Introduction

This is a brief survey on some generalized class of sets and continuous functions preferred by some remarkable research works done by some topologists. We have studied some papers in this context such as [4], [8], [9], [14]

In 1963, Norman Levine first introduced a general class of open sets and continuous functions namely semi-open sets and semi-continuous functions. In1965, Thomas was motivated by Levin's work. The notion of ideals in general topological space was treated in the classical book by Kuratowski and R.Vaidyarathaswamy .But unfortunately ideals are not considered in any modern text .But the theory of ideals has application in other branches of mathematics.

To progress our motivation firstly we have made two precise separation of initial theory by introducing two chapters ,one deals with semi-open set and other with semi-continuity motivated by Levin and Thomas .Our advanced part starts from the 3rd chapter preferred by [4].It defines semi-open sets through the idea of Ideal in topology .Our last chapter has extended all the ideas to semi-I_s -open set and semi-I_s-continuous function ,for this we mainly consult-[12].Just like all of the previous paper it characterized those special sets but the reader must take special interest to study this chapter to motivate themselves enough for further works .Throughout our paper we have used notation in standard way and anyone, who is familiar with elementary topology course ,can enjoy the paper.

Some notations:

(X,τ) = Topology on a given set X.

$P(X)$ = Power set of X.

φ = Empty set.

Int (A) = Interior of the set A.

$cA/cl(A)$ = Closure of A.

$C(A)/A^c$ = Complement of A.

(X,d) = Metric space defined on X with the metric d.

T_0-space = Kolmogorov space

T_1-space = Fréchet space

T_2-space = Hausdorff space

T_3-space = regular Hausdorff space

T_4-space = normal Hausdorff space

T_5-space = completely normal Hausdorff space

CHAPTER 1: SEMI OPEN SETS

In various topological class we know about open sets under different topology. But in this content we go forward with semi open sets instead of open sets. In our full content we use the usual definition of well known terms and so we skip that definitions. We will only give the required unusual definitions.

Definition 1.1[9]**:** A set A in a topological space (X, τ) will be called semi open (written as s.o) iff there exists an open set O such that $O \subseteq A \subseteq cO$.

Remark 1.1 : A set A in a topological space (X, τ) is s.o iff $A \subset cIntA$.

First we see two following examples:

Example 1.1 : Let , X be the real's with the usual topology and let E be the set consisting of the open interval $(0,1)$. Then if cE is the set consisting of the closed interval $[0,1]$. Then if A is either one of the half intervals $(0,1]$ or $[0,1)$ or if A is E or cE, then A is semi open .

Example 1.2 : Let , X be the space of the real's with the usual topology and let, $A = \left(\frac{1}{2}, 1\right) \cup \left(\frac{1}{4}, \frac{1}{2}\right) \cup \ldots \cup \left(\frac{1}{2^{m+1}}, \frac{1}{2^m}\right) \cup \ldots$ and $B = \{0\} \cup \left(\frac{1}{2}, 1\right) \cup \left(\frac{1}{4}, \frac{1}{2}\right) \cup \ldots \cup \left(\frac{1}{2^{m+1}}, \frac{1}{2^m}\right) \cup \ldots$ Here $cA = [0,1]$ and $A \subseteq B \subseteq cA$ shows that B is s.o in X . But B is neither open nor closed .

Remark 1.2: Any open set is s.o but not conversely.

Remark 1.3: A closed set may be an s.o set. But singleton sets is not s.o.

Result 1.1[14]**:** Let , $\{A_\alpha\}_{\alpha \in \Lambda}$ be a collection of s.o sets in a topological vector space X . Then $\bigcup_{\alpha \in \Lambda} A_\alpha$ is s.o in X .

Remark 1.4 : Intersection of two semi open sets may not be s.o for instance take $A = [1,2], B = [2,3]$. A and B are s.o , but $A \cup B = \{2\}$ is not s.o .

Definition 1.2: Complement of a semi open set is called semi closed set. And the semi closure of A in (X,τ) is denoted by the intersection of all semi closed sets containing A and is denoted by $l(A)$.

It is to be noted that intersection of any two semi closed sets is also semi closed.

Definition 1.3 : $\mathrm{S.O(X)}$ will denoted the class of all semi open sets in X.

Result 1.2 : Let , $A \subseteq B \subseteq cA$, where A is open set . If $B-A$ is non-empty , the points of $B-A$ are limit points of A. Moreover all of the limit points of $B-A$ are contained in c(A) .

Theorem 1.1[9]**:** Let, $\mathcal{B}=\{B_\alpha\}$ be a collection of sets in X such that (i) $\tau \subset \mathcal{B}$ (ii) $B \in \mathcal{B}$ and $\subset D \subset cB$, then $D \in \mathcal{B}$. Then $S.O(X) \subset \mathcal{B}$. Thus $S.O(X)$ is the smallest class of sets in X satisfying (i) and (ii) .

Proof : Let $A \in S.O(X)$ .Then $O \subset A \subset cO$ for some $O \in \tau$.Then $O \in \mathcal{B}$ by (i) and thus $A \in \mathcal{B}$ by (ii) .

Theorem 1.2[9]**:** Let $A \subset Y \subset X$ where X is a topological space and Y is a subspace . Let $A \in S.O(X)$. Then $A \in S.O(Y)$.

Proof :$O \subset A \subset c_X O$,where O is open in X . Now$O \subset Y$ and thus $O = O \cap Y \subset A \cap Y \subset \ Y \cap c_X O$ or $O \subset A \subset c_Y O$. Since $O = O \cap Y$, O is open in Y and the theorem is proved .

Result 1.3[9]**:** Let , O be open in X . Then $cO - O$ is nowhere dense in X .

Theorem 1.3 : Let $A \in S.O(X)$ where X is a topological space . Then $A = O \cup B$ where (i) $O \in \tau$ (ii) $O \cap B = \phi$ and (iii) B is nowhere dense .

Proof :$O \subset A \subset cO$ for some O open in X . But $A = O \cup (A - O)$. Let $B = A - O$. Then $B \subset cO - O$ and thus is nowhere dense by Result 1.3 . Then $A = O \cup B$ and (i) and (ii) immediately follows .

Remark 1.5: Converse of Theorem 1.2 is not true in general. For instance take $Y = \{0\}$ and $A = \{0\}$. So A is open in Y and hence $A \in S.O(Y)$. But $A \notin S.O(\mathbb{R})$.

Remark 1.6 : Converse of Theorem 1.3 is not true in general even when connectedness is imposed upon A . For example let X be the plane and $= \{(x,y)|0 < x < 1, 0 < y < 1\} \cup \{(x,0)|1 \le x \le 2\}$. Clearly , $A \notin S.O(X)$ although (i),(ii) and (iii) of Theorem 1.3 are satisfied .

Theorem 1.4[9]**:** Let X be a topological space and $A = O \cup B$ where (i)O($\neq \phi$) is open . (ii) A is connected (iii) $B' = \phi$. Then $A \in S.O(X)$.

Remark 1.7: It is not true that components of semi open sets are semi open. In example 1.2 B is semi open and $\{0\}$ is a component of B, but $\{0\}$ is not semi open in X .

Theorem 1.5[14]**:** Let $f: X \to X^*$ be continuous and interior where X and X^* are topological spaces. Let $A \in S.O(X)$. Then $f(A) \in S.O(X^*)$.

Proof : Let , $A = O \cup B$, where O is open and $B \subset cO - O$. Then , $f(0) \subset f(A) = f(0) \cup f(B) \subset f(0) \cup cf(0) = cf(0)$. Hence , $f(0) \subset f(A) \subset cf(0)$ and $f(0)$ is open in X^* , as $f: X \to X^*$ is interior .

Remark 1.8: If 'interior' is removed from Theorem 1.5 , then the theorem is not true in general. For instance, take $f: X \to X^*$ is continuous map and $f(x) \equiv 1 \forall x \in X$. Then X is semi open in X but $f(X)$ is not semi open in X^* .

Definition 1.4 : Let X be a topological space and $\mathcal{B} = \{B_\alpha\}$ a collection of subsets . Then $Int\ \mathcal{B}$ will denote $\{Int\ B_\alpha\}$.

Result 1.4 : Let τ be the class of open sets in the topological space X . Then $\tau \equiv Int\ S.O(X)$.

Theorem 1.6 : Let τ and τ^* be two topologies for X . Suppose $S.O(X,\tau) \subset S.O(X,\tau^*)$. Then $\tau \subset \tau^*$.

Corollary 1.1 : Let τ and τ^* be two topologies for X . Suppose $S.O(X,\tau) = S.O(X,\tau^*)$. Then $\tau = \tau^*$.

Remark 1.9[14]**:** Converse of Theorem 1.6 may not true in general. For example , we take $X = \mathbb{R}$ and τ be topology generated by sets of the form (x,y) where $x < y$. Let τ^* be the topology generated by the sets of the form $[x,y)$ where $x < y$.

Clearly $\tau \subset \tau^*$, but $S.O(X,\tau) \not\subset S.O(X,\tau^*)$ as $(x,y] \in S.O(X,\tau)$ but $(c,y] \notin S.O(X,\tau^*)$.

<u>Theorem 1.7</u>[9]**:** Let X_1 and X_2 be topological spaces and $X = X_1 \times X_2$ be topological product . Let , $A_1 \in S.O(X_1)$ and $A_2 \in S.O(X_2)$. Then , $A_1 \times A_2 \in S.O(X_1 \times X_2)$.

Proof : We have $A_i = O_i \cup B_i$, for $i = 1,2$ where O_i is open in X_i and $B_i \subset c_i O_i - O_i$, by Theorem 1.3 . Then $A_1 \times A_2 = (O_1 \times O_2) \cup (B_1 \times O_2) \cup (O_1 \times B_2) \cup (B_1 \times B_2)$. But , $O_1 \times O_2$ is open in $X_1 \times X_2$and $(B_1 \times O_2) \cup (O_1 \times B_2) \cup (B_1 \times B_2) \subset c_{X_1} O_1 \times c_{X_2} O_1 = c_{X_1 \times X_2} O_1 \times O_2$. It follows that , $A_1 \times A_2 \in S.O(X_1 \times X_2)$.

<u>Remark 1.10</u> : If $X = X_1 \times X_2$, X_i being topological spaces and $A \in S.O(X)$, then it is not true in general that A is a union of sets of the form $A_1 \times A_2$ where $A_i \in S.O(X_i), i = 1,2$. For instance, take
$A = \{(x,y) | 0 < x < 1, 0 < y < 1\} \cup (1,1)$.

<u>Definition 1.5</u> : A topological space X is said to be semi-connected if X cannot be expressed as the union of two disjoin non-empty semi open sets in X .

<u>Theorem 1.8</u>[14]**:** Semi-connectedness is a topological property.
Proof : Let , $f: X \to X^*$ be a homeomorphism , where X and X^* are topological spaces . Let $C \subseteq E \subseteq c(E) \subseteq X$, where C is an open separated set and E is s.o and connected . So , C is semi-connected . Then $f(C) = C^*$ implies C^* is an open separated set . $f(E) = E^*$ implies that E^* is semi open , since semi-openness is topological property by Theorem 1.5 and since connectedness is a topological property E^* must be connected . So, we get , C^* is semi-connected , since f is a homeomorphism and $C^* \subseteq E^* \subseteq c(E^*) \subseteq X^*$.

<u>Semi-open sets on some topological space</u> [14]

(I) Let (Y, τ) be a topological space and τ be an co-finite topology. Since all finite sets are closed sets, the only open set they contain is empty set.So, there is no finite semi-open sets in this topology.

<u>Result 1.5</u> : In the topological space Y every semi open set is an open set .

<u>Result 1.6</u>: In the topological space Y every point of a finite set A is a limit point of its compliment .

<u>Theorem 1.9</u> : In a topological space Y , if an open set is compact , each of its semi-open subsets is compact .

Proof : Let CA be a compact open subset of Y and E a s.o set of Y defined by CA . Now give any open covering $G$of Ethere exist a finite no . of sets of G that cover CA since $CA \subseteq E$. $E - CA$ is a finite subset of E and thus a finite no. of elements of G which cover $E - CA$. Hence E is covered by the union of the finite coverings of CA and the union of these two finite coverings . Therefore , E is compact .

(II) Let X be a topological space with discrete topology . Then, every set is an open set and since every open set is s.o then all sets are s.o.

(III) Let X be the topological space with the indiscrete topology . Here ϕ and X are only semi-open sets i.e $\phi \subseteq \phi \subseteq c\phi$ and $X \subseteq X \subseteq cX$.

(IV) Let X be a T_0 space and $x, y \in X$ with $x \neq y$. Let G be an open set with $\in G$. If y is not contained in any open set other than the entire space or if y is contained in a nowhere dense set disjoint from , then $G \cup y$ may be semi open provided $y \in cG$. However , unless more is known about a space , the property of being T_0 is not sufficient to investigate semi-open sets .

(V) Let X be a T_1 space and $x, y \in X$ with $x \neq y$. First we state a theorem

<u>Theorem 1.10</u> : In a T_1 space X , a point x is a limit point of a set E iff every open set containing x contains an infinite number of distinct points of E .

By Theorem 1.10 , it follows that no finite set E is a T_1 space can have a limit point , as no open set containing such a limit point can possibly contain a infinite number of points of E . Therefore, every finite set is a closed set. Thus it would seem that no finite set can define a semi-open set in a T_1 space unless the set is both open and closed in the topological space .

(VI) T_2 space , T_3 space , T_4 space and T_5 spaces are T_1 spaces and follows from the rule of T_1 spaces . In regular and normal spaces, semi-open sets may be contained in closed sets.

A semi open set contains an open set. Therefore , any semi open set B is a neighbor hood of a point x if$x \in A \subseteq B \subseteq cA$ where A is an open set in the topological space . The semi open set B satisfies all the axioms foe neighborhood, but since the intersections of semi open sets need not to be semi open , not all neighbor hoods are semi open . Thus, there appears to be little value in applying the neighborhood concept to semi open sets.

Semi open sets may be a base for a topology, but as the intersection of any two members of a base must be a union of members of the base, it is necessary to form the base of s.o sets whose intersections are s.o.

Finally, a topology in terms of semi open sets is considered. Let X be a non empty set of points and S be the family of s.o sets of X such that the intersection of any finite no of elements of S in s.o. Then (X, S) is a topological space .

<u>Example 1.3</u> : Let X be an Euclidean plane . Let S be the family of semi open sets which are unions of sets $\{(x,y)|a \leq x < b, c \leq y < d\}$. Then this family satisfies the axioms of topological spaces. If the interval $a \leq x < b$ were closed on the right instead of the left , or if the interval $c \leq y < d$ were cosed above instead of below , the members of S will satisfy the axioms . However , if some of the intervals $a \leq x < b$ were closed on the right and some on the left and/or if some of the intervals $c \leq y < d$ were closed above and some below , then the intersections would not necessarily be semi open .

So, a topology formed from s.o sets is possible but since it would have to exclude that s.o sets whose intersections are not s.o, it may be impractical.

CHAPTER 2: SEMI CONTINUITY

We have already study the structure of R via open sets in R, the set of reals. Now we interested to study semi continuity, semi connectedness via semi open sets which is nothing but the parallel concepts we read in topology. The open and closed intervals are basically open sets and closed sets in R with respect to the usual topology. Semi open sets are weaker concepts in a topological space. Levine has developed the concept of semi continuity and semi connectedness in a formal way. From previous section it is clear that the empty set does not define any semi open sets except itself, it is vacuously true. In mathematical analysis semi continuity is a property of extended real valued function. Though Semi continuous functions has numerous application in Analysis. Like connectedness in a topological space, we now interested to study semi connectedness in topological space, in which open sets that defines the semi open sets plays an important role, we will discuss the topic further, for the basics we assume that the preliminary and basic definitions that used here are known to the reader. We denote the sets as clopen which is both open and closed. Finally, will characterize semi continuous functions and derive the concept of continuous functions from here.

Definition 2.1 : Let $f: X \to X^*$ be single valued (continuity not assumed) where X and X^* are topological spaces. Then $f: X \to X^*$ is termed *semi-continuous (written s.c)* if and only if, for O^* open in X*, then $f^{-1}\ (O^*)\ \in S.O(\ X^*)$.

Remark 2.1: Continuity implies semi-continuity, of course, but not conversely; Consider, X =X^* = [0, 1]. Let $f: X \to X^*$as follows:
$f(x)\ =\ 1$ if $x\ \in [\ 0,\frac{1}{2}\ )$ and $f(x)\ =\ 0$ if$x\ \in [\ \frac{1}{2}, 1\ ]$, f is semi open .

Theorem 2.1:[10]Let $f: X \to X^*$ be a single valued function, X and X^* being topological spaces. Then $f: X \to X^*$ is s.c if and only if for $f(p) \in\ \ O^*$. there exists an $A\ \in\ S.O.(X)$ such that $p \in A$ and $f(A)\ \subset O^*$.

Proof. Sufficiency- Let O^* be open in X^* and let $p \in f^{-1}(O^*)$. Then $f(p) \in O^*$ and thus there exists an $A_p \in S.O.(X)$ such that $p \in A$, and $f(A_p) \subset O^*$. Then $p \in A_p \subset f^{-1}(O^*)$ and $f^{-1}(O^*) = \cup A_p$. where $p \in f^{-1}(O^*)$ and union of s.o sets is s.o , $f^{-1}(O^*) \in S.O.(X)$.
Necessity-Let $f(p) \in O^*$. Then $p \in f^{-1}(O^*) \subset S.O.(X)$ since $f : X \to X^*$is s.c. Let $A = f^{-1}(O^*)$ Then $p \in A$ and $f(A) \subset O^*$.

Example 2.1: Let $X = X^* = R$, $f: X \to X^*$defined as follows
$f(x) = 1 \ \forall x \in X$ then if $\emptyset \neq A \in S.O.(X)$ then $f(A) = \{1\} \subseteq X^*$ but it is not s.o in X^*

Theorem 2.2:[10] Let $f: X \to X^*$ be s.c. and X^* is a 2nd axiom space. Let P be the set of points of discontinuity off. Then P is of first category.
Proof: Let $p \in P$. Then there exists an $O_{i_p}^*$in the countable open basis for X^* such that $p \in O$ open in X implies that $f(O) \notin O_{i_p}^*$.Now there exists an $A_{i_p} \in S.0.(X)$ such that $p \in A_{i_p}^*$ and $f(A_{i_p}) \subset O_{i_p}^*$by previous theorem.
But A_{i_p}=$O_{i_p} \cup B_{i_p}$whereB_{i_p} *is no where dense*. Hence $p \notin B_{i_p}$ and thus $p \in B_{i_p}$, a nowhere dense set.$\cup B_{i_p}$is nowhere dense, and P is a subset of B_{i_p}it followsthat P is of first category.

Remark 2.2: The converse is generally false, as shown by
Let $X = (0,1]$ and $X^* = [0,1]$.
$f(x) = 0$ if x is irrational and
$f(x) = 1/q$ if $x = p/q$ and p and q are relatively prime.
Then f is continuous at the irrationals and discontinuous at the rationals. Hence the set of discontinuities is ofthe first category.
But $f: X \to X*$ is not s.c. because $f^{-1}(\frac{1}{2}, 1]$ is a subset of the rationals and thus is not s.o.

Theorem 2.3:[16]Let $f_i: X_i \to X_i^*$be s.c. for $i = 1,2$. Let $f: X_1 \times X_2 \to X_1^* \times X_2^*$ as$f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then $f: X_1 \times X_2 \to X_1^* \times X_2^*$is s.c.
Proof: Let $O_1^* \times O_2^* \subset X_1^* \times X_2^*$where $0_i^*$is open in X_i^* for $i = 1,2$. Then

$f^{-1}(O_1^* \times O_2^*) = f^{-1}(O_1^*) \times f^{-1}(O_2^*)$. But $f_1^{-1}(O_1^*)$ and $f_2^{-1}(O_2^*)$ are s.o. in X_1 and X_2 respectively and thus $f_1{}^{-1}(O_1^*) \times f_2{}^{-1}(O_2^*)$ is s.o. in $X_1 \times X_2$ .Now if O^* is any open set in $X_1^* \times X_2^*$ then $f^{-1}(O^*) = f^{-1}(\cup\ O_\alpha^*)$ where O_α^* is of the form $O_{\alpha 1}^* \times O_{\alpha 2}^*$.Then $f^{-1}(O^*) = \cup\ f^{-1}(O_\alpha^*)$ which is s.o.

Theorem 2.4 :[10] Let $h: X \to X_1 \times X_2$ be s.c. where X, X_1 and X_2 are topological spaces. Let $f_i: X \to X_i$ as follows: for $x \in X$, $h(x) = (x_1, x_2)$. Let $f_i(x) = x_i$. Then $f_i: X \to X_i$ is s.c for $i = 1, 2$.

Proof: We shall show only that $f_1: X \to X_1$ is s.c. Let O_1 be open in X_1. Then $O_1 \times X_2$ is open in $X_1 \times X_2$ and $h^{-1}(O_1 \times X_2)$ is s.o. in X. But $f^{-1}(0_1) = h^{-1}(O_1 \times X_2)$ and thus $f_1: X \to X_1$ is s.c.

Remark 2.3: The converse of Theorem is generally false, as shown.

Let $X = X_1 = X_2$= [0, 1]. Let $f_i: X \to X_i$ for i=1,2 as follows:

$f_1(x) = 1$ if $x \in \left[0, \frac{1}{2}\right]$ and

$f_1(x) = 0$ if $x \in (\frac{1}{2}, 1]$

$f_2(x) = 1$ if $x \in \left[0, \frac{1}{2}\right)$ and

$f_2(x) = 0$ if $x \in [\frac{1}{2}, 1]$

then $f_i: X \to X_i$ is s.c

but $h(x) = \big(f_1(x), f_2(x)\big): X \to X_1 \times X_2$ is not s.c. for $S_{1/2}$ (1, 0) is open in $X_1 \times X_2$, but h^{-1} ($S_{1/2}$ $(1, 0)$)= ($\frac{1}{2}$) which is not s.o. in X.$S_{1/2}$ (1, 0) denotes the spherical neighbourhood of $(1, 0)$ of radius $\frac{1}{2}$

Remark 2.4: The limit of a sequence of s.c functions is not in general s.c, as shown by following example

Let $X = X^* = [0, 1]$ and $f_n: X \to X^*$ be defined as follows: $f_n(x) = x_n$ for $n = 1, 2$. Then $f_0: X \to X^*$ is the limit of the sequence where $f_0(x) = 0$ if x ∈ [0,1)and $f_0(x) = 1$ if $x = 1$.But f_0 is not s.c. For ($\frac{1}{2}$, 1] is open in X^*, but $f_0^{-1}(\frac{1}{2}, 1]$ = (1) which is not s.o. in X. We can, however, prove the following theorem.

Theorem 2.5:[10] Let $f_n: M \to M^*$, where M and M* are metric spaces with metric d andd^*, be s.c. for $n = 1, 2, ...$ and let $f_0: M \to M^*$ be the uniform limit of $\{f_n\}$ Then$f_0: M \to M^*$ is s.c.

Proof:$f_0 \in O^*$ then $f_0(x) \in S^*_\eta(f_0(x) \subset O^*$ for some $\eta > 0$. $\exists m$ such that $d^*\big(f_m(y), f_0(x)\big) < \frac{\eta}{2} \forall y \in M$ then $d^*\big(f_m(x), f_0(x)\big) < \frac{\eta}{2}$ and thus $f_m(x) \in S^*_{\eta/2}(f_0(x) \subset O^*$since f_m is s.c there exist an A such that x$\in A$ $f_m(x) \in S^*_{\eta/2}(f_0(x)$ if we shown $f_0(x) \subset O^*$we are done,

$d^*\big(f_0(y), f_0(x)\big) < d^*\big(f_0(y), f_m(x)\big) + d^*\big(f_m(x), f_0(x)\big) < \frac{\eta}{2} + \frac{\eta}{2} = \eta$

Which proves our theorem.

Theorem2.6:[16] If C is an open connected set and $C \subseteq E \subseteq cl(C)$ then E is semi open and connected

Proof: Clearly E is semi open by definition.

If E is not connected, then E must have a separation A and B. then C must have contained in either A or B. W.L.O.G we may assume $C \subseteq A$ then $C \subseteq c(A)$, hence $c(C) \cap B \subseteq c(C) \cap A = \emptyset$, but $B \subseteq E \subseteq c(C)$, $c(C) \cap B = \emptyset$ so $B = \emptyset$ which contradicts that E must have a separation A and B , so E must be connected.

Remark 2.5: The converse is not true $A = (0,1) \cup (1,2), E = (0,2), c(A) = [0,2]$ A is an open set not connected,E is semi open and connected , finally since A and E are both open sets and $c(E)$ is s.o , it is clear that same semi open set E can be defined by more than one open set i.e $E \subseteq c(E) \subseteq c(E), A \subseteq c(E) \subseteq c(A)$ where $c(A) = c(E) = [0,2]$

We now give more definitions and results and define Separation axioms in semi open space topology parallel to our known concepts in topology: -

Definition 2.2 :[15]A function $f: X \to Y$is said to be

(i) *Semi-continuous* if the inverse image of each open subset of Y is Semi open in X. (already known).

(ii) *Totally continuous* if the inverse image of every open subset of Y isclopen(both closed and open) subset of X.

(iii*Strongly continuous* if the inverse image of every subset of Y is aclopen subset of X.

(iv) *Totally semi-continuous* if the inverse image of every open subset ofY is semi-clopen in X.
(v) *Strongly semi-continuous* if the inverse image of every subset of Y is semi-clopen in X.
(vi) *Pre semi-open* if the image of every semi-open set in X is semi-openin Y .

We now discuss the separation axioms.

Definition 2.3 :[15]A topological space X is said to be
(i*) Semi-*T_0 if for each pair of distinct points in X, there exists a semi-openset containing one point but not the other.
(ii*) Semi-*T_1 if for each pair of distinct points $x$and $y$of X, there exist semi-open sets U and V containing x and yrespectively such that $x \in U, y \in U$ and $x \in V , y \in V$.
(iii) *Semi-*T_2if every two distinct points of X can be separated by disjoint semi-opensets.
(iv) *s-norma*l if each pair of non-empty disjointclosed sets can be separated by disjoint semi-open sets.
(v) *s-regular* if for each closed set F of X andeach $x \notin F$, there exist disjoint semi-open sets U and V suchthat $F \subset U$and $x \in V$.
(vi) *Semi-normal* if for each pair of disjoint semi closed sets U and V of X, there exist two disjoint semi-open sets $G$and Hsuch that $U \subset G$and $V \subset H$.
(vii) *Semi- regular* if for each semi-closed set F of $X$and each $x \notin F$, there exist disjoint semi-open sets $U$and Vsuch that $F \subset U$and $x \in V$
(viii) *Locally indiscrete* if every open set of X is closed in X.
(ix) s*-connected* if X is not the union of two nonempty disjoint semi-open subsets of X.
(x) *Semi-open* if $f(U)$is semi-open in Y for each open set Uin X.
(xi) *Semi-closed* if $f(F)$is semi-closed in Y for each closed set Fin X.

Definition 2.4*:*[15] Let Xbe a topological space and $x \in X$. Then the set of all points $y$in Xsuch that x and y cannot be separated by semi-separation of X is said to be the *quasi semi-component* of x.

We now give some theorems and results which establishes the relation between them defined above: -

Theorem 2.7:[15] Every semi totally continuous function is totally continuous.
Proof: Suppose $f: X \rightarrow Y$ is semi totally continuous function and U isany open set in Y. Since every open set is semi open U is semi open , and f is semi totallycontinuous, it follows that $f^{-1}(U)$is clopen in X . Thus the inverse image of each open set in Y is clopen in X. Therefore, f is totally semi continuous.

Remark 2.6- Every strongly continuous function is semi totally continuous since inverse image of each semi open set in Y is clopen in X.

Theorem 2.8: If $f : X \rightarrow Y$is semi-totally continuous function from an s-connected space X onto any space Y , then Y is an indiscrete space.
Proof: Suppose $f : X \rightarrow Y$is a semi-totally continuous function from an s-connected space X onto any space Y. If possible, suppose $Y$is not indiscrete. Let A be a proper non empty semi-open subset of Y . Then $f^{-1}(A)$is a proper non-empty clopen and hence semi-clopen subset of X. This implies $f^{-1}(A)$ is a proper non-empty semi-open subset of X, which is a contradiction to the fact that X is s-connected. Therefore, Ymust be indiscrete.

Theorem 2.9:[15]Every semi totally continuous function is totally semi continuous.
Proof: Suppose$f : X \rightarrow Y$ is semi totally continuous function and A is any open set in Y. Since every open set is semi open and f is semi totally continuous, it follows that $f^{-1}(A)$is clopen and hence semi clopen in X. Thus the inverse image of each open set in Y is semi clopen in X. Therefore is totally semi continuous.

Theorem 2.10:[15] Every semi totally continuous function is semi continuous.
Proof: Suppose $f : X \rightarrow Y$is a semi totally continuous function and A is any open set in Y. Since $A$is semi totally continuous function, $f^{-1}(A)$is clopen and hence semi clopen in X. This implies $f^{-1}(A)$is semiopen in X. Thus the inverse image of an open set in $Y$is semi open in X.

Therefore f is semi continuous function.

Remark 2.7:The converse of the above theorem need not be true ingeneral.Let $X = Y = \{a, b, c\}, \tau = \{X, \emptyset, \{a\}\}, \sigma = \{X, \emptyset, \{a\}, \{a, b\}\}$be topologies on X and Y respectively. Then $S.O(X) = S.O(Y) = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$.Define a function $f: X \to Y$as $f(a) = a, f(b) = b$ & $f(c) = c$, Clearly f is a semi continuous function. But it is not semitotally continuous, because for the set $\{a\}$in Y,$f^{-1}\{a\} = \{a\}$ is not clopenin X.
Thus the following interrelationship.

Strong continuity → Semi total continuity → Total continuity → Total semi continuity → Semi continuity→ Upper and Lower semi-continuous→ Continuous.

The converses except the last two are not true in general.

Theorem 2.11:[15] If $f : X \to Y$is semi totally continuous surjection and X is connected then $Y$is s-connected.
Proof: Suppose $Y$is not s-connected. Let A and B form disconnection of Y. Then A and B are semi open sets in Y and$Y = AUB$where $A \cap B = \emptyset$, Also $X = f^{-1}(Y) = f^{-1}(A\ U\ B) = f^{-1}(A)\ U\ f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$are nonempty clopen sets in X, because f is semi totally continuous. Further $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = \emptyset$.This implies X is not connected, which is a contradiction. Hence $Y$is s-connected.

Theorem 2.12:[15] Every semi-totally continuous function in to a finite T_1 space is strongly continuous.
Proof:Suppose $f : X \to Y$is a semi-totally continuous function, Y be a finite T_1space and $B \subset Y$. Since Y is finite T_1, Ymust be a discrete space. Therefore B is an open set and hence semi-open set in Y. Since$f$ is semi-totally continuous, $f^{-1}(B)$ is clopen in$X$. Thus the inverse image of every subset B of $Y$is clopen in X. Therefore f is strongly continuous.
We now establish a relation between Semi continuous and Continuous function [6], actually a function f is continuous iff it is both upper and lower semi continuous.

Definition 2.5:Let (X, d) be a metric space$f : X \to \mathbb{R}$and a point $y \in \mathbb{R}$, the upper contour set defined by *y* is

$$U(y) = f^{-1}([y, \infty)) = \{x \in X : f(x) \geq y\}$$

The lower contour set defined by *y* is

$$L(y) = f^{-1}([-\infty, y)) = \{x \in X : f(x) \leq y\}$$

The next result establishes that a number of properties are equivalent.

Theorem 2.13 :[6]Let$f : X \to \mathbb{R}$,

1. The following are equivalent.

(a) For any $y \in \mathbb{R}$, $U(y)$is closed.

(b) For any $y \in \mathbb{R}$, $f^{-1}(-\infty, y) = [U(y)]^c$is open.

(c) For any $x \in X$, if the sequence (x_t)in X converges to x, then for any $\varepsilon > 0$there is a T such that for all $t > T, f(x) > f(x_t) - \varepsilon$.

2. The following are equivalent.

(a) For any $y \in \mathbb{R}$, $L(y)$is closed.

(b) For any $y \in \mathbb{R}, f^{-1}((y, \infty)) = [L(y)]^c$is open.

(c) For any $x \in X$, if the sequence (x_t)in Xconverges to $x$then for any $\varepsilon > 0$there is aTsuch that for all$t > T, f(x) < f(x_t) + \varepsilon$.

Proof: We provide the proof of equivalence for the first set of conditions. The prooffor the second set of conditions is analogous.

• $1(a) \Rightarrow 1(b)$ Almost immediate, since $U(y)$ is closed iff$[U(y)]^c$is open.

• $1(b) \Rightarrow 1(c)$ By contraposition. Suppose that there is an $x \in X$and a sequence (x_t)in *X* that converges to *x* such that for some $\varepsilon > 0$ there are infinitely many *t* such that $f(x) \leq f(x_t) - \varepsilon$ Choose any$y \in (f(x), f(x) + \varepsilon)$. Then there are infinitely many *t* such that $x_t \in U(y)$. These $x_t$constitute a sequence in $U(y)$ that converges to *x*, but $x \notin U(y)$,hence $U(y)$ is not closed, hence $[U(y)]^c$is not open.

• $1(c) \Rightarrow 1(a)$ Take any $y \in \mathbb{R}$. If $U(y) = \emptyset$;then we are done. Otherwise, take anyconvergent sequence (x_t) in $U(y)$ let $x = \lim x_t$. We need to show that $x \in U(y)$. By 1(c), for any $\varepsilon > 0$there is a *T* such that for all $t > T, f(x) > f(x_t) - \varepsilon$ Since $x_t \in U(y), f(x_t) \geq y$, hence $f(x) > y - \varepsilon$.

Since this must hold for any $\varepsilon > 0$, $f(x) \geq y$, which implies $x \in U(y)$.

Since conditions listed under 1 and 2 are equivalent, we can choose any pair of them to define upper and lower semi continuity. To underscore the analogy withcontinuity, we use the “b" conditions.

Definition 2.6: [6]Let $f : X \to \mathbb{R}$,
1. f is *upper semi continuous* (USC) iff for any $y \in \mathbb{R}$, $f^{-1}\big((-\infty\ y)\big)$ is open.
2. f is *lower semi continuous* (LSC) iff for any $y \in \mathbb{R}$, $f^{-1}\big((y, \infty)\big)$ is open
Informally, a function is upper semi continuous if it is continuous or, if not, it only jumps up; a function is lower semi continuous if it is continuous or, if not, itonly jumps down.

Example 2.2: Define $f\colon \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} \frac{1}{\mathrm{x}} & if\ x < 0 \\ 0 & if\ x = 0 \\ -\frac{1}{x} & if\ x > 0 \end{cases}$$

Then f is upper semi-continuous.
In particular, if $x_t \to 0$, then $f(x_t) \to -\infty < 0 = f(0)$

Theorem 2.14:[6]*f*is continuous iff it is both upper and lower semi-continuous.
Proof. Almost immediate form property 1(c) and 2(c), which together are equivalent to requiring that if (x_t) converges to $x$then for any $\varepsilon > 0$, there is a T suchthat for all $t > T$, $f(x_t) \in N_\varepsilon(f(x))$, hence $f(x_t)$ converges to $f(x)$ which is the sequential criterion of continuity of f .
We finally derive a relation between semi continuous and continuous function.

CHAPTER 3 : SEMI-I-OPEN SETS

In the year 1963, Norman Levine introduced the precise idea of semi open sets and generalized the idea of continuity through semi continuity. Here we define a more generalized class of Semi open sets by defining I -semi open set and this chapter is devoted to characterize I - semi open set in a given Topology. There is no restriction on Separation Axioms which is another advantage to study this paper.

Definition 3.1 [2] **:**A nonempty collection $I$of subsets of X in a Topological space (X, τ) satisfies the following properties;
(a)If $A \in I$ & $B \subset A$ then $B \in I$
(b)If $A, B \in I$, then $A \cup B \in I$
then I is called an ideal on (X, τ).

Definition 3. 2 [3]**:** A subset A of X is said to be Semi-open with respect to an Ideal I (written as I -semi open or I -s.o or semi- I-open set) if there exists an Open set U such that $U - A \in I$ an $A - cl(U) \in I$.

Proposition 3.1 : If $A \in I$ then A is I - semi open.
Proof: $int(A) - A = \phi \in I$ and $A - cl\big(int(A)\big) \subset A$ & $A \in I$.
By definition 3.1 (a) $A - cl\big(int(A)\big) \in I$
Hence A is I -semi open.

Proposition 3.2: If A is s.o then A is $I - s.o$ also.
Proof: Since A is S.O [1] $\exists$ an Open set U such that $U \subset A \subset cl(U)$.
i.e. $U - A \in I$ and $A - cl(U) \in I$
Hence A is I -semi open also; so it is obvious that A is open set implies $I - s.o$ also according as [9], any open sets are semi open. But the converse of the above proposition is not true in general and so we observe the following example.

Example 3.1: Consider a Topological Space(X, τ).
$X = \{a, b, c\}$&τ = $\{\phi, \{a\}, \{a, c\}, X\}$ .Choose I = $\{\phi, \{b\}, \{c\}, \{b, c\}\}$& observe that $\{b\}$ is $I - s.o$ but {b} is not $s.o$.

Now a question arises “In which case s.o and I -s.o are equivalent?” So we are motivated enough to read the 1st Theorem.

Theorem 3.1[4]**:**If an ideal Ithat is not closed under countable additivity, the concept of semi-openness &I -semi openness coincides iff $I = \{\phi\}$.

Proof: Let $I = \{\phi\}$.In view of proposition 3.2 it is enough to prove $A \in I - S.O$ then $A \in S.O$.

As $A \in I - S.O$,$\exists$ an open set U such that $U - A \in I = \{\phi\}$ i.e. $U \subset A$ and $A - cl(U) \in I = \{\phi\}\ i.e. A \subset cl(U)$

Hence, $U \subset A \subset cl(U)\ \ i.e. A \in S.O$.

Conversely,

Let if A is $I - s.o$ then it is s.o.

Let $B \in I$ then B is I -semi open and by assuming B is s.o $\exists$ an open set V_1 such that $V_1 \subset B \subset cl(V_1)$. Since $B \in I$ &$V_1 \subset B$, we have that $V_1 \in I$ and so $B \cup V_1 \in I$ i.e. $B \cup V_1 \in I$-semi open. By our assumption it is s.o also. Hence $\exists\ V_2 \in \tau$ such that $V_2 \subset (B \cup V_1) \subset cl(V_2)$. Similarly, there is an open set V_3 such that $V_3 \subset (B \cup V_1 \cup V_2) \subset cl(V_3)$ .Continuing this way we have the an infinite collection $V_1, V_2, V_3, V_4, \dots$ such that $B \cup V_1 \cup V_2 \cup V_3 \cup \dots \in I$. But we assumed I as not closed under countable additivity so necessarily $B = \phi$. Hence $I = \{\phi\}$.

Theorem 3.2 [4] **:** If I_1, I_2 be two ideals on (X, τ) such that

(a) If $I_1 \subset I_2$, then every I_1-s.o set A is I_2s.o.

(b) If A is $(I_1 \cap I_2)$ s.o, then it is both I_1-s.o and I_2-s.o.

Proof :(a) let A is I_1-s.o then $\exists$ an open set U such that

$$U - A \in I_1\ ,\ \ A - cl(U) \in I_1\ .$$

As $I_1 \subset I_2, U - A \in I_2\ ,\ \ A - cl(U) \in I_2$

Hence, A is I_2-s.o.

Proof: (b)If A is $(I_1 \cap I_2)$-s.o then $\exists$ an open set U such that

$$U - A \in I_1 \cap I_2\ i.e. U - A \in I_1 \&\ U - A \in I_2$$

$$A - cl(U) \in I_1 \cap I_2\ i.e. A - cl(U) \in I_1 \&\ A - cl(U) \in I_2$$

Hence,A is both I_1-s.o &A is I_2-s.o.

Theorem 3.3 [4]**:**$I_A = \{A \cap S\ |S \in I\}$ forms an ideal on (X, τ)

Where $A \subset X$ and I is an ideal .Moreover if a set B is I_A-S.O then it is I-s.o also.

Proof: If $A \cap S \in I_A$ for some $S \in I$.
Let $Y \subset A \cap S$ then $Y \subset A$ and $Y \subset S$ i.e. $Y \in I$. For convenience we rename Y as S^*. Clearly $Y = S^* = A \cap S^* \in I_A$.
If $A \cap S_1, A \cap S_2 \in I_A$ then$(A \cap S_1) \cup (A \cap S_2) = A \cap (S_1 \cup S_2) \in I_A$
As $S_1 \cup S_2 \in I$,from definition 3.1(b). Thus I_A forms an Ideal.
We choose any element of I_A ,$A \cap S$ (say) .As $A \cap S \subset S \in I$.
So$A \cap S \in I$. It proves $I_A \subset I$. Now we prove our desired result by using theorem 3.2(a).

It's a trivial fact knowing the definition of I that union of finite number of members of I belongs to I. Does the same result holds in more general class of sets, I- s.o sets?

<u>Theorem 3.4</u> [4]**:** If A, B are both I-s.o. then $A \cup B$ is also I-s.o.
Proof: If A is I-s.o. then $\exists$ an open set U such that
$U - A \in I$ & $A - cl(U) \in I$.
And similarly, Open set V such that
$V - A \in I$ & $A - cl(V) \in I$.
Let $W = U \cup V$,then $(U \cup V) - (A \cup B) = [(U - A) - B] \cup [V - B) - A] \in I$.
Also,$(A \cup B) - cl(U \cup V)[\{A - cl(U)\} - cl(V)] \cup [\{B - cl(V)\} - cl(U)] \in I$.
By the definition 3.2 it can be said $A \cup B$ is I-s.o.

<u>Theorem 3.5</u>[4]**:** Let (X, τ) be a topological space in which there is an open singleton subset $\{a\}$ satisfying $cl(\{a\}) = X$ .for any ideal I on X with $\{a\} \in I$, we have that

(a) Every singleton subset of X is I-s.o.
(b) Every finite subset of X is I-s.o.

Proof :(a) Let $\{a\} - \{s\} = \{a\} \in I$ &$\{s\} - cl(\{a\}) = \{s\} - X = \phi \in I$.
Hence {a} is I-s.o.
(b) Proof of this part is obvious followed by theorem 3.4 and theorem 3.5(a).

<u>Example 3.2</u>[4]**:** Consider the finite set $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$. We observe that $cl(\{a\}) = X$, if we choose the minimal ideal $I = \{\phi\}$ on X, then the singleton subset $\{b\}$ is not s.o. and hence by theorem 3.1 not I-s.o.

Theorem 3.6 [4]**:** Let A, B are two subsets of a topological space (X, τ) such that A is open, $A \subset B$ and A is dense in B. Then B is I-S.O for any ideal I on X.

Proof: clearly, $A - B = \phi \in I$ & $B - cl(A) = \phi \in I$.

Thus B is I-semi open.

Example 3.3 [4]**:** If two sets A, B are I-S.O then $A \cap B$ need not be I-S.O. Let X=$\{a, b, c\}$ with the topology $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$. Hence $cl(\{a\}) = \{a, b\}$& $cl(\{c\}) = \{b, c\}$. These are s.o and hence I-s.o if we take I as minimal ideal $\{\phi\}$ .Their intersection is $\{b\}$ not s.o and hence not I-s.o by theorem 3.1.

Obviously, if$A, B \in I$ then $A \cap B \in I$. In this case intersection of two I-s.o sets is again I-s.o. Following this argument and example 3.3 we claim that if $A \cap B$ is not I-s.o then necessarily A, B both are not in I. Next theorem has characterised such A, B two I-s.o sets such that $A \cap B$ is I-s.o.

Theorem 3.7 [3] **:** Let I be an ideal on a topology (X, τ), where any non empty open subset of X is dense, and the collection of open subsets of X satisfies the finite intersection property.

(a) If A is I-s.o. &$A \subset B$, then B is I-s.o.
(b) If A,B are I-s.o. then $A \cap B$ are I-s.o.
(c) A is I-s.o iff $cl(A)$ is I-s.o.

Proof: (a) Let A is I-s.o &$\subset B$. As A is I-s.o $\exists$ an open set U such that $U - A \in I$ & $A - cl(U) \in I$. As we are interested in those case where A is not in I we take $U \neq \phi$. By the given condition $cl(U) = X$

Now, $A \subset B$ $i.e. U - B \subset U - A \in I$.

Hence $U - B \in I$ & $B - cl(U)$ $i.e. B - X = \phi \in I$.

Thus B is I-S.O

(b) let A, B both are I-s.o. Then we have following two cases.

Case 1: If $A \cap B = \phi$, we are done.

Case 2: If $A \cap B$ not null .

By definition 3.2 $\exists$ $U \in \tau$,$V \in \tau$ such that $U - A \in I, A - cl(U) \in I$ and $V - B \in I, B - cl(V) \in I$.

Now,$(U \cap V) - (A \cap B) = \big((U - A) \cap V\big) \cup \big(U \cap (V - B)\big) \in I$
and $(A \cap B) - cl(U \cap V) = (A \cap B) - X = \phi \in I$.
These prove $A \cap B$ is I-s.o.

(c) Let A be I-s.o. and as $A \subset cl(A)$ by theorem 5.7 (a) $cl(A)$ is also I-s.o.
Conversely, by the similar argument of part (a) $\exists$, $U \in \tau$ & $U \neq \phi$
Such that , $U - cl(A) \in I$ & $cl(A) - cl(U) \in I$.
We consider the open set $V = U - cl(A) = U \cap \big(cl(A)\big)^c \in I$.
By definition 3.1 (a) $V - A \in I$ & $A - cl(V) = A - X = \phi \in I$.
Thus A is I -S.O if $cl(A)$ is I-s.o.

Theorem 3.9 :The following statements are equivalent for $A \subset X$.

(a) $X - A$ is I-s.o.

(b) $\exists$ a closed set F such that $int(F) - A \in I$ & $A - F \in I$.

Proof : Let (a) holds, then $\exists$,$U \in \tau$ such that $U - (X - A) \in I$ &$(X - A) - cl(U) = int(X - U) - A \in I$, Now we put $X - U = F$ to obtain (b).
Conversely we assume the statement (b) holds .We shall consider the open set $X - F = U$, to define $X - A$ as I-s.o.
Before we proceed to the next definition recall the definition of closed set on any arbitrary topology (X, τ). A subset A of X is said to be closed iff $X - A$ is Open. The next definition is analogous to it.

Definition 3.3[4]**:** A subset A of X is said to be semi-closed w.r.t. an ideal I (written as I-semi closed) iff $X - A$ is I-semi open.

Clearly, all closed sets and semi closed sets are I-semi closed from proposition 3.2. Now recall the fact that intersection of two semi closed sets is semi closed from chapter 1 and then investigate to our final claim.

Theorem 3.10: If both A & B are I-semi closed, then $A \cap B$ is I-semi closed.

Proof : From definition 3.3 it follows that both $X - A$ & $X - B$ are I-semi open.
Using theorem 3.9 $\exists$ F_1, F_2 two closed sets s.t.
$int(F_1) - A \in I, A - F_1 \in I$ & $int(F_2) - A \in I, A - F_2 \in I$.
We choose , $F_1 \cup F_2 = F$ closed in (X, τ) to fulfil our purpose.

$int(F_1 \cap F_2) - (A \cap B) = \quad \{(int(F_1) - \{(int(F_1) - A) \cap int(F_2)\} \cup$
$\{int(F_2) \cap (int(F_2) - B)\} \in I$ and
$(A \cap B) - (F_1 \cap F_2) = \{(A - F_1) \cap B\} \cup \{A \cap (B - F_2)\} \in I$.
Above two sets belong to I from definition 3.1
i.e.$int(F) - (A \cap B) \in I$ &$(A \cap B) - F \in I$. From theorem 3.9 and definition 3.3 we conclude that $A \cap B$ is I-semi closed.

SEMI-I_S -CONTINUOUS FUNCTION

Ideal in topological space have been considered since 1930 by Kuratowski[ ] and Vaidyanathaswamy[ ].In 1990 Jankovic and Hamlett [ ] investigated further properties of ideal topological spaces.In 2010, Khan and Noiri[ ] introduced and studied the concept of semi-local functions .The notion of semi-open sets and semi-continuity was first introduced and investigated by Levine [10] in 1963.In 2005, Hatir and Noiri [ ] introduced the notion of semi-$\mathcal{I}$-open sets and semi-$\mathcal{I}$-continuity in ideal topological spaces .In this contend we obtain several characterizations of semi-I_S -open sets and semi-I_S -continuous function.

Let P(X) be the power set of X.Then the operator $(\ )^*: P(X) \to P(X)$is called a local function [ ] of A with respect to τ and I is defined as follows :for $A \subseteq X, A^*(I,\tau) = \{x\ \in X: U \cap A\ \notin\ I\ for\ every\ U\ \in\ \tau(X,x)\}$We simply denote $A^*(I,\tau)$ as A^*.For every ideal topological space (X,τ,I), there exists topology τ^* finer than τ generated by $\beta(I,\tau) = \{U \backslash J: U \in \tau\ and\ J \in I\}$ but in general $\beta(I,\tau)$ is not always a topology .Additionally $cl^*(A) = A \cup A^*$ defines Kuratowski closure operator for a topology τ^* finer than τ . Similarly $int^*(A)$ denotes the interior of A with respect to τ^* .

Definition 4.1: Let (X,τ,I) be an ideal topological space and A be a subspace of X and A be a subset of X.Then $A_*(I,\tau) = \{x \in X: A \cap U \notin I\ for\ every\ U \in SO(X,x)\}$ is called the semi-local function of A with respect to I and τ where $SO(X,x) = \{U \in SO(X): x \in U\}$.

We will use A_* in the place $A_*(I,\tau)$.

Remark 4.1: Let (X,τ,I) be an ideal topological space and A be a subspace of X.Then

I. $A_*(I,\tau) \subseteq A^*(I,\tau)$ for every subset A of X.
II. $A_*(I,\tau) = A^*(I,\tau)$ for every subset A of X.
III. The simplest ideals are $\{\phi\}$ and $P(X) = \{A: A \subseteq X\}$.We observe that $A_*\{\phi\} = scl(A) \neq cl(A)$ and $A^*(P(X)) =\ \phi$ gives $A_*(P(X)) = \phi$ for every subset A of X.
IV. If $A \in I$then $A_* =\ \phi$.

V. Neither $A \subseteq A_*$ nor $A_* \subseteq A$ in general

Example 4.1: Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, X, \{a, b\}\}$ with $I = \{\phi, \{a\}, \{b\}, \{a, b\}\}$.

Let $A = \{a, d\}$

Therefore,$A^*(I) = \{x \in X: A \cap U \notin I, U \in \tau(X, x)\} = \{c, d\} = cl(A^*)$

Also$A_*(I) = \{x \in\ A \cap U \notin I, U \in SO(X, x)\}$

$= \{d\}\ \ [Take\ U = \{b, d\}]$

$= scl(A_*)$

Example 4.1 shows that converse of remark 4.1(i) may not be true.

Theorem 4.1: Let (X, τ, I) be an ideal topological space and A, B be a subspace of X.Then for semi local functions ,the following properties hold:

I. If $A\ \subset B$ then$A_* \subset B_*$

II. $A_* = scl(A_*)\ \subset scl(A)$ and A_* is semi-closed in (X, τ)

III. $(A_*)_* \subset A_*$

IV. $(A \cup B)_* = A_* \cup B_*$

V. $A_* - B_* = (A - B)_* - B_* \subset (A - B)_*$

VI. $If\ U \in \tau\ then\ U \cap A_* = U \cap (U \cap A)_* \subset (U \cap A)_*$

VII. $If\ I0 \in I\ then\ (A - I_0)_* \subset A_* = (A \cup I_0)_*$

Proof: (i)Suppose that $A \subset B$ and $x\ \notin\ B_*$

Therefore, there exists $U \in SO(X, x)$ such that $U \cap B \in\ I$

Since $A \subset B, U \cap A \in\ I$ and $x \notin\ A_*$

Hence $A_* \subset B_*$

(ii) We have $A_* \subset\ scl(A_*)$ in general.

Let,$x \in\ scl(A_*)$.Then $A_* \cap U \neq \phi$ for every $U \in SO(X, y)$

Since $y \in A_*$, $A \cap U \notin\ I$ and hence $x \in A_*$.

Hence we have $scl(A_*) \subset\ A_*$

Again, let $x \in\ scl(A_*) =\ A_*$.Then $A \cap U \notin\ I$ for every $U \in SO(X, x)$.

This implies $U \cap A \neq \phi$ for every $U \in SO(X, x)$.

Therefore $x \in\ scl(A)$.

This proves$A_* =\ scl(A_*) \subset\ scl(A)$.

(iii)Let,$x \in (A_*)_*$.Then for every $U \in SO(X, x)$ $U \cap\ A_* \notin I$and hence $U \cap A_* \neq \phi$

Let, $y \in\ U \cap A_*$.

Then $U \in SO(X, y)$ and $y \in\ A_*$Hence we have $U \cap\ A \notin\ I$and $x \in A_*$.This shows that $(A_*)_* \subset A_*$

(iv) We know that $A \subseteq (A \cup B)$ and $B \subseteq (A \cup B)$.So, we have by (i) that $A_* \cup B_* \subseteq (A \cup B)_*$

Let $x \in (A \cup B)$.Then for every $U \in SO(X, x)$, $(U \cap A) \cup (U \cap B) = U \cap (A \cup B)\ \notin I$and so, $A \cap U \notin I\ or, B \cap U \notin I$.This implies that $x \in A_* or\ x \in B_*$.

So, we have $(A \cup B)_*\ \subseteq A_* \cup A_*$.

Consequently we obtain, $(A \cup B)_* = A_* \cup B_*$.

(v)Since $A = (A \backslash B)\ \cup (B \cap A)$

So, $A_* = (A \backslash B)_*\ \cup (B \cap A)\ _*$[By (iv)]

Hence,$A_* \backslash B_* =\ A_* \cap (X \backslash B_*) = [(A \backslash B)_*\ \cup (B \cap A)_*]\ \cap (X \backslash B_*)$

$$= \ [(A \backslash B)_*\ \cap (X \backslash B_*)] \cup [(B \cap A)_*\ \cap (X \backslash B_*)]$$

$$= [[(A \backslash B)_* \backslash\ B_*] \cup \phi \subset (A \backslash B)_*$$

(vi)Let $U \in \tau$ and $x \in U \cap A_*$.

Then $x \in\ U$ and $x \in A_*$

Let V be any semi-open set containing x,Then $V \cap U \in SO(X, x)$ and $V \cap (U \cap A) = (V \cap U)\ \cap A \notin I$.

This shows that $x \in (U \cap A)_*$ and hence obtain $(U \cap A_*) \subset (U \cap A)_*$.Moreover $(U \cap A_*) \subset U \cap (U \cap A)_*$and by $(i)\ A_* \supset (U \cap A)_*$and $U \cap (U \cap A)_* \subset\ (U \cap A_*)$.

So, $U \cap A_* = U \cap (U \cap A)_*$

(vii) Since $(A \backslash I_0) \subset A$, by (i)$(A \backslash I_0 I_0 I_0)_* \subset A_*$

So by (iv) and remark 4.1(iv) $(A \cup\ I_0) *= A_* \cup (I_0)_* = A_* \cup\ \phi =\ A_*$

Therefore we obtain,$(A - I_0)_* \subset A_* = (A \cup I_0)_*$

<u>**Theorem 4.2**</u>**:**Let (X, τ) be a topological space with idealsI_1 and I_2 on X and A be a subset of X.Then

I. If $I_1 \subset I_2$then $A_*(\ I_2)\ \subset A_*(I_1)$
II. $A_*(I_1 \cap I_2) =\ A_*(I_1) \cup\ A_*(I_2)$

Proof:(i) Let $I_1 \subset I_2$ and $x \in\ A_*(I_2)$.

Then $U \cap A \notin I_2$ for every $U \in SO(X,x)$.Since $I_1 \subset I_2, U \cap A \notin I_1$ and hence $x \in A_*(I_1)$.

So, $A_*(I_2) \ \subset A(I_1)$

(ii) By theorem 4.2(i) we have $A_*(\ I_1) \ \subset A_*(I_1 \cap I_2)$ and $A_*(\ I_2) \ \subset A_*(I_1 \cap I_2)$ follows

$$A_*(I_1) \ \cap \ A_*(\ I_2) \ \ \subset A_*(I_1 \cap I_2)$$

Now let, $x \in \ A_*(I_1 \cap I_2)$

Then for every $U \in SO(X,x), U \cap A \notin I_1 \cap I_2$ and hence $U \cap A \notin I_1 or\ U \cap A \notin I_2$.

So, $x \in \ A_*(I_1)\ or\ x \in \ A_*(I_2)$

Therefore $x \in \ A_*(I_1) \cup \ A_*(I_2)$

This shows that $A_*(I_1 \cap I_2) \ \subset A_*(\ I_1) \ \cap \ A_*(\ I_2)$

Consequently it follows that $A_*(I_1 \cap I_2) = A_*(\ I_1) \cup \ A_*(I_2)$

Remark 4.2: Let (X, τ, I) be an ideal topological space and $A \subseteq B$.If $I = \{\phi\}$, then $A_* = scl(A)$

Definition 4.2: A subset A of a topological space (X, τ, I) is said to be

I. α-open [ ] if $A \ \subseteq \ int(cl(int(A)))$
II. pre-open [ ] if $A \ \subseteq \ int(cl(A))$
III. β-open[ ] if $A \ \subseteq \ cl(int(cl(A)))$

The complement of a pre-open set (α-open set) is called pre-closed set(α-closed

set). The intersection of all pre-closed set(α-closed set) containing A is called pre-closure(α-closure) of A.

Definition 4.3: A subset A of an ideal topological space (X, τ, I) is said to be

I. α - I -open[ ] if $A \ \subseteq \ int(cl^*(int(A)))$
II. semi-I -open [ ] if $A \ \subseteq \ cl^*(int(A))$
III. pre-I -open [ ] if $A \ \subseteq \ int(cl^*(A))$

Definition 4.4: A subset A of an ideal topological space (X, τ, I) is said to be

I. α -I_s -open[ ] if $A \ \subseteq \ int(cl^{*s}(int(A)))$
II. semi-I_s -open [ ] if $A \ \subseteq \ cl^{*s}(int(A))$
III. pre-I_s -open [ ] if $A \ \subseteq \ int(cl^{*s}(A))$

Remark 4.3: Considering the above definition we can construct the following diagram [ ]:

$$\text{open} \rightarrow \alpha\text{-}I_s\text{-open} \rightarrow \text{semi-}I_s\text{-open} \rightarrow \text{semi open}$$

$$\downarrow\downarrow$$

$$\text{pre-}I_s\text{-open} \rightarrow \text{pre open} \rightarrow \beta\text{-open}$$

By $SISO(X,\tau)$ we denote the family of all semi-I_s -open sets of a space (X,τ,I).

Theorem 4.3: A subset A of a space (X,τ,I) is semi-I_s -open if and only if $cl^{*s}(A) = cl^{*s}(int(A))$.

Proof: Let A be semi-I_s -open, we have $A \subseteq cl^{*s}(int(A))$.
Then $cl^{*s}(A) \subseteq cl^{*s}(int(A))$.
Clearly, $cl^{*s}(int(A))\ ) \subseteq cl^{*s}(A)$.
Hence ,$cl^{*s}(A) = cl^{*s}(int(A))$.
Converse part is obvious.

Theorem 4.4: A subset A of a space (X,τ,I) is semi-I_s -open if and only if there exists $U \in \tau$ such that $U \subseteq A \subseteq cl^{*s}(U)$.

Proof: Let A be semi-I_s -open, we have $A \subseteq cl^{*s}(int(A))$.
Take $U = int(A)$
Then we have $U \subseteq A \subseteq cl^{*s}(U)$.
Conversly let $U \subseteq A \subseteq cl^{*s}(U)$ for some $U \in \tau$.
Since $U \subseteq A$ we have $U \subseteq int(A)$ and hence $cl^{*s}(U) \subseteq cl^{*s}(int(A))$.
Thus we obtain $A \subseteq cl^{*s}(int(A))$.

Theorem 4.5: If A is semi-I_s -open set in a space (X,τ,I) and $A \subseteq B \subseteq cl^{*s}(A)$, then B is semi-I_s -open in (X,τ,I)

Proof: Since A is semi-I_s -open, by theorem 4.4 there exists an open set U such that

$$U \subseteq A \subseteq cl^{*s}(U)$$

Then we have $U \subseteq A \subseteq B \subseteq cl^{*s}(A) \subseteq cl^{*s}(cl^{*s}(U)) = cl^{*s}(U)$
i.e, $U \subseteq B \subseteq cl^{*s}(U)$

i.e, B is semi-I_s -open.

Theorem 4.6: Let (X, τ, I) be an ideal topological space and A, B are subsets of X.

I. If $U\alpha \in SISO(X,\tau)$ for each $\alpha \in \Delta$ then $\cup \{ U\alpha: \alpha \in \Delta\} \in SISO(X,\tau)$

II. If $A \in SISO(X,\tau)$ and $B \in \tau$ then $A \cap B \in SISO(X,\tau)$

Proof:(i) Since $U\alpha \in SISO(X,\tau)$ we have $U\alpha \subseteq cl * s(int(U\alpha))$ for each $\alpha \in \Delta$.

Now by using theorem 4.1 we obtain,

$$\bigcup_{\alpha\in\Delta} U_\alpha \subseteq \bigcup_{\alpha\in\Delta} cl^{*s}\big(int(U_\alpha)\big)$$

$$\subseteq \bigcup_{\alpha\in\Delta}\{\big(int(U_\alpha)\big)_* \cup (int(U_\alpha))\}$$

$$\subseteq (\bigcup_{\alpha\in\Delta}(int(U_\alpha))\ _* \cup int(\bigcup_{\alpha\in\Delta} U_\alpha)$$

$$\subseteq (int(\bigcup_{\alpha\in\Delta} U_\alpha))_* \cup int(\bigcup_{\alpha\in\Delta} U_\alpha)$$

$$= cl^{*s}(int(\bigcup_{\alpha\in\Delta} U_\alpha))$$

This shows that $\bigcup_{\alpha\in\Delta} U_\alpha \in SISO(X,\tau)$

(ii) Let, $A \in SISO(X,\tau)$ and $B \in \tau$

Then $A \subseteq cl^{*s}(int(A))$ and using theorem 4.1 we have

$$A \cap B \subseteq cl^{*s}(int(A)) \cap B = ((int(A))_* \cup (int(A))) \cap B$$

$$= ((int(A))_* \cap B) \cup (int(A) \cap B)$$

$$\subseteq (int(A) \cap B)_* \cup int(A \cap B)$$

$$= \big(int(A \cap B)\big)_* \cup int(A \cap B)$$

$$= cl^{*s}(int(A \cap B))$$

This shows that $A \cap B \in SISO(X,\tau)$

Definition 4.5: A subset F of an ideal topological space (X, τ, I) is said to be semi-I_s -closed if its complement is semi-I_s -open.

Theorem 4.7: If a subset A of a topological space (X, τ, I) is semi-I_s -closed then $int(cl^{*s}(A)) \subseteq A$.

Proof: Since$A$ is semi-I_s -closed, $X \backslash A \in SISO(X, \tau)$.

Since τ*(I) is finer than τ, we have

$X \backslash A \subseteq cl^{*s}(int(X \backslash A)) \subseteq cl(int(X \backslash A) = X \backslash int(cl(A)) \subseteq X \backslash int(cl^{*s}(A))$.

Therefore we get $int(cl^{*s}(A)) \subseteq A$.

Corollary 4.1: Let A be a subset of a space (X, τ, I) such that $X \backslash int(cl^{*s}(A)) = cl^{*s}(int(X \backslash A))$.

Then A is semi-I_s -closed if and only if $int(cl^{*s}(A)) \subseteq A$.

Theorem 4.8: Let (X, τ, I) be an ideal topological space.If $Y \in \tau$ and $W \in SISO(X)$,then

$$Y \cap W \in SISO(Y, \tau|_Y, I_Y)$$

Proof: Since Y is open, we have $int_Y(A) = int(A)$ for any subset A of Y.

Now, $Y \cap W \subseteq Y \cap cl^{*s}(int(W)) = Y \cap (int(W))_* \cup int(W)$

$$= ((Y \cap (int(W))_*) \cup (Y \cap (int(W))) \cap Y$$
$$= (Y \cap (Y \cap (int(W)))_*) \cup (Y \cap (int(W))) \cap Y$$
$$= Y \cap (int_Y(Y \cap W))_* \cup (Y \cap (int_Y(W)))$$
$$= (int_Y(Y \cap W))_*(I_Y, \tau|_Y) \cup (int_Y(Y \cap W))$$
$$= cl^{*s}{}_Y(int_Y(Y \cap W))$$

This shows that $Y \cap W \in SISO(Y, \tau|_Y, I_Y)$

Definition 4.6: A function $f: (X, \tau, I) \rightarrow (Y, \sigma)$ is said to be semi-I_s -continuous [ ] (semi-I-continuous [ ]) if $f^{-1}(V)$ is semi-I_s -open (semi-I-open) in (X, τ, I) for each open set V of (Y, σ).

Definition 4.7: A function $f: (X, \tau, I) \rightarrow (Y, \sigma, \mathcal{J})$ is said to be semi-I_s -irresolute (I-irresolute [ ]) if $f^{-1}(V)$ is semi-I_s -open (semi- I-open) in (X, τ, I) for each semi-$\mathcal{J}_s$-open set (semi- $\mathcal{J}$-open set) V of $(Y, \sigma, \mathcal{J})$.

Remark 4.4: Continuity implies semi-I_s -continuous, semi-I_s -continuous implies semi- I-continuous and finally semi-I-continuous implies semi continuous.

Theorem 4.9: For a function $f: (X, \tau, I) \rightarrow (Y, \sigma)$ the following are equivalent :

I. f is semi-I_s -continuous

II. For each $x \in X$ and each $V \in \sigma$ containing $f(x)$, there exists $W \in SISO(X)$ containing x such that $f(W) \subseteq V$

III. The inverse image of each closed set in Y is semi-I_s -closed.

Proof: (i)⟹(ii)

Let, $x \in X$ and V be any open set of Y containing $f(x)$.
Set $W = f^{-1}(V)$, then by definition 4.6, W is a semi-I_s -open set containing x and $f(W) \subseteq V$

(ii) ⟹(iii)

Let,F be a closed set of Y.
Set $V = Y \backslash F$,then V is open in Y.
Let $x \in f^{-1}(V)$
So there exists a semi-I_s -open set W of X containing x such that $f(W) \subseteq V$
Thus $x \in W \subseteq cl^{*s}(int(W)) \subseteq cl^{*s}(int(f-1(V)))$
So, $f^{-1}(V) \subseteq cl^{*s}(int(f^{-1}(V)))$
This shows that $f^{-1}(V)$ is semi-I_s -open in X.
Hence $f^{-1}(F) = X \backslash f^{-1}(Y \backslash F) = X \backslash f^{-1}(V)$ is semi-I_s -closed in X.

(iii) ⟹(i)

Let, V be an open set of Y.
Set, $F = Y \backslash V$, then F is closed in Y.

$$f^{-1}(V) = X \backslash f^{-1}(Y \backslash V) = X \backslash f^{-1}(F).$$

So $f^{-1}(V)$ is semi-I_s -open in X.

<u>**Theorem 4.10**</u>**:** Let $f: (X, \tau, I) \to (Y, \sigma)$ be semi-I_s -continuous and $U \in \tau$.Then the restriction $f|U: (U, \tau|_U, I_U) \to (Y, \sigma)$ is semi-I_s -continuous.

Proof: Let V be any open set of (Y, σ).
Since f is semi-I_s -continuous, $f^{-1}(V) \in SISO(X, \tau)$ and bt theorem 4.8,

$$(f|_U) - 1(V) = f^{-1}(V) \cap U \in SISO(X, \tau|_U).$$

This shows that $f|_U: (U, \tau|_U, I_U) \to (Y, \sigma)$ is semi-I_s -continuous.

<u>**Remark 4.5**</u>**:** For a function $f: (X, \tau, I) \to (Y, \sigma, \mathcal{J})$ and $g: (Y, \sigma, \mathcal{J}) \longrightarrow (Z, \eta)$, the following hold:

I. $g \circ f$ is semi-I_s -continuous if f is semi-I_s -continuous and g is continuous.
II. $g \circ f$ is semi-I_s -continuous if f is semi-I_s -irresolute and g is semi-I_s -continuous.

Theorem 4.11: Let $f: (X, \tau, I) \to (Y, \sigma, \mathcal{J})$ be semi-I_s -continuous and $f^{-1}(V_*) \subseteq \left(f^{-1}(V)\right)_*$ for each $V \in \tau$.Then f is I_s -irresolute.

Proof: Let B be a semi-$\mathcal{J}$-open set of $(Y, \sigma, \mathcal{J})$.
By theorem 4.4, there exists $V \in \sigma$ such that $V \subseteq B \subseteq cl^{*s}(V)$.
So ,$f^{-1}(V) \subseteq f^{-1}(B) \subseteq f^{-1}(cl^{*s}(V)) \subseteq cl^{*s}\left(f^{-1}(V)\right)$.
Since f is semi-I_s -continuous and $V \in \sigma, f^{-1}(V) \in SISO(X, \tau)$ and hence by theorem 4.5 $f^{-1}(B)$ is semi-I_s -open in (X, τ, I).
So f isI_s -irresolute.

Definition 4.8: A function $f: (X, \tau) \to (Y, \sigma, \mathcal{J})$ is said to be semi-I_s -open (semi-I_s -closed)if for each $U \in \tau$ (U is closed) $f(U) \in SISO(Y, \sigma, \mathcal{J})(f(U)$ is semi-I_s -closed set).

Definition 4.9:[ ] A function $f: (X, \tau) \to (Y, \sigma, \mathcal{J})$ is said to be semi-I-open (semi-I -closed)if for
each $U \in \tau$ (U is closed) $f(U)$ is semi-I -open (semi-I -closed) set in $(Y, \sigma, \mathcal{J})$.

Remark 4.6:
I. Every semi-I_s -open function is semi open.
II. Every semi-I_s -open function is semi-I -open.
III. every open function is semi-I_s -open.

Let $X = \{a, b, c, d\}, \tau = \{\phi, X, \{a, b\}\}, \sigma = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $I = \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$.Considering this we follow the following examples:

Example 4.2: Define a function $f: (X, \tau) \to (Y, \sigma, \mathcal{J})$ as follows:
$$f(a) = b, f(b) = c, f(c) = f(d) = a$$
Then f is semi open but not semi-I_s -open.

This example shows that converse of remark 4.6(i) is not true in general.

Example 4.3: Define a function $f: (X, \tau) \to (Y, \sigma, \mathcal{J})$ as follows:

$$f(a) = a, f(b) = c, f(c) = f(d) = d$$

Then f is semi-I -open but not semi-I_s -open.

This example shows that converse of remark 4.6(ii) is not true in general.

Example 4.4: Let $X = \{a, b, c, d\}, \tau = \{\phi, X, \{c\}, \{a, b, c\}\}, \sigma = \{\ \phi, X, \{c\}, \{a, b\}, \{a, b, c\}\}$ and $I = \{\ \phi, \{a\}\}$.

The identity function $f: (\ X, \tau) \to (Y, \sigma, \mathcal{I})$ is semi-I_s -open but not open..

This example shows that converse of remark 4.6(iii) is not true in general.

Theorem 4.12: A function $f: (X, \tau) \to (Y, \sigma, \mathcal{I})$ is semi-I_s -open if and only if for each $x \in X$ and each neighbourhood U of x,there exists $V \in SISO(Y, \sigma)$ containing $f(x)$ such that $V \subseteq f(U)$.

Proof: Let f is semi-I_s -open function.

For each $x \in X$ and each neighbourhood $U0 \in \tau$ such that $x \in\ U0 \subseteq U$.

Since, f is semi-I_s -open, $V = f(U_0)\ \in SISO(Y, \sigma)$ and $f(x)\ \in\ V \subseteq f(U)$.

Conversely, let U be an open set of (X, τ).

For each $x \in U$, there exists $Vx \in SISO(Y, \sigma)$ such that $f(x)\ \in Vx \subseteq f(U)$.

So, $f(U) = \cup \{Vx : x \in U\}$ and $f(U)\ \in SISO(Y, \sigma)$.

Hence f is semi-I_s -open.

Remark 4.7: For any bijection function $f: (X, \tau) \to (Y, \sigma, \mathcal{I})$, the following are equivalent:

I. $f - 1: (\ Y, \sigma, \mathcal{I})\ \to (X, \tau)$ is semi-I_s-continuous.

II. f is semi-I_s -open.

III. f is semi-I_s -closed.

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