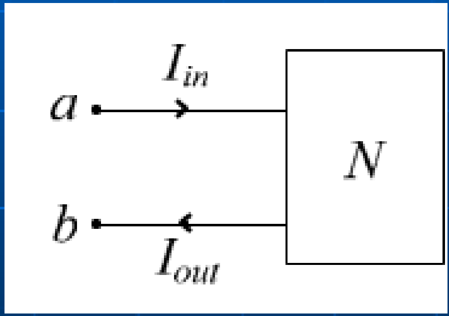
TWO-PORT CIRCUITS

CONTENTS

Definition of Two-Port Circuits

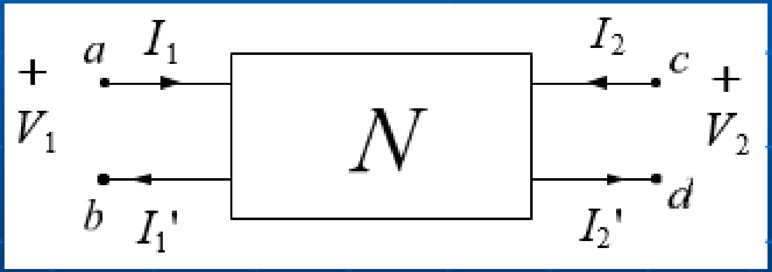
Consider a linear two-terminal circuit N consisting of no independent sources as follows :



For a, b two terminals, if $I_{in} = I_{out}$, then it constitutes a port.

Definition of Two-Port Circuits

Now consider the following linear four-terminal circuit containing no independent sources.

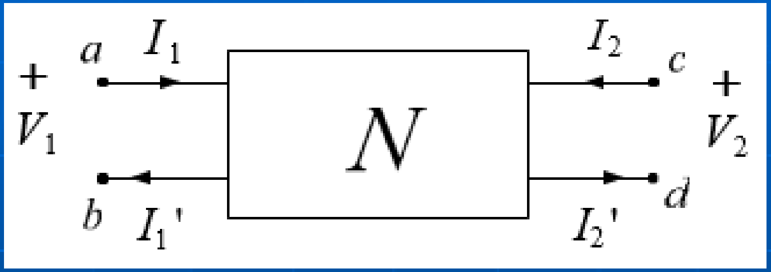


with $I_1 = I_1'$

$I_2 = I_2'$

Then terminals a , b constitute the input port and terminals c , d constitute the output port.

Definition of Two-Port Circuits



No external connections exist between the input and output ports.

The two-port model is used to describe the performance of a circuit in terms of the voltage and current at its input and output ports.

Purpose of the two-port model

- A **two-port network** (**four-terminals network**) is an electrical network (circuit) or device with two *pairs* of terminals to connect to external circuits.

- Two terminals constitute a port if the currents applied to them satisfy the port condition:

❑ the electric current entering one terminal must equal to the current emerging from the other terminal on the same port.

- The ports constitute interfaces where:

❑ the network connects to other networks,

❑ the points where signals are applied or outputs are taken.

- In a two-port network, often port 1 is considered the input port and port 2 is considered the output port.

- The two-port network model is used in mathematical circuit analysis techniques to isolate portions of larger circuits.

- A two-port network is regarded as a **"black box"** with its properties specified by a matrix of numbers.

- Advantages; Response calculation, Comparing circuits

❑ The response of the network to signals applied to the ports can be calculated easily, without solving for all the internal voltages and currents in the network.

❑ It also allows similar circuits or devices to be compared easily.

- Any linear circuit with four terminals can be regarded as a two-port network provided that it does not contain an independent source and satisfies the port conditions.

- Examples of circuits analyzed as two-ports are filters, matching networks, transmission lines, transformers, and small-signal models for transistors (such as the hybrid-pi model).

- In two-port mathematical models, the network is described by a 2 by 2 square matrix of complex numbers.

- The common models that are used are referred to as *z-parameters*, *y-parameters*, *h-parameters*, *g-parameters*, and *ABCD-parameters*.

- These are usually expressed in matrix notation, and they establish relations between the variables

  - ❑ V1= voltage across port 1
  - ❑ I1= current into port 1
  - ❑ V2 = voltage across port 2
  - ❑ I2=current into port 2

Definition of Two-Port Circuits

Two-port circuits are useful in communications, control systems, power systems, and electronic systems.

They are also useful for facilitating cascaded design of more complex systems.

Classification of Two-Port Parameters

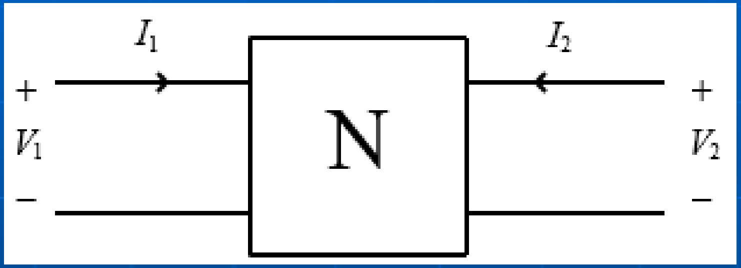
There are four terminal variables , namely V_1 ,V_2 , I_1 , I_2 , only two of them are independent.

Hence , there are only six possible sets of two-port parameters.

$$ {}_4C_2 = \frac{4\times 3}{2\times 1} = 6 $$

Classification of Two-Port Parameters

(1) The impedance , or Z , parameters   Z Parameters

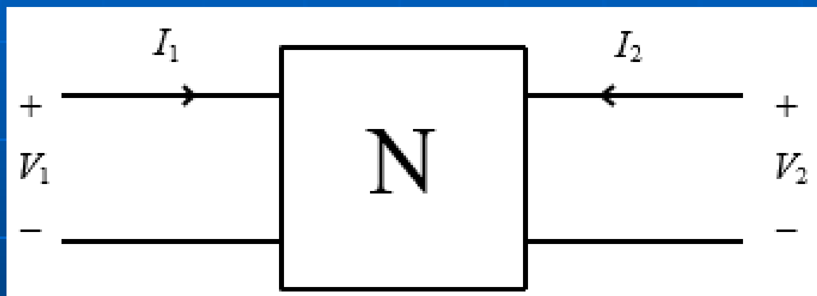


$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}, Z_{ij} \; in \; \Omega$$

For two-port networks , four parameters are generally required to represent the circuit.

Classification of Two-Port Parameters

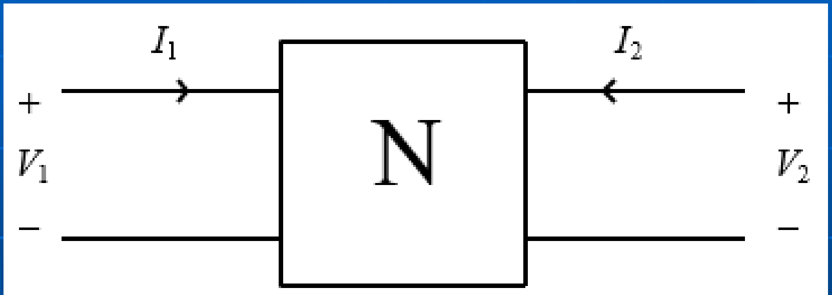
(2) The admittance , or Y , parameters   Y Parameters



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, \; Y_{ij} \; in \; S$$

Classification of Two-Port Parameters

(3) The hybrid , or h , parameters H Parameters



$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix},$$

h_{11} : *in* Ω

h_{22} : *in* S

h_{12} & h_{22} *scalars*

Finding Two-Port Parameters

Method 1: Calculate or measure by invoking appropriate short-circuit and open-circuit conditions at the input and output ports.

Method 2: Derive the parameters from another set of two-port parameters.

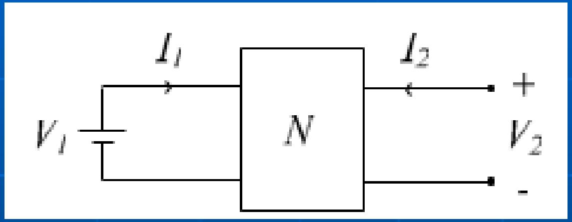
Open-circuit the Port 1
Short-cct Output

Finding Two-Port Parameters

Method 1 : Choose Z parameters as an illustration.

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = z_{21} I_1 + z_{22} I_2$$



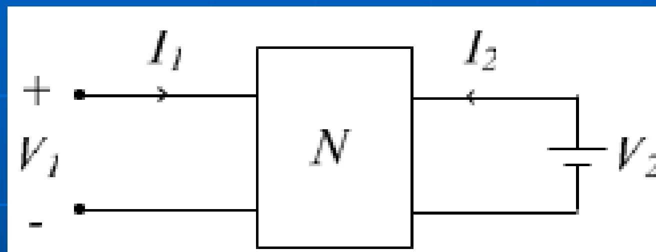
when $I_2 = 0$, output port is open

$$V_1 = z_{11} I_1 , \ \therefore z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$ *, input impedance*

$$V_2 = z_{21} I_1 , \ \therefore z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$ *, transfer impedance*

Finding Two-Port Parameters

When $I_1 = 0$, input port is open



$$V_1 = z_{12} I_2 \therefore z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} , \text{ transfer impedance}$$

$$V_2 = z_{22} I_2 \therefore z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} , \text{ output impedance}$$

When the two-port does not contain any dependent source, then $z_{12}=z_{21}$.

Finding Two-Port Parameters

z_{11} & z_{22} are called driving-point impedances.

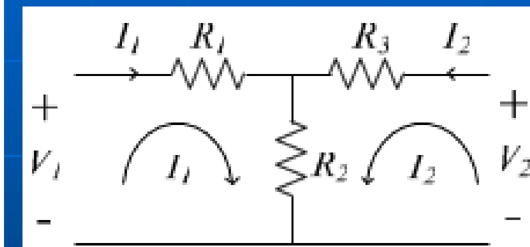
z_{12} & z_{21} are called transfer impedances.

When $z_{11}=z_{22}$, the two-port circuit is said to be symmetrical.

When $z_{12}=z_{21}$, the two-port circuit is called a reciprocal circuit.

Finding Two-Port Parameters

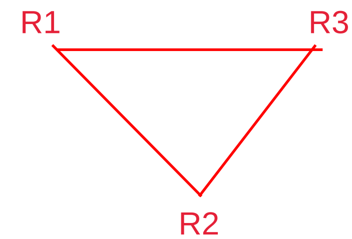
Example 1 : Find the Z parameters of the T-network



Assign mesh currents as shown :

$$\begin{bmatrix} R_1+R_2 & R_2 \\ R_2 & R_2+R_3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}=\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

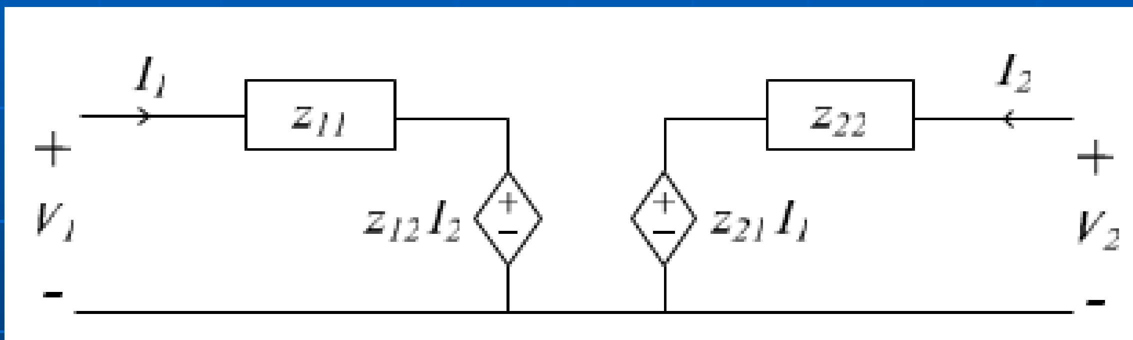
$$\therefore \boxed{\begin{aligned} z_{11} &= R_1+R_2 \\ z_{12} &= z_{21} = R_2 \\ z_{22} &= R_2+R_3 \end{aligned}}$$



Finding Two-Port Parameters

A two-port can be replaced by the following equivalent circuit.

Equivalent Circuit

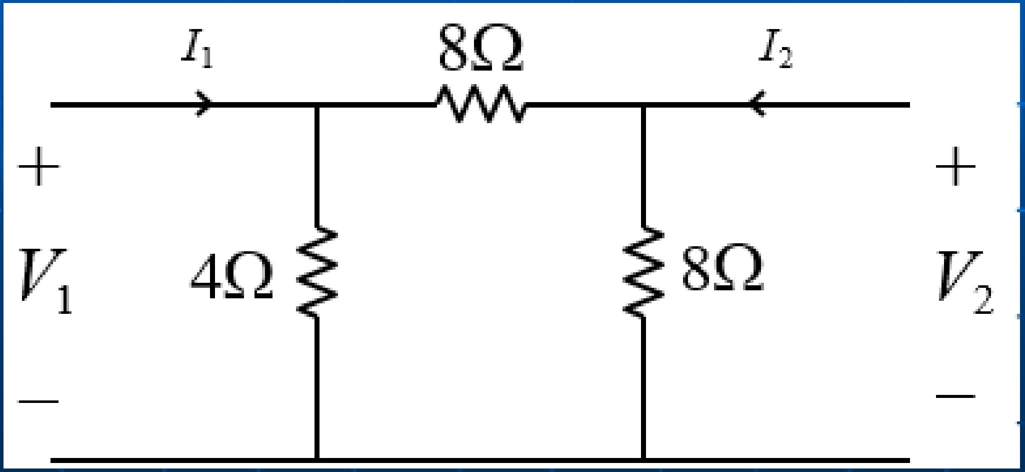


$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

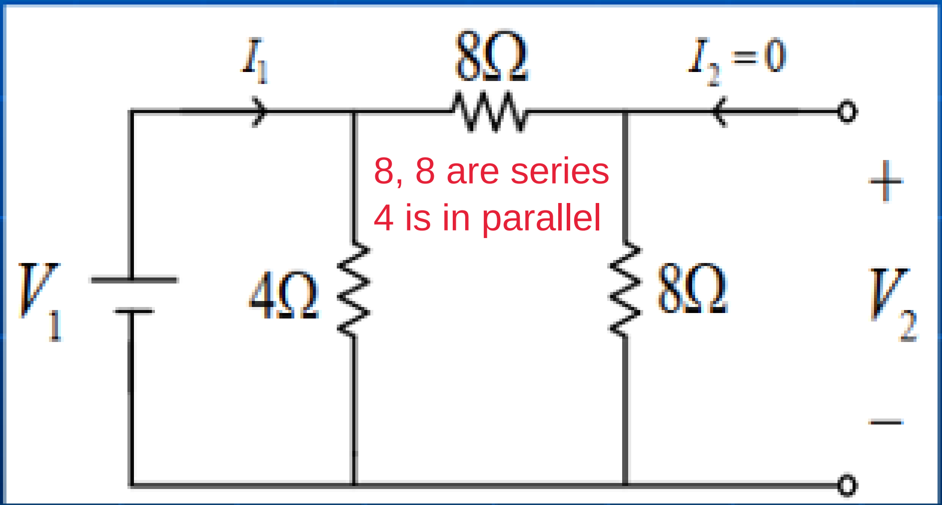
Finding Two-Port Parameters

Example 2: Find the Z parameters of the following circuit by definition of Z parameters.



Finding Two-Port Parameters

Step1: Let $I_2 = 0$ and apply V_1



$$I_1 = V_1 / (4\Omega // (8+8)\Omega) = \frac{20}{64} V_1$$

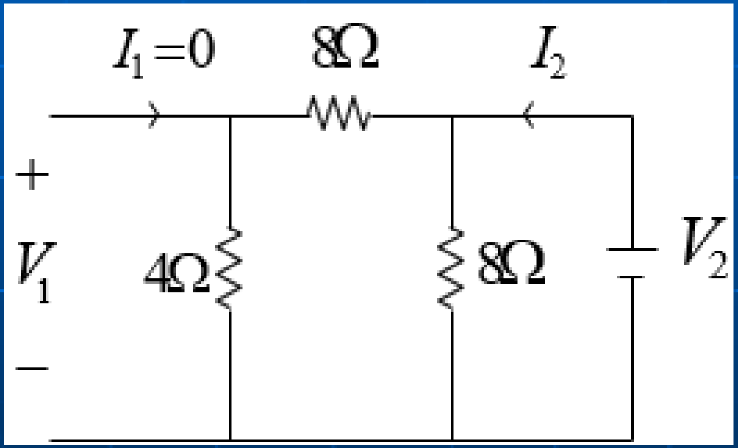
$$V_2 = \frac{8}{8+8} V_1 = \frac{1}{2} V_1$$

$$\therefore z_{11} = \frac{V_1}{I_1} = \frac{64}{20} = \frac{16}{5}\Omega$$

$$z_{21} = \frac{V_2}{I_1} = \frac{\frac{1}{2}V_1}{I_1} = \frac{1}{2} z_{11} = \frac{8}{5}\Omega$$

Finding Two-Port Parameters

Step2: Let $I_1 = 0$ and apply V_2



$$I_2 = V_2 / (8\Omega // (8+4)\Omega) = \frac{5}{24} V_2$$

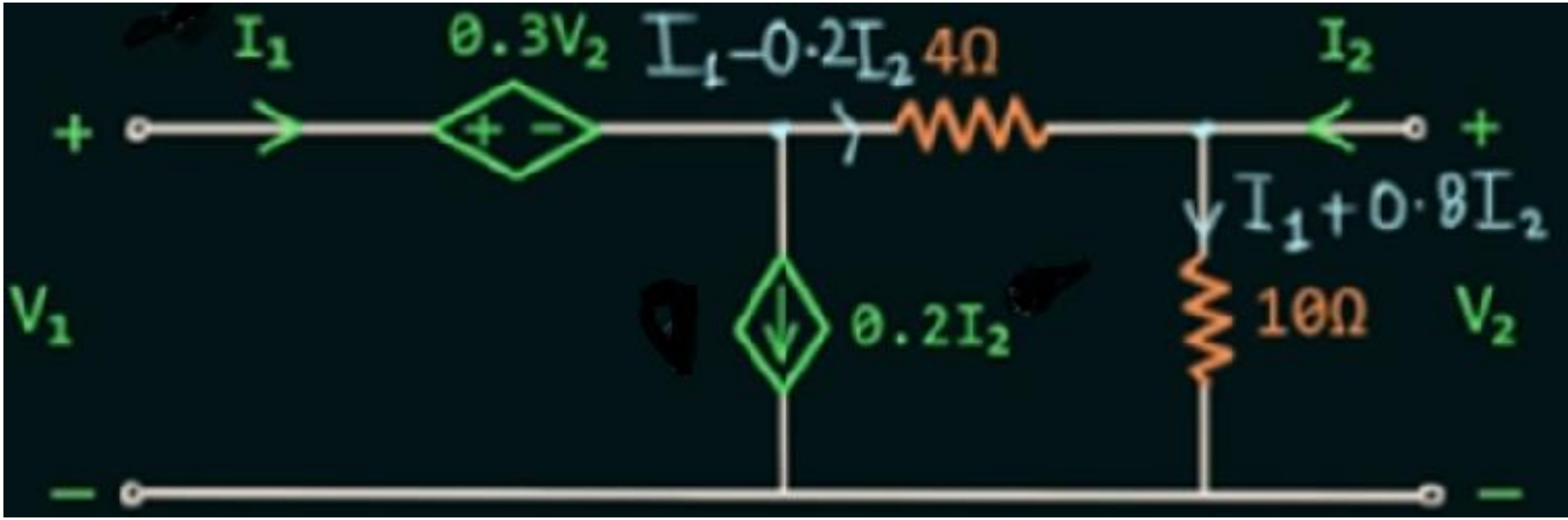
$$V_1 = \frac{4}{4+8} V_2 = \frac{1}{3} V_2$$

$$\therefore z_{22} = \frac{V_2}{I_2} = \frac{24}{5}\Omega$$

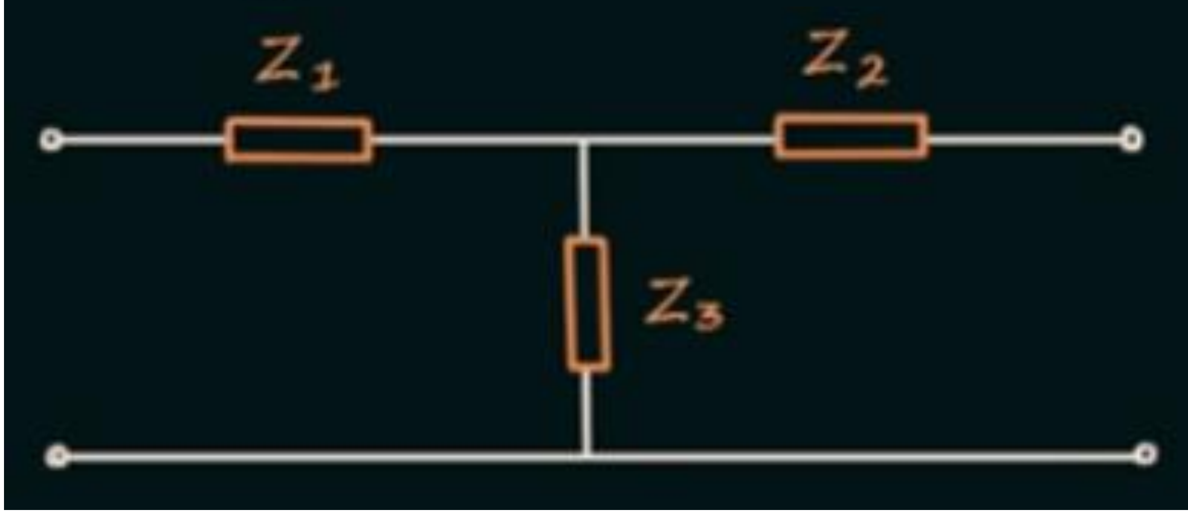
$$z_{12} = \frac{V_1}{I_2} = \frac{\frac{1}{3}V_2}{I_2} = \frac{1}{3} z_{22} = \frac{8}{5}\Omega$$

$$\therefore [Z] = \begin{bmatrix} 16/5 & 8/5 \\ 8/5 & 24/5 \end{bmatrix}\Omega$$

Detemine the Z parameter matrix of the port networks below



Homework: If the Z-parameters for the T-network as shown below are $Z_{11} = 40\Omega$, $Z_{22} = 50\Omega$ and $Z_{12} = Z_{21} = 30\Omega$, then what are the values of Z_1, Z_2 and Z_3?



Z11 = Z1 + Z3
Z12 = Z21 = Z3
Z22 = Z3 + Z2

Z3 = Z12 = Z21 = 30 ohms
Z2 = Z22 - Z3 = 50 - 30 = 20 ohms
Z1 = Z11 - Z3 = 40 - 30 = 10 ohms

a) 10 ohm, 20 and 30
b) 20 ohm, 30 and 20
c) 30 ohm, 40 and 10
d) 40 ohm, 50 and 10