Linear Programming Graphical Solutions of LP Problems

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Subject: Optimisation & Data Analytics by
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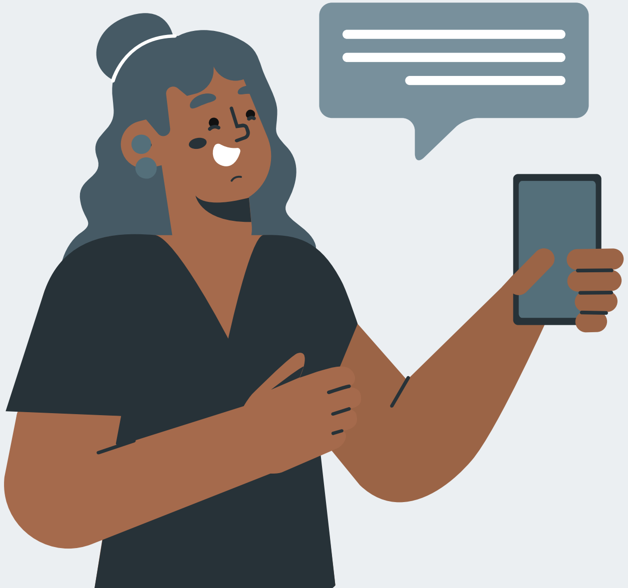

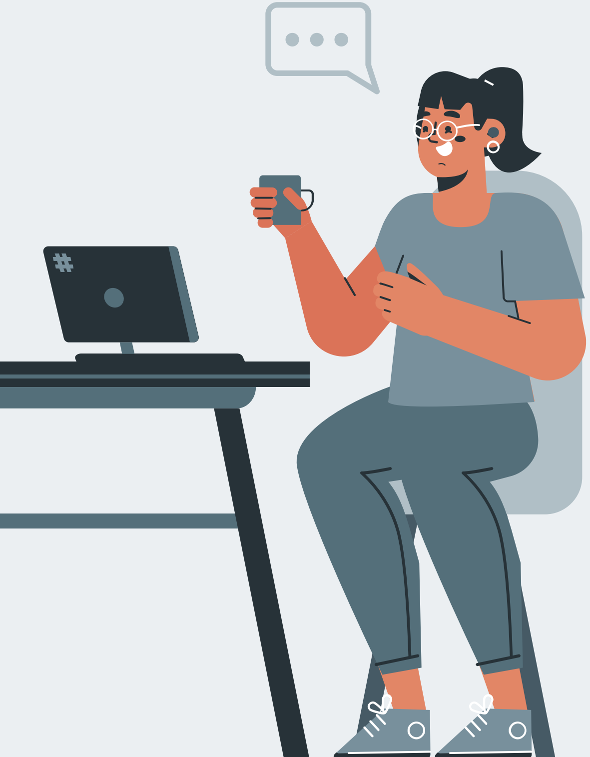

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Theoretical Foundations

Our Topic

- Implement a **method for the graphical solution** of LP problems with **two decision variables**
- **Input:** objective function, two variables and the constraints
- Generate a **graphical representation** of the **feasible region**
- Represent **objective function** as a line demonstrating the **optimal solution**
- **Output:** optimal solution

Introduction to Linear Programming

Definition: LP is a mathematical method to optimize a target variable (e.g., profit, cost)

Goal: Maximize or minimize an objective function subject to linear constraints

Components:

- Objective function: Mathematical expression to optimize
- Constraints: Linear equations or inequalities defining feasible solutions

Applications:

- Production planning
- Inventory management
- Resource allocation in logistics

Reference: (Matthäus and Matthäus, 2014, p.353)

Motivation

- Linear Programming is widely used in various industries to **optimize resources**
- Practical applications include:
  - Maximizing profits
  - Minimizing costs
  - Adhering to constraints like budget, production capacities, workforce availability

- Graphical solution provides visualization of feasible region
- Better understanding of **constraint impact**
- Useful for **small-scale** optimization problems

Manual Approach

1) Set Constraints and solve for intercepts

- For each constraint, set one variable to zero and solve for the other

2) Draw the graphs

- Plot graphs for constraints

3) Find area of feasible solutions

- **Maximization:** Identify feasible region by shading area beneath overlapping constraint lines
- **Minimization:** Shade area above constraint lines

4) Find optimal solution

- List corner points of constraints
- Substitute into objective function
- Plot graph for objective function
- Optimal point is at the vertex (corner point) of feasible region

Reference: (Matthäus and Matthäus, 2014, p.353)

Our Graphical Solution Method

Input Handling & Validation

- Takes input for objective function and constraints
- Validates input to ensure correct formatting and logic
- Handle errors

Model Creation (Using PuLP)

- Define decision variables and formulate objective function
- Solve LP problem
- Handle different solutions (Optimal, Infeasible, Unbounded)

Analyze solution

- Retrieve optimal values
- Identify corner points of feasible region

Graphical Visualization

- Plot constraints and feasible region
- Display optimal solution on graph
- Highlight multiple solutions (if applicable)

02

Code Implementation

Let’s move on to our notebook on:

python + colab

03

Real-World Application

Optimization Problem: Furniture Factory Production

Dataset: IndustryOR Nr.: 9
Optimum Profit: 9,800.00 USD

Factory Production Scenario:

A **furniture factory** needs to determine how many **tables**, **chairs**, and **bookshelves** to produce in order to **maximize profit**. The factory can sell **tables** for **200 Dollar** each, **chairs** for **50 Dollar** each, and **bookshelves** for **150 Dollar** each. The **manufacturing costs for each** table, chair, and bookshelf are **120 Dollar**, **20 Dollar**, and **90 Dollar**, respectively. Profit is the difference between the selling price and the manufacturing cost. Tables, chairs, and bookshelves each occupy **5, 2, and 3 square meters** of warehouse space, respectively. Due to limited warehouse space, the **total space cannot exceed 500 square meters**. Additionally, due to market demand, the factory needs to **produce at least 10 tables and 20 bookshelves**. Finally, the **total number of items produced** by the furniture factory **cannot exceed 200**.

Reference: (vgl. IndustryOR.json · CardinalOperations/IndustryOR at main o. D.)

Optimization Problem: Furniture Factory Production

Variables:

x_1: Number of tables produced
x_2: Number of chairs produced
x_3: Number of bookshelves produced

Objective function:

$Z = (200 - 120)x_1 + (50 - 20)x_2 + (150 - 90)x_3$
$\mathbf{Z = 80x_1 + 30x_2 + 60x_3}$

Constraints:

1. Space Constraint: $\mathbf{5x_1 + 2x_2 + 3x_3 \leq 500}$

2. Minimum Production (Market Demand): $\mathbf{x_1 \geq 10, x_3 \geq 20}$

3. Maximum Total Production: $\mathbf{x_1 + x_2 + x_3 \leq 200}$

4. Non-Negativity: $\mathbf{x_1, x_2, x_3 \geq 0}$

Optimization Problem: Furniture Factory Production

Variables:

x_1: Number of tables produced
~~x_2: Number of chairs produced~~
x_3: Number of bookshelves produced

Objective function:

$Z = (200 - 120)x_1 + (150 - 90)x_3$
$\mathbf{Z = 80x_1 + 60x_3}$

Constraints:

1. Space Constraint (with $x_2 = 0$): $\mathbf{5x_1 + 3x_3 \leq 500}$

2. Minimum Production (Market Demand): $\mathbf{x_1 \geq 10, x_3 \geq 20}$

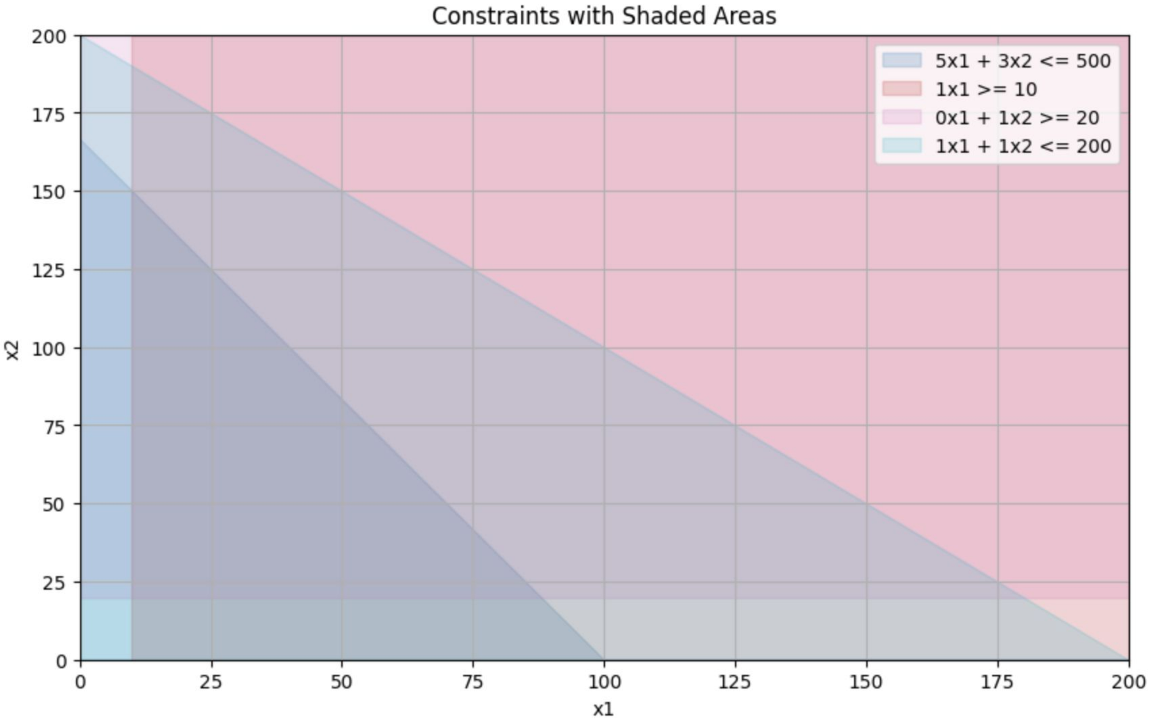
3. Maximum Total Production (with $x_2 = 0$): $\mathbf{x_1 + x_3 \leq 200}$

4. Non-Negativity: $\mathbf{x_1, x_3 \geq 0}$

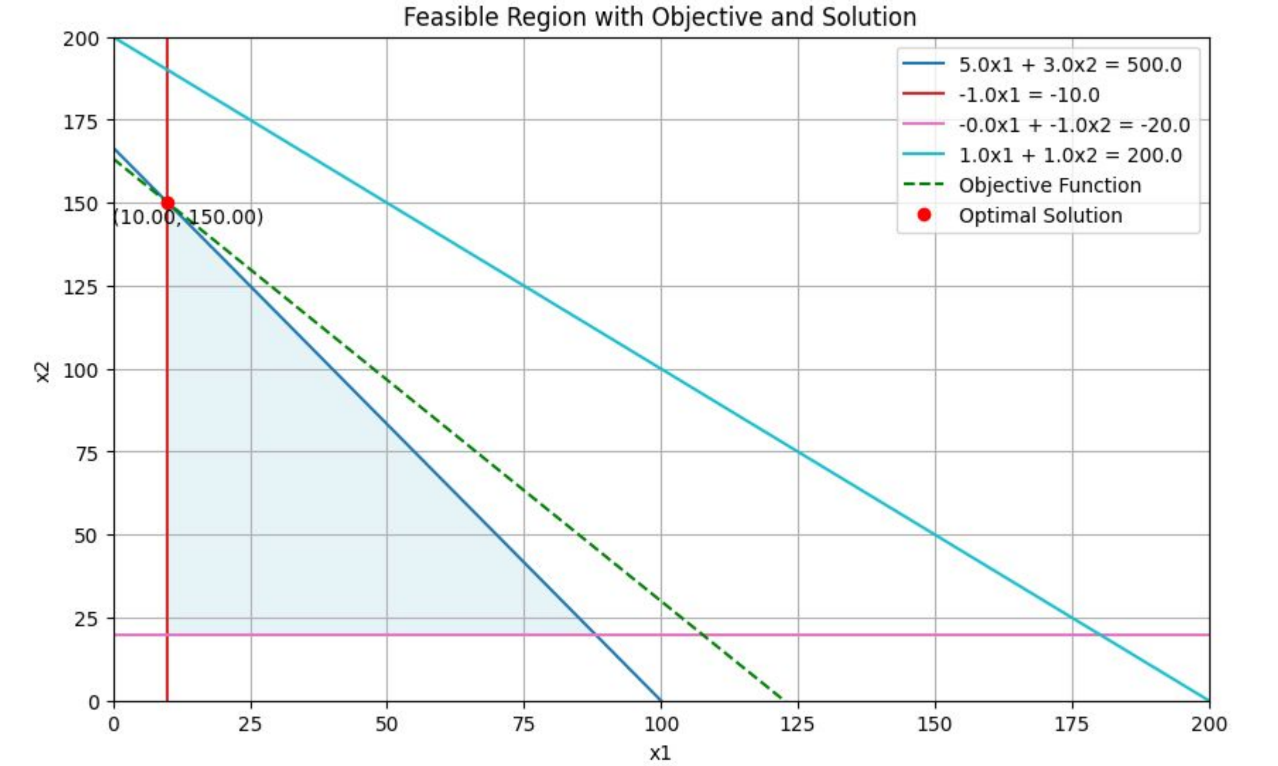
→ **In our case we only need two variables!**

Optimization Problem: Furniture Factory Production

```
Reduced Problem:
No multiple optimal solutions detected.
```



Optimization Problem: Furniture Factory Production




```
{'Optimal Point': (10.0, 150.0), 'Optimal Value': 9800.0, 'Solver Status': 'Optimal'}
```

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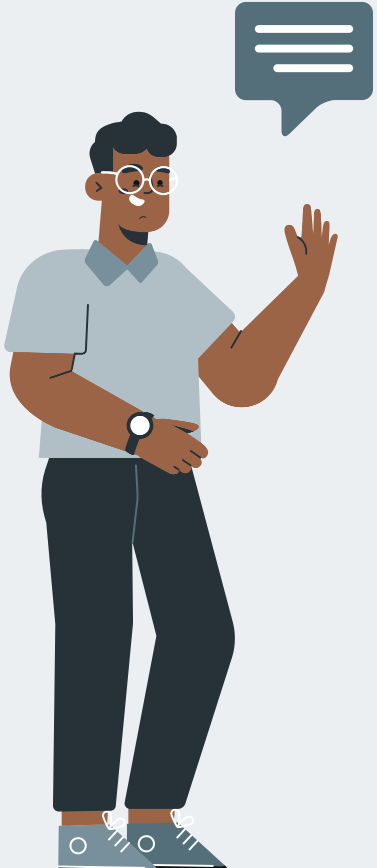
Conclusion

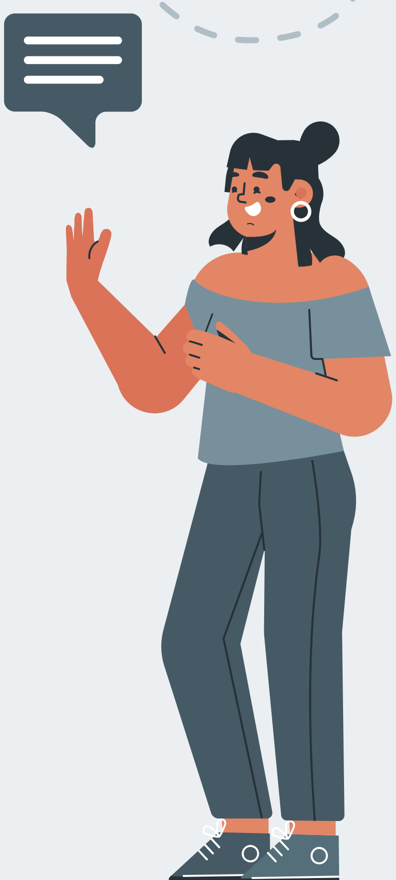
Key Takeaways

- Testing **input** to conform to **LP requirements** to **prevent errors**
- Converting **inequalities** to a **standardized form** for **consistency**
- Utilizing PuLP solver to detect **infeasibility** or **unboundedness**
- Identifying situations where multiple solutions exist
- Visual representation of **constraint, objective function** and **feasible region**



Thanks!

Does anyone have any questions?



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References

- Ye, Y. (2023) Feasible Regions, Feasible and Improving Directions, and Optimality Test for LP. MS&E211 Lecture Notes, Stanford University, [online] Available at: https://web.stanford.edu/class/msande211x/Lecture042023.pdf (Accessed: 20 January 2025).
- Matthäus, H. and Matthäus, W.-G. (2014) Mathematik für BWL-Bachelor: Schritt für Schritt mit ausführlichen Lösungen. 4th edn. Wiesbaden: Springer Gabler.
- InustryOR.json · CardinalOperations/IndustryOR at main (o. D.): huggingface.co, [online] Available at: https://huggingface.co/datasets/CardinalOperations/IndustryOR/blob/main/IndustryOR.json (Accessed: 19 January 2025).