

Instrument types and performance characteristics

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2.1 Introduction

Two important aspects of measurement that were covered in the opening chapter concerned how to choose appropriate instruments for a particular application and a review of the main applications of measurement. Both of these activities require knowledge of the characteristics of different classes of instruments and, in particular, how these different classes of instrument perform in different applications and operating environments. Therefore, we start this chapter by reviewing the various classes of instruments that exist. All instruments can be divided into active and passive ones according to whether they have an energy source contained within them. The next distinction is between null-type instruments that require adjustment until a datum level is reached and deflection-type instruments that give an output measurement in the form of either a deflection of a pointer against a scale or a numerical display. The third distinction covered is between analog and digital instruments, which differ according to whether the output varies continuously

(analog instrument) or in discrete steps (digital instrument). Fourth, we shall look at the distinction between instruments that are merely indicators and those that have a signal output. Indicators give some visual or audio indication of the magnitude of the measured quantity and are commonly found in the process industries. Instruments with a signal output are commonly found as part of automatic control systems. The final distinction we will consider is between smart and nonsmart instruments. Smart, often known as intelligent, instruments are very important today and predominate in most measurement applications. Because of their importance, they are given more detailed consideration later in Chapter 11.

The second part of this chapter looks at the various attributes of instruments that determine their performance and suitability for different measurement requirements and applications. We will first look at the static characteristics of instruments. These are their steady-state attributes (when the output measurement value has settled to a constant reading after any initial varying output) such as accuracy, measurement sensitivity, and resistance to errors caused by variations in their operating environment. We will then examine the dynamic characteristics of instruments. These describe their behavior following the time that the measured quantity changes value up until the time when the output reading attains a steady value. Various kinds of dynamic behavior can be observed in different instruments ranging from an output that varies slowly until it reaches a final constant value to an output that oscillates about the final value until a steady reading is obtained. The dynamic characteristics are an important factor in deciding on the suitability of an instrument for a particular measurement application. Finally, at the end of the chapter, we will briefly consider the issue of instrument calibration, although this is considered in much greater detail later in Chapter 5.

2.2 Review of instrument types

Instruments can be subdivided into separate classes according to several criteria. These subclassifications are useful for broadly establishing several attributes of particular instruments such as accuracy, cost, and general relevance for different applications.

2.2.1 Active and passive instruments

Instruments are divided into active or passive ones according to whether the instrument output is entirely produced by the quantity being measured or whether the quantity being measured simply modulates the magnitude of some external power source. This is illustrated by examples.

An example of a passive instrument is the pressure-measuring device shown in [Fig. 2.1](#). The pressure of the fluid is translated into a movement of a pointer against a scale.

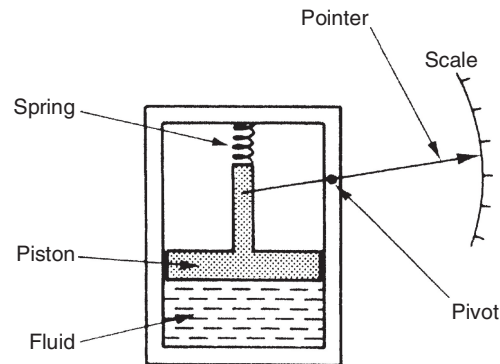


Figure 2.1
Passive pressure gauge.

The energy expended in moving the pointer is derived entirely from the change in pressure measured: there are no other energy inputs to the system.

An example of an active instrument is a float-type fuel tank level indicator as shown in [Fig. 2.2](#). Here, the change in fuel level moves a potentiometer arm, and the output signal consists of a proportion of the external voltage source applied across the two ends of the potentiometer. The energy in the output signal comes from the external power source: the primary transducer float system is merely modulating the value of the voltage from this external power source.

In active instruments, the external power source is usually in electrical form, but in some cases, it can be other forms of energy such as a pneumatic or hydraulic one.

One important difference between active and passive instruments is the level of measurement resolution that can be obtained. With the simple pressure gauge shown, the amount of movement made by the pointer for a particular pressure change is closely

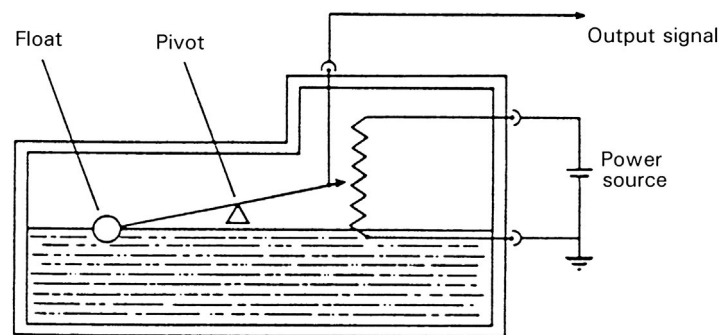


Figure 2.2
Fuel tank level indicator.

defined by the nature of the instrument. Although it is possible to increase measurement resolution by making the pointer longer, such that the pointer tip moves through a longer arc, the scope for such improvement is clearly restricted by the practical limit of how long the pointer can conveniently be. In an active instrument, however, adjustment of the magnitude of the external energy input allows much greater control over measurement resolution. Although the scope for improving measurement resolution is much greater incidentally, it is not infinite because of limitations placed on the magnitude of the external energy input, in consideration of heating effects and for safety reasons.

In terms of cost, passive instruments are normally of a simpler construction than active ones and are therefore cheaper to manufacture. Therefore, the choice between active and passive instruments for a particular application involves carefully balancing the measurement resolution requirements against cost.

2.2.2 Null-type and deflection-type instruments

The pressure gauge just mentioned is a good example of a deflection type of instrument, in which the value of the quantity being measured is displayed in terms of the amount of movement of a pointer. An alternative type of pressure gauge is the dead-weight gauge shown in Fig. 2.3, which is a null-type instrument. Here, weights are put on top of the piston until the downward force balances the fluid pressure. Weights are added until the piston reaches a datum level, known as the null point. Pressure measurement is made in terms of the value of the weights needed to reach this null position.

The accuracy of these two instruments depends on different things. The accuracy of the first type depends on the linearity and calibration of the spring, while the accuracy of the second type relies on the calibration of the weights. Because calibration of weights is much easier than careful choice and calibration of a linear-characteristic spring, the second type of instrument will normally be more accurate. This is in accordance with the general rule that null-type instruments are more accurate than deflection types.

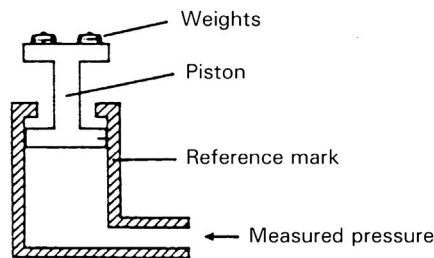


Figure 2.3
Dead-weight pressure gauge.

In terms of use, the deflection-type instrument is clearly more convenient. It is far simpler to read the position of a pointer against a scale than to add and subtract weights until a null point is reached. A deflection-type instrument is therefore the one that would normally be used in the workplace. However, for calibration duties, the null-type instrument is preferable because of its superior accuracy. The extra effort required to use such an instrument is perfectly acceptable in this case because of the infrequent nature of calibration operations.

2.2.3 Analog and digital instruments

An analog instrument gives an output that varies continuously as the quantity being measured changes. The output can have an infinite number of values within the range that the instrument is designed to measure. The deflection-type of pressure gauge described earlier in this chapter (Fig. 2.1) is an example of an analog instrument. As the input value changes, the pointer moves with a smooth continuous motion. Although the pointer can therefore be in an infinite number of positions within its range of movement, the number of different positions that the eye can discriminate between is strictly limited. This discrimination depends on how large the scale is and how finely it is divided.

A digital instrument has an output that varies in discrete steps and so can have only a finite number of values. The rev-counter in Fig. 2.4 is an example of a digital instrument. A cam is attached to the revolving body whose motion is being measured, and on each revolution the cam opens and closes a switch. The switching operations are counted by an electronic counter. This system can count only whole revolutions and cannot discriminate any motion that is less than a full revolution.

The distinction between analog and digital instruments has become particularly important with the rapid growth in the application of microcomputers to automatic control systems. Any digital computer system, of which the microcomputer is but one example, performs

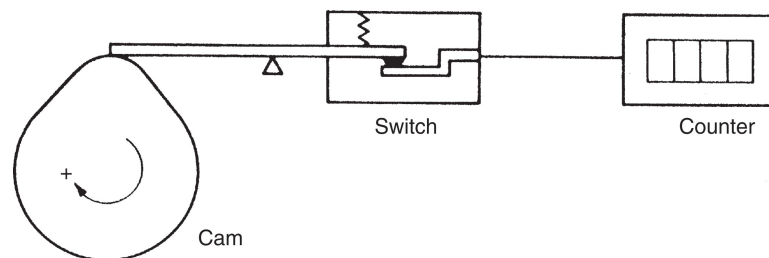


Figure 2.4
Rev-counter.

its computations in digital form. An instrument whose output is in digital form is therefore particularly advantageous in such applications, because it can be interfaced directly to the control computer. Analog instruments must be interfaced to the microcomputer by an analog-to-digital (A/D) converter, which converts the analog output signal from the instrument into an equivalent digital quantity that can be read into the computer. This conversion has several disadvantages. First, the A/D converter adds a significant cost to the system. Second, a finite time is involved in the process of converting an analog signal to a digital quantity, and this time can be critical in the control of fast processes in which the accuracy of control depends on the speed of the controlling computer. Degrading the speed of operation of the control computer by imposing a requirement for A/D conversion thus impairs the accuracy by which the process is controlled.

2.2.4 Indicating instruments and instruments with a signal output

The final way in which instruments can be divided is between those that merely give an audio or visual indication of the magnitude of the physical quantity measured and those that give an output in the form of a measurement signal whose magnitude is proportional to the measured quantity.

The class of indicating instruments normally includes all null-type instruments and most passive ones. Indicators can also be further divided into those that have an analog output and those that have a digital display. A common analogue indicator is the liquid-in-glass thermometer. Another common indicating device, which exists in both analogue and digital forms, is the bathroom scale. The older mechanical form of this is an analogue type of instrument that gives an output consisting of a rotating pointer moving against a scale (or sometimes a rotating scale moving against a pointer). More recent electronic forms of bathroom scale have a digital output consisting of numbers presented on an electronic display. One major drawback with indicating devices is that human intervention is required to read and record a measurement. This process is particularly prone to error in the case of analog output displays, although digital displays are not prone to error unless the human reader is careless.

Instruments that have a signal-type output are commonly used as part of automatic control systems. In other circumstances, they can also be found in measurement systems in which the output measurement signal is recorded in some way for later use. This subject is covered in later chapters. Usually, the measurement signal involved is an electrical voltage, but it can take other forms in some systems such as an electrical current, an optical signal, or a pneumatic signal.

2.2.5 Smart and nonsmart instruments

The advent of the microprocessor has created a new division in instruments between those that incorporate a microprocessor (smart) and those that do not. Smart devices are considered in detail in Chapter 11.

2.3 Static characteristics of instruments

If we have a thermometer in a room and its reading shows a temperature of 20°C, it does not really matter whether the true temperature of the room is 19.5°C or 20.5°C. Such small variations around 20°C are too small to affect whether we feel warm enough or not. Our bodies cannot discriminate between such close levels of temperature, and therefore a thermometer with an inaccuracy of $\pm 0.5^\circ\text{C}$ is perfectly adequate. If we had to measure the temperature of certain chemical processes, however, a variation of 0.5°C might have a significant effect on the rate of reaction or even the products of a process. A measurement inaccuracy much less than $\pm 0.5^\circ\text{C}$ is therefore clearly required.

Accuracy of measurement is thus one consideration in the choice of instrument for a particular application. Other parameters such as sensitivity, linearity, and the reaction to ambient temperature changes are further considerations. These attributes are collectively known as the static characteristics of instruments and are given in the data sheet for a particular instrument. Values quoted for instrument characteristics in such a data sheet apply only when the instrument is used under specified standard calibration conditions. Due allowance must be made for variations in the characteristics when the instrument is used under other conditions.

The various static characteristics are defined in the following paragraphs.

Accuracy and inaccuracy (measurement uncertainty): The *accuracy* of an instrument is a measure of how close the output reading of the instrument is to the correct value. In practice, it is more usual to quote the *inaccuracy* or *measurement uncertainty* value rather than the accuracy value for an instrument. Inaccuracy or measurement uncertainty is the extent to which a reading might be wrong and is often quoted as a percentage of the full-scale reading of an instrument.

■ Example 2.1

A pressure gauge with a measurement range of 0–10 bar has a quoted inaccuracy of $\pm 1.0\%$ of the full-scale reading.

- (a) What is the maximum measurement error expected for this instrument?
- (b) What is the likely measurement error expressed as a percentage of the output reading if this pressure gauge is measuring a pressure of 1 bar?



■ Solution

(a) The maximum error expected in any measurement reading is 1.0% of the full scale reading, which is 10 bar for this particular instrument. Hence, the maximum likely error is $1.0\% \times 10 \text{ bar} = 0.1 \text{ bar}$.

(b) The maximum measurement error is a constant value related to the full-scale reading of the instrument, irrespective of the magnitude of the quantity that the instrument is actually measuring. In this case, as worked out earlier, the magnitude of the error is 0.1 bar. Thus, when measuring a pressure of 1 bar, the maximum possible error of 0.1 bar is 10% of the measurement value.

■

This example carries a very important message. Because the maximum measurement error in an instrument is usually related to the full-scale reading of the instrument, measuring quantities that are substantially less than the full-scale reading means that the possible measurement error is amplified. For this reason, it is an important system design rule that instruments are chosen such that their range is appropriate to the spread of values being measured, so that the best possible accuracy is maintained in instrument readings. Clearly, if we are measuring pressures with expected values between 0 and 1 bar, we would not use an instrument with a measurement range of 0–10 bar.

Precision/repeatability/reproducibility: *Precision* is a term that describes an instrument's degree of freedom from random errors. If a large number of readings are taken of the same quantity by a high-precision instrument, the spread of readings will be very small. Precision is often, although incorrectly, confused with accuracy. High precision does not imply anything about measurement accuracy. A high-precision instrument may have a low accuracy. Low-accuracy measurements from a high-precision instrument are normally caused by a bias in the measurements, which is removable by recalibration.

The terms “repeatability” and “reproducibility” mean approximately the same but are applied in different contexts as discussed later. *Repeatability* describes the closeness of output readings when the same input is applied repetitively over a short time, with the same measurement conditions, same instrument and observer, same location, and same conditions of use maintained throughout. *Reproducibility* describes the closeness of output readings for the same input when there are changes in the method of measurement, observer, measuring instrument, location, conditions of use, and time of measurement. Both terms thus describe the spread of output readings for the same input. This spread is referred to as repeatability if the measurement conditions are constant and as reproducibility if the measurement conditions vary.

The degree of repeatability or reproducibility in measurements from an instrument is an alternative way to express its precision. Fig. 2.5 illustrates this more clearly. The figure shows the results of tests on three industrial robots that were programmed to place components at a particular point on a table. The target point was at the center of the concentric circles shown, and the black dots represent the points where each robot actually deposited components at each attempt. Both the accuracy and precision of Robot 1 is shown to be low in this trial. Robot 2 consistently puts the component down at approximately the same place, but this is the wrong point. Therefore, it has high precision but low accuracy. Finally, Robot 3 has both high precision and high accuracy because it consistently places the component at the correct target position.

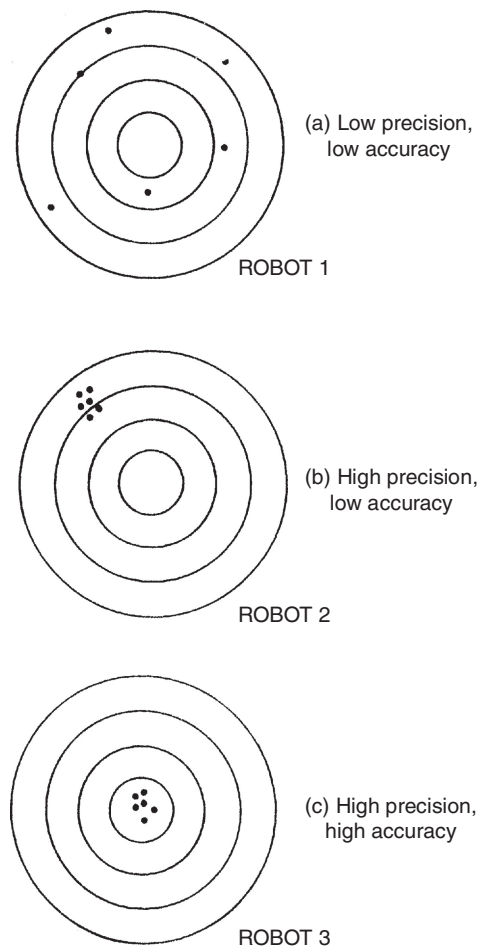


Figure 2.5

Comparison of accuracy and precision.

■ Example 2.2

The width of a room is measured 10 times by an ultrasonic rule and the following measurements are obtained (in meters): 5.381, 5.379, 5.378, 5.382, 5.380, 5.383, 5.379, 5.377, 5.380, and 5.381.

The width of the same room is then measured by a calibrated steel tape that gives a reading of 5.374 m, which can be taken as the correct value for the width of the room.

- (a) What is the measurement precision of the ultrasonic rule?
 - (b) What is the maximum measurement inaccuracy of the ultrasonic rule?
-

■ Solution

- (a) The mean (average) value of the 10 measurements made with the ultrasonic rule is 5.380 m.

The maximum deviation below this mean value is -0.003 m and the maximum deviation above the mean value is $+0.003$ m. Thus, the precision of the ultrasonic rule can be expressed as ± 0.003 m (± 3 mm).

- (b) The correct value of the room width has been measured as 5.374 m by the calibrated steel rule. All ultrasonic rule measurements are above this, with the largest value being 5.383 m. This last measurement is the one that exhibits the largest measurement error. This maximum measurement error can be calculated as $5.383 - 5.374 = 0.009$ m (9 mm). Thus, the maximum measurement inaccuracy can be expressed as $+9$ mm.
-

This example illustrates nicely that although the ultrasonic rule has fairly high precision, its actual measurement inaccuracy is substantially inferior.

Tolerance: Tolerance is a term closely related to accuracy and defines the maximum error that is to be expected in some value. Although, strictly speaking, it is not a static characteristic of measuring instruments, it is mentioned here because the accuracy of some instruments is sometimes quoted as a tolerance value. When used correctly, tolerance describes the maximum deviation of a manufactured component from some specified value. For instance, crankshafts are machined with a diameter tolerance quoted as so many microns (10^{-6} m), and electric circuit components such as resistors have tolerances of perhaps 5%.

■ Example 2.3

A packet of resistors bought in an electronics component shop gives the nominal resistance value as $1000\ \Omega$ and the manufacturing tolerance as $\pm 5\%$. If one resistor is chosen at random from the packet, what is the minimum and maximum resistance value that this particular resistor is likely to have?

■ Solution

The minimum likely value is $1000\ \Omega - 5\% = 950\ \Omega$.

The maximum likely value is $1000\ \Omega + 5\% = 1050\ \Omega$.

Range or span: The range or span of an instrument defines the minimum and maximum values of a quantity that the instrument is designed to measure.

■ Example 2.4

A particular micrometer is designed to measure dimensions between 50 and 75 mm. What is its measurement range?

■ Solution

The measurement range is simply the difference between the maximum and minimum measurements. Thus, in this case the range is $75 - 50 = 25\ \text{mm}$.

Threshold: If the input to an instrument is gradually increased from zero, the input will have to reach a certain minimum level before the change in the instrument output reading is of a large enough magnitude to be detectable. This minimum level of input is known as the threshold of the instrument. Manufacturers vary in the way that they specify threshold for instruments. Some quote absolute values, whereas others quote threshold as a percentage of full-scale readings. As an illustration, a car speedometer typically has a threshold of about 15 miles/h. This means that if the vehicle starts from rest and accelerates, no output reading is observed on the speedometer until the speed reaches 15 miles/h.

Resolution: When an instrument is showing a particular output reading, there is a lower limit on the magnitude of the change in the input measured quantity that produces an observable change in the instrument output. Like threshold, resolution is sometimes specified as an absolute value and sometimes as a percentage of the full-scale deflection. A major factor influencing the resolution of an instrument is how finely its output scale is divided into subdivisions. Using a car speedometer as an example again, this has subdivisions of typically 10 miles/h. This means that when the needle is between the scale markings, we cannot estimate speed more accurately than to the nearest 5 miles/h. This value of 5 miles/h thus represents the resolution of the instrument.

Linearity: It is normally desirable for the output reading of an instrument to be linearly proportional to the quantity being measured. The X's marked on Fig. 2.6 show a plot of the typical output readings of an instrument when a sequence of input quantities are applied to it. Normal procedure is to draw a good fit straight line through the X's, as

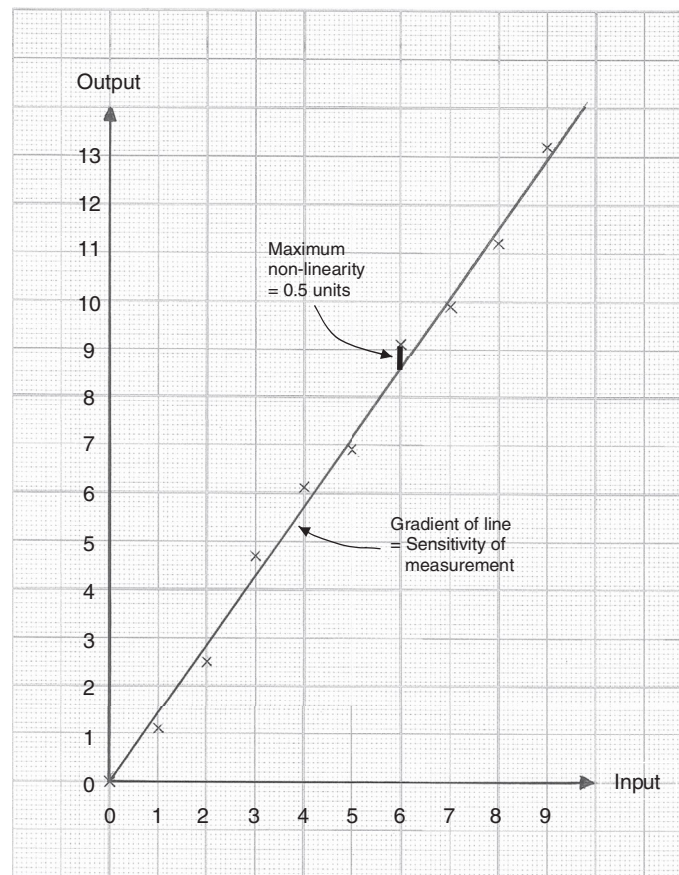


Figure 2.6
Instrument output characteristic.

shown in Fig. 2.6. (Although this can often be done with reasonable accuracy by eye, it is always preferable to apply a mathematical least-squares line-fitting technique, as described in Chapter 10.) The nonlinearity is then defined as the maximum deviation of any output reading marked X from this straight line. Nonlinearity is usually expressed as a percentage of full-scale reading.

■ Example 2.5

Suppose the instrument characteristic shown in Fig. 2.6 is that of a pressure sensor, in which the input units are expressed in bars from 1 to 9 bars and the output units are expressed in volts from 1 to 13 V.

- (a) What is the maximum nonlinearity expressed as a percentage of the full-scale deflection?
- (b) What is the resolution of the sensor as determined by the instrument characteristic given?

■ Solution

(a) The maximum nonlinearity is the maximum deviation of any data point on Fig. 2.6 away from a straight line drawn through the data points. This is shown by the thick vertical line drawn in Fig. 2.6. The length of this line is 0.5 units, which translates to 0.5 V. The full-scale deflection (calculated for the fitted straight line) is 13.0 units, which translates to 13.0 V.

The maximum nonlinearity can therefore be expressed as $\frac{0.5}{13} \times 100 = 3.8\%$ of the full-scale deflection.

(b) The resolution of the sensor as determined from the graph in Fig. 2.6 is the smallest change in input that is detectable. For the graph paper illustrated, the naked eye cannot determine anything smaller than one small square, which is one-tenth of a unit or 0.1 bar. This figure of 0.1 bar pressure is therefore the resolution of the sensor as determined from the graph.

Sensitivity of measurement: The sensitivity of measurement is a measure of the change in instrument output that occurs when the quantity being measured changes by a given amount. Thus, sensitivity is the ratio:

$$\frac{\text{scale deflection}}{\text{value of measurand producing deflection}}$$

The sensitivity of measurement is therefore the slope of the straight line drawn in Fig. 2.6. If, for example, a pressure of 2 bar produces a deflection of 10 degrees in a pressure transducer, the sensitivity of the instrument is 5 degrees/bar (assuming that the deflection is zero with zero pressure applied).

■ Example 2.6

The following resistance values of a platinum resistance thermometer were measured at a range of temperatures. Determine the measurement sensitivity of the instrument in ohms/°C.

Resistance (Ω)	Temperature ($^{\circ}\text{C}$)
307	200
314	230
321	260
328	290

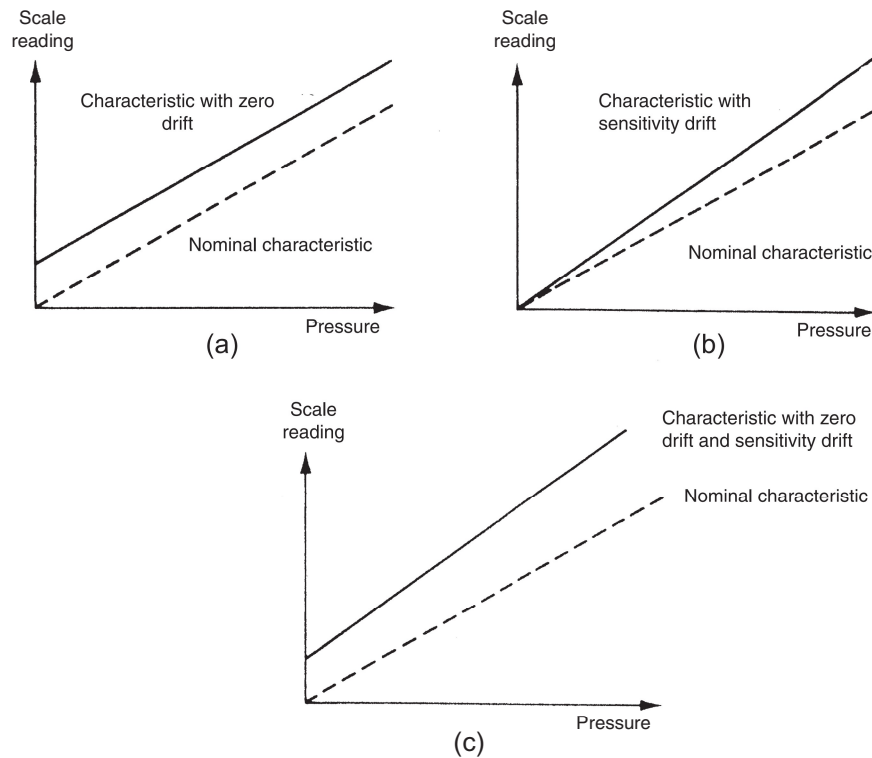
■ Solution

If these values are plotted on a graph, the straight-line relationship between resistance change and temperature change is obvious.

For a change in temperature of 30°C, the change in resistance is 7 Ω . Hence the measurement sensitivity = $7/30 = 0.233 \text{ } \Omega/^{\circ}\text{C}$.

Sensitivity to disturbance: All calibrations and specifications of an instrument are valid only under controlled conditions of temperature, pressure, and so on. These standard ambient conditions are usually defined in the instrument specification. As variations occur in the ambient temperature, and so forth, certain static instrument characteristics change, and the sensitivity to disturbance is a measure of the magnitude of this change. Such environmental changes affect instruments in two main ways, known as *zero drift* and *sensitivity drift*. Zero drift is sometimes known by the alternative term, *bias*.

Zero drift or bias describes the effect in which the zero reading of an instrument is modified by a change in ambient conditions. This causes a constant error that exists over the full range of measurement of the instrument. The mechanical form of bathroom scale

**Figure 2.7**

Effects of disturbance: (A) Zero drift. (B) Sensitivity drift. (C) Zero drift plus sensitivity drift.

is a common example of an instrument that is prone to zero drift. It is usual to find that there is a reading of perhaps 1 kg with no one standing on the scale. If someone known to weigh 70 kg were to get on the scale, the reading would be 71 kg, and if someone known to weigh 100 kg were to get on the scale, the reading would be 101 kg. Zero drift is normally removable by calibration. In the case of the bathroom scale just described, a thumbwheel is usually provided that can be turned until the reading is zero with the scales unloaded, thus removing the zero drift.

The typical unit by which such zero drift is measured is volts/°C. This is often called the **zero drift coefficient** related to temperature changes. If the characteristic of an instrument is sensitive to several environmental parameters, it will have several zero drift coefficients, one for each environmental parameter. A typical change in the output characteristic of a pressure gauge subject to zero drift is shown in [Fig. 2.7A](#).

■ Example 2.7

This table shows the output measurements of a voltmeter under two sets of conditions:

- (a) use in an environment kept at 20°C, which is the temperature at which it was calibrated; and
- (b) use in an environment at 50°C.

Voltage readings at calibration temperature of 20°C (assumed correct)	Voltage readings at temperature of 50°C
10.2	10.5
20.3	20.6
30.7	31.0
40.8	41.1

Determine the zero drift when it is used in the 50°C environment, assuming that the measurement values when it was used in the 20°C environment are correct. Also calculate the zero drift coefficient.

■ Solution

The zero drift at 50°C is the constant difference between the pairs of output readings (i.e., 0.3 V).

The zero drift coefficient is the magnitude of drift (0.3 V) divided by the magnitude of the temperature change causing the drift (30°C). Thus, the zero drift coefficient is $0.3/30 = 0.01 \text{ V/}^\circ\text{C}$.

Sensitivity drift (also known as *scale factor drift*) defines the amount by which an instrument's sensitivity of measurement varies as ambient conditions change. It is quantified by sensitivity drift coefficients that define how much drift there is for a unit change in each environmental parameter to which the instrument characteristics are sensitive. Many components within an instrument are affected by environmental fluctuations, such as temperature changes: for instance, the modulus of elasticity of a spring is temperature-dependent. Fig. 2.7B shows what effect sensitivity drift can have on the output characteristic of an instrument. Sensitivity drift is measured in units of the form (angular degree/bar)/°C. If an instrument suffers both zero drift and sensitivity drift at the same time, the typical modification of the output characteristic is as shown in Fig. 2.7C.

■ Example 2.8

A spring balance is calibrated in an environment at 20°C and has the following deflection/load characteristic.

Load (kg)	0	1	2	3
Deflection (degrees)	0	20	40	60

It is then used in an environment at 30°C and the following deflection/load characteristic is measured.

Load (kg)	0	1	2	3
Deflection (degrees)	5	27	49	71

Determine the zero drift and sensitivity drift per degrees Celsius change in ambient temperature.

■ Solution

At 20°C, the deflection/load characteristic is a straight line. Sensitivity = 20 degrees/kg.

At 30°C, the deflection/load characteristic is still a straight line. Sensitivity = 22 degrees/kg.

Zero drift (bias) = 5 degrees (the no-load deflection).

Sensitivity drift = 2 degrees/kg.

Zero drift/°C = $5/10 = 0.5$ degrees/°C.

Sensitivity drift/°C = $2/10 = 0.2$ (degrees per kg)/°C.

Hysteresis effects: Fig. 2.8 illustrates the output characteristic of an instrument that exhibits *hysteresis*. If the input-measured quantity to the instrument is steadily increased from a negative value, the output reading varies in the manner shown in curve (a). If the input variable is then steadily decreased, the output varies in the manner shown in curve (b). The noncoincidence between these loading and unloading curves is known as *hysteresis*. Two quantities are defined, *maximum input hysteresis* and *maximum output hysteresis*, as shown in Fig. 2.8. These are normally expressed as a percentage of the full-scale input or output reading, respectively.

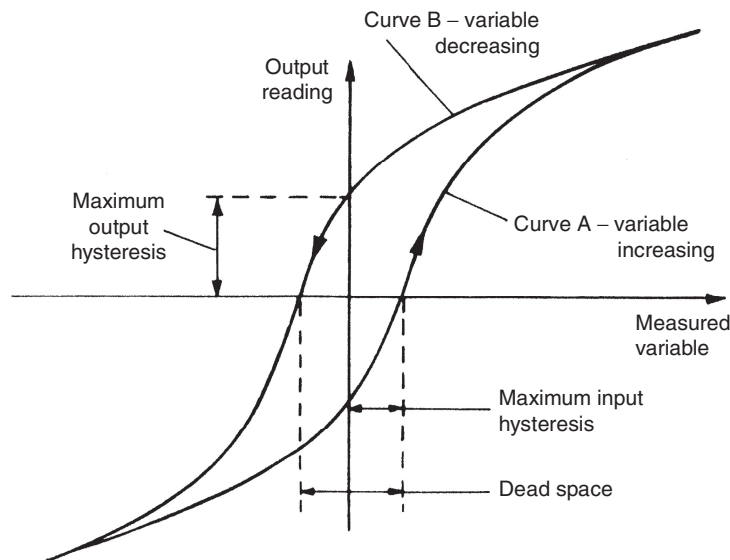


Figure 2.8
Instrument characteristic with hysteresis.

Hysteresis is most commonly found in instruments that contain springs, such as the passive pressure gauge (Fig. 2.1). It is also evident when friction forces in a system have different magnitudes depending on the direction of movement, such as in the pendulum-scale¹ mass-measuring device. Devices such as the mechanical flyball¹ (a device for measuring rotational velocity) suffer hysteresis from both of these sources because they have friction in moving parts and contain a spring. Hysteresis can also occur in instruments that contain electrical windings formed round an iron core, because of magnetic hysteresis in the iron. This occurs in devices such as the variable inductance displacement transducer, the linear variable differential transformer (LVDT) and the rotary differential transformer.

Dead space: Dead space is defined as the range of different input values over which there is no change in output value. Any instrument that exhibits hysteresis also displays dead space, as marked in Fig. 2.8. Some instruments that do not suffer from any significant hysteresis can still exhibit a dead space in their output characteristics, however. Backlash in gears is a typical cause of dead space and results in the sort of instrument output characteristic shown in Fig. 2.9. Backlash is commonly experienced in gear sets used to convert between translational and rotational motion (which is a common technique used to measure translational velocity).

¹ Both the pendulum scale and the mechanical flyball are old devices that are now rarely used.

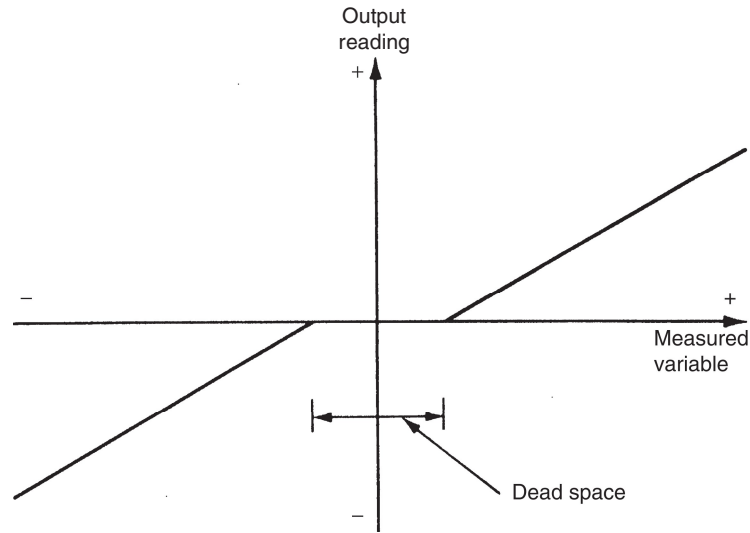


Figure 2.9
Instrument characteristic with dead space.

2.4 Dynamic characteristics of instruments

The static characteristics of measuring instruments are concerned only with the steady-state reading to which the instrument settles, such as the accuracy of the reading.

The dynamic characteristics of a measuring instrument describe its behavior between the time a measured quantity changes value and when the instrument output attains a steady value in response. As with static characteristics, any values for dynamic characteristics quoted in instrument data sheets apply only when the instrument is used under specified environmental conditions. Outside these calibration conditions, some variation in the dynamic parameters can be expected.

In any linear, time-invariant measuring system, the following general relation can be written between input and output for time $(t) > 0$:

$$a_n \frac{d^n q_o}{dt^n} + a_{n-1} \frac{d^{n-1} q_o}{dt^{n-1}} + \cdots + a_1 \frac{dq_o}{dt} + a_0 q_o = b_m \frac{d^m q_i}{dt^m} + b_{m-1} \frac{d^{m-1} q_i}{dt^{m-1}} + \cdots + b_1 \frac{dq_i}{dt} + b_0 q_i \quad (2.1)$$

where q_i is the measured quantity, q_o is the output reading, and $a_o \dots a_n, b_o \dots b_m$ are constants.

The reader whose mathematical background is such that this equation appears daunting should not worry unduly, because only certain special, simplified cases of it are applicable under normal measurement situations. The major point of importance is to have a practical

appreciation of the manner in which various different types of instrument respond when the measurand applied to them varies.

If we limit consideration only to that of step changes in the measured quantity, Eq. (2.1) reduces to:

$$a_n \frac{d^n q_o}{dt^n} + a_{n-1} \frac{d^{n-1} q_o}{dt^{n-1}} + \cdots + a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i \quad (2.2)$$

Further simplification can be made by taking certain special cases of Eq. (2.2), which collectively apply to nearly all measurement systems.

2.4.1 Zero-order instrument

If all coefficients $a_1 \dots a_n$ other than a_0 in Eq. (2.2) are assumed to be zero:

$$a_0 q_o = b_0 q_i \quad \text{or} \quad q_o = b_0 q_i / a_0 = K q_i \quad (2.3)$$

where K is a constant known as the instrument sensitivity, as defined earlier.

Any instrument that behaves according to Eq. (2.3) is said to be of zero-order type. After a step change in the measured quantity at time t , the instrument output moves immediately to a new value at the same time instant t , as shown in Fig. 2.10. A potentiometer, which measures motion, is a good example of such an instrument, in which the output voltage changes instantaneously as the slider is displaced along the potentiometer track.

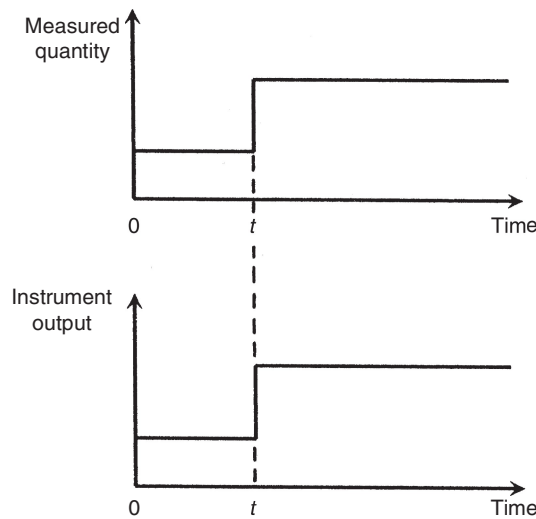


Figure 2.10
Zero-order instrument characteristic.

2.4.2 First-order instrument

If all coefficients $a_2 \dots a_n$ except for a_o and a_1 are assumed to be zero in Eq. (2.2) then:

$$a_1 \frac{dq_o}{dt} + a_o q_o = b_o q_i \quad (2.4)$$

Any instrument that behaves according to Eq. (2.4) is known as a first-order instrument. If d/dt is replaced by the D operator in Eq. (2.4), we get:

$$a_1 D q_o + a_o q_o = b_o q_i$$

and rearranging this then gives:

$$q_o = \frac{(b_o/a_o) q_i}{[1 + (a_1/a_o) D]} \quad (2.5)$$

Defining $K = b_o/a_o$ as the static sensitivity and $\tau = a_1/a_o$ as the time constant of the system, Eq. (2.5) becomes:

$$q_o = \frac{K q_i}{1 + \tau D} \quad (2.6)$$

If Eq. (2.6) is solved analytically, the output quantity q_o in response to a step change in q_i at time t varies with time in the manner shown in Fig. 2.11. The time constant τ of the step response is the time taken for the output quantity q_o to reach 63% of its final value.

The thermocouple (see Chapter 14) is a good example of a first-order instrument. It is well-known that if a thermocouple at room temperature is plunged into boiling water, the output voltage does not rise instantaneously to a level indicating 100°C but instead approaches a reading indicating 100°C in a manner similar to that shown in Fig. 2.11.

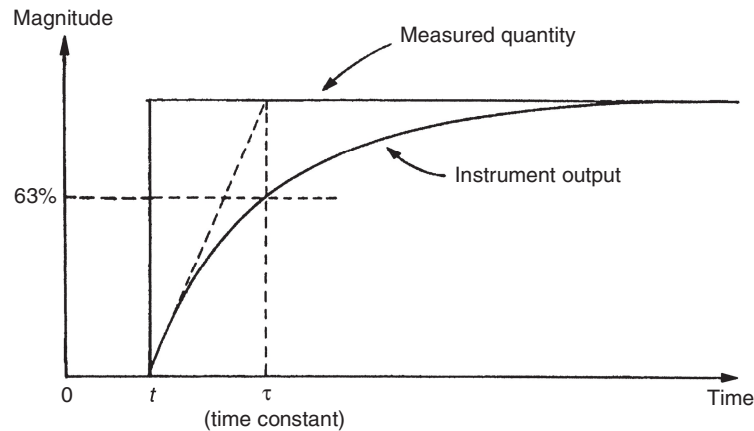


Figure 2.11
First-order instrument characteristic.

A large number of other instruments also belong to this first-order class. This is particularly important in control systems, in which it is necessary to consider the time lag that occurs between a measured quantity changing in value and the measuring instrument indicating the change. Fortunately, the time constant of many first-order instruments is small relative to the dynamics of the process being measured, and so no serious problems are created.

■ Example 2.9

A balloon is equipped with temperature- and altitude-measuring instruments and has radio equipment that can transmit the output readings of these instruments back to ground. The balloon is initially anchored to the ground with the instrument output readings in steady-state. The altitude-measuring instrument is approximately zero-order and the temperature transducer first-order with a time constant of 15 seconds. The temperature on the ground, T_0 , is 10°C and the temperature T_x at an altitude of x meters is given by the relation: $T_x = T_0 - 0.01x$

(a) If the balloon is released at time 0, and thereafter rises upward at a velocity of 5 m/s, draw a table showing the temperature and altitude measurements reported at intervals of 10 seconds over the first 50 seconds of travel. Show also in the table the error in each temperature reading.

(b) What temperature does the balloon report at an altitude of 5000 m?

■ Solution

To answer this question, it is assumed that the solution of a first-order differential equation has been presented to the reader in a mathematics course. If the reader is not so equipped, the solution presented next will be difficult to follow.

Let the temperature reported by the balloon at some general time t be T_r . Then, T_x is related to T_r by the relation:

$$T_r = \frac{T_x}{1 + \tau D} = \frac{T_0 - 0.01x}{1 + \tau D} = \frac{10 - 0.01x}{1 + 15D}$$

The balloon rises at 5 m/s; hence, $x = 5t$, and thus

$$T_r = \frac{10 - 0.05t}{1 + 15D}$$

The transient or complementary function part of the solution ($T_x = 0$) is given by:

$$T_{r_{cf}} = Ce^{-t/15}.$$

The particular integral part of the solution is given by: $T_{r_{pi}} = 10 - 0.05(t - 15)$.

Thus, the whole solution is given by: $T_r = T_{r_{cf}} + T_{r_{pi}} = Ce^{-t/15} + 10 - 0.05(t - 15)$.

Applying initial conditions: At $t = 0$, $T_r = 10$, i.e., $10 = Ce^{-0} + 10 - 0.05(-15)$.

Thus $C = -0.75$ and the solution can be written as:

$$T_r = 10 - 0.75e^{-t/15} - 0.05(t - 15).$$

Using this expression to calculate T_r for various values of t , this table can be constructed:

Time	Altitude	Temperature reading	Temperature error
0	0	10	0
10	50	9.86	0.36
20	100	9.55	0.55
30	150	9.15	0.65
40	200	8.70	0.70
50	250	8.22	0.72

(b) At 5000 m, $t = 1000$ s. Calculating T_r from this expression:

$$T_r = 10 - 0.75e^{-1000/15} - 0.05(1000 - 15)$$

The exponential term approximates to zero, and so T_r can be written as: $T_r \approx 10 - 0.05(985) = -39.25^\circ\text{C}$.

This result might have been inferred from this table, where it can be seen that the error is converging toward a value of 0.75. For large values of t , the transducer reading lags the true temperature value by a period of time equal to the time constant of 15 seconds. In this time, the balloon travels a distance of 75 m and the temperature falls by 0.75°C . Thus, for large values of t , the output reading is always 0.75°C less than it should be.

■

2.4.3 Second-order instrument

If all coefficients $a_3 \dots a_n$ other than a_0 , a_1 , and a_2 in Eq. (2.2) are assumed to be zero, we get:

$$a_2 \frac{d^2 q_o}{dt^2} + a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i \quad (2.7)$$

Applying the D operator again: $a_2 D^2 q_o + a_1 D q_o + a_0 q_o = b_0 q_i$, and rearranging:

$$q_o = \frac{b_0 q_i}{a_0 + a_1 D + a_2 D^2} \quad (2.8)$$

It is convenient to reexpress the variables a_0 , a_1 , a_2 , and b_0 in Eq. (2.8) in terms of three parameters: K (static sensitivity), ω (undamped natural frequency), and ξ (damping ratio), where:

$$K = b_0/a_0 \quad ; \quad \omega = \sqrt{a_0/a_2} \quad ; \quad \xi = a_1/2\sqrt{a_0 a_2}$$

ξ can be written as $\xi = \frac{a_1}{2a_0\sqrt{a_2/a_0}} = \frac{a_1\omega}{2a_0}$.

If Eq. (2.8) is now divided by a_0 , we get:

$$q_o = \frac{(b_0/a_0)q_i}{1 + (a_1/a_0)D + (a_2/a_0)D^2} \quad (2.9)$$

The terms in Eq. (2.9) can be written in terms of ω and ξ as follows:

$$\frac{b_0}{a_0} = K \quad ; \quad \left(\frac{a_1}{a_0}\right) D = \frac{2\xi D}{\omega} \quad ; \quad \left(\frac{a_2}{a_0}\right) D^2 = \frac{D^2}{\omega^2}$$

Hence, dividing Eq. (2.9) by q_i and substituting for a_0 , a_1 , and a_2 gives:

$$\frac{q_o}{q_i} = \frac{K}{D^2/\omega^2 + 2\xi D/\omega + 1} \quad (2.10)$$

This is the standard equation for a second-order system, and any instrument whose response can be described by it is known as a second-order instrument. If Eq. (2.10) is solved analytically, the shape of the step response obtained depends on the value of the damping ratio parameter ξ . The output responses of a second-order instrument for various values of ξ after a step change in the value of the measured quantity at time t are shown in Fig. 2.12. For case (A) in which $\xi = 0$, there is no damping and the instrument output exhibits constant amplitude oscillations when disturbed by any change in the physical quantity measured. For light damping of $\xi = 0.2$, represented by case (B), the response to a step change in input is still oscillatory, but the oscillations gradually die down. A further increase in the value of ξ reduces oscillations and overshoot still more, as shown by curves (C) and (D), and finally the response becomes overdamped as shown by curve (E), in which the output reading creeps slowly toward the correct reading. Clearly, the extreme response curves (A) and (E) are grossly unsuitable for any measuring instrument. If an instrument were to be only subjected to step inputs, the design strategy would be to aim toward a damping ratio of 0.707, which gives the critically damped response (C). Unfortunately, most of the physical quantities that instruments are required to measure do not change in the mathematically convenient form of steps, but rather in the form of ramps of varying slopes. As the form of the input variable changes, so the best value for ξ varies

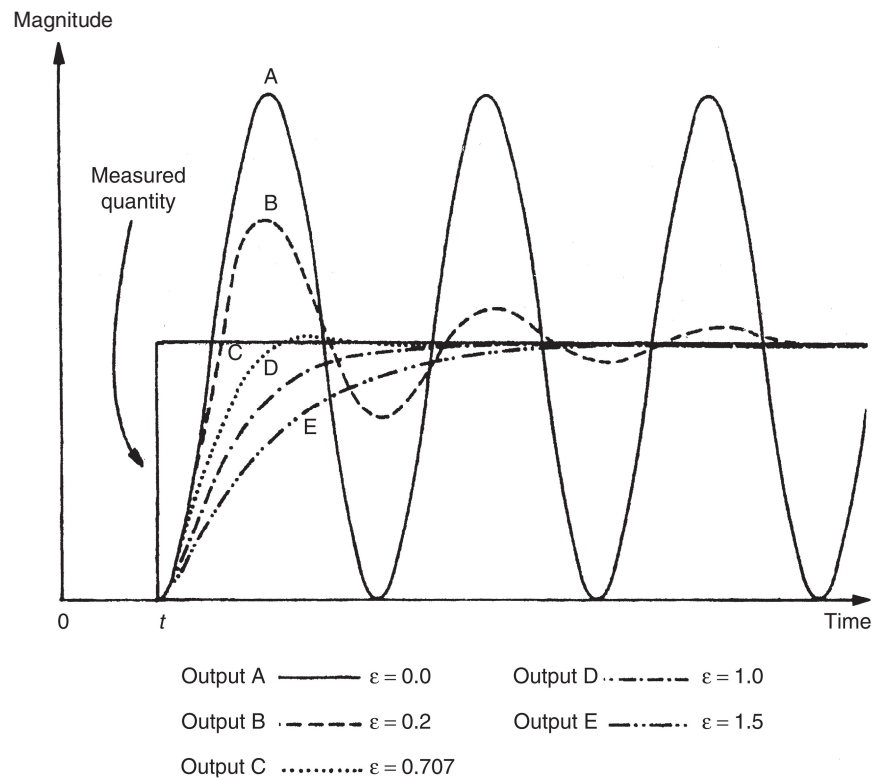


Figure 2.12
Response characteristics of second-order instruments.

and the choice of ξ becomes one of compromise between values that are best for each type of input variable behavior anticipated. Commercial second-order instruments, of which the accelerometer is a common example, are generally designed to have a damping ratio (ξ) somewhere in the range of 0.6–0.8.

2.5 Necessity for calibration

This discussion has described the static and dynamic characteristics of measuring instruments in some detail. However, an important qualification that has been omitted is that an instrument conforms to stated static and dynamic patterns of behavior only after it has been calibrated. It can normally be assumed that a new instrument will have been calibrated when it is obtained from an instrument manufacturer and will therefore initially behave according to the characteristics stated in the specifications. During use, however, its behavior will gradually diverge from the stated specification for a variety of reasons, including mechanical wear and the effects of dirt, dust, fumes, and chemicals in the operating environment. The rate of divergence from standard specifications varies

according to the type of instrument, the frequency of use, and the severity of the operating conditions. However, there will come a time, determined by practical knowledge, when the characteristics of the instrument will have drifted from the standard specification by an unacceptable amount. When this situation is reached, it is necessary to recalibrate the instrument back to the standard specifications. Such recalibration is performed by adjusting the instrument at each point in its output range until its output readings are the same as those of a second standard instrument to which the same inputs are applied. This second instrument is one kept solely for calibration purposes whose specifications are accurately known. Calibration procedures are discussed more fully in Chapter 5.

2.6 Summary

This chapter started by reviewing various different classes of instruments and considering how these differences affect their typical use. We saw, for example, that null-type instruments are favored for calibration duties because of their superior accuracy, whereas deflection-type instruments are easier to use for routine measurements. We also looked at the distinction between active and passive instruments, analog and digital instruments, indicators and signal-output type instruments and, finally, smart and nonsmart instruments. After this, we examined the various static characteristics of instruments. These define the quality of measurements when an instrument output has settled to a steady reading. Several important lessons arose from this coverage. In particular, we saw the important distinction between accuracy and precision, which are often incorrectly taken to mean the same thing. We saw that high precision does not promise anything about measurement accuracy, and in fact a high-precision instrument can sometimes give poor measurement accuracy. The final topic covered in this chapter was the dynamic characteristics of instruments. We saw that there are three kinds of dynamic characteristic, zero order, first order, and second order. Analysis of these showed that both first- and second-order instruments take time to settle to a steady-state reading when the measured quantity changes. It is therefore necessary to wait until the dynamic motion has ended before a reading is recorded. This places a serious limitation on the use of first- and second-order instruments to make repeated measurements. Clearly, the frequency of repeated measurements is limited by the time taken by the instrument to settle to a steady-state reading.

2.7 Problems

- 2.1 Briefly explain four ways in which measuring instruments can be subdivided into different classes according to their mode of operation, giving examples of instruments that fall into each class.

- 2.2 Explain what is meant by:
- (a) active instruments
 - (b) passive instruments
- Give examples of each and discuss the relative merits of these two classes of instruments.
- 2.3 Discuss the advantages and disadvantages of null and deflection types of measuring instrument. What are null types of instrument mainly used for, and why?
- 2.4 Explain the differences between analog and digital instruments and give examples of each type. What advantages do digital instruments have over analog ones?
- 2.5 Explain the difference between static and dynamic characteristics of measuring instruments.
- 2.6 Briefly define and explain all of the static characteristics of measuring instruments.
- 2.7 How is the accuracy of an instrument usually defined? Explain the difference between accuracy and precision, using sketches as appropriate to illustrate the differences.
- 2.8 Draw sketches to illustrate the dynamic characteristics of:
- (a) zero-order instrument
 - (b) first-order instrument
 - (c) second-order instrument
- In the case of a second-order instrument, indicate the effect of different degrees of damping on the time response.
- 2.9 State briefly how the dynamic characteristics of an instrument affect its use.
- 2.10 Define the measurement inaccuracy of a measuring instrument. If a tungsten resistance thermometer with a range of -270°C to $+1100^{\circ}\text{C}$ has a quoted inaccuracy of $\pm 1.5\%$ of full-scale reading, what is the likely measurement error when it is reading a temperature of 950°C ?
- 2.11 What is meant by the term “tolerance” in manufacturing systems? If a batch of steel rods is manufactured to a nominal length of 5 m with a quoted tolerance of $\pm 2\%$, what is the longest and shortest length of rod to be expected in the batch?
- 2.12 A manganin wire pressure sensor has a measurement range of 0–20,000 bar and a quoted inaccuracy of $\pm 1\%$ of full-scale deflection. What is the maximum measurement error when the instrument is reading a pressure of 15,000 bar?
- 2.13 What do you understand by the word “tolerance”? If a batch of ceramic bricks is manufactured with a tolerance of $\pm 1.5\%$, what is the length of the shortest and longest bricks to be expected in the batch?
- 2.14 What is the measurement range for a micrometer designed to measure diameters between 7.5 and 10.0 cm?

- 2.15 A pressure gauge is designed to measure absolute pressures between 0 and 50 bar. If its inaccuracy is quoted as $\pm 0.5\%$ of full-scale reading, calculate (a) its measurement range and (b) the possible measurement error when it is reading a pressure of 10 bar.
- 2.16 A packet of resistors bought in an electronics component shop gives the nominal resistance value as $5000\ \Omega$ and the manufacturing tolerance as $\pm 3\%$. If one resistor is chosen at random from the packet, what is the minimum and maximum resistance value that this particular resistor is likely to have?
- 2.17 A pressure gauge with a measurement range of 0–30 bar has a quoted inaccuracy of $\pm 0.5\%$ of the full-scale reading.
- What is the maximum measurement error expected for this instrument?
 - What is the likely measurement error expressed as a percentage of the output reading if this pressure gauge is measuring a pressure of 5 bar?
 - If the measurement error when reading a pressure of 5 bar is deemed to be too high, what are the two main options for reducing the measurement error? Which of these two options would you recommend, and why would you recommend it?
- 2.18 The width of a room is measured 10 times by an ultrasonic rule and the following measurements are obtained (in meters): 4.292, 4.295, 4.296, 4.293, 4.292, 4.294, 4.293, 4.290, 4.294, and 4.291.
- The width of the same room is then measured by a calibrated steel tape that gives a reading of 4.276 m, which can be taken as the correct value for the width of the room.
- What is the measurement precision of the ultrasonic rule?
 - What is the maximum measurement inaccuracy of the ultrasonic rule?
- 2.19 (a) Explain the difference between precision and accuracy, illustrating your answer with sketches as appropriate.
- (b) A person uses a set of spring-based (analog) bathroom scales to measure his weight. He gets on and off the scales 10 times and obtains the following measurements (in kilograms): 70.1, 69.6, 69.4, 70.4, 69.9, 69.6, 70.2, 70.7, 69.4, and 69.7.
- He then uses a set of calibrated electronic scales to measure his weight that, for the purposes of this exercise, are assumed to have zero error. These scales give a reading of 70.5 kg.
- Calculate (i) the precision of the spring-based scales and (ii) their inaccuracy.
- 2.20 A tungsten/5% rhenium–tungsten/26% rhenium thermocouple has an output electromagnetic field as shown in the following table when its hot (measuring) junction is at the temperatures shown. Determine the sensitivity of measurement for the thermocouple in mV/°C.

mV	4.37	8.74	13.11	17.48
°C	250	500	750	1000

- 2.21 A set of known pressures was applied to a pressure gauge and the output voltages were recorded as shown in the following table:

Pressure (bar)	10	30	50	70	100	120	140	160	180	200
Output (volts)	0.45	1.44	2.37	3.25	4.77	5.58	6.49	7.45	8.50	9.46

What is the sensitivity of measurement of the pressure gauge?

- 2.22 The calibration of a pressure sensor with a range of 0–16 bar is checked by applying known inputs to it in steps of 2 bar. The following measurements were obtained:

Known input pressure (bar)	0	2	4	6	8	10	12	14	16
Output reading (volts)	0.00	0.55	1.25	1.70	2.35	3.15	3.65	4.05	4.85

- (a) Draw a graph by plotting the data points on the graph paper provided and draw a best-fit straight line through the data points. Use this graph to calculate:
 - (b) the measurement sensitivity of the pressure sensor.
 - (c) the maximum nonlinearity apparent in the measurements as a percentage of the full-scale deflection.
- 2.23 Define sensitivity drift and zero drift. What factors can cause sensitivity drift and zero drift in instrument characteristics?
- 2.24 The calibration of a spring balance is checked by applying a set of known masses to it, with the following results:

Known input (grams)	10	20	30	40	50	60	70	80
Output reading (grams)	15.5	25.0	32.0	45.0	52.0	61.0	74.5	81.0

- (a) Draw a graph by plotting the data points on the graph paper provided and draw a best-fit straight line through the data points.
 - (b) Which of the following static measurement characteristics are evident in the input–output graph drawn: nonlinearity, dead space, sensitivity drift, threshold, zero drift, hysteresis?
 - (c) Calculate the magnitude of the static measurement characteristics that you have just identified.
- 2.25 Explain the terms “zero drift” and “zero-drift coefficient.”
The following table shows the output measurements of a pressure sensor under two sets of conditions:
- (a) when used in an environment kept at 20°C, which is the temperature at which it was calibrated; and

- (b) when used in an environment at 30°C.

Pressure readings at calibration temperature of 20°C (assumed correct) in units of pascal	Pressure readings at 30°C in units of pascal
1.31	1.50
2.46	2.65
3.63	3.82
4.82	5.01
6.27	6.46

Determine the zero drift when it is used in the 30°C environment, assuming that the measurement values when it was used in the 20°C environment are correct. Also calculate the zero-drift coefficient.

- 2.26 (a) An instrument is calibrated in an environment at 20°C and the following output readings y are obtained for various input values x :

y	13.1	26.2	39.3	52.4	65.5	78.6
x	5	10	15	20	25	30

Determine the measurement sensitivity, expressed as the ratio y/x .

- (b) When the instrument is subsequently used in an environment at 50°C, the input–output characteristic changes to the following:

y	14.7	29.4	44.1	58.8	73.5	88.2
x	5	10	15	20	25	30

Determine the new measurement sensitivity. Then determine the sensitivity drift due to the change in ambient temperature of 30°C.

- 2.27 A spring balance is calibrated in an environment at 20°C and has the following deflection/load characteristic:

Load (kg)	0	0.4	0.8	1.2	1.6	2.0
Deflection (degrees)	0	36	72	108	144	180

It is then used in an environment at 27°C and the following deflection/load characteristic is measured:

Load (kg)	0	0.40	0.8	1.2	1.6	2.0
Deflection (degrees)	6	43	80	117	154	191

Determine the zero drift and sensitivity drift per degrees Celsius change in ambient temperature.

- 2.28 The following temperature measurements were taken with an infrared thermometer that produced biased measurements owing to the instrument being out of calibration. Calculate the bias in the measurements.

Values measured by uncalibrated instrument (°C)	Correct value of temperature (°C)
20	21.5
35	36.5
50	51.5
65	66.5

- 2.29 A spiral Bourdon tube was found to be damaged. When its calibration was checked, the following readings were obtained:

Indicated pressure (bar)	100	200	300	400	500
Correct pressure (bar)	103	203	303	403	503

What is the bias in the readings?

- 2.30 A sensor was calibrated at a temperature of 20°C and the following input (x) and output (y) data were recorded:

Input (x)	0	20	40	60	80	100
Output (y)	0	31	62	93	124	155

It was then recalibrated at a temperature of 40°C and the following input (x) and output (y) data were recorded:

Input (x)	0	20	40	60	80	100
Output (y)	0.5	32.0	63.5	95.0	126.5	158.0

Calculate the zero drift in the sensor caused by the change in temperature from 20°C to 40°C. Also, calculate the measurement sensitivity of the sensor at the two temperatures and the total sensitivity drift.

- 2.31 A load cell is calibrated in an environment at a temperature of 21°C and has the following deflection/load characteristic:

Load (kg)	0	50	100	150	200
Deflection (mm)	0.0	1.0	2.0	3.0	4.0

When used in an environment at 35°C, its characteristic changes to the following:

Load (kg)	0	50	100	150	200
Deflection (mm)	0.2	1.3	2.4	3.5	4.6

- Determine the sensitivity at 21°C and 35°C
- Calculate the total zero drift and sensitivity drift at 35°C.

- (c) Then determine the zero drift and sensitivity drift coefficients (in units of $\mu\text{m}/^\circ\text{C}$ and $(\mu\text{m per kg})/^\circ\text{C}$).
- 2.32 A balloon is equipped with temperature- and altitude-measuring instruments and has radio equipment that can transmit the output readings of these instruments back to ground. The balloon is initially anchored to the ground with the instrument output readings in steady-state. The altitude-measuring instrument is approximately zero order and the temperature transducer is approximately first order with a time constant of 10 seconds. The temperature on the ground, T_0 , is 20°C and the temperature T_h at an altitude of x meters is given by the relation: $T_h = T_0 - 0.012 h$
- (a) If the balloon is released at time 0, and thereafter rises upward at a velocity of 6 m/s, draw a table showing the temperature and altitude measurements reported at intervals of 10 seconds over the first 100 seconds of travel. Show also in the table the error in each temperature reading.
- (b) What temperature does the balloon report at an altitude of 8000 m?
- (c) What is the error in the temperature reading at this altitude of 8000 m?
- 2.33 The time response of a liquid-in-glass thermometer was tested by plunging it into a container of boiling water and recording the temperature at intervals of 10 seconds over 2 minutes. The thermometer was initially at 0°C and the following readings were recorded:

Time (seconds)	0	10	20	30	40	50	60	70	80	90	100	110	120
Temp ($^\circ\text{C}$)	0	31	52	67	77	84	89	92	95	96	97	98	99

Draw a graph of the output readings against time. Using this graph, estimate the time constant of the thermometer.

- 2.34 An unmanned submarine is equipped with temperature- and depth-measuring instruments and has radio equipment that can transmit the output readings of these instruments back to the surface. The submarine is initially floating on the surface of the sea with the instrument output readings in steady-state. The depth-measuring instrument is approximately zero order and the temperature transducer first order with a time constant of 50 seconds. The water temperature on the sea surface, T_0 , is 20°C and the temperature T_x at a depth of x meters is given by the relation:

$$T_x = T_0 - 0.01x$$

- (a) If the submarine starts diving at time 0 and thereafter goes down at a velocity of 0.5 m/s, draw a table showing the temperature and depth measurements reported at intervals of 100 seconds over the first 500 seconds of travel. Show also in the table the error in each temperature reading.
- (b) What temperature does the submarine report at a depth of 1000 m?
- 2.35 Write the general differential equation describing the dynamic response of a second-order measuring instrument and state the expressions relating the static sensitivity,

undamped natural frequency, and damping ratio to the parameters in this differential equation. Sketch the instrument response for the cases of heavy damping, critical damping, and light damping, and state which of these is the usual target when a second-order instrument is being designed.

- 2.36 A sheathed thermocouple with a time constant of 4.0 seconds is initially at a room temperature of 20°C. It is then plunged into a beaker of boiling water at 100°C. Assuming that the thermocouple has a first-order characteristic, what temperature would it indicate when it had been in the water for 8.0 seconds?