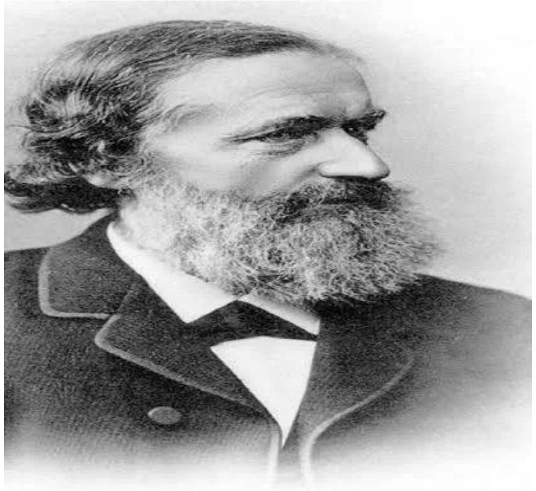
KIRCHHOFF LAWS

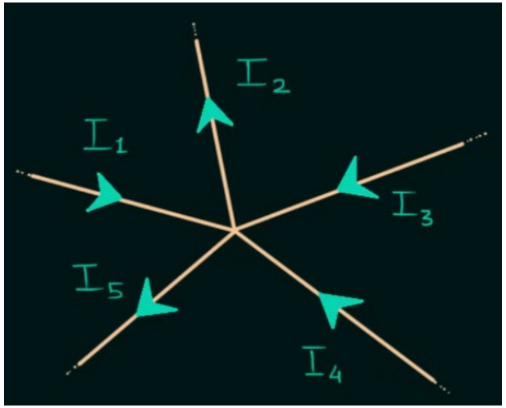
INTRODUCTION OF KIRCHHOFF LAWS

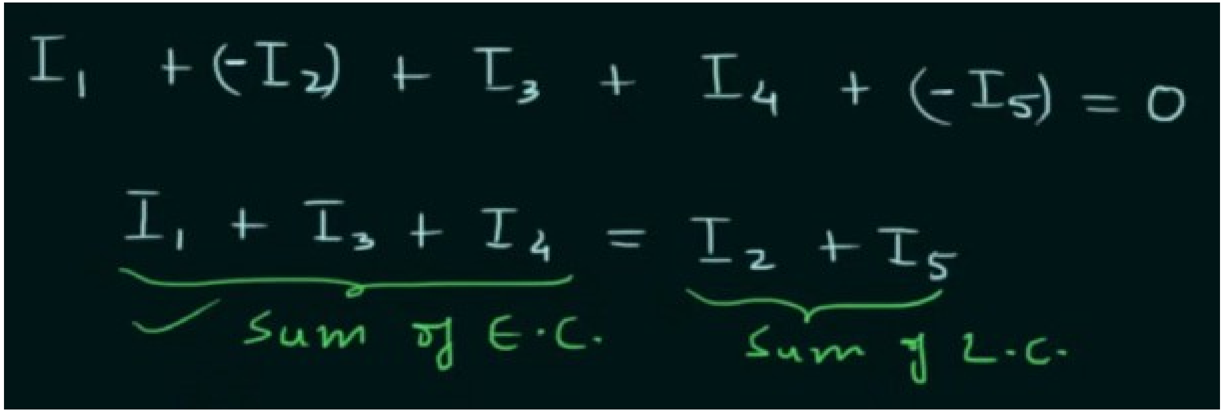
- Are two fundamental principles for analyzing electrical circuits
- These are Kirchhoff’s current law and Kirchhoff’s voltage law
- They were first described in 1845 by German physicist Gustav Kirchhoff



Kirchhoff’s Current Law

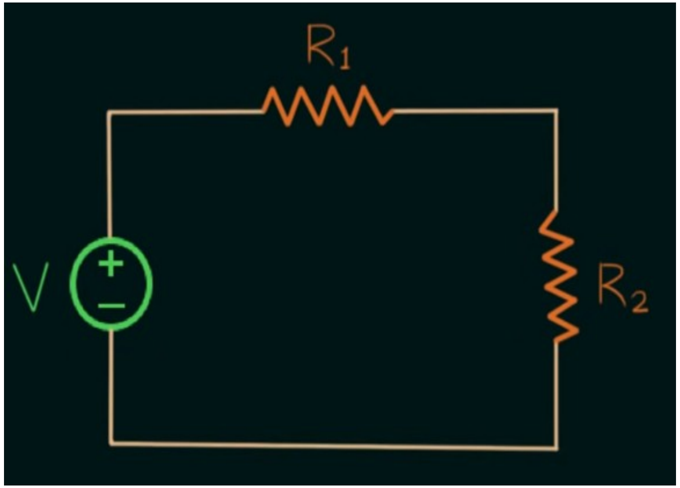
- State that “the algebraic sum of currents entering any node is zero”
- To understand this you need to consider to things:- entering currents is positive, leaving currents is negative.





Kirchhoff Voltage Law

- State that “the algebraic sum of voltages in a closed loop is zero”
- Considering in rising potential will give you the positive sign and considering in dropping potential will give you negative sign



$$\Rightarrow V - IR_1 - IR_2 = 0$$

Procedures for Analyzing Circuit

1)Label the circuit

- Identify all nodes
- Assign a variable name and a direction to the current in every branch

2)Apply Kirchhoff's Current Law (KCL)

- At each independent node, write an equation stating that the sum of currents entering the node equals the sum of currents leaving it

3)Apply Kirchhoff's Voltage Law (Kvl)

- Identify the loops in the circuit. For each loop, choose a direction (clokwise or counter-clockwise) and write an equation stating the algebraic sum of all voltages around the loop is zero.

Procedures for Analyzing Circuit

4) Establish sign conventions

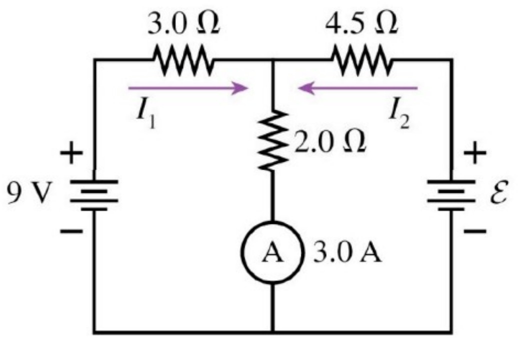
- When moving across a resistor in the same direction as the current, the voltage change is negative. If moving against the current, it is positive. For a battery or voltage sources, if you move from the negative terminal to the positive terminal, it is a voltage rise +.

5) Solve the simulataneous Equations

- Combine the KCL and KVL equations into a system of linear equations. Use algebraic methods such as substitution, elimination, or matrix algebra to solve for the unknown currents and voltages.

EXAMPLE 1

- Find the currents and voltages across each resistor and cell in the following circuit



$$I_1 + I_2 = 3.0\ \text{A}$$

$$9.0\ \text{V} - (3.0\ \Omega \cdot I_1) - (2.0\ \Omega \cdot 3.0\ \text{A}) = 0$$

$$9.0 - 3.0I_1 - 6.0 = 0$$

$$3.0 = 3.0I_1 \implies \mathbf{I_1 = 1.0\ A}$$

$$\mathcal{E} - (4.5\ \Omega \cdot I_2) - (2.0\ \Omega \cdot 3.0\ \text{A}) = 0$$

$$\mathcal{E} - (4.5 \cdot 2.0) - 6.0 = 0$$

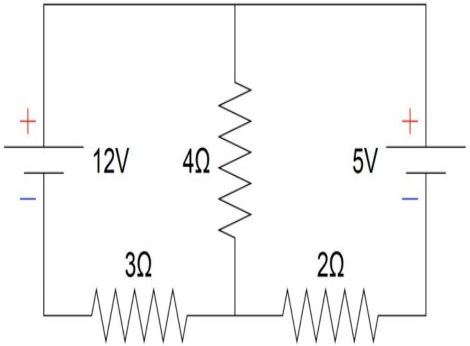
$$\mathcal{E} - 9.0 - 6.0 = 0 \implies \mathcal{E} = \mathbf{15.0}\ \text{V}$$

Now we use I_1 to find I_2:

- **Current I_2:** $I_2 = 3.0\ \text{A} - I_1 = 3.0 - 1.0 = \mathbf{2.0}\ \text{A}$
- **Voltage across $3.0\ \Omega$ resistor:** $V = 1.0\ \text{A} \cdot 3.0\ \Omega = \mathbf{3.0}\ \text{V}$
- **Voltage across $4.5\ \Omega$ resistor:** $V = 2.0\ \text{A} \cdot 4.5\ \Omega = \mathbf{9.0}\ \text{V}$
- **Voltage across $2.0\ \Omega$ resistor:** $V = 3.0\ \text{A} \cdot 2.0\ \Omega = \mathbf{6.0}\ \text{V}$

EXAMPLE 2

- Find the currents across each resistor in the following circuit



$I_1 + I_3 = I_2 \implies I_1 - I_2 + I_3 = 0$ Left Loop: $12 - 3I_1 - 4I_2 = 0 \implies 3I_1 + 4I_2 = 12$ (Equation 2)

Right Loop: $5 - 2I_3 - 4I_2 = 0 \implies 4I_2 + 2I_3 = 5$ (Equation 3)

Replace I_2 with $(I_1 + I_3)$ in Eq 2: $3I_1 + 4(I_1 + I_3) = 12 \implies 7I_1 + 4I_3 = 12$ (**Eq 4**)

Replace I_2 with $(I_1 + I_3)$ in Eq 3: $4(I_1 + I_3) + 2I_3 = 5 \implies 4I_1 + 6I_3 = 5$ (**Eq 5**)

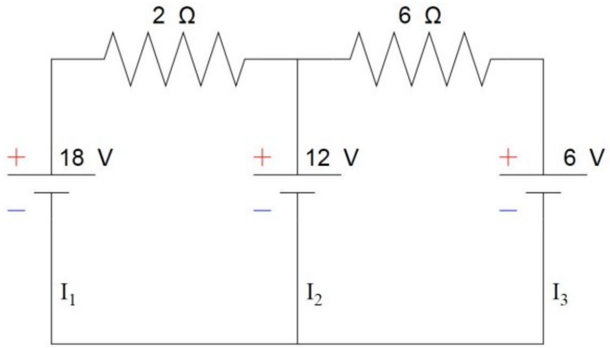
$$I_1 = 2.00 \text{ A}$$

$I_3 = -0.50 \text{ A}$ (The negative sign means actual flow is opposite to our guess).

$$I_2 = I_1 + I_3 = 2.00 + (-0.50) = 1.50 \text{ A}$$

EXAMPLE 3

- Find the currents across each resistor in the following circuit



Equation 1: $I_1 + I_3 = I_2$

Equation 2: $18 - (1 \cdot I_1) - (2 \cdot I_2) = 0.$

Equation 3: $24 - (2 \cdot I_3) - (2 \cdot I_2) = 0.$

From Eq 2: $18 - I_1 - 2(I_1 + I_3) = 0 \implies 3I_1 + 2I_3 = 18.$

From Eq 3: $24 - 2I_3 - 2(I_1 + I_3) = 0 \implies 2I_1 + 4I_3 = 24.$

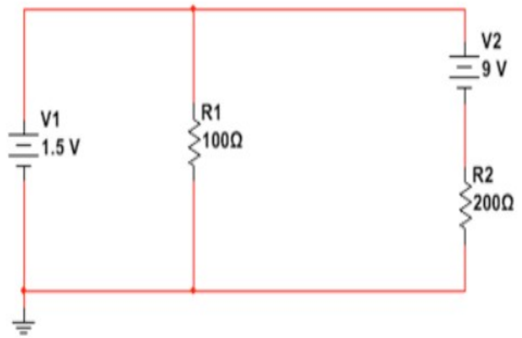
$$I_1 = 3.0 \text{ A.}$$

$$I_3 = 4.5 \text{ A.}$$

$$I_2 = I_1 + I_3 = \mathbf{7.5} \text{ A.}$$

EXAMPLE 4

- Find the currents and voltage across each resistor in the following circuit



$I = I_1 + I_2$ (equation 1)

Loop A: (start from the upper left corner and move clockwise)

$-I_1 \times (100\ \Omega) + 1.5V = 0$ (equation 2)

Therefore: $\mathbf{I_1 = 0.015\ A}$

Loop B:

$-9V - I_2 \times (200\ \Omega) + I_1 \times (100\ \Omega) = 0$ (equation 3)

Substituting the value of I_1 into equation 3 yields:

$-9 - I_2 \times (200\ \Omega) + (0.015)(100\ \Omega) = 0$

$-7.5 = (200) \times I_2$ therefore: $\mathbf{I_2 = -0.0375\ A}$

And then $\mathbf{I = -0.0225\ A}$

$\mathbf{V_{R1} = I_1 \times R_1} = (0.015\ A) \times (100\ \Omega)$ therefore $\mathbf{V_{R1} = 1.5V}$

$\mathbf{V_{R2} = I_2 \times R_2} = (-0.0375\ A) \times (200\ \Omega)$ therefore $\mathbf{V_{R2} = -7.5V}$

APPLICATIONS OF KIRCHHOFF LAWS

- Electronic Circuit Design
- Power Grid and Distribution
- Modern Transportation Systems
- Advanced Circuit Simulation Software
- Electronic Signal Processing (In telecommunications and audio electronics)

SHORTCOMINGS

1) **High-Frequency Failure**

- Kirchhoff’s laws work best when the wavelength of the signal is much larger than the physical size of the circuit.
- The Issue: At very high frequencies (like those in GHz computer processors or microwave circuits), the time it takes for a signal to travel from one side of a board to the other becomes significant.
- Result: In these cases, current may not be the same at both ends of a single wire at the exact same moment, which violates the basic premise of KCL. Engineers must switch to Maxwell’s Equations or use Transmission Line Theory instead.

SHORTCOMINGS

2) Effects of Parasitic Capacitance and Inductance

- KCL assumes that charge cannot be "stored" at a node—it must enter and leave immediately.
- The Issue: In reality, every wire and junction has a tiny amount of "stray" or parasitic capacitance.
- Result: At high speeds, some charge can briefly "pile up" at a junction or leak into the surrounding air/components. This means the sum of currents entering a node might not exactly equal the sum leaving it if you look at a nanosecond timescale.

SHORTCOMINGS

3) **Fluctuating Magnetic Fields (Non-Conservative Fields)**

- KVL assumes that the sum of voltages around a loop is zero, which is only true if the electric field is conservative (meaning the energy gained/lost depends only on the start and end points).
- The Issue: If a circuit is placed in a rapidly changing external magnetic field (like near a large motor or a transformer), an EMF is induced directly into the wires of the loop itself.
- Result: Because the "source" of energy is distributed throughout the wire rather than located in a specific battery, the voltages you measure across the resistors will not sum to zero. This is a famous experimental paradox often demonstrated in advanced physics labs.

SHORTCOMINGS

4) **Complexity with Non-Linear Components**

- Kirchhoff’s Laws result in linear equations.
- The Issue: While they work perfectly for resistors, they become much harder to solve manually when you include non-linear components like diodes or transistors, where the resistance changes depending on the voltage.
- Result: You often end up with complex transcendental equations that require iterative numerical methods or computer simulations (like SPICE) to solve, rather than simple pencil-and-paper algebra.

SHORTCOMINGS

5) Radiation Losses

- KCL and KVL assume that all energy is either stored in components or dissipated as heat.
- The Issue: In systems like radio antennas, energy is intentionally "lost" to the environment as electromagnetic radiation.
- Result: Kirchhoff’s Laws do not account for energy leaving the circuit as a wave. To analyze an antenna properly, you must model the radiation as a "radiation resistance," which is a workaround to make the laws still apply.