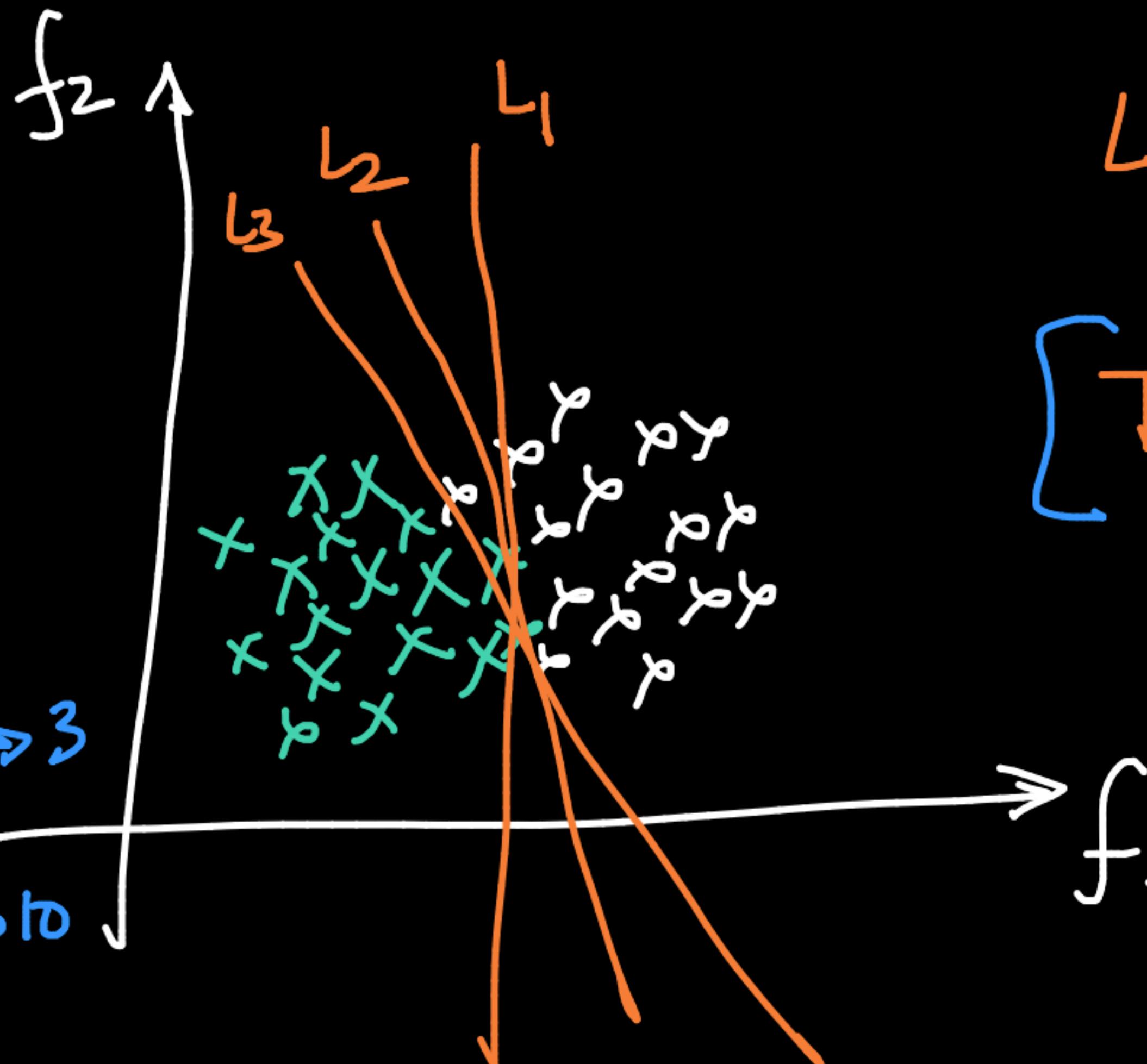
2D

$$\checkmark M_1 = 10; C_1 = 2 \rightarrow 3$$

$$M_2 = 12; C_2 = 4 \rightarrow 10$$

.....

→



$$L_i : M_i x + C_i$$

[Train &amp; error]

 $M_i$  &  $C_i$ 

number

(Q)

$$\Pi: w_1 \downarrow \quad w_2 \downarrow \quad w_3 \downarrow \quad w_4 \downarrow \quad w_5 \nearrow^D \\ 3x_1 + 2x_2 + 4x_3 + 5x_4 + 0 = 0$$

4D-space  
(Separatrl)

how-dim-space →

$$\Pi_2: w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4 = 0$$

exists in  
3D  
NOT a Separation

Line → 3D

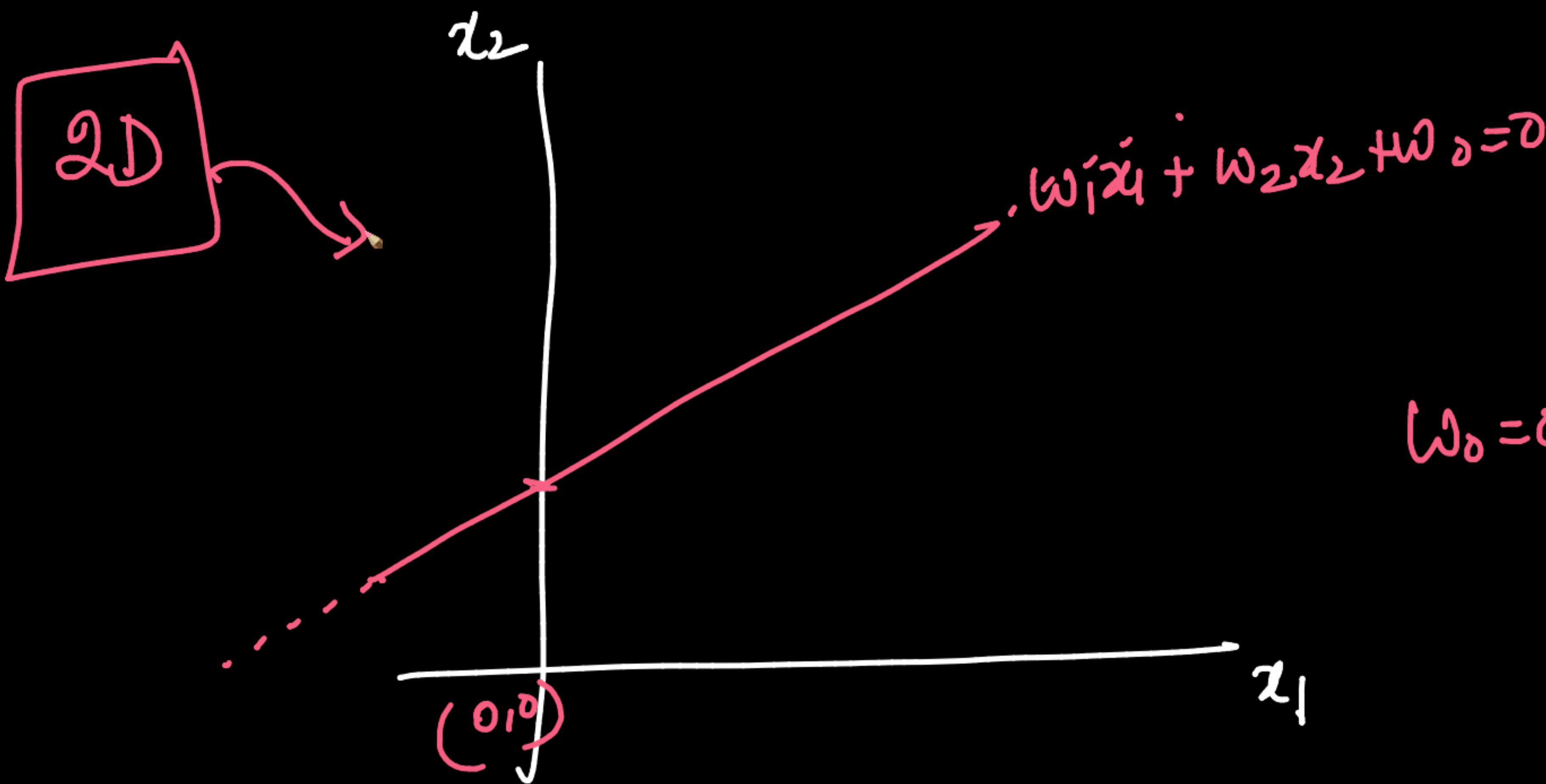
{ 3D-plane → 4D  
(dim)

NOT a Sep - .

✓  $\left\{ \begin{array}{l} \Pi: 3x_1^0 + 2x_2^0 + 4x_3^0 + 5x_4^0 = 0 \\ w_0 = 0 \end{array} \right. \rightarrow 4D \text{ (separatn)} \right.$

(Q) does it pass through  in 4D-space

$$O = (0, 0, 0, 0) \text{ lies on } \Pi$$



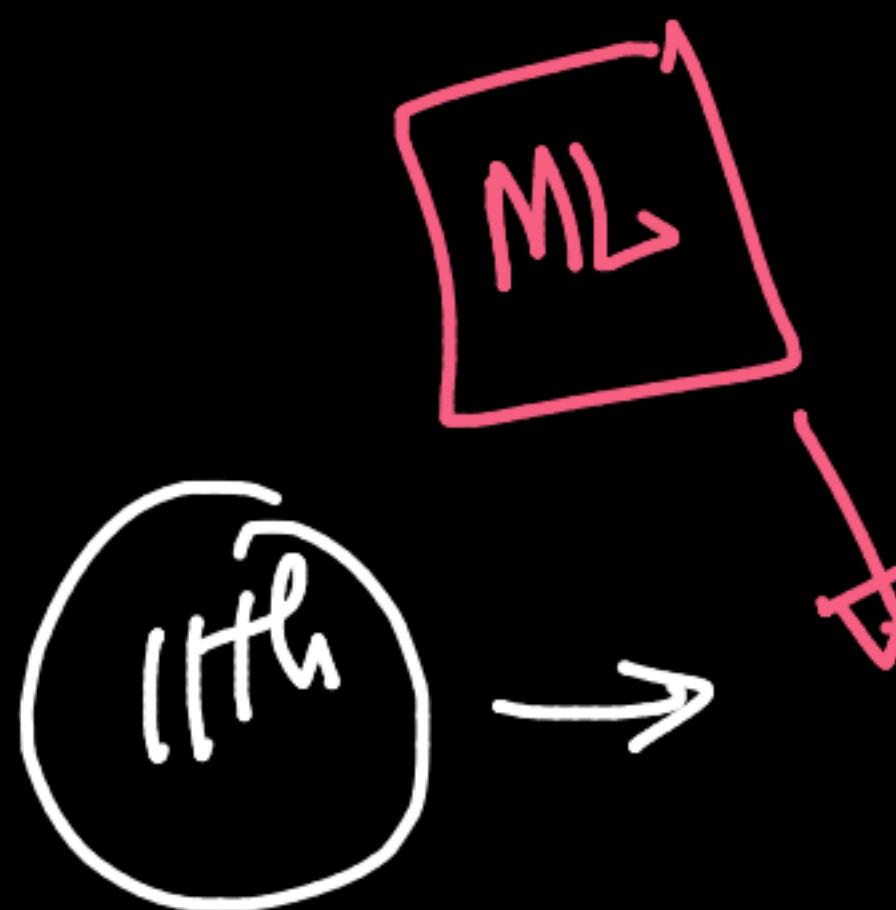
$w_0 = 0 \Rightarrow L \text{ passes through origin}$



TT<sub>d</sub><sup>sep</sup>:

$$\sum_{i=1}^d w_i x_i + w_0 = 0 \rightarrow \text{summation}$$

Cumbersome



Notational [Linear-Algebra]

$$[w_1 \ w_2 \ w_3 \ \dots \ w_d] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix} + w_0 = 0 \quad \text{scalar}$$

$$\{ w_1 x_1 + w_2 x_2 + \dots + w_d x_d + w_0 = 0$$

~~X Physics: Velocity~~

✓ Math:

$$\begin{bmatrix} 1.1 \\ 2 \\ 2.6 \end{bmatrix}$$

~~2.3~~ scalar

✓ CS array

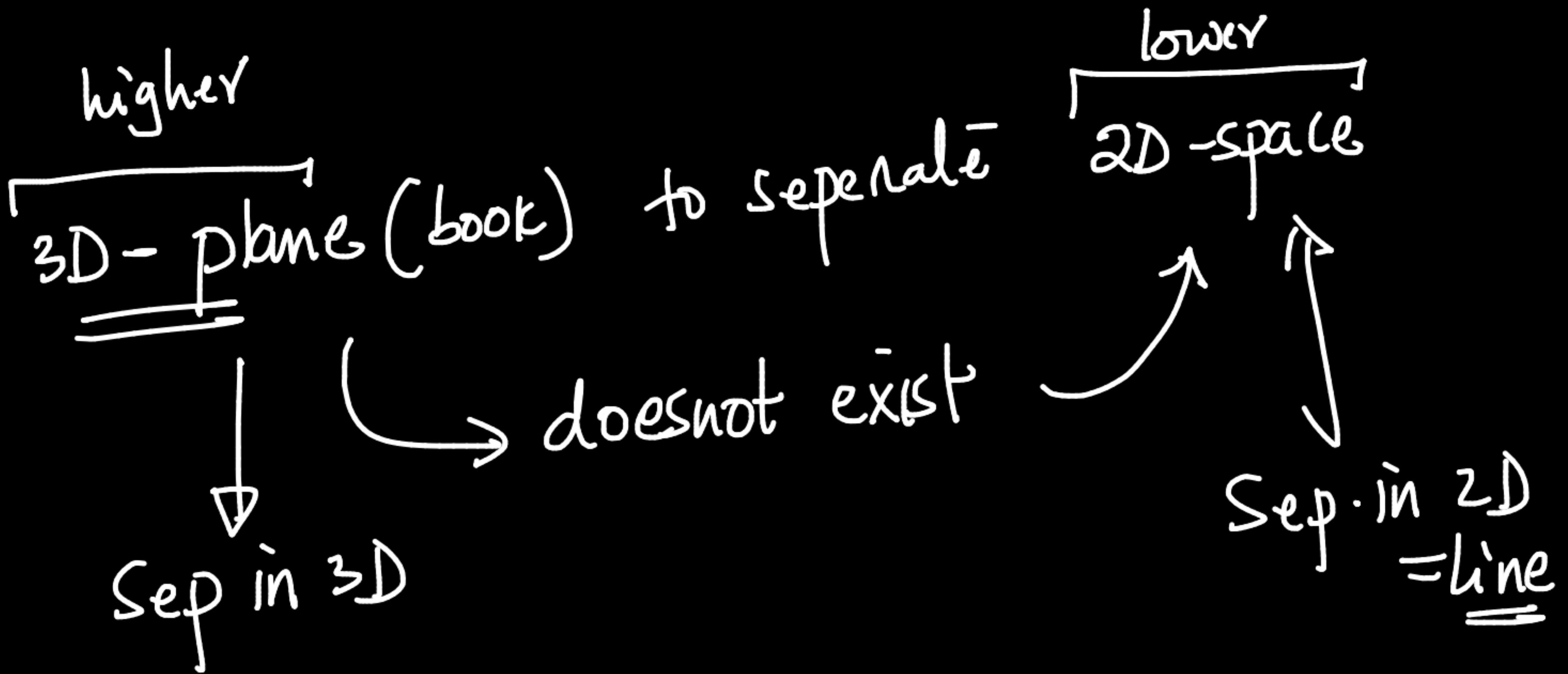
$$\Pi_3: 2x_1 + 3x_2 + 0x_3 + \underline{5} = 0 \rightarrow \text{SUMMation}$$

$$\Pi_d: \sum_{i=1}^d w_i x_i + w_0$$

$$\begin{bmatrix} w_1 & w_2 & \dots & w_d \end{bmatrix} \quad \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix} + w_0 = 0$$

row-vector      column-vector

vector  $w$



$Td \rightarrow d\text{-dim Space}$

d-dim → vector → default convention → Col. vector

[ ]  $\otimes$  [ ]  
row  
d-dim row-vector  
col

$w : d\text{-dim}$

$x : d\text{-dim}$

$$\pi_d : \left[ w_1 \quad w_2 \quad \dots \quad w_d \right] \xrightarrow{\quad} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix} + w_0 = 0$$

$$\underline{\pi_d} : \boxed{w^T \cdot x + w_0 = 0}$$

Notational  
Simple

vector -  $w$  d-dim:

d-dim:  $x$

$$w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_d \end{bmatrix}$$

d-tuple  
tuple

( , , , , )

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix}$$

Transpose:

(operation)

d-dim vec

$$w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_d \end{bmatrix}$$

$d \times 1$

(matrices)

$$w^T = [w_1 \ w_2 \ \dots \ w_d] \quad l \times d$$

$$(w^T)^T = w$$

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

2x2

$$A^T = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

2x2

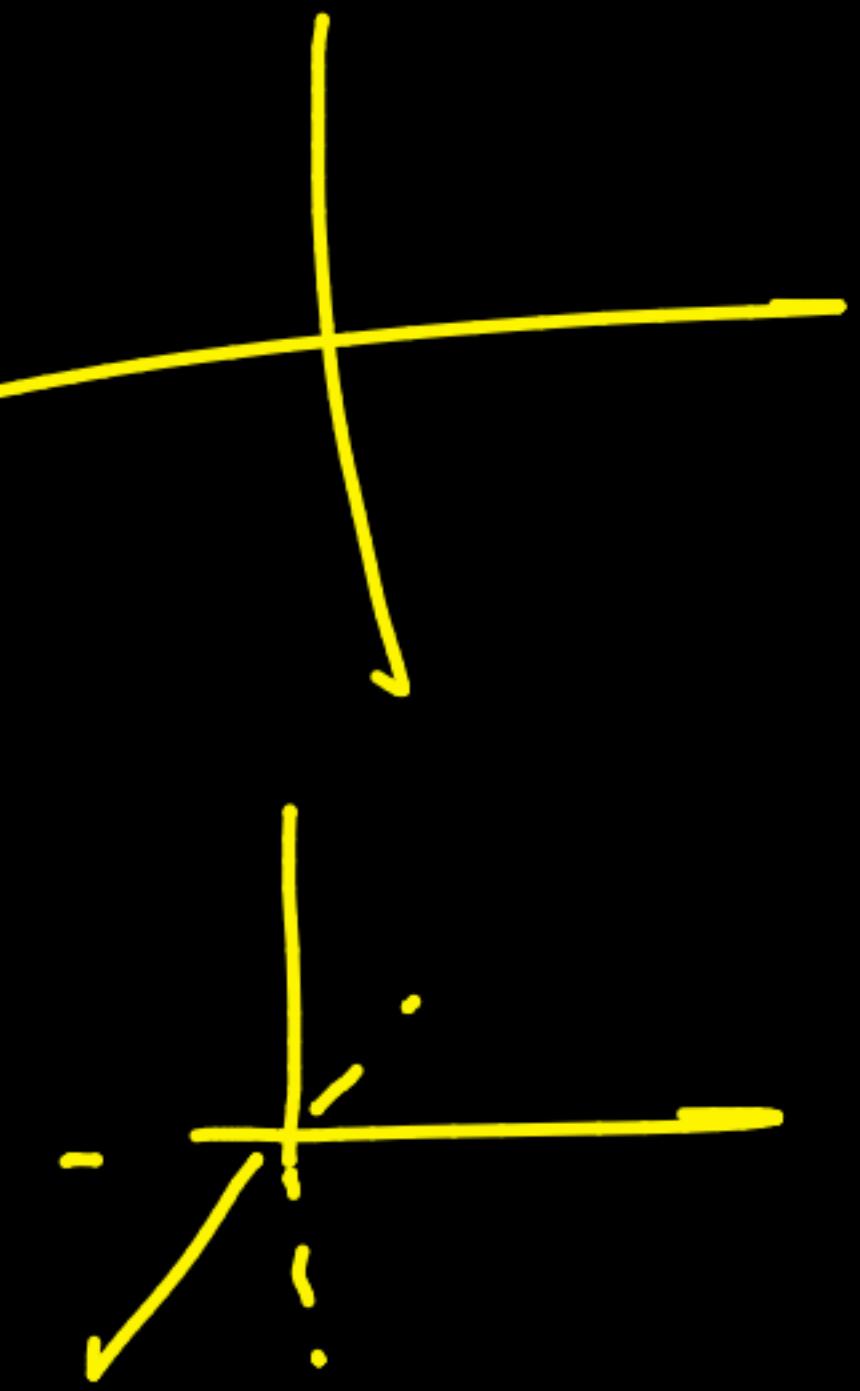
$d$ -components

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_d \end{bmatrix}$$

$$\vec{w}_d : \underline{\omega}^T x + w_0 = 0$$

$\omega$  &  $x$  are 2-dim vectors

$\omega \in \mathbb{R}^d$



real-valued Space

Line  
3D-plane  
2D-plane

$$\text{Eq: } w^T x + w_0 = 0$$
$$w \in \mathbb{R}^d ; x \in \mathbb{R}^d ; w_0 \in \mathbb{R}$$

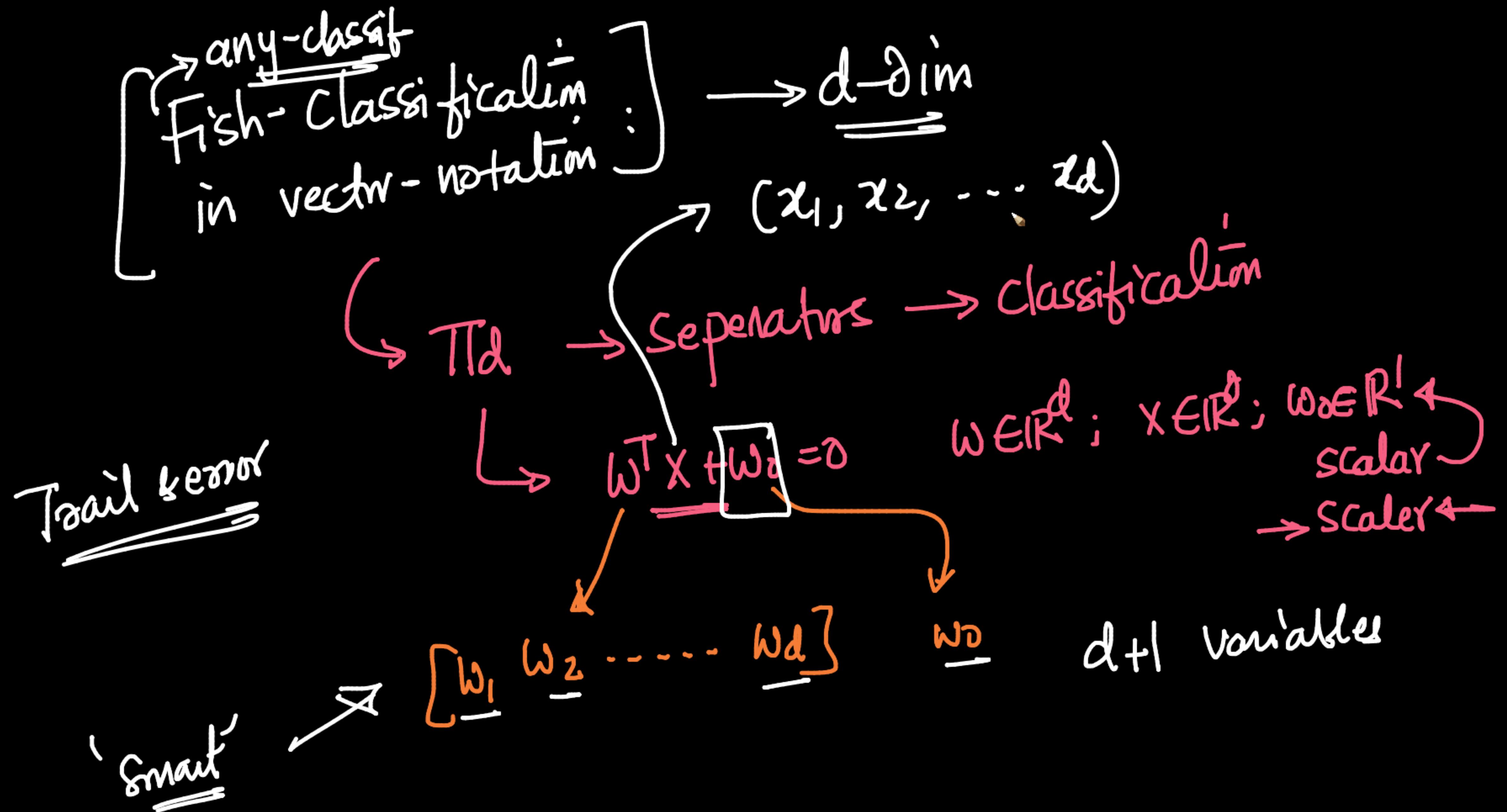
R: real

C: complex

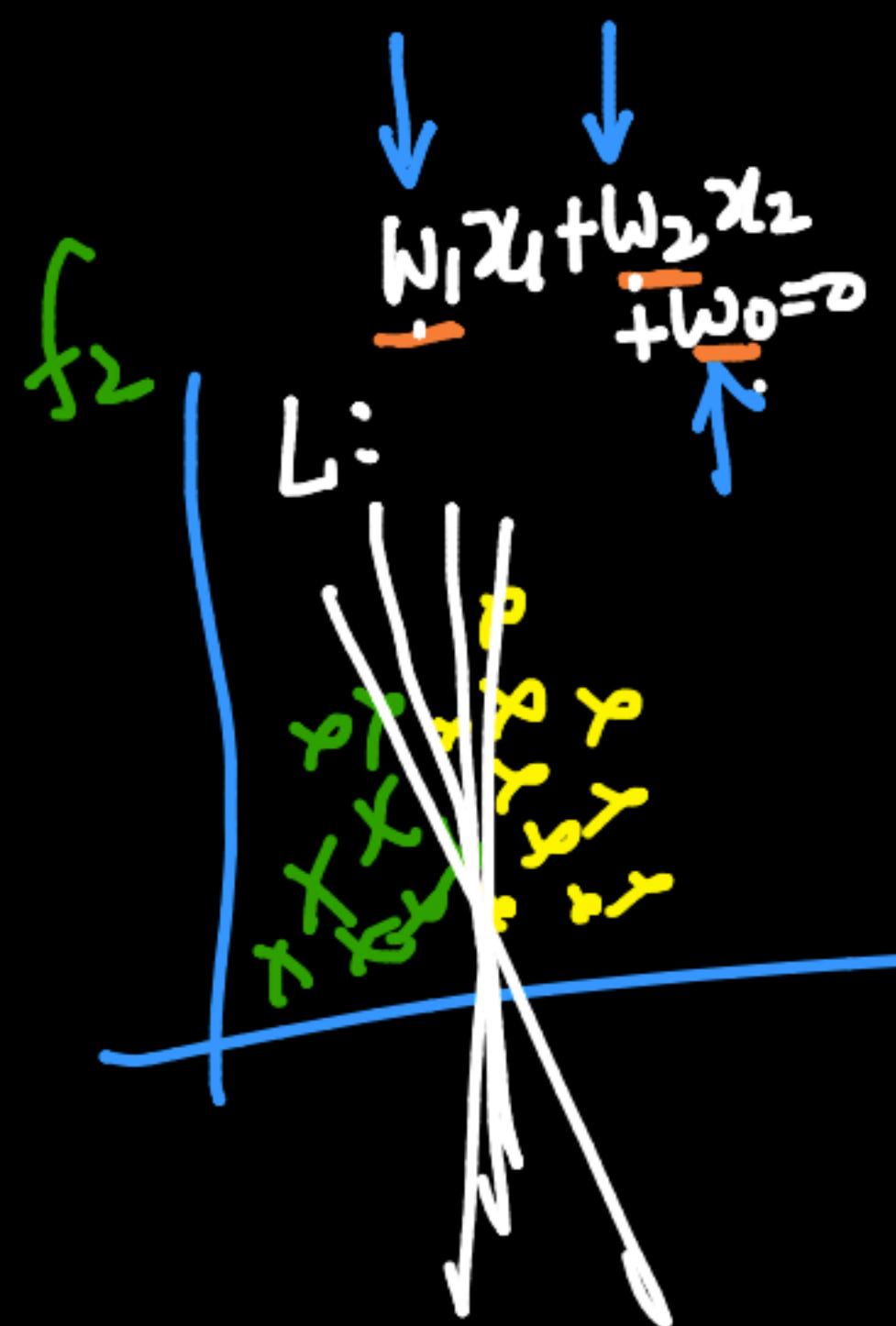
(Q)  $[2 \leq 3 \leq 1]$

NOT -valid

23:07      5 min break



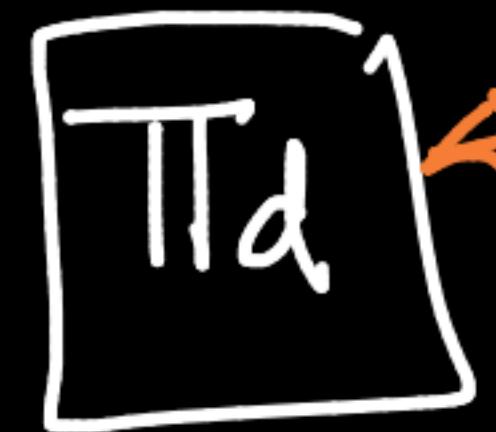
Troll sensor



d-dim -space

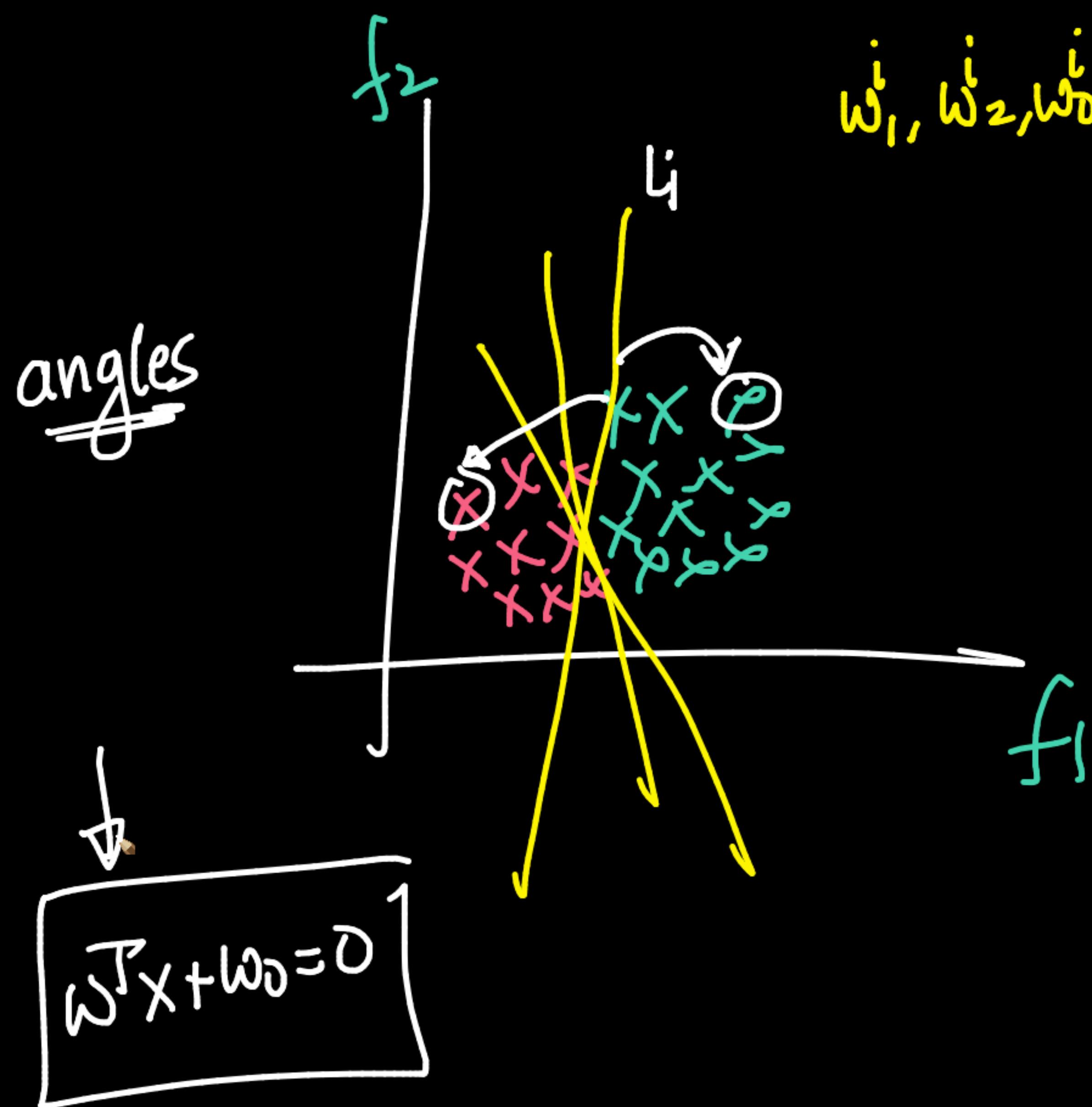
each fish has  $d=10$  features

$d$ -dim space



best hy-plane  
that min  
miscalcification  
error

$x_1 \ x_2 \ x_3 \ \dots \ x_d$   
fish  $\rightarrow (l, w, w_g, fl, MW, \dots)$   
 $d$ -dim point  
scanners



Boule tree

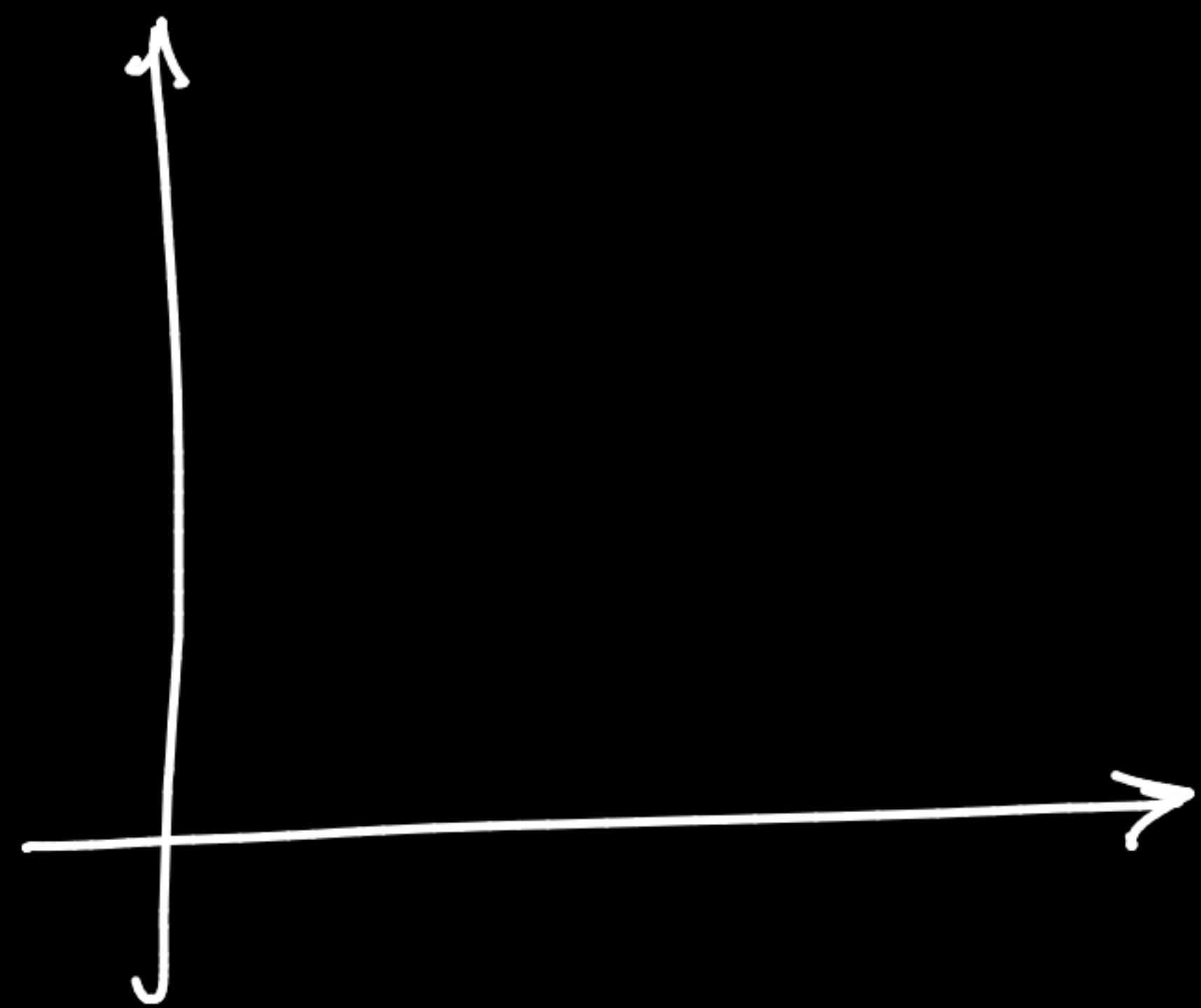
for

$w_1^i : 1 \text{ to } 100 :$

$f_w \quad w_2^i : -100 \text{ to } 100$

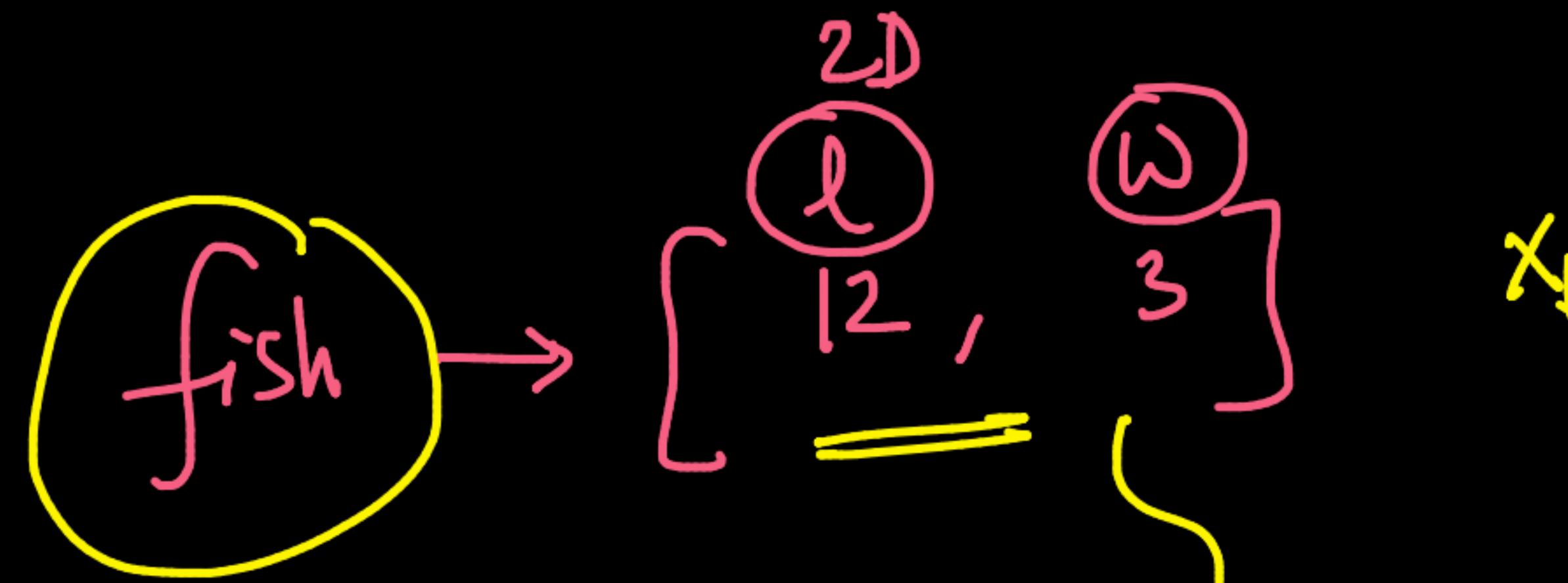
$f_w \quad w_0^i : -100 \text{ to } 100$





classfn:

- ①  $\rightarrow$  Line/plane
  - ② dist  $\pi$  to P
  - ③ which side
- A yellow bracket groups the first two items, and another yellow bracket groups all three items.

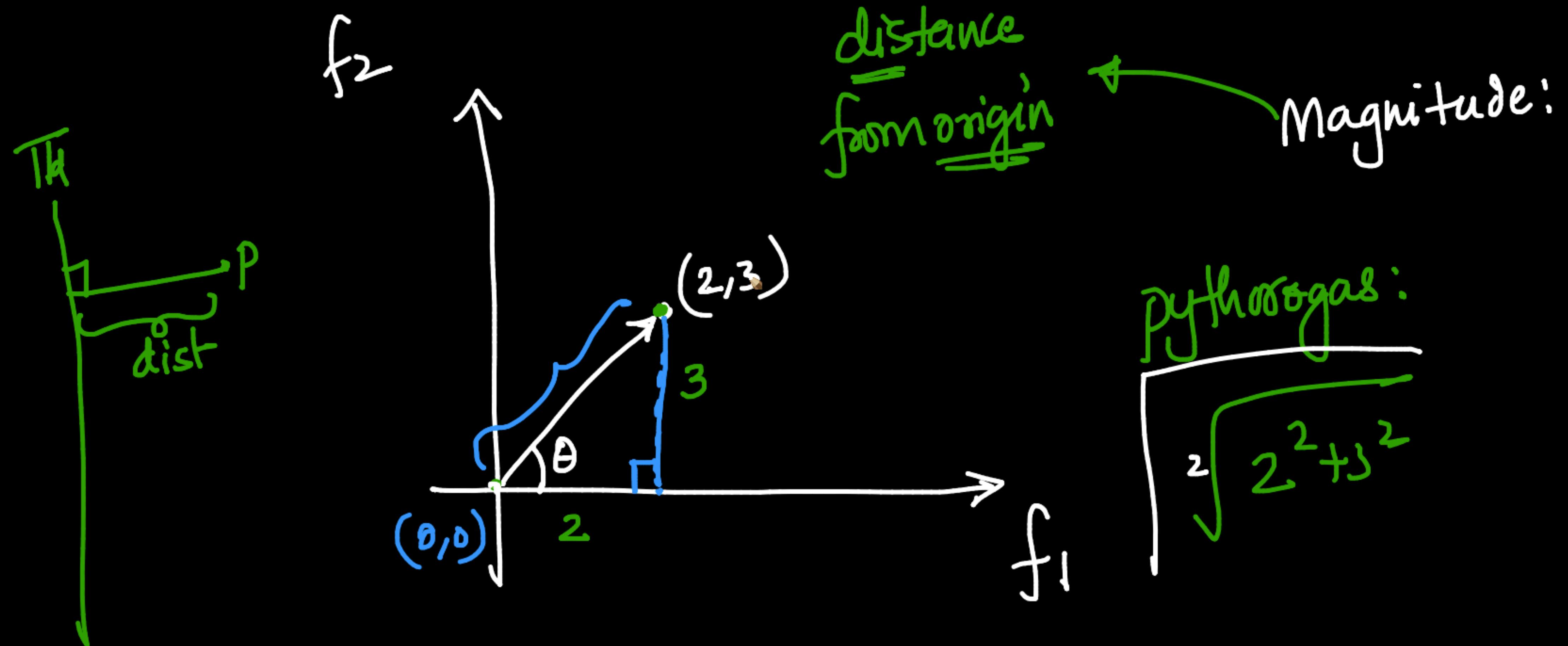


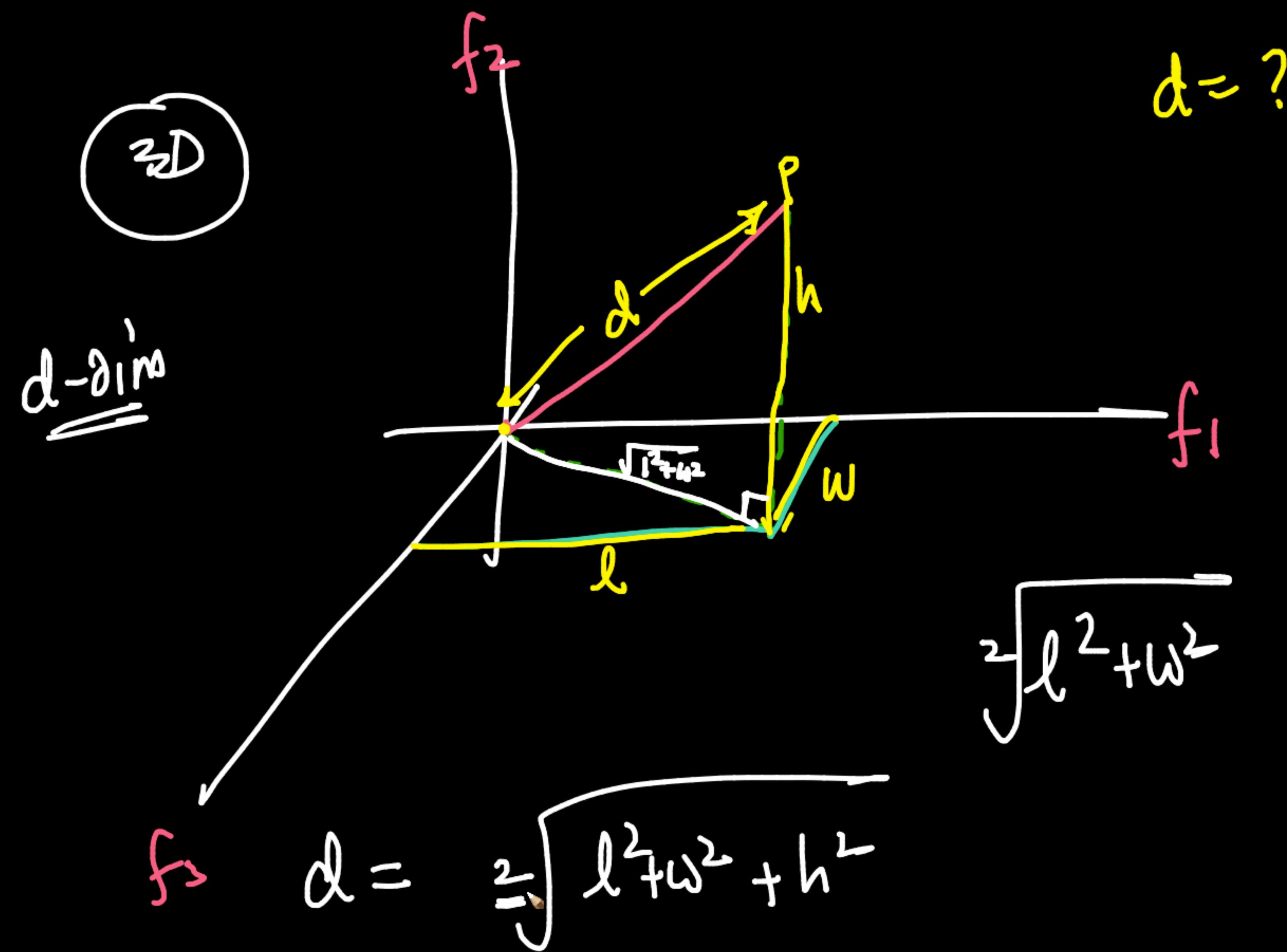
$$\underline{w} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$
$$w_0 = 10$$

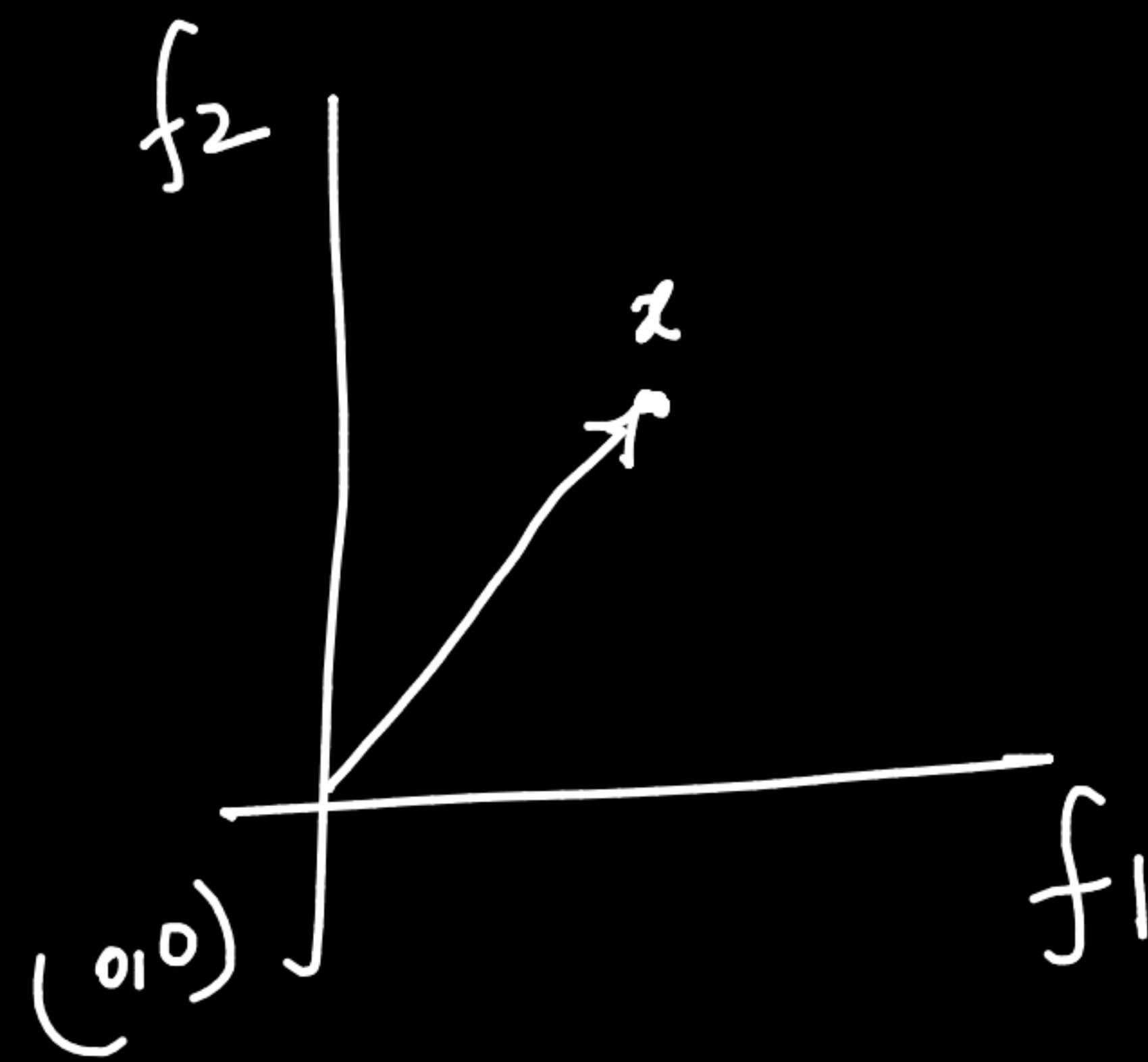
✓ line-sep :  $\underline{w}^T \underline{x} + w_0 = 0$

$w \in \mathbb{R}^d$   
 $x \in \mathbb{R}^d$   
 $w_0: \text{scalar}$

$$\begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} 12 \\ 3 \end{bmatrix} + 10$$







$$x \in \mathbb{R}^2$$

Magnitude of  $x = \underline{\|x\|}$

absolute value

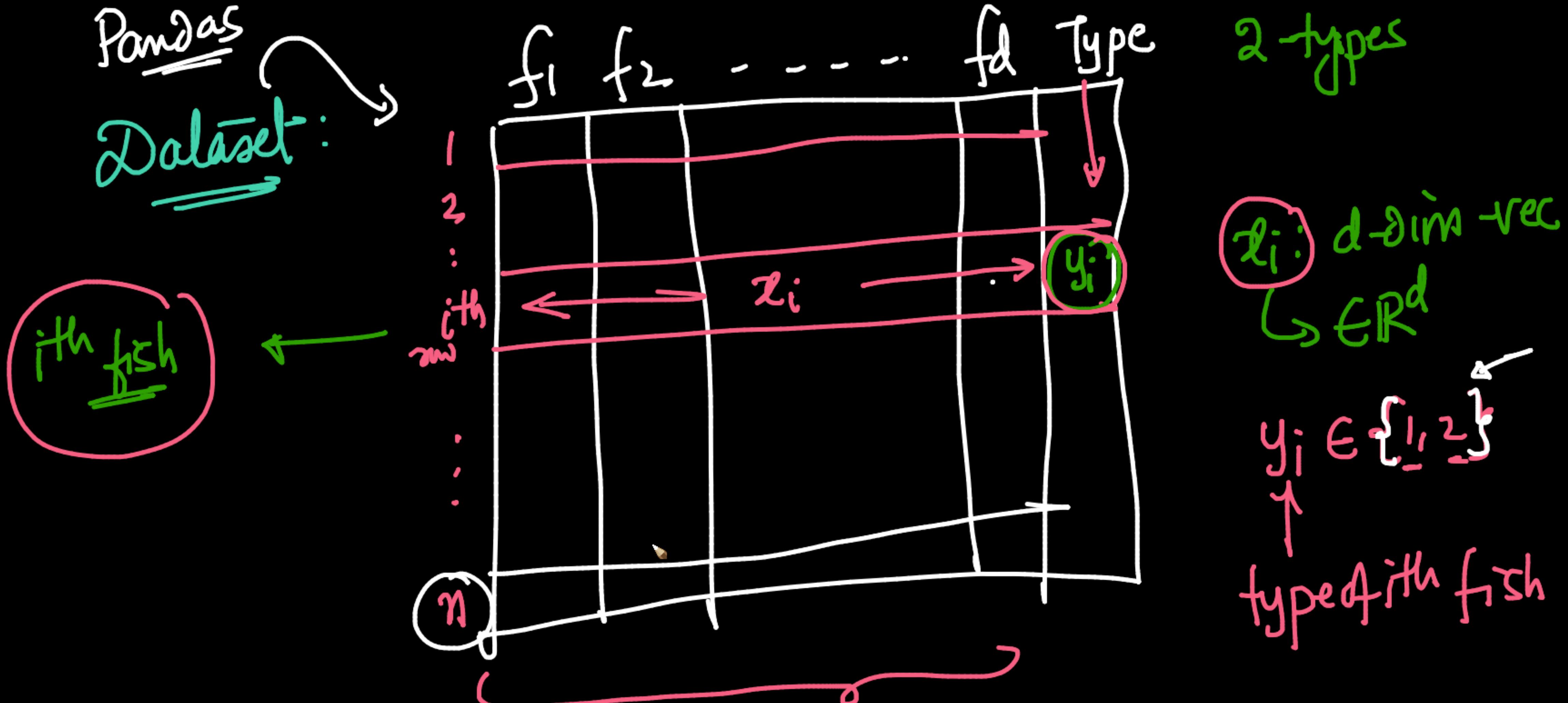
$\|x\|$   $\rightarrow L_2$ -norm

$\|x\|$   $\rightarrow$  norm

plane  
vector →

ML / CG:  
data-point in 2-dim Space

↓  
displacement in physical



dataset

$$\mathcal{D} = \left\{ \underbrace{\left( \underline{x}_i, \underline{y}_i \right)}_{i=1}^n ; \quad \begin{array}{l} \text{tuple} \\ \text{vector} \end{array} ; \quad \underline{x}_i \in \mathbb{R}^d ; \quad \underline{y}_i \in \{1, 2\} \right\}$$

Notation

sklearn



read it in english

{ } : Set

( , ) : tuple

eqn. of plane:  $\pi_d:$

$$\underline{w}^T \underline{x} + w_0 = 0$$

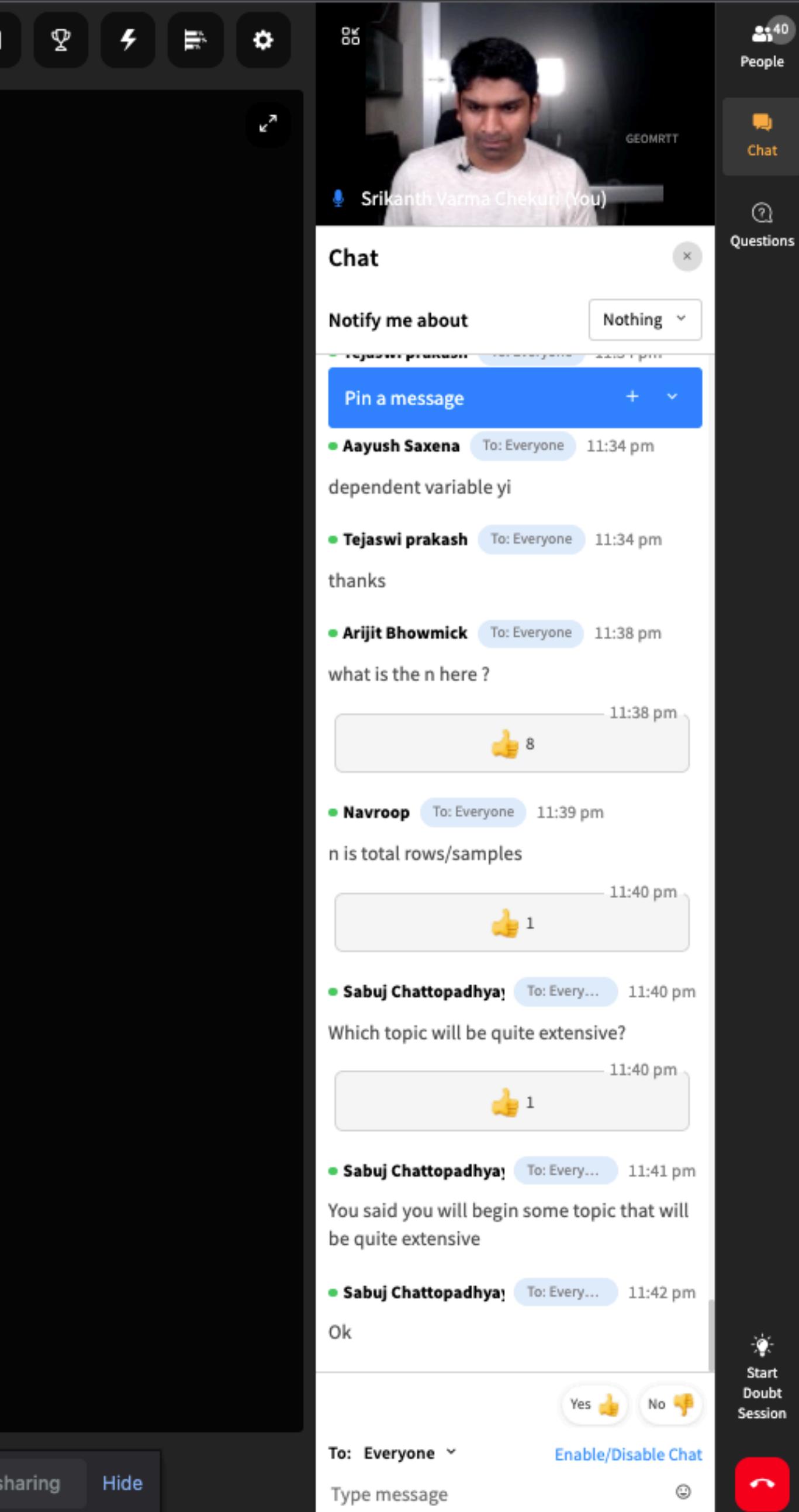
$\left\{ \begin{array}{l} 2D: w_1 x_1 + w_2 x_2 + w_0 = 0 \\ 3D: w_1 x_1 + w_2 x_2 + w_3 x_3 + w_0 = 0 \\ 4D: \end{array} \right.$

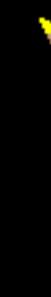


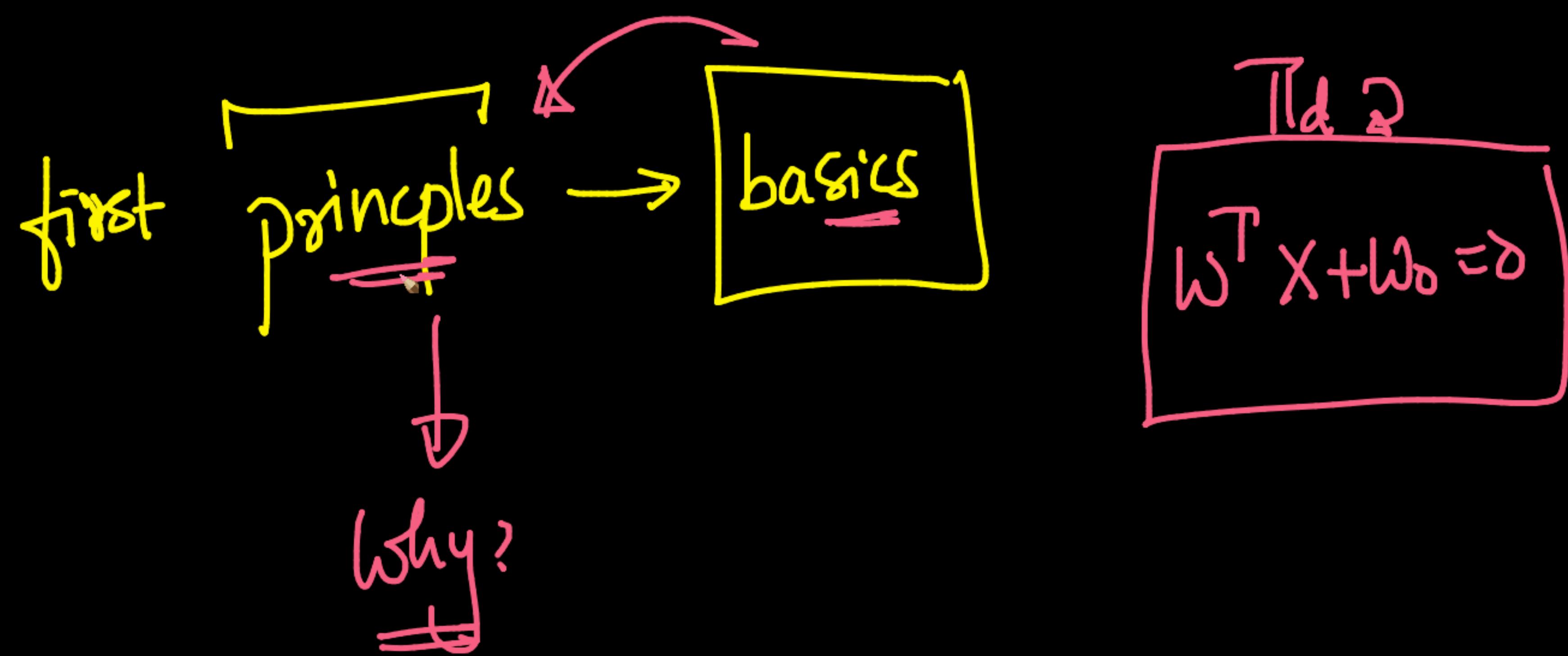


Live | DSML Advanced : Lin

 [scaler.com/meetings/i/dsml-advanced-linear-algebra-2-3/live](https://scaler.com/meetings/i/dsml-advanced-linear-algebra-2-3/live)







Id 2

$$w^T x + w_0 = d$$

Live | DSML Advanced : Lin

scaler.com/meetings/i/dsml-advanced-linear-algebra-2-3/live

DSML Advanced : Linear Algebra 2 | Lecture

People: 30

Chat

Notify me about: Nothing

This total rows/samples

Pin a message

Sabuj Chattopadhyay To: Everyone 11:40 pm Which topic will be quite extensive? 11:40 pm 1

Sabuj Chattopadhyay To: Everyone 11:41 pm You said you will begin some topic that will be quite extensive

Sabuj Chattopadhyay To: Everyone 11:42 pm Ok

Subham Roy To: Everyone 11:42 pm are we going through integral calculus or differential calculus ??

Tejaswi prakash To: Everyone 11:50 pm cdf

Ankit Gupta To: Everyone 11:50 pm CDFs?

Rishav Kumar To: Everyone 11:50 pm integration

Subham Roy To: Everyone 11:50 pm integration

Start Doubt Session

To: Everyone

Enable/Disable Chat

Type message

70 / 70

Prob & Stats:

PDF

$f(x)$

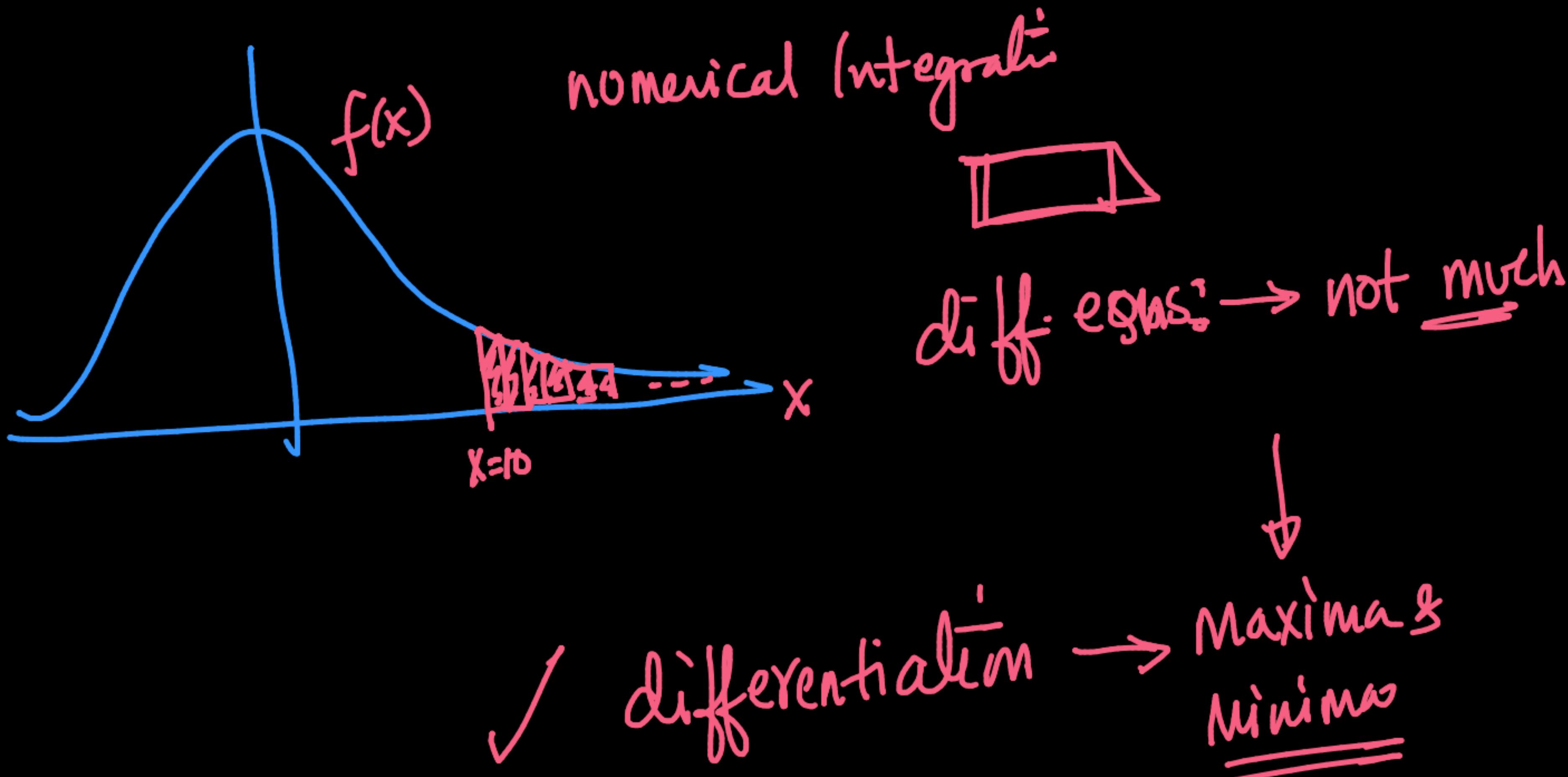
$\int_{-\infty}^{\infty} f(x) dx$

$x = l$

You are sharing your screen now

Stop Sharing

AUC  $\rightarrow$  ROC



Live | DSML Advanced : Linear X +

scaler.com/meetings/i/dsml-advanced-linear-algebra-2-3/live

You have left the meeting

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- 1 Scan the QR code with your iPad  
Scanner should be present in the top menu on your iPad
- 2 Upload Notes on the generated link  
All notes uploaded will be visible in the saved version of this session

OR

Drag and drop files or [click here to upload](#)

Files Uploaded from your computer appear here

72 / 72