Portfolio Optimization

Results

Introduction

- In this project, we consider a portfolio containing 7 stocks:
  - ADANIPORTS, GAIL, HDFCBANK, INFY, KOTAKBANK, RELIANCE, TATASTEEL
- We study the data from 30th April 2020 - 30th April 2021
- For the above data, we do the following:
  - Compute the *minimum variance portfolio*
  - Draw the *efficient frontier*
  - Calculate *Beta*
  - Compute the *security market line*
- We perform these tasks using **Quadratic Regression** *and* **Monte Carlo** methods
- We then compare the results obtained from both these methods
- Finally, we summarise the results and draw certain conclusions from the results obtained

Method Overview - Quadratic Programming

- Quadratic Programming is a mathematical tool used for optimizing (here minimizing the portfolio risk) a quadratic function under some given linear constraints. Let there be a portfolio of n assets with rate of return of asset i given by a random variable with expectation m_i . We find the fraction or weight x_i one should invest in each asset i so as risk is minimal. Assuming C to be a covariance matrix of rates of asset returns, the mean-variance portfolio consists of minimizing portfolio risk given by,
- Portfolio risk = ½ $x^T Cx$
- The investor expects a minimal rate portfolio return r such that $\sum m_i x_i \geq r$; i= 1, 2,...,n
- $\sum x_i = 1$ as the total probability or in other words the sum of weights must be 1 ; $0 \leq x_i \leq 1$
- As the portfolio risk is a quadratic and the constraints are linear, this optimization problem is an example of quadratic programming.

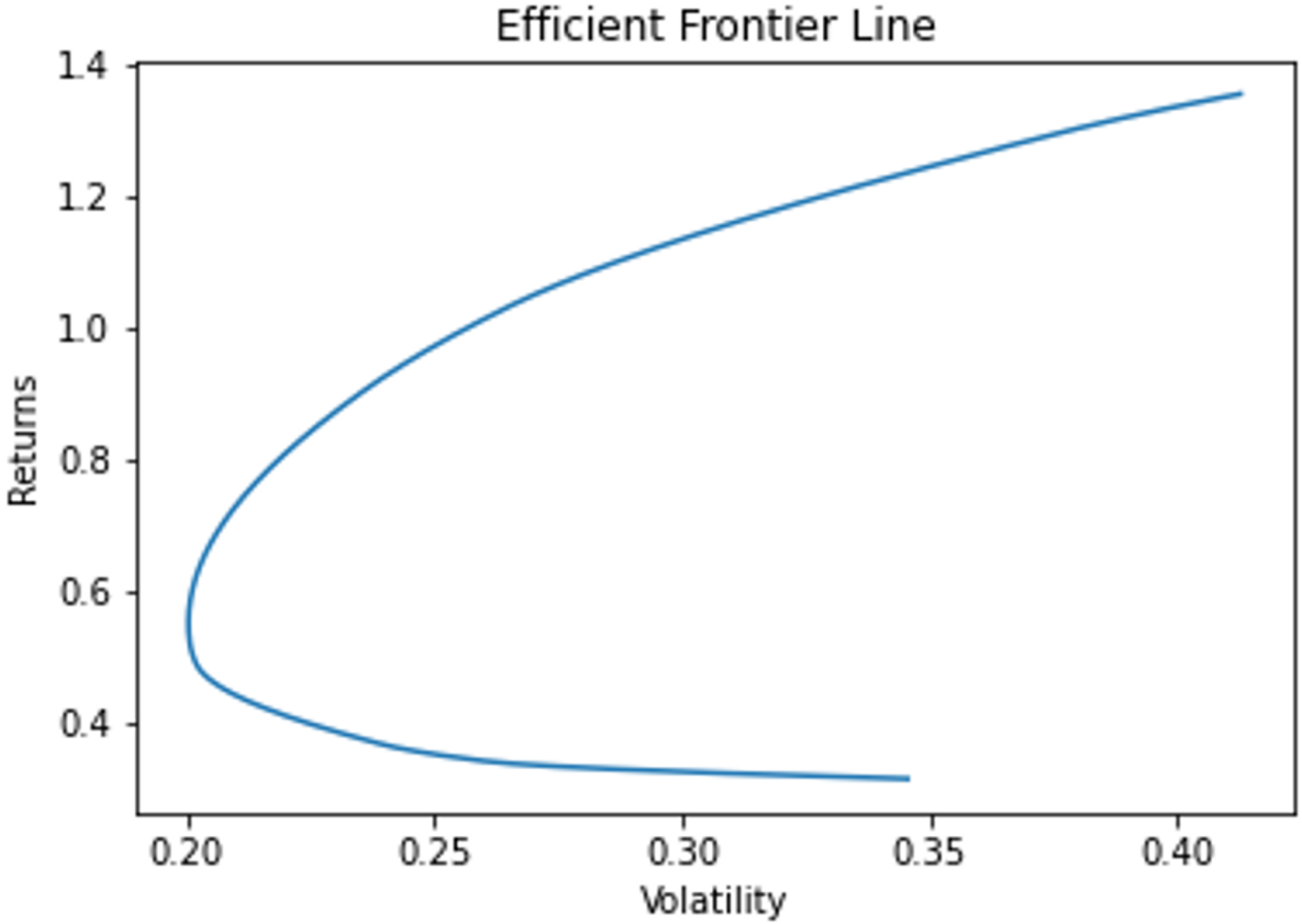
Method Overview - Monte Carlo

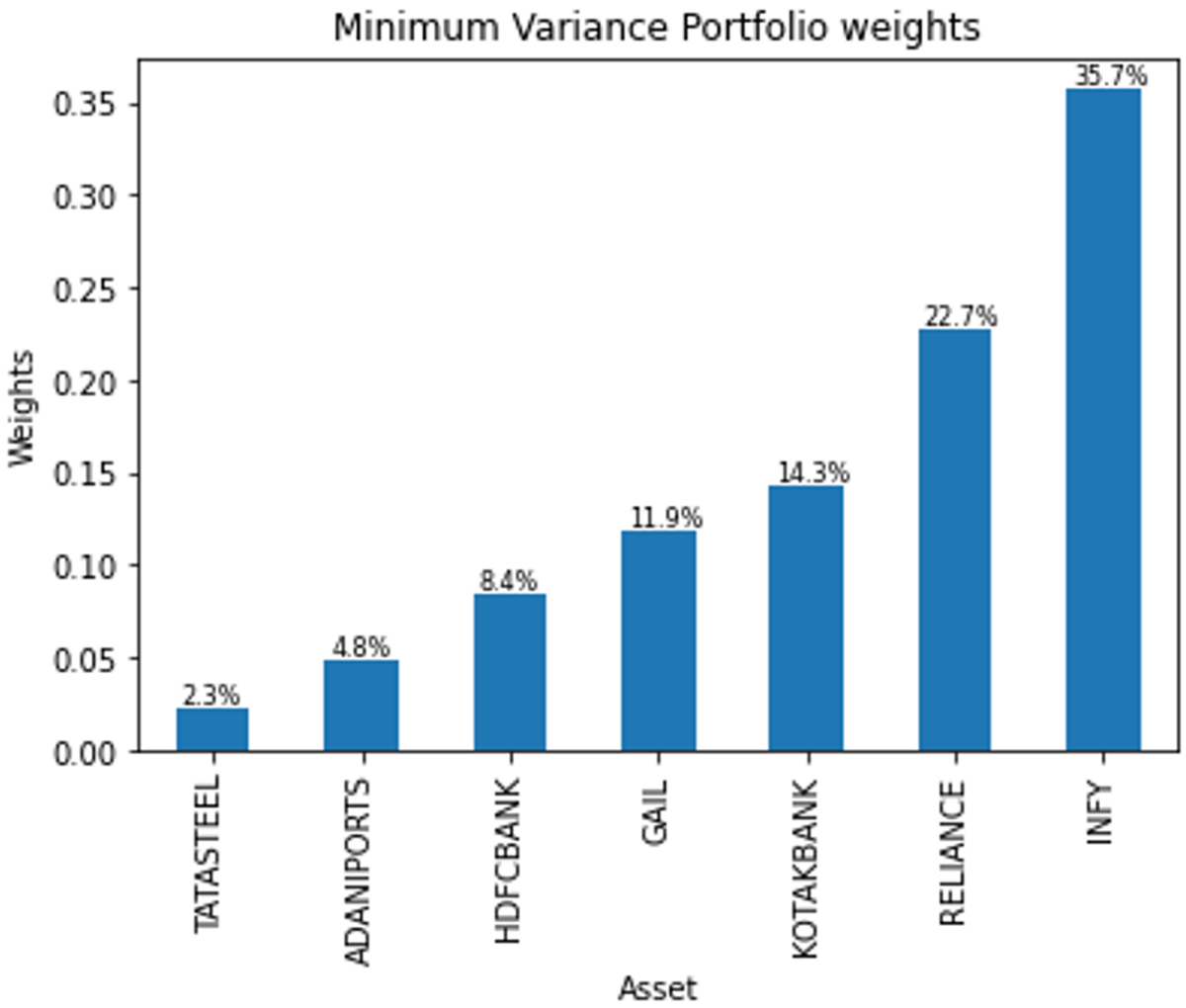
- Unlike deterministic mathematics, Monte Carlo method employs random sampling of data to obtain the result.
- Let's take a basic example to understand the procedure. The problem is to estimate the value of π.
- Consider a 2*2 square centred at the centre of a circle of unit radius. The side of the square is two units. Now, Let Z= 1 if $\sqrt{(X^2 + Y^2)} \leq 0$ and Z= 0 otherwise. Assume that we randomly pick points from inside the square m times (with X and Y being chosen randomly) with Z_i being the value of Z at the i-th run.
- If $W=\sum Z_i$ then, $E[W] = E[\sum Z_i] = \sum E[Z_i] = m\pi/4$
- Now, W' = (4/m)W gives a natural estimates of π.
- Monte Carlo method is usually applied in corporate finance by analysts who prefer a probabilistic approach to a given problem.
- They model the various sources of uncertainty in the portfolio to estimate a rough numerical value under some given inputs.

QUADRATIC PROGRAMMING

Code and Parameters

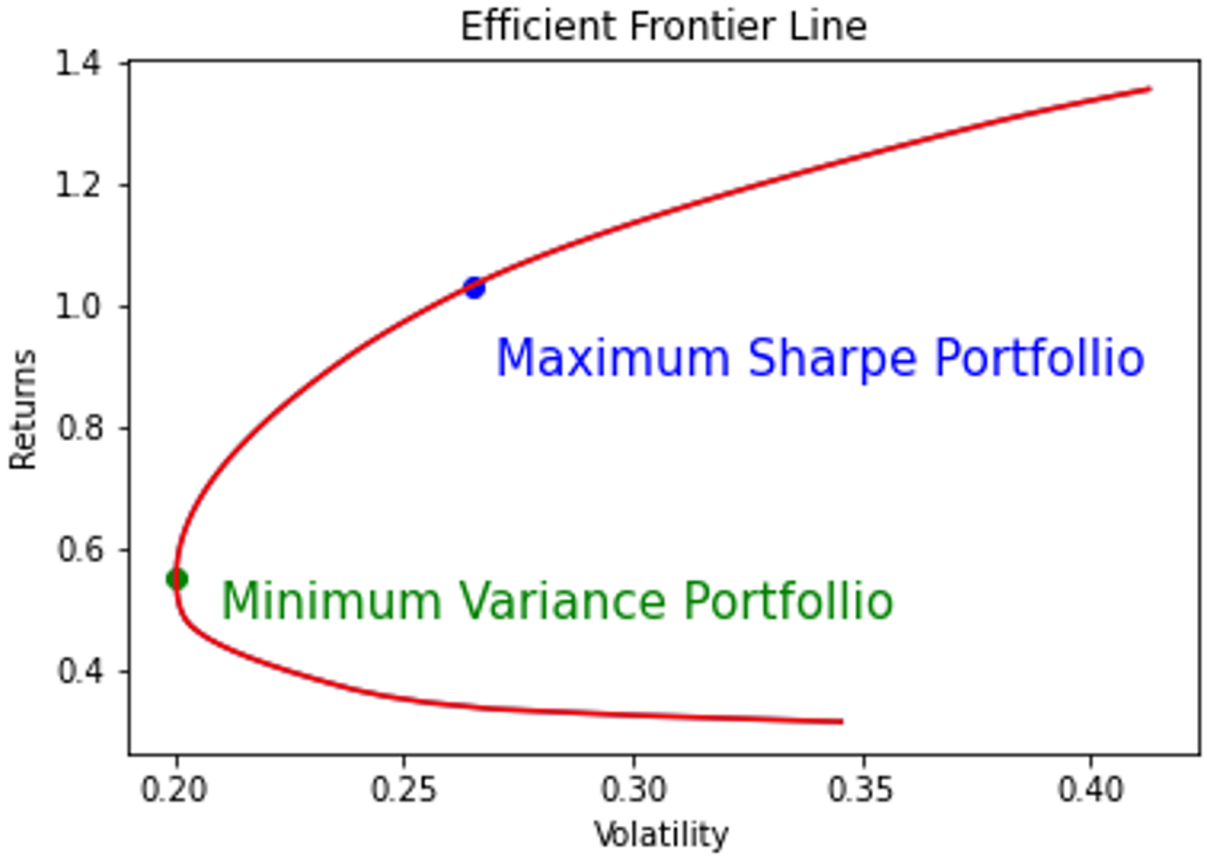
- The main segment of our program is divided into four tasks
- **Efficient Frontier Line:** The main task is performed by the function minimize_vol(). This function takes an initial guess and with the help of the given constraints (weights_sum_to_1 & return_is_target) & the portfolio_vol data. We use the minimize function (SLSQP) in order to minimise the risk for the portfolio and we plot return (vs) volatility graph.
- **Minimum variance portfolio:** The main function here is to plot the minimum variance portfolio weights for the given stocks (weights in % (vs) assets). We take min_var as the input (`min_var = df.Weights[df.Volatility.argmin()]`)
- **Beta assets:** We calculate the beta value of the stocks using the formula
  Beta = Covariance of return of asset wrt index/ Variance of index.
- **Security Market Line:** We take assetReturns(rf+(rm-rf)*beta) and betas as input and plot the graph of assetReturns (vs) betas. Here rm is return of the market. Rf is the risk free rate.
- **Tangency line:** We plot volatility (vs) returns tangent at point of maximum Sharpe ratio (where the capital allocation market line is tangent to the efficient frontier line). Intercepts at Rf


Efficient Frontier Line
1.4
1.2
1.0
0.8
0.6
0.4
Returns
0.20
0.25
0.30
0.35
0.40
Volatility



Minimum Variance Portfolio Percentages (Graph Attached)

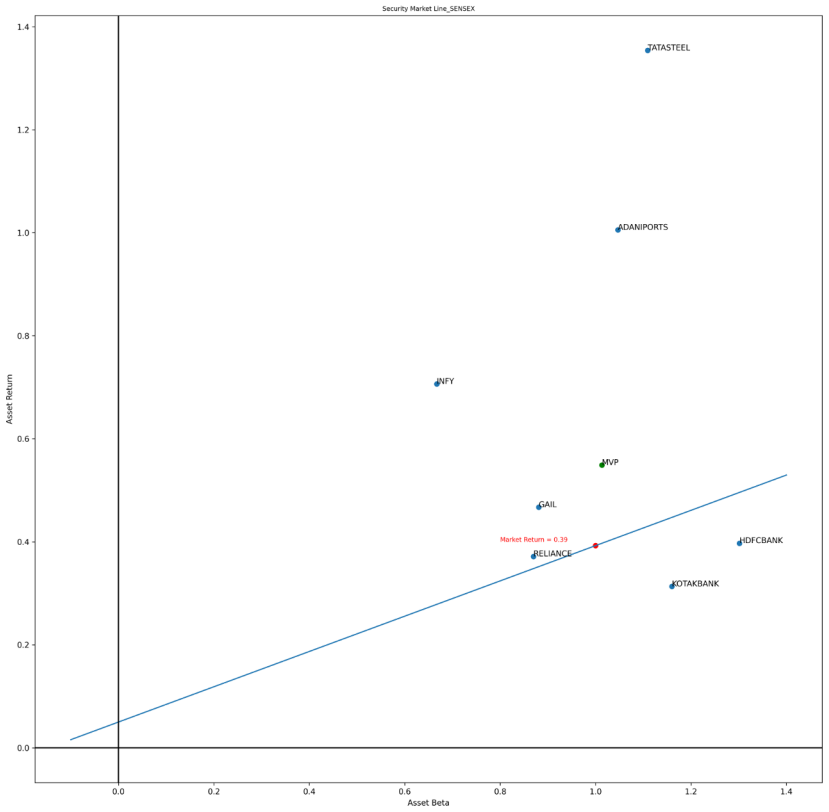
* TATA STEEL : 2.3%
* ADANIPORTS : 4.8%
* HDFCBANK : 8.4%
* GAIL : 11.9%
* KOTABANK : 14.3%
* RELIANCE : 22.7%
* INFY : 35.7%



- **Minimum Variance Portfolio :**
  TATASTEEL 0.022649 ADANIPORTS 0.048180 HDFCBANK 0.083842 GAIL 0.118748 KOTAKBANK 0.142834 RELIANCE 0.226849 INFY 0.356898

- **Maximum Sharpe Portfolio** : [2.00617697e-01, 2.91108283e-17, 0.00000000e+00, 3.91150398e-01,1.00830802e-17, 0.00000000e+00, 4.08231905e-01] in alphabetical order of stocks
- **Efficiency Frontier Line**
  - * Minimum Volatility (Variance) : 0.200
  - * Return at minimum Volatility: 0.548
  - * Maximum Sharpe Ratio : 3.699 (Return-Rf/Risk)
  - * Volatility at maximum Sharpe Ratio : 0.26
  - * Return at maximum Sharpe Ratio : 1.03

Market SML BSE Sensex



Return of Sensex 0.39

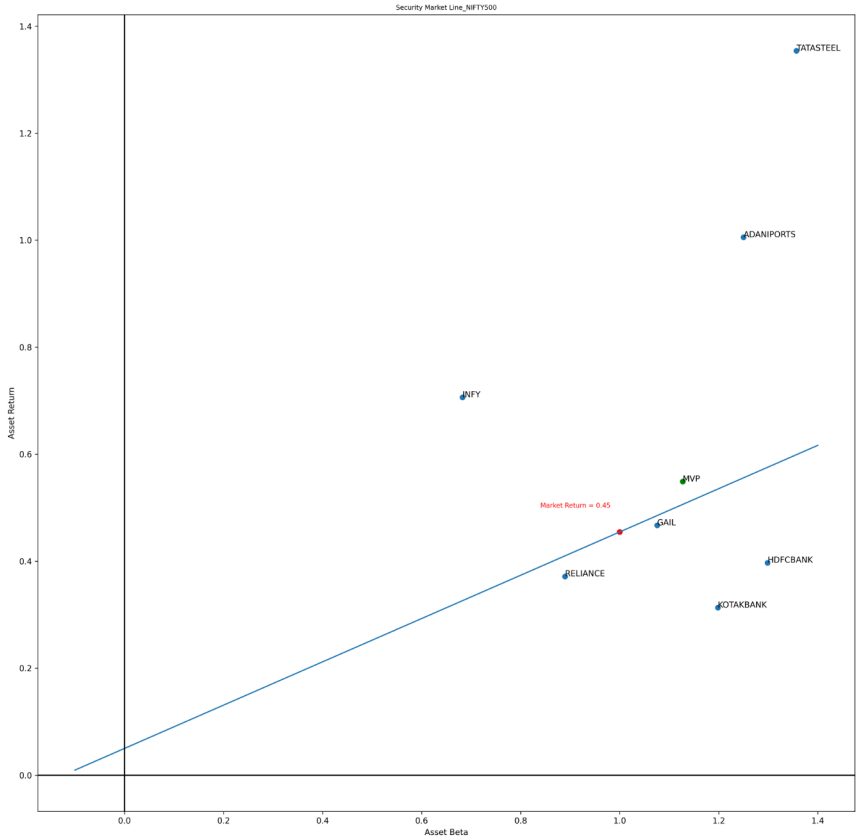
Beta of each stock in alphabetical order wrt Sensex: [1.04, 0.88, 1.30, 0.67, 1.15, 0.87, 1.10]

Beta of the Minimum Variance Portfolio (Sensex) : 1.0

Security Market Line : Sensex

* Slope : 0.34
* Y-Intercept : 0.050 (Risk Free Rate)

Market SML Nifty 500



Return of Nifty 0.45

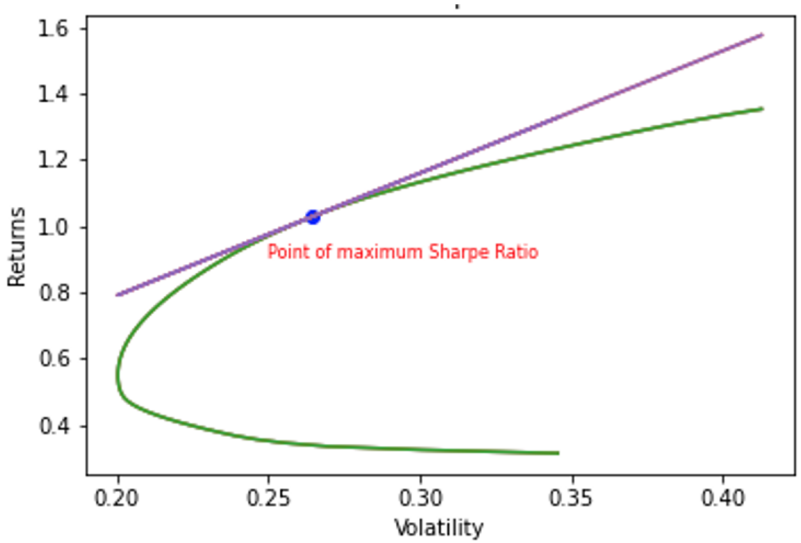
Beta of each stock in alphabetical order wrt Nifty500: [1.25, 1.07, 1.30, 0.68, 1.19, 0.88, 1.35]

Beta of the Minimum Variance Portfolio (Nifty500) : 1.1

Security Market Line : Nifty500

* Slope : 0.40
* Y-Intercept : 0.050 (Risk Free Rate)

Tangency Portfolio at Max Sharpe Ratio



Maximum Sharpe Ratio : 3.699
Volatility at maximum Sharpe Ratio : 0.26
Return at maximum Sharpe Ratio : 1.03

Summary of Results

- **Minimum Variance Portfollio** : TATASTEEL 0.022649 ADANIPORTS 0.048180 HDFCBANK 0.083842 GAIL 0.118748 KOTAKBANK 0.142834 RELIANCE 0.226849 INFY 0.356898
- **Minimum Variance Portfollio Percentages** (Graph Attached)

    * TATA STEEL : 2.2%
    * ADANIPORTS : 4.8%
    * HDFCBANK : 8.3%
    * GAIL : 11.8%
    * KOTABANK : 14.2%
    * RELIANCE : 22.6%
    * INFY : 35.6%

- **Maximum Sharpe Portfollio** : [2.00617697e-01, 2.91108283e-17, 0.00000000e+00, 3.91150398e-01,1.00830802e-17, 0.00000000e+00, 4.08231905e-01] in alphabetical order of stocks

- **Efficiency Frontier Line**

    * Minimum Volatility (Variance) : 0.200
    * Return at minimum Volatility: 0.548
    * Maximum Sharpe Ratio : 3.699
    * Volatility at maximum Sharpe Ratio : 0.26
    * Return at maximum Sharpe Ratio : 1.03

- **Beta** of each stock in alphabatical order wrt Sensex: [1.04, 0.88, 1.30, 0.67, 1.15, 0.87, 1.10]
- **Beta** of each stock in alphabatical order wrt Nifty500: [1.25, 1.07, 1.30, 0.68, 1.19, 0.88, 1.35]
- **Beta of the Minimum Variance Portfollio (Sensex)** : 1.0
- **Beta of the Minimum Variance Portfollio (Nifty500)** : 1.1
- **Security Market Line : Sensex**

    * Slope : 0.34
    * Y-Intercept : 0.050 (Risk Free Rate)

- **Security Market Line : Nifty500**

    * Slope : 0.40
    ik Free Rate)

-

    * Slope : 3.353

Inferences

- We calculated the minimum variance portfolio for each stock and it turns out that maximum weight in the stock INFY and minimum in the stock TATA STEEL gives the lowest risk.
- In the efficient frontier line, the minimum volatility(variance) is at the leftmost end of the curve (vertex of the hyperbola- shape obtained from the plot) and the return at that point is 0.548
- Return at maximum sharpe ratio is 1.03
- Beta value is a measure of a stock's volatility in relation to the overall market. Beta greater than unity indicates greater volatility with time. According to the data analysis 4 stocks showed beta value greater than 1 wrt Sensex and 5 wrt Nifty. Beta value of minimum variance portfolio is 1 for Sensex and 1.1 for Nifty.
- The slope of the security market line is obtained as 0.34 wrt Sensex and 0.4 wrt Nifty. It is equal to the market risk premium and it signifies the risk return trade off at a given time.
- Slope of the capital allocation market line is 3.353

MONTE CARLO METHOD

Code and Parameters

- We use closing price of the given stocks (df) and the Nifty 500 closing prices (df 2)
- We calculate beta by taking the covariance of matrix df2 and using the formula:
  Beta = (Covariance*)/(Variance*)
- Now, for randomization, we:
  A. Randomize w.r.t. Uniform distribution of weights
  B. Tweak the uniform distribution by taking the cubes of weights obtained from the uniform distribution and then normalising

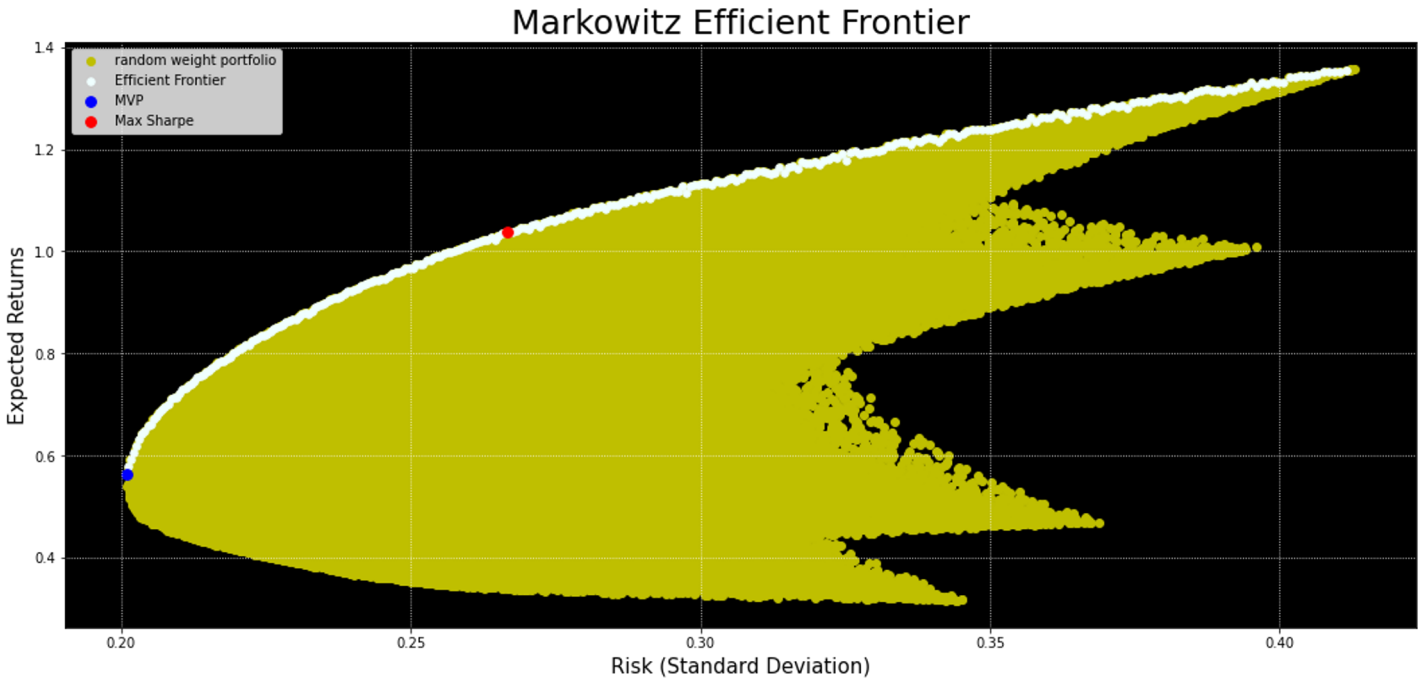
```
for portf in range(n):
    weights = np.random.random(7)
    #weights = weights**(3)
    weights /= np.sum(weights)
    returns = np.dot(weights,annual_return)
    risk = np.sqrt(np.dot(weights.T,np.dot(annual_cov,weights)))

    portfolio_reutrn.append(returns)
    portfolio_risk.append(risk)
    stock_weights.append(weights)
```

* = w.r.t. Nifty 500

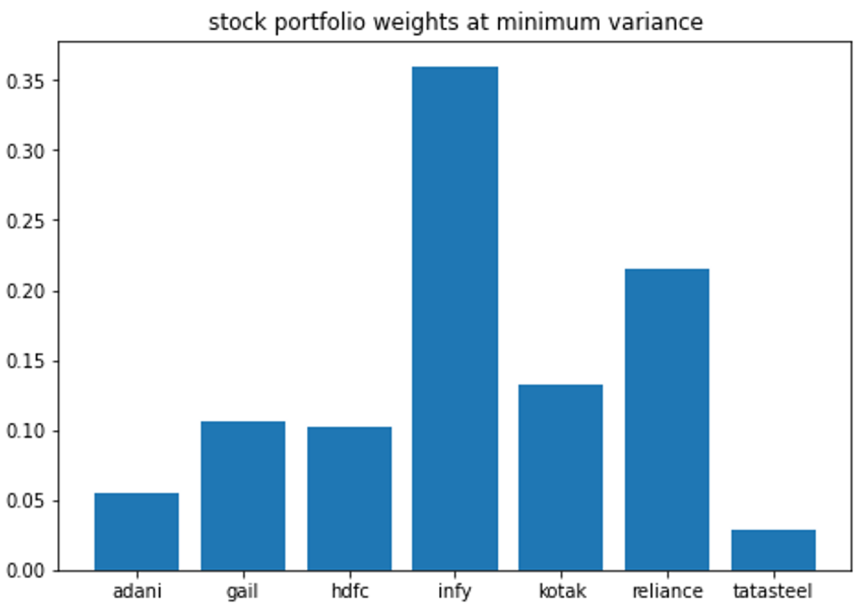
Code and Parameters

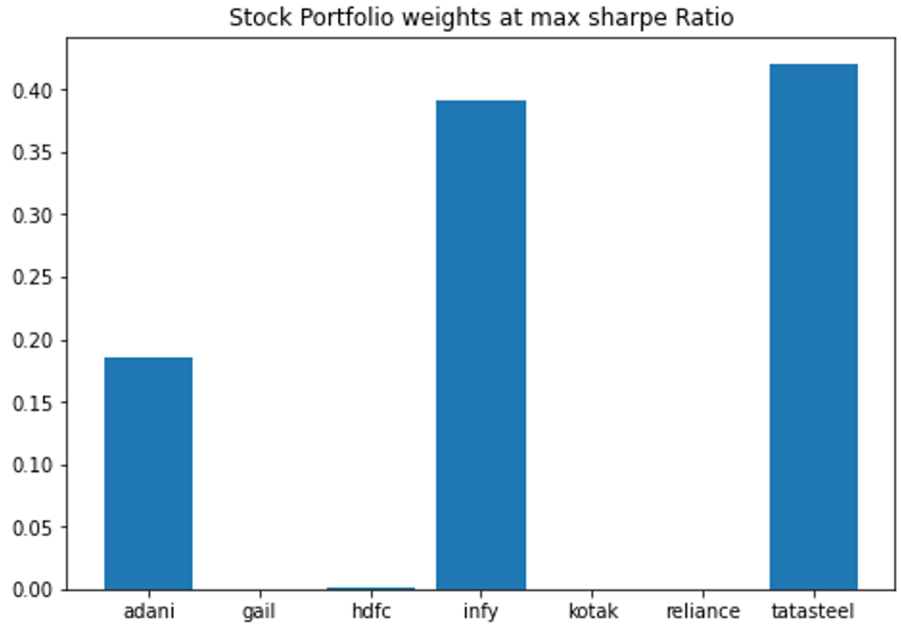
- For the **Efficient Frontier**
  According to the two randomizations, we plot the Efficient Frontier as follows:
  - Divide the risk spectrum into increasingly bigger intervals (300)
  - For each of these small intervals, we take the point with max return and in Scatter plot, we colour it white (rest of the random weight portfolio is yellow)
- For the **Security Market Line (SML)**, we consider two cases:
  1. r_m = Annual Return of Nifty 500
  2. r_m = Annual Return of Max Sharpe Ratio
  - We take Risk free rate to be 5% (as mentioned in the problem)
  - We plot SML for both the above assumptions

Results (with the cube modification)



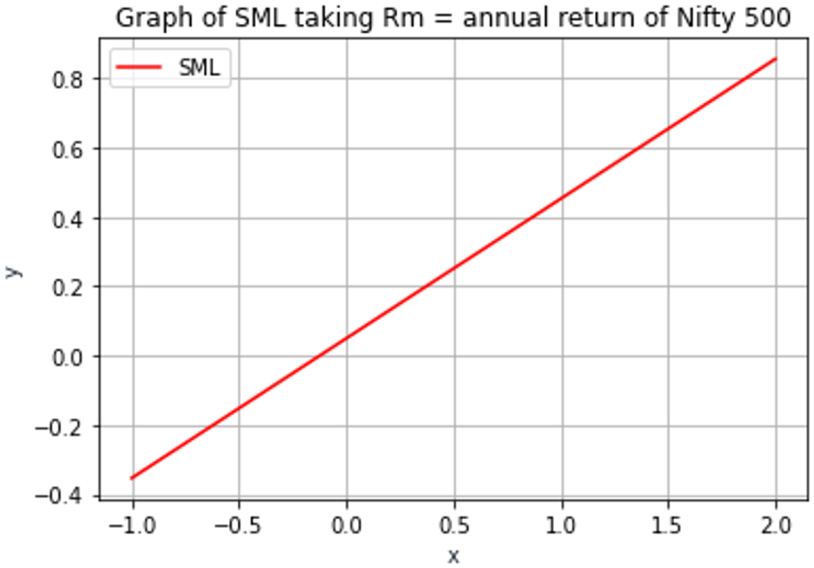
Results (with the cube modification)

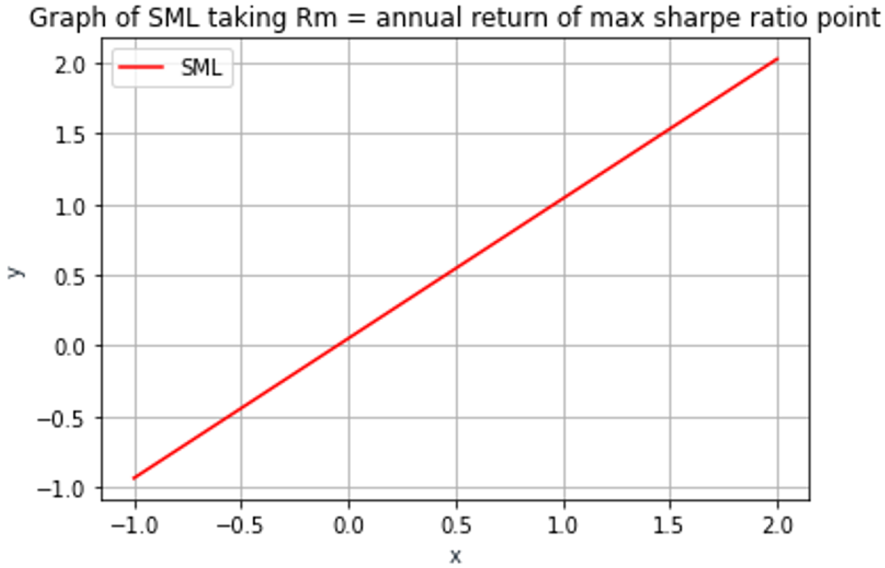




Results (with the cube modification)

Security Market Line





Summary of Results*

Minimum Variance portfolio:

adani	5.49%
gail	10.59%
hdfc	10.27%
infy	36.02%
kotak	13.19%
reliance	21.51%
tatasteel	2.9%
Return	56.27%
Risk	20.09%
Beta	0.95

Max Sharpe Ratio portfolio:

adani	18.55%
gail	0.00%
hdfc	0.11%
infy	39.17%
kotak	0.01%
reliance	0.03%
tatasteel	42.13%
Risk	26.66%
Return	103.82%
Beta	1.07
sharpe ratio	3.71

Security Market Line:

Case 1 (taking the Nifty 500 return as the market return)

Slope of SML Line: 0.40

Y intercept: 0.05

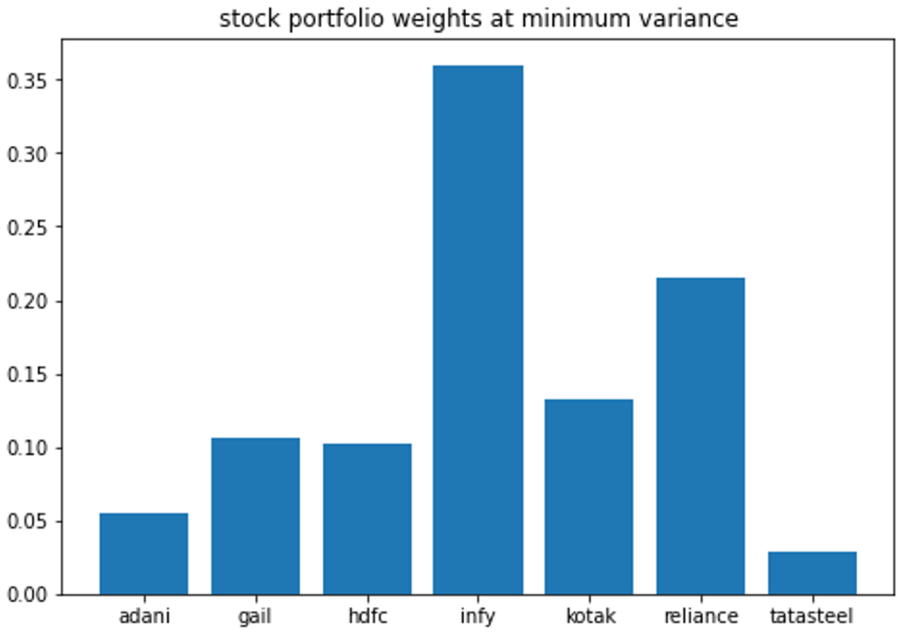
Case 2 (taking the return corresponding to max sharpe ratio as market return)

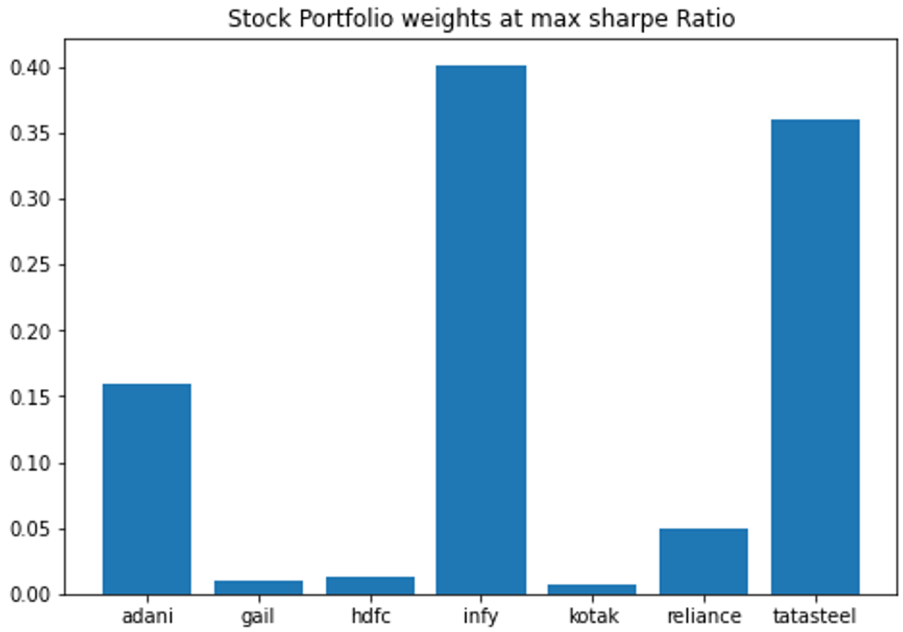
Slope of SML line = 0.99 (in terms of fraction and not in percentage)

Y intercept: 0.05

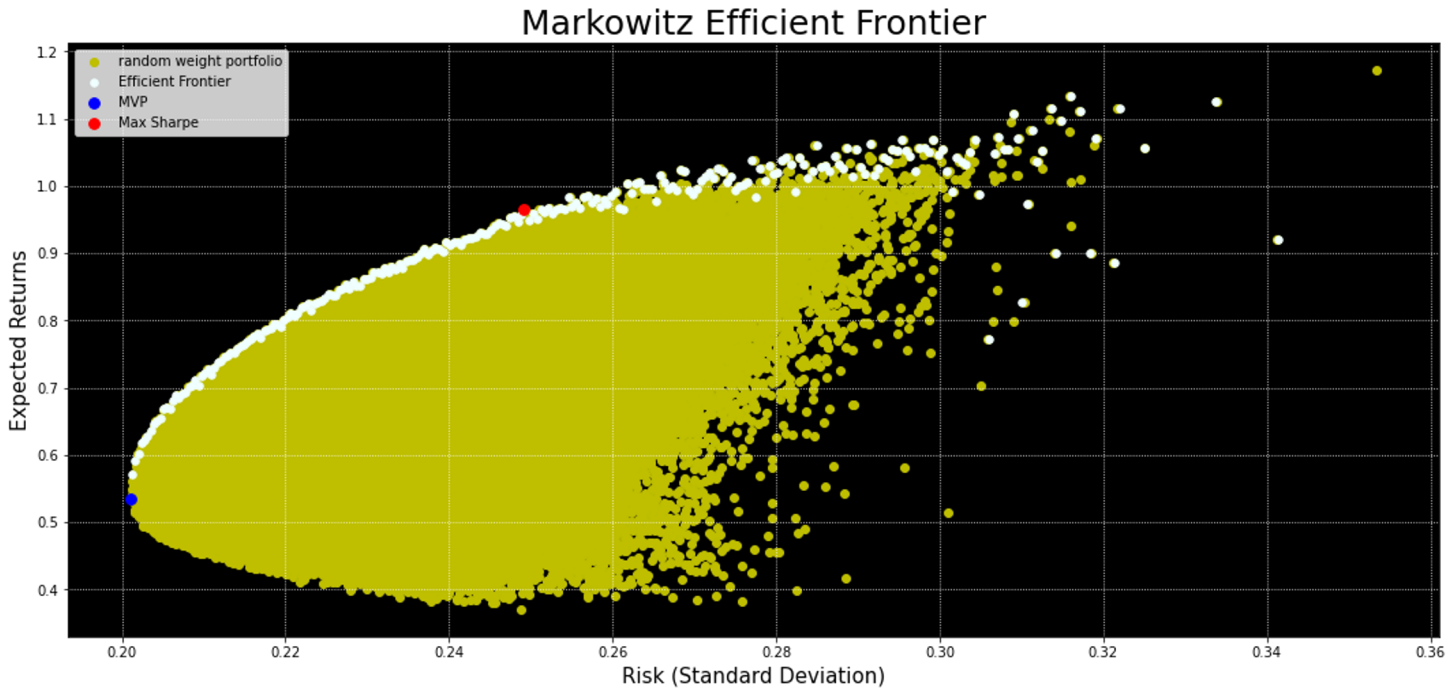
*with cube modification

Results (for uniform distribution)



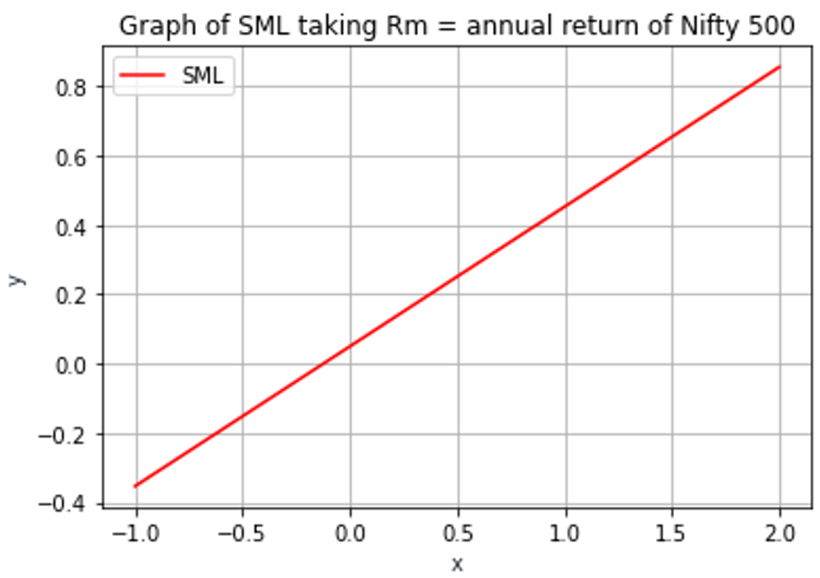


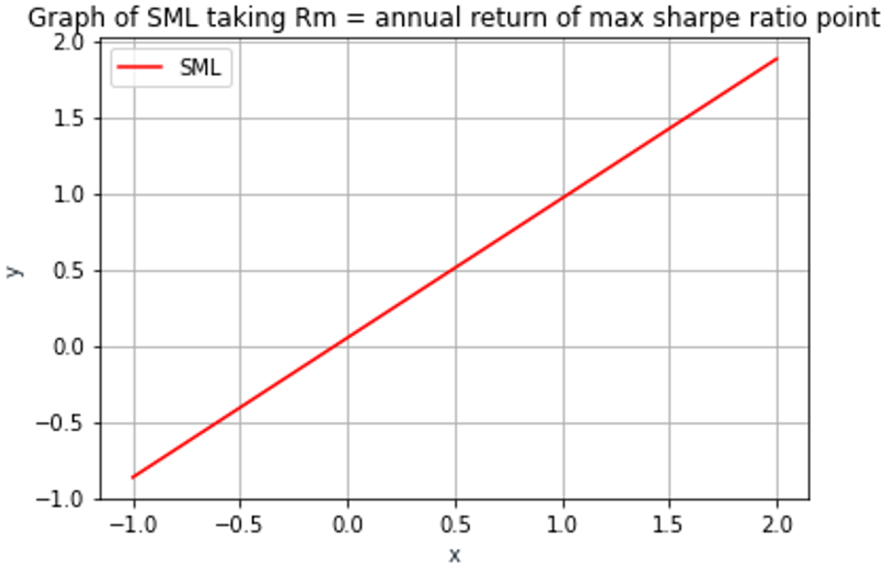
Results (for uniform distribution)



Results (for uniform distribution)

Security Market Line





Summary of Results*

Minimum Variance portfolio:

adani	5.49%
gail	10.59%
hdfc	10.27%
infy	36.02%
kotak	13.19%
reliance	21.51%
tatasteel	2.9%
Return	56.27%
Risk	20.09%
Beta	0.95

Max Sharpe Ratio portfolio:

adani	18.55%
gail	0.00%
hdfc	0.11%
infy	39.17%
kotak	0.01%
reliance	0.03%
tatasteel	42.13%
Risk	26.66%
Return	103.82%
Beta	1.07
sharpe ratio	3.71

Security Market Line:

Case 1 (taking the Nifty 500 return as the market return)

Slope of SML Line: 0.40

Y intercept: 0.05

Case 2 (taking the return corresponding to max sharpe ratio as market return)

Slope of SML line = 0.99 (in terms of fraction and not in percentage)

Y intercept: 0.05

*with cube modification

Inferences

- According to the minimum variance portfolio, the lowest risk is obtained when the weightage is maximum is INFY and minimum in ADANIPORTS
- The return at the minimum volatility point is 0.562 and that at the max sharpe ratio is 1.038
- According to the data analysis 5 stocks showed beta value greater than 1 wrt Nifty 500. Beta value of minimum variance portfolio is 1.1 for Nifty 500, indicating greater volatility with time for these 5 stocks, and the MVP.
- The slope of SML is 0.40 when r_m is taken as annual return of Nifty 500, and 0.99 when r_m is taken as the return corresponding to maximum sharpe point

Conclusion

- As we can see from the results, both methods come to very similar conclusions.
- The efficient frontier calculated by Monte Carlo becomes closer to the efficient frontier calculated by Analytical Methods like Quadratic Programming as we increase the number of Points(portfolios).
- There are always more points clustered Near the MVP and less points near the Area of High risk-High return
- The Efficient frontier generated from this method is Always inside (Below and on the right) the true efficient frontier calculated be Methods like Quadratic Programming
- If there are many more stocks in the portfolio, then the Monte Carlo method will get prohibitively time consuming, in that case we would have to calculate the Efficient Frontier through Quadratic Programming only.
- Some limitations of the QP include:
  - Unstable solutions
  - Reallocation Costs
  - Effects of estimation error
- We can curb these challenges using constrained optimization, sensitivity analysis, etc.

Thank you!