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Cover Sheet

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Dynamic Pricing, Scheduling and Energy Management for Profit Maximization in PHEV Charging Stations

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***Abstract*—Recently, as plug-in hybrid electric vehicles (PHEVs) take center stage for the eco-friendly and cost-effective transportation, commercial PHEV charging stations would be widely prevalent in the future. However, previous studies in the fields of the management of PHEV charging stations did not synthetically take practical charging systems into account. In this paper, we study the profit-optimal management of a PHEV charging station under the realistic environment with addressing not only various types of vehicles but also waiting time guarantee for PHEV customers. This paper is first to jointly take into account pricing for charging services, scheduling of reserved vehicles to PHEV chargers, dropping of reserved vehicles and management of the energy storage in a unified framework which contains key features of a practical PHEV charging station. Based on this framework, we develop an algorithm to find parameters required for the charging management by invoking "Lyapunov drift-plus-penalty" technique. Through theoretical analysis, we prove that the proposed algorithm achieves close-to-optimal performance under particular conditions by exploiting opportunism of time varying arrival of charging vehicles, price of electricity and renewable energy generation while it does not require any probabilistic future information. Finally, we find several significant messages via trace-driven simulation of the proposed algorithm.**

***Index terms*—Plug-in hybrid electric vehicle (PHEV), pricing, scheduling, dropping, energy management, profit maximization, charging service provider (CSP).**

NOMENCLATURE

ϵ_m	Constant arrival of virtual queue $Z_m(t)$.
λ	Offset energy level of energy storage.
$\mathcal{M}$	Set of vehicle-type.
ω_m	Required power to charge unit type-m vehicle.
τ_m	Required time to charge unit type-m vehicle.
$a_m(t)$	# of type-m vehicle arrives at time t.
$c(t)$	Price of electricity at time t.
$d_m(t)$	# of dropped type-m vehicles at time t.
$E(t)$	Remaining energy level of energy storage at time t.
$e(t)$	Amount of charging/discharging of energy storage at time t.
l_m	Worst-case waiting time of type-m customers that the CSP guarantees.
$n_m(t)$	# of type-m vehicles out of $a_m(t)$ use the charging station at time t.
$p_m(t)$	Price to charge unit type-m vehicle at time t.
q_m	Penalty fee the CSP has to pay for type-m vehicle.
$Q_m(t)$	Remaining workloads of type-m vehicles at time t.
$r(t)$	Amount of renewable energy harvested at time t.
S	Total # of battery chargers.
$s_m(t)$	# of newly scheduled type-m vehicles at time t.
$s_m(t^-)$	# of unfinished type-m vehicles scheduled before time t.
T_f	Interval of a time frame.
$U_{m,t}(\cdot)$	Utility of type-m representative user at time t.
V	Profit-delay tradeoff parameter.
$Z_m(t)$	Virtual queue of type-m vehicle.

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I. INTRODUCTION

Recently, with increasing awareness of environmental contamination, there has been a consensus on the need to limit CO_2 and exhaust emissions globally. For example, EU (European Union) passed a bill to reduce CO_2 emissions from 130g/km in 2015 to 95g/km in 2020 [1]. Also, the government of United States regulated to reduce exhaust emissions and fine dust up to 81% and 70% than before, respectively [2]. It leads to a drastic increase in the number of PHEVs (Plug-in Hybrid Electrical Vehicles) which are partially powered by electric energy. This tendency would not only reduce CO_2 emissions, but also help to solve oil depletion problem [3].

The increasing number of PHEVs may result in the proliferation of the commercial PHEV charging stations in several locations, e.g., shopping centers and companies. The management of commercial charging stations is similar to that of commercial gas stations in a perspective of buying resources from wholesale markets and reselling them to drivers. However, the management of PHEV charging stations for profit maximization of charging service provider (CSP) is more complex and tricky than that of existing gas stations because of the fundamental differences: the existence of charging time and the characteristic of electricity resources. *(i)* The existence of charging time (about 30 minutes to 3 hours [4]) brings an issue that the CSP should guarantee the appointed service completion time of the reserved vehicles. It is connected with

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scheduling to determine the order of the vehicles to be charged. According to the scheduling policy, the CSP sometimes may not guarantee the completion time and drop the reserved vehicles by paying some penalty fee to the customers. Moreover, a price to charge the vehicles varies depending on the electricity price, the completion time which the CSP guarantees and the number of vehicles which are waiting and being charged. *(ii)* The energy resources are more flexible for charging stations than for gas stations in a sense that the CSP can buy the electric energy resources from the electric utility grid in real-time. The CSP opportunistically stores energy resources in a private energy storage from not only the electric utility grid with payment, but also complementary renewable energy, e.g., solar or wind, which can be utilized for PHEV charging. Therefore, the scheduling to allocate the reserved PHEVs in each battery charger, the dropping of reserved PHEVs with penalty fees, the pricing for the PHEV charging and the charging/discharging in the energy storage should be carefully determined in light of the profit maximization of the CSP.

It is challenging to jointly control pricing, scheduling, dropping and energy management (charging/discharging) because each decision affects the other decisions. For example, if the CSP determines the charging price too low, many PHEVs may be requested to charge and the CSP have to schedule the reserved vehicles as much as possible by maximally using the energy in the private storage even though the electricity price is high. Moreover, the CSP may drop some vehicles with penalty fee due to the excessive admission of the vehicle caused by low charging price. However, a simultaneous consideration of all the coupled control parameters to optimize CSP's profit may require higher complexity due to the higher dimension of searching spaces. Therefore, we need to develop a low-complex, but close to optimal algorithm.

In this paper, we suggest a dynamic pricing, scheduling, dropping and energy management, called PCSM algorithm for profit maximization of a CSP in a commercial PHEV charging system. The contributions of this paper are summarized as follows.

- The proposed PCSM algorithm is the first to jointly optimize pricing for charging service, scheduling reserved vehicles, dropping reserved vehicles and energy management in real PHEV charging system environment by invoking the Lyapunov optimization. It is easily applied to real charging station management in a sense that the proposed algorithm not only has low computational complexity, but also reflects practical features of the charging station, e.g., strict waiting time guarantee and the finite capacity of energy storage.
- We theoretically prove following four theorems related to the conditions of a practical charging station when the CSP adopts the PCSM algorithm: *(i)* The CSP is able to guarantee the appointed waiting time of customers for a given profit-delay tradeoff parameter. *(ii)* The CSP does not need to drop any reserved PHEVs if certain system conditions are satisfied. *(iii)* The energy storage does not overflow if certain system conditions are satisfied. *(iv)* Even though our system has a dependency among time slots, it achieves near-optimal performance with a known optimality gap in a sense that it maximizes the profit of the CSP. A theoretical novelty of our paper is first to present a proper profit-delay tradeoff region, which "jointly" satisfies above four theorems within a unified framework, which is absolutely not shown in any previous work to deal with Lyapunov optimization [5]–[10].
- We demonstrate the proposed algorithm via trace-driven simulation with real data sets in a PHEV charging system while most of previous studies demonstrated the performance of their algorithms by numerical results. We use the real data sets of Lithium-ion battery for commercial PHEVs and traces of electricity price and renewable energy generations to demonstrate how the PCSM algorithm works well in real world scenarios.

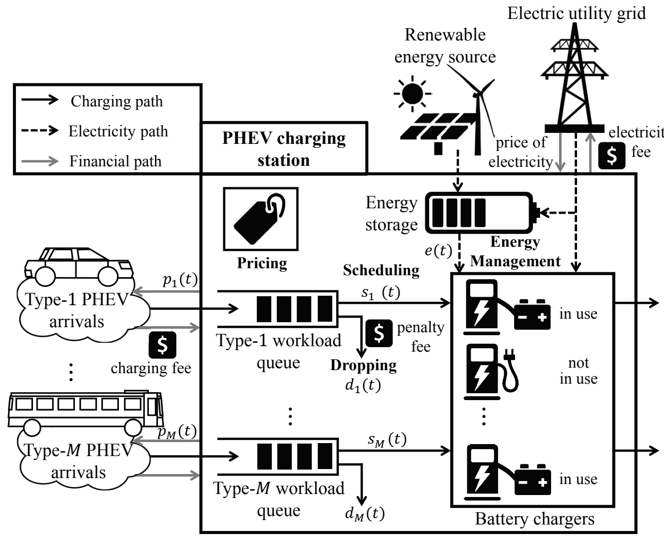

Fig. 1: A framework for management of a PHEV charging station.

The rest of the paper is organized as follows. We begin with describing system model in Section II. In Section III, we propose PHEV charging station management algorithm called PCSM. Next, in Section IV, we theoretically analyze the PCSM algorithm. We evaluate the PCSM algorithm by trace-driven simulation based on real data set in Section V. Next, related work is reviewed in Section VI. Finally, we conclude this paper in Section VII.

II. System Model

A. Charging vehicle model.

Fig. 1 illustrates a framework for management of a PHEV charging station. We consider a time-slotted system indexed by $t \in \{0, 1, ...\}$. We consider one charging station and M types of charging vehicles where each type is indexed by $m \in \mathcal{M}$. Because a battery characteristic depends on the model of vehicle (e.g., car, truck and bus), the charging power is not the same. Even though the models of two vehicles are the same, required times to fully charge the battery might be different when the remaining energy of the two vehicles are different. Therefore, the type-m vehicles can be differentiated by power

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(energy per unit time) to charge ω_m[1] and time to charge τ_m. From now on, we call the time to charge as a workload[2]. For example, a type-1 vehicle requires not only 100 Joule of energy every time slot, but also 20 time slots for charging completion.

Every time slot, $a_m(t)$ vehicles (of which type is m) arrive at the charging station and we assume that $a_m(t)$ is i.i.d. and[3] upper-bounded by $a_m(t) \leq a_m^{\max}$ for all $m \in \mathcal{M}$. Under the price $p_m(t)$ for charging unit type-m vehicle given by CSP, each type-m vehicle decides whether to charge its battery by paying $p_m(t)$ or to leave the charging station. We use representative user model (it is as widely used in literature (see [12] and references therein)): all type-m vehicles are considered as one representative user. We denote $U_{m,t}(n_m(t))$ as a utility function of type-m representative user when the user decides to charge $n_m(t)$ out of $a_m(t)$ vehicles at the charging station. In general, the utility function is a concave[4], differentiable and non-decreasing function on $n_m(t)$, and $U_{m,t}(0) = 0$ for all m, t. Also $U_{m,t}(\cdot)$ is time-varying and independent over time. Then, type-m representative user decides the number of vehicles being charged $n_m(t) \in [0, a_m(t)]$ for a given price $p_m(t)$ as follows [13].

$$\begin{aligned} \max_{n_m(t)} \quad & U_{m,t}(n_m(t)) - n_m(t)p_m(t), \\ \text{subject to} \quad & n_m(t) \in [0, a_m(t)], \forall m \in \mathcal{M}, \end{aligned} \tag{1}$$

where the second term means the total charging fee that type-m vehicles, of which representative user decides to charge, have to pay.

B. Charging station model.

Workload queue model. Every time slot, the CSP decides a price for charging one type-m vehicle, $p_m(t) \in [0, p_m^{\max}]$ (in \$) for all $m \in \mathcal{M}$ where $p_m^{\max}$ is high enough for customers (users) to make $n_m(t) = 0$. We consider M queues waiting for charging requested by representative users of all types of vehicles. We denote $Q_m(t)$ by the remaining workloads (i.e., the required time slots to charge) of type-m vehicles at time slot t where $Q_m(0) = 0$ for all $m \in \mathcal{M}$. For customers' convenience, the CSP makes a contract with type-m customers that all requested vehicles should be completely charged until l_m time slots for all $m \in \mathcal{M}$. Otherwise, the CSP has to pay the penalty fee q_m for type-m vehicle to the customer which is high enough to compensate the customer's dissatisfaction, $q_m \geq p_m^{\max}$ (This constraint also can be seen in [10]). In order to consider fairness, we set the penalty fee to be the same for the same type customers.

We consider total S battery chargers in the charging station, which are powered by a electric utility grid and a private energy storage as shown in Fig. 1. Every time slot, the CSP determines the number of type-m vehicles to be charged for the first time[5], $s_m(t)$ (we call it as scheduling) for all $m \in \mathcal{M}$. Because frequent change of scheduling (e.g., charging vehicle A at t, B at $t+1$ and A at $t+2$ again using same battery charger) leads to increase of additional charging delay, we assume that each vehicle charging is never stopped until it is fully charged once it is scheduled, i.e., if type-m vehicle is scheduled firstly at time slot t, it occupies one battery charger during time interval $[t, t+\tau_m-1]$. Then, we denote $s_m(t^-)$ as the number of type-m vehicles which are scheduled before time slot t, but not completely charged yet. Because both $s_m(t)$ and $s_m(t^-)$ vehicles occupy battery chargers at time slot t, following condition should be satisfied.

$$\sum_{m \in \mathcal{M}} (s_m(t) + s_m(t^-)) \leq S, \forall t. \tag{2}$$

The CSP has an option to drop waiting type-m vehicles with penalty fee q_m. We denote $d_m(t) \in [0, d_m^{\max}]$ as the number of type-m vehicles dropped by the CSP at time slot t where $d_m^{\max} \geq a_m^{\max}$. Because the dropping occurs only when it is impossible to guarantee worst-case waiting time, the customer who waits for the longest time in the workload queue (at the front of the queue) and has not been scheduled yet, is dropped. Then, the queueing dynamics of $Q_m(t)$ can be described as follows.

$$\begin{aligned} & Q_m(t+1) \\ & = \Big[Q_m(t) - s_m(t) - s_m(t^-) - \tau_m d_m(t) + \tau_m n_m(t)\Big]^+, \end{aligned} \tag{3}$$

where $[x]^+ = \max(x, 0)$. The amount of workload arrival to the type-m workload queue is determined by the representative user's decision $n_m(t)$ which is controlled by the price $p_m(t)$. The departure is controlled by last and current scheduling, $s_m(t^-)$ and $s_m(t)$, and dropping, $d_m(t)$.

Energy storage queue model. The CSP has a private energy storage for storing electric energy where the remaining energy state at time slot t is denoted by $E(t)$. Because the energy storage has finite capacity, the remaining energy state is upper-bounded by $E(t) \leq E^{\max}$. The energy storage can be charged by electric utility grid and be discharged by battery chargers. The amount of energy charged to or discharged from the storage can be represented by $e(t) \in [e^{\min}, e^{\max}]$ (in Joule) where $e^{\min} < 0$ and $e^{\max} > 0$. Note that $e(t)$ can be a positive or a negative value according to the CSP's decision.

Additionally, the CSP has renewable energy generators which harvest solar and wind energy. Every time slot, $r(t) \in [0, r^{\max}]$ amount of renewable energy is harvested and stored in the energy storage (in Joule) and we assume that $r(t)$ is an i.i.d. process and upper-bounded by $r^{\max} \leq e^{\max}$. We denote $c(t)$ as a price of electricity at time slot t (in \$/Joule) which is time-varying due to the fact that it depends on peak demands of electricity at the grid [14]. We assume that $c(t)$ is an i.i.d. process and bounded by $c(t) \in [c^{\min}, c^{\max}]$. Then, we the

[1] Although the required power to charge some PHEV battery may not constant over time and depends on its SoC (state of charge) because of the chemical characteristics of Lithium-ion battery, we assume that the charging power is constant in our model for simplicity. This assumption is as widely accepted in recent literature (see [11] and references therein).

[2] Even though the time is seldom called as the workload in general, the time to charge is modeled to be stacked on the queue in our system, which is the reason why we call the time to charge as the workload.

[3] Independent and identically distributed.

[4] Concavity is a reasonable assumption to reflect heterogeneity of customers' behavior for given charging price.

[5] Only the type-m vehicles which have not been scheduled before t can be counted in $s_m(t)$.

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dynamics of energy storage $E(t)$ can be described as follows.

$$E(t+1) = \Big[E(t) - e(t) + r(t)\Big]^+, \tag{4}$$

where the energy arrival is determined by $r(t)$ and $e(t)$, and the energy departure is determined by $e(t)$.

We summarize the assumptions in our system model as follows.

- The vehicle arrival $a_m(t)$ is an i.i.d. process and upper-bounded by $a_m^{\max}$ for all $m \in \mathcal{M}$ and t.
- The harvested renewable energy $r(t)$ is an i.i.d. process and upper-bounded by $e^{\max}$ for all t.
- The electricity price $c(t)$ is an i.i.d. process and bounded by $e^{\min}$ and $e^{\max}$ for all t.
- Each vehicle type-m requires the constant charging power ω_m.
- The vehicle charging is never stopped until it is fully charged once it is scheduled.

III. PHEV Charging Station Management Algorithm

In this section, we formulate an optimization problem considering profit maximization with strict delay constraints for a PHEV charging station. Then, we develop a management algorithm (called PCSM) for profit maximization of a PHEV charging service provider (CSP).

A. Problem Formulation

First, we define the profit of the charging station at time slot t, $h(t)$.

$$\begin{aligned} h(t) = &\sum_{m\in\mathcal{M}} n_m(t)p_m(t) - \sum_{m\in\mathcal{M}} d_m(t)q_m \\ &- \Big[\textstyle\sum_{m\in\mathcal{M}} \omega_m\big(s_m(t)+s_m(t^-)\big) - e(t)\Big]c(t), \end{aligned} \tag{5}$$

where the first term of the right hand side (RHS) represents the total charging fee paid by customers. The second term means the sum of penalty fees that the CSP has to pay for the customers whose workloads are dropped. The last term of the RHS means the overall electricity fee that the station has to pay to the electric utility. Because the energy storage has limited charging/discharging times, the cost of energy storage also should be contained in the profit function. We can consider the cost of energy storage as a form of amortized time-invariant function in our system model [8][6].

Our objective of the framework shown in Fig. 1 is to develop profit-maximal algorithm for the CSP by jointly controlling the charging price per one vehicle $p_m(t)$, the number of vehicles to be scheduled $s_m(t)$, the number of vehicles to be dropped $d_m(t)$ for each type of vehicles and charging/discharging of the energy storage $e(t)$ under the strict delay constraints. The arrivals of reserved vehicles and renewable energy are within the capacity region which is defined as the set of all arrival rates of charging vehicles and renewable energy that the CSP can serve within finite time. We formally state the long-term average profit optimization problem as follows.

$$(\mathbf{P}): \max_{(\boldsymbol{p},\boldsymbol{s},\boldsymbol{d},\boldsymbol{e})} \lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\big[h(t)\big], \tag{6}$$

subject to

$$0 \le p_m(t) \le p_m^{\max}, \forall m \in \mathcal{M}, t\in[1,T], \tag{7}$$

$$0 \le d_m(t) \le d_m^{\max}, \forall m \in \mathcal{M}, t\in[1,T], \tag{8}$$

$$\sum_{m\in\mathcal{M}} \big(s_m(t)+s_m(t^-)\big) \le S, \forall t\in[1,T], \tag{9}$$

$$e^{\min} \le e(t) \le e^{\max}, \forall t\in[1,T], \tag{10}$$

$$E(t) \le E^{\max}, \forall t\in[1,T], \tag{11}$$

$$\begin{aligned} &\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\big[\tau_m n_m(t)\big] \le \\ &\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\big[s_m(t)+s_m(t^-)+\tau_m d_m(t)\big], \forall m\in\mathcal{M}, \end{aligned} \tag{12}$$

$$\text{Strict delay constraint: } l_m \text{ slots for all } m\in\mathcal{M}, \tag{13}$$

where the control parameters $(\boldsymbol{p},\boldsymbol{s},\boldsymbol{d},\boldsymbol{e})$ denote as follows.

$$\begin{aligned} &\boldsymbol{p} = (p_1(t), p_2(t), ..., p_M(t))_{t=0}^T, \boldsymbol{s} = (s_1(t), s_2(t), ..., s_M(t))_{t=0}^T, \\ &\boldsymbol{d} = (d_1(t), d_2(t), ..., d_M(t))_{t=0}^T, \boldsymbol{e} = (e(t))_{t=0}^T. \end{aligned} \tag{14}$$

Constraint (12) means the stability condition of queue $Q_m(t)$, by ensuring that the average departure rate is higher than or equal to the average arrival rate.

B. Algorithm Design

We obtain a solution of our problem (**P**) under the unknown future information on vehicle arrivals, utility functions of representative users, renewable energy arrival and price of electricity by invoking "Lyapunov drift-plus-penalty" framework [5]. The theoretical meaning of this framework is to minimize cost (or maximize profit) by trading delay without loss of capacity (satisfying queueing stability). Moreover, in order to guarantee strict delay constraints, we design virtual queues for all types of vehicles.

Virtual queue design. To guarantee worst-case waiting time l_m for all vehicle type $m \in \mathcal{M}$, we define a virtual queue $Z_m(t)$ for type-m vehicle. The queueing dynamics of $Z_m(t)$ can be described as follows.

$$\begin{aligned} Z_m(t+1) = \Big[&Z_m(t) + \mathbb{1}_{Q_m(t)>0}\big(\epsilon_m - s_m(t) - s_m(t^-)\big) \\ &-\tau_m d_m(t) - \mathbb{1}_{Q_m(t)=0}S\Big]^+, \forall m\in\mathcal{M}, \end{aligned} \tag{15}$$

where the indicator function $\mathbb{1}_{\{X\}}$ is 1 when the proposition $\{X\}$ is satisfied and 0 otherwise. ϵ_m is a constant value which is no larger than the maximum workload arrival of type-m vehicle $\tau_m a_m^{\max}$. The departure S can be interpreted as the maximum number of type-m vehicles than can be scheduled simultaneously at the charging station using the fact $s_m(t) + s_m(t^-) \le S$. $Z_m(t)$ is based on the ϵ-persistent service queue technique for worst-case delay bound [7] where $Z_m(0) = 0$ for all $m \in \mathcal{M}$.

Making short-term objective. First, we define Lyapunov function and one slot conditional Lyapunov drift function as

[6]However, we omit this term because the issue for the cost of battery life may be important at much longer time scale than our problem.

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follows.

$$L(t)=\frac{1}{2}\Big[\sum_{m\in\mathcal{M}}\Big\{Q_m(t)^2+Z_m(t)^2\Big\}+\Big\{E(t)-\lambda\Big\}^2\Big], \quad (16)$$

$$\Delta L(t)=\mathbb{E}\big[L(t+1)-L(t)|\boldsymbol{K}(t)\big], \quad (17)$$

where $\boldsymbol{K}(t)=(\boldsymbol{Q}(t),\boldsymbol{Z}(t),E(t))$, and $\boldsymbol{Q}(t)$ and $\boldsymbol{Z}(t)$ are vectors of $Q_m(t)$ and $Z_m(t)$, respectively. The Lyapunov function (16) is designed to stabilize workload queue $Q_m(t)$ and virtual queue $Z_m(t)$ for all types $m\in\mathcal{M}$, and energy storage queue $E(t)$ with offset energy level λ.

Next, we define Lyapunov drift-plus-penalty function where the penalty function is the expected profit during time slot t, as follows.

$$\Delta L(t)-V\mathbb{E}\big[h(t)|\boldsymbol{K}(t)\big], \quad (18)$$

where V is a non-negative tradeoff parameter between profit and (workload and virtual) queueing delay, i.e., how much we care about the profit increment compared to the queueing delay. Then, our objective is to minimize the short-term function (18) by controlling $(\boldsymbol{p}(t),\boldsymbol{s}(t),\boldsymbol{d}(t),e(t))$ every time slot t. The key derivation step is to obtain an upper bound to the Lyapunov drift-plus-penalty function (18).

Deriving an upper bound. We derive an upper bound of (18) using queueing dynamics (3), (4) and (15), and upper bounds of vehicle arrivals, the number of scheduled vehicles, the number of dropped vehicles and variation of energy storage queue in Section II.

Lemma 1. *Under any possible control variables $\boldsymbol{p}(t),\boldsymbol{s}(t),\boldsymbol{d}(t)$ and $e(t)$, we have:*

$$\begin{aligned}&\Delta L(t)-V\mathbb{E}\big[h(t)|\boldsymbol{K}(t)\big]\le B_1-V\mathbb{E}\big[h(t)|\boldsymbol{K}(t)\big]\\&+\mathbb{E}\Big[\sum_{m\in\mathcal{M}}Q_m(t)\big\{\tau_m n_m(t)-s_m(t)-s_m(t^-)-\tau_m d_m(t)\big\}|\boldsymbol{K}(t)\Big]\\&+\mathbb{E}\Big[\sum_{m\in\mathcal{M}}Z_m(t)\big\{\epsilon_m-s_m(t)-s_m(t^-)-\tau_m d_m(t)\big\}|\boldsymbol{K}(t)\Big]\\&+\mathbb{E}\Big[\big(E(t)-\lambda\big)\big(-e(t)+r(t)\big)|\boldsymbol{K}(t)\Big],\end{aligned} \quad (19)$$

where $B_1=\frac{1}{2}\Big[\sum_{m\in\mathcal{M}}\big\{(\tau_m a_m^{\max})^2+2(\tau_m d_m^{\max}+S)^2+(\epsilon_m)^2\big\}+\big\{\max(e^{\max},r^{\max}-e^{\min})\big\}^2\Big]$.

Proof. Please refer to the Appendix VIII-A. □

Deriving Algorithm. We develop the PCSM algorithm by finding $(\boldsymbol{p}(t),\boldsymbol{s}(t),\boldsymbol{d}(t),e(t))$ which minimizes the RHS of (19) every time slot. The minimization problem can be decomposed into several problems where each problem independently has one control variable. Then we can solve each problem as follows.

(a) Pricing $\boldsymbol{p}(t)$: The original pricing problem can be written as follows.

$$\begin{aligned}\min_{\boldsymbol{p}(t)}&\sum_{m\in\mathcal{M}}n_m(t)\big[\tau_m Q_m(t)-Vp_m(t)\big],\\ \text{subject to}\quad&0\le p_m(t)\le p_m^{\max},\forall m\in\mathcal{M}.\end{aligned} \quad (20)$$

The problem (20) can be decomposed into each type. Because $U_{m,t}$ is a concave, differentiable and non-decreasing function of $p_m(t)$, type-m user determines $n_m(t)$ as follows.

$$n_m(t)=(\dot{U}_{m,t})^{-1}(p_m(t)). \quad (21)$$

We define $p_m^0(t)=\dot{U}_{m,t}(0)$ as the threshold price that makes type-m user does not assign any vehicle to the charging station ($n_m(t)=0$). Then, we solve a transformed problem for each vehicle type-m as follows.

$$\min_{p_m(t)\le\min(p_m^0(t),p_m^{\max})}(\dot{U}_{m,t})^{-1}(p_m(t))\big[\tau_m Q_m(t)-Vp_m(t)\big]. \quad (22)$$

The operation mechanism of the above policy can be explained as follows: *(i)* If the CSP decides a price too low, the revenue from customers decreases even though many vehicles are assigned. On the other hand, if the CSP decides a price too high, the profit decreases again due to few assignments of vehicles. *(ii)* If the CSP accepts too many vehicles by lowering the price, the charging delay may increase, which leads to dissatisfaction of customers due to the excess of maximum waiting time. Moreover, during the morning/evening rush hour, the CSP increases the charging price to regulate the admissions $n_m(t)$ to take a balance among temporal load differences.

(b) Scheduling $\boldsymbol{s}(t)$: The original scheduling problem can be written as follows.

$$\begin{aligned}\min_{\boldsymbol{s}(t)}&\sum_{m\in\mathcal{M}}s_m(t)\Big[V\omega_m c(t)-\big(Q_m(t)+Z_m(t)\big)\Big],\\ \text{subject to}\quad&\sum_{m\in\mathcal{M}}\big(s_m(t)+s_m(t^-)\big)\le S.\end{aligned} \quad (23)$$

To solve the problem, we define a new set of types, $\mathcal{M}_1(t)=\{m\in\mathcal{M}|V\omega_m c(t)-\big(Q_m(t)+Z_m(t)\big)<0\}$ which represents the set of charging vehicle-types that are preferred to be charged at time slot t. We can see that more types are preferred to be charged as the price of electricity gets lower. We define a type $m_1^{\min}(t)=\arg\min_{m\in\mathcal{M}_1(t)}\Big[V\omega_m c(t)-\big(Q_m(t)+Z_m(t)\big)\Big]$ which is the most urgent type that has to be scheduled in the set $\mathcal{M}_1(t)$. Then, the scheduling algorithm for each type can be decided as follows.

$$s_m(t)=\begin{cases}S-\sum_{m'\in\mathcal{M}}s_m(t^-), & \text{if } m=m_1^{\min}(t),\\ 0, & \text{otherwise.}\end{cases} \quad (24)$$

The above policy indicates that all vacant battery chargers after completing the previous charging are allocated to the most urgent type. Note that (24) does not mean that only one type is charged every time slot, because other types can be charged by the past scheduling $s_{m'}(t^-)$ for all $m'\in\mathcal{M}-\{m_1^{\min}(t)\}$. As the price of electricity becomes cheaper ($c(t)$) and waiting vehicles increase ($Q_m(t)$), the number of schedules to charge the vehicles becomes increasing.

(c) Dropping $\boldsymbol{d}(t)$: The original dropping problem can be written as follows.

$$\begin{aligned}\min_{\boldsymbol{d}(t)}&\sum_{m\in\mathcal{M}}d_m(t)\Big[Vq_m-\tau_m\big(Q_m(t)+Z_m(t)\big)\Big],\\ \text{subject to}\quad&0\le d_m(t)\le d_m^{\max},\forall m\in\mathcal{M}.\end{aligned} \quad (25)$$

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The problem (25) also can be decomposed into each vehicle-type similar to the pricing case. Then, the dropping algorithm for each type can be decided as follows.

$$d_m(t) = \begin{cases} d_m^{\max}, \text{ if } \dfrac{Vq_m}{\tau_m} < Q_m(t) + Z_m(t), \\ 0, \text{ otherwise.} \end{cases} \quad (26)$$

The above policy demonstrates a tendency to drop type-m vehicle if the penalty fee per workload q_m/τ_m is not expensive enough and many loads (for $Q_m(t)$ and $Z_m(t)$) are left. However, in a practical PHEV charging station, it is preferable to do not drop any vehicle because the dropping of vehicles leads to dissatisfaction of customers and monetary loss of the CSP. In Theorem 2 of Section IV, we will present no-vehicle dropping conditions.

(d) Managing energy storage $e(t)$: The original energy storage managing problem can be written as follows.

$$\min_{e(t)} e(t)\big[\lambda - Vc(t) - E(t)\big], \quad \text{subject to} \quad e^{\min} \le e(t) \le e^{\max}. \quad (27)$$

We can directly derive the energy management algorithm.

$$e(t) = \begin{cases} e^{\max}, \text{ if } Vc(t) > \lambda - E(t), \\ e^{\min}, \text{ otherwise.} \end{cases} \quad (28)$$

The above algorithm can be intuitively explained as follows. The algorithm (28) tries to store energy through an electric grid when the price of electricity is low and the remaining energy is less than threshold λ. The stored energy is used to charge scheduled battery during the period of high electricity price. Choosing appropriate λ is another important issue to manage the energy storage efficiently[7]. We can avoid energy overcharging problems if we set λ appropriately, which will be shown in Theorem 3 of Section IV.

We can summarize the PCSM algorithm in Fig. 2. Even though the PCSM algorithm solve the problem (6) with constraints (7)-(13), there exist some conditions for profit-delay tradeoff parameter V to guarantee the worst-case waiting time for each vehicle type m (13). These conditions can be derived from a theoretical analysis of the PSCM algorithm, which is shown in Theorem 1 of Section IV.

IV. Theoretical Analysis

In this section, we theoretically analyze the performance of a PCSM algorithm. Via analysis of the PCSM algorithm, we give a CSP an intuition to find feasible conditions for pre-determined and control parameters to jointly satisfy both of the optimality of the CSP and the reality of a PHEV charging station. We prove four theorems to answer the following questions: *(i)* How to guarantee worst-case waiting time for customers by designing virtual queue $Z_m(t)$? *(ii)* What are the conditions that we do not drop any PHEV vehicle? *(iii)* How to determine an offset energy level λ to avoid overflow of energy storage? *(iv)* How much the performance gap is shown between the PCSM algorithm and the offline optimal algorithm?

[7]If we set λ to be too low, the charging station excessively discharges the energy storage even though the price of electricity is low. On the other hand, if we set λ to be too high, the charging station excessively charges the energy storage even though the price of electricity is high and the capacity of the energy storage is full.

PHEV Charging Station Management (PCSM) algorithm

Every time slot t,

Input (fixed): $S, V, \lambda, e^{\min}, e^{\max}, \tau_m, \omega_m, \epsilon_m, q_m, p_m^{\max}, d_m^{\max}, \forall m \in \mathcal{M}$.
Input (varying): $E(t), c(t), r(t), Q_m(t), Z_m(t), a_m(t), s_m(t^-), U_{m,t}(\cdot), \forall m \in \mathcal{M}$.
Output: $\boldsymbol{p}(t), \boldsymbol{s}(t), \boldsymbol{d}(t), e(t)$.

1: **for** each vehicle-type $m \in \mathcal{M}$,
2: Select the price for charging type-m vehicle, $p_m(t)$ by solving (22).
3: Schedule type-m vehicles, $s_m(t)$ by following (24).
4: Drop type-m vehicles, $d_m(t)$ by following (26).
5: **end for**
6: Manage the energy storage, $e(t)$ by following (28).

Update the queues $\boldsymbol{Q}_m(t), \boldsymbol{Z}_m(t), E(t)$ according to the queue dynamics (3), (15) and (4). Update $s_m(t^-)$.

Fig. 2: Algorithm description

A. Worst-case waiting time guarantee for each vehicle type

Theorem 1. *For a fixed V, we have:*

(i) $Q_m(t)$ is bounded by $Q_m^{\max} = \dfrac{Vp_m^{\max}}{\tau_m} + \tau_m a_m^{\max}, \forall m \in \mathcal{M}$,

(ii) $Z_m(t)$ is bounded by $Z_m^{\max} = \dfrac{Vq_m}{\tau_m} + \epsilon_m, \forall m \in \mathcal{M}$,

(iii) We can guarantee worst-case waiting time l_m under the condition, $l_m = \lceil \dfrac{Q_m^{\max} + Z_m^{\max}}{\epsilon_m} \rceil, \forall m \in \mathcal{M}$.

Proof. Please refer to the Appendix VIII-B. □

According to the statement of Theorem 1, we can guarantee the worst-case waiting time l_m of type-m customers by adjusting profit-delay tradeoff parameter V and persistent virtual arrival ϵ_m. Note that $p_m^{\max}$, q_m, τ_m and $a_m^{\max}$ are different per each type of vehicles. Because each vehicle type requires different time to charge ω_m, the CSP guarantees different worst-case waiting time for each type-m. We can guarantee it by regulating ϵ_m for all $m \in \mathcal{M}$ under given V.

B. No dropping conditions

Theorem 2. *Reserved vehicles are not dropped from the workload queues for all time slots if the following three conditions are satisfied.*

(i) $S \ge \big(\sum_{m' \in \mathcal{M}} \tau_{m'}\big)\big(\tau^{\max} a^{\max} + \epsilon^{\max}\big)$,

(ii) $\tau_{m_1} = \tau_{m_2}, \forall m_1, m_2 \in \mathcal{M}$,

(iii) $V\dfrac{q_m}{\tau_m} \ge V\omega^{\max} c^{\max} + \big(\sum_{m' \in \mathcal{M}} \tau_{m'}\big)\big(\tau^{max} a^{max} + \epsilon^{max}\big), \forall m \in \mathcal{M}$,

where $\tau^{\max} = \max_{m \in \mathcal{M}} \tau_m$, $\omega^{\max} = \max_{m \in \mathcal{M}} \omega_m$, $\epsilon^{\max} = \max_{m \in \mathcal{M}} \epsilon_m$, $a^{\max} = \max_{m \in \mathcal{M}} a_m^{max}$.

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The condition *(i)* means that the maximum workload service rate S is big enough to cover the maximum instantaneous arrival of workloads. The condition *(ii)* means that the workloads of all vehicle-types are the same. The condition *(iii)* means that the drop penalty fee q_m is sufficiently expensive, so as that the CSP is reluctant to drop the reserved vehicles, even if it should schedule the vehicles by paying the highest electricity fee.

Proof sketch of Theorem 2 is as follows. We first show that the sum of workloads and virtual queue lengths $Q_m(t)+Z_m(t)$ is upper-bounded by $V\omega^{\max}c^{\max}+(\sum_{m'\in\mathcal{M}}\tau_{m'})(\tau^{\max}a^{\max}+\epsilon^{\max})$ for all $m\in\mathcal{M}$ by contradiction using the conditions *(i)* and *(ii)*. Then, we can easily prove $Q_m(t)+Z_m(t)\le Vq_m/\tau_m, \forall m\in\mathcal{M}$ using the condition *(iii)* which implies that the CSP does not drop any vehicle by the dropping algorithm (26). The detailed proof is presented in the Appendix VIII-C.

C. No overcharging conditions for energy storage

Because the energy storage has a finite capacity $E^{\max}$, energy loss could be occurred if the energy storage is overcharged by renewable energy. To increase the profit, the CSP should prevent to overflow the energy storage by appropriate management of charging/discharging.

Theorem 3. *We can guarantee $E(t)\le E^{max}$ for all time slots if the following two conditions are satisfied.*

(i) $\lambda = Vc^{\max}+e^{\max}$,

(ii) $0\le V\le \dfrac{E^{\max}-r^{\max}-e^{\max}+e^{\min}}{c^{\max}-c^{\min}}$.

In the condition *(i)*, the offset energy level λ can be easily determined in a sense that it only requires knowledge of the maximum price of electricity and discharging rate of the energy storage. The condition also can be interpreted that the storage capacity should be $O(V)$ in order to do not overflow the storage when the PCSM algorithm is adopted. The condition *(ii)* means that the profit-delay tradeoff parameter V should not be determined too high because it makes the PCSM algorithm do not charge any vehicle for a long time to reduce the electricity costs. We can easily prove Theorem 3 as follows.

Proof. Please refer to the Appendix VIII-D □

D. Optimality gap

To demonstrate the optimality gap between proposed algorithm and offline optimal algorithm, original Lyapunov optimization technique uses the fact that minimizing 1-slot Lyapunov drift-plus-penalty every time slot leads to an optimization of the original long-term objective within $O(1/V)$ gap. However, the technique has a strong assumption about independence of 1-slot objective over time slots. In our system model, the workloads of charging vehicles are not a unit time slot and we consider a practical scenario that battery charging will be never stopped once it is scheduled until the charging ends. This cannot satisfy the previous 1-slot independence assumption because our scheduling policy $s_m(t)$ influences to the next τ_m-1 slots, which make the algorithm based on the original Lyapunov optimization be not able to derive the performance bound as it is.

To find performance bound under inter-time slot dependency, we modify our scheduling algorithm using the technique motivated by previous study [10]. First, we group T_{f} $(\ge\tau^{\max})$ time slots into one time frame. Suppose that the time slot t is in $(\alpha+1)$-th time frame, $t\in[\alpha T_{\mathrm{f}},(\alpha+1)T_{\mathrm{f}}-1]$ where α is a non-negative integer. Then we define a new set of vehicle-types $\mathcal{M}_2(t)=\{m\in\mathcal{M}|\tau_m\le(\alpha+1)T_{\mathrm{f}}-t\}$ and define a vehicle-type $m^{\min}_{1\cap2}(t)=\arg\min_{m'\in\mathcal{M}_1(t)\cap\mathcal{M}_2(t)}\Big[V\omega_{m'}c(t)-\big(Q_{m'}(t)+Z_{m'}(t)\big)\Big]$. Then our scheduling policy is changed as follows. For all types $m\in\mathcal{M}$,

$$s_m(t)=\begin{cases} S-\sum\limits_{m'\in\mathcal{M}} s_m(t^-), & \text{if } m=m^{\min}_{1\cap2}(t),\\ 0, & \text{otherwise.}\end{cases} \tag{29}$$

Now, our scheduling algorithm (24) is replaced by (29). We can see that the scheduling (29) behaves the same as (24) when $\tau^{\max}\le(\alpha+1)T_{\mathrm{f}}-t$. The key concept is to make the decisions in the α-th time frame do not influence to the $(\alpha+1)$-th time frame. Then we can apply the same technique for bounding original Lyapunov optimization in an inter-time frame scale by making $s_m((\alpha+1)^-)=0$. However, there would be a profit loss which cannot be bounded by $O(1/V)$ in an intra-time frame scale, and we will find it in Theorem 4. First, we define $(1+\delta)$-optimal profit as follows.

Definition 1 ($(1+\delta)$-optimal profit)**.** *Suppose the charging vehicles' arrival rate vector $\boldsymbol{x}$ and renewable energy's arrival rate y satisfy $(1+\delta)\boldsymbol{x}\in\boldsymbol{\mathcal{X}}$ and $(1+\delta)y\in\mathcal{Y}$, where $x_m=\lim_{T\to\infty}\frac{1}{T}\sum_{\tau=0}^{T-1}\mathbb{E}\big[\tau_m n_m(\tau)\big]$ for all $m\in\mathcal{M}$, $y=\lim_{T\to\infty}\frac{1}{T}\sum_{\tau=0}^{T-1}\mathbb{E}\big[r(\tau)\big]$ and $(\boldsymbol{\mathcal{X}},\mathcal{Y})$ is the capacity region without dropping vehicles and violating worst-case waiting time guarantee. Then, $h^{(1+\delta)}$ is the $(1+\delta)$-optimal profit that offline optimal algorithm can achieve with satisfying no-vehicle dropping and worst-case waiting time guarantee condition.*

Theorem 4. *When PHEV charging system parameters satisfy no dropping conditions in Theorem 2, there exists some $\delta>0$, such that supportable arrival vector $\boldsymbol{x},y$ by PCSM algorithm satisfies $\frac{(1+\delta)T_f}{T_f-\tau^{\max}}\boldsymbol{x}\in\boldsymbol{\mathcal{X}}$ and $\frac{(1+\delta)T_f}{T_f-\tau^{\max}}y\in\mathcal{Y}$, the average profit achieved by PCSM has a constant and $O(1/V)$ gap from $\frac{T_f}{T_f-\tau^{\max}}$-optimum:*

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\big[h(t)\big]\ge h^{\frac{(1+\delta)T_{\mathrm{f}}}{T_{\mathrm{f}}-\tau^{\max}}}-\frac{\mathbf{B}}{V}-\mathbf{D}, \tag{30}$$

where

$$\begin{aligned}\mathbf{B}=B_1&+\frac{(T_{\mathrm{f}}-\tau^{\max})(T_{\mathrm{f}}-\tau^{\max}-1)}{2T_{\mathrm{f}}}\big[\tau_m a_m^{\max}+2S+\epsilon_m\big]S\\&+\frac{T_{\mathrm{f}}-1}{2}\sum_{m\in\mathcal{M}}\big[(\epsilon_m)^2+(\tau_m a_m^{\max})^2\big]+\lambda e^{\max},\end{aligned} \tag{31}$$

$$\begin{aligned}\mathbf{D}=&\frac{(T_{\mathrm{f}}-\tau^{\max})(T_{\mathrm{f}}-\tau^{\max}-1)}{2T_{\mathrm{f}}}\omega^{\max}[c^{\max}-c^{\min}]S\\&+\frac{1}{T_{\mathrm{f}}}\tau^{\max}\omega^{\max}c^{\max}S.\end{aligned} \tag{32}$$

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$h^{\frac{(1+\delta)T_\mathrm{f}}{T_\mathrm{f}-\tau^{\max}}}$ is the $\frac{(1+\delta)T_\mathrm{f}}{T_\mathrm{f}-\tau^{\max}}$-optimal profit in Definition 1. Note that as δ approaches close to 0, the PCSM algorithm can achieve a gap from $\frac{T_\mathrm{f}}{T_\mathrm{f}-\tau^{max}}$-optimum. Additionally, as V and T_f goes to infinity while satisfing finite T_f/V, PCSM has a constant gap from 1-optimum which is the offline optimal profit of the original problem (**P**) in Section III. A performance bound of the average profit cannot be derived in the original Lyapunov optimization technique, which does not consider the time frame, due to the inter-time slot dependency. However, because the scheduling within one time frame does not affect other time frames in the modified scheduling policy (29), we can derive the performance bound with $\mathbf{D}$ which means the loss due to the inter-time slot dependency within a time frame.

Proof. Please refer to the Appendix VIII-E □

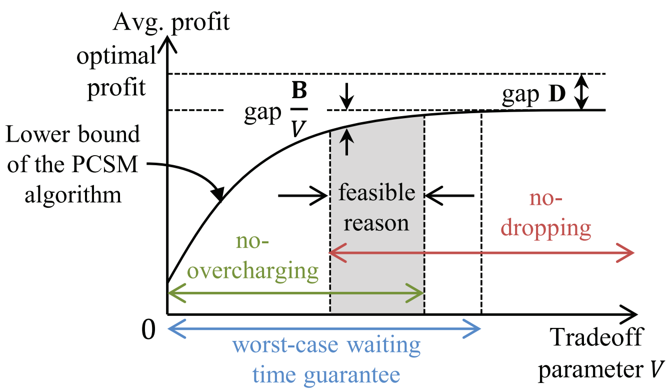

Fig. 3: Visualization of theoretical analysis.

We summarize the key results of four theorems for all tradeoff parameter Vs. We find feasible region of Vs to satisfy worst-case waiting time (Theorem 1) which is the constraint of our PHEV charging system, no dropping of vehicles (Theorem 2) and no overcharging for the energy storage (Theorem 3) with achieving the objective of the CSP. Finally, we demonstrate the constant optimality gap of the PCSM algorithm (Theorem 4). In other word, the feasible region of Vs to simultaneously satisfy three theorems (Theorem 1 $\sim$ Theorem 3) in the PHEV charging system is represented by gray area in Fig. 3 which is the intersection of three theorems' conditions[8]. We conclude that the optimal V for the CSP is the rightmost point in the feasible region.

V. Performance Evaluation

In this section, we evaluate proposed PCSM algorithm via trace-driven simulations based on real data sets.

A. Datasets, Traces and Simulation Setup

Vehicle-type. We investigate the specifications of commercial batteries of Hitachi [15] and Mitsubishi [16] which are the representative battery manufacturers for PHEVs. We choose three kinds (two from [15] and one from [16]) of Lithium-ion batteries for PHEVs and extract power to charge ω_m and time to charge τ_m from the specifications. Table I describes the specifications of each vehicle-type.

[8]Note that the feasible regions for no overcharging sometimes can be wider than that of worst-case waiting time guarantee.

TABLE I: Types of vehicle.

Vehicle-type	Battery model	Charging time (sec)	Charging power (Joule/sec)
1	115V Lithum-ion [15]	1800	253
2		3600	
3	173V Lithum-ion [15]	1800	475
4		3600	
5	300V Lithum-ion [16]	1800	2400
6		3600	

Electricity price & renewable energy. For the data set of electricity price ($/Joule), we use the trace of California Independent System Operator (CAISO) [17] where the time granularity is 5 minutes. Next, we consider two kinds of renewable energy sources, i.e., solar and wind energy. We use the trace of average solar irradiance (W/m^2) every 5 minutes from Measurement and Instrumentation Data Center (MIDC) where the harvested solar energy can be calculated by solar irradiance (W/m^2) times the area of the solar power generator (m^2). We use the wind energy trace gathered by CAISO and interpolate the trace into 5 minutes because a granularity of the trace is longer than 5 minutes (1 hour). All the electricity price and renewable energy generation traces are measured in Los Angeles and we pick them during the same date (from 06/01/2015 to 06/20/2015) and time (from 10:00 to 17:00).

Simulation setup. In our simulation, we set the interval of unit time slot to be 5 minutes. We consider a scenario that a PHEV charging station has 100 battery chargers (S=100), energy storage with 12kWh capacity, and solar/wind power generators with 10 m^2 area. We set the vehicle arrival $a_m(t)$ to be 5 for all $m \in \mathcal{M}$ for ease of analysis. The utility function of the representative user of type-m is $U_{m,t}(n_m(t)) = \beta_{m,t}\log(1 + n_m(t))$ in which the scale variable $\beta_{m,t}$ is uniformly distributed between $[\tau_m\omega_m a_m^{\max}\bar{c}, 2\tau_m\omega_m a_m^{\max}\bar{c}]$ for all t where $\bar{c}$ is the average price of electricity [12]. We decide the range of $\beta_{m,t}$ based on two philosophies. First, the worthy of battery energy for customers is higher enough than the average electricity price to charge that amount of energy. Second, some customers may not assign charging the battery and leave the station because the charging fee is a heavy burden for them. The range of $\beta_{m,t}$ we set is reasonable to show the dynamic behaviors of the customer under the given charging fee. For each vehicle-type, we set the upper bound of charging price as 10 times of the average electricity fee to completely charge the vehicle. The maximum drops for each vehicle-type is set to be 5 which is the same as the vehicle arrives and the penalty fee is the same as the maximum charging fee. Detailed simulation parameters are summarized in Table II.

B. Simulation Results

Profit and delay tradeoff. We compare our PCSM algorithm with three baseline algorithms which are derived from the PCSM algorithm. The algorithms behave the same as the PCSM algorithm except for one control variable, respectively: *(i)* A price for charging unit energy is the same for all types. *(ii)* The energy storage is charged by only renewable energy not electric grid and the stored energy is maximally utilized for

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TABLE II: Simulation settings.

Number of vehicle-types M	6
Avg. electricity price (\$/Joule)	9.1×10^{-9}
Avg. solar energy arrival (Joule/sec)	5.9×10^3
Avg. wind energy arrival (Joule/sec)	2.4×10^3
Vehicle arrival $a_m(t)$	5
$e^{\min}$, $e^{\max}$ (Joule/sec)	-4.8×10^4, 4.8×10^4
Maximum charging fee $p_m^{\max}$ (\$)	$\tau_m \omega_m \bar{c} \times 10$
Drop penalty fee q_m (\$)	$p_m^{\max}$
Maximum number of vehicle drop $d_m^{\max}$	5
Capacity of energy storage $E^{\max}$ (kWh)	12
Virtual queue arrival ϵ_m (time slots)	$\tau_m a_m^{\max}/6$
Offset of energy storage λ (Joule)	$E^{\max}/2$

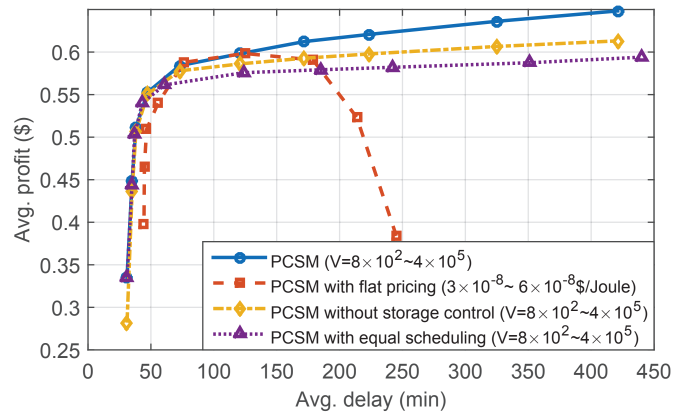

Fig. 4: Avg. delay of charging vehicles and avg. profit of a PHEV charging station.

charging scheduled vehicles. *(iii)* Each vehicle-type is equally scheduled every time slot.

Fig. 4 depicts the average delay of charging vehicles and the average profit of the CSP. We can see the profit-delay tradeoff curve (solid line) of the PCSM algorithm controlled by a parameter V. As V becomes higher, the PCSM algorithm tries to accommodate more vehicle arrivals in (22), to do not schedule waiting vehicles in (23) and to do not drop the waiting vehicles in (25), which results in increasing of waiting time (i.e., delay). However, as the delay gets longer, there are more rooms for admitting and processing the charging vehicles and exploiting variations of renewable energy arrival and price of electricity by managing energy storage. We observe 57% and 63% of profit increments by trading 42 minutes and 141 minutes of delay, respectively.

We observe the profit and delay performances (thick dotted line) for the case *(i)* under different charging prices for unit energy. Although the average profit of flat pricing is almost the same as that of the PCSM algorithm during the average delay interval $[70, 120]$ minutes, the performance gap becomes larger when the average delay gets out of the range. It is mainly due to the tradeoff between charging price and the number of vehicles requested for charging as we mentioned in pricing algorithm description of Section III. Therefore, time-dependent pricing for charging vehicles in consideration of the customer's willingness to pay is a crucial factor to increase the CSP's profit.

We can see the tradeoff curves for the case *(ii)* (broken line) and *(iii)* (thin dotted line) of which profits are 6% and 10% lower than the PCSM algorithm, respectively in 7 hours of average delay. The first profit gain from the PCSM without storage control to the PCSM algorithm demonstrates the potentiality of energy storage used for storing and releasing not only renewable energy but also energy from the electric grid opportunistically. The second profit gain from the PCSM with equal scheduling to the PCSM algorithm represents that a scheduling in the PCSM algorithm works well to increase CSP's profit depending on the vehicle-types.

Operation of PCSM algorithm. Fig. 5 depicts slot-by-slot operation of the PCSM algorithm when $V = 10^5$. We set virtual queue arrival ϵ_m based on Theorem 1 and make worst-case waiting time to be the same for all $m \in \mathcal{M}$ for simplicity. Fig. 5(a) presents profit, charging fee, electricity fee and penalty fee due to vehicle drops in the charging station every time slot. The instantaneous profit is dynamically changing over times (and sometimes it can be minus values). The profit gain of the PCSM algorithm comes from the fact that the algorithm exploits opportunism of electricity price, PHEV arrivals and renewable energy. For example, when the electricity price is low, it may charge vehicles as much as possible to fully exploit electric utility grid than other algorithms. We can observe that electricity fee sometimes can be zero when the electricity price is extremely high. At that time, the PCSM algorithm is able to use the saved energy in the storage and minimally scheduling the reserved vehicles as long as the strict waiting time is guaranteed. Also, we observe that the dropping event does not occur (zero penalty fee) in our simulation because we carry out the simulation under no-dropping conditions in Theorem 2 of Section IV.

Fig. 5(b) depicts the workload and virtual queue lengths for type-1 ($\omega_1 = 253$ Joule/sec and $\tau_1 = 1800$ sec) and type-5 ($\omega_5 = 2400$ Joule/sec and $\tau_5 = 1800$ sec) every time slot.[9] We can see that the two queues are stabilized within their own range, respectively. It is because, in our scheduling policy (23), the vehicle-type requiring higher power for charging one vehicle has lower priority to be scheduled than other types.

Fig. 5(c) demonstrates admitted arrivals and departures of vehicle for type-1 and type-5 every time slot. We observe that the vehicle-types which require high charging power are highly admitted and scheduled than other types. In our PCSM algorithm, worst-case waiting time can be similarly guaranteed for all vehicle-types (175 $\sim$ 190 minutes for the six vehicle-types), by regulating the number of vehicle arrivals per each type, which can be indirectly controlled by pricing. Also, it is a natural behavior of the CSP to admit more vehicles which require higher charging power in our system model, because it brings mostly higher profit per unit time.

VI. Related Work

Because electric vehicles (EVs) need to be charged frequently with limited capacity of battery and each charge takes a long time, EV charging station has been studied into two

[9]We only draw two curves out of six vehicle-types to clearly show the tendency.

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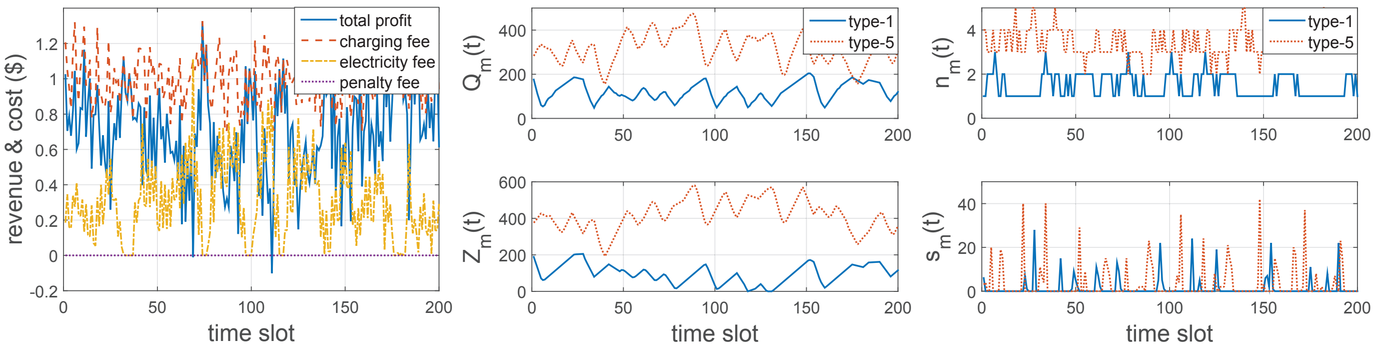

(a) Revenue and cost. (b) $Q_m(t)$ and $Z_m(t)$ for type-1 and type-5. (c) $n_m(t)$ and $s_m(t)$ for type-1 and type-5.

Fig. 5: Time traces of the result by the PCSM algorithm during $t \in [1001, 1200]$ when $V = 10^5$ and ϵ_m is set to make worst-case waiting time be the same for all vehicle-types.

representative branches: *(i)* deployment and *(ii)* management. In this section, we introduce previous studies about deployment and management of EV charging stations, and Lyapunov optimization where we use it as a technical tool.

Deployment of PHEV charging systems. There have been several studies dealing with deployment issues of charging stations. Lam *et al.* [18] proposed a charging station placement algorithm focused on human factors to make EV drivers be able to access a charging station within their driving capacities. They introduced four placement algorithms which have different solution quality, algorithmic efficiency, problem size, nature of the algorithm and existence of system prerequisite. Song *et al.* [19] proposed the prediction algorithm of the number of EVs in a planned area and the demand for the EV charging station. Based on the prediction, they found the site of a new charging station using Voronoi diagram. Liu *et al.* [20] jointly considered the location and the size of the charging station by a modified primal-dual interior point algorithm where the objective is to minimize the total cost associated with charging stations. In an another recent study, Zhang *et al.* [21] proposed a joint charger placement and power allocation algorithm to optimize charging quality in a wireless charging scenario.

Management of PHEV charging systems. There have been several studies to manage PHEV charging stations. Qin *et al.* [22] and Zhang *et al.* [11] proposed electric vehicle (EV) scheduling algorithms of charging stations to minimize waiting time of customers. To reduce the waiting time of customers and increase the charging efficiency, Dong *et al.* [23] considered battery replacement where the extra batteries are charged before PHEVs arrive. They proposed charging decision of extra batteries in the light of tradeoff between energy cost reduction and service quality improvement. However, the interest of commercial charging service provider (CSP) is more likely to maximize its financial profit. Moreover, [11] and [23] did not take into account practical features of charging stations such as pricing control for charging energy or dropping of charging works. Zhao *et al.* [24] proposed a pricing algorithm for revenue maximization of the CSP when both of battery charging and replacement services coexist. In [23] and [24], the authors assumed that all the vehicles have the same type of batteries to enable battery replacement which might not be applicable in real world[10]. Lee *et al.* [25] studied the pricing competition among neighboring and independent EV charging stations using game theory by considering the transmission line capacity, the distance between EV and charging station and the number of charging outlets. Bayram *et al.* [26] considered multi-class of EV customers characterized by charging preference, need and technology, and proposed dynamic pricing of charging stations to prevent grid failures and efficiently distribute the charging resources. In our work, we deal with battery charging service to maximize commercial CSP's profit. This paper is novel compared to existing works [11], [23]–[26] which dealt with the management of PHEV charging stations in the following point of views. *(i)* We take account of all the variables that the CSP can practically control such as pricing, scheduling, dropping and energy management policies. *(ii)* We theoretically demonstrate that our algorithm jointly guarantees worst case waiting time, no dropping and no overcharging for energy storage in certain conditions.

Lyapunov optimization. Since the seminal work of Neely [5], there have been extensive studies on various application fields, applying "Lyapunov drift-plus-penalty optimization" technique, e.g., mobile device [6], networking [7], smart grid [8], data center [9], [10] and so on. The theoretical meaning of this technique is to optimize the objective by trading delay with satisfying queueing stability. The advantage of this framework is to achieving optimality without any knowledge of future information. For example, Kwak *et al.* [6] proposed a joint CPU speed scaling and network selection algorithm in a smartphone using original Lyapunov drift-plus-penalty framework where the processing and networking workloads are modeled by queues. They demonstrated the upper bounds of energy consumption and processing and networking delay of workloads in an average sense. However, they could not guarantee the delay of each workload, which means some workloads might have an extremely huge processing delay.

Neely [7] proposed a scheduling algorithm in multi-hop wireless networks that guarantee all flows have a bounded worst-case delay where the key concepts are as follows: *(i)* The waiting data can be dropped with enduring penalty situation-

[10]There exist various types of PHEV batteries depending on output power and capacity, and the battery price depends on brand even though the battery specifications are the same.

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ally. *(ii)* The system has to stabilize not only real data queue, but also additional virtual queue where the arrival is persistent. Guo *et al.* [8] proposed an energy management algorithm for residential households in a smart grid to minimize total energy cost. They modeled not only waiting energy requests for elastic home job, but also remaining energy in the private household battery as a queue. The battery queue has a different characteristic with other type of queues in a sense that the battery queue is not a workload to serve but a resource that assists the system. The battery queue is used for efficiently storing and releasing the energy controlled by the system.

Zhao *et al.* [10] proposed a pricing, scheduling and server provisioning algorithm for cloud data centers. They dealt with a practical system model where the current scheduling decision affects the future system cost which makes the system be hard to guarantee the constant performance gap using original Lyapunov optimization. They demonstrated the constant and $O(1/V)$ optimality gap by modifying a scheduling algorithm based on time frame which is longer than charging time. Our work is to optimize a PHEV management system by jointly considering all of advanced mathematical techniques [7], [8], [10] in a unified framework.

VII. Conclusion

In this paper, we first modeled the practical environment with not only arrivals of various types of vehicles, but also waiting time guarantee for PHEV customers in a commercial PHEV charging station. For the given PHEV charging framework, we developed a close-to-optimal algorithm by controlling charging price, the number of vehicles to be scheduled, the number of vehicles to be dropped and charging/discharging of energy storage for profit maximization of a PHEV charging service provider (CSP). Through extensive theoretical analysis based on the Lyapunov mathematics, we proved following important theorems which demonstrate that the proposed algorithm achieves a close to optimal performance with satisfying practical aspects of a charging station. *(i)* The CSP can guarantee the strict delay constraint. *(ii)* The CSP does not need to drop any reserved PHEVs for given conditions. *(iii)* The energy storage does not overflow for given conditions. Finally, we demonstrate the effect of each control parameter on the performance of the proposed algorithm via trace-driven simulation. These results give us a message that there practically exists a sweet spot (which have not been presented in the past Lyapunov optimization studies) to maximize the profit of CSP with considering all of the practical aspects of future PHEV charging stations.

VIII. Appendix

A. Proof of Lemma 1.

Proof. Let us consider queueing dynamics of type-m workload, (3). By taking square on (3) and using the fact that $([X]^+)^2 \le X^2$, we have:

$$\begin{aligned} &Q_m(t+1)^2 - Q_m(t)^2 \\ &\le 2Q_m(t)\Big\{\tau_m n_m(t) - s_m(t) - s_m(t^-) - \tau_m d_m(t)\Big\} \\ &\quad + \big(\tau_m a_m^{\max}\big)^2 + \big(S + \tau_m d_m^{\max}\big)^2, \end{aligned} \tag{33}$$

where the inequality of (33) is from the queueing dynamics (3), and the bounds of vehicle arrivals, the number of scheduled and dropped vehicles.

Similarly, we obtain the following by repeating for the dynamics of type-m virtual queue, (15).

$$\begin{aligned} &Z_m(t+1)^2 - Z_m(t)^2 \\ &\le 2Z_m(t)\Big\{\epsilon_m - s_m(t) - s_m(t^-) - \tau_m d_m(t)\Big\} \\ &\quad + \big(\epsilon_m\big)^2 + \big(\tau_m d_m^{\max} + S\big)^2, \end{aligned} \tag{34}$$

where the inequality of (34) is from the queueing dynamics (15) and the bounds of the number of scheduled and dropped vehicles.

Similarly, we obtain the following by repeating for the dynamics of energy storage queue, (4).

$$\begin{aligned} &\big(E(t+1) - \lambda\big)^2 - \big(E(t) - \lambda\big)^2 \\ &\le E(t+1)^2 - E(t)^2 - 2\lambda\big(E(t+1) - E(t)\big) \\ &\le 2E(t)\big(-e(t) + r(t)\big) \\ &\quad -2\lambda\big(E(t+1) - E(t)\big) + \big(-e(t) + r(t)\big)^2 \\ &\le 2\big(-e(t) + r(t)\big)\big(E(t) - \lambda\big) + \big(\max(e^{\max}, r^{\max} - e^{\min})\big)^2. \end{aligned} \tag{35}$$

By $\frac{1}{2}\big[\sum_{m\in\mathcal{M}}\{(33) + (34)\} + (35)\big]$, we obtain the upper bound of Lyapunov drift in Lemma 1. □

B. Proof of Theorem 1.

Proof of (i):

(a) If $Vp_m^{\max}/\tau_m < Q_m(t) \le Vp_m^{\max}/\tau_m + \tau_m a_m^{\max}$, then $\tau_m Q_m(t) - Vp_m^{\max} > 0$. By the pricing algorithm (22), the CSP selects $p_m(t) = p_m^{\max}$ to make type-m user select $n_m(t) = 0$. So $Q_m(t+1) \le Q_m(t) \le Vp_m^{\max}/\tau_m + \tau_m a_m^{\max} = Q_m^{\max}$.

(b) Else if $Q_m(t) \le Vp_m^{\max}/\tau_m$, the maximum vehicle arrival is $\tau_m a_m^{\max}$, therefore $Q_m(t+1) \le Q_m(t) + \tau_m a_m^{\max} \le Vp_m^{\max}/\tau_m + \tau_m a_m^{\max} = Q_m^{\max}$.

Because $Q_m(0) = 0$, *(i)* can be proved by induction.

Proof of (ii):

(a) If $Vq_m/\tau_m < Z_m(t) \le Vq_m/\tau_m + \epsilon_m$, then $Vq_m - \tau_m Z_m(t) < 0$. By the dropping algorithm (26), the CSP drops the vehicles maximally, $d_m(t) = d_m^{\max}$. By queueing dynamics of $Z_m(t)$ (15) and the bound of ϵ_m, $\epsilon_m \le \tau_m a_m^{\max} \le \tau_m d_m^{\max}$, $Z_m(t+1) \le Z_m(t) \le Vq_m/\tau_m + \epsilon_m = Z_m^{\max}$.

(b) Else if $Z_m(t) \le Vq_m/\tau_m$, the virtual arrival is ϵ_m, therefore $Z_m(t+1) \le Z_m(t) + \epsilon_m \le Vq_m/\tau_m + \epsilon_m = Z_m^{\max}$.

Because $Z_m(0) = 0$, *(ii)* can be proved by induction.

Proof of (iii): Suppose that type-m vehicles arrive at the charging station at time slot t.

(a) If Q_m becomes zero before time slot $t + l_m$, then all vehicles arrived at time slot t are charged or dropped within l_m time slots.

(b) Else, $Q_m > 0$ during $[t, t + l_m]$. Then, there are $\epsilon_m l_m$ arrivals and $\sum_{\tau=t+1}^{t+l_m}\big(s_m(\tau) + s_m(\tau^-) + d_m(\tau)\tau_m\big)$ departures at the virtual queue Z_m during $[t, t + l_m]$. Because Z_m is bounded by $Z_m^{\max}$, the remaining workloads which have arrived after time slot t are bounded by

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$\epsilon_m l_m - \sum_{\tau=t+1}^{t+l_m} \big(s_m(\tau)+s_m(\tau^-)+d_m(\tau)\tau_m\big) \le Z_m^{\max}$. If we define $l_m = \lceil Q_m^{\max} + Z_m^{\max}/\epsilon_m \rceil$, then $\epsilon_m l_m - Z_m^{\max} \ge Q_m^{\max}$. Then we can derive $\sum_{\tau=t+1}^{t+l_m} \big(s_m(\tau) + s_m(\tau^-) + d_m(\tau)\tau_m\big) \ge \epsilon_m l_m - Z_m^{\max} \ge Q_m^{\max}$, which means the total amount of departures from Q_m during $[t, t+l_m]$ is greater than or equal to $Q_m^{\max}$. This shows all type-m vehicles arrived at time slot t are charged or dropped within l_m time slots due to $Q_m(t) \le Q_m^{\max}$. □

C. Proof of Theorem 2.

Proof. We first prove that $Q_m(t) + Z_m(t) \le V\omega^{\max}c^{\max} + (\sum_{m'\in\mathcal{M}} \tau_{m'})(\tau^{\max}a^{\max} + \epsilon^{\max}), \forall m \in \mathcal{M}$ using conditions *(i)* and *(ii)*. We prove it that by contradiction. Generally, we suppose that $Q_{m_0}(t) + Z_{m_0}(t) > V\omega^{\max}c^{\max} + (\sum_{m'\in\mathcal{M}} \tau_{m'})(\tau^{\max}a^{\max} + \epsilon^{\max})$. under conditions *(i)* and *(ii)*. Let $Q_{m_0}(0) = Z_{m_0}(0) = 0$ and $t_0 = \min\{t | Q_{m_0}(t) + Z_{m_0}(t) > V\omega^{\max}c^{\max} + (\sum_{m'\in\mathcal{M}})(\tau^{\max}a^{\max} + \epsilon^{\max})\}$. Then type-$m_0$ vehicle should not be scheduled during $[t_0 - \sum_{m\in\mathcal{M}} \tau_m, t_0 - 1]$ because of the reason as follows. If type-m_0 vehicle is scheduled in time slot $t' \in [t_0 - \sum_{m\in\mathcal{M}} \tau_m, t_0 - 1]$,

$$\begin{aligned}
&Q_{m_0}(t'+1) + Z_{m_0}(t'+1) \\
&\le Q_{m_0}(t) + Z_{m_0}(t) + (\tau^{\max}a^{\max} + \epsilon^{\max}) - S \\
&\le V\omega^{\max}c^{\max} + (\sum_{m'\in\mathcal{M}} \tau_{m'})(\tau^{\max}a^{\max} + \epsilon^{\max}) \\
&\quad + (\tau^{\max}a^{\max} + \epsilon^{\max}) - (\sum_{m'\in\mathcal{M}} \tau_{m'})(\tau^{\max}a^{\max} + \epsilon^{\max}) \\
&= V\omega^{\max}c^{\max} + (\tau^{\max}a^{\max} + \epsilon^{\max}).
\end{aligned} \tag{36}$$

Then, for time slot t_0,

$$\begin{aligned}
&Q_{m_0}(t_0) + Z_{m_0}(t_0) \\
&\le Q_{m_0}(t'+1) + Z_{m_0}(t'+1) + (t_0 - t' - 1)(\tau^{\max}a^{\max} + \epsilon^{\max}) \\
&\le V\omega^{\max}c^{\max} + (t_0 - t')(\tau^{\max}a^{\max} + \epsilon^{\max}) \\
&\le V\omega^{\max}c^{\max} + (\sum_{m'\in\mathcal{M}} \tau_{m'})(\tau^{\max}a^{\max} + \epsilon^{\max}),
\end{aligned} \tag{37}$$

which contradicts our assumption. Therefore type-m_0 vehicle should not be scheduled during $[t_0 - \sum_{m\in\mathcal{M}} \tau_m, t_0 - 1]$. Also, $Q_{m_0}(t) + Z_{m_0}(t) \ge V\omega^{\max}c^{\max}, \forall t \in [t_0 - \sum_{m\in\mathcal{M}} \tau_m, t_0 - 1]$ which implies $m_0 \in \mathcal{M}_1(t)$. To not be scheduled during that interval, other type of vehicle should be scheduled. However, the length of interval is $\sum_{m'\in\mathcal{M}} \tau_{m'}$ which is greater than $M-1$ which is the total number of vehicle-types except m_0. Then, there exists a type of vehicle m_i which is scheduled twice during interval $[t_0 - \sum_{m\in\mathcal{M}} \tau_m, t_0 - 1]$. We denote t_{i1} and t_{i2} as the time slots that type-m_i is scheduled where $t_0 - \sum_{m'\in\mathcal{M}} \tau_{m'} \le t_{i1} < t_{i2} \le t_0 - 1$.

$$\begin{aligned}
&(Q_{m_0}(t_{i2}) + Z_{m_0}(t_{i2})) \\
&\ge (Q_{m_0}(t_0) + Z_{m_0}(t_0)) - (t_0 - t_{i2})(\tau^{\max}a^{\max} + \epsilon^{\max}) \\
&> V\omega^{\max}c^{\max} + (\sum_{m'\in\mathcal{M}} \tau_{m'})(\tau^{\max}a^{\max} + \epsilon^{\max}) \\
&- (t_0 - t_{i2})(\tau^{\max}a^{\max} + \epsilon^{\max}).
\end{aligned} \tag{38}$$

Because type-m_i is scheduled at time slot t_{i2} and by scheduling algorithm (24),

$$\begin{aligned}
&V\omega_{m_0}c(t_{i2}) - (Q_{m_0}(t_{i2}) + Z_{m_0}(t_{i2})) \\
&\ge V\omega_{m_i}c(t_{i2}) - (Q_{m_i}(t_{i2}) + Z_{m_i}(t_{i2})).
\end{aligned} \tag{39}$$

Then, by (38), (39) and condition *(ii)*,

$$\begin{aligned}
&(Q_{m_i}(t_{i2}) + Z_{m_i}(t_{i2})) \\
&\ge V(\omega_{m_i} - \omega_{m_0})c(t_{i2}) + (Q_0(t_{i2}) + Z_0(t_{i2})) \\
&\ge V\omega^{\max}c^{\max} + Vc(t_{i2})(\omega_{m_i} - \omega_{m_0}) \\
&+ (\sum_{m'\in\mathcal{M}} \tau_{m'})(\tau^{\max}a^{\max} + \epsilon^{\max}) - (t_0 - t_{i2})(\tau^{\max}a^{\max} + \epsilon^{\max}) \\
&= V\omega^{\max}c^{\max} + (\sum_{m'\in\mathcal{M}} \tau_{m'})(\tau^{\max}a^{\max} + \epsilon^{\max}) \\
&\quad - (t_0 - t_{i2})(\tau^{\max}a^{\max} + \epsilon^{\max}).
\end{aligned} \tag{40}$$

Meanwhile, type-m_i vehicle is also scheduled at t_{i1}. By (36),

$$Q_{m_i}(t_{i1}+1) + Z_{m_i}(t_{i1}+1) \le V\omega^{\max}c^{\max} + (\tau^{\max}a^{\max} + \epsilon^{\max}). \tag{41}$$

Then, by (41)

$$\begin{aligned}
&Q_{m_i}(t_{i2}) + Z_{m_i}(t_{i2}) \\
&\le Q_{m_i}(t_{i1}+1) + Z_{m_i}(t_{i1}+1) + (t_{i2} - t_{i1} - 1)(\tau^{\max}a^{\max} + \epsilon^{\max}) \\
&\le V\omega^{\max}c^{\max} + (t_{i2} - t_{i1})(\tau^{\max}a^{\max} + \epsilon^{\max}) \\
&\le V\omega^{\max}c^{\max} + (\sum_{m'\in\mathcal{M}} \tau_{m'})(\tau^{\max}a^{\max} + \epsilon^{\max}) \\
&\quad - (t_0 - t_{i2})(\tau^{\max}a^{\max} + \epsilon^{\max}).
\end{aligned} \tag{42}$$

which contradicts (40). Therefore all the types can be scheduled at most once. Then it is impossible that type-m_0 vehicle is not scheduled during $[t_0 - \sum_{m\in\mathcal{M}} \tau_m, t_0 - 1]$. Therefore $Q_m(t) + Z_m(t) \le V\omega^{\max}c^{\max} + (\sum_{m'\in\mathcal{M}} \tau_{m'})(\tau^{\max}a^{\max} + \epsilon^{\max}), \forall m \in \mathcal{M}$. Then, we can prove $Q_m(t) + Z_m(t) \le Vq_m/\tau_m, \forall m \in \mathcal{M}$ easily by condition (iii) which implies there is no drop in Eq. (26). □

D. Proof of Theorem 3.

Proof. *(i)* If $\lambda - Vc(t) < E(t) \le E^{\max}$, by energy storage managing algorithm (28), $e(t) = e^{\max}$.
Then, $E(t+1) = E(t) + r(t) - e^{\max} \le E(t) + (r^{\max} - e^{\max}) \le E(t) \le E^{\max}$.

(ii) else if $E(t) \ge \lambda - Vc(t)$, $e(t) = e^{\min}$.
Then, $E(t+1) = E(t) + r(t) - e^{\min} \le \lambda - Vc(t) + (r^{\max} - e^{\min}) \le \lambda - Vc^{\min} + (r^{\max} - e^{\min})$
$\le V(g^{\max} - g^{\min}) + (e_{\text{in}}^{\max} + e_{\text{out}}^{\max} - e_{\text{out}}^{\min}) \le E^{\max}$

Because $E(0) = 0$, Theorem 3 can be proved by induction. □

E. Proof of Theorem 4.

Proof. We define $J(t) = Vc(t)\sum_{m\in\mathcal{M}} \omega_m [s_m(t) + s_m(t^-)] - \sum_{m\in\mathcal{M}} [s_m(t) + s_m(t^-)][Q_m(t) + Z_m(t)]$. Derive the bound of $J(t)$ for $t \in [\alpha T_{\text{f}} + 1, (\alpha+1)T_{\text{f}} - \tau^{\max} - 1]$.

$$\begin{aligned}
J(t) &\le Vc(t) \sum_{m\in\mathcal{M}} \omega_m [s_m(t-1) + s_m((t-1)^-)] \\
&\quad - \sum_{m\in\mathcal{M}} [s_m(t-1) + s_m((t-1)^-)][Q_m(t) + Z_m(t)].
\end{aligned} \tag{43}$$

The inequality is due to the fact that $s_m(t^-)$ is included in $s_m(t-1) + s_m((t-1)^-)$ and $s_m(t)$ is the optimal policy minimizing (43). Also, we can show that

$$\begin{cases}
|Q_m(t) + Z_m(t) - Q_m(t-1) - Z_m(t-1)| \\
\le |Q_m(t) - Q_m(t-1)| + |Z_m(t) - Z_m(t-1)| \\
\le \tau_m a_m^{\max} + \epsilon_m + 2S, \\
|c(t) - c(t-1)| \le c^{\max} - c^{\min}.
\end{cases} \tag{44}$$

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Using (43) and (44),

$$
\begin{aligned}
&J(t) - J(t-1) \\
&\le V\big[c(t) - c(t-1)\big] \sum_{m\in\mathcal{M}} \omega_m \big[s_m(t-1) + s_m((t-1)^-)\big] \\
&\quad - \sum_{m\in\mathcal{M}} \big[s_m(t-1) + s_m((t-1)^-)\big] \\
&\qquad \big[Q_m(t) + Z_m(t) - Q_m(t-1) - Z_m(t-1)\big] \\
&\le B_2 + V\omega^{\max}\big[c^{\max} - c^{\min}\big]S.
\end{aligned} \tag{45}
$$

where $B_2 = S\big[\tau^{\max}a^{\max} + 2S + \epsilon_m\big]$.

$$
\therefore J(t) \le J(t-1) + B_2 + V\omega^{\max}\big[c^{\max} - c^{\min}\big]S. \tag{46}
$$

Derive the bound of $J(t)$ for $t \in [(\alpha+1)T_\mathrm{f} - \tau^{\max}, (\alpha+1)T_\mathrm{f} - 1]$.

$$
J(t) \le Vc(t) \sum_{m\in\mathcal{M}} \omega_m \big[s_m(t) + s_m(t^-)\big] \le V\omega^{\max}c^{\max}S. \tag{47}
$$

Derive the bound of $\mathbb{E}\big[L(t+1) - L(t) - Vh(t)|\boldsymbol{K}(\alpha T_\mathrm{f})\big]$ for $t \in [\alpha T_\mathrm{f}, (\alpha+1)T_\mathrm{f} - 1]$. From Lemma 1,

$$
\begin{aligned}
&\mathbb{E}\big[L(t+1) - L(t) - Vh(t)|\boldsymbol{K}(\alpha T_\mathrm{f})\big] \\
&\le B_1 + \sum_{m\in\mathcal{M}} \mathbb{E}\big[Z_m(t)\epsilon_m|\boldsymbol{K}(\alpha T_\mathrm{f})\big] + \mathbb{E}\big[J(t)|\boldsymbol{K}(\alpha T_\mathrm{f})\big] \\
&\quad + \sum_{m\in\mathcal{M}} \mathbb{E}\big[\tau_m n_m(t)Q_m(t) - Vn_m(t)p_m(t)|\boldsymbol{K}(\alpha T_\mathrm{f})\big] \\
&\quad + \mathbb{E}\big[-Ve(t)c(t) - [E(t) - \lambda][e(t) - r(t)]|\boldsymbol{K}(\alpha T_\mathrm{f})\big].
\end{aligned} \tag{48}
$$

Derive the bound of $\mathbb{E}\big[L((\alpha+1)T_\mathrm{f}) - L(\alpha T_\mathrm{f}) - V\sum_{t=\alpha T_\mathrm{f}}^{(\alpha+1)T_\mathrm{f}-1} h(t)|\boldsymbol{K}(\alpha T_\mathrm{f})\big]$ by summation (48) from $t = \alpha T_\mathrm{f}$ to $t = (\alpha+1)T_\mathrm{f} - 1$ and substitution (46) and (47).

$$
\begin{aligned}
&\mathbb{E}\Big[L((\alpha+1)T_\mathrm{f}) - L(\alpha T_\mathrm{f}) - V\sum_{t=\alpha T_\mathrm{f}}^{(\alpha+1)T_\mathrm{f}-1} h(t)|\boldsymbol{K}(\alpha T_\mathrm{f})\Big] \\
&\le T_\mathrm{f}B_1 + \sum_{t=\alpha T_\mathrm{f}}^{(\alpha+1)T_\mathrm{f}-1} \sum_{m\in\mathcal{M}} \mathbb{E}\Big[Z_m(t)\epsilon_m|\boldsymbol{K}(\alpha T_\mathrm{f})\Big] \\
&\quad + \sum_{t=\alpha T_\mathrm{f}}^{(\alpha+1)T_\mathrm{f}-1} \sum_{m\in\mathcal{M}} \mathbb{E}\Big[n_m(t)\big[\tau_m Q_m(t) - Vp_m(t)\big]|\boldsymbol{K}(\alpha T_\mathrm{f})\Big] \\
&\quad + \sum_{\tau=0}^{T_\mathrm{f}-\tau^{\max}-1} \tau V\omega^{\max}[c^{\max} - c^{\min}]S \\
&\quad + \sum_{\tau=0}^{T_\mathrm{f}-\tau^{\max}-1} \tau B_2 + \tau^{\max}V\omega^{\max}C^{\max}S \\
&\quad + (T_\mathrm{f} - \tau^{\max})\Big[Vc(\alpha T_\mathrm{f}) \sum_{m\in\mathcal{M}} \omega_m \big[s_m(\alpha T_\mathrm{f}) + s_m((\alpha T_\mathrm{f})^-)\big] \\
&\quad - \sum_{m\in\mathcal{M}} \big[s_m(\alpha T_\mathrm{f}) + s_m((\alpha T_\mathrm{f})^-)\big]\big[Q_m(\alpha T_\mathrm{f}) + Z_m(\alpha T_\mathrm{f})\big]\Big] \\
&\quad + \sum_{t=\alpha T_\mathrm{f}}^{(\alpha+1)T_\mathrm{f}-1} \mathbb{E}\Big[-Ve(t)c(t) - \big[E(t) - \lambda\big]\big[e(t) - r(t)\big]|\boldsymbol{K}(\alpha T_\mathrm{f})\Big].
\end{aligned} \tag{49}
$$

By Caratheodory theorem [27], there exists an offline algorithm that achieves optimal profit under supportable arrival rate vector $\boldsymbol{x} = (x_1, x_2, ..., x_M) \in \boldsymbol{\mathcal{X}}$ and $y \in \mathcal{Y}$ without dropping any vehicle and violating worst-case delay guarantee. Consider the vehicle arrival rate vector $\boldsymbol{x}$ that satisfies $\frac{T_\mathrm{f}}{T_\mathrm{f} - \tau^{\max}}\boldsymbol{x} \in \boldsymbol{\mathcal{X}}$ and renewable energy arrival rate y that satisfies $\frac{T_\mathrm{f}}{T_\mathrm{f} - \tau^{\max}} y \in \mathcal{Y}$, then there exists $\delta \ge 0$ which satisfies $\frac{(1+\delta)T_\mathrm{f}}{T_\mathrm{f} - \tau^{\max}}\boldsymbol{x} \in \boldsymbol{\mathcal{X}}$ and $\frac{(1+\delta)T_\mathrm{f}}{T_\mathrm{f} - \tau^{\max}} y \in \mathcal{Y}$. Under these arrival conditions, we denote $h^{\frac{(1+\delta)T_\mathrm{f}}{T_\mathrm{f} - \tau^{\max}}}$ as the offline optimal profit can be achieved and $p_m^*(t), s_m^*(t)$ and $e^*(t)$ are the optimal policies. Our PCSM algorithm controls $p_m(t), s_m(t), e(t)$ to minimize the bound of 1-slot Lyapunov drift-plus-penalty. Moreover, due to new PCSM scheduling algorithm (29), $s_m((\alpha T_\mathrm{f})^-)$ is always zero for all $\alpha \in \{0, 1, 2, ...\}$.

$$
\begin{aligned}
&\text{RHS of (49)} \\
&\le T_\mathrm{f}B_1 + \sum_{t=\alpha T_\mathrm{f}}^{(\alpha+1)T_\mathrm{f}-1} \sum_{m\in\mathcal{M}} \mathbb{E}\Big[Z_m(t)\epsilon_m|\boldsymbol{K}(\alpha T_\mathrm{f})\Big] \\
&\quad + \sum_{t=\alpha T_\mathrm{f}}^{(\alpha+1)T_\mathrm{f}-1} \sum_{m\in\mathcal{M}} \mathbb{E}\Big[n_m^*(t)\big[\tau_m Q_m(t) - Vp_m^*(t)\big]|\boldsymbol{K}(\alpha T_\mathrm{f})\Big] \\
&\quad + \sum_{\tau=0}^{T_\mathrm{f}-\tau^{\max}-1} \tau V\omega^{\max}[c^{\max} - c^{\min}]S \\
&\quad + \sum_{\tau=0}^{T_\mathrm{f}-\tau^{\max}-1} \tau B_2 + \tau^{\max}V\omega^{\max}c^{\max}S \\
&\quad + (T_\mathrm{f} - \tau^{\max})\Big[Vc(\alpha T_\mathrm{f}) \sum_{m\in\mathcal{M}} \omega_m s_m^*(\alpha T_\mathrm{f}) \\
&\qquad - \sum_{m\in\mathcal{M}} s_m^*(\alpha T_\mathrm{f})\big[Q_m(\alpha T_\mathrm{f}) + Z_m(\alpha T_\mathrm{f})\big]\Big] \\
&\quad + \sum_{t=\alpha T_\mathrm{f}}^{(\alpha+1)T_\mathrm{f}-1} \mathbb{E}\Big[-Ve^*(t)c(t) - \big[E(t) - \lambda\big]\big[e^*(t) - r(t)\big]|\boldsymbol{K}(\alpha T_\mathrm{f})\Big].
\end{aligned} \tag{50}
$$

Substitute the fact that $Z_m(t) \le Z_m(\alpha T_\mathrm{f}) + (t - \alpha T_\mathrm{f})\epsilon_m$ and $Q_m(t) \le Q_m(\alpha T_\mathrm{f}) + (t - \alpha T_\mathrm{f})\tau_m a_m^{\max}$ to (50).

$$
\begin{aligned}
&\text{RHS of (50)} \\
&\le T_\mathrm{f}B_1 - \sum_{t=\alpha T_\mathrm{f}}^{(\alpha+1)T_\mathrm{f}-1} \sum_{m\in\mathcal{M}} \mathbb{E}\Big[n_m^*(t)Vp_m^*(t)|\boldsymbol{K}(\alpha T_\mathrm{f})\Big] \\
&\quad + \sum_{t=\alpha T_\mathrm{f}}^{(\alpha+1)T_\mathrm{f}-1} \sum_{m\in\mathcal{M}} \mathbb{E}\Big[\big[Z_m(\alpha T_\mathrm{f}) + (t - \alpha T_\mathrm{f})\epsilon_m\big]\epsilon_m|\boldsymbol{K}(\alpha T_\mathrm{f})\Big] \\
&\quad + \sum_{t=\alpha T_\mathrm{f}}^{(\alpha+1)T_\mathrm{f}-1} \sum_{m\in\mathcal{M}} \mathbb{E}\Big[n_m^*(t)\tau_m\big[Q_m(\alpha T_\mathrm{f}) \\
&\qquad + (t - \alpha T_\mathrm{f})\tau_m a_m^{\max}\big]|\boldsymbol{K}(\alpha T_\mathrm{f})\Big] \\
&\quad + \sum_{\tau=0}^{T_\mathrm{f}-\tau^{\max}-1} \tau V\omega^{\max}[c^{\max} - c^{\min}]S \\
&\quad + \sum_{\tau=0}^{T_\mathrm{f}-\tau^{\max}-1} \tau B_2 + \tau^{\max}V\omega^{\max}c^{\max}S \\
&\quad + (T_\mathrm{f} - \tau^{\max})\Big[Vc(\alpha T_\mathrm{f}) \sum_{m\in\mathcal{M}} \omega_m s_m^*(\alpha T_\mathrm{f}) \\
&\qquad - \sum_{m\in\mathcal{M}} s_m^*(\alpha T_\mathrm{f})\big[Q_m(\alpha T_\mathrm{f}) + Z_m(\alpha T_\mathrm{f})\big]\Big] \\
&\quad + \sum_{t=\alpha T_\mathrm{f}}^{(\alpha+1)T_\mathrm{f}-1} \mathbb{E}\Big[-Ve^*(t)c(t) - \big[E(t) - \lambda\big]\big[e^*(t) - r(t)\big]|\boldsymbol{K}(\alpha T_\mathrm{f})\Big] \\
&\le T_\mathrm{f}B_1 - \sum_{t=\alpha T_\mathrm{f}}^{(\alpha+1)T_\mathrm{f}-1} \sum_{m\in\mathcal{M}} \mathbb{E}\Big[n_m^*(t)Vp_m^*(t)|\boldsymbol{K}(\alpha T_\mathrm{f})\Big] \\
&\quad + \sum_{\tau=0}^{T_\mathrm{f}-1} \sum_{m\in\mathcal{M}} \tau\big[(\epsilon_m)^2 + (\tau_m a_m^{\max})^2\big] \\
&\quad + \sum_{\tau=0}^{T_\mathrm{f}-\tau^{\max}-1} \tau V\omega^{\max}[c^{\max} - c^{\min}]S + \sum_{\tau=0}^{T_\mathrm{f}-\tau^{\max}-1} \tau B_2 \\
&\quad + \tau^{\max}V\omega^{\max}c^{\max}S + (T_\mathrm{f} - \tau^{\max})Vc(\alpha T_\mathrm{f}) \sum_{m\in\mathcal{M}} \omega_m s_m^*(\alpha T_\mathrm{f}) \\
&\quad - \sum_{m\in\mathcal{M}} Q_m(\alpha T_\mathrm{f})\Big[(T_\mathrm{f} - \tau^{\max})s_m^*(\alpha T_\mathrm{f})
\end{aligned}
$$

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$$
\begin{aligned}
&- \sum_{t=\alpha T_{\mathrm{f}}}^{(\alpha+1)T_{\mathrm{f}}-1} \tau_m \mathbb{E}\big[n_m^*(t)|\boldsymbol{K}(\alpha T_{\mathrm{f}})\big]\Big] \\
&- \sum_{m\in\mathcal{M}} Z_m(\alpha T_{\mathrm{f}})\Big[(T_{\mathrm{f}}-\tau^{\max})s_m^*(\alpha T_{\mathrm{f}}) - T_{\mathrm{f}}\epsilon_m\Big] \\
&+ \sum_{t=\alpha T_{\mathrm{f}}}^{(\alpha+1)T_{\mathrm{f}}-1} \mathbb{E}\Big[-Ve^*(t)g(t) - \big[E(t)-\lambda\big]\big[e^*(t)-r(t)\big]|\boldsymbol{K}(\alpha T_{\mathrm{f}})\Big].
\end{aligned} \tag{51}
$$

Take expectation and average on (51) from $\alpha = 0$ to $\alpha = \pi-1$,

$$
\begin{aligned}
&\frac{\mathbb{E}\big[L(\pi T_{\mathrm{f}})\big]}{\pi V T_{\mathrm{f}}} - \frac{\mathbb{E}\big[L(0)\big]}{\pi V T_{\mathrm{f}}} - \frac{1}{\pi T_{\mathrm{f}}}\sum_{\alpha=0}^{\pi-1}\sum_{t=\alpha T_{\mathrm{f}}}^{(\alpha+1)T_{\mathrm{f}}-1} \mathbb{E}\big[h(t)\big] \\
&\le \frac{B_1}{V} + \frac{(T_{\mathrm{f}}-\tau^{\max})(T_{\mathrm{f}}-\tau^{\max}-1)}{2VT_{\mathrm{f}}}B_2 \\
&\quad + \frac{T_{\mathrm{f}}-1}{2V}\sum_{m\in\mathcal{M}}\big[(\epsilon_m)^2 + (\tau_m a_m^{\max})^2\big] + \frac{1}{T_{\mathrm{f}}}\tau^{\max}\omega^{\max}c^{\max}S \\
&\quad + \frac{(T_{\mathrm{f}}-\tau^{\max})(T_{\mathrm{f}}-\tau^{\max}-1)}{2T_{\mathrm{f}}}\omega^{\max}[c^{\max}-c^{\min}]S \\
&\quad - \frac{1}{\pi T_{\mathrm{f}}}\sum_{\alpha=0}^{\pi-1}\sum_{t=\alpha T_{\mathrm{f}}}^{(\alpha+1)T_{\mathrm{f}}-1}\mathbb{E}\big[n_m^*(t)p_m^*(t)\big] \\
&\quad + \frac{1}{\pi T_{\mathrm{f}}}(T_{\mathrm{f}}-\tau^{\max})\sum_{\alpha=0}^{\pi-1}\mathbb{E}\Big[c(\alpha T_{\mathrm{f}})\sum_{m\in\mathcal{M}}\omega_m s_m^*(\alpha T_{\mathrm{f}})\Big] \\
&\quad - \frac{1}{\pi T_{\mathrm{f}}}\sum_{\alpha=0}^{\pi-1}\sum_{m\in\mathcal{M}}\mathbb{E}\big[Q_m(\alpha T_{\mathrm{f}})\big] \\
&\qquad \frac{(T_{\mathrm{f}}-\tau^{\max})\mathbb{E}[s_m^*(\alpha T_{\mathrm{f}})] - \sum_{t=\alpha T_{\mathrm{f}}}^{(\alpha+1)T_{\mathrm{f}}-1}\tau_m\mathbb{E}[n_m^*(t)]}{V} \\
&\quad - \frac{1}{\pi T_{\mathrm{f}}}\sum_{\alpha=0}^{\pi-1}\sum_{m\in\mathcal{M}}\mathbb{E}\big[Z_m(\alpha T_{\mathrm{f}})\big]\frac{(T_{\mathrm{f}}-\tau^{\max})\mathbb{E}[s_m^*(\alpha T_{\mathrm{f}})] - T_{\mathrm{f}}\epsilon_m}{V} \\
&\quad - \frac{1}{\pi T_{\mathrm{f}}}\sum_{\alpha=0}^{\pi-1}\sum_{t=\alpha T_{\mathrm{f}}}^{(\alpha+1)T_{\mathrm{f}}-1}\mathbb{E}\big[e^*(t)c(t)\big] \\
&\quad - \frac{1}{\pi T_{\mathrm{f}}}\sum_{\alpha=0}^{\pi-1}\sum_{t=\alpha T_{\mathrm{f}}}^{(\alpha+1)T_{\mathrm{f}}-1}\frac{\mathbb{E}\big[[E(t)-\lambda][e^*(t)-r(t)]\big]}{V}.
\end{aligned} \tag{52}
$$

Because we consider the capacity region $\frac{T_{\mathrm{f}}-\tau^{\max}}{T_{\mathrm{f}}}\boldsymbol{\mathcal{X}}$ and $\frac{T_{\mathrm{f}}-\tau^{\max}}{T_{\mathrm{f}}}\mathcal{Y}$, for all $m \in \mathcal{M}$,

$$
\begin{aligned}
&\text{a) } \frac{T_{\mathrm{f}}-\tau^{\max}}{T_{\mathrm{f}}}\mathbb{E}[s_m^*(\alpha T_{\mathrm{f}})] \ge \frac{(1+\delta)}{T_{\mathrm{f}}}\sum_{t=\alpha T_{\mathrm{f}}}^{(\alpha+1)T_{\mathrm{f}}-1}\tau_m\mathbb{E}[n_m^*(t)], \\
&\text{b) } \frac{T_{\mathrm{f}}-\tau^{\max}}{T_{\mathrm{f}}}\mathbb{E}[s_m^*(\alpha T_{\mathrm{f}})] \ge (1+\delta)\epsilon_m, \\
&\text{c) } \frac{T_{\mathrm{f}}-\tau^{\max}}{T_{\mathrm{f}}}\sum_{t=\alpha T_{\mathrm{f}}}^{(\alpha+1)T_{\mathrm{f}}-1}\mathbb{E}[e^*(t)] \ge \frac{(1+\delta)}{T_{\mathrm{f}}}\sum_{t=\alpha T_{\mathrm{f}}}^{(\alpha+1)T_{\mathrm{f}}-1}\mathbb{E}[r(t)].
\end{aligned} \tag{53}
$$

The equation (53)-a) comes from the workload queue Q_m, b) comes from the virtual queue Z_m and c) comes from the energy storage queue E. By rearranging (53),

$$
\begin{aligned}
&\text{a) } \frac{1}{T_{\mathrm{f}}}\sum_{t=\alpha T_{\mathrm{f}}}^{(\alpha+1)T_{\mathrm{f}}-1}\tau_m\mathbb{E}[n_m^*(t)] - \frac{T_{\mathrm{f}}-\tau^{\max}}{T_{\mathrm{f}}}\mathbb{E}[s_m^*(\alpha T_{\mathrm{f}})] \\
&\qquad\qquad \le -\frac{\delta}{T_{\mathrm{f}}}\sum_{t=\alpha T_{\mathrm{f}}}^{(\alpha+1)T_{\mathrm{f}}-1}\tau_m\mathbb{E}[n_m^*(t)], \\
&\text{b) } \epsilon_m - \frac{T_{\mathrm{f}}-\tau^{\max}}{T_{\mathrm{f}}}\mathbb{E}[s_m^*(\alpha T_{\mathrm{f}})] \le -\delta\epsilon_m, \\
&\text{c) } \frac{1}{T_{\mathrm{f}}}\sum_{t=\alpha T_{\mathrm{f}}}^{(\alpha+1)T_{\mathrm{f}}-1}\mathbb{E}[r(t)] - \frac{1}{T_{\mathrm{f}}}\sum_{t=\alpha T_{\mathrm{f}}}^{(\alpha+1)T_{\mathrm{f}}-1}\mathbb{E}[e^*(t)] \\
&\qquad\qquad \le -\frac{\delta}{T_{\mathrm{f}}}\sum_{t=\alpha T_{\mathrm{f}}}^{(\alpha+1)T_{\mathrm{f}}-1}\mathbb{E}[r(t)].
\end{aligned} \tag{54}
$$

Substitute (54) to (52),

$$
\begin{aligned}
&\text{RHS of (52)} \\
&\le \frac{B_1}{V} + \frac{(T_{\mathrm{f}}-\tau^{\max})(T_{\mathrm{f}}-\tau^{\max}-1)}{2VT_{\mathrm{f}}}B_2 \\
&\quad + \frac{T_{\mathrm{f}}-1}{2V}\sum_{m\in\mathcal{M}}\big[(\epsilon_m)^2 + (\tau_m a_m^{\max})^2\big] + \frac{1}{T_{\mathrm{f}}}\tau^{\max}\omega^{\max}c^{\max}S \\
&\quad + \frac{(T_{\mathrm{f}}-\tau^{\max})(T_{\mathrm{f}}-\tau^{\max}-1)}{2T_{\mathrm{f}}}\omega^{\max}[c^{\max}-c^{\min}]S \\
&\quad + \frac{1}{\pi T_{\mathrm{f}}}(T_{\mathrm{f}}-\tau^{\max})\sum_{\alpha=0}^{\pi-1}\mathbb{E}\Big[c(\alpha T_{\mathrm{f}})\sum_{m\in\mathcal{M}}\omega_m s_m^*(\alpha T_{\mathrm{f}})\Big] \\
&\quad - \frac{1}{\pi T_{\mathrm{f}}}\sum_{\alpha=0}^{\pi-1}\sum_{t=\alpha T_{\mathrm{f}}}^{(\alpha+1)T_{\mathrm{f}}-1}\mathbb{E}\big[n_m^*(t)p_m^*(t)\big] \\
&\quad - \frac{1}{\pi T_{\mathrm{f}}}\sum_{\alpha=0}^{\pi-1}\sum_{m\in\mathcal{M}}\mathbb{E}\big[Q_m(\alpha T_{\mathrm{f}})\big]\frac{\delta\sum_{t=\alpha T_{\mathrm{f}}}^{(\alpha+1)T_{\mathrm{f}}-1}\tau_m\mathbb{E}[n_m^*(t)]}{V} \\
&\quad - \frac{1}{\pi T_{\mathrm{f}}}\sum_{\alpha=0}^{\pi-1}\sum_{m\in\mathcal{M}}\mathbb{E}\big[Z_m(\alpha T_{\mathrm{f}})\big]\frac{\delta\epsilon_m}{V} \\
&\quad - \frac{1}{\pi T_{\mathrm{f}}}\sum_{\alpha=0}^{\pi-1}\sum_{t=\alpha T_{\mathrm{f}}}^{(\alpha+1)T_{\mathrm{f}}-1}\mathbb{E}\big[e^*(t)c(t)\big] \\
&\quad - \frac{1}{\pi T_{\mathrm{f}}}\sum_{\alpha=0}^{\pi-1}\sum_{t=\alpha T_{\mathrm{f}}}^{(\alpha+1)T_{\mathrm{f}}-1}\mathbb{E}[E(t)]\frac{\delta\mathbb{E}[r(t)]}{V} + \frac{\lambda e^{\max}}{V}.
\end{aligned} \tag{55}
$$

By the inequality as follows,

$$
\begin{aligned}
&\lim_{\pi\to\infty}\Bigg[\frac{1}{\pi T_{\mathrm{f}}}\sum_{\alpha=0}^{\pi-1}\sum_{t=\alpha T_{\mathrm{f}}}^{(\alpha+1)T_{\mathrm{f}}-1}\mathbb{E}\big[n_m^*(t)p_m^*(t)\big] \\
&\qquad - \frac{1}{\pi T_{\mathrm{f}}}(T_{\mathrm{f}}-\tau^{\max})\sum_{\alpha=0}^{\pi-1}\mathbb{E}\Big[c(\alpha T_{\mathrm{f}})\sum_{m\in\mathcal{M}}\omega_m s_m^*(\alpha T_{\mathrm{f}})\Big] \\
&\qquad + \frac{1}{\pi T_{\mathrm{f}}}\sum_{\alpha=0}^{\pi-1}\sum_{t=\alpha T_{\mathrm{f}}}^{(\alpha+1)T_{\mathrm{f}}-1}\mathbb{E}\big[e^*(t)c(t)\big]\Bigg] \\
&= h^{\frac{(1+\delta)T_{\mathrm{f}}}{T_{\mathrm{f}}-\tau^{\max}}} + \lim_{\pi\to\infty}\frac{1}{\pi T_{\mathrm{f}}}\tau^{\max}\sum_{\alpha=0}^{\pi-1}\mathbb{E}\Big[c(\alpha T_{\mathrm{f}})\sum_{m\in\mathcal{M}}\omega_m s_m^*(\alpha T_{\mathrm{f}})\Big] \\
&\ge h^{\frac{(1+\delta)T_{\mathrm{f}}}{T_{\mathrm{f}}-\tau^{\max}}},
\end{aligned} \tag{56}
$$

The average profit of PCSM algorithm can be lower-bounded as follows,

$$
\begin{aligned}
&\lim_{\pi\to\infty}\frac{1}{\pi T_{\mathrm{f}}}\sum_{\alpha=0}^{\pi-1}\sum_{t=\alpha T_{\mathrm{f}}}^{(\alpha+1)T_{\mathrm{f}}-1}\mathbb{E}[h(t)] \\
&\ge h^{\frac{(1+\delta)T_{\mathrm{f}}}{T_{\mathrm{f}}-\tau^{\max}}} - \frac{B_1}{V} - \frac{(T_{\mathrm{f}}-\tau^{\max})(T_{\mathrm{f}}-\tau^{\max}-1)}{2T_{\mathrm{f}}V}B_2 \\
&\quad - \frac{T_{\mathrm{f}}-1}{2V}\sum_{m\in\mathcal{M}}\big[(\epsilon_m)^2 + (\tau_m a_m^{\max})^2\big] \\
&\quad - \frac{\lambda e^{\max}}{V} - \frac{(T_{\mathrm{f}}-\tau^{\max})(T_{\mathrm{f}}-\tau^{\max}-1)}{2T_{\mathrm{f}}}\omega^{\max}[c^{\max}-c^{\min}]S \\
&\quad - \frac{1}{T_{\mathrm{f}}}\tau^{\max}\omega^{\max}c^{\max}S.
\end{aligned} \tag{57}
$$

□

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