A Finite-Volume, ~~Semi-Implicit~~ Multifluid Algorithm for the Simulation of Counterstreaming Plasma Dynamics

SIAM Annual Meeting

Debojyoti Ghosh[1], Christos Kavouklis[2], Thomas Chapman[3], Richard Berger[3]

[1]*Center for Applied Scientific Computing*
[2]*Computational Engineering Division*
[3]*Weapons and Complex Integration*

CASC
Center for Applied Scientific Computing

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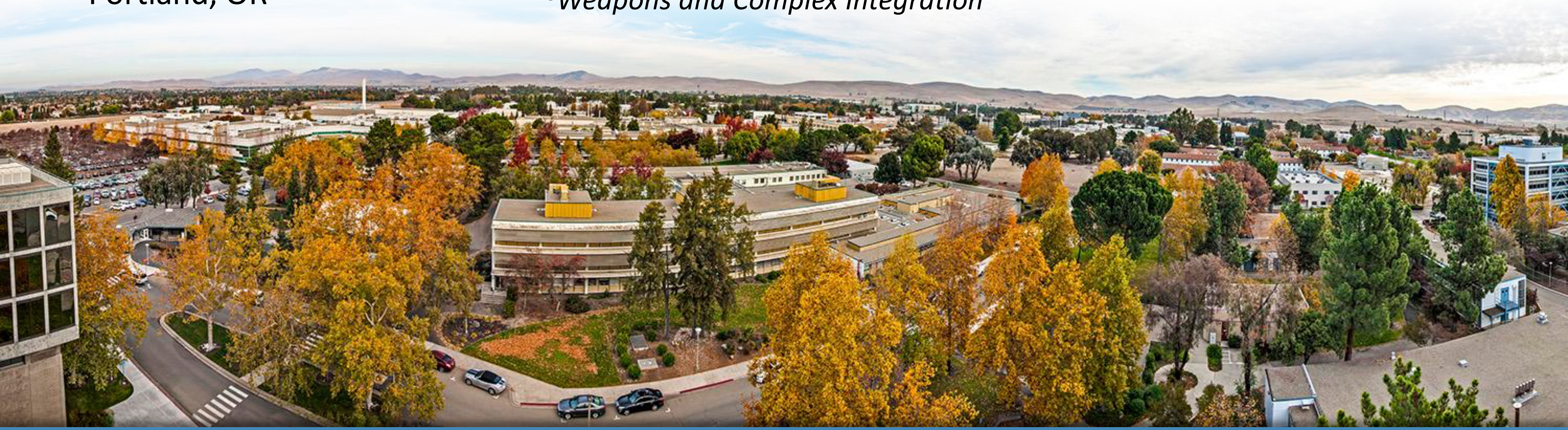

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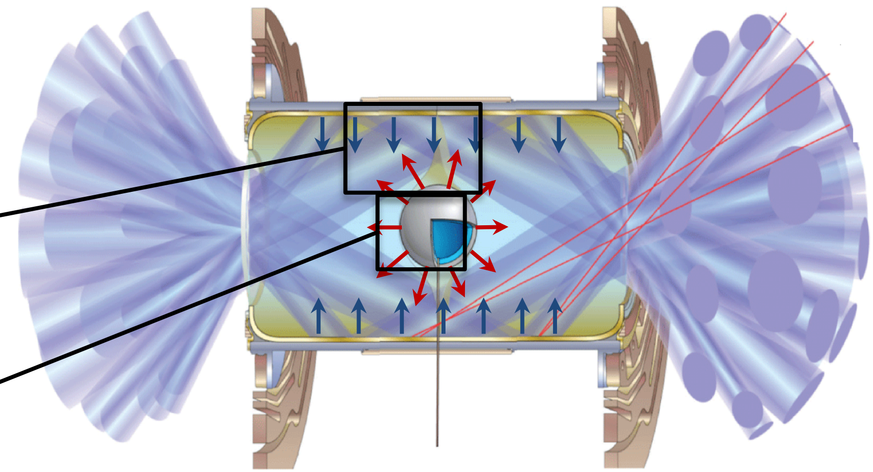
Background and Motivation

Inertial Confinement Fusion: **Colliding plasmas from hohlraum wall and capsule**

Interpenetration of plasma flows from capsule and hohlraum wall
- Large range of Z: $2 \le Z \le 60$
- Supersonic flows ($\Delta u \approx 10^8$ cm/s)

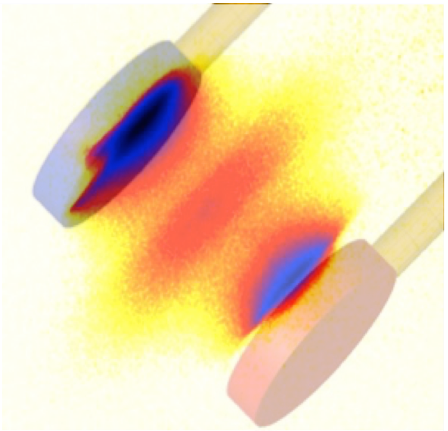
Species separation inside target capsule



Source: https://csdl-images.computer.org/mags/cs/2014/06/figures/mcs20140600421.gif

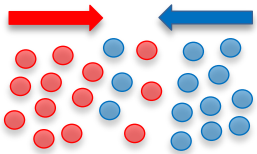
High Energy Density Physics (HEDP) Experiments

Carbon plasma streams ablating off paddles hit by laser beams and colliding with each other

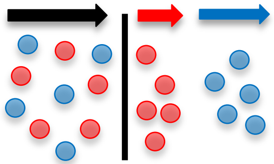


Source: H. S. Park et al., 2012

Multifluid phenomena that we want to model



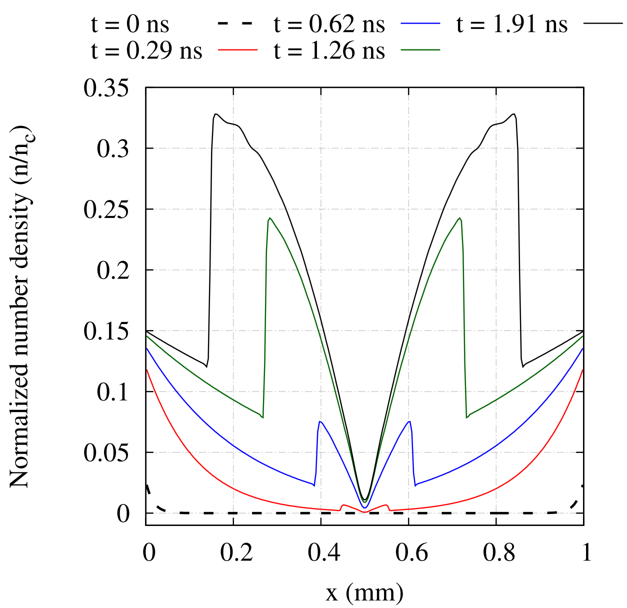
Interpenetrating plasmas

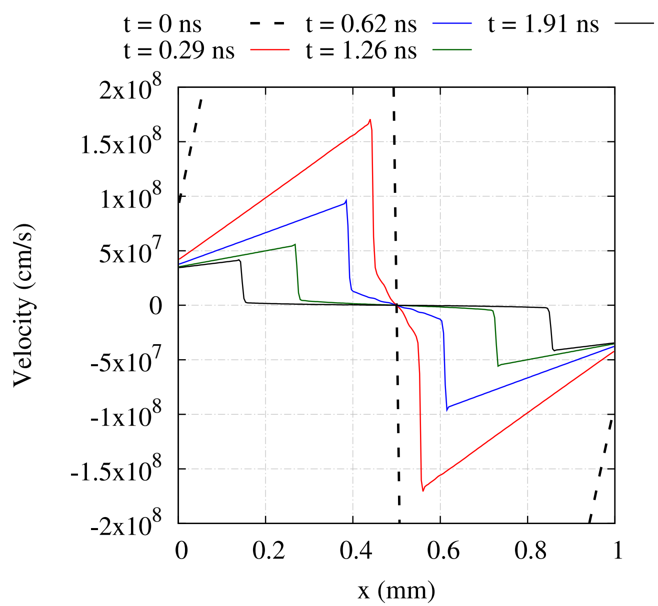


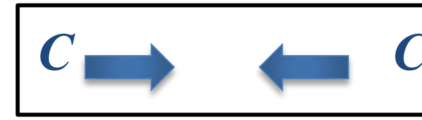
Plasma species separation

Why Not Single Fluid? Lacks Key Physics

Single-fluid simulation of *colliding carbon plasma streams*







Initial and boundary conditions: Expansion fan inflows at x = 0, 1; Vacuum inside the domain

Unphysical solution

- Lack of distinct velocity fields for each species
- **Stagnation and density pile-up, shocks**
- Push-back of incoming plasma stream

Needs: physics beyond single-fluid theory – *multi-fluid* solver

Governing Equations

We solve the inviscid Euler equations for each ion species

$$\alpha = 1, \cdots, n_s$$

$$\frac{\partial \rho_\alpha}{\partial t} + \nabla \cdot (\rho_\alpha \mathbf{u}_\alpha) = 0$$

$$\frac{\partial \rho_\alpha \mathbf{u}_\alpha}{\partial t} + \nabla \left(P_\alpha + \rho_\alpha \mathbf{u}_\alpha \otimes \mathbf{u}_\alpha \right) = -Z_\alpha e n_\alpha \nabla \phi + \sum_{\beta \neq \alpha} \mathbf{R}_{\alpha,\beta}$$

$$\frac{\partial \mathcal{E}_\alpha}{\partial t} + \nabla \cdot \left[(\mathcal{E}_\alpha + P_\alpha) \mathbf{u}_\alpha \right] = -Z_\alpha e n_\alpha \mathbf{u}_\alpha \cdot \nabla \phi + \sum_{\beta \neq \alpha} \left(\mathbf{R}_{\alpha,\beta} \cdot \mathbf{u}_\alpha + Q_{\alpha,\beta} \right)$$

Interaction between species

Assuming quasineutral, isothermal electrons*

$$\nabla \phi = \frac{T_e}{n_e} \nabla n_e + \frac{1}{n_e} \sum_\alpha R_{e,\alpha}$$

Electron momentum equation neglecting inertia terms and assuming

$$P_e = n_e T_e$$

Frictional drag

$$\mathbf{R}_{\alpha,\beta} = m_\alpha n_\alpha \nu_{\alpha,\beta} \left(\mathbf{u}_\beta - \mathbf{u}_\alpha \right)$$

Frictional heating and thermal equilibration

$$Q_{\alpha,\beta} = Q^{\mathrm{fric}}_{\alpha,\beta} + Q^{\mathrm{eq}}_{\alpha,\beta}$$

$$Q^{\mathrm{fric}}_{\alpha,\beta} = m_{\alpha,\beta} n_\alpha \nu_{\alpha,\beta} \left(\mathbf{u}_\beta - \mathbf{u}_\alpha \right)^2$$

$$Q^{\mathrm{eq}}_{\alpha,\beta} = -3 m_\alpha n_\alpha \frac{\nu_{\alpha,\beta}}{m_\alpha + m_\beta} \left(T_\alpha - T_\beta \right)$$

Governing Equations

Reformulated Governing Equations

Ion Euler equations with isothermal, quasineutral e⁻

Advective nature of electrostatic force

- Included **electron pressure** on LHS with hydrodynamic pressure
- Derived the **eigenstructure** for **characteristic-based discretization**

Effect of discretization error in dense species on **dynamics of sparse species**

Reformulation of electrostatic source terms to *avoid sums/differences of terms of disparate scales*

$$\frac{\partial \rho_\alpha}{\partial t} + \nabla \cdot (\rho_\alpha \mathbf{u}_\alpha) = 0,$$

$$\frac{\partial \rho_\alpha \mathbf{u}_\alpha}{\partial t} + \nabla \left(\rho_\alpha \mathbf{u}_\alpha \otimes \mathbf{u}_\alpha + P^*_\alpha\right) = Z_\alpha T_e n_e \nabla \left(\frac{n_\alpha}{n_e}\right) + \frac{Z_\alpha n_\alpha}{n_e} \sum_\beta \mathbf{R}_{e,\beta} + \mathbf{R}_{\alpha,e} + \sum_{\beta \neq \alpha} \mathbf{R}_{\alpha,\beta},$$

$$\frac{\partial \mathcal{E}_\alpha}{\partial t} + \nabla \cdot \{(\mathcal{E}_\alpha + P^*_\alpha)\mathbf{u}_\alpha\} = Z_\alpha T_e n_e \nabla \left(\frac{\mathbf{u}_\alpha n_\alpha}{n_e}\right) + \frac{Z_\alpha n_\alpha}{n_e} \sum_\beta \mathbf{u}_\alpha \cdot \mathbf{R}_{e,\beta} + \sum_{\beta \neq \alpha} (\mathbf{R}_{\alpha,\beta} \cdot \mathbf{u}_\alpha + Q_{\alpha,\beta}) + \mathbf{R}_{\alpha,e} \cdot \mathbf{u}_\alpha + Q^{\mathrm{eq}}_{\alpha,e},$$

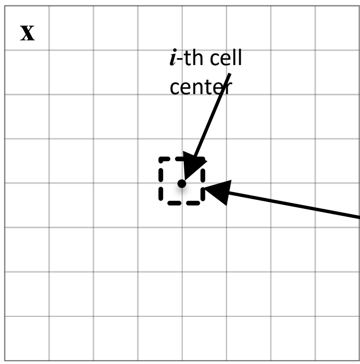
Electron pressure

where $P^*_\alpha = P_\alpha + Z_\alpha T_e n_\alpha$
is the "**augmented pressure**" (hydro + *e*⁻)

Wavespeeds (eigenvalues) : $\mathbf{v}, \mathbf{v} \pm \sqrt{\frac{\gamma_\alpha P^*_\alpha}{\rho_\alpha}}$

Summary of Numerical Method

4th order finite-volume discretization (using the *CHOMBO* library) **with AMR**



3D Domain $\Omega \equiv \{\mathbf{x} : 0 \leq \mathbf{x} \cdot \mathbf{e}_d \leq L_d, 1 \leq d \leq 3\}$

$$\mathbf{u} = \begin{bmatrix} \vdots \\ \rho_\alpha \\ \rho_\alpha \mathbf{v}_\alpha \\ \mathcal{E}_\alpha \\ \vdots \end{bmatrix}$$

discretized into computational cells

$$\omega_{\mathbf{i}} = \prod_{d=1}^{3} \left[\left(\mathbf{i} - \frac{1}{2}\mathbf{e}_d \right) h, \left(\mathbf{i} + \frac{1}{2}\mathbf{e}_d \right) h \right]$$

$\mathbf{i}$: 3-dimensional integer index (i, j, k)
h: grid spacing

Spatially-discretized ODE in time (integrated in time using **4th order Runge-Kutta method**)

$$\frac{\partial \bar{\mathbf{u}}_{\mathbf{i}}}{\partial t} = \frac{1}{h} \sum_{d=1}^{3} \left(\left\langle \hat{\mathbf{F}}_{\mathbf{i}+\frac{1}{2}\mathbf{e}_d} \right\rangle - \left\langle \hat{\mathbf{F}}_{\mathbf{i}-\frac{1}{2}\mathbf{e}_d} \right\rangle \right)$$

Cell-averaged solution

Face-averaged fluxes

Strong shocks and gradients
O(1) to O(1e-14)

- **Characteristic-based discretization**
- Implemented **5th-order WENO scheme** with **Monotonicity-Preserving limiting**

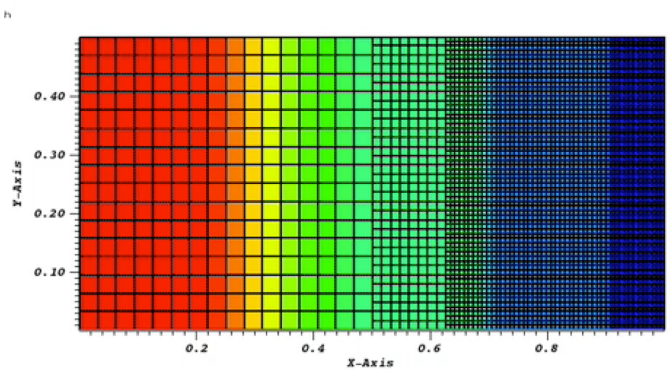
Example: Collisionless Electrostatic Single Fluid Shock Tube

Extension of the Sod's shock tube test case

- **Initial solution:** Riemann problem
- Helium gas specified
- Essentially a **1D problem**
- Inviscid wall along *x*, periodic along *y* and *z.*

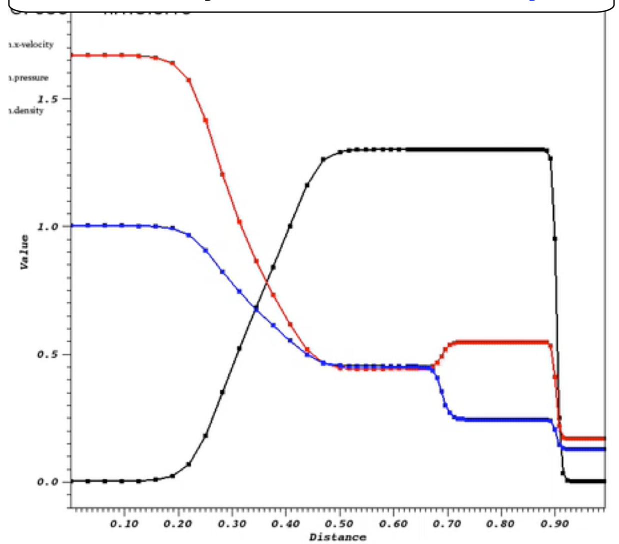
Since **Debye length much smaller than domain**, dynamics *similar to neutral gas dynamics* with *hydro pressure augmented by the electron pressure*

Density color plot (0.125 to 1.0)



$0 \leq x \leq 1$

Velocity Pressure Density

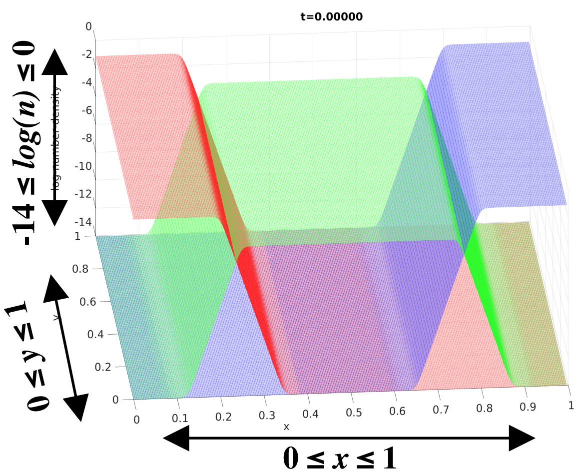


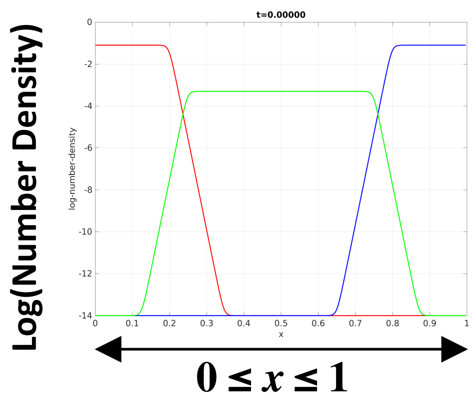
Initial Riemann discontinuity decomposes into a shock, a contact, and a rarefaction

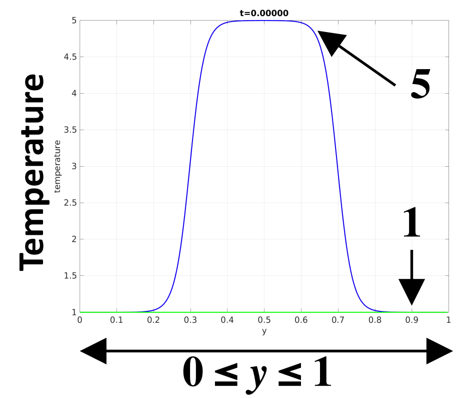
Example: Two Species Interpenetration with Gas Fill
Problem Setup

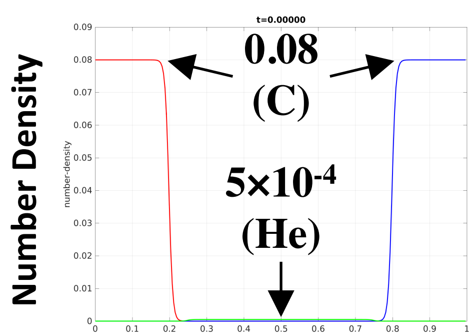
Interpenetration of two *carbon* streams in the presence of *helium* gas fill (2D)

- **Initial solution:** two species piled up on either end (*smoothed slab* density); gas fill present in the space in between.
- **Temperature variation along *y*** – the plasmas are hotter in the center of the domain









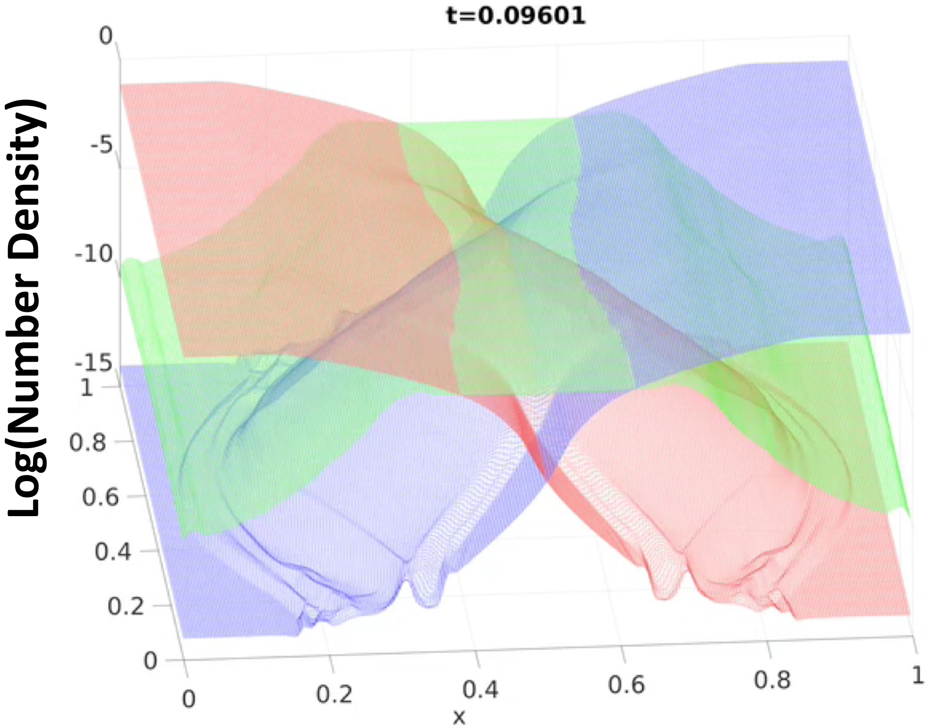
Boundary conditions:

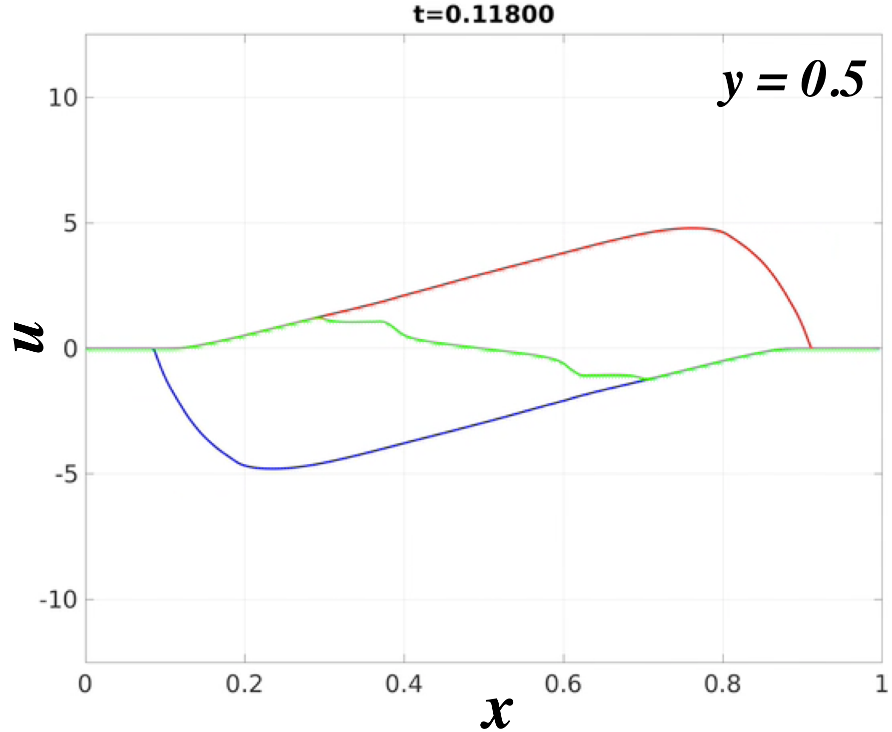
- Solid wall BCs along *x*
- Periodic along *y*

Reference quantities:

Mass: *proton mass* (1.6730e-24 g)
Number density: n_{crit} (9.0320e+21 cm^{-3})
Length: 1 mm
Temperature: 1 keV (1.6022e-09 ergs)

Example: Two Species Interpenetration with Gas Fill





Final time: ~1.3 ns

- **Species interaction** prevents one species from reaching the other end of the domain along *x*
- The fill gas is pushed towards the center of the domain by the carbon streams.

Reference quantities:

Number density: n_{crit} (9.0320e+21 cm^{-3});
Length: 1 mm; Time: 3.2314e-09 s;
Velocity: 3.0946e+07 cm/s

Conclusions and Future Work

Summary

EUCLID: EUlerian Code for pLasma Interpenetration Dynamics

- Developed a **3D, parallel, AMR-capable multifluid flow solver**
- Implemented the *quasineutral, isothermal electron model* as a computationally tractable electron model for our target applications.
- *Verified EUCLID for accuracy and convergence* (benchmark cases, manufactured solutions)
- *Simulated flows motivated by laboratory astrophysics experiments and ICF hohlraums.*

Current and Future Work

- Conduct **simulations of plasma interpenetration experiments** (e.g. *Ross et al., 2013, Le Pape et al., ongoing*)
- Investigate the *use of IMEX time integrators* for *stiff collisional terms involving high-Z species*.
- Investigate *higher-fidelity electron models*, for example, adding an electron energy equation.
- Add *source terms to energy equations to simulate heating*

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