

# A Scalable Particle-Based Microphysics Model for Atmospheric Flow Simulations

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# Modeling Microphysics



**Cloud Microphysics:** Dynamics and interactions of aerosol, cloud, and precipitating particles in atmospheric flows

$q_v$ : vapour  
 $q_c$ : cloud  
 $q_r$ : rain

} Microphysics variables (density fractions)

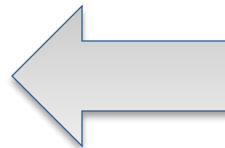
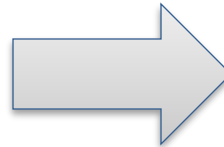
**At each time step:**  $t^n \rightarrow t^{n+1}$

## Fluid Solver

- Update the state variables based on the moist Euler/Navier-Stokes equations

$$\begin{bmatrix} \rho \\ \rho \mathbf{u} \\ \rho \theta \end{bmatrix}^n \rightarrow \begin{bmatrix} \rho \\ \rho \mathbf{u} \\ \rho \theta \end{bmatrix}^{n+1}$$

- Advect moisture variables with the current flow velocity ( $\mathbf{u}^n$ )



## Microphysics Model

- Update moisture variables as specified by the model

$$\begin{bmatrix} q_v \\ q_c \\ q_r \\ \vdots \end{bmatrix}^n \rightarrow \begin{bmatrix} q_v \\ q_c \\ q_r \\ \vdots \end{bmatrix}^{n+1}$$

- Update state variables ( $\theta$ )

# Types of Microphysics Models

Computational Expense

## **Bulk Models:**

Evolve averaged moisture quantities ( $q_v, q_c, \dots$ )

- Computationally cheap – evolve ODEs along with flow equations
- Limited accuracy due to empirical models of droplets dynamics
- Examples: *Kessler*, *Single-Moment*

$$\frac{d}{dt} \begin{bmatrix} q_v \\ q_c \\ q_r \\ \vdots \end{bmatrix}^n = F(q_v, q_c, \dots)$$

## **Super-Droplets Method (SDM):**

Particle-based model for simulating cloud & rain

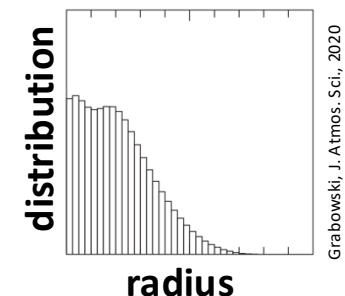
- Include fundamental droplet dynamics
- “Super-particle” approach for acceptable cost
- Examples: *PySDM*, *libcloudph++*, *SCALE-SDM*



## **Bin Methods:**

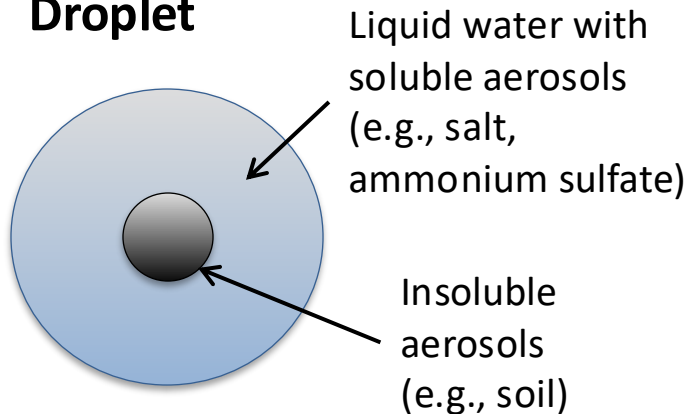
Evolve droplet density distributions at each grid point based on dynamics

- Evolve the spectral density function discretized in droplet size
- Potentially very accurate since they model droplet dynamics
- Computational expense is prohibitive for practical applications



# What is a Super-Droplet?

## Droplet

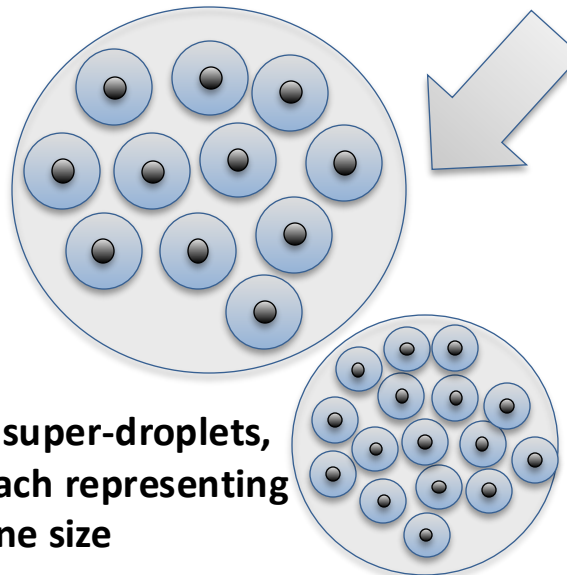
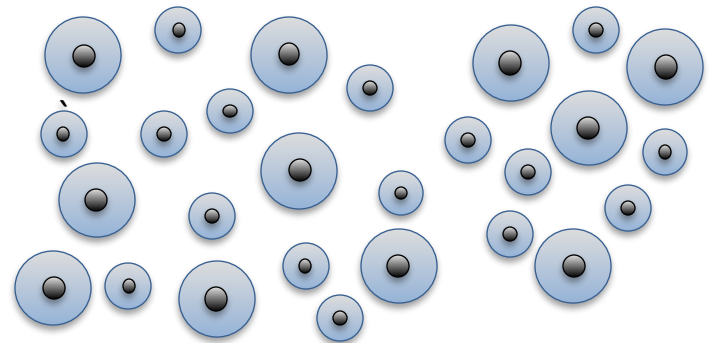


*Assumed to be a sphere*

### Physical Attributes

- Position
- Velocity
- Terminal velocity
- Radius
- Aerosols and their masses

## Droplets of 2 different sizes



## “Super-droplet”

- Represents multiple droplets of the same size
- All physical attributes assumed to be the same
- Computational attribute: **multiplicity** (number of physical droplets a super-droplet represents)

# Physical Processes

**Particle Motion:** New position computed with first-order update

$$\mathbf{x}_p^{n+1} = \mathbf{x}_p^n + \Delta t \left( \mathbf{u}_p - v_t \hat{\mathbf{k}} \right)$$

**Advection:**  $\mathbf{u}_p$  is computed at particle location from flow velocity

**Terminal Velocity:**  $v_t$  computed from particle size and flow conditions

**Droplet Growth/Shrinking:**

Size change due to

condensation and evaporation

*Stiff ODE solved implicitly (Backward Euler + Newton) for each super-droplet*

The diagram illustrates the droplet growth/shrinking equation with the following components:

- Saturation ratio:**  $\frac{q_v}{q_{\text{sat}}}$  (Annotated with "Saturation ratio" in a blue box)
- Curvature effect:**  $\frac{a}{R_i T}$  (Annotated with "Curvature effect" in a blue box)
- Solute effect:**  $\frac{b}{R_i^3}$  (Annotated with "Solute effect" in a blue box)
- Solute mass:**  $i M_i$  (Annotated with "Solute mass" in a blue box)
- Solute mol. weight:**  $m_s$  (Annotated with "Solute mol. weight" in a blue box)
- Radius:**  $R_i$  (Annotated with "Radius" in a blue box)
- Equation:** 
$$R_i \frac{dR_i}{dt} = \frac{(S - 1) - \frac{a}{R_i T} + \frac{b}{R_i^3}}{F_k + F_d}$$
- Forces:**
  - $F_k = \left( \frac{L}{R_v T} - 1 \right) \frac{L \rho_l}{K T}$
  - $F_d = \frac{\rho_l R_v T}{De_s(T)}$

**Coalescence:** due to random collisions between particles

Key process for rain formation from cloud particles

→ Computationally efficient **Monte-Carlo algorithm**

Probability of **collision between two physical droplets**

The diagram illustrates the collision probability equation with the following components:

- Collision kernel:**  $C(r_i, r_j)$  (Annotated with "Collision kernel (e.g., Hall, 1980)" in a blue box)
- Velocity difference:**  $|v_i - v_j|$  (Annotated with "Velocity difference" in a blue box)
- Time interval and volume:**  $\frac{\Delta t}{\Delta v}$  (Annotated with "Time interval and volume" in a blue box)
- Equation:** 
$$P_{ij} = C(r_i, r_j) |v_i - v_j| \frac{\Delta t}{\Delta v}$$

# Fluid Solver (“DyCore”): ERF

## Energy Research & Forecasting (ERF)

- **Nonhydrostatic atmospheric flow simulation** code:  
Solves the compressible Navier-Stokes equations
- **Built on AMReX** for *scalability* and *portability*: Unified implementation on CPUs and GPUs (NVIDIA, AMD, Intel)
  - C++ with MPI and OpenMP/CUDA/HIP/SYCL
- **Multirate time integration** and **high-order** (2<sup>nd</sup> to 6<sup>th</sup>) **spatial discretization**
- Block-structured grids with **AMR**
  - **Terrain-conforming** coordinates
  - **Embedded boundaries** for urban geometries



<https://github.com/erf-model/ERF>



# Super-Droplet Method in ERF

**Implementation:** Super-droplets using **AMReX Particle** and **ParticleContainer** classes & functions

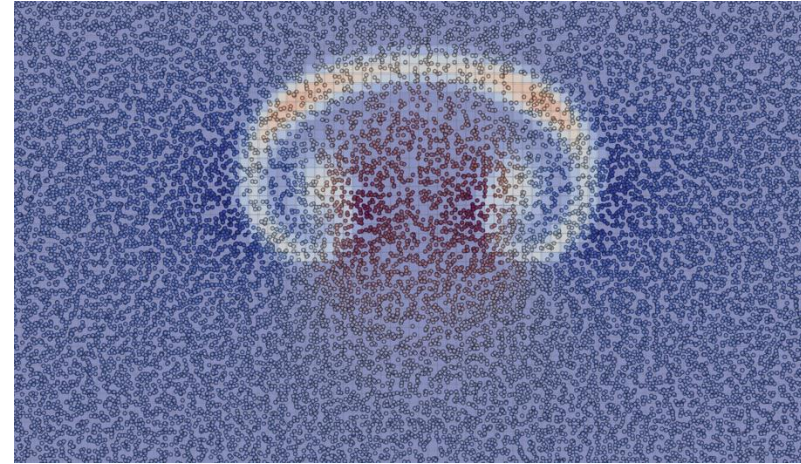
## Time Evolution

### Grid

- Update the state variables based on the moist Navier-Stokes equations
- Advect  $q_v$  with the current flow velocity ( $\mathbf{u}^n$ )

### Particles to Grid

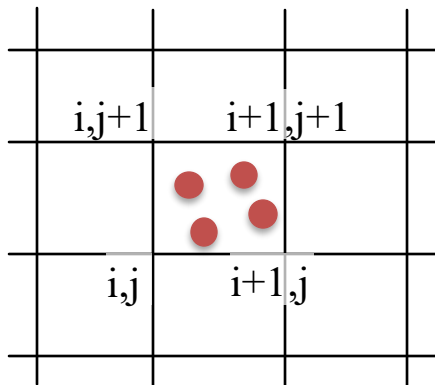
- Update moisture variables based on particles
- Compute  $q_c(\mathbf{x})$  and  $q_r(\mathbf{x})$  from particle positions and masses
  - Update by  $q_v$  subtracting  $q_c + q_r$
  - Update  $\theta$  due to latent heat of vaporization



### Particles

- Update super-droplets attributes based on droplet dynamics
- **Advection** and **terminal velocity** – position update
  - **Condensation** and **evaporation** – radius/mass
  - **Coalescence** – radius/mass

# Initialization of Super-Droplets



**Initial position:** Super-droplets are placed randomly within each grid cell with **zero initial velocity**

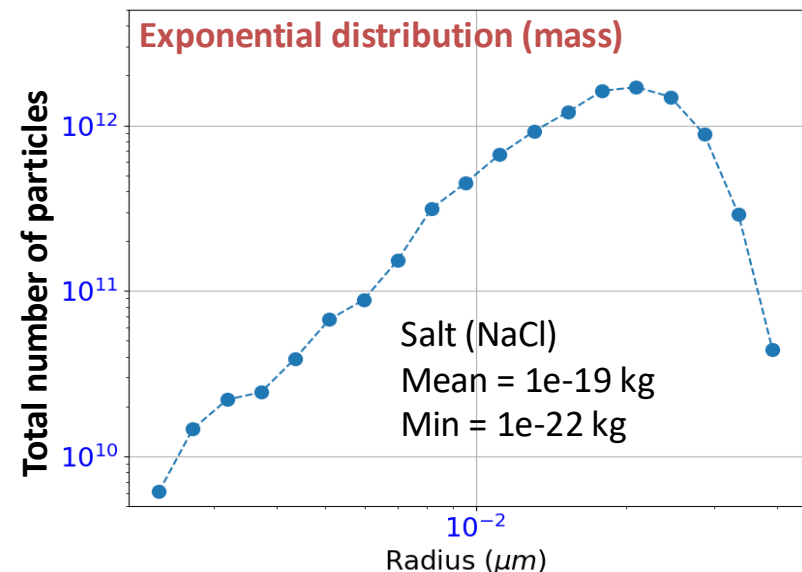
Physical number density  
(may vary spatially)

Initial number of  
super-droplets per cell

**Initial multiplicity**

**Aerosol masses** and **droplet radius** for each super-droplet are sampled from a specified distribution

- **Aerosol Species:** Salt, Ammonium Sulfate, Soil
- **Exponential distribution** for mass
- **Log-normal distribution** for radius
- Sum of multiple distributions (for example, **bimodal distribution**)





# Example: 2D Rising Bubble

## 2D Rising Bubble in Moist Atmosphere

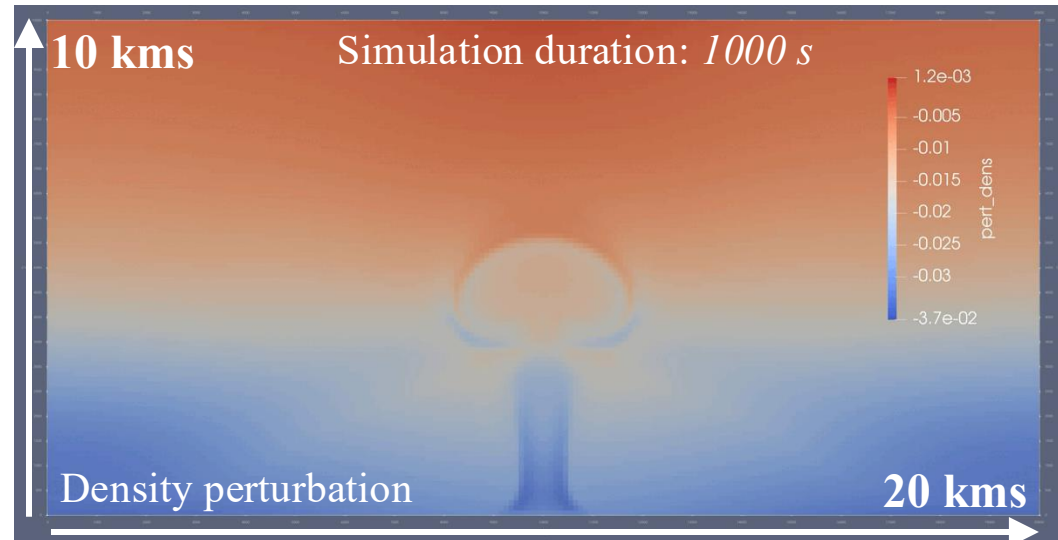
As the bubble rises, moisture is convected upwards and cools down to form clouds and rain

- Domain:  $20\text{ km} \times 10\text{ km}$
- “Slip wall” BCs on all sides
- Warm bubble with radius  $2\text{ km}$  initially located at  $(10\text{ km}, 2\text{ km})$
- Bubble temperature perturbation:  $2\text{ K}$

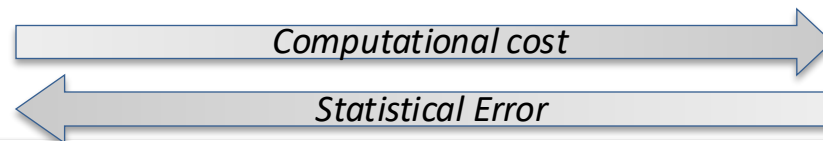
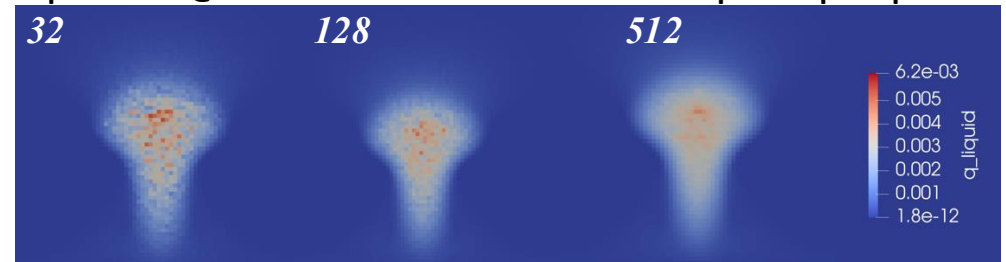
## Computational Setup:

- Grid:  $200 \times 4 \times 100$  ( $100\text{ m}$  resolution)
- Aerosol species: salt (NaCl) - Exponential distribution with mean mass  $10^{-19}\text{ kg}$
- Initial physical concentration:  $1e7\text{ m}^{-3}$
- Initial number of super-droplets per cell: 256

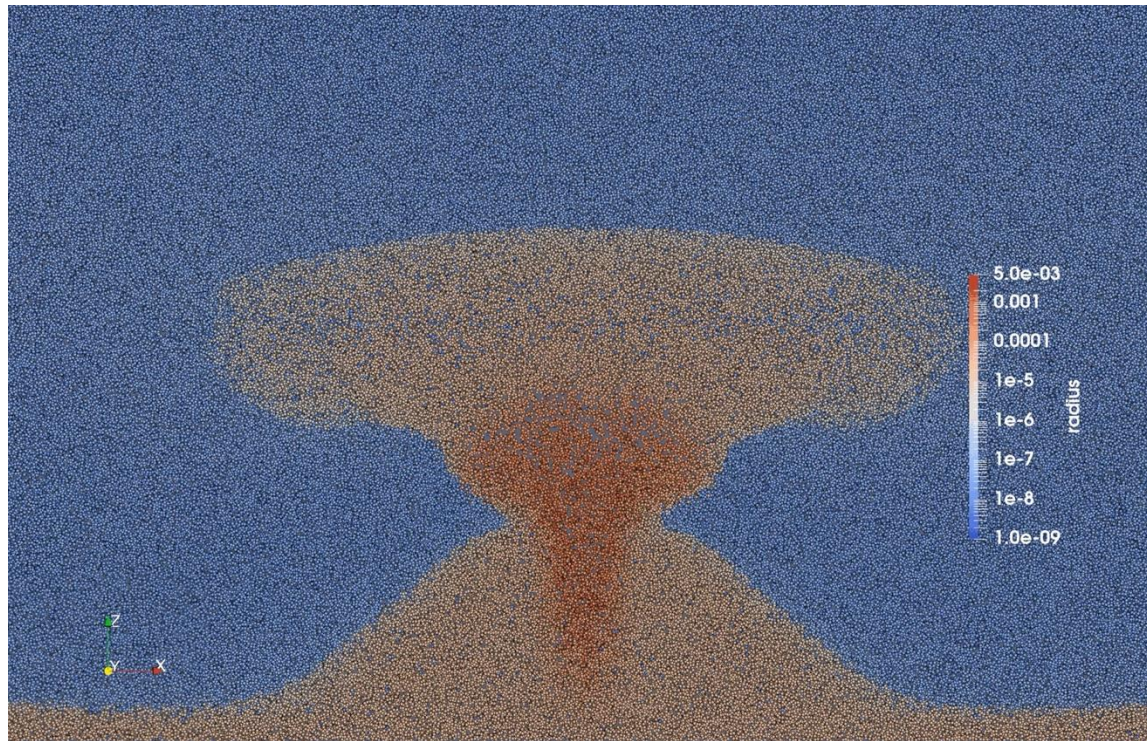
➔ Approx. **20 million super-droplets** representing  $8 \times 10^{17}$  physical particles



Liquid water @500s with various initial number of super-droplets per cell



# Example: 2D Rising Bubble

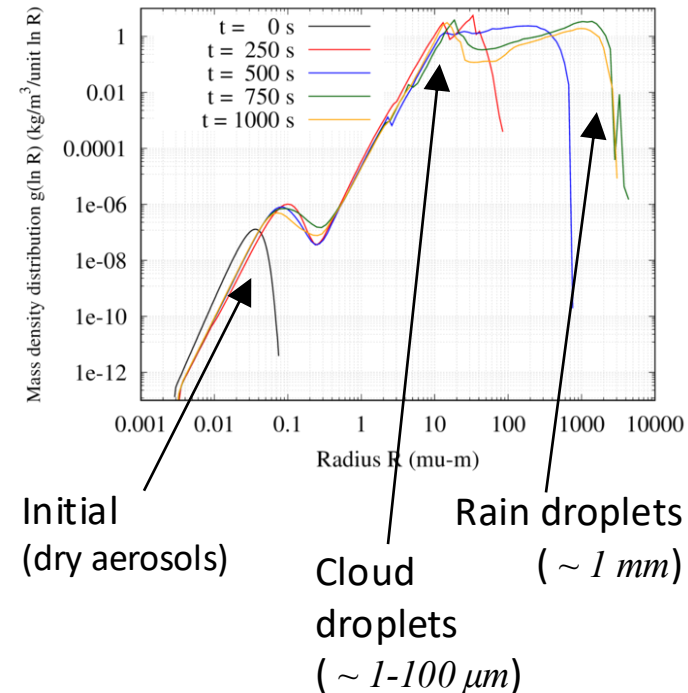


## Visualization of the super-droplets (colored by radius)

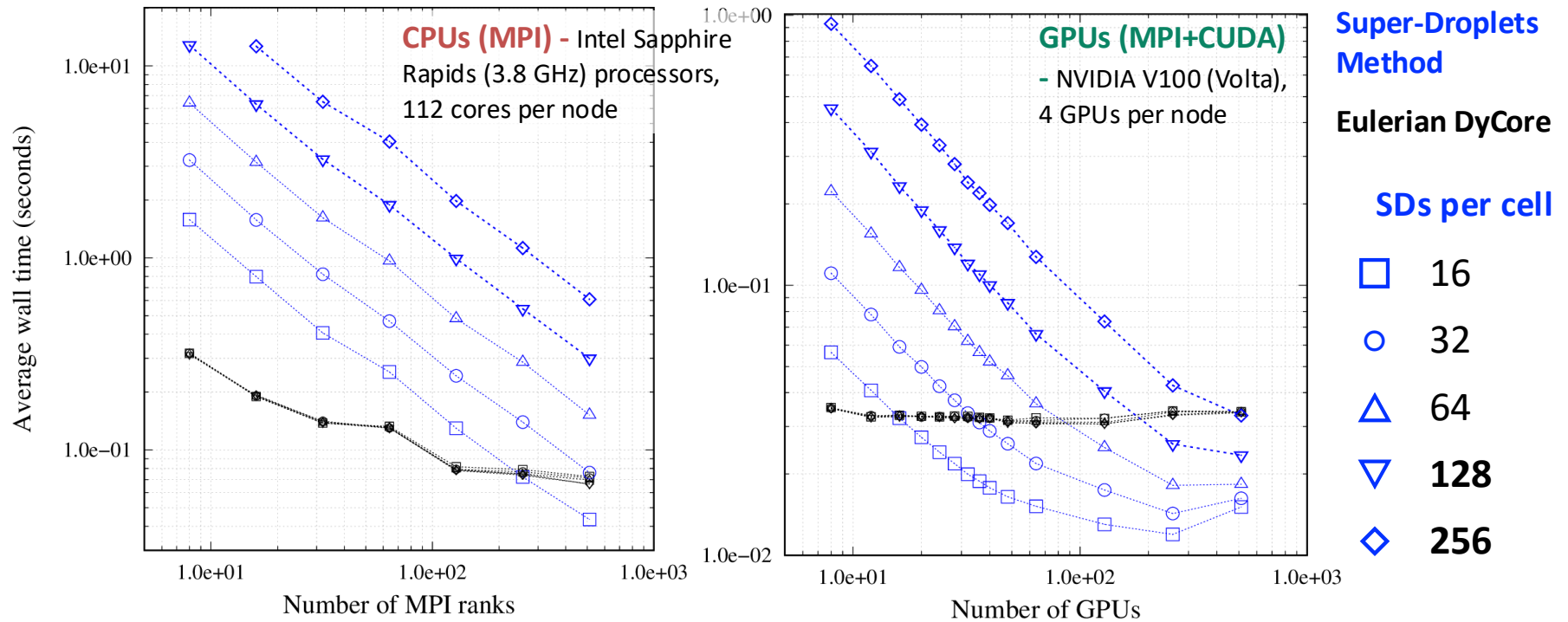
- Simulated with 4 super-droplets per cell to allow plotting
- Super-droplets convect upwards with the flow and grow due to condensation
- Coalescence causes formation of rain that precipitates

Mass distribution evolves from **unimodal** (dry aerosols) to **bimodal** (aerosol + cloud) and **trimodal** (aerosol, cloud, rain)

## Mass distribution evolution



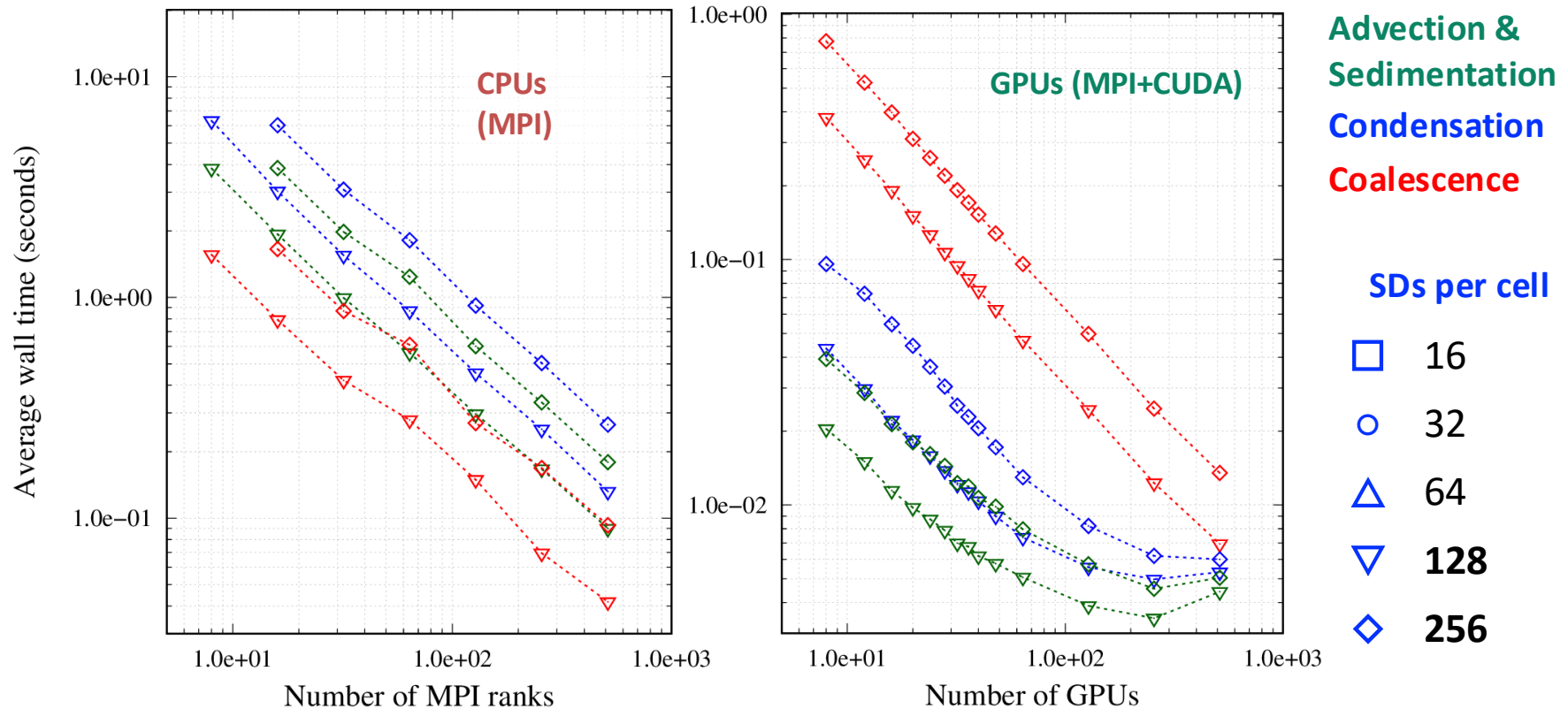
# Strong Scaling on CPUs and GPUs



- Grid: **512 x 4 x 512** points (*~17 million to ~2.4 billion particles*)
- Number of CPUs (MPI ranks) or GPUs: **8 to 512**
- **Good strong scaling** observed for SDM on both CPUs and GPUs; note that Eulerian DyCore doesn't scale well in this setup since it is *over-decomposed on CPUs and doesn't fill the GPUs*.



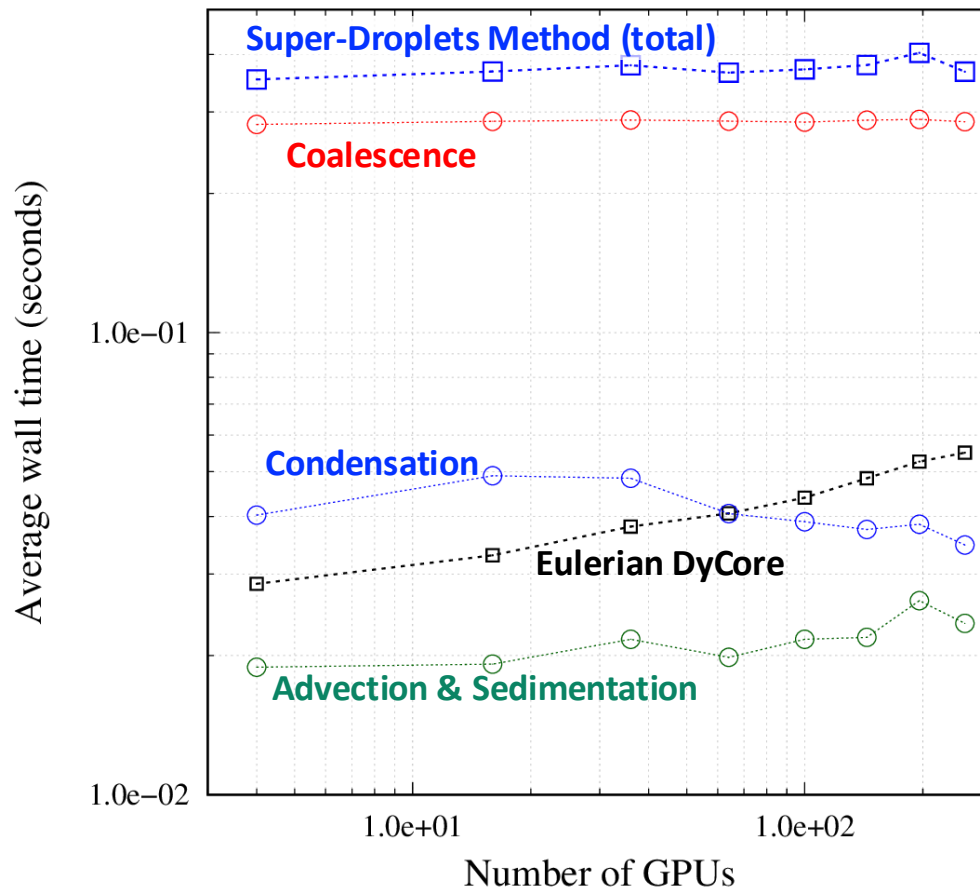
# Scaling of SDM Processes



**CPU-only:** Condensation is the most expensive since it involves implicit solution of ODE for each super-droplet

**With GPUs:** Coalescence is the most expensive on GPUs since it involves the Monte-Carlo collisions algorithm

# Weak Scaling on GPUs (MPI + CUDA)



- **On each GPU:** 128 x 4 x 128 grid with 256 superdroplets per cell  
→ ~16.8 million particles per GPU
- **Number of GPUs:** 4 to 256  
→ ~67 million to 4.3 billion total particles
- Most MPI communications *in Eulerian DyCore* (filling ghost cells); *in SDM, only particle redistribution*
- Excellent weak scaling observed for the SDM and all component processes.

# Work-in-Progress...

Implemented a **Lagrangian moisture model in ERF** based on the **super-droplets method**

- Limited to simulation of flows under warm conditions (**no ice/snow**)
- Computationally *more expensive than bulk models*
  - Incorporates higher fidelity droplet dynamics
  - Does not rely on empirical models of phase change
  - With GPUs, computational expense is acceptable
- **Excellent scalability** from using **AMReX's particle implementation**
- Currently working on **verifying/validating implementation** for various cases (Congestus clouds, cloud chamber, etc.)

## Future plans:

- Implement cold processes (simulate formation of ice/snow/graupe)l)
- Incorporate terrain into super-droplets dynamics



**Thank you. Questions?**

# Implementation and Parallelism



**Super-droplets** are implemented using the **Particle** and **ParticleContainer** classes and utilities in **AMReX**



**Portable** and **scalable** on various heterogenous architectures

- MPI is used for domain decomposition over multiple CPUs/nodes
- On-node parallelism using CUDA/HIP on GPUs or OpenMP on CPUs

Advection

Condensation & Evaporation



Independent for each particle  
→  $O(N_p)$  parallelizable

$N_p$ : number of particles

$N_g$ : number of grid cells

$$N_p \gg N_g$$

Coalescence (Monte-Carlo Algorithm)

Shuffling & pairing

Independent for each grid cell →  $O(N_g)$  parallelizable

Attribute update

Independent for each particle →  $O(N_p)$  parallelizable

Computing Eulerian moisture variables from particles

Independent for each grid cell →  $O(N_g)$  parallelizable

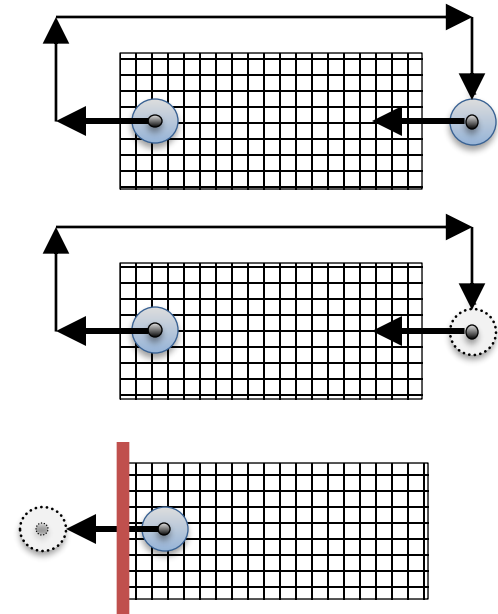
# Boundary Treatment

**Periodic Boundary:** Super-droplet re-enters domain from the other side with attributes preserved

**Inflow/Outflow:** Super-droplet re-enters domain from the other side *as dry aerosol*

**Side and Top Walls:** Super-droplet gets “deactivated” - velocities set to 0, multiplicities set to 0, *does not participate in the simulation anymore*

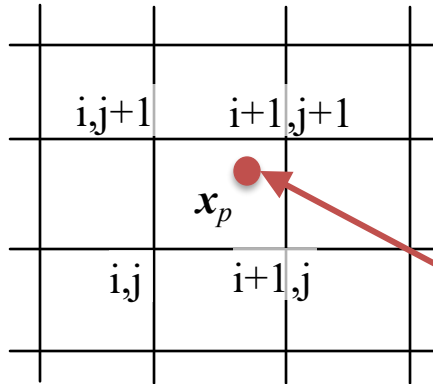
**Ground:** Same as side/top walls, but *rain accumulation on ground is updated* based on super-droplet mass and multiplicity



## Recycling:

Put back deactivated super-droplet as dry aerosol at a random location in domain

# Advection & Terminal Velocity



**New position** computed with first-order update

$$\mathbf{x}_p^{n+1} = \mathbf{x}_p^n + \Delta t \left( \mathbf{u}_p - v_t \hat{\mathbf{k}} \right)$$

**Advection:**  $\mathbf{u}_p$  is computed at particle location from flow velocity

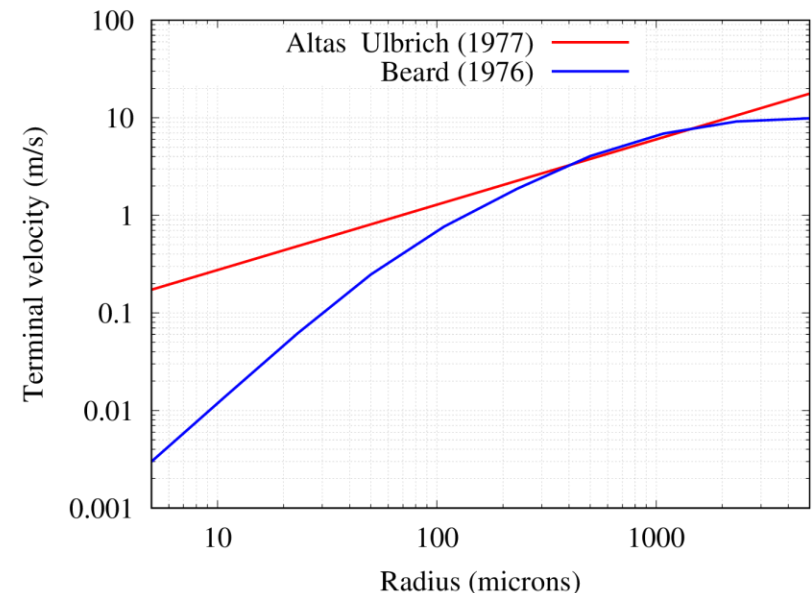
Eulerian flow variables are computed at particle location by **linear interpolation**

## Terminal Velocity Models:

**Atlas & Ulbrich (1977)** function of particle size  $v_t = 3.778D^{0.67}$

**Beard (1976):** Considers three regimes

- **Stoke's:** diameter less than 20 microns
- **Transitional:** 20 microns to 1 mm
- **Newton's:** larger than 1 mm



# Condensation/Evaporation

**Droplet size equation**

$$R_i \frac{dR_i}{dt} = \frac{(S-1) - \frac{a}{R_i T} + \frac{b}{R_i^3}}{F_k + F_d}$$

Droplet radius  $R_i$   
 Saturation ratio  $\frac{q_v}{q_{sat}}$   
 Curvature effect  $\frac{a}{R_i T}$   
 Solute effect  $\frac{b}{R_i^3}$   
 where  $F_k = \left( \frac{L}{R_v T} - 1 \right) \frac{L \rho_l}{K T}$ ,  $F_d = \frac{\rho_l R_v T}{De_s(T)}$

$b = 4.3 \times 10^{-6} \frac{i M_i}{m_s}$   
 Solute mass  $i M_i$   
 Solute molecular weight  $m_s$

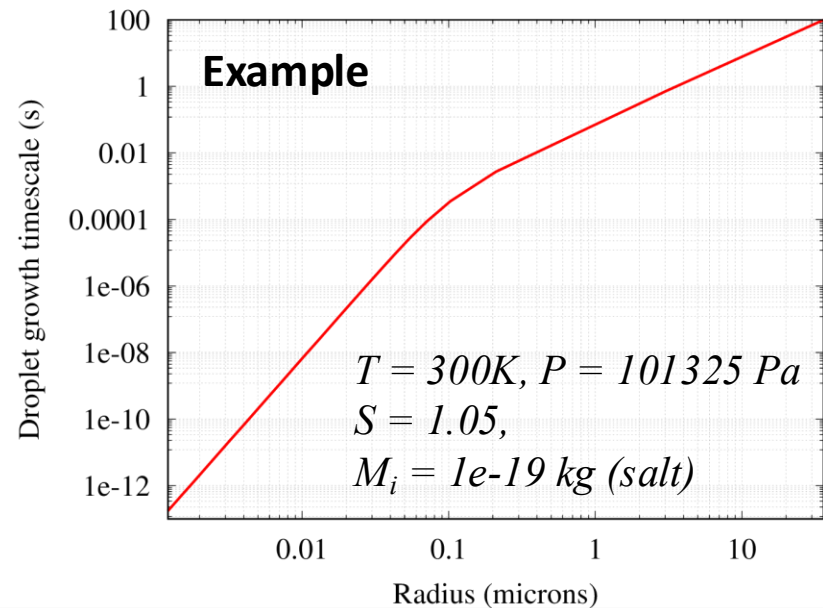
Growth timescales are **much smaller** than fluid convection/acoustic timescales

$$\tau^{-1} = \frac{1}{2} \left| \frac{1}{F_k + F_d} \left( -\frac{2a}{R_i^3 T} + \frac{6b}{R_i^5} \right) \right|$$

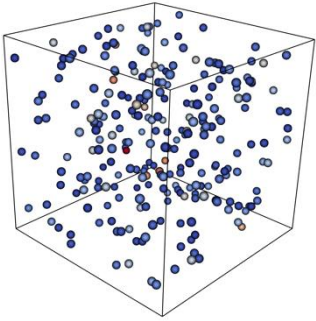


**Sub-stepping** within each ERF timestep

- Solve the ODE for each super-droplet independently
- Backward-Euler time integration with CFL 100
- Newton method to solve the nonlinear equation



# Stochastic Coalescence



**Random collisions of droplets** near each other resulting in coalescence  
**Key process** forming rain droplets from cloud

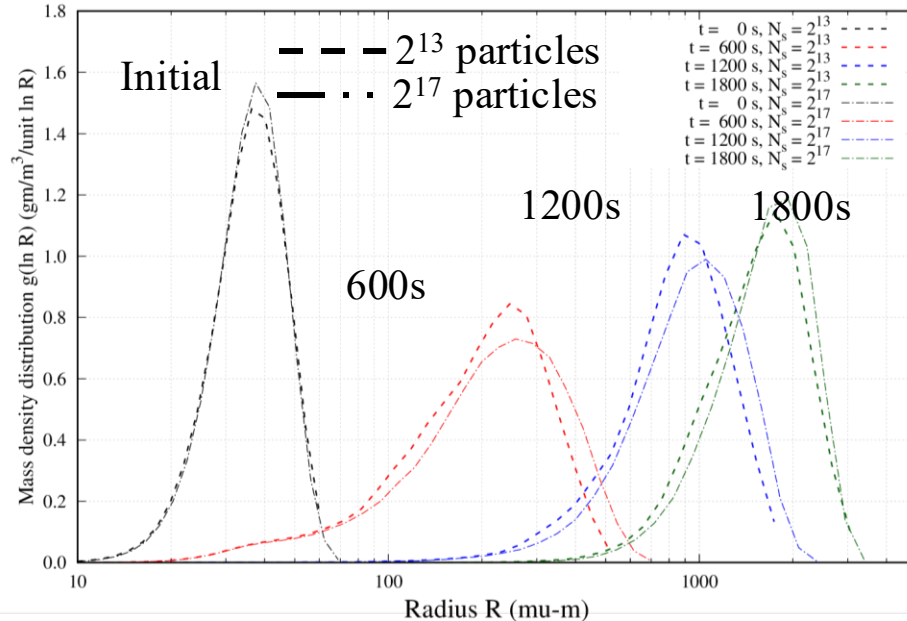
Probability of collision  
 between two physical droplets

$$P_{ij} = C(r_i, r_j) |v_i - v_j| \frac{\Delta t}{\Delta v}$$

Collision kernel  
(e.g., Hall, 1980)
Velocity difference
Time interval and volume

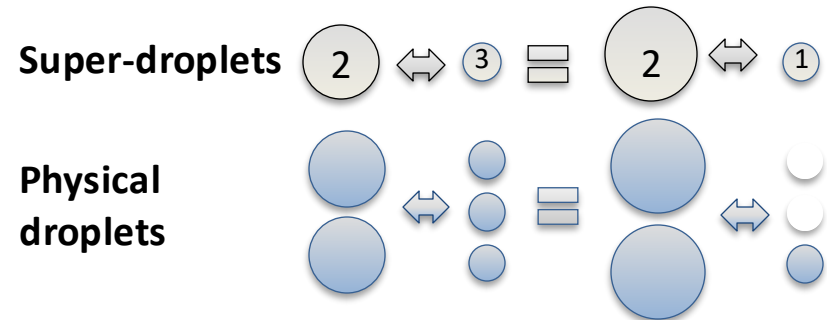
**Validation** in a box (no flow) – Hall kernel

Good agreement with results in *Shima, et al, 2009*



**Monte-Carlo algorithm** for super-droplets:

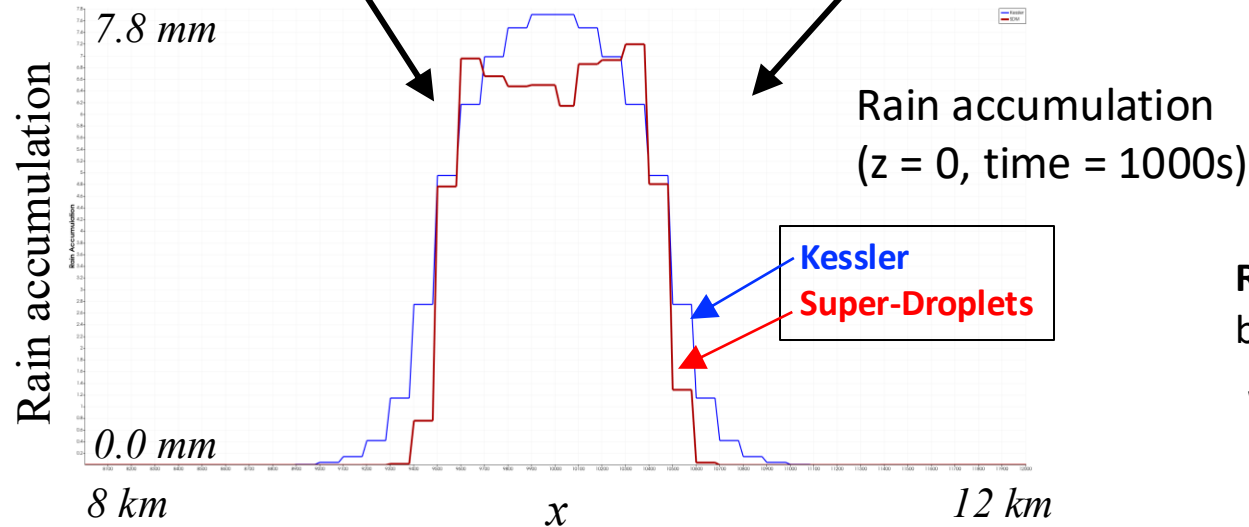
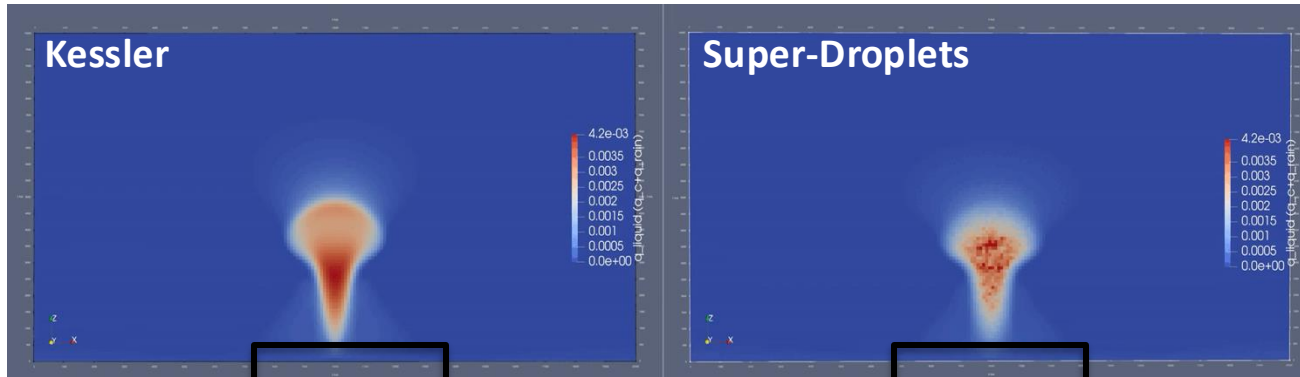
- In each grid cell, *shuffle particles*, split into two groups, and *create pairs*
- Compute *probability of collision* for each pair
- If they collide, *update super-droplets attributes*



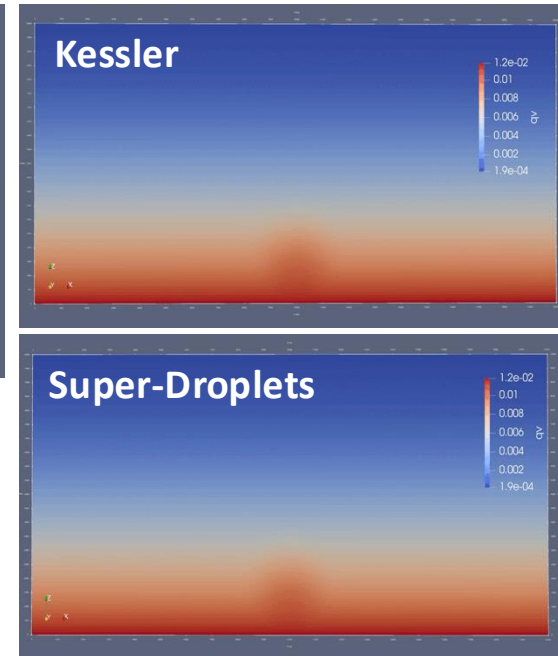


# Example: 2D Rising Bubble

Total liquid water fraction  $q_c + q_r$



Vapour fraction  $q_v$



**Reasonable agreement** observed between Kessler and super-droplets

**Wall times** (on 4 V100 GPUs):

- Kessler: 63 seconds
- Super-droplets: 381 seconds