

A Multispecies, Multifluid Model for Laser–Induced Counterstreaming Plasma Simulations[☆]

D. Ghosh¹, T. D. Chapman², R. L. Berger², A. Dimits³

Lawrence Livermore National Laboratory, Livermore, CA 94550

J. W. Banks⁴

Rensselaer Polytechnic Institute, Troy, NY 12180

Abstract

The interpenetration of counterstreaming plasmas is an important phenomenon in several application areas, such as astrophysical flows, design of controlled fusion devices, and laser–induced plasma experiments. Multispecies “single-fluid” codes are unable to model this phenomenon due to the single velocity representation for all the species/fluids. Kinetic codes, though capable of modeling interpenetration, are computationally prohibitive for at-scale simulations. In this paper, we propose a multifluid model that solves the fluid equations for each ion fluid or stream. This allows distinct flows that interact with each other through electrostatic and collisional forces. We introduce and describe our code,

[☆]This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract No. DE-AC52-07NA27344 and funded by the LDRD Program at LLNL under project tracking code 17-ERD-081. This document was prepared as an account of work sponsored by an agency of the United States government. Neither the United States government nor Lawrence Livermore National Security, LLC, nor any of their employees makes any warranty, expressed or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States government or Lawrence Livermore National Security, LLC. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States government or Lawrence Livermore National Security, LLC, and shall not be used for advertising or product endorsement purposes. LLNL-JRNL-764617

¹Center for Applied Scientific Computing, Computation Directorate

²Design Physics, Weapons & Complex Integration Directorate

³Physics Division, Physical & Life Sciences Directorate

⁴Department of Mathematical Sciences

EUCLID, that uses a conservative finite-difference formulation to discretize the governing equations in space. The 5th-order Monotonicity-Preserving WENO scheme is used for the upwind approximation of the hyperbolic flux, and the explicit 4th-order Runge-Kutta scheme is used for time integration. The code is verified for several benchmark cases and manufactured solutions. We simulate one- and two-dimensional interactions of counterstreaming plasmas in vacuum as well as in the presence of gas fill, where the setups are representative of laser-induced plasma experiments.

Keywords: Plasma interpenetration, Multifluid plasma, Conservative finite-difference method, Laser-induced plasmas

2010 MSC: 76X05, 35L04, 35L65, 65M06

1. Introduction

Counterstreaming plasmas often exhibit interpenetration, where they flow through each other, and localized regions may exist where plasma species or populations inhabiting the same physical space may have different velocities.

- 5 This dynamics plays a significant role in applications such as astrophysical flows, controlled fusion devices [1, 2], and experiments involving the interaction of laser-induced plasma streams [3–7]. Multispecies, single-fluid codes [8] fail to capture the interpenetration; the assumption of a single velocity field results in counterstreaming plasmas stagnating and causing density pile-ups.
- 10 Alternatively, collisional, kinetic codes that solve the Boltzmann equations for each species [9–13] are able to represent this phenomenon; however, they are computationally expensive due to their high-dimensionality, and therefore, impractical for experimental-scale simulations. An effort has also been made to derive species diffusion models [14, 15] for multispecies, single-fluid codes. These
- 15 are empirical in nature, and the introduction of a diffusion term results in a stiff system of partial differential equations (PDEs). The drawbacks of these current approaches motivate the use of a multifluid model.

The multifluid equations are derived by taking the velocity-space moments

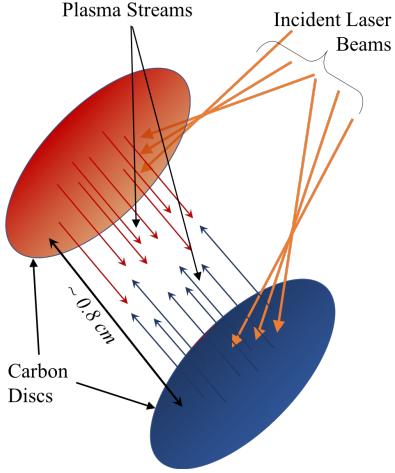


Figure 1: Illustration of the laser-induced plasma interactions in HEDP experiments with counterstreaming plasmas ablating off the carbon discs.

of the Boltzmann equation for each species [16]. This results in a distinct set of fluid equations for each stream or fluid that is solved along with the Maxwell's equations or some simplification thereof. Over the last few decades, multifluid models have been successfully applied to several plasma applications. In the context of magnetically-confined fusion device simulations, these methods have been applied to θ -pinch [17] and Z-pinch [18, 19] simulations. The multifluid approach has been applied to device edge plasma simulations where the modeling of impurities introduced from the device wall is essential [20–25]. In astrophysics, the multifluid approach has been used to study the interactions of the solar wind with the magnetosphere [26] and the local interstellar cloud [22, 27], collisionless reconnection [18, 19, 28–30], and the interactions of weakly/partially-ionized plasmas in the heliosphere and the atmosphere [23, 31, 32]. Other applications include plasma glow discharge [33, 34], ICF capsule detonation [35, 36], and plasma-enhanced combustion [37]. A majority of these multifluid approaches use a predefined number of fluids, for example, two-fluid formulations with one ion species and one electron [18, 19, 25, 28, 36, 38–40], two-fluid formulations with one ion and one neutral species [22, 23], and three- and four-fluid formu-

lations with one ion species, one electron species, and neutral species including dust [22, 29, 32]. Formulations with an arbitrary number of species have also been proposed [21, 27, 34, 41].

This paper focuses on the interactions of laser-induced counterstreaming plasmas in high energy density physics (HEDP) experiments [3–7]. Figure 1 illustrates a typical setup of such experiments. Two foils/discs made of carbon or a carbon-based compound are irradiated with high-energy laser beams, and the interaction of the counterstreaming plasmas ablating off them is studied. Multifluid models for these applications were proposed [42–44] that solved the fluid equations for two ion species in a one-dimensional domain. A kinetic model was incorporated [42] to represent the friction and heating resulting from plasma instabilities. Overall, these studies showed that a multifluid model was essential to model the interpenetration, and a good agreement with kinetic simulations was observed [44]. In this paper, we present an Eulerian Code for pLasma Interpenetration Dynamics (EUCLID) that extends these previous efforts in developing and implementing a computationally efficient multifluid model for the applications described above. EUCLID solves an electrostatic multispecies, multifluid model that comprises the inviscid Euler equations for each ion fluid or stream, where the number of ion fluidsstreams is arbitrary. The fluids interact with each other through the electrostatic forces as well as collisional friction and thermal equilibration terms [44]. It should be noted that an arbitrary number of fluids may be of the same species; in other words, each species may be divided into distinct populations/flows, for example, the carbon stream ablating off the top disc in Fig. 1 is distinct from the one ablating off the bottom disc. The Debye length in our applications is much smaller than the characteristic length scales (by a factor of the order of 10^5 – 10^6), and therefore, we use a reduced model for the electrons that simplifies the electrostatic force term. The nondimensionalized governing equations are discretized in a Cartesian domain with the conservative finite-difference formulation [45, 46]. The basic data structures in EUCLID are implemented using Chombo [47], a parallel, block-structured adaptive mesh refinement (AMR) library for PDEs, and therefore, EUCLID is

multidimensional and supports distributed-memory parallelism.

The primary numerical challenge in our applications stems from flows involving the presence of a fluid that exists only in a localized part of the domain; it
70 doesn't exist elsewhere. Since the fluid density cannot be numerically zero anywhere in the domain (even if other fluids with nonzero densities exist in the same region), we specify the species density corresponding to "numerical vacuum" as 10^{-14} . This results in very steep gradients, where the species density and pressure may transition from $O(1)$ to $O(10^{-14})$ within a few grid cells. Therefore,
75 we use the 5th-order weighted essentially nonoscillatory (WENO) scheme [48] and the characteristic-based reconstruction procedure, which has been shown to be more robust than reconstructing the conserved or the primitive variables for supersonic flows involving strong shocks and gradients. In addition, because the WENO scheme may result in $O(\epsilon)$ oscillations, where ϵ is a WENO parameter
80 that is typically set to 10^{-6} , we implement the monotonicity-preserving (MP) limits [49] to avoid any numerical oscillations. The equations are evolved in time using the four-stage, 4th-order explicit Runge-Kutta (RK4) method.

The outline of the paper is as follows. The multifluid model that EUCLID solves is described in Sec. 2, and Sec. 3 describes the numerical method. Section
85 4 outlines the code verification using exact solutions, manufactured solutions, and benchmark cases, and the simulation of flows representative of the application areas described above are shown in Sec. 5. Conclusions are summarized in Sec. 6.

2. Governing Equations

The inviscid Euler equations for each ion fluid are obtained by taking the moments of the collisionless Boltzmann equation and neglecting viscosity and

heat conduction [16], and can be expressed in their nondimensional form as:

$$\frac{\partial \rho_\alpha}{\partial t} + \nabla \cdot (\rho_\alpha \mathbf{u}_\alpha) = 0, \quad (1a)$$

$$\frac{\partial \rho_\alpha \mathbf{u}_\alpha}{\partial t} + \nabla \cdot (\rho_\alpha \mathbf{u}_\alpha \otimes \mathbf{u}_\alpha + P_\alpha) = -Z_\alpha n_\alpha \nabla \phi + \sum_{\beta \neq \alpha} \mathbf{R}_{\alpha,\beta} + \mathbf{R}_{\alpha,e}, \quad (1b)$$

$$\begin{aligned} \frac{\partial \mathcal{E}_\alpha}{\partial t} + \nabla \cdot \{(\mathcal{E}_\alpha + P_\alpha) \mathbf{u}_\alpha\} &= -Z_\alpha n_\alpha \mathbf{u}_\alpha \cdot \nabla \phi + \sum_{\beta \neq \alpha} (\mathbf{R}_{\alpha,\beta} \cdot \mathbf{u}_\alpha + Q_{\alpha,\beta}) \\ &\quad + \mathbf{R}_{\alpha,e} \cdot \mathbf{u}_\alpha + Q_{\alpha,e}, \end{aligned} \quad (1c)$$

where $\alpha = 1, \dots, n_f$ is the fluid index, n_f is the number of fluids, ρ_α is the density, \mathbf{u}_α is the velocity vector, P_α is the pressure, Z_α is the ionization number, ϕ is the electrostatic potential, $n_\alpha = \rho_\alpha/m_\alpha$ is the number density, m_α is the atomic mass, and

$$\mathcal{E}_\alpha = \frac{P_\alpha}{\gamma_\alpha - 1} + \frac{1}{2} \rho_\alpha \mathbf{u}_\alpha \cdot \mathbf{u}_\alpha \quad (2)$$

is the internal energy. The ideal gas law $P_\alpha = n_\alpha T_\alpha$ is assumed, and the specific heat ratio is specified as $\gamma_\alpha = 5/3$ for all fluids. On the right-hand side (RHS) of (1), the terms $-Z_\alpha n_\alpha \nabla \phi$ and $-Z_\alpha n_\alpha \mathbf{u}_\alpha \cdot \nabla \phi$ represent the electrostatic force and the work done by it, respectively, and the remaining terms represent the collisional interactions between the ion fluids and electrons, as described below. We reiterate here that (1) is solved for each “fluid” or distinct population of ion species; a single species can be partitioned into multiple populations and modeled as separate fluids.

Friction and Thermal Equilibration Terms. The frictional force on fluid α due to fluid β is proportional to the velocity of β relative to that of α , and this can be expressed as

$$\mathbf{R}_{\alpha,\beta} = m_\alpha n_\alpha \nu_{\alpha,\beta} (\mathbf{u}_\beta - \mathbf{u}_\alpha) = -\mathbf{R}_{\beta,\alpha}, \quad (3)$$

and the total heating of fluid α due to fluid β is

$$Q_{\alpha,\beta} = Q_{\alpha,\beta}^{\text{fric}} + Q_{\alpha,\beta}^{\text{eq}}, \quad (4)$$

where

$$Q_{\alpha,\beta}^{\text{fric}} = m_{\alpha,\beta} n_\alpha \nu_{\alpha,\beta} (\mathbf{u}_\alpha - \mathbf{u}_\beta) \cdot (\mathbf{u}_\alpha - \mathbf{u}_\beta) \quad (5)$$

is the frictional heating between two fluids that is proportional to the square of the relative velocity, and

$$Q_{\alpha,\beta}^{\text{eq}} = 3m_\alpha n_\alpha \frac{\nu_{\alpha,\beta}}{m_\alpha + m_\beta} (T_\beta - T_\alpha) = -Q_{\beta,\alpha}^{\text{eq}} \quad (6)$$

is the thermal equilibration due to a temperature difference between the two fluids. The collisional coefficient $\nu_{\alpha,\beta}$ is given by

$$\nu_{\alpha,\beta} = \frac{4\sqrt{2\pi}}{3} \frac{Z_\alpha^2 Z_\beta^2 e^4 n_\beta \Lambda_{\alpha,\beta}}{m_\alpha m_{\alpha,\beta}} \left[r (\mathbf{u}_\alpha - \mathbf{u}_\beta) \cdot (\mathbf{u}_\alpha - \mathbf{u}_\beta) + \frac{T_\alpha}{m_\alpha} + \frac{T_\beta}{m_\beta} \right]^{-\frac{3}{2}} \left(\frac{n_{\text{ref}} x_{\text{ref}}}{T_{\text{ref}}^2} \right), \quad (7)$$

where $m_{\alpha,\beta} = m_\alpha m_\beta / (m_\alpha + m_\beta)$. Similarly, the electron–ion friction and thermal equilibration terms are

$$\mathbf{R}_{\alpha,e} = -\mathbf{R}_{e,\alpha} = -m_e n_e \nu_{e,\alpha} (\mathbf{u}_\alpha - \mathbf{u}_e), \quad (8a)$$

$$Q_{\alpha,e} = Q_{\alpha,e}^{\text{eq}} = -Q_{e,\alpha}^{\text{eq}} = -3m_e n_e \frac{\nu_{e,\alpha}}{m_\alpha + m_e} (T_\alpha - T_e), \quad (8b)$$

where the electron–ion friction heating $Q_{\alpha,e}^{\text{fric}}$ has been ignored. Since $m_e \ll m_\alpha \Rightarrow m_{\alpha,e} \approx m_e$, we have

$$\nu_{e,\alpha} = \frac{4\sqrt{2\pi}}{3} \frac{Z_\alpha^2 e^4 n_\alpha \Lambda_{ei}}{m_e^2 (T_e/m_e)^{\frac{3}{2}}} \left(\frac{n_{\text{ref}} x_{\text{ref}}}{T_{\text{ref}}^2} \right). \quad (9)$$

as the simplified form for the electron–ion collisional coefficient. In the simulations reported in this paper, the ion–ion log-lambda is $\Lambda_{\alpha,\beta} = 5$ and the electron–ion log-lambda is $\Lambda_{e,\alpha} = 10$, unless otherwise mentioned, and $r = [2/(9\pi)]^{1/3}$ is a constant.

Equation (1) is expressed in terms of nondimensional variables, and these can be multiplied with the corresponding reference quantity to obtain their physical values. The primary reference quantities and their CGS units are mass m_{ref} (g), number density n_{ref} (cm^{-3}), length x_{ref} (cm), temperature T_{ref} (ergs), and charge e (elementary charge in statcoulomb). The derived reference quantities

and their units are:

$$\begin{aligned}
u_{\text{ref}} &= \sqrt{T_{\text{ref}}/m_{\text{ref}}} & (\text{cm/s}), \\
\rho_{\text{ref}} &= m_{\text{ref}} n_{\text{ref}} & (\text{g/cm}^3), \\
t_{\text{ref}} &= x_{\text{ref}}/u_{\text{ref}} = x_{\text{ref}} \sqrt{m_{\text{ref}}/T_{\text{ref}}} & (\text{s}), \\
P_{\text{ref}} &= n_{\text{ref}} T_{\text{ref}} & (\text{Ba}), \\
\phi_{\text{ref}} &= T_{\text{ref}}/e & (\text{V}).
\end{aligned} \tag{10}$$

Electron Model. The fluid equations for electrons can be expressed by removing the $\mathbf{R}_{\alpha,e}$ and $Q_{\alpha,e}$ terms in (1) and replacing the subscript α with the subscript e in the remaining terms, and these equations can be solved along with the ion fluid equations to resolve the electron dynamics, where the Poisson's equation is solved for the electrostatic potential. However, to avoid the stiff time scales arising from the electron thermal velocity, we make the following assumptions:

$$\frac{\partial \rho_e \mathbf{u}_e}{\partial t} + \nabla (\rho_e \mathbf{u}_e \otimes \mathbf{u}_e) \rightarrow 0 \Rightarrow \nabla P_e = n_e \nabla \phi + \sum_{\alpha} \mathbf{R}_{e,\alpha}, \tag{11a}$$

$$P_e = n_e T_e \quad (T_e \text{ constant}), \tag{11b}$$

$$n_e = \sum_{\alpha} n_{\alpha} Z_{\alpha}, \quad \mathbf{u}_e = \frac{1}{n_e} \sum_{\alpha} Z_{\alpha} n_{\alpha} \mathbf{u}_{\alpha}. \tag{11c}$$

Equation (11a) and (11b) assume that the electrons are inertialess and isothermal, respectively. Equation (11c) assumes that the plasma is quasineutral with zero currents, which is a consequence of the Debye lengths being much smaller than the length scales of interest in our applications. Consequently, the electric field is given by

$$\nabla \phi = \frac{T_e}{n_e} \nabla n_e - \frac{1}{n_e} \sum_{\alpha} \mathbf{R}_{e,\alpha}. \tag{12}$$

Substituting (12) in (1) and adding $\nabla \cdot (Z_\alpha T_e n_\alpha)$ and $\nabla \cdot (Z_\alpha T_e n_\alpha \mathbf{u}_\alpha)$ on both sides of (1b) and (1c), respectively, the equations for the ion fluids are:

$$\frac{\partial \rho_\alpha}{\partial t} + \nabla \cdot (\rho_\alpha \mathbf{u}_\alpha) = 0, \quad (13a)$$

$$\begin{aligned} \frac{\partial \rho_\alpha \mathbf{u}_\alpha}{\partial t} + \nabla \cdot (\rho_\alpha \mathbf{u}_\alpha \otimes \mathbf{u}_\alpha + P_\alpha^*) &= \frac{Z_\alpha T_e}{n_e} [n_e \nabla n_\alpha - n_\alpha \nabla n_e] \\ &\quad + \frac{Z_\alpha n_\alpha}{n_e} \sum_{\beta} \mathbf{R}_{e,\beta} \\ &\quad + \mathbf{R}_{\alpha,e} + \sum_{\beta \neq \alpha} \mathbf{R}_{\alpha,\beta}, \end{aligned} \quad (13b)$$

$$\begin{aligned} \frac{\partial \mathcal{E}_\alpha}{\partial t} + \nabla \cdot \{(\mathcal{E}_\alpha + P_\alpha^*) \mathbf{u}_\alpha\} &= \frac{Z_\alpha T_e}{n_e} [n_e \nabla (\mathbf{u}_\alpha n_\alpha) - (\mathbf{u}_\alpha n_\alpha) \nabla n_e] \\ &\quad + \frac{Z_\alpha n_\alpha}{n_e} \sum_{\beta} \mathbf{u}_\alpha \cdot \mathbf{R}_{e,\beta} \\ &\quad + \sum_{\beta \neq \alpha} (\mathbf{R}_{\alpha,\beta} \cdot \mathbf{u}_\alpha + Q_{\alpha,\beta}) \\ &\quad + \mathbf{R}_{\alpha,e} \cdot \mathbf{u}_\alpha + Q_{\alpha,e}, \end{aligned} \quad (13c)$$

where

$$P_\alpha^* = P_\alpha + Z_\alpha T_e n_\alpha \quad (14)$$

is the augmented pressure, and $Z_\alpha T_e n_\alpha$ is the “electron pressure”. The main motivation behind including the electron pressure on the left-hand-side (LHS) of (13) is to allow its inclusion in the upwinding of the spatial discretization operator described in the next section. The electrostatic force acting through
115 this term is advective in nature, and therefore it should be included along with the hydrodynamic pressure P_α and the convective velocity \mathbf{u}_α in the calculation of the wavespeeds.

Thus, to summarize, (13) with $\alpha = 1, \dots, n_f$ represents the complete system of equations solved by EUCLID. We note that (13) reduces to the inviscid,
120 compressible Euler equations governing neutral gas dynamics [50] for $n_f = 1, Z_\alpha = 0$.

3. Numerical Method

Equation (13) constitutes a system of N hyperbolic PDEs with source terms that can be expressed as

$$\partial_t \mathbf{U} + \nabla \cdot \mathbf{F}(\mathbf{U}) = \mathbf{S}(\mathbf{U}), \quad (15)$$

where

$$\begin{aligned} \mathbf{U} &\equiv \left[\mathbf{U}_1, \dots, \mathbf{U}_\alpha, \dots, \mathbf{U}_{n_f} \right]^T, \\ \mathbf{F}(\mathbf{U}) &\equiv \left[\mathbf{F}_1(\mathbf{U}_1), \dots, \mathbf{F}_\alpha(\mathbf{U}_\alpha), \dots, \mathbf{F}_{n_f}(\mathbf{U}_{n_f}) \right]^T, \\ \mathbf{S}(\mathbf{U}) &\equiv \left[\mathbf{S}_1(\mathbf{U}), \dots, \mathbf{S}_\alpha(\mathbf{U}), \dots, \mathbf{S}_{n_f}(\mathbf{U}) \right]^T, \end{aligned}$$

are the state vector, flux tensor, and source vector for all the fluids with $N = 5n_f$ components (5 components for each set of three-dimensional fluid equations), and

$$\mathbf{U}_\alpha(\mathbf{U}_\alpha) = \left[\rho_\alpha, \rho_\alpha \mathbf{u}_\alpha, \mathcal{E}_\alpha \right]^T, \quad (16a)$$

$$\mathbf{F}_\alpha(\mathbf{U}_\alpha) = \left[\rho_\alpha \mathbf{u}_\alpha, \rho_\alpha \mathbf{u}_\alpha \mathbf{u}_\alpha + P_\alpha^*, (\mathcal{E}_\alpha + P_\alpha^*) \mathbf{u}_\alpha \right]^T \quad (16b)$$

are the state vector and flux tensor for fluid α , respectively, and

$$\mathbf{S}_\alpha(\mathbf{U}) = \mathbf{S}_\alpha^{\text{elec}}(\mathbf{U}) + \mathbf{S}_{\alpha,ii}^{\text{coll}}(\mathbf{U}) + \mathbf{S}_{\alpha,ei}^{\text{coll}}(\mathbf{U}),$$

is its source term comprising the electrostatic and collisional terms:

$$\mathbf{S}_\alpha^{\text{elec}}(\mathbf{U}) = \frac{Z_\alpha T_e}{n_e} \left[0, n_e \nabla n_\alpha - n_\alpha \nabla n_e, n_e \nabla \cdot (n_\alpha \mathbf{u}_\alpha) - n_\alpha \mathbf{u}_\alpha \cdot \nabla n_e \right]^T, \quad (17a)$$

$$\mathbf{S}_{\alpha,ii}^{\text{coll}}(\mathbf{U}) = \sum_{\beta \neq \alpha} m_\alpha n_\alpha \nu_{\alpha,\beta} \begin{bmatrix} 0 \\ (\mathbf{u}_\beta - \mathbf{u}_\alpha) \\ (\mathbf{u}_\beta - \mathbf{u}_\alpha) \cdot \mathbf{u}_\alpha + \frac{m_{\alpha,\beta}(\mathbf{u}_\beta - \mathbf{u}_\alpha)^2}{m_\alpha} + \frac{3(T_\beta - T_\alpha)}{m_\alpha + m_\beta} \end{bmatrix}, \quad (17b)$$

$$\mathbf{S}_{\alpha,ei}^{\text{coll}}(\mathbf{U}) = m_e n_e \begin{bmatrix} 0 \\ \nu_{e,\alpha} (\mathbf{u}_e - \mathbf{u}_\alpha) + \frac{Z_\alpha n_\alpha}{n_e} \sum_\beta \nu_{e,\beta} (\mathbf{u}_\beta - \mathbf{u}_e) \\ \left\{ \nu_{e,\alpha} (\mathbf{u}_e - \mathbf{u}_\alpha) + \frac{Z_\alpha n_\alpha}{n_e} \sum_\beta \nu_{e,\beta} (\mathbf{u}_\beta - \mathbf{u}_e) \right\} \cdot \mathbf{u}_\alpha + \frac{3\nu_{e,\alpha}(T_e - T_\alpha)}{m_\alpha + m_e} \end{bmatrix}. \quad (17c)$$

We note that the flux tensor of fluid α depends only on the state vector of fluid α , while the source term for each fluid is a function of the other fluids as well since it contains the inter-fluid interactions. In subsequent discussions, we refer to the following equation:

$$\partial_t \mathbf{U} + \nabla \cdot \mathbf{F}(\mathbf{U}) = \mathbf{S}^{\text{elec}}(\mathbf{U}), \quad \mathbf{S}^{\text{elec}}(\mathbf{U}) \equiv [\mathbf{S}_\alpha^{\text{elec}}(\mathbf{U}) \mid \alpha = 1, \dots, n_f]^T \quad (18)$$

as the “collisionless” multifluid equations, where the collisional source terms have been excluded.

Spatial Discretization. Equation (15) is discretized on a three-dimensional Cartesian grid, with $\mathbf{i} \equiv \{i, j, k\}$ as the grid index, and the conservative finite-difference formulation [45, 46] can be written at a grid point as:

$$\frac{d\mathbf{U}_i}{dt} = \mathbf{L}(\mathbf{U}_i) \equiv - \sum_{d=1}^3 \left(\frac{\hat{\mathbf{f}}_{i+\frac{1}{2}\mathbf{e}_d}^{(d)} - \hat{\mathbf{f}}_{i-\frac{1}{2}\mathbf{e}_d}^{(d)}}{\Delta x} \right) + \mathbf{S}_i, \quad (19)$$

where \mathbf{U}_i and $\mathbf{S}_i \equiv \mathbf{S}(\mathbf{U}_i)$ are the cell-centered values of the state and source vectors, d is the dimension index, and \mathbf{e}_d is the unit vector along d (its d -th component is 1 and the other components are 0). The grid spacing, Δx , is taken as uniform in all dimensions in our implementation. The high-order approximation to the flux primitive, $\hat{\mathbf{f}}^{(d)}$, is

$$\hat{\mathbf{f}}_{i+\frac{1}{2}\mathbf{e}_d}^{(d)} = \mathcal{S} \left(\left\{ \mathbf{f}_{i+k\mathbf{e}_d}^{(d)} \mid -w_L \leq k \leq w_R \right\} \right) = \mathbf{h}_{i+\frac{1}{2}\mathbf{e}_d}^{(d)} + \mathcal{O}(\Delta x^p), \quad (20)$$

where p is the order of the discretization scheme, \mathcal{S} is the finite-difference reconstruction operator, w_L and w_R define its stencil width, and $\mathbf{h}^{(d)}$ is the flux primitive satisfying:

$$\mathbf{f}^{(d)}(\mathbf{x}) \equiv \mathbf{f}^{(d)}(\mathbf{U}(\mathbf{x})) = \frac{1}{\Delta x} \int_{x_d - \frac{\Delta x}{2}}^{x_d + \frac{\Delta x}{2}} \mathbf{h}^{(d)}(\boldsymbol{\xi}) d\boldsymbol{\xi} \quad (21)$$

and $\mathbf{f}^{(d)}(\mathbf{U}) \equiv \mathbf{F}(\mathbf{U}) \cdot \mathbf{e}_d$ is the d -th component of the flux tensor. The numerical flux approximation $\hat{\mathbf{f}}_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{(d)}$ is computed at the cell interfaces $\mathbf{i} \pm \frac{1}{2}\mathbf{e}_d$ independently along each dimension; the following discussion describes this for one spatial dimension.

The vector flux approximation is reconstructed at the cell interface using a characteristic-based approach [45, 46, 48], where it is computed as:

$$\hat{\mathbf{f}}_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d} = \left[R^{(d)} \left(\mathbf{U}_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{\text{Roe}} \right) \right] \boldsymbol{\varphi}_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}, \quad (22)$$

where $R^{(d)}$ is the matrix whose columns are the right-eigenvectors of the flux Jacobian evaluated at the Roe-averaged state $\mathbf{U}_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{\text{Roe}}$ [50]. The Roe-fixed scheme [48] is used to compute the components of the characteristic flux vector, $\boldsymbol{\varphi}_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}$, at the cell interfaces:

$$\boldsymbol{\varphi}_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{(k)} = \begin{cases} \varphi_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{(k),\text{L}} & \text{if } \lambda_{k,\mathbf{i}}^{(d)}, \lambda_{k,\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{(d)}, \lambda_{k,\mathbf{i} + \mathbf{e}_d}^{(d)} > 0 \\ \varphi_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{(k),\text{R}} & \text{if } \lambda_{k,\mathbf{i}}^{(d)}, \lambda_{k,\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{(d)}, \lambda_{k,\mathbf{i} + \mathbf{e}_d}^{(d)} < 0 \\ \frac{1}{2} \left[\varphi_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{(k),\text{L}} + \varphi_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{(k),\text{R}} - \lambda_{k,\max}^{(d)} \left(\varrho_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{(k),\text{R}} - \varrho_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{(k),\text{L}} \right) \right] & \text{otherwise} \end{cases}, \quad (23)$$

$$\boldsymbol{\varphi}_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d} = \left[\varphi_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{(k)} \mid k = 1, \dots, N \right],$$

where $\lambda_{k,\{\mathbf{i}, \mathbf{i} + \mathbf{e}_d\}}^{(d)}$ are the k -th eigenvalue (characteristic wavespeed) along dimension d evaluated at the cell-centered solution $\mathbf{U}_{\{\mathbf{i}, \mathbf{i} + \mathbf{e}_d\}}$, $\lambda_{k,\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{(d)}$ is the k -th eigenvalue evaluated at the Roe-averaged state $\mathbf{U}_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{\text{Roe}}$. The left- and right-biased characteristic flux and solution components at the cell interfaces are computed as:

$$\varphi_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{(k),\text{L}} = \text{MPWENO5} \left(\varphi_{\mathbf{i} - 2\mathbf{e}_d}^{(k)}, \varphi_{\mathbf{i} - \mathbf{e}_d}^{(k)}, \varphi_{\mathbf{i}}^{(k)}, \varphi_{\mathbf{i} + \mathbf{e}_d}^{(k)}, \varphi_{\mathbf{i} + 2\mathbf{e}_d}^{(k)} \right), \quad (24a)$$

$$\varphi_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{(k),\text{R}} = \text{MPWENO5} \left(\varphi_{\mathbf{i} + 3\mathbf{e}_d}^{(k)}, \varphi_{\mathbf{i} + 2\mathbf{e}_d}^{(k)}, \varphi_{\mathbf{i} + \mathbf{e}_d}^{(k)}, \varphi_{\mathbf{i}}^{(k)}, \varphi_{\mathbf{i} - \mathbf{e}_d}^{(k)} \right), \quad (24b)$$

$$\varrho_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{(k),\text{L}} = \text{MPWENO5} \left(\varrho_{\mathbf{i} - 2\mathbf{e}_d}^{(k)}, \varrho_{\mathbf{i} - \mathbf{e}_d}^{(k)}, \varrho_{\mathbf{i}}^{(k)}, \varrho_{\mathbf{i} + \mathbf{e}_d}^{(k)}, \varrho_{\mathbf{i} + 2\mathbf{e}_d}^{(k)} \right), \quad (24c)$$

$$\varrho_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{(k),\text{R}} = \text{MPWENO5} \left(\varrho_{\mathbf{i} + 3\mathbf{e}_d}^{(k)}, \varrho_{\mathbf{i} + 2\mathbf{e}_d}^{(k)}, \varrho_{\mathbf{i} + \mathbf{e}_d}^{(k)}, \varrho_{\mathbf{i}}^{(k)}, \varrho_{\mathbf{i} - \mathbf{e}_d}^{(k)} \right), \quad (24d)$$

where $\varphi_{(\cdot)}^{(k)}, \varrho_{(\cdot)}^{(k)}$ represent the k -th component of the cell-centered characteristic flux and solution vectors $\boldsymbol{\varphi}_{(\cdot)}, \boldsymbol{\varrho}_{(\cdot)}$, respectively, and MPWENO5(\cdot) is the 5th-

order monotonicity-preserving WENO scheme [48, 49] summarized in Appendix
140 A. The cell-centered characteristic flux and solution vectors are calculated as

$$\varphi_{(\cdot)} = \left[L^{(d)} \left(\mathbf{U}_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{\text{Roe}} \right) \right] \mathbf{f}_{(\cdot)}, \quad \boldsymbol{\varrho}_{(\cdot)} = \left[L^{(d)} \left(\mathbf{U}_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{\text{Roe}} \right) \right] \mathbf{U}_{(\cdot)}, \quad (25)$$

where $L^{(d)} \left(\mathbf{U}_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{\text{Roe}} \right)$ is the matrix whose rows are the left-eigenvectors of the flux Jacobian evaluated at the Roe-averaged state $\mathbf{U}_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{\text{Roe}}$.

In the above method, the Roe-averaged state is computed independently for each fluid at the cell interfaces, i.e., the state vectors of each fluid \mathbf{U}_α is extracted
145 from \mathbf{U} , and the Roe-averaged state is calculated. The complete eigenstructure
 $(\lambda^{(d)}, L^{(d)}, R^{(d)}; d \in \{x, y, z\})$ for the multifluid system (15) required by the
characteristic-based procedure outlined above is provided in Appendix B. We
reiterate here that the contribution of the electron pressure, included on the LHS
of (13), is incorporated in to the derivation of the characteristic wavespeeds, and
consequently, the upwind discretization. Thus, to summarize, the following are
150 the steps in the discretization of the hyperbolic flux term in (15) along each
dimension d :

1. At each cell interface $\mathbf{i} + \frac{1}{2}\mathbf{e}_d$, compute the Roe-averaged state $\mathbf{U}_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{\text{Roe}}$ [50],
its eigenvalues $\lambda^{(d)} \left(\mathbf{U}_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{\text{Roe}} \right)$, and its left and right eigenvector matrices
155 $L^{(d)} \left(\mathbf{U}_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{\text{Roe}} \right), R^{(d)} \left(\mathbf{U}_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{\text{Roe}} \right)$ using the expressions given in Appendix
B.
 2. Compute the characteristic flux and solution vectors $\varphi_{(\cdot)}, \boldsymbol{\varrho}_{(\cdot)}$ at all the grid points needed by the MPWENO5 operator as (25).
 3. Reconstruct the left- and right-biased characteristic flux and solution vectors at the cell interface $\mathbf{i} + \frac{1}{2}\mathbf{e}_d$ as given by (24) using the MPWENO5
160 scheme in Appendix A.
 4. Apply the Roe-fixed upwinding (23) procedure to compute the upwind characteristic flux vector at the interface $\varphi_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}$, and then compute the conserved flux vector using (22).
- 165 The source term in (15) comprises the electrostatic and collisional source terms, as given in (17). While the collisional source term does not contain any derivatives, the electrostatic source term (17a) contains gradient and divergence terms.

These are computed using the fourth-order central difference operator along each dimension, which can be expressed for an arbitrary grid variable ψ as:

$$\frac{\partial \psi}{\partial x^{(d)}} \Big|_{\mathbf{i}} = \frac{1}{12\Delta x} (\psi_{\mathbf{i}-2\mathbf{e}_d} - 8\psi_{\mathbf{i}-\mathbf{e}_d} + 8\psi_{\mathbf{i}+\mathbf{e}_d} - \psi_{\mathbf{i}+2\mathbf{e}_d}) + \mathcal{O}(\Delta x^4). \quad (26)$$

¹⁷⁰ This differencing is applied component-wise for vector quantities.

Time Integration. Equation (19) is a ordinary differential equation (ODE) in time, and it is integrated using the fourth-order, four-stage, explicit Runge-Kutta method. Since the time integration method is explicit, the time step Δt is restricted by the CFL criterion, where the CFL number is defined as

$$\sigma = \Delta t \frac{\max_{\Omega, \alpha} (|\mathbf{u}_\alpha| + c_\alpha^*)}{\Delta x}, \quad (27)$$

¹⁷⁵ where Ω is the total domain, and α is the fluid index. Linear stability typically requires $\sigma < 1$, although this is specific to the spatial discretization method, and it is not a sufficient condition for stability for nonlinear simulations. Equation (27) does not take into account the time scales of the collisional source terms, (17b) and (17c); consequently, a lower time step that resolves the collisional time scales is needed, and this is reported in the simulations presented in ¹⁸⁰ subsequent sections.

4. Verification

EUCLID is verified through several benchmark test problems. Since (13) reduces to the inviscid Euler equations for gasdynamics for $Z_\alpha = 0, n_f = 1$, ¹⁸⁵ standard compressible flow test cases are solved and the results verified. Since few benchmark problems exist for our specific applications, we verify our code for charged fluids using the following approach. An analytical, easily-differentiable form is assumed for the solution \mathbf{U} , and (19) is modified as

$$\frac{d\mathbf{U}_i}{dt} = \mathbf{L}(\mathbf{U}_i) - \mathbf{L}_{ex}(\mathbf{U}_i), \quad (28)$$

where $\mathbf{L}_{ex}(\mathbf{U}_i)$ is the exact form of $\mathbf{L}(\mathbf{U}_i)$ obtained by analytically evaluating ¹⁹⁰ its constituent terms. We then verify that the numerical error in computing

$\mathbf{L}(\mathbf{U})$, quantified as

$$\varepsilon = \|\mathbf{L}(\mathbf{U}) - \mathbf{L}_{\text{ex}}(\mathbf{U})\|, \quad (29)$$

converges to zero at the theoretical order of the spatial discretization scheme, where $\|(\cdot)\|$ is an appropriate norm over the entire grid. In all the cases described in this section, we take the discrete 2-norm $\|(\cdot)\|_2$. The analytical 195 solutions used here are the ones that could be derived by Mathematica [51] in a reasonable time while testing as many terms in the governing equations as possible. EUCLID is implemented for the three-dimensional equations; lower-dimensional flows are simulated by specifying a small number of grid points (the minimum required by the finite-difference stencil) and periodic boundary 200 conditions along the inactive dimension(s). These tests are repeated along each dimension to verify the code; for example, a two-dimensional test is repeated in the $x-y$, $y-z$, and $x-z$ planes.

4.1. Collisionless Multifluid Equations

The collisionless multifluid equations are given by (18), and the verification 205 tests in this section demonstrate the accuracy, convergence, and nonoscillatory properties of our algorithm.

Accuracy and Convergence. Our implementation is verified for a single charged fluid ($n_f = 1$) by generating a two-dimensional solution that results in all terms in (18) being nonzero (except those along the third dimension). The governing equations, (13), contain terms that are either uni-dimensional (for example, ρu , ρv , ρu^2 , etc.) or bi-dimensional (for example, ρuv , ρuw , ρvw , etc.), and therefore, a two-dimensional test applied in all three planes ($x-y$, $y-z$, and $x-z$) is sufficient. Since the number of fluids is 1, the fluid index subscript is omitted.

The species is set to helium ($m = 4, Z = 2$), and the solution is prescribed as

$$\rho(x, y) = 1 + \frac{1}{5} \cos(2\pi x) \cos(2\pi y), \quad (30a)$$

$$u(x, y) = \frac{1}{2} \cos(2\pi x) \cos(4\pi y), \quad (30b)$$

$$v(x, y) = \frac{1}{4} \cos(4\pi x) \cos(2\pi y), \quad (30c)$$

$$w(x, y) = 0, \quad (30d)$$

$$P(x, y) = 2 + \frac{1}{4} \cos(2\pi x) \cos(2\pi y). \quad (30e)$$

The electron temperature T_e is 1, and the specific heat ratio is taken as $\gamma = 5/3$. The analytical term $\mathbf{L}_{\text{ex}}(\mathbf{U}_i)$ is obtained using Mathematica. A periodic, two-dimensional domain with unit length along x and y is considered. Figure 2(a)
²¹⁰ shows the error as a function of the grid spacing Δx for the number of grid points varying from 16×16 to 128×128 ; the number of grid points is the same along x and y for all the cases. The order convergence is observed to be greater than 4; the solutions are almost 5th order, except between the two finest grids (64 \times 64 and 128 \times 128). Examining the electrostatic source term, (17a), we
²¹⁵ note that $n_e \equiv \sum Z_\alpha n_\alpha = Zn$ for $n_f = 1$, and thus it reduces to

$$\mathbf{S}^{\text{elec}}(\mathbf{U}) = \begin{bmatrix} 0, & \mathbf{0}, & ZT_e n \nabla \cdot \mathbf{u} \end{bmatrix}^T. \quad (31)$$

Thus, the error is dominated by the flux-divergence term $\nabla \cdot \mathbf{F}$ in (18), which is discretized using the 5th-order MPWENO scheme described in the previous section.

We now consider a two-fluid case that verifies the implementation of terms
²²⁰ that involve multiple fluids. Smooth, two-dimensional equilibria for the two-fluid equations ($n_f = 2$) are derived as follows. The two fluids are two distinct species, and the initial number densities are specified functions while the initial velocities are zero for both the fluids:

$$n_{1,2}(x, y, t = 0) \equiv n_{1,2}^{(0)}(x, y), \quad \mathbf{u}_{1,2}^{(0)} = 0, \quad (32)$$

The pressures $P_{1,2}(x, y, t = 0) \equiv P_{1,2}^{(0)}(x, y)$ are derived using Mathematica such

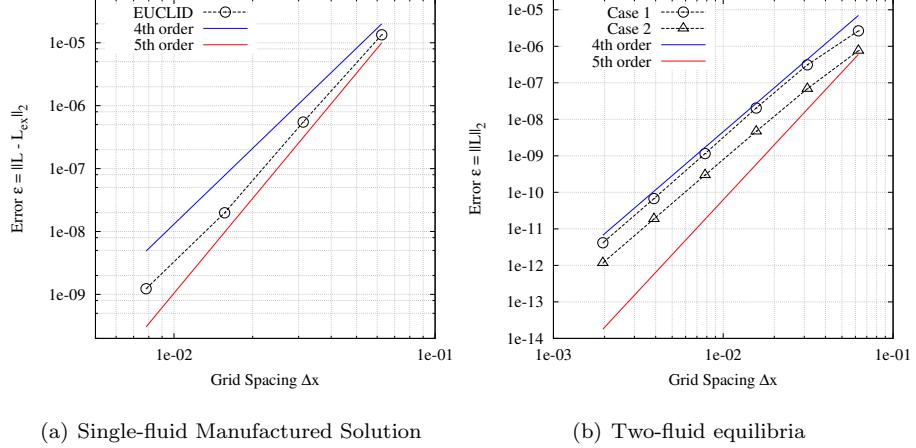


Figure 2: Error vs. grid spacing for (a) a two-dimensional solution with a single fluid, and (b) two two-dimensional 2-fluid equilibria. The order of convergence is at least 4 in both cases; the order is closer to 5 for the single-fluid case because the electrostatic source terms, which are discretized using a 4th order discretization, are very small for this case, and the flux divergence term, discretized by the 5th-order MPWENO scheme, is dominant.

that they satisfy the equilibrium conditions:

$$\begin{bmatrix} \partial_x \left(P_\alpha^{(0)} + Z_\alpha T_e n_\alpha^{(0)} \right) \\ \partial_y \left(P_\alpha^{(0)} + Z_\alpha T_e n_\alpha^{(0)} \right) \end{bmatrix} = \frac{Z_\alpha T_e}{n_e^{(0)}} \begin{bmatrix} n_e^{(0)} \partial_x n_\alpha^{(0)} - n_\alpha^{(0)} \partial_x n_e^{(0)} \\ n_e^{(0)} \partial_y n_\alpha^{(0)} - n_\alpha^{(0)} \partial_y n_e^{(0)} \end{bmatrix}, \quad \alpha = 1, 2, \quad (33)$$

where $n_e^{(0)} = Z_1 n_1^{(0)} + Z_2 n_2^{(0)}$. Derivation of the pressure functions involves solving the PDE given by (33), and the constants of integration are taken such that the resulting functions have no nonpositive values. We consider two cases, where the species are helium ($Z_1 = 2, m_1 = 4$) and carbon ($Z_2 = 6, m_2 = 12$), the domain is $x, y \in [0, 1]$ with periodic boundary conditions, and the number densities are

$$n_1^{(0)}(x, y) = \begin{cases} 1 + \cos(2\pi x) \cos(4\pi y) & \text{Case 1} \\ 1 & \text{Case 2} \end{cases}, \quad (34a)$$

$$n_2^{(0)}(x, y) = \begin{cases} \frac{1}{2} & \text{Case 1} \\ \frac{1}{2} + \frac{1}{4} \cos(4\pi x) \cos(2\pi y) & \text{Case 2} \end{cases}. \quad (34b)$$

We were unable to obtain real-valued pressure functions for a case where the number densities for both the fluids varied spatially, and therefore, these two cases, where one of the number densities is constant, are necessary to verify our implementation. The electron temperature T_e and specific heat ratios for both fluids are 1 and 5/3, respectively. The pressure functions that satisfy (33) are

$$P_1^{(0)}(x, y) = \begin{cases} -2 \cos(2\pi x) \cos(4\pi y) + 3 \log[5 + 2 \cos(2\pi x) \cos(4\pi y)] & \text{Case 1} \\ -2[-5 + \log\{10 + 3 \cos(4\pi x) \cos(2\pi y)\}] & \text{Case 2} \end{cases}, \quad (35a)$$

$$P_2^{(0)}(x, y) = \begin{cases} 10 - 3 \log[5 + 2 \cos(2\pi x) \cos(4\pi y)] & \text{Case 1} \\ \frac{1}{2}[-3 \cos(4\pi x) \cos(2\pi y)] + 2 \log[10 + 3 \cos(4\pi x) \cos(2\pi y)] & \text{Case 2} \end{cases}. \quad (35b)$$

Since these are equilibrium cases, $\mathbf{L}_{\text{ex}}(\mathbf{U}_i) \equiv \mathbf{0}$, and the error can be measured by advancing (19) by one time step with the forward Euler scheme:

$$\varepsilon = \|\mathbf{L}(\mathbf{U})\|_2 = \frac{1}{\Delta t} \|\mathbf{U}^{n+1} - \mathbf{U}^n\|_2. \quad (36)$$

Figure 2(b) shows the error as a function of the grid spacing Δx , starting from a grid with 16×16 points to a grid with 256×256 points. Since the electrostatic source term (17a) has the same magnitude as the flux divergence term $\nabla \cdot \mathbf{F}$, 4th-order convergence is observed.

The two tests discussed above are sufficient to verify our discretization of the multifluid equations (13), except the collisional terms, and demonstrate the high-order convergence of the spatial discretization schemes. The single-fluid test verifies the discretization of the flux term, while the two-fluid cases verify the discretization of the electrostatic source term. These tests are repeated in the $y-z$ and $x-z$ planes to complete the verification of all non-collisional terms in (13).

Nonoscillatory Behavior. The Sod shock tube test [50, 52] is a one-dimensional Riemann problem that is used to test the nonoscillatory behavior of compressible gasdynamics codes. The initial solution is a discontinuity that has a high-density, high-pressure fluid on one side and a low-density, low-pressure fluid on

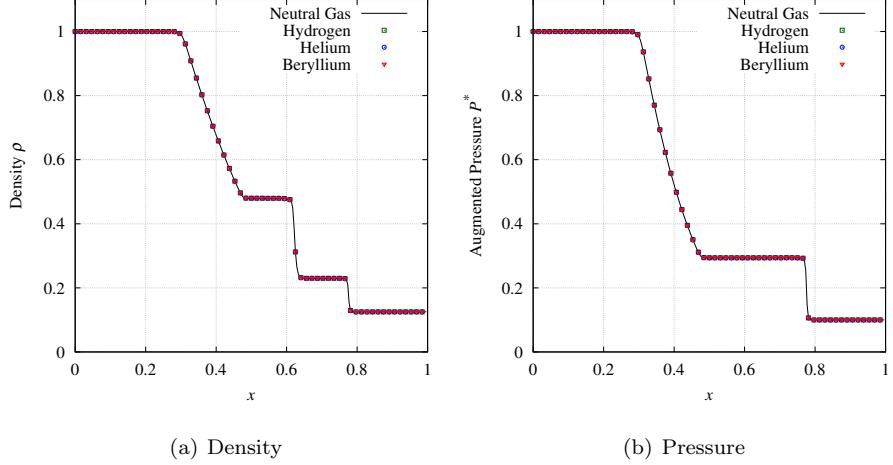


Figure 3: Solution of the electrostatic shock tube problem: The solutions for the charged species (hydrogen, helium, and beryllium) agree well with the neutral gas solution.

the other side. As the simulation progresses, the initial discontinuity decomposes in to a rarefaction wave, a contact discontinuity, and a shock wave. While the smooth problems discussed above test the accuracy and convergence properties of the spatial discretization, the Riemann problem tests the ability of an upwind discretization scheme to resolve the characteristic wavespeeds correctly and yield nonoscillatory solutions. In this section, we consider an electrostatic shock tube problem by noting that the LHS of (13) is identical to the compressible Euler equations for neutral gases [50], if the hydrodynamic pressure in the latter is replaced by the augmented pressure (sum of the hydrodynamic pressure and electron pressure). Equation (18) is thus solved with a zero RHS on a one-dimensional domain $x \in [0, 1]$ with inviscid wall boundary conditions at both ends. The number of fluids is $n_f = 1$, and the initial solution is specified as

$$\{\rho, P^*\} = \begin{cases} \{1, 1\} & x \leq 0.5 \\ \{0.125, 0.1\} & x > 0.5 \end{cases}, \quad u = 0, \quad P = P^* - \frac{Z T_e}{m} \rho. \quad (37)$$

255 We consider four cases, each with a different species:

$$\begin{aligned}
 \text{Neutral gas} & \quad Z = 0, m \in \mathbb{R}^+ \\
 \text{Hydrogen} & \quad Z = 1, m = 1 \\
 \text{Helium} & \quad Z = 2, m = 4 \\
 \text{Beryllium} & \quad Z = 4, m = 9
 \end{aligned} \quad , \quad (38)$$

260 and the electron temperature T_e and the specific heat ratio γ are set to 0.01 and $5/3$, respectively. Figure 3 shows the density and pressure at $t = 0.15$ for solutions obtained on a grid with 256 points. The CFL number for all the simulations are set to $\sigma = 0.9$. The solutions obtained with the charged species (hydrogen, helium, and beryllium) agree well with the neutral gas solution, as expected. This case demonstrates that the characteristic-based MPWENO5 spatial discretization described in the previous section is able to yield nonoscillatory solutions for flows with strong discontinuities.

4.2. Collisional Terms

We now verify our implementation of the collisional source terms by comparing them with previous results in the literature. Since these terms do not involve any spatial gradients, “zero-dimensional” cases with uniform flow variables are sufficient for their verification. We consider two setups that are identical to those solved by Rambo & Procassini [44]: the thermal equilibration of two fluids at initially different temperatures, and the slowing down of a fluid stream due to the collisional interactions with a background fluid. The results presented in Rambo & Procassini are in terms of the physical variables (with dimensions), and therefore, the discussion and figures in this section involve these variables (obtained by multiplying the nondimensional variables solved by EUCLID by the corresponding reference quantity). The primary reference quantities for the cases reported here are

$$\begin{aligned}
 m_{\text{ref}} &= 1.67 \times 10^{-24} \text{ g} \quad (\text{proton mass}), \quad n_{\text{ref}} = 1 \times 10^{20} \text{ cm}^{-3}, \\
 x_{\text{ref}} &= 0.1 \text{ cm} \quad (1 \text{ mm}), \quad T_{\text{ref}} = 1.602 \times 10^{-9} \text{ ergs} \quad (1 \text{ keV}), \\
 e &= 4.803 \times 10^{-10} \text{ statcoulomb} \quad (\text{elementary charge}),
 \end{aligned}$$

and, therefore, the derived reference quantities are obtained from (10) are:

$$u_{\text{ref}} = 3.095 \times 10^7 \text{ cm/s}, \quad \rho_{\text{ref}} = 1.67 \times 10^{-4} \text{ g/cm}^{-3}, \\ t_{\text{ref}} = 3.23 \times 10^{-9} \text{ s}, \quad P_{\text{ref}} = 1.602 \times 10^{11} \text{ Ba.}$$

²⁶⁵ The specific heat ratio for all fluids is specified as $\gamma_\alpha = 5/3$.

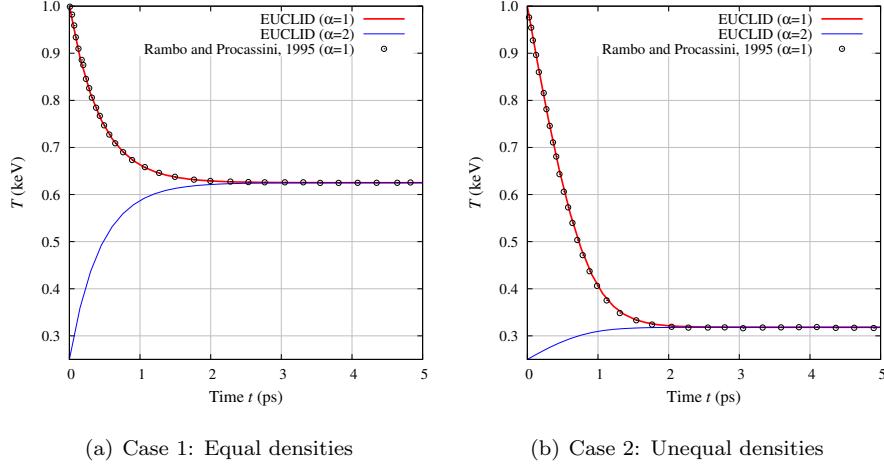


Figure 4: Thermal equilibration of two fluids at initially different temperatures. An excellent agreement is observed with results in the literature [44].

The first test considers two carbon fluids ($n_f = 2$, $Z_{\{1,2\}} = 6$, and $m_{\{1,2\}} = 12$), and two cases with the following initial solutions (in terms of the nondimensional variables):

$$\begin{aligned} \text{Case 1 : } & n_1 = n_2 = 1, & u_1 = u_2 = 0, & T_1 = 1, & T_2 = 0.25 \\ \text{Case 2 : } & n_1 = 0.1, & n_2 = 1, & u_1 = u_2 = 0, & T_1 = 1, & T_2 = 0.25 \end{aligned} . \quad (39)$$

The domain is $x = [0, 1]$ with periodic boundary conditions, and it is discretized by 8 grid points. The final time for the simulation is specified as $t_f = 0.00155$, which corresponds to a physical time of ~ 5 ps. A CFL of $\sigma = 0.0005$ is specified to resolve the equilibration time scale. Figure 4 shows the evolution of the temperatures of both the fluids: in both cases, the two fluids equilibrate to the density-weighted mean of the initial temperatures $(n_1 T_1 + n_2 T_2) / (n_1 + n_2)$. An excellent agreement is observed with prior results [44].

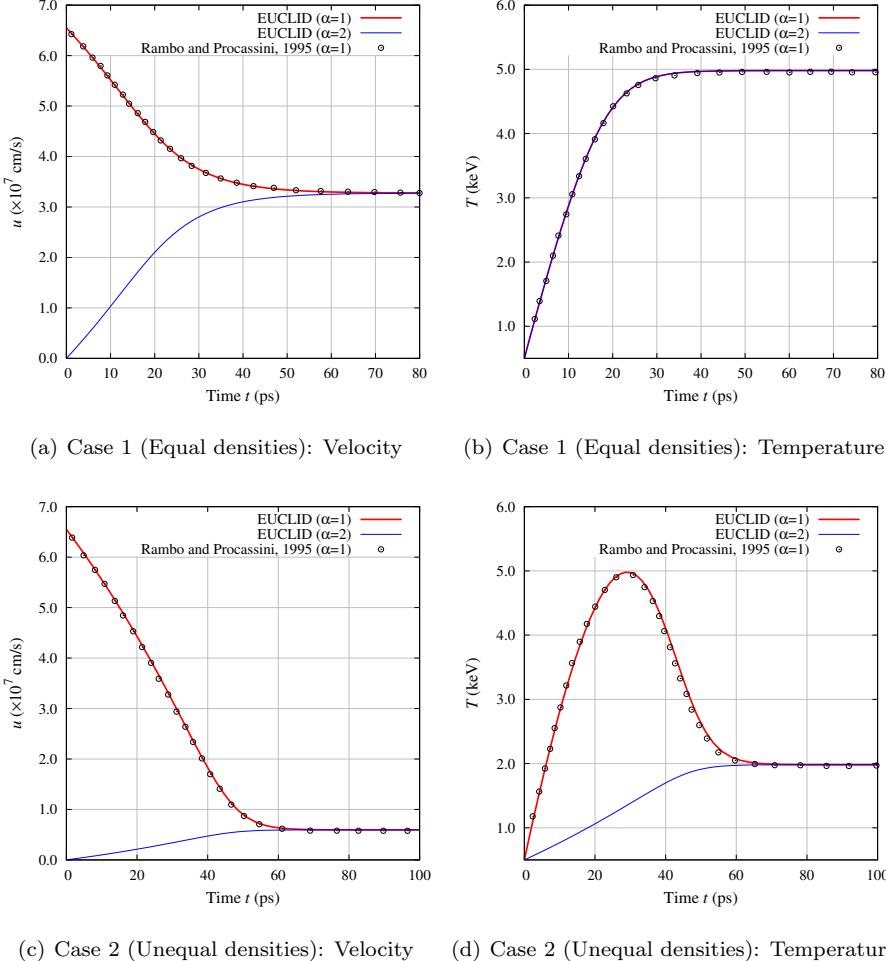


Figure 5: Frictional slowing of one fluid with respect to a background fluid with an initially nonzero relative velocity. An excellent agreement is observed with results in the literature [44].

The second test considers two carbon fluids as well, and two cases with the following initial solutions (in terms of the nondimensional variables) are solved:

$$\begin{aligned} \text{Case 1 : } & n_1 = n_2 = 1, \quad u_1 = 2.1166, \quad u_2 = 0, \quad T_1 = T_2 = 0.5 \\ \text{Case 2 : } & n_1 = 0.1, \quad n_2 = 1, \quad u_1 = 2.1166, \quad u_2 = 0, \quad T_1 = T_2 = 0.5 \end{aligned} . \quad (40)$$

The nondimensional u specified above corresponds to a physical velocity of $6.55 \times 10^7 \text{ cm/s}$. The domain is $x = [0, 1]$ with periodic boundary conditions,

275 and it is discretized by 8 grid points. The final time is specified as $t_f = 0.031$,
 which corresponds to a physical time of ~ 100 ps. A CFL of $\sigma = 0.01$ is specified
 to resolve the frictional time scale. Figures 5(a) and 5(b) show the evolution
 of the velocities and temperatures of both the fluids for Case 1. The first fluid
 $\alpha = 1$ slows down and accelerates the second fluid $\alpha = 2$ due to the inter-
 280 fluid friction; the velocities for both the fluids converge to the mean value. The
 frictional forces also result in heating both the fluids, as is observed by the
 increase in the temperatures by a factor of ~ 10 . Figures 5(c) and 5(d) shows
 the evolution of the velocities and temperatures for Case 2, where the density
 of the first fluid is 10 times lower. The velocities of both the fluids converge to
 285 the density-weighted mean. The evolution of the temperatures show the rapid
 heating of the first (lower density) fluid due to the friction, followed by cooling
 due to thermal equilibration, while the second (higher density) fluid experiences
 a more gradual heating due to friction and thermal equilibration. In both these
 cases, our results agree well with those previously published [44].

290 **5. Results**

In this section, we use EUCLID to simulate multifluid flows that are representative of laser-induced plasma experiments. We consider the interactions of counterstreaming plasmas in vacuum and in the presence of a gas fill. Both one-dimensional and two-dimensional cases are simulated. The simulations reported in these section use the following primary reference quantities that correspond to the physical setup of HEDP experiments [4, 6, 7, 53]:

$$\begin{aligned}
 m_{\text{ref}} &= 1.67 \times 10^{-24} \text{ g } (\text{proton mass}), & n_{\text{ref}} &= 9.03 \times 10^{21} \text{ cm}^{-3} \ (n_{\text{crit}}), \\
 x_{\text{ref}} &= 0.1 \text{ cm } (1 \text{ mm}), & T_{\text{ref}} &= 1.602 \times 10^{-9} \text{ ergs } (1 \text{ keV}), \\
 e &= 4.803 \times 10^{-10} \text{ statcoulomb } (\text{elementary charge}),
 \end{aligned}$$

where n_{crit} is the critical plasma density above which the fluid becomes opaque to electromagnetic radiation [54]. The derived reference quantities are obtained

from (10), and they are:

$$u_{\text{ref}} = 3.095 \times 10^7 \text{ cm/s}, \quad \rho_{\text{ref}} = 1.51 \times 10^{-2} \text{ g/cm}^{-3}, \\ t_{\text{ref}} = 3.23 \times 10^{-9} \text{ s}, \quad P_{\text{ref}} = 1.447 \times 10^{13} \text{ Ba.}$$

The subsequent discussions in this section describe the nondimensional variables that are solved for by EUCLID; they can be multiplied by these reference quantities to obtain their physical values with units. The specific heat ratio for all species in the simulations presented in this section is specified as $\gamma_\alpha = 5/3$.

²⁹⁵ The initial solution in the cases discussed here involve fluids that exist with a constant density only within a part of the domain. This is implemented using a “smoothed slab” function that is defined in one spatial dimension as

$$\Upsilon(x; \delta_x, x_{\min}, x_{\max}) = e^{\frac{x}{\delta_x}} \left[\left(e^{\frac{x_{\min}}{\delta_x}} + e^{\frac{x}{\delta_x}} \right)^{-1} - \left(e^{\frac{x_{\max}}{\delta_x}} + e^{\frac{x}{\delta_x}} \right)^{-1} \right]. \quad (41)$$

³⁰⁰ This function attains unit value inside (x_{\min}, x_{\max}) , smoothly transitions to 0 with a width δ_x , and is zero outside $[x_{\min}, x_{\max}]$. The transition width δ_x is specified as a small number; however we must caution the reader that very low values for δ_x results in numerical overflow/underflow while evaluating $\Upsilon(x)$.

The first example we consider is the one-dimensional interaction of two counterstreaming plasmas. The domain is $x \in [0, 1]$, and we consider two fluids $n_f = 2$. The initial solution is as follows:

$$n_\alpha(x, t=0) = n_{\text{vac}} + \tilde{n}_\alpha \Upsilon(x; \delta_{x,\alpha}, x_{\min,\alpha}, x_{\max,\alpha}), \quad \alpha = 1, 2, \quad (42) \\ T_\alpha(x, t=0) = 1, \quad u_\alpha(x, 0) = 0$$

where

$$x_{\min,1} = -1, \quad x_{\max,1} = 0.2, \quad x_{\min,2} = 0.8, \quad x_{\max,2} = 2, \quad \delta_{x,\{1,2\}} = 0.005,$$

and $n_{\text{vac}} = 10^{-14}$ is the numerical “vacuum”. This represents a setup where fluid $\alpha = 1$ is a localized slab in $x \in [0, 0.2]$, and fluid $\alpha = 2$ is a localized slab in $x \in [0.8, 1]$ with vacuum in between (see the top left plots in Figures 6 and 7). The electron temperature is $T_e = 0.5$, and we consider two symmetric cases (hydrogen–hydrogen and carbon–carbon interactions):

Case 1 (hydrogen – hydrogen) : $m_{\{1,2\}} = 1, Z_{\{1,2\}} = 2, \tilde{n}_{\{1,2\}} = 0.5$

Case 2 (carbon – carbon) : $m_{\{1,2\}} = 12, Z_{\{1,2\}} = 6, \tilde{n}_{\{1,2\}} = 0.1$

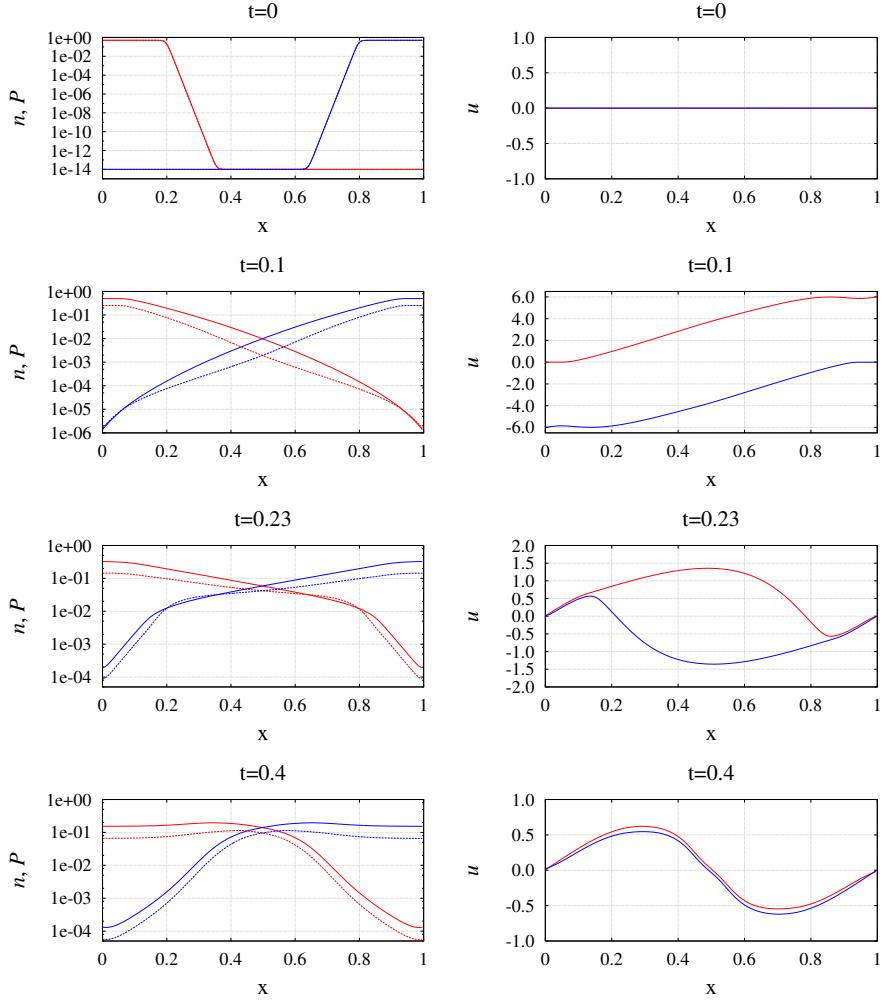


Figure 6: Interpenetration of two hydrogen streams: The left column shows the number density n_α (solid lines) and the pressure P_α (dashed line), and the right column shows the velocity u_α . The red lines correspond to $\alpha = 1$, and the blue lines correspond to $\alpha = 2$.

The boundary conditions are as follows: An inviscid wall boundary is imposed at $x = 0$, and outflow is imposed at $x = 1$ for fluid $\alpha = 1$, while outflow is imposed at $x = 0$, and inviscid wall boundary is imposed at $x = 1$ for fluid $\alpha = 2$.
 305 The domain is discretized by a grid with 256 points, and the final time for the simulations is $t_f = 0.4$. The CFL number for Case 1 (hydrogen–hydrogen) was

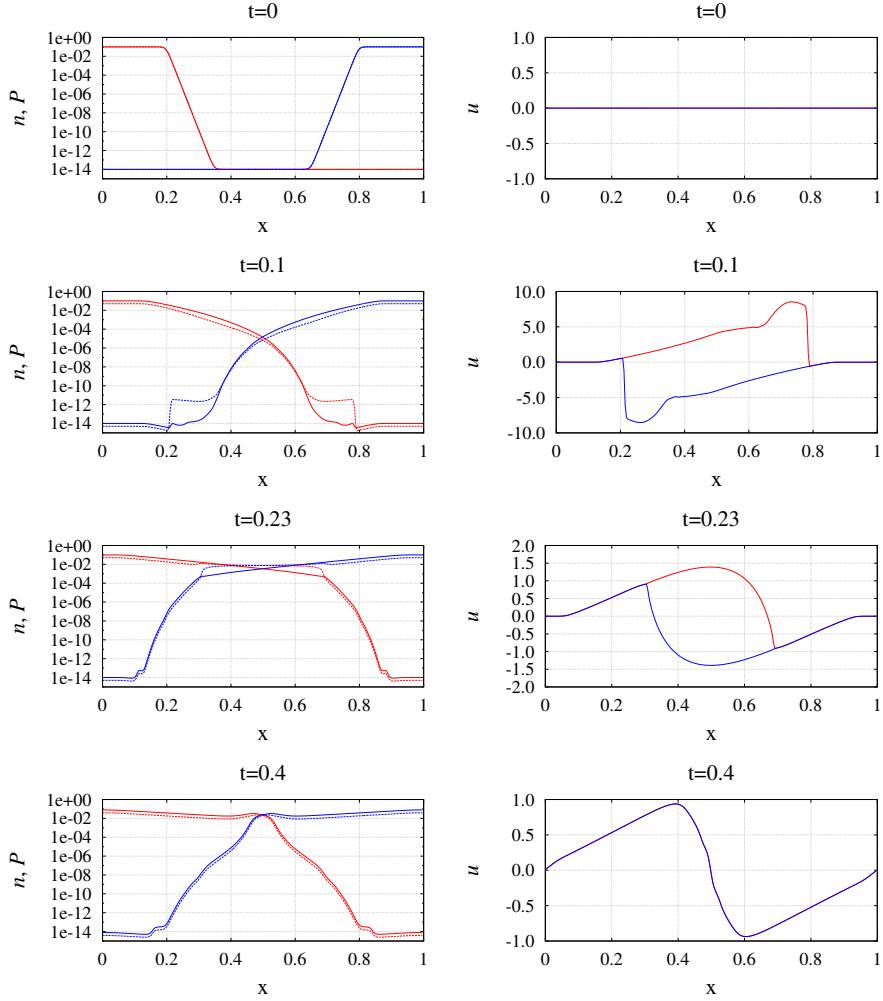


Figure 7: Interpenetration of two carbon streams: The left column shows the number density n_α (solid lines) and the pressure P_α (dashed line), and the right column shows the velocity u_α . The red lines correspond to $\alpha = 1$, and the blue lines correspond to $\alpha = 2$.

specified as 0.5, while that for Case 2 (carbon–carbon) was specified as 0.05; these values ensure that the collisional source terms are also linearly stable at the computed time step size.

³¹⁰ Figure 6 shows number density $n_{\{1,2\}}$ and pressure $P_{\{1,2\}}$ in the left column and the velocity $u_{\{1,2\}}$ in the right column for the hydrogen–hydrogen inter-

action (Case 1), where successive rows show the solution at simulation times of 0 (initial solution), 0.1, 0.23, and 0.4 (final time). The interaction can be described as follows. At $t = 0$ (initial solution), each fluid is piled up at either
 315 end of the domain. As the simulation progresses, the fluids expand from either side of the domain, and by $t = 0.1$, they interpenetrate each other. At this time, the velocity plot shows unimpeded expansion for each hydrogen fluid as they leave the domain. However, by $t = 0.23$, the friction between the fluids results in the outflowing fluid at each end of the domain getting pulled back
 320 by the dominant fluid that is flow in the opposite direction. For example, at $x \rightarrow 0$, the fluid $\alpha = 2$ has assumed the positive (rightward) velocity of the dominant fluid $\alpha = 1$ because it is being pulled back by it. At this time, the region between $x \sim 0.3$ and $x \sim 0.7$ exhibits a multifluid counterstreaming flow. The pressure and density plots show a sharper gradient compared to a smooth
 325 expansion at $t = 0.1$ (in the region $x < 0.2$ for $\alpha = 2$ and $x > 0.8$ for $\alpha = 1$) indicating the push-back from the dominant fluid. Finally, at $t = 0.4$, while each fluid has been pushed back by the other at either end of the domain, the friction results in the velocities converging to a single value at each spatial location, and thus, this becomes a “single-fluid” flow that can be solved by a multispecies,
 330 single-fluid code.

Figure 7 shows the solution for the carbon–carbon interaction (Case 2), where the same quantities are plotted at the same simulation times. Qualitatively, a similar behavior is observed; however, since carbon has a higher atomic number Z_α , the interactions (electrostatic and collisional forces) are significantly
 335 stronger. As a result, the fluids at either end of the domain are unable to reach and exit the other end of the domain. Even at an early time of $t = 0.1$, while each fluid interpenetrates each other and flow in opposite directions in the region between $x \sim 0.2$ and $x \sim 0.8$, the friction forces result in a small region adjacent to the expansion front where the expanding fluid is getting pushed back by the dominant fluid (for example, in the region $0.15 < x < 0.21$ for the fluid $\alpha = 2$). At $t = 0.23$, the pressure and number density gradients and the velocity show the push-back each fluid is facing from the other. Finally, by $t = 0.4$, the fluids

have converged to a “single fluid” with identical velocities.

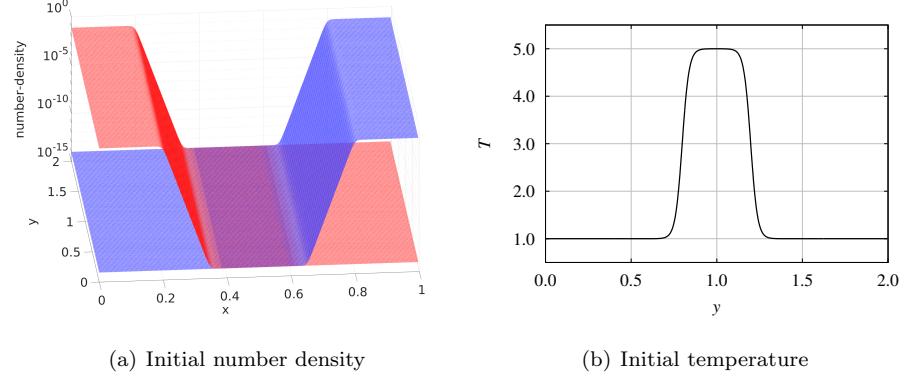


Figure 8: Initial solution for the two-dimensional two-fluid (carbon–carbon) interpenetration case: The red mesh in the left figure is the number density of the first carbon fluid $\alpha = 1$ (*carbon-left*), while the blue mesh is the number density of the second carbon fluid $\alpha = 2$ (*carbon-right*). The right figure shows the initial temperature variation along y for both the fluids.

We now consider a two-dimensional interaction of two counterstreaming fluids. The domain is $x \in [0, 1], y \in [0, 2]$, and the initial solution for the two fluids is:

$$\begin{aligned} n_\alpha(x, y, t=0) &= n_{\text{vac}} + \tilde{n}_\alpha \Upsilon(x; \delta_{x,\alpha}, x_{\min,\alpha}, x_{\max,\alpha}) \\ T_\alpha(x, y, t=0) &= T_0 + \tilde{T}_\alpha \Upsilon(y; \delta_{y,\alpha}, y_{\min,\alpha}, y_{\max,\alpha}) \quad , \quad \alpha = 1, 2, \\ u_\alpha(x, y, t=0) &= v_\alpha(x, y, t=0) = 0 \end{aligned} \quad (43)$$

where

$$\begin{aligned} x_{\min,1} &= -1, \quad x_{\max,1} = 0.2, \quad x_{\min,2} = 0.8, \quad x_{\max,2} = 2, \quad \delta_{x,\{1,2\}} = 0.005, \\ y_{\min,1} &= y_{\min,2} = 0.8, \quad y_{\max,1} = y_{\max,2} = 1.2, \quad \delta_{y,\{1,2\}} = 0.02, \\ \tilde{n}_{\{1,2\}} &= 0.08, \quad T_0 = 1, \quad \tilde{T}_{\{1,2\}} = 4, \end{aligned}$$

and $n_{\text{vac}} = 10^{-14}$ is the numerical “vacuum”. Along the x dimension, the density variation is identical to that in the one-dimensional examples discussed above, and this is shown in Figure 8(a). Both the fluids are carbon ($Z_{\{1,2\}} = 6$ and $m_{\{1,2\}} = 12$), and we refer to the first fluid $\alpha = 1$ as “*carbon-left*” (red

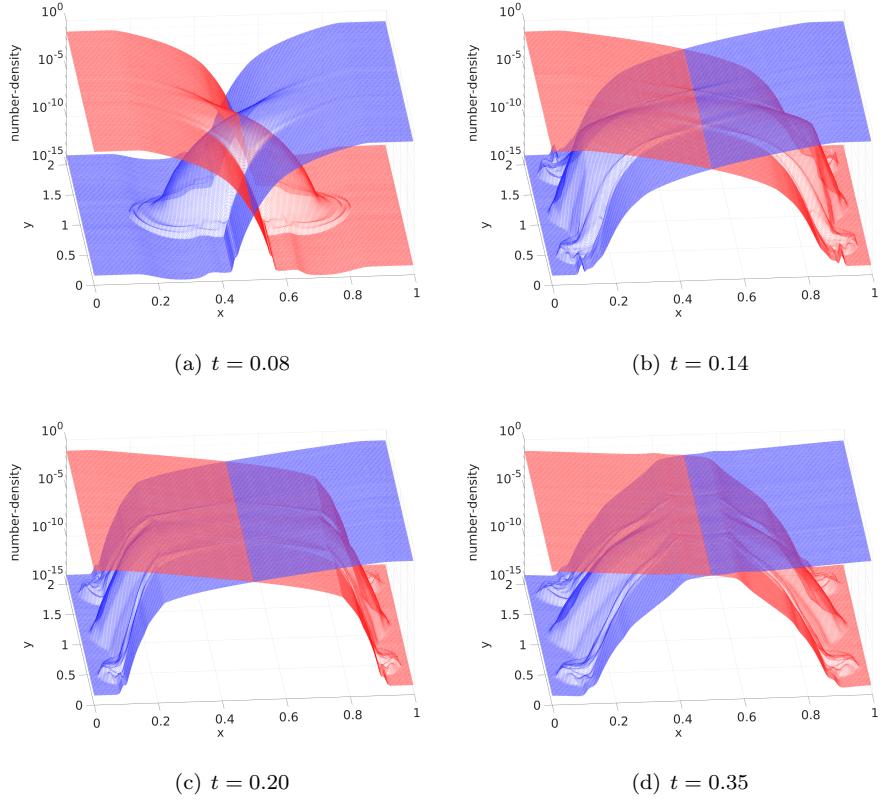


Figure 9: Evolution of the number density for the two-dimensional carbon–carbon interpénétration case: The red mesh is *carbon-left*, and the blue mesh is *carbon-right*.

curve) and the second fluid $\alpha = 2$ as “*carbon-right*” (blue curve). Along the y dimension, the temperature towards the center of the domain in this dimension ($0.8 \leq y \leq 1.2$) is specified to be 5 times hotter than the temperature near the boundaries ($y < 0.8, y > 1.2$). Figure 8(b) shows this temperature variation, which is identical for both fluids. This represents a situation where the two localized slabs of plasmas at each end of the domain are hotter in the middle than at the edges. This case is representative of the laser-induced plasma experiments where high-energy laser beams heat the carbon foils around their middle, while the edges remain cooler. The electron temperature is specified as $T_e = 0.5$. The boundary conditions are similar to that of the one-dimensional cases: Inviscid

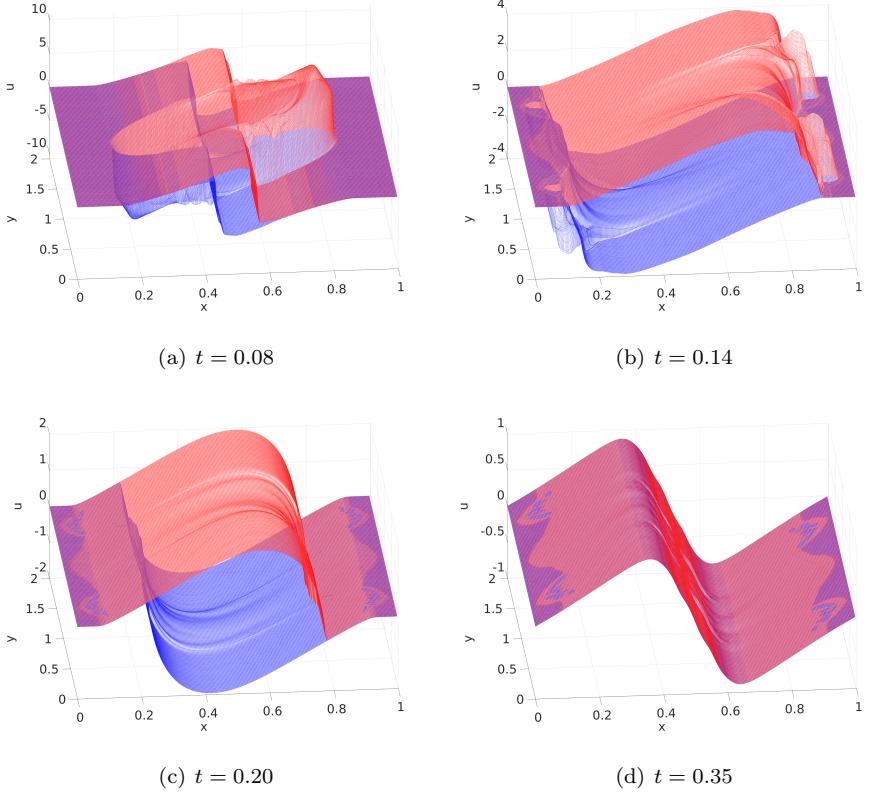


Figure 10: Evolution of the x -velocity (u) for the two-dimensional carbon–carbon interpenetration case: The red mesh is *carbon-left*, and the blue mesh is *carbon-right*.

wall boundary is imposed at $x = 0$ for *carbon-left* and at $x = 1$ for *carbon-right*, and outflow is specified at $x = 1$ for *carbon-left* and at $x = 0$ for *carbon-right*.

³⁶⁰ Outflow boundaries are specified for both fluids at $y = 0, 2$.

The final time for the simulation is $t_f = 0.35$, and CFL is specified as $\sigma = 0.05$; this ensures that the time step size resolves the collisional terms. Figure 9 shows the evolution of the number densities for the two fluids at $t = \{0.08, 0.14, 0.20, 0.35\}$, while Figure 10 shows the evolution of u , the x -component of the velocity. In these figures, the red surfaces correspond to *carbon-left*, and the blue surfaces correspond to *carbon-right*. Along the x -dimension, the overall dynamics is similar to that in the one-dimensional ex-

ample discussed above. The two fluids start expanding towards each other, as can be observed at $t = 0.08$. The expansion is faster near the center of the domain along y because of the higher temperature specified in that region. In Figures 9(a) and 10(a), the expansion front in $0.8 \leq y \leq 1.2$ leads that near the y -boundaries. At $t = 1.4$, the two fluids have interpenetrated into each other, and this is shown in Figure 9(b). Figure 10(b) shows that both the fluids are expanding in the large majority of the domain; however, they are also starting to get pushed back by the other fluid. For example, *carbon-left* is being pushed back by *carbon-right* near $x = 1$, and this is more pronounced for $0.8 \leq y \leq 1.2$ due to the higher temperature. The inter-fluid friction prevents each fluid from reaching the other end of the domain, and by $t = 0.2$, they get pushed back. Figure 9(c) shows the sharp gradient in the number density that develops as a result of this push-back, and Figure 10(c) shows the velocities converging to the same value near the x -boundaries, while counterstreaming flows exist towards the center of the domain $0.2 < x < 0.8$. For example, near $x = 0$, *carbon-right* is being carried right-ward (towards $x = 1$) by *carbon-left*. The two fluids assume identical velocities by $t = 0.35$, and they essentially become a single fluid.

Finally, we consider the two-dimensional interaction of two counterstreaming fluids in the presence of a gas fill. The details of the setup is identical to that in the previous example of the two-dimensional interaction of two carbon fluids with the addition of helium as the gas fill. Thus, the number of fluids is $n_f = 3$, and the properties of the third fluid are $Z_3 = 2$ and $m_3 = 4$. The initial solution for the third fluid, which we refer to as *helium-fill*, is as follows:

$$\begin{aligned} n_3(x, y, t=0) &= n_{\text{vac}} + \tilde{n}_3 \Upsilon(x; \delta_{x,3}, x_{\min,3}, x_{\max,3}), \\ T_3(x, y, t=0) &= T_{0,3} = 0.25, \quad u_3(x, y, t=0) = v_3(x, y, t=0) = 0, \end{aligned} \tag{44}$$

where $\tilde{n}_3 = 10^{-4}$ is the fill gas density, and

$$\delta_{x,3} = 0.005, \quad x_{\min,3} = 0.22, \quad x_{\max,3} = 0.78.$$

Thus, compared with the previous setup, the vacuum in between the two carbon fluids in the initial solution is filled by a low-density helium gas. The initial

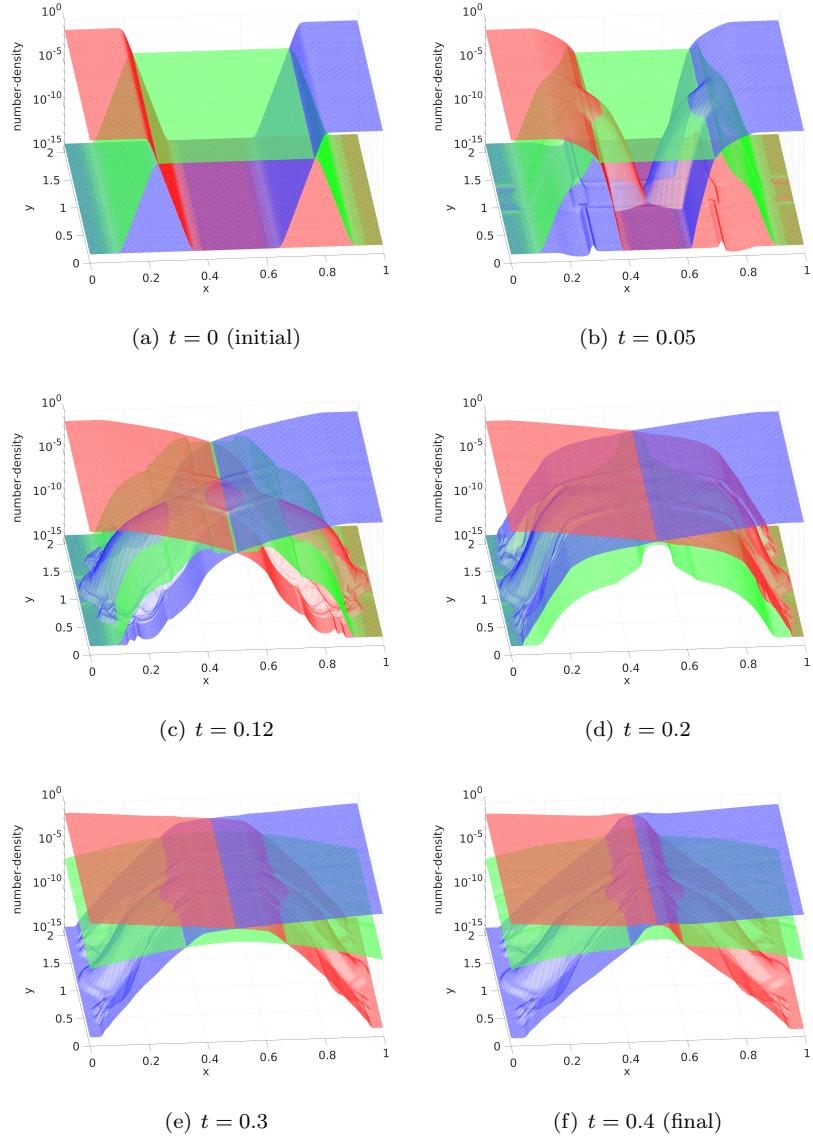


Figure 11: Evolution of the number density for the two-dimensional interaction of two carbon fluids in the presence of helium gas fill: The red and blue meshes show the number densities for *carbon-left* ($\alpha = 1$) and *carbon-right* ($\alpha = 2$), respectively, while the green mesh shows the number density for *helium-fill* ($\alpha = 3$).

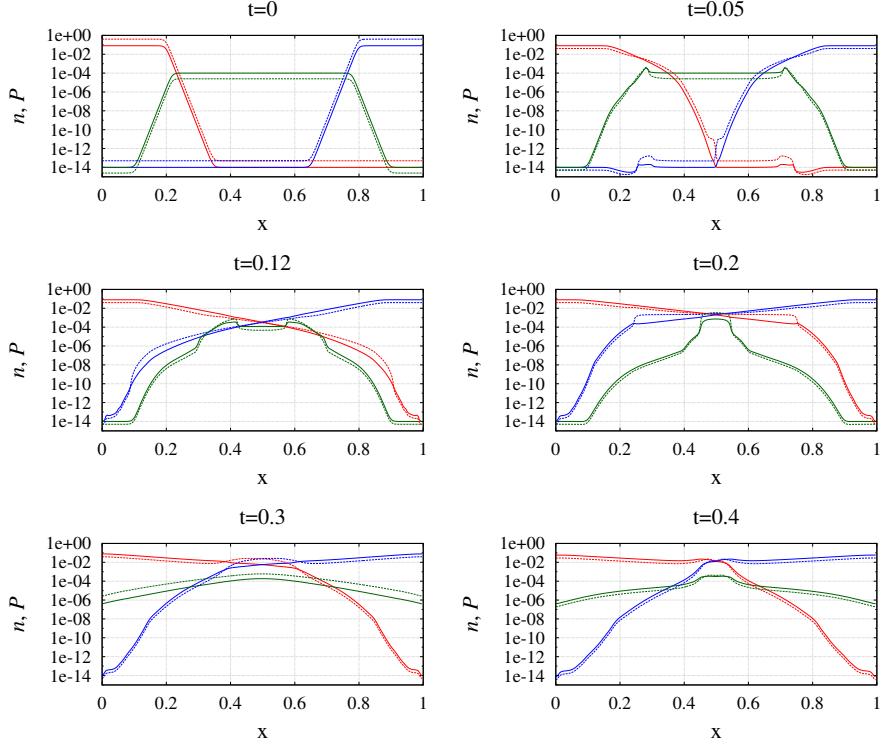


Figure 12: Evolution of the pressure and number density for the two-dimensional interaction of two carbon fluids in the presence of helium gas fill along the x -dimension at $y = 1$: The red and blue curves correspond to *carbon-left* and *carbon-right*, respectively, and the green curve corresponds to *helium-fill*. The solid lines show the number density, and the dashed lines show the pressure.

temperature for the two carbon fluids vary along the y -dimension as before, as shown in Figure 8(b), and the initial temperature for the helium fluid is 390 uniform. Figure 11(a) shows the number density at $t = 0$, where the red and blue meshes correspond to *carbon-left* and *carbon-right*, respectively, and the green mesh corresponds to *helium-fill*. The final time for the simulation is specified as $t_f = 0.4$, and the CFL is specified as $\sigma = 0.05$. The boundary conditions for *carbon-left* and *carbon-right* are same as before; outflow boundaries are specified 395 for *helium-fill* everywhere.

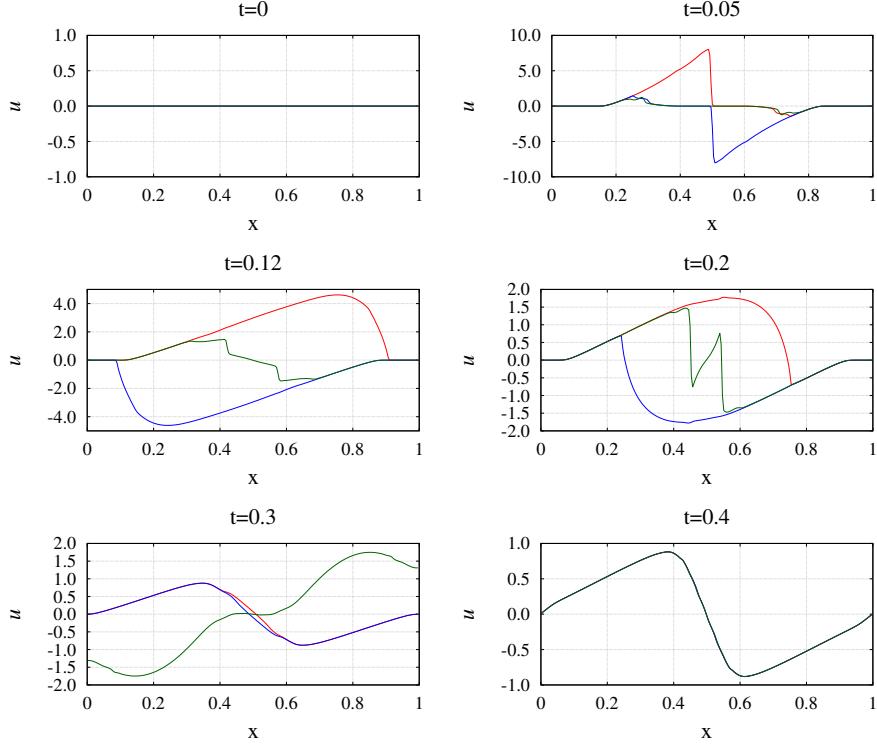


Figure 13: Evolution of the x -velocity (u) for the two-dimensional interaction of two carbon fluids in the presence of helium gas fill along the x -dimension at $y = 1$: The red and blue curves correspond to *carbon-left* and *carbon-right*, respectively, and the green curve corresponds to *helium-fill*.

Figure 11 shows the number density for the three fluids at various solution times, while Figure 12 shows the number density and pressure variation along x at $y = 1$. As the two carbon fluids expand from either end of the domain, they drag the helium gas fill along with them, and this is observed in the solutions at $t = 0.05$ and $t = 0.12$. Figure 13 shows the velocity along the x -dimension (u) at $y = 1$, and the solution at $t = 0.12$ shows that *helium-fill* assumes the velocity of the dominant carbon fluid (*carbon-left* in $x < 0.5$ and *carbon-right* in $x > 0.5$), while the two carbon fluids are expanding and interpenetrating. By $t = 0.2$, a large amount of *helium-fill* has accumulated in the center of the domain, and the

405 velocity variation indicates that it has started expanding outwards through the
two carbon fluids. Meanwhile, similar to the behavior observed in the previous
two-fluid cases, the carbon fluids from either end are pushed back from the
other boundary by the dominant fluid at that boundary, and although they are
counterstreaming around the center of the domain, they have converged to a
410 single velocity near the boundaries ($x < 0.3$ and $x > 0.7$). The two carbon
fluids converge completely to a single velocity by $t = 0.3$, and each fluid has
been pushed back by the other at each end of the domain; however, *helium-fill*
streams through the carbon fluids to expand outward, and the velocity variation
at $t = 0.3$ shows significant helium–carbon counterstreaming. Finally, at $t = 0.4$,
415 all three fluids converge to a single velocity field and essentially become one
fluid.

6. Conclusions

420 This paper reports the initial development of EUCLID, a multifluid code for
simulating plasma interpenetration. The governing equations are the inviscid
Euler equations, expressed as the conservation of mass, momentum, and energy,
425 for each ion fluid. The formulation allows the partitioning of the same species
into multiple fluids or streams. The fluids interact through electrostatic forces
and collisional terms that include friction and thermal equilibration (both ion–
ion and electron–ion). The electrons are assumed to be isothermal and inertialess,
430 and the plasma is assumed to be quasi-neutral; thus, the fluid equations for
the electrons and the Poisson equation for the electrostatic potential need not
be solved. The governing equations are discretized using a conservative finite-
difference method, and the 5th-order Monotonicity-Preserving WENO scheme
is used for the upwind discretization of the hyperbolic flux. Currently, the ex-
435 plicit 4th-order Runge-Kutta method is used for time integration. EUCLID is
verified and its accuracy, convergence, and nonoscillatory behavior is demon-
strated using several benchmark flow problems and the method of manufac-
tured solutions, and these are reported in this paper. Finally, we simulate

one- and two-dimensional flows that are representative of laser-induced plasma
435 experiments—the interactions of counterstreaming fluids in vacuum as well as in
the presence of a gas fill.

EUCLID is a “work-in-progress”, and this paper is intended as a description
of the basic algorithm. One of the challenges with our current approach results
from using an explicit time integration method. The time scales of the colli-
440 sional terms (both friction and thermal equilibration) are often faster than the
convective and acoustic time scales for our applications. In the results reported
in this paper, a very low CFL number was needed to resolve these time scales,
especially for interactions between heavier species like carbon. More physically-
relevant simulations will involve heavier species such as gold ($Z = 40, m = 197$)
445 resulting in faster collisional time scales. Thus, the implementation of implicit-
explicit (IMEX) time integration methods [55–57] to integrate the source terms
implicitly is being investigated. In addition, several aspects of the physical
model and their improvements are areas of active research. Examples include
removing the assumption of isothermal electrons and solving the electron en-
450 ergy equation along with the ion fluid equations and the inclusion of heat source
terms in the energy equations to simulate the continued localized heating due to
laser beams. Finally, counterstreaming plasmas result in the growth of kinetic
instabilities that have a significant effect on the solution. A kinetic simulation
455 of these instabilities within a fluid code is computationally intractable; however,
we are investigating the development of collisional kinetic models [58] that will
appear as kinetic friction and heating terms in the fluid equations.

Acknowledgments

The authors wish to acknowledge helpful discussions with J. A. F. Hittinger,
M. Dorr, G. V. Vogman, I. Joseph, and M. Dorf at the Lawrence Livermore
460 National Laboratory.

Appendix A. Monotonicity-Preserving WENO5 (MPWENO5) Scheme

This section summarizes the reconstruction procedure along dimension d ; the purpose is to provide a brief but sufficient description that allows the exact reproducibility of our algorithm. The readers are encouraged to refer to the original publications [48, 49] for a detailed description of this method. The fifth-order monotonicity-preserving WENO (MPWENO5) scheme described here computes a left-biased approximation to the flux primitive at the interface $\mathbf{i} + \frac{1}{2}\mathbf{e}_d$, given the values of the flux function at grid points $\mathbf{i} - 2\mathbf{e}_d, \dots, \mathbf{i} + 2\mathbf{e}_d$. This same procedure can be used to compute the right-biased approximation by providing the flux functions at the grid points $\mathbf{i} + 3\mathbf{e}_d, \dots, \mathbf{i} - \mathbf{e}_d$ as the inputs (note the reverse ordering). The fifth-order WENO approximation can be computed as:

$$f_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{\text{WENO5}} = \sum_{l=1}^3 \omega_l f_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{(l)}; \quad \omega_l = \frac{\zeta_l}{\sum_{m=1}^3 \zeta_m}, \quad l = 1, 2, 3, \quad (\text{A.1})$$

where

$$f_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{(1)} = \frac{1}{3}f_{\mathbf{i}-2\mathbf{e}_d} - \frac{7}{6}f_{\mathbf{i}-\mathbf{e}_d} + \frac{11}{6}f_{\mathbf{i}}, \quad (\text{A.2a})$$

$$f_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{(2)} = -\frac{1}{6}f_{\mathbf{i}-\mathbf{e}_d} + \frac{5}{6}f_{\mathbf{i}} + \frac{1}{3}f_{\mathbf{i}+\mathbf{e}_d}, \quad (\text{A.2b})$$

$$f_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{(3)} = \frac{1}{3}f_{\mathbf{i}} + \frac{5}{6}f_{\mathbf{i}+\mathbf{e}_d} - \frac{1}{6}f_{\mathbf{i}+2\mathbf{e}_d} \quad (\text{A.2c})$$

are the three third-order approximations of $f_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}$, and

$$\zeta_l = \frac{c_l}{\epsilon + \mathcal{S}_l}, \quad l = 1, 2, 3; \quad c_1 = \frac{1}{10}, \quad c_2 = \frac{6}{10}, \quad c_3 = \frac{3}{10}; \quad (\text{A.3a})$$

$$\mathcal{S}_1 = \frac{13}{12} (f_{\mathbf{i}-2\mathbf{e}_d} - 2f_{\mathbf{i}-\mathbf{e}_d} + f_{\mathbf{i}})^2 + \frac{1}{4} (f_{\mathbf{i}-2\mathbf{e}_d} - 4f_{\mathbf{i}-\mathbf{e}_d} + 3f_{\mathbf{i}})^2, \quad (\text{A.3b})$$

$$\mathcal{S}_2 = \frac{13}{12} (f_{\mathbf{i}-\mathbf{e}_d} - 2f_{\mathbf{i}} + f_{\mathbf{i}+\mathbf{e}_d})^2 + \frac{1}{4} (f_{\mathbf{i}-\mathbf{e}_d} - f_{\mathbf{i}+\mathbf{e}_d})^2, \quad (\text{A.3c})$$

$$\mathcal{S}_3 = \frac{13}{12} (f_{\mathbf{i}} - 2f_{\mathbf{i}+\mathbf{e}_d} + f_{\mathbf{i}+2\mathbf{e}_d})^2 + \frac{1}{4} (f_{\mathbf{i}} - 4f_{\mathbf{i}+\mathbf{e}_d} + 3f_{\mathbf{i}+2\mathbf{e}_d})^2, \quad (\text{A.3d})$$

where ζ_l are the WENO weights that are computed by scaling the optimal weights c_l by the smoothness indicators \mathcal{S}_l , and $\epsilon = 10^{-6}$ is a small parameter to prevent division by zero. In the regions where the solution is smooth, $\omega_l \rightarrow c_l$,

⁴⁶⁵ and (A.1) becomes

$$f_{\mathbf{i}+\frac{1}{2}\mathbf{e}_d}^{\text{WENO5}} = \sum_{l=1}^3 c_l f_{\mathbf{i}+\frac{1}{2}\mathbf{e}_d}^{(l)} = \frac{1}{30} f_{\mathbf{i}-2\mathbf{e}_d} - \frac{13}{60} f_{\mathbf{i}-\mathbf{e}_d} + \frac{47}{60} f_{\mathbf{i}} + \frac{27}{60} f_{\mathbf{i}+\mathbf{e}_d} - \frac{1}{20} f_{\mathbf{i}+2\mathbf{e}_d}, \quad (\text{A.4})$$

which is the fifth-order approximation of $f_{\mathbf{i}+\frac{1}{2}\mathbf{e}_d}$. Near discontinuities, the weights ω_l corresponding to the stencils containing the discontinuity go to zero, and a nonoscillatory approximation biased away from the discontinuity is computed.

The MPWENO5 approximation is obtained from $f_{\mathbf{i}+\frac{1}{2}\mathbf{e}_d}^{\text{WENO5}}$ by ensuring that the final approximation is within the monotonicity-preserving bounds. Defining the following functions:

$$\text{sign}(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}, \quad (\text{A.5a})$$

$$\text{minmod}(x, y) = \frac{1}{2} (\text{sign}(x) + \text{sign}(y)) \min(|x|, |y|), \quad (\text{A.5b})$$

$$\text{median}(x, y, z) = x + \text{minmod}(y - x, z - x), \quad (\text{A.5c})$$

⁴⁷⁰ the monotonicity-preserving approximation is expressed as:

$$f_{\mathbf{i}+\frac{1}{2}\mathbf{e}_d} = \text{median} \left(f_{\mathbf{i}+\frac{1}{2}\mathbf{e}_d}^{\text{WENO5}}, f_{\mathbf{i}+\frac{1}{2}\mathbf{e}_d}^{\text{min}}, f_{\mathbf{i}+\frac{1}{2}\mathbf{e}_d}^{\text{max}} \right). \quad (\text{A.6})$$

The minimum and maximum bounds are computed as

$$f_{\mathbf{i}+\frac{1}{2}\mathbf{e}_d}^{\text{min}} = \max \left[\min \left(f_{\mathbf{i}}, f_{\mathbf{i}+\mathbf{e}_d}, f_{\mathbf{i}+\frac{1}{2}\mathbf{e}_d}^{\text{med}} \right), \min \left(f_{\mathbf{i}}, f_{\mathbf{i}+\frac{1}{2}\mathbf{e}_d}^{\text{UL}}, f_{\mathbf{i}+\frac{1}{2}\mathbf{e}_d}^{\text{LC}} \right) \right], \quad (\text{A.7a})$$

$$f_{\mathbf{i}+\frac{1}{2}\mathbf{e}_d}^{\text{max}} = \min \left[\max \left(f_{\mathbf{i}}, f_{\mathbf{i}+\mathbf{e}_d}, f_{\mathbf{i}+\frac{1}{2}\mathbf{e}_d}^{\text{med}} \right), \max \left(f_{\mathbf{i}}, f_{\mathbf{i}+\frac{1}{2}\mathbf{e}_d}^{\text{UL}}, f_{\mathbf{i}+\frac{1}{2}\mathbf{e}_d}^{\text{LC}} \right) \right], \quad (\text{A.7b})$$

where

$$f_{\mathbf{i}+\frac{1}{2}\mathbf{e}_d}^{\text{med}} = \frac{1}{2} \left(f_{\mathbf{i}} + f_{\mathbf{i}+\mathbf{e}_d} - d_{\mathbf{i}+\frac{1}{2}\mathbf{e}_d}^{\text{med}} \right), \quad (\text{A.8a})$$

$$f_{\mathbf{i}+\frac{1}{2}\mathbf{e}_d}^{\text{UL}} = f_{\mathbf{i}} + v (f_{\mathbf{i}} - f_{\mathbf{i}-\mathbf{e}_d}), \quad (\text{A.8b})$$

$$f_{\mathbf{i}+\frac{1}{2}\mathbf{e}_d}^{\text{LC}} = f_{\mathbf{i}} + \frac{1}{2} (f_{\mathbf{i}} - f_{\mathbf{i}-\mathbf{e}_d}) + \frac{\beta}{3} d_{\mathbf{i}-\frac{1}{2}\mathbf{e}_d}^{\text{LC}} \quad (\text{A.8c})$$

are the median value of the flux, an upper limit based on v , and an estimate allowing for a large curvature (LC), respectively. In our implementation, we set

$v = 2, \beta = 4$, and $d_{(\cdot)}^{\text{LC}} = d_{(\cdot)}^{\text{med}} = d_{(\cdot)}^{\text{MM}}$, where

$$d_{\mathbf{i} + \frac{1}{2}\mathbf{e}_d}^{\text{MM}} = \text{minmod}(d_{\mathbf{i}}, d_{\mathbf{i} + \mathbf{e}_d}); \quad d_{\mathbf{i}} = f_{\mathbf{i}-\mathbf{e}_d} - 2f_{\mathbf{i}} + f_{\mathbf{i}+\mathbf{e}_d}. \quad (\text{A.9})$$

These choices for the monotonicity-preserving scheme work well for our applications, which involves density and pressure varying from $\mathcal{O}(1)$ to $\mathcal{O}(10^{-14})$, based on the discussion on the role of these parameters and their effect on the solution [49].

Appendix B. Eigenstructure

The complete eigenstructure of (13), derived using Mathematica [51], is provided in this section. The eigenvalues of the flux Jacobian $\partial \mathbf{f}_{\alpha}^{(d)} / \partial \mathbf{U}_{\alpha}$; $d \in \{x, y, z\}$ along each dimension for the fluid with index α are:

$$\Lambda_{\alpha}^{(x)} = \{u_{\alpha}, u_{\alpha}, u_{\alpha}, u_{\alpha} - c_{\alpha}^*, u_{\alpha} + c_{\alpha}^*\}, \quad (\text{B.1a})$$

$$\Lambda_{\alpha}^{(y)} = \{v_{\alpha}, v_{\alpha}, v_{\alpha}, v_{\alpha} - c_{\alpha}^*, v_{\alpha} + c_{\alpha}^*\}, \quad (\text{B.1b})$$

$$\Lambda_{\alpha}^{(z)} = \{w_{\alpha}, w_{\alpha}, w_{\alpha}, w_{\alpha} - c_{\alpha}^*, w_{\alpha} + c_{\alpha}^*\}, \quad (\text{B.1c})$$

where $c_{\alpha}^* = \sqrt{\gamma_{\alpha} P_{\alpha}^*/\rho_{\alpha}}$ is the augmented speed of sound, and $u_{\alpha}, v_{\alpha}, w_{\alpha}$ are the Cartesian components of the velocity vector \mathbf{u}_{α} . The right and left eigenvectors along the three dimensions are provided below. In these expressions,

$$s_{\alpha}^2 = u_{\alpha}^2 + v_{\alpha}^2 + w_{\alpha}^2, \quad \tau_{\alpha} = \frac{Z_{\alpha} T_e}{m_{\alpha}}, \quad \text{and} \quad \mu_{\alpha} = \gamma_{\alpha} - 1.$$

Along X .

$$R_\alpha^{(x)} = \begin{pmatrix} 0 & 0 & 1 & \frac{1}{c_\alpha^{*2}} & \frac{1}{c_\alpha^{*2}} \\ 0 & 0 & u_\alpha & \frac{u_\alpha - c_\alpha^*}{c_\alpha^{*2}} & \frac{u_\alpha + c_\alpha^*}{c_\alpha^{*2}} \\ 0 & \rho_\alpha & v_\alpha & \frac{v_\alpha}{c_\alpha^{*2}} & \frac{v_\alpha}{c_\alpha^{*2}} \\ \rho_\alpha & 0 & w_\alpha & \frac{w_\alpha}{c_\alpha^{*2}} & \frac{w_\alpha}{c_\alpha^{*2}} \\ \rho_\alpha w_\alpha & \rho_\alpha v_\alpha & \frac{1}{2} \left(s_\alpha^2 - \frac{2\tau_\alpha}{\mu_\alpha} \right) & \frac{(s_\alpha^2 - \frac{2\tau_\alpha}{\mu_\alpha})}{2c_\alpha^{*2}} - \frac{u_\alpha}{c_\alpha^*} + \frac{1}{\mu_\alpha} & \frac{(s_\alpha^2 - \frac{2\tau_\alpha}{\mu_\alpha})}{2c_\alpha^{*2}} + \frac{u_\alpha}{c_\alpha^*} + \frac{1}{\mu_\alpha} \end{pmatrix}$$

$$L_\alpha^{(x)} = \begin{pmatrix} -\frac{w_\alpha}{\rho_\alpha} & 0 & 0 & \frac{1}{\rho_\alpha} & 0 \\ -\frac{v_\alpha}{\rho_\alpha} & 0 & \frac{1}{\rho_\alpha} & 0 & 0 \\ \frac{(s_\alpha^2 - 2\tau_\alpha - s_\alpha^2 \gamma_\alpha)}{2c_\alpha^{*2}} + 1 & \frac{\mu_\alpha u_\alpha}{c_\alpha^{*2}} & \frac{\mu_\alpha v_\alpha}{c_\alpha^{*2}} & \frac{\mu_\alpha w_\alpha}{c_\alpha^{*2}} & -\frac{\mu_\alpha}{c_\alpha^*} \\ \frac{\mu_\alpha s_\alpha^2}{4} + \frac{\tau_\alpha}{2} + \frac{u_\alpha c_\alpha^*}{2} & -\frac{1}{2}(\mu_\alpha u_\alpha + c_\alpha^*) & -\frac{1}{2}(\mu_\alpha v_\alpha) & -\frac{1}{2}(\mu_\alpha w_\alpha) & \frac{\mu_\alpha}{2} \\ \frac{\mu_\alpha s_\alpha^2}{4} + \frac{\tau_\alpha}{2} - \frac{u_\alpha c_\alpha^*}{2} & -\frac{1}{2}(\mu_\alpha u_\alpha - c_\alpha^*) & -\frac{1}{2}(\mu_\alpha v_\alpha) & -\frac{1}{2}(\mu_\alpha w_\alpha) & \frac{\mu_\alpha}{2} \end{pmatrix}$$

Along Y .

$$R_\alpha^{(y)} = \begin{pmatrix} 0 & 0 & 1 & \frac{1}{c_\alpha^{*2}} & \frac{1}{c_\alpha^{*2}} \\ 0 & \rho_\alpha & u_\alpha & \frac{u_\alpha}{c_\alpha^{*2}} & \frac{u_\alpha}{c_\alpha^{*2}} \\ 0 & 0 & v_\alpha & \frac{v_\alpha - c_\alpha^*}{c_\alpha^{*2}} & \frac{v_\alpha + c_\alpha^*}{c_\alpha^{*2}} \\ \rho_\alpha & 0 & w_\alpha & \frac{w_\alpha}{c_\alpha^{*2}} & \frac{w_\alpha}{c_\alpha^{*2}} \\ \rho_\alpha w_\alpha & \rho_\alpha u_\alpha & \frac{1}{2} \left(s_\alpha^2 - \frac{2\tau_\alpha}{\mu_\alpha} \right) & \frac{(s_\alpha^2 - \frac{2\tau_\alpha}{\mu_\alpha})}{2c_\alpha^{*2}} - \frac{v_\alpha}{c_\alpha^*} + \frac{1}{\mu_\alpha} & \frac{(s_\alpha^2 - \frac{2\tau_\alpha}{\mu_\alpha})}{2c_\alpha^{*2}} + \frac{v_\alpha}{c_\alpha^*} + \frac{1}{\mu_\alpha} \end{pmatrix}$$

$$L_\alpha^{(y)} = \begin{pmatrix} -\frac{w_\alpha}{\rho_\alpha} & 0 & 0 & \frac{1}{\rho_\alpha} & 0 \\ -\frac{u_\alpha}{\rho_\alpha} & \frac{1}{\rho_\alpha} & 0 & 0 & 0 \\ \frac{(s^2 - 2\tau_\alpha - s_\alpha^2 \gamma_\alpha)}{2c_\alpha^{*2}} + 1 & \frac{\mu_\alpha u_\alpha}{c_\alpha^{*2}} & \frac{\mu_\alpha v_\alpha}{c_\alpha^{*2}} & \frac{\mu_\alpha w_\alpha}{c_\alpha^{*2}} & -\frac{\mu_\alpha}{c_\alpha^*} \\ \frac{\mu_\alpha s_\alpha^2}{4} + \frac{\tau_\alpha}{2} + \frac{v_\alpha c_\alpha^*}{2} & -\frac{1}{2}(\mu_\alpha u_\alpha) & -\frac{1}{2}(\mu_\alpha v_\alpha + c_\alpha^*) & -\frac{1}{2}(\mu_\alpha w_\alpha) & \frac{\mu_\alpha}{2} \\ \frac{\mu_\alpha s_\alpha^2}{4} + \frac{\tau_\alpha}{2} - \frac{v_\alpha c_\alpha^*}{2} & -\frac{1}{2}(\mu_\alpha u_\alpha) & -\frac{1}{2}(\mu_\alpha v_\alpha - c_\alpha^*) & -\frac{1}{2}(\mu_\alpha w_\alpha) & \frac{\mu_\alpha}{2} \end{pmatrix}$$

Along Z .

$$R_\alpha^{(z)} = \begin{pmatrix} 0 & 0 & 1 & \frac{1}{c_\alpha^{*2}} & \frac{1}{c_\alpha^{*2}} \\ 0 & \rho_\alpha & u_\alpha & \frac{u_\alpha}{c_\alpha^{*2}} & \frac{u_\alpha}{c_\alpha^{*2}} \\ \rho_\alpha & 0 & v_\alpha & \frac{v_\alpha}{c_\alpha^{*2}} & \frac{v_\alpha}{c_\alpha^{*2}} \\ 0 & 0 & w_\alpha & \frac{w_\alpha - c_\alpha^*}{c_\alpha^{*2}} & \frac{w_\alpha + c_\alpha^*}{c_\alpha^{*2}} \\ \rho_\alpha v_\alpha & \rho_\alpha u_\alpha & \frac{1}{2} \left(s_\alpha^2 - \frac{2\tau_\alpha}{\mu_\alpha} \right) & \frac{(s_\alpha^2 - \frac{2\tau_\alpha}{\mu_\alpha})}{2c_\alpha^{*2}} - \frac{w_\alpha}{c_\alpha^*} + \frac{1}{\mu_\alpha} & \frac{(s_\alpha^2 - \frac{2\tau_\alpha}{\mu_\alpha})}{2c_\alpha^{*2}} + \frac{w_\alpha}{c_\alpha^*} + \frac{1}{\mu_\alpha} \end{pmatrix}$$

$$L_\alpha^{(z)} = \begin{pmatrix} -\frac{v_\alpha}{\rho_\alpha} & 0 & \frac{1}{\rho_\alpha} & 0 & 0 \\ -\frac{u_\alpha}{\rho_\alpha} & \frac{1}{\rho_\alpha} & 0 & 0 & 0 \\ \frac{(s_\alpha^2 - 2\tau_\alpha - s_\alpha^2 \gamma_\alpha)}{2c_\alpha^{*2}} + 1 & \frac{\mu_\alpha u_\alpha}{c_\alpha^{*2}} & \frac{\mu_\alpha v_\alpha}{c_\alpha^{*2}} & \frac{\mu_\alpha w_\alpha}{c_\alpha^{*2}} & -\frac{\mu_\alpha}{c_\alpha^{*2}} \\ \frac{\mu_\alpha s_\alpha^2}{4} + \frac{\tau_\alpha}{2} + \frac{w_\alpha c_\alpha^*}{2} & -\frac{1}{2}(\mu_\alpha u_\alpha) & -\frac{1}{2}(\mu_\alpha v_\alpha) & -\frac{1}{2}(\mu_\alpha w_\alpha + c_\alpha^*) & \frac{\mu_\alpha}{2} \\ \frac{\mu_\alpha s_\alpha^2}{4} + \frac{\tau_\alpha}{2} - \frac{w_\alpha c_\alpha^*}{2} & -\frac{1}{2}(\mu_\alpha u_\alpha) & -\frac{1}{2}(\mu_\alpha v_\alpha) & -\frac{1}{2}(\mu_\alpha w_\alpha - c_\alpha^*) & \frac{\mu_\alpha}{2} \end{pmatrix}$$

The eigenvalues and eigenvector matrices for the entire system (15) are obtained by assembling the eigenvalues and eigenvector matrices for each fluid in a block-wise manner:

$$\Lambda^{(d)} = \left\{ \Lambda_1^{(d)}, \dots, \Lambda_\alpha^{(d)}, \dots, \Lambda_{n_f}^{(d)} \right\},$$

$$R^{(d)} = \begin{bmatrix} R_1^{(d)} & & & & \\ & \ddots & & & \\ & & R_\alpha^{(d)} & & \\ & & & \ddots & \\ & & & & R_{n_f}^{(d)} \end{bmatrix}, L^{(d)} = \begin{bmatrix} L_1^{(d)} & & & & \\ & \ddots & & & \\ & & L_\alpha^{(d)} & & \\ & & & \ddots & \\ & & & & L_{n_f}^{(d)} \end{bmatrix}$$

for $d \in \{x, y, z\}$.

References

- [1] L. F. Berzak Hopkins, S. Le Pape, L. Divol, N. B. Meezan, A. J. Mackinnon, D. D. Ho, O. S. Jones, S. Khan, J. L. Milovich, J. S. Ross, P. Amendt, D. Casey, P. M. Celliers, A. Pak, J. L. Peterson, J. Ralph, J. R. Rygg, Near-vacuum hohlraums for driving fusion implosions with high density carbon ablatars, Physics of Plasmas 22 (5) (2015) 056318. doi:10.1063/1.4921151.

- [2] S. Le Pape, L. F. Berzak Hopkins, L. Divol, N. Meezan, D. Turnbull, A. J. Mackinnon, D. Ho, J. S. Ross, S. Khan, A. Pak, E. Dewald, L. R. Benedetti, S. Nagel, J. Biener, D. A. Callahan, C. Yeamans, P. Michel, M. Schneider, 490 B. Kozioziemski, T. Ma, A. G. Macphee, S. Haan, N. Izumi, R. Hatarik, P. Sterne, P. Celliers, J. Ralph, R. Rygg, D. Strozzi, J. Kilkenny, M. Rosenberg, H. Rinderknecht, H. Sio, M. Gatun-Johnson, J. Frenje, R. Petrasso, A. Zylstra, R. Town, O. Hurricane, A. Nikroo, M. J. Edwards, The near vacuum hohlraum campaign at the NIF: A new approach, *Physics of Plasmas* 23 (5) (2016) 056311. doi:10.1063/1.4950843.
- [3] R. A. Bosch, R. L. Berger, B. H. Failor, N. D. Delamater, G. Charatis, R. L. Kauffman, Collision and interpenetration of plasmas created by laserilluminated disks, *Physics of Fluids B: Plasma Physics* 4 (4) (1992) 979–988. doi:10.1063/1.860114.
- [4] Y. Kuramitsu, Y. Sakawa, T. Morita, C. D. Gregory, J. N. Waugh, S. Dono, H. Aoki, H. Tanji, M. Koenig, N. Woolsey, H. Takabe, Time evolution of collisionless shock in counterstreaming laser-produced plasmas, *Phys. Rev. Lett.* 106 (2011) 175002. doi:10.1103/PhysRevLett.106.175002.
- [5] H.-S. Park, D. Ryutov, J. Ross, N. Kugland, S. Glenzer, C. Plechaty, 505 S. Pollaine, B. Remington, A. Spitkovsky, L. Gargate, G. Gregori, A. Bell, C. Murphy, Y. Sakawa, Y. Kuramitsu, T. Morita, H. Takabe, D. Froula, G. Fiksel, F. Miniati, M. Koenig, A. Ravasio, A. Pelka, E. Liang, N. Woolsey, C. Kuranz, R. Drake, M. Grosskopf, Studying astrophysical collisionless shocks with counterstreaming plasmas from high power lasers, 510 High Energy Density Physics 8 (1) (2012) 38–45. doi:10.1016/j.hedp. 2011.11.001.
- [6] J. S. Ross, S. H. Glenzer, P. Amendt, R. Berger, L. Divol, N. L. Kugland, O. L. Landen, C. Plechaty, B. Remington, D. Ryutov, W. Rozmus, D. H. Froula, G. Fiksel, C. Sorce, Y. Kuramitsu, T. Morita, Y. Sakawa, H. Takabe, R. P. Drake, M. Grosskopf, C. Kuranz, G. Gregori, J. Mei-

- necke, C. D. Murphy, M. Koenig, A. Pelka, A. Ravasio, T. Vinci, E. Liang, R. Presura, A. Spitkovsky, F. Miniati, H.-S. Park, Characterizing counter-streaming interpenetrating plasmas relevant to astrophysical collisionless shocks, *Physics of Plasmas* 19 (5) (2012) 056501. doi:[10.1063/1.3694124](https://doi.org/10.1063/1.3694124).
- [7] D. D. Ryutov, N. L. Kugland, H.-S. Park, C. Plechaty, B. A. Remington, J. S. Ross, Intra-jet shocks in two counter-streaming, weakly collisional plasma jets, *Physics of Plasmas* 19 (7) (2012) 074501. doi:[10.1063/1.4736973](https://doi.org/10.1063/1.4736973).
- [8] M. M. Marinak, S. W. Haan, T. R. Dittrich, R. E. Tipton, G. B. Zimmerman, A comparison of three-dimensional multimode hydrodynamic instability growth on various National Ignition Facility capsule designs with HYDRA simulations, *Physics of Plasmas* 5 (4) (1998) 1125–1132. doi:[10.1063/1.872643](https://doi.org/10.1063/1.872643).
- [9] J. M. Dawson, Particle simulation of plasmas, *Rev. Mod. Phys.* 55 (1983) 403–447. doi:[10.1103/RevModPhys.55.403](https://doi.org/10.1103/RevModPhys.55.403).
- [10] O. Larroche, Kinetic simulation of a plasma collision experiment, *Physics of Fluids B* 5 (8) (1993) 2816–2840. doi:[10.1063/1.860670](https://doi.org/10.1063/1.860670).
- [11] E. Sonnendrücker, J. Roche, P. Bertrand, A. Ghizzo, The semi-Lagrangian method for the numerical resolution of the Vlasov equation, *Journal of Computational Physics* 149 (2) (1999) 201–220. doi:[10.1006/jcph.1998.6148](https://doi.org/10.1006/jcph.1998.6148).
- [12] S. Markidis, G. Lapenta, Rizwan-uddin, Multi-scale simulations of plasma with iPIC3D, *Mathematics and Computers in Simulation* 80 (7) (2010) 1509–1519, multiscale modeling of moving interfaces in materials. doi:[10.1016/j.matcom.2009.08.038](https://doi.org/10.1016/j.matcom.2009.08.038).
- [13] F. Filbet, E. Sonnendrücker, Comparison of Eulerian Vlasov solvers, *Computer Physics Communications* 150 (3) (2003) 247–266. doi:[10.1016/S0010-4655\(02\)00694-X](https://doi.org/10.1016/S0010-4655(02)00694-X).

- [14] G. Kagan, X.-Z. Tang, Electro-diffusion in a plasma with two ion species,
545 Physics of Plasmas 19 (8) (2012) 082709. doi:[10.1063/1.4745869](https://doi.org/10.1063/1.4745869).
- [15] G. Kagan, X.-Z. Tang, Thermo-diffusion in inertially confined plasmas,
Physics Letters A 378 (21) (2014) 1531–1535. doi:[10.1016/j.physleta.2014.04.005](https://doi.org/10.1016/j.physleta.2014.04.005).
- [16] S. I. Braginskii, Transport processes in a plasma, Reviews of Plasma
550 Physics 1 (1965) 205.
- [17] D. Anderson, Axisymmetric multifluid simulation of high beta plasmas with anisotropic transport using a moving flux coordinate grid, Journal of Computational Physics 17 (3) (1975) 246–275.
- [18] J. Loverich, A. Hakim, U. Shumlak, A discontinuous Galerkin method
555 for ideal two-fluid plasma equations, Communications in Computational Physics 9 (2) (2011) 240–268. doi:[10.4208/cicp.250509.210610a](https://doi.org/10.4208/cicp.250509.210610a).
- [19] B. Srinivasan, U. Shumlak, Analytical and computational study of the ideal full two-fluid plasma model and asymptotic approximations for Hall-magnetohydrodynamics, Physics of Plasmas 18 (9) (2011) 092113. doi:
560 [10.1063/1.3640811](https://doi.org/10.1063/1.3640811).
- [20] E. L. Vold, Multidimensional and multifluid plasma edge modelling: Status and new directions, Contributions to Plasma Physics 32 (34) (1992) 404–421. doi:[10.1002/ctpp.2150320336](https://doi.org/10.1002/ctpp.2150320336).
- [21] R. Simonini, G. Corrigan, G. Radford, J. Spence, A. Taroni, Models and
565 numerics in the multi-fluid 2-D edge plasma code EDGE2D/U, Contributions to Plasma Physics 34 (23) (1994) 368–373. doi:[10.1002/ctpp.2150340242](https://doi.org/10.1002/ctpp.2150340242).
- [22] E. T. Meier, U. Shumlak, A general nonlinear fluid model for reacting plasma-neutral mixtures, Physics of Plasmas 19 (7) (2012) 072508. doi:
570 [10.1063/1.4736975](https://doi.org/10.1063/1.4736975).

- [23] A. A. Laguna, A. Lani, H. Deconinck, N. Mansour, S. Poedts, A fully-implicit finite-volume method for multi-fluid reactive and collisional magnetized plasmas on unstructured meshes, *Journal of Computational Physics* 318 (2016) 252–276. doi:[10.1016/j.jcp.2016.04.058](https://doi.org/10.1016/j.jcp.2016.04.058).
- ⁵⁷⁵ [24] P. Cagas, A. Hakim, J. Juno, B. Srinivasan, Continuum kinetic and multi-fluid simulations of classical sheaths, *Physics of Plasmas* 24 (2) (2017) 022118. doi:[10.1063/1.4976544](https://doi.org/10.1063/1.4976544).
- ⁵⁸⁰ [25] A. Alvarez-Laguna, N. Ozak, A. Lani, N. N. Mansour, H. Deconinck, S. Poedts, A versatile numerical method for the multi-fluid plasma model in partially- and fully-ionized plasmas, *Journal of Physics: Conference Series* 1031 (1) (2018) 012015.
- [26] R. M. Winglee, Multi-fluid simulations of the magnetosphere: The identification of the geopause and its variation with IMF, *Geophysical Research Letters* 25 (24) (1998) 4441–4444. doi:[10.1029/1998GL900217](https://doi.org/10.1029/1998GL900217).
- ⁵⁸⁵ [27] Alexashov, D., Izmodenov, V., Kinetic vs. multi-fluid models of H-atoms in the heliospheric interface: a comparison, *Astronomy & Astrophysics* 439 (3) (2005) 1171–1181. doi:[10.1051/0004-6361:20052821](https://doi.org/10.1051/0004-6361:20052821).
- ⁵⁹⁰ [28] A. Hakim, J. Loverich, U. Shumlak, A high resolution wave propagation scheme for ideal two-fluid plasma equations, *Journal of Computational Physics* 219 (1) (2006) 418–442. doi:[10.1016/j.jcp.2006.03.036](https://doi.org/10.1016/j.jcp.2006.03.036).
- [29] J. E. Leake, V. S. Lukin, M. G. Linton, E. T. Meier, Multi-fluid simulations of chromospheric magnetic reconnection in a weakly ionized reacting plasma, *The Astrophysical Journal* 760 (2) (2012) 109.
- ⁵⁹⁵ [30] L. Wang, A. H. Hakim, A. Bhattacharjee, K. Germaschewski, Comparison of multi-fluid moment models with particle-in-cell simulations of collisionless magnetic reconnection, *Physics of Plasmas* 22 (1) (2015) 012108. doi:[10.1063/1.4906063](https://doi.org/10.1063/1.4906063).

- [31] S. O'Sullivan, T. P. Downes, A three-dimensional numerical method for modelling weakly ionized plasmas, *Monthly Notices of the Royal Astronomical Society* 376 (4) (2007) 1648–1658. doi:[10.1111/j.1365-2966.2007.11429.x](https://doi.org/10.1111/j.1365-2966.2007.11429.x).
- [32] A. C. Jones, T. P. Downes, The Kelvin–Helmholtz instability in weakly ionized plasmas – II. multifluid effects in molecular clouds, *Monthly Notices of the Royal Astronomical Society* 420 (1) (2012) 817–828. doi:[10.1111/j.1365-2966.2011.20095.x](https://doi.org/10.1111/j.1365-2966.2011.20095.x).
- [33] M. S. Benilov, Multifluid equations of a plasma with various species of positive ions and the Bohm criterion, *Journal of Physics D: Applied Physics* 29 (2) (1996) 364.
- [34] M. S. Benilov, Multifluid description of nonequilibrium plasmas, in: IEEE International Conference on Plasma Science, 1996, pp. 163–164. doi:[10.1109/PLASMA.1996.550702](https://doi.org/10.1109/PLASMA.1996.550702).
- [35] E. Sousa, U. Shumlak, A blended continuous–discontinuous finite element method for solving the multi-fluid plasma model, *Journal of Computational Physics* 326 (2016) 56–75. doi:[10.1016/j.jcp.2016.08.044](https://doi.org/10.1016/j.jcp.2016.08.044).
- [36] B. Srinivasan, G. Kagan, C. S. Adams, Multi-fluid studies of plasma shocks relevant to inertial confinement fusion, *Journal of Physics: Conference Series* 717 (1) (2016) 012054.
- [37] S. Polak, X. Gao, A Fourth-Order Finite-Volume Method with Adaptive Mesh Refinement for the Multifluid Plasma Model, *American Institute of Aeronautics and Astronautics*, 2018. doi:[10.2514/6.2018-2195](https://doi.org/10.2514/6.2018-2195).
- [38] U. Shumlak, J. Loverich, Approximate Riemann solver for the two-fluid plasma model, *Journal of Computational Physics* 187 (2) (2003) 620–638. doi:[10.1016/S0021-9991\(03\)00151-7](https://doi.org/10.1016/S0021-9991(03)00151-7).
- [39] S. Baboolal, Finite-difference modeling of solitons induced by a density hump in a plasma multi-fluid, *Mathematics and Computers in Simulation*

- 55 (4) (2001) 309–316, nonlinear Waves: Computation and Theory. doi:
[10.1016/S0378-4754\(00\)00310-4](https://doi.org/10.1016/S0378-4754(00)00310-4).
- [40] H. Kumar, S. Mishra, Entropy stable numerical schemes for two-fluid plasma equations, *Journal of Scientific Computing* 52 (2) (2012) 401–425.
630 doi:[10.1007/s10915-011-9554-7](https://doi.org/10.1007/s10915-011-9554-7).
- [41] U. Shumlak, R. Lilly, N. Reddell, E. Sousa, B. Srinivasan, Advanced physics calculations using a multi-fluid plasma model, *Computer Physics Communications* 182 (9) (2011) 1767–1770. doi:[10.1016/j.cpc.2010.12.048](https://doi.org/10.1016/j.cpc.2010.12.048).
- [42] R. L. Berger, J. R. Albritton, C. J. Randall, E. A. Williams, W. L. Kruer,
635 A. B. Langdon, C. J. Hanna, Stopping and thermalization of interpenetrating plasma streams, *Physics of Fluids B: Plasma Physics* 3 (1) (1991) 3–12.
doi:[10.1063/1.859954](https://doi.org/10.1063/1.859954).
- [43] P. W. Rambo, J. Denavit, Interpenetration and ion separation in colliding plasmas, *Physics of Plasmas* 1 (12) (1994) 4050–4060. doi:[10.1063/1.870875](https://doi.org/10.1063/1.870875).
640
- [44] P. W. Rambo, R. J. Procassini, A comparison of kinetic and multifluid simulations of laserproduced colliding plasmas, *Physics of Plasmas* 2 (8) (1995) 3130–3145. doi:[10.1063/1.871145](https://doi.org/10.1063/1.871145).
- [45] C.-W. Shu, S. Osher, Efficient implementation of essentially non-oscillatory shock-capturing schemes, *Journal of Computational Physics* 77 (2) (1988)
645 439–471. doi:[10.1016/0021-9991\(88\)90177-5](https://doi.org/10.1016/0021-9991(88)90177-5).
- [46] C.-W. Shu, S. Osher, Efficient implementation of essentially non-oscillatory shock-capturing schemes, II, *Journal of Computational Physics* 83 (1) (1989) 32–78. doi:[10.1016/0021-9991\(89\)90222-2](https://doi.org/10.1016/0021-9991(89)90222-2).
- [47] M. Adams, P. Colella, D. T. Graves, J. Johnson, N. Keen, T. J. Ligocki,
650 D. F. Martin, P. McCorquodale, D. Modiano, P. Schwartz, T. Sternberg,

- B. V. Straalen, Chombo software package for amr applications - design document, Tech. Rep. LBNL-6616E, Lawrence Berkeley National Laboratory (2015).
- [48] G.-S. Jiang, C.-W. Shu, Efficient implementation of weighted ENO schemes, *Journal of Computational Physics* 126 (1) (1996) 202–228. doi: [10.1006/jcph.1996.0130](https://doi.org/10.1006/jcph.1996.0130).
- [49] D. S. Balsara, C.-W. Shu, Monotonicity preserving weighted essentially non-oscillatory schemes with increasingly high order of accuracy, *Journal of Computational Physics* 160 (2) (2000) 405–452. doi: [10.1006/jcph.2000.6443](https://doi.org/10.1006/jcph.2000.6443).
- [50] C. B. Laney, *Computational Gasdynamics*, Cambridge University Press, 1998.
- [51] Wolfram Research, Inc., Mathematica, Version 11.3, champaign, IL, 2018.
- [52] G. A. Sod, A survey of several finite difference methods for systems of nonlinear hyperbolic conservation laws, *Journal of Computational Physics* 27 (1) (1978) 1–31. doi: [10.1016/0021-9991\(78\)90023-2](https://doi.org/10.1016/0021-9991(78)90023-2).
- [53] J. S. Ross, H.-S. Park, R. Berger, L. Divol, N. L. Kugland, W. Rozmus, D. Ryutov, S. H. Glenzer, Collisionless coupling of ion and electron temperatures in counterstreaming plasma flows, *Phys. Rev. Lett.* 110 (2013) 145005. doi: [10.1103/PhysRevLett.110.145005](https://doi.org/10.1103/PhysRevLett.110.145005).
- [54] F. F. Chen, *Introduction to Plasma Physics and Controlled Fusion*, 2nd Edition, Springer International Publishing, Switzerland, 1984. doi: [10.1007/978-3-319-22309-4](https://doi.org/10.1007/978-3-319-22309-4).
- [55] U. M. Ascher, S. J. Ruuth, R. J. Spiteri, Implicit-explicit Runge-Kutta methods for time-dependent partial differential equations, *Applied Numerical Mathematics* 25 (2-3) (1997) 151–167. doi: [10.1016/S0168-9274\(97\)00056-1](https://doi.org/10.1016/S0168-9274(97)00056-1).

- [56] C. A. Kennedy, M. H. Carpenter, Additive Runge-Kutta schemes for
680 convection-diffusion-reaction equations, Applied Numerical Mathematics
44 (1-2) (2003) 139–181. doi:10.1016/S0168-9274(02)00138-1.
- [57] L. Pareschi, G. Russo, Implicit-explicit Runge-Kutta schemes and applica-
tions to hyperbolic systems with relaxation, Journal of Scientific Comput-
ing 25 (1-2) (2005) 129–155. doi:10.1007/BF02728986.
- 685 [58] A. Dimits, J. W. Banks, R. L. Berger, S. Brunner, T. Chapman,
D. Copeland, D. Ghosh, W. J. Arrighi, J. Hittinger, I. Joseph, Linearized
Coulomb collision operator for simulation of interpenetrating plasma
streams, Submitted.