J **Renormalisation**:

$$\begin{aligned}\Delta J &= -\frac{J^2}{2}\Big[\frac{1}{w-\frac{D}{2}+\frac{J}{8}+t_\perp}+\frac{1}{w-\frac{D}{2}+\frac{J}{8}-t_\perp}\Big]\\ &= \frac{J^2}{2}\Big[\frac{1}{d_0+t_\perp}+\frac{1}{d_0-t_\perp}\Big]\\ \Delta J &= J^2\frac{d_0}{d_0^2-t_\perp^2}\end{aligned}$$

We have taken $d_0 = -w+\frac{D}{2}-\frac{J}{8}$.
For MCK type Case:

$$\begin{aligned}\Delta J &= \frac{J^2}{2(d_0-t_\perp)}(1+\frac{d_0-t_\perp}{2d_0})\\ &= \frac{J^2}{2(d_0-t_\perp)}(\frac{3}{2}-\frac{t_\perp}{2d_0})\end{aligned}$$

Here $d_0 - t_\perp$ is small compare to d_0 and ΔJ will change sign due to denominator $(d_0 - t_\perp)$ before the first bracket starts.

For e-SIAM type case:

$$\Delta J = -J^2\frac{d_0}{t_\perp^2}(1+\frac{d_0^2}{t_\perp^2})$$

Here d_0 is small compare to $t_\perp$ and ΔJ will change sign due to numerator d_0 before the first bracket starts.
V **Renormalisation**:

$$\begin{aligned}\Delta V &= \frac{3VJ}{8}\Big[\frac{d_0}{d_0^2-t_\perp^2}+\frac{d_0-\frac{U}{2}}{(d_0-\frac{U}{2})^2-t_\perp^2}\Big]\\ &= \frac{3VJ}{8}\Big[\frac{\Delta J}{J^2}+\frac{d_0-\frac{U}{2}}{(d_0-\frac{U}{2})^2-t_\perp^2}\Big]\end{aligned}$$

U Renormalisation:

$$\begin{aligned}\Delta U &= 4V^2\Big[\frac{d_0+\frac{J}{8}+\frac{U}{2}}{(d_0+\frac{J}{8}+\frac{U}{2})^2-t_\perp^2}-\frac{d_0-\frac{U}{2}}{(d_0-\frac{U}{2})^2-t_\perp^2}\Big]+J^2\frac{d_0}{d_0^2-t_\perp^2}\\ &= 4V^2\Big[\frac{d_0+\frac{J}{8}+\frac{U}{2}}{(d_0+\frac{J}{8}+\frac{U}{2})^2-t_\perp^2}-\frac{8\Delta V}{3VJ}+\frac{\Delta J}{J^2}\Big]+\Delta J\\ &= 4V^2\frac{d_0+\frac{J}{8}+\frac{U}{2}}{(d_0+\frac{J}{8}+\frac{U}{2})^2-t_\perp^2}-\frac{32V}{3J}\Delta V+(1+4\frac{V^2}{J^2})\Delta J\end{aligned}$$

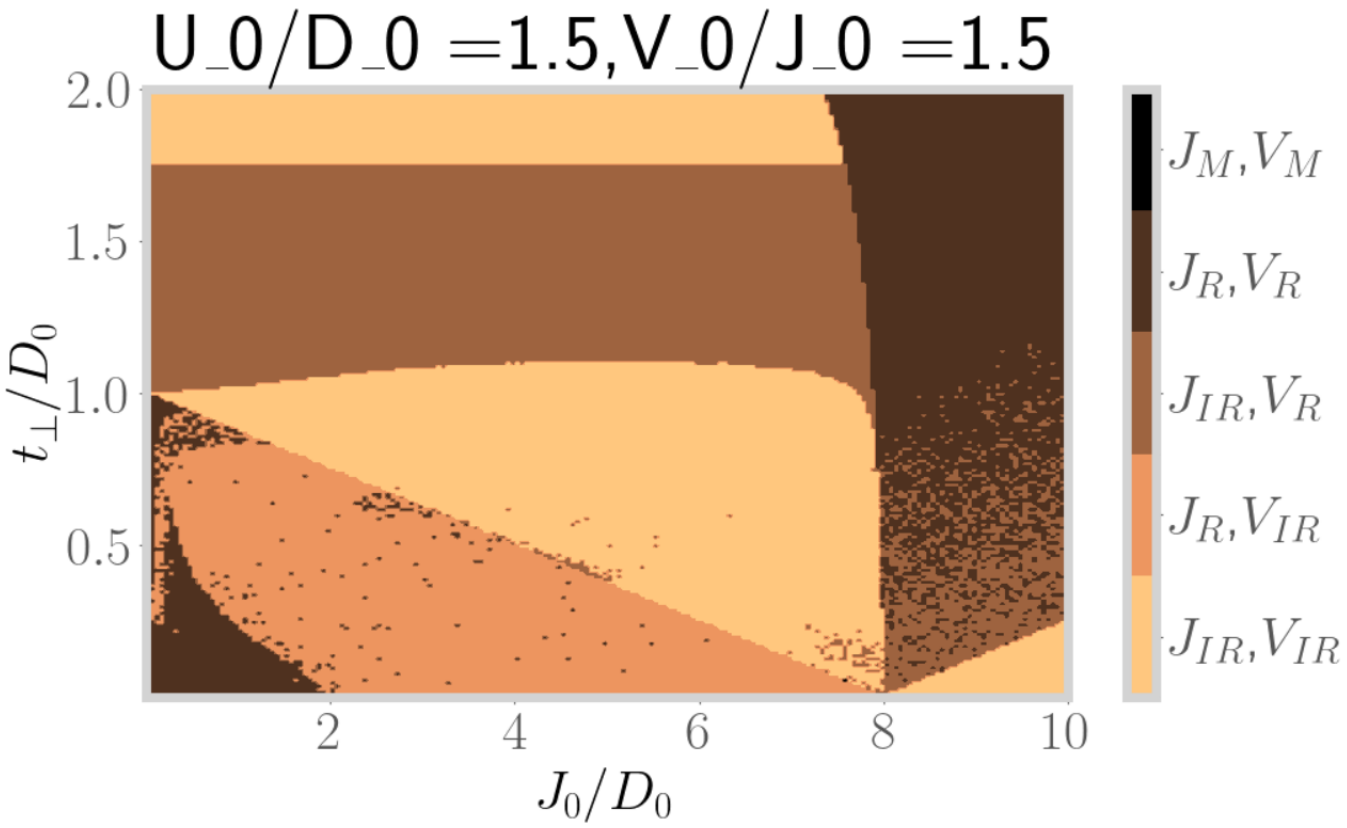


Figure 1: D0=10

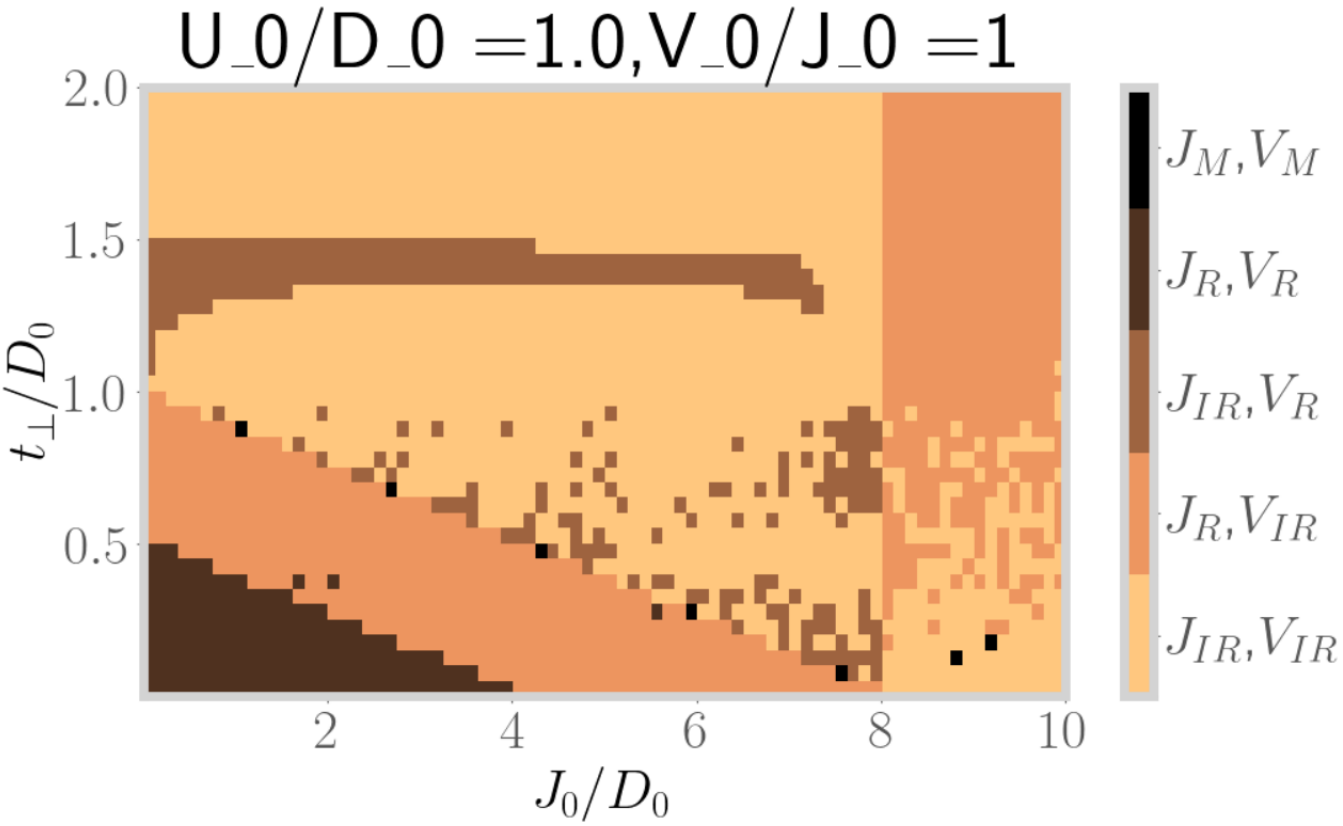


Figure 2: D0=1000

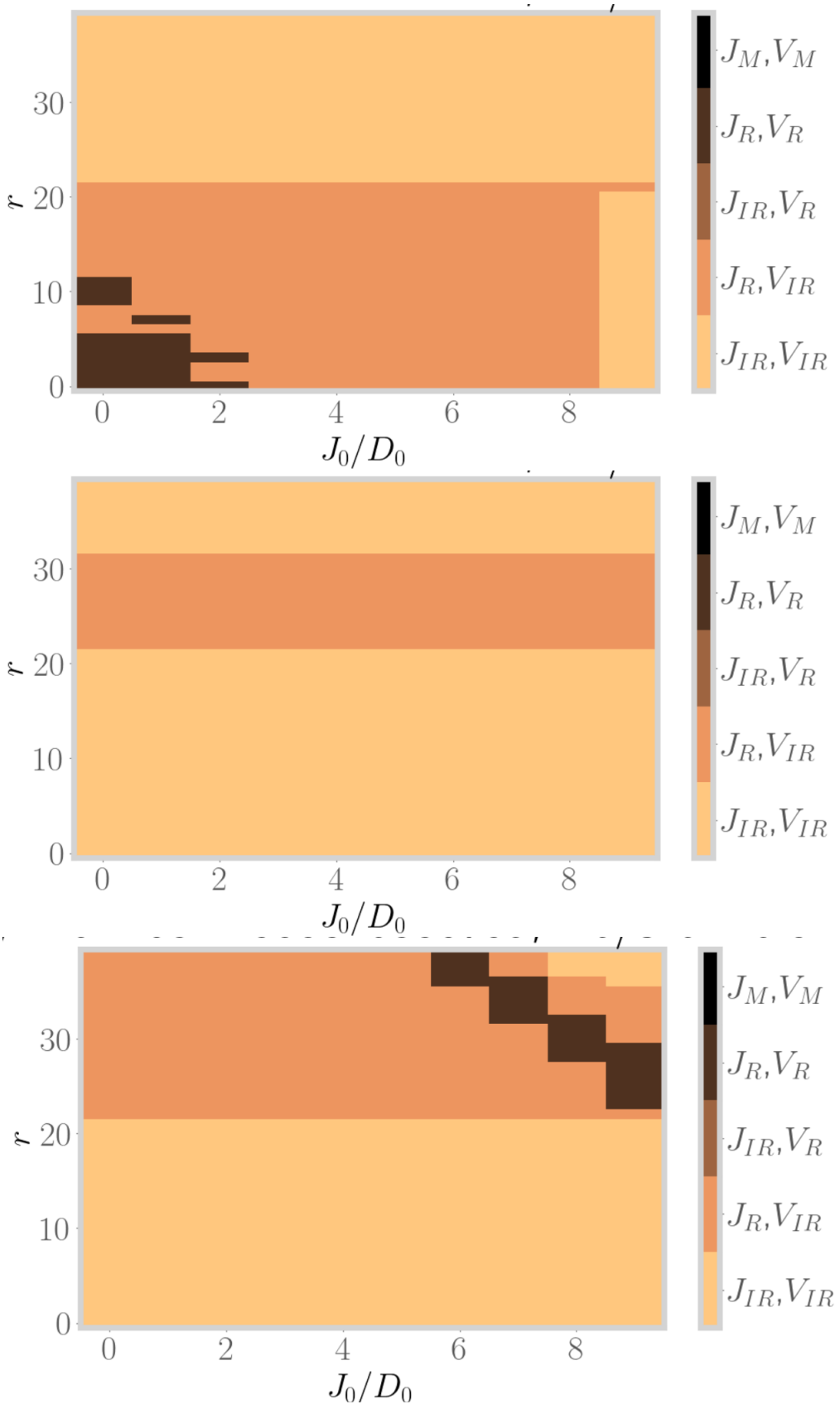
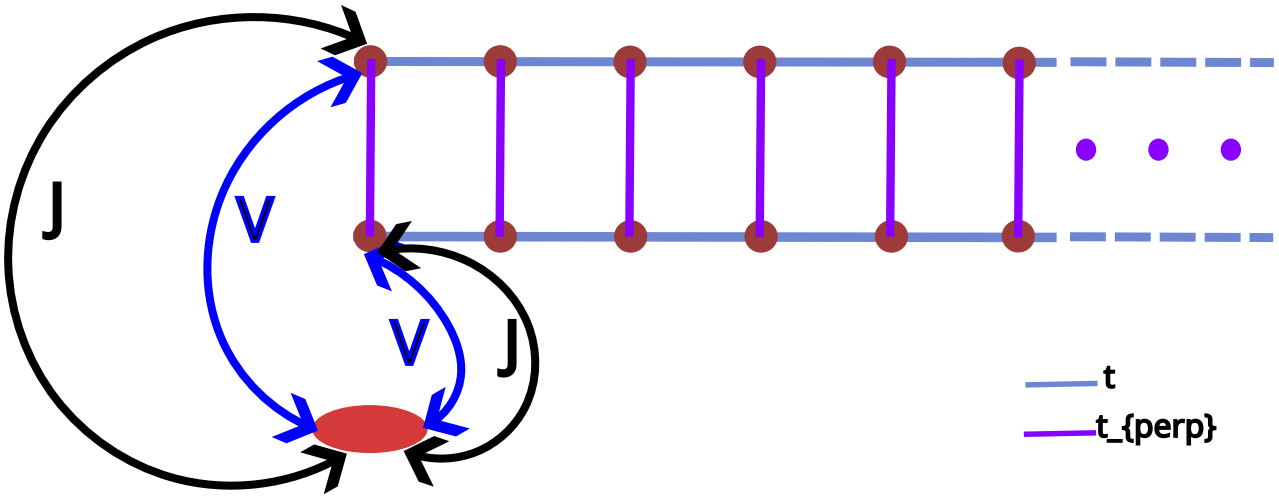


Figure 3: With ^4U_b like e-SIAM


J
V
V
J
t
t_{perp}