High-order Implicit-Explicit Time Integration for the Kinetic Simulation of Magnetized Plasmas

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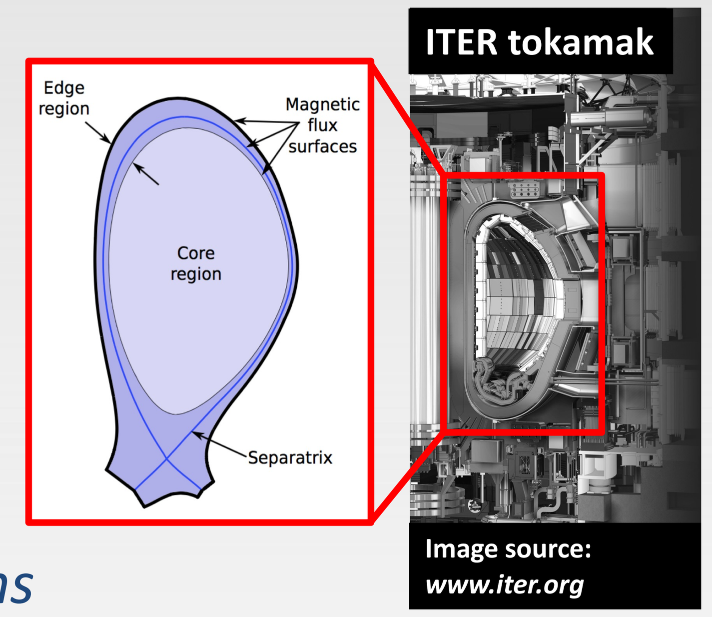
Challenges in Simulating Tokamak-Edge Plasma Dynamics



Kinetic effects are essential

- Strong deviations from the Maxwellian distribution
- Large poloidal variation in the electrostatic potential

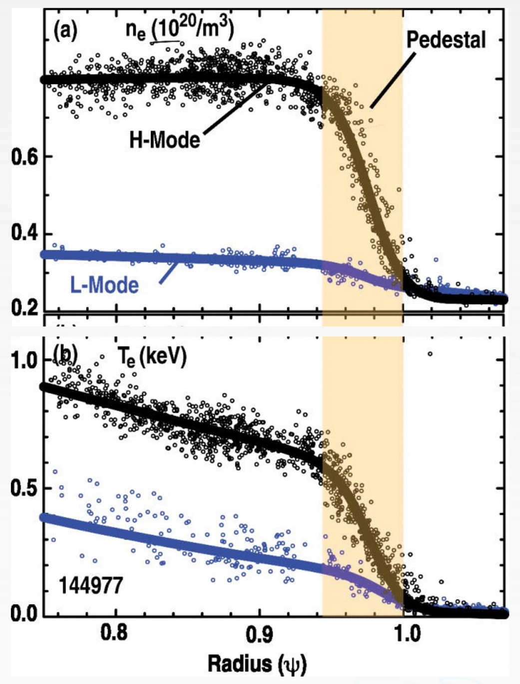
High-dimensional governing equations

Complicated geometry & anisotropy

- Magnetic separatrix and X-point
- Physical boundaries
- **Strong magnetic field** implies parallel advection larger than perpendicular drifts

Collision regimes vary rapidly

- Weakly-collisional in the hot core
- Strongly-collisional in the cold edge



A. W. Leonard, Phys. Plasmas 21, 090501 (2014)

Plasma Timescales and Time Integration

Tokamak edge plasma dynamics is characterized by a **large range of time scales**

Perpendicular Ion Transport | *Ion Acoustic Modes* | *Electrostatic Sound Wave* | *Parallel Electron Transport* | *Plasma Frequency*

Slow ⟵ ⟶ **Fast**

Parallel Ion Transport | *Ion-Ion Collisions* | *Electrostatic Alfven Waves*

Explicit time-integration

constrained by *fastest time scale in the model*

- *Inefficient when resolving slow dynamics*

Implicit time-integration

requires solution to *nonlinear system of equations*

- *Unconditional stability*
- *Pay for inverting terms we want to resolve?*

Implicit-Explicit (IMEX) Time Integration

Resolve scales of interest; Treat implicitly faster scales

$$\frac{d\mathbf{y}}{dt} = \mathcal{R}(\mathbf{y}) \qquad \boxed{\mathcal{R}(\mathbf{y}) = \mathcal{R}_{\text{nonstiff}}(\mathbf{y}) + \mathcal{R}_{\text{stiff}}(\mathbf{y})}$$

Tokamak: Toroidal magnetic confinement fusion device

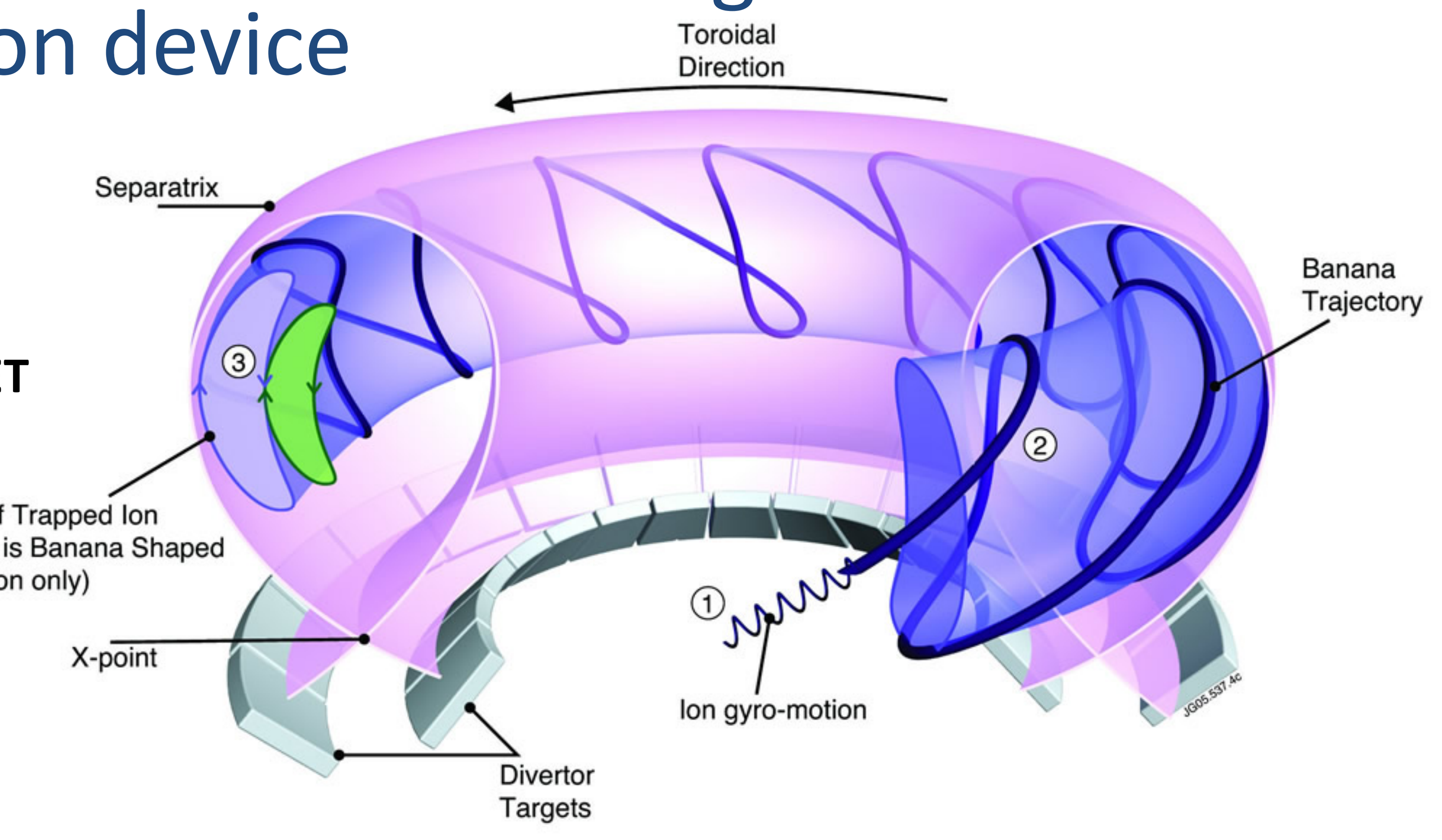
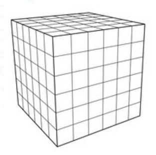


Image: EFDA-JET

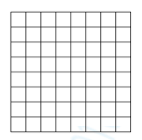
Governing Equations: *Cross-separatrix Transport Model with Self—Consistent Electric Fields*

Phase-space collisional drift-kinetic model (4D/5D) – ***ion species***



$$\frac{\partial\left(B^*_{\parallel\alpha}f_\alpha\right)}{\partial t} + \nabla_{\mathbf{X}}\cdot\left(\dot{\mathbf{X}}_\alpha B^*_{\parallel\alpha}f_\alpha\right) + \frac{\partial}{\partial v_\parallel}\left(\dot{v}_{\parallel\alpha}B^*_{\parallel\alpha}f_\alpha\right) = \mathcal{C}\left[B^*_{\parallel\alpha}f_\alpha\right]$$

Configuration-space self-consistent, quasineutral model (2D/3D) – ***electrostatic potential***



$$\frac{\partial}{\partial t}\left[\nabla_\perp\cdot\left(\frac{e^2 n_i}{m_i\Omega_i^2}\nabla_\perp\phi\right)\right] = \nabla_\perp\cdot\mathbf{j}_{i,\perp} + \nabla_\parallel\left[\sigma_\parallel\left(\frac{1}{e n_i}\nabla_\parallel P_e - \nabla_\parallel\phi + \frac{0.71}{e}\nabla_\parallel T_e\right)\right] - \nabla_\perp\cdot\left(\frac{c^2 m_i n_i \nu_{ex}}{B^2}\nabla_\perp\phi\right)$$

Partitioned system of ODEs for IMEX time integration

$$\frac{d}{dt}\left[\mathbb{M}(\mathbf{U})\right] = \mathcal{R}_{\text{nonstiff}}(\mathbf{U}) + \mathcal{R}_{\text{stiff}}(\mathbf{U})$$

where

$$\mathbf{U} \equiv \begin{bmatrix}\mathbf{f}\\ \boldsymbol{\Phi}\end{bmatrix},\quad \mathbb{M} \equiv \begin{bmatrix}\mathcal{I} & 0\\ 0 & \mathcal{M}\end{bmatrix},\quad \mathcal{R}_{\text{nonstiff}} \equiv \begin{bmatrix}\mathcal{V}(\mathbf{f},\boldsymbol{\Phi})\\ \mathcal{R}_\perp(\mathbf{f},\boldsymbol{\Phi})\end{bmatrix},\quad \mathcal{R}_{\text{stiff}} \equiv \begin{bmatrix}\mathcal{C}(\mathbf{f})\\ \mathcal{R}_\parallel(\mathbf{f},\boldsymbol{\Phi})\end{bmatrix}$$

$$\mathbf{f} = \begin{bmatrix}\vdots\\ B^*_{\parallel\alpha}f_\alpha\\ \vdots\end{bmatrix},\quad \boldsymbol{\Phi} = \begin{bmatrix}\vdots\\ \phi\\ \vdots\end{bmatrix}$$

(vectors of solution at grid points)

Fast timescales: ***kinetic collisions and parallel current divergence***

Additive Runge-Kutta (ARK) Methods: *Modified for nonlinear LHS term*

$$\mathbb{M}\left(\mathbf{U}^{(i)}\right) = \mathbb{M}\left(\mathbf{U}^n\right) + \Delta t\left[\sum_{j=1}^{i-1} a_{ij}\mathcal{R}_{\text{nonstiff}}\left(\mathbf{U}^{(j)}\right) + \sum_{j=1}^{i}\tilde{a}_{ij}\mathcal{R}_{\text{stiff}}\left(\mathbf{U}^{(j)}\right)\right]$$

$$\mathbb{M}\left(\mathbf{U}^{n+1}\right) = \mathbb{M}\left(\mathbf{U}^n\right) + \Delta t\sum_{i=1}^{s} b_i\left[\mathcal{R}_{\text{nonstiff}}\left(\mathbf{U}^{(i)}\right) + \mathcal{R}_{\text{stiff}}\left(\mathbf{U}^{(i)}\right)\right]$$

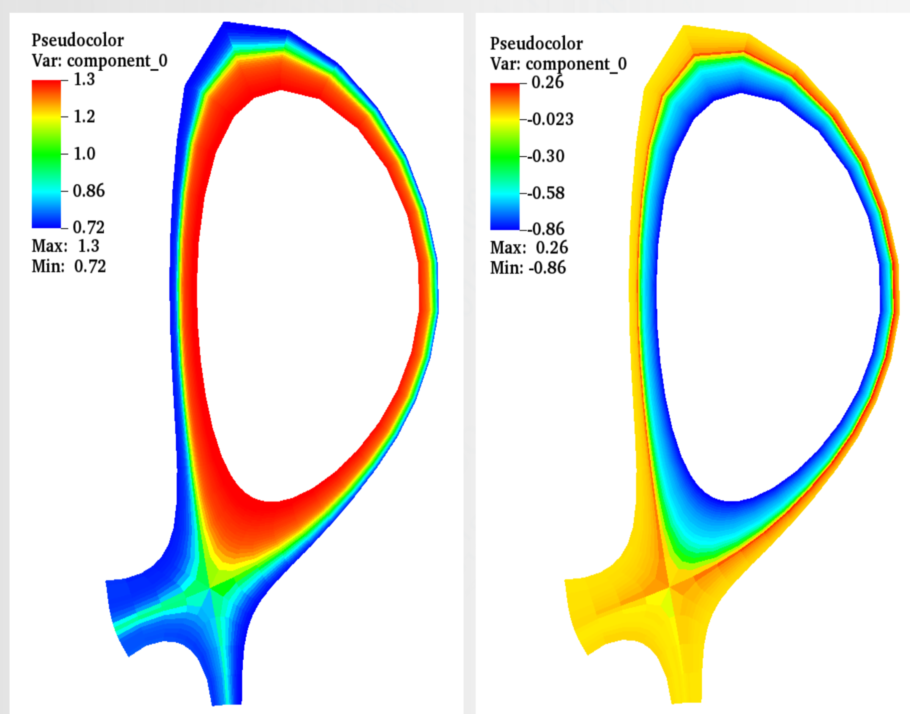
Note: *"Explicit" stages and step completion* ***also require solution to nonlinear system of equations***

System of equations solved by the ***preconditioned Jacobian-free Newton-Krylov (JFNK)*** method

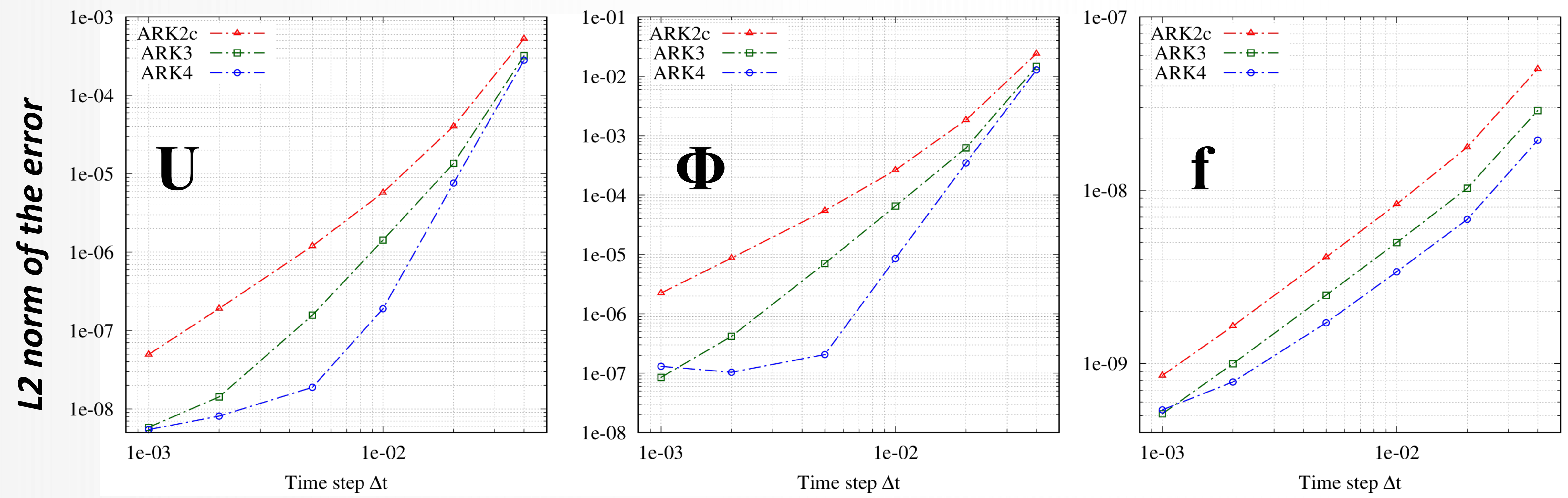
Plasma equilibration under H-mode parameters in DIII-D tokamak

Reference: *Dorf & Dorr, 2018, Contrib. Plasma Phys.*

Solution at $t = 2.8ms$



Ion Temperature **Potential**



Electrostatic potential ($\boldsymbol{\Phi}$) converges at the theoretical orders (*semi-implicit in time*, with ***nonlinear LHS operator***); **Distribution function** ($\mathbf{f}$) converges at ~***1st order*** (*completely explicit* in time)

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