Heterogeneous Expectation Formation

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Abstract

The aim of this dissertation was to explore heterogeneous expectation formation in an asset pricing model environment. We reviewed literature comparing two approaches to expectation formation: the established Rational Expectation hypothesis and the more contemporary approach, Heterogeneous Expectation hypothesis. We found there were validity issues with the Rational Expectation hypothesis assumptions, which meant, empirically the assumptions failed to hold. Moreover, there was significant evidence proving the existence of heterogeneity, in expectation formation. For heterogeneity, we allowed the use of *two* statistical methods (simple linear regressions and autoregressions) for expectation formation in our model. Agents had two objectives: *i)* to predict the future asset market price and, *ii)* keep, sell or buy an asset, alternatively they could choose inactivity. We found agents using both statistical methods had developed a preference to keep assets, rather than to sell. Then, we investigated how observability could impact agents' decisions and found it had a deterring effect on agents, they were more likely to choose inactivity than to keep, sell or buy. Our model has many limitations, the first being that it is a cursory and oversimplified exploration of the complexities involved in decision-making. Ergo, this model captures complexities within an exogenous data generating process, it fails to provide a complete model by excluding endogenous heterogeneity. In addition to the above, our model uses a synthetic population, rather than human subjects, as do most Heterogeneous Expectation Formation models. Thus, our results are limited in terms of generalisations and extrapolations to real life populations.

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Chapter 1

Introduction

Macro-economic models are used to describe the processes of an economy, to predict the affects of policy changes or to produce economic forecasts (Abel, Bernanke, and Croushore, 2014). These models examine comparative statics and aggregate quantities such as Gross Domestic Product (GDP), Income, Employment and Price Levels (Mas-Colell, Whinston, and Green, 1995). Macro-economic models study decisions with uncertainty and over time, therefore, the expectations of consumers (individuals), businesses and governments are a fundamental part of the model. With models that include any uncertainty, some assume that these expectations are formed rationally. To assume rational expectations means any expectations formed by agents (consumers, businesses or governments) are correct on average, but not all the time. Given that the objective for any agent is to maximise utility of future consumption, (Muth, 1961); (Lucas, 1972) proposed that under the rational expectations hypothesis (RE), agents used three factors to make their decisions: rationality, information and experiences.

RE is a prominent feature in most contemporary macro-economic models, however, they underestimate the importance of heterogeneity. Brock and C. H. Hommes (1997) outlined the heterogeneous expectations hypothesis (HEH) which suggests that agents that switch between different forecasting models performed better. C. Hommes, Sonnemans, et al. (2005) performed a laboratory experiment that used a simple asset model to test the existence of HEH and found only 25% used rational expectations to form predictions whereas 75% used another set of adaptive rules. In further work, C. Hommes (2011) provided evidence in support of their heuristic switching model outlined in previous work, they proposed four ways agents could form their expectations: RE, Naive Expectation, Sample Average, or using an autoregressive model. Over the past 15 years a large body of work has been developed in the support of HEH, the inclusion of heterogeneity paired with agent-based models allows for a deeper understanding aggregate phenomena.

This dissertation aims to recreate a Heterogeneous agent model (HAM) using a simple asset pricing model that will allow agents to form expectations differently; however, will not be given the opportunity to switch between forecasting rules as proposed under HEH. Considering the natural scope a dissertation, agents will be limited to a choice of just two expectation formation methods, under a adaptive learning forecasting rule. This method introduces heterogeneity through the bounding of agents analytical capabilities, this will mirror the differences in skill/foresight that consumers may exhibit in real life. Furthermore, this model will explore the impact that observability could have on decision-making.

In this dissertation, we will undertake the following:

1. *Introduce heterogeneity into the rational expectation hypothesis*

   Under REH agents use all the information available to form expectations and on average make correct predictions. Expectations in this model will be formed in one of two ways: simple linear regression (SLR) or autoregressive model (ARIMA). The model will use an adaptive learning technique which means agents will training their models on the previous asset market prices, to predict future asset market prices. The difference between each predictive tool is *how* information is utilised, as proposed in the HEH. ARIMA uses previous prices to predict the future, whereas SLR does not. Agents still utilise all available information, as in REH, but in different ways to allow for heterogeneity.

2. *Investigate the impact observability may have on agent's decision-making*

   Agents will make decisions based on their expectations regardless of the accuracy of their predictive tool. There are four decisions an agent can make each new period; they can *buy*, *sell* or *keep* an asset, or *do nothing*. Given the assumption that agents using the ARIMA model will make better decisions that maximise their utility (which is to own and keep assets); we will then compare these decisions to those made by agents using SLR, but also allow for SLR agents to observe the decision of others. These agents able to observe, will have no insight on the predictive rule used to form expectations and decisions.

This dissertation is organised as follows. Chapter two is a review of expectation formation literature, it evaluates both the rational expectations and the heterogeneous expectation hypotheses. Chapter three outlines the agent-based model framework as our methodology. In chapter four we discuss the results and findings. Finally, chapter five concludes.

Chapter 2

Literature Review

This chapter will review expectation formation literature. We will begin by outlining and evaluating the well-established Rational Expectation hypothesis, then we will describe the alternative expectation formation approach, heterogeneous expectation. The final subsection will briefly summarise the agent-based computational economics methodology, a now widely accepted tool used when modelling economic phenomenon.

2.1 Rational Expectation Hypothesis

Muth (1961) proposed the theory of rational expectations (RE) which describes market outcomes as partly dependent on what people expect to happen. This idea of expectations is not novel, Keynes (1936) established the connection between people's expectations and the future, calling it "waves of optimism and pessimism" (p.247) - which plays a central role in determining the business cycle[1]. It was not until the 1970's neoclassical revolution where Lucas (1972) placed the spotlight on rational expectations, proposing it as a theory of consumer behaviour.

RE proposes that agents within a model become fully aware its processes, so the models predictions are considered to be valid most of the time (Snowdon, Vane, and Wynarczyk, 1994). By assuming "model-consistent expectations" this ensures consistency when modelling uncertainty. In other words, on average, agent's expectations of the future are correct. For this reason, RE gained popularity when modelling macroeconomic scenarios, as they investigate decisions under future uncertainty. What individuals, businesses and governments expect of the future is crucial to the model; RE allows for agents to make accurate decisions but occasionally make the wrong ones. And so, considering that there are limits to predicting the future, agent's expectations are not systematically biased and all information is utilised during the formation of expectations.

McCloskey (1998) accentuated that RE was an "expression of intellectual modesty" (p.53):

[1]Business cycle refers to the fluctuations of an economy's gross domestic product around the long-term growth trend (Madhani, 2010)

Muth's notion was that the professors [of economics], even if correct in their model of man, could do no better in predicting than could the hog farmer or steelmaker or insurance company. The notion is one of intellectual modesty. The common sense is "rationality": therefore Muth called the argument "rational expectations"

It is important to distinguish the RE assumption from Rational Choice assumptions of individual rationality. RE assumes aggregate consistency in dynamic models, Rational Choice theory explores the decision-making of individuals - this theory is used extensively in game theory and other areas (Levine, n.d.).

RE defines expectations as one's best guess of the future that uses all relevant information; therefore, forecasted outcomes do not contradict market equilibrium results (Gao, Song, and Wang, 2013). This suggests that individuals do not make systematic errors when predicting, so divergence from perfect foresight are random (Muth, 1961). As an equation we would model this as the expected value of a variable equating to the expected value predicted (Fair, 1993). Lets consider the following example, the equilibrium price of a simple market is P and is determined by supply and demand. RE says the real price will only diverge from expectations if an 'information shock'[2] occurs (Lane and Sophister, 1994), which suggests price should usually equal RE:

$$P = P^* + \epsilon$$

$$E[P] = P^*$$

P^* as the RE prediction, ϵ the random error with the expected value of 0.

RE is a theory considered fundamental to macroeconomic studies, notwithstanding its significance, it has been criticised theoretically and empirically. Theoretically, Arrow (1978) criticised the assumption that: standard individuals skills for predicting should equate to the skills of a trained professional, who in turn are more successful at predicting. Lane and Sophister (1994) rebutted that criticism by arguing, RE assumes individuals with a standard set of skills and trained professionals would form the same expectation, just not using the same methods. From Arrow (1978) criticism came the common misunderstanding that assuming rationality was implausible. In other words, to assume all agents have the capability to fully comprehend and utilise all available information to make, on average correct decisions, is unrealistic. However, Von Mises (1998) studied human action and argued humans always act rational, meaning individuals would always choose the means that enable them to fulfill their targets. For this reason, it is in fact the discussion of irrationality that is inexplicable.

Another popular criticism of RE was the ambiguity surrounding the information element. B. M. Friedman (1979) highlighted that there was little clarity on how agents' derived the information used to form expectations. Lane and Sophister (1994) argued that insight on learning

[2]Information shock refers to any information that was not unavailable when expectations were formed (Muth, 1961)

processes would create concise model output, but was not essential to the fulfillment of the RE hypothesis. Not only was information extraction a problem, but so was agent's accessibility. Stigler (1961) found that RE failed to accommodate for the cost and availability of information, therefore failed to consider that agents would have varying qualities of information. Further, Frömmel (2017) posed that individuals would form expectations with imperfect knowledge which "represented their optimal demand for information". This proposed the idea that for some agents access to all the information would be nonoptimal.

Likewise, the assumption that agents could utilise all available information was investigated by Tversky and Kahneman (1989) and Rubinstein (1998). They found that, as humans have natural cognitive limitations with memory and processing capabilities, it was improbable to assume that all the information could be used at all. Therefore, individuals would choose pieces of information they considered useful and important.

Empirically, the validity of has also been heavily criticised. Lovell (1986) reviewed evidence from a number of papers challenging the validity of RE as an hypothesis, questioning whether it could be investigated both on a macro-level and a micro-level. As, a theory only able to explain on aggregate terms, only provides half the picture and cannot properly explain system processes as a whole. In addition, Sargent (1982) who made substantial contributions to the development of RE, did not claim RE was realistic, nor that it could be proved through any empirical analysis. Prescott (1971) also stated that RE could not be directly observed, in the same way that some economic phenomena are acknowledged to exist, but remain unquantifiable. In spite of these criticisms Lovell (1986) suggested the use of survey data for expectational variables, could be useful when modelling RE empirically.

Following this suggestion, there are a number of studies investigating the validity of rationality, through the use of survey data. Engel (1996) surveyed literature to investigating forward discount (when the expected future price of a currency is less than the spot price foreign exchange)as a predictor in the foreign exchange market. He concluded that the forward discount was a biased predictor. J. A. Frankel and Froot (1987), Froot and J. A. Frankel (1989), Cavaglia, Verschoor, and Wolff (1993a), Cavaglia, Verschoor, and Wolff (1993b), Chinn and J. Frankel (1994) then used survey data to investigate the bias and found that both risk and irrationality[3] were the source of bias. Similarly, Dominguez (1986) tested the RE hypothesis on foreign exchange expectations survey data of different currency markets, using seemingly unrelated regressions (SUR). The analysis led them to reject the RE hypothesis[4]. Miah et al. (2003) examined RE in foreign exchange markets using currency data that spanned over 12 years. Using ADF and DF-GLS unit root tests they compared actual and expected rates and assessed the relationship. They found that 2/3 currencies investigated 1-month expectations were rational, but at 6 and 12-months ahead none of the currency forecasts were rational.

In studies investigating foreign market prices, RE failed to hold, even with the use of survey data. However, Lovell (1986) explained using survey data as proxies would suggest that all

[3] Irrationality here refers to the divergence from the RE hypothesis

[4] For similar results, Peter C. Liu and G. Maddala (1992) and P. C. Liu and G. S. Maddala (1992)

individuals within that market were forming rational expectations. Though RE states that on average, rational expectations would be correct, it does not insinuate every individual would form rational expectation. Consequently, the results of empirical analysis using survey data would not describe the forecasts of market behaviour; thus any irrationality found in survey forecasts would not imply market forecasts were irrational Lane and Sophister (1994). Though, the RE hypothesis failed to hold in many of these currency market studies, the reason is likely related to the ambiguity stemming from the central elements of the theorem. Miller, Nelson, and Supel (1975) concluded though RE may not provide a thorough explanation of expectation formation, it identified a "blind spot" in the behavioural aspect of macro models, that made significant changes to the industry.

2.2 Heterogeneous Expectation Hypothesis

Although the rational expectation hypothesis has its shortcoming, it is still widely accepted amongst researchers. However, when discussing both empirical and laboratory experiments, RE model outcomes remain inconclusive. The validity issue could be the result of ambiguity around the information assumption, as indicated in the previous section. To assume knowledge in the intricate processes of economic systems is common knowledge remains unrealistic, as even trained professionals *approximate*. A real world example that RE failed to explain was the 2007/2008 contraction of financial markets. The economic crises that ensued for years diverged completely from the RE assumptions, behaviours displayed were not rational.

Around the same time that RE was introduced, an alternative approach, Adaptive learning had also been proposed. Contrary to RE, Simon (1957) proposed that agents were bounded rationally, which assumed they have no true understanding of economic processes. Therefore, rather than forming expectations on all available information at that time, they formed expectations on timeseries observations and they would be able to adapt their behaviour over time.

Bounded rationality allows for the organic development of insight into economic processes, rather than unrealistically assuming everyone natural had this understanding. Bounded rationality, allows room for more error, unlike RE. Despite this assumption, Adaptive Learning still had shortcomings, the prevalent criticism was: there was simply to *too many* non-rational behaviour could be modelled, excluding heterogeneity (Sims, 1980).

Nonetheless, as RE developed, so did adaptive learning and bounded rationality; and from this came, Heterogeneous Agent Models (HAM). In HAM, agents would employ the use of heterogeneous strategies to form future expectations, by using the computational method Agent-based models (ABMs)[5]. However, capturing agent's decision-making processes (also referred to as the belief-outcome interplay) remains a challenge in both rational and adaptive scenarios. The standard approach is laboratory experiments and the use of human subjects; were the subjects choose expectations through an action, or are encouraged through incentives to choose

[5] For more on Agent-based models in economics see (LeBaron, 2006) and (C. H. Hommes, 2006)

accurately. Schmalensee (1976) experimented with expectation formation in a laboratory setting using timeseries data and adaptive expectations; he found subjects took longer to adjust in periods where actual prices changed. Similarly, Dwyer et al. (1993) compared the results of human subjects to RE, testing if the subjects would use the information they are given to maximise their earnings. They found no evidence of inefficient use of information, still the random deviations from RE still remained unpredictable. Other papers successfully using laboratory experiments to test expectation formation include, Smith, Suchanek, and Williams (1988), and Kelley and D. Friedman (2002).

Hey (1994) created an early experiment were human subjects had to forecast a random variable from timeseries observations. Subjects had no knowledge of the processes used to generate the data and could choose the quantity of information they had to form expectations, with the maximum of 50 previous values. Following observations of previous prices, subjects would then make a prediction of the future value of the random variable. Hey (1994) found that only $4\%$of subjects formed rational expectations, whereas 66% were adaptive learners, but the remaining 30% were neither rational or adaptive learners. Hey's research and findings provided a deeper understanding of expectation formation. Rather than assuming there could be only *one* way to form expectations, Hey's proved that there were multiple ways. From this, three methods to explore expectations were proposed: asset price model, positive v. negative feedback and cobweb model experiments. In this section, just as we studied RE, we will focus on asset price models.

The Hey's experiment was an exogenous data generating process, C. Hommes, Sonnemans, et al. (2005) captured the endogenous data generating process through the use of the asset price model. C. Hommes, Sonnemans, et al. (2005) studied whether subjects could estimate the price of a risky asset consistent with RE prediction. C. Hommes, Sonnemans, et al. (2005) found that a few groups converged to the RE prediction, but subjects also exhibited coordination forecasts within groups, almost like a expectation formation norm. C. Hommes, Sonnemans, et al. (2005) also found some groups expectations showed an oscillatory pattern, which is a attribute of trend extrapolation behaviour. Finally, C. Hommes, Sonnemans, et al. (2005) found that 75% of subjects used linear adaptive rules that diverged from RE predictions; they departed completely from the rational expectations equilibrium prediction.

In a similar study, C. Hommes, Sonnemans, et al. (2008) investigated the existence of "rational bubbles" and found no evidence of them. C. Hommes, Sonnemans, et al. (2008) concluded that agent expectation formation patterns exhibited that of trend-following behaviour.

And so, having established the existence of multiple expectation formation behaviours, Anufriev and C. Hommes (2012) created a heuristics switching model. They used Brock and C. H. Hommes (1997) heterogeneous expectations model, where economic agents moved to strategies with better predictive power, as their foundation. The heuristics switching model used fitted asset pricing from learning-to-forecast experiments (LtFEs), and to counteract the "wilderness of bounded rationality" (Sims, 1980), agents could only choose from a limited number of forecasting heuristics. Anufriev and C. Hommes (2012) showed the importance of heterogeneity in expectation formation, as it described individual forecasting and aggregate price behavioral patterns.

Additionally, C. Hommes (2011) used LtFEs that included humans subjects to test the heterogeneous expectations model (Brock and C. H. Hommes, 1997). They used of 2 individual learning techniques: adaptive leaning and evolutionary selection, paired with 4 forecasting heuristics. The adaptive learning technique updated heuristics over time; in evolutionary selection agents observed the heuristics performance then would switch to the most successful rule. The four heuristics used in this paper were: The *first* was adaptive expectations which predicts the price as a weighted average of the last observed price and last forecasted price. The *second* and *third* rules are trend-following: weak trend (WTR) and strong trend (STR). Trend-following rules are the strategies used to decide the buying and selling assets when their prices go up or down respectively (Menkhoff and Taylor, 2007). The *fourth* rule is a learning anchor and adjustment rule, which uses a sample of average past prices as a proxy for the unknown price.

C. Hommes (2011) found, when compared to experimental price data, the heuristics switching model fit perfectly. Where the experimental data was used to forecast future prices in the heuristics switching model, they found it was able to adapt and track different patterns of price behaviour: slow monotonic convergence, sustained and dampened oscillations. In the monotonic convergence group the 4 rules made the same impact, but then adaptive expectations later dominates. In the sustained oscillations (where prices continually increase and decrease) the learning anchor and adjustment rule dominated. For dampened oscillations (were the increasing and decreases effects slowly fade) the strong trend rule, learning anchor and adjustment rules and adaptive expectations, all dominated at different times. When subjects observed strong trends in prices, the strong trend rules were implemented, but once it's predictive powers failed once the rules was never used again. The learning anchor and adjustment rule made a more accurate prediction of periods with trend reversal, so became the dominant rule. Yet, when the oscillation effect dampened, adaptive expectations became the better predictor but performed the worst other times.

From Hommes's extensive research and experiments, we can conclude heterogeneity plays an integral part in the forming of expectations. The papers explained the path dependence of price behaviours all within a single heterogeneous expectations model; which also explained different outcomes in different settings. This survey of literature on based on bounded rationality, suggests that a coherent and general theory of heterogeneous expectations is essential to the understanding of decision-making behaviour.

Furthermore, Arifovic and Duffy (2018) surveyed a vast collection of literature investigating heterogeneity in expectation formation; they concluded heterogeneity was useful when investigating decision-making. However, HAM has some disadvantages; the first being, all the heterogeneity captured were from individuals. In effect, only endogenous heterogeneity was captured successfully, to build the bigger picture HAM models must capture heterogeneity both endogenously and exogenously (Arifovic and Duffy, 2018). Furthermore, Arifovic and Duffy (2018) found contrasting evidence on how agent-based modelling researchers and macro-economists solve heterogeneity computationally. Macro-economists replicate heterogeneity observed from aggregate data, thus empirical validity comes from emergent outcomes generated from their model, the heterogeneous characteristics observed from individuals. This inconsistency in the definition of heterogeneity and its capturing, needs further clarification. Heterogeneity is occasionally a "persistent phenomenon", seen in different degree of cognitive abilities or types, but then on other occasions, heterogeneity is a transient phenomenon, a process by which agents learn a strong and stable rational expectation equilibrium (Arifovic and Duffy, 2018). Notwithstanding the criticisms above, research into the heterogeneity of expectation formation is essential to the development of economics.

2.3 Agent-based Computational Economics

Tesfatsion (2002) introduced a new branch of economic modelling called Agent-based computational economics (ACE), where economies are modelled as complex adaptive systems. ACE uses the Agent-based modelling framework with application to economics and finance. Agent-based models (ABMs) were developed in the 1970's, the earliest model was a Segregation model created by Schelling (1971) which produced significant findings. Since then, ABMs as a computational method, has developed through an abundance of research and contribution. Because of this, there is no widely accepted definition for ABMs nor are there particularly strict characteristics and features that it must exhibit. Nonetheless, Crooks et al. (2012) was able to suggest some standard characteristics ABMs may have, these were summarised in Table 2.1.

Characteristic	**Description**
Autonomy	Agents are self-governed and they pursue their own interests without any external influence. They freely interact, consume, and exchange information.
Heterogeneity	Agents can have different attributes and preferences which lead to different outcomes and displays of behaviour.
Actions	The differences in agents create different outcomes, all independent of each other. Agents will have features, such as: pro-activity/goal -orientated, reactive/perceptive, bounded rationality, interactive, mobility and adaptive learning.

Table 2.1: Standard Agent-based model (ABM) features

Furthermore, Tesfatsion (2002) argued that traditional economic models applied a "topdown approach" to model construction, which restricted the emergence of dynamic structures and behaviours. Evidence for this was provided by Epstein and Axtell (1996) who advocated for a "bottom-up approach" where social structures were built on *individual agents interactions*. As a result, collective structures could grow from the bottom up. Tesfatsion added, traditional models relied too heavily on four elements: fixed decision rules, common knowledge assumptions, representative agents, and market equilibrium constraint. Similarly, Farmer and Foley (2009) proposed agent-based modelling was the finer approach to economic modelling, as it tries to understand the system as a whole, not just in isolated pieces as in traditional economic models. Furthermore, traditional economic models do not attempt to understand or capture the different interactions between economic entities, whereas interactions are exactly what ACE models attempt to illustrate. In recent times, economists have expanded their tools and can now attempt to quantify complex phenomena. Thus, ACE allows for the study of economies as evolving systems of autonomous agent interactions.

Tefatsion (2002) then briefly outlined the laboratory experiment process, as the following:

1. *Build an economy*: create a population of agents, agents can represent any economic entity.

2. *Initialise the economy*: specify agents attributes, these attributes can be characteristics, behavioural norms, information stores and anything applicable to your research question.

3. *Evolution*: allow the model to run over time, the economy will evolve over-time without any intervention.

Compared to traditional economic models, ACE models are not constructed to meet an aim, they are built free of restrictions to allow for the observation of emergent behaviour. In ACE models, the *arrangement of the parts* outweighs the importance of individual parts (Christen and Franklin, 2004). In summary, ACE models accommodates for Heterogeneity, through key characteristics, such as, bounded rationality and agent interactions. These features and characteristics allow for the exploration of the behavioural elements, often neglected by DSGE models. However, Bonabeau (2002) outlined three issues with ABMs: i) models are built for a purpose, therefore cannot be generalised, ii) interpretation can be tricky, especially when quantifying complexities and, iii) ABMs are computationally demanding, this is a problem when modelling large scale systems.

Chapter 3

Methods

3.1 ACE model framework

Agent-based models study the processes of dynamic systems, when the method is adapted to study economic processes, these are considered as agent-based computational economic models (ACE) (Tesfatsion, 2002). Agents are modelled as objects that uses rules to interact. These rules can be based on incentives and information. Agent-based models have three important features *agents*, a *environment* and *behavioural rules and relationships* (Kennedy, 2012), this model only utilises two features, see Table 3.1 for model framework (See Appendix A for code).

	Characteristic	Description
Agents	Money	Agents disposable income, at $t = 0$ they start with nothing
	Assets	A good or service that agents have a choice of acquiring
	Expectation Formation	To predict the price of the asset in $t + 1$, agents implore one of two statistical methods: a simple linear regression or an autoregressive model
Rules	Earn money	Each timestep, agents have a likelihood of earning money
	Predict asset price	Agents predict future asset prices using one of the statistical methods outlined above
	Make a decision	Agents can choose to sell, keep or buy assets based on their future prediction and money. To sell: agents must own *atleast* 1 asset and the actual asset price must be *greater* than the agents' prediction. To buy: the actual asset price must be *greater* than agents predictions, and they must have enough money to buy the asset. To keep: actual asset price must *equal* agents predictions, and own *atleast* 1 asset.

Table 3.1: Model Framework

In order to fulfill the aim of this dissertation, the following hypotheses have been established:

Hypothesis 1: How available information is utilised, impacts the accuracy of forecasting rules

Hypothesis 2: The observability will improve agent's decision-making

To satisfy **Hypothesis 1** agents utilise one of the two statistical methods to predict the asset price in the next period, which is denoted here as P^e_{t+1}. The first forecasting rule is a SLR, which uses a linear combination of observations:

$$P^e_{t+1} = \beta_0 + \beta_1 P + \epsilon$$

The SLR is optimised on actual market prices then predicts the next period, in this model we include an error term, denoted as ϵ. Thus, the expectation of the next period is dependent on market prices.

$$E[P_{t+1}|P]$$

The second forecasting rule is ARIMA, which uses a combination of previous observations to predict the next period.

$$P^e_{t+1} = \beta_0 + \beta_1 Pt + \beta_1 Pt - 1 + \epsilon_{1t}$$

Expectations of the next time period consider all the available information as variables when forecasting. Therefore, the expectation of the next period is dependent on the information available at the end of a period, in this example $t - 1$.

$$E[P_{t+1}|I_{t,t-1}]$$

And finally, the actual market price is taken from normal distribution and updates every period.

$$P = X \sim N(47, 3)$$

For **Hypothesis 2** we add an additional agent characteristic and behavioural rule to the model framework, see Table 3.2.

	Characteristic	Description
Agents	Confidence	Agents are given a level of confidence between $(0, 1)$, with 0 as no confidence and 1 as the highest level confidence. Confidence can increase and decrease according to their predictive accuracy and money.
Rules	Observe	If agents confidence falls below 0.5, they become easily influenced by the decisions of other agents. Agents are forced to sell when: the majority of the population must have a history of selling more buying or keeping. When forced to buy: the majority of the population must be buying more than keeping and selling. When forced to keep: the majority of the population must be keeping rather than buying and selling.

Table 3.2: Hypothesis 2, additional characteristics

Chapter 4

Results

4.1 Asset Price

The first 26 asset prices were generated randomly and were used as the historical data that agents used to make predictions.

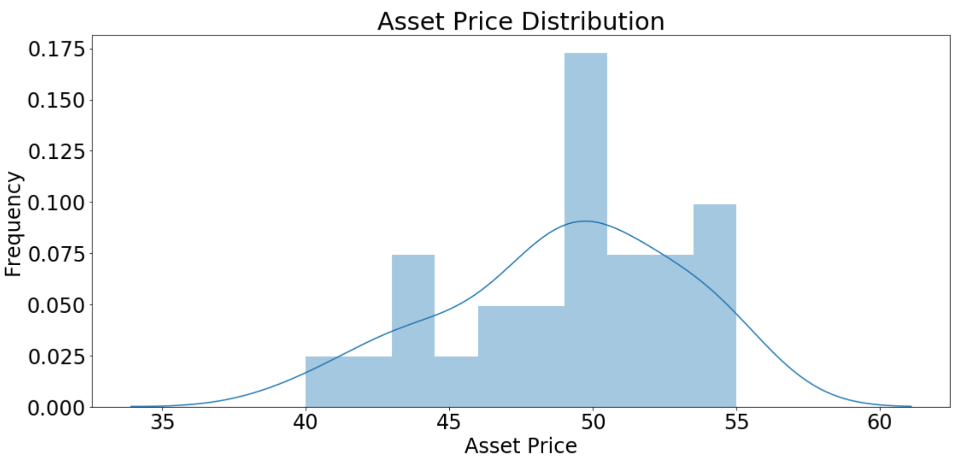

Figure 4.1: Historical asset price distribution

Figure 4.1 illustrates the distribution of the historical asset prices. Where the price peaks, here at 50, this indicates the common price throughout the 26 previous periods. Though faint, the distribution is right-skewed, indicating that it is not normally distributed. From the numerical summary (see Table 4.1), we can observe that the data has the mean 50.230, with a standard deviation of 3.952. Figure 4.2 is the boxplot of the the minimum, maximum and quartiles values given in Table 4.1 also. Figure 4.2 further supports our observation of a right-skewed distribution.

	Summary
Count	26
Mean	50.230
Std	3.952
Min	40.0
25%	49.0
50%	50.5
75%	53.0
Max	55.0

Table 4.1: Summary of historical asset prices

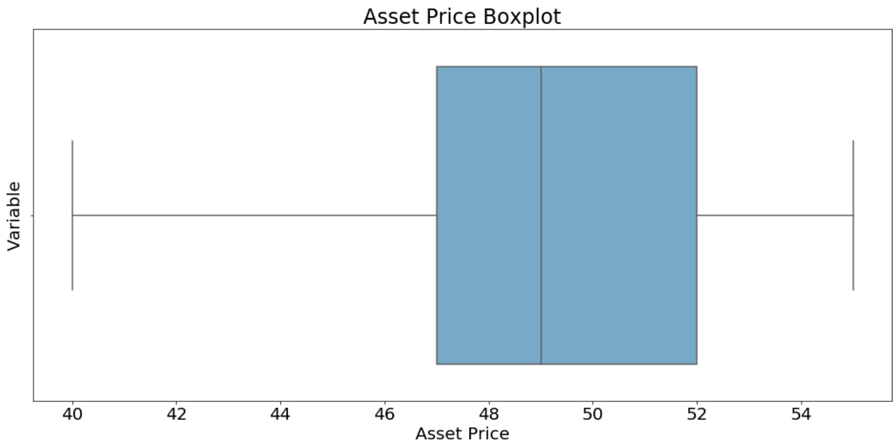

Figure 4.2: Boxplot of Historical asset prices

Figure 4.3: Historical asset prices

Figure 4.3 is the timeseries, we can observe the behaviour of the asset price through the previous 26 periods. Figure 4.3 indicates that the data is not stationary with both: the data spread varying and a non-constant covariance over time. To test for stationarity, we used the Augmented Dickey Fuller test, see Table 4.2 for the numerical results. The p-value, 0.0000030, is less than the significance level 0.05%, and the ADF statistic is less than the critical value at 5%, therefore, we can conclude that the timeseries is non-stationary.

	Summary
ADF Statistic:	-5.424
p-value:	3.010e-06
Critical Values:	
1%:	-3.723
5%:	-2.986
10%:	-2.632

Table 4.2: ADF output

4.2 Hypothesis 1

Hypothesis 1: How available information is utilised, impacts the accuracy of forecasting rules

4.2.1 Simple Linear Regression

The simple linear regression (SLR) was used to estimate the relationship time, denoted here as $Xtime$, had with the dependent variable as the $Y_{assetprice}$, the asset market prices. Equation 4.1 is the SLR formula with the statistical estimates, presented in Table 4.3.

$$\hat{Y_{assetprice}} = 49.523 - 0.047X_{time} + \epsilon \tag{4.1}$$

	coef	**std err**	**t**	**P>t**	**[0.025**	**0.975]**
const	49.523	1.538	32.675	0.000	47.074	53.422
Time	-0.047	0.105	-0.013	0.990	-0.219	0.216

Table 4.3: SLR output (a)

We can interpret equation 4.1 as, with every one unit increase in time, there is a 0.047 unit decrease in asset prices. The standard error is 0.105, which suggests the standard distance between the regression line and the data points is 0.105%. The test statistic is very small, -0.013, this suggests the results were more likely to have occurred by chance. The p-value 0.990 is larger than the significance level (0.05), therefore, we fail to reject the null hypothesis that conclude that time does not have a statistically significant effect on asset price.

Table 4.4 is the evaluation of residual characteristics. To test if errors are normally distributed, the result of the omnibus statistic is 5.121. The statistic is far from zero, which does not indicate normalcy. The Prob (Omnibus) statistic supports this finding with the value 0.077, if there was normalcy the statistic would be much closer to 1. Skew, suggests that the data is not symmetric as it is far from zero and negative. The Durbin-watson statistic tests autocorrelation, with a value between 2 and 4, this indicates the presence of negative autocorrelation. Finally, the condition number tests for multicollinearity, with the number lower than 30 this suggests that there are higher fluctuations to small changes in the data.

Omnibus:	5.121	**Durbin-Watson:**	2.204
Prob(Omnibus):	0.077	**Jarque-Bera (JB):**	3.577
Skew:	-0.886	**Prob(JB):**	0.167
Kurtosis:	3.399	**Cond. No.**	28.4

Table 4.4: SLR output (b)

These results suggest that SLR fails to capture the true dynamic of the dataset, this is because the data did not meet the OLS conditions. The is evident in the regression plot (see Figure 4.4). The R-squared value is 0.0106, this suggests that only 1.06% of the data fits the regression model. This low r-squared value indicates a poor fit for the model.

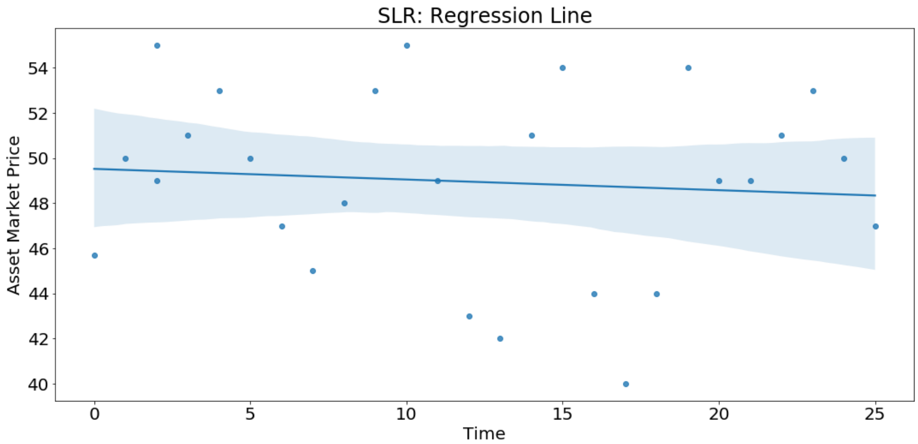

Figure 4.4: Regression Line

R-squared	0.0106	**MAE:**	3.2446
MSE:	15.8288	**RMSE:**	3.9785

Table 4.5: SLR output (c)

Given all of the summaries and outputs produced by our statistical analysis, we can assume that the agents allocated the SLR as its predictor, will make poor forecasts. We will evaluate this assumption in the later sections.

4.2.2 Autoregressive Models

The alternative statistical tool agents could utilise to predict the asset market price was the ARIMA model. As outlined in the methods section ARIMA models allow for predictions to be made on historical observations. ARIMA models produce more accurate forecasts in the case where we predict the value of a Y when dependent on time.

To apply a statistical method to any timeseries data, we test for stationarity. This was done Section 4.1, using the ADF test; we found our data to be non-stationary. The ARIMA model includes a differencing that transforms the data from non-stationary to stationary. To parametrize the ARIMA model, we need three integers, as outlined in Methods section of this dissertation (Section 3). These integers are:

$$(p, d, q)$$

- p is the AR order, which is the number of autoregressive terms
- d is known as the differencing order
- q is the MA order, the number of moving-averages

To determine the AR order, i.e. the number of historical asset prices to include, we use the PACF. PACF calculates the correlation between asset prices but at two points in time.

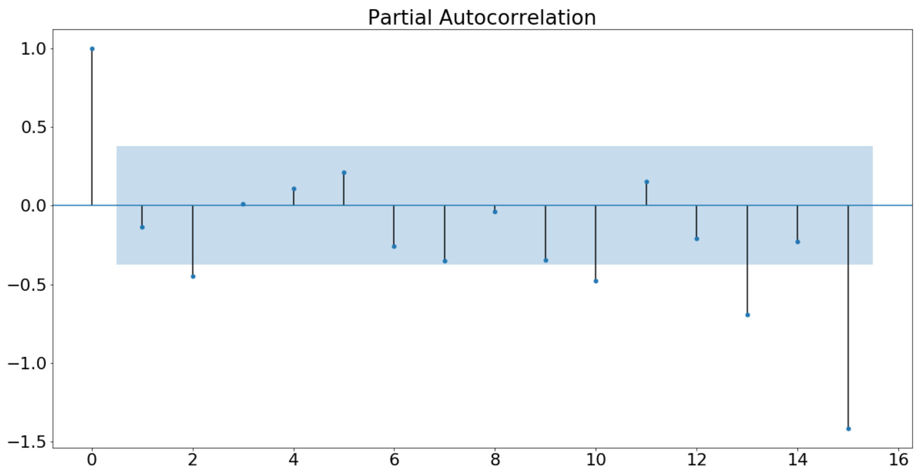

Figure 4.5: Partial Auto Correlation Function (PACF)

Figure 4.5 is the PACF, with the x-axis illustrating the lags and the y-axis as the PACF values. The AR order is determined by the number of vertical lines that surpass the light shaded rectangle; from observation we can see this happens 4 times. However, the PACF spikes significantly at lag 13 and 15, so we will use AR(2).

For the MA order, we will use the ACF. The MA order is the number of error terms to include the model, as it assumes asset prices at the current period is dependent on the error terms of the previous period.

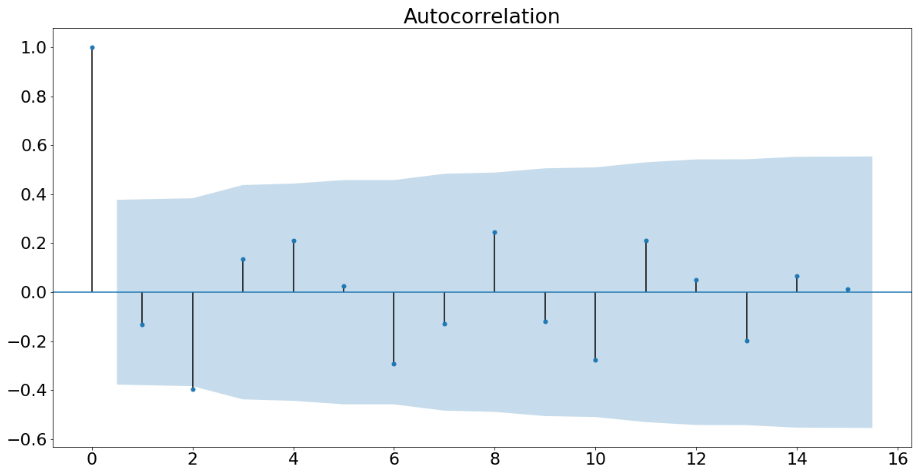

Figure 4.6: Auto Correlation Function (ACF)

Figure 4.6 is the ACF plot, with the x-axis illustrating the lags and the y-axis as the ACF values. The plot suggests there is only 1 significant spike to consider, this is at lag 2. Therefore, we will use MA(2).

With the integers $(2, 1, 2)$, we then compared the model to the original time series. Figure 4.7 compares both the historical asset prices with the ARIMA predictions, and although not perfect, we can infer that the ARIMA model produced more accurate predictions than the SLR.

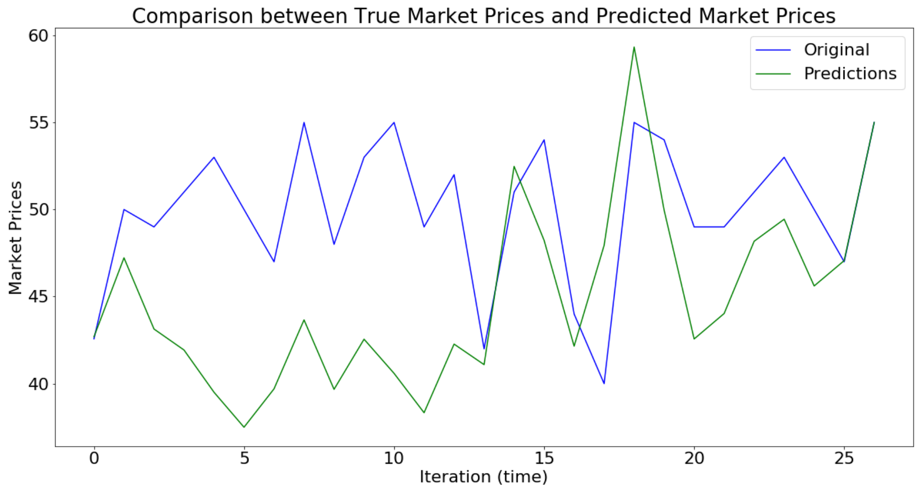

Figure 4.7: ARIMA model predictions and historical asset price comparison

4.2.3 ARIMA and SLR Forecasts Comparison

With 2 agents, we compared the ARIMA and SLR predictions for 25 periods. In Figure 4.8, we find that Agent(AR) is predicting more accurately than Agent(LR), as there is more overlap between predictions. Agent(LR) seems to overestimate the market price entirely, at certain points we can notice predicted prices moving in the opposite direction of the true market price.

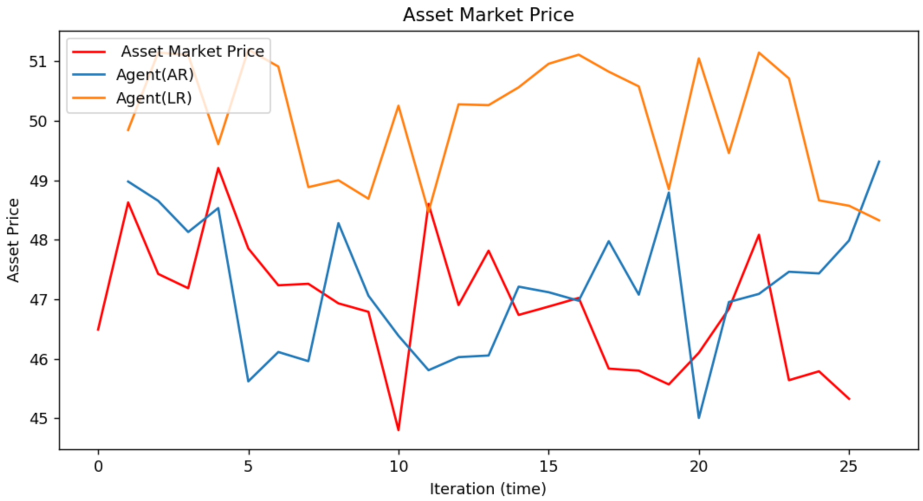

Figure 4.8: SLR and ARIMA predictions, compared to true asset market price

To commute some accuracy rate, we calculated the difference between the predictions and the asset market price. We used this difference to calculate percentage changes at each point then totalled each rate.

Total % Change of Agent(LR) prediction	**Total % Change of Agent(AR) prediction**
-6.0959	-0.4541

Table 4.6: Total percentage changes (a)

Table 4.6 shows the percentage changes for both predictive tools. Though we both predictions indicate a decreasing percentage change, Agent(LR) percentage change is less than that of Agent(AR). This smaller number suggests there were larger differences between the actual market price and that predicted, therefore SLR proves to be an insufficient tool.

To test the validity of this finding, we compared it to a scenario with 10 agents, predicting 50 periods.

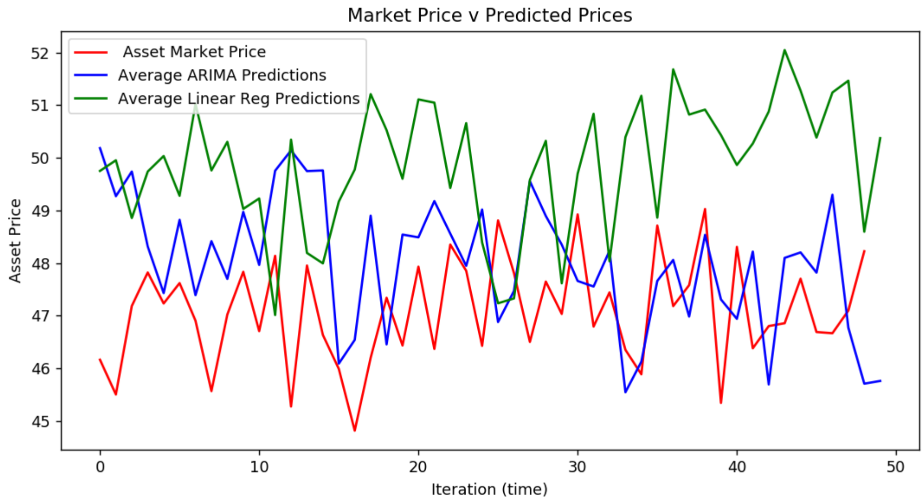

Figure 4.9: SLR and ARIMA, with 10 agents predicting 50 periods

Figure 4.9 compares the average predictions of both SLR and ARIMA over 50 periods. Compared to Figure 4.8, we still observe an overestimation. Table 4.7 are the respective percentage change calculations, when compared to together there is a smaller difference than in Table 4.6. However, average SLR predictions still perform worse than the ARIMA predictions, further supporting our above findings.

Total % Change of LR prediction	**Total % Change of AR prediction**
-5.9224	-1.9634

Table 4.7: Total percentage changes (b)

Hypothesis 1 asks whether the differences in *how* available information is utilised affects the accuracy of predictive tools. SLR is an optimization problem, it analyses the relationship between the asset market price with time. However, being that timeseries data does not satisfy the OLS conditions, the true effect that time has on asset market prices cannot be directly observed. On the contrary, the ARIMA model does a seemly better job at capturing market price time dynamics. In the real world, we have individuals who may have access to information to make a "prediction" on their future, however, they may not have the analytical capability to utilise the information correctly. This finding is supported if we consider SLR as a analytical tool individuals may use to form future expectations, the same dynamic is evident in the behaviour we observe in Figure 4.8.

4.3 Hypothesis 2

Hypothesis 2: The observability will improve agent's decision-making

We found that agents using the ARIMA model predicted more accurately than agents using the SLR. Although SLR is not the standard tool or methodology used to explore timeseries, we used this tool to demonstrate how the differences in analytical skill could impact how individuals form expectations. To explore **hypothesis 2** we will first examine agents' decisions when they are unaware of the decisions of others, then compare the results to the scenario with observability.

Agents make the decision on whether to *buy*, *sell* or *keep* assets using their predictions; as the ARIMA model produced the more accurate results, we will use their decisions as the benchmark for comparison. Agents that used SLR to predict will be given the ability to observe. We used the same 10 agents to investigate this hypothesis, as we did in the previous section.

	Buy	*Sell*	*Keep*	*No decision*	*Total*
Total	44	25	156	25	250

Table 4.8: Aggregate results of agent decisions using ARIMA predictions

Table 4.8 are the results of $5/10$ agents that used ARIMA predictions to make their decisions. We found by the 50^{th} time-step agents had bought 44 and sold 25 assets respectively, but had chosen to keep 156 times. This suggests that on average, each agents bought approximately 8 assets, that were kept around 19 times. We also found that roughly half of the assets bought by agents were sold; we can surmise that of the agents who owned an asset, they were 6 times more likely to keep than sell their asset. We can therefore identify a preference for keeping, rather than selling. Lastly, agents chose to be inactive and made no decisions 25 times.

	Buy	*Sell*	*Keep*	*No decision*	*Total*
Total	38	20	172	20	250

Table 4.9: Aggregate results of agent decisions using SLR predictions

Table 4.9 are the results of $5/10$ agents that used the SLR as their predictor. In comparison to ARIMA decisions, less agents bought and sold assets but more kept. On average, agents bought approximately 7 assets that were kept 22 times; also agents chose to sell less times than they bought. Of the agents that bought assets, they were 8 times more likely to keep than sell. These results suggest that these agents have a stronger preference to keep than the ARIMA agents. Interestingly, agents were more active than ARIMA by making just 20 no decisions. This further supports our theory that agents adopting the SLR predictor had a stronger preference to keep assets than agents using ARIMA. In summary, we found that agents using SLR predictions, were 13% less likely to buy, 20% less likely to sell, and 10% more likely to keep assets than ARIMA agents. Further, they were 20% less likely to choose inactivity.

To test whether observability affected the quality of the decisions agents using SLR made, we introduced a confidence function and a new decision-making rules (see chapter 3). We increased the number of agents in the model to 20, with 10 agents using the ARIMA and SLR each; of the SLR agents 5 had less confidence than the others and therefore used thew new decision-making rules. The results of the agents using ARIMA as their predictor are in table 4.10.

	Buy	*Sell*	*Keep*	*No decision*	*Total*
Total	99	59	286	56	500

Table 4.10: Aggregate results of agent decisions using ARIMA predictions

With 10 agents, the results still follow the trend we established above. On average, agents possessed roughly 9 assets, which were kept approximately 28 times. We can also suggest, agents who owned atleast 1 asset 4 times likely to keep than sell. The results above fall just below/above the results obtained with 5 agents in Table 4.8.

	Buy	*Sell*	*Keep*	*No decision*	*Total*
Low	35	18	105	92	250
High	48	31	144	27	250
Total	83	49	249	119	500

Table 4.11: Aggregate results of agent decisions using SLR prediction

With agents using the SLR as predictors, we summarised the results above. Agents with *low* confidence, were less active than those with *high* confidence, indicated by the 'no decision' values 92 and 27 respectively. Furthermore, on average each agent owned 7 assets which were kept 15 times. As found above, agents with assets, were approximately 4.6 times more likely to keep than sell. In comparison to agents with *high* confidence, they were 37 times more likely to buy assets, 1.72 times more likely to sell and 1.37 times more likely to keep.

In summary, we found that agents using SLR predictions, were 16% less likely to buy or sell, and 12% less likely to keep assets. This was because, they were 112% more likely to choose inactivity. With just 5 agents, we assumed that ones using SLR had a stronger preference for keeping, although this was not evident from the agents with *low* confidence, the results of agents with *high* confidence who kept, is approximately half of the keep decisions by ARIMA. We can therefore suggest, that a lack of confidence impacts agents desire/ability to make a decision. Therefore, when given the opportunity they would choose inactivity.

Chapter 5

Conclusion

The aim of this dissertation was to recreate a heterogeneous agent model using a simple asset pricing model. We introduced *heterogeneity into the rational expectations* hypothesis, by allowing our agents to use different predictive tools (SLR and ARIMA) to predict future asset prices. We found after 25 time periods, agents using ARIMA made more accurate price predictions than agents using SLR. After 50 periods, though ARIMA continued to perform better, its accuracy decreased significantly, whilst SLR increased. These result suggest that the SLR predictive power could improve over time, although, ARIMA may not always performs better.

Then, we investigated the impact *observability* had on agent's decision-making. We found that naturally, with both SLR and ARIMA: agents who bought assets, had a preference to keep them rather than to sell. Interestingly, we found agents using the SLR had a *stronger preference* to keep, than agents using ARIMA. To test observability, we used the decisions made with ARIMA as a benchmark comparison, then introduced confidence and observability to the agents using SLR. We found though there was still a preference to keep, agents with *low confidence* were more likely to choose inactivity than to follow the behaviour of the wider population. Also, agents with *low confidence* were less likely to buy and sell, compared to agents with *high confidence* or agents using ARIMA.

This model was able to successfully grow an economy from the "bottom-up" and arrive at the intuitive outcome: when faced with uncertainty in asset markets, there is a subgroup of the population that may exhibit *risk-averse* behavioral patterns and would choose the most comfortable (less risky) option, in our case that was to keep assets they owned, rather than to sell or buy more. In the same way, there is a subgroup of the population, less effected by this uncertainty, who are able to sell assets or remain completely inactive. In addition, we found that observability only deterred agents from making a decision.

As insightful as this model is, it has some significant limitations. Firstly, this model lacks completeness through the exclusion of endogenous data generating process. A good example being, instead of randomly generating our asset market price, we would replace it with the aggregate of all the agents predicted asset market prices. Then, every so often allow for exogenous shocks that could change the asset market price. In addition, choosing to randomly generate the asset market price is a poor choice, as: *i)* does not provide an accurate depiction of real asset price dynamics and, *ii)* it excludes the behavioural patterns that asset pricing exhibits (e.g. trends and seasonality). Hence, it is nearly (or is) impossible to make accurate predictions, thus we must question the accuracy of our results. Standard ABMs and ACE models, give agents human attributes so they can be somewhat representative of population; in our model we, fail to give agents any attributes outside of confidence, that could directly impact their decision-making process. And so, our results are limited in terms of generalisations and extrapolations to real life populations. Finally, the use of a synthetic population in our model, compared to the standard use of survey data/laboratory experiments with human subjects in HAM, are indicative of the lack of completeness in our model.

In light of the above analysis, we were able to recreate a very simplistic asset pricing model that allowed for the observation of behaviours that naturally emerged. However, further research into the true impact of observability on expectation formation is essential.

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Appendices

Appendix A

Model code

Full code is available on my GitHub: https://github.com/deborah-O/Dissertation-2020