Service Report

Company Information

Name: none

Address: 13335 85th Ave N.

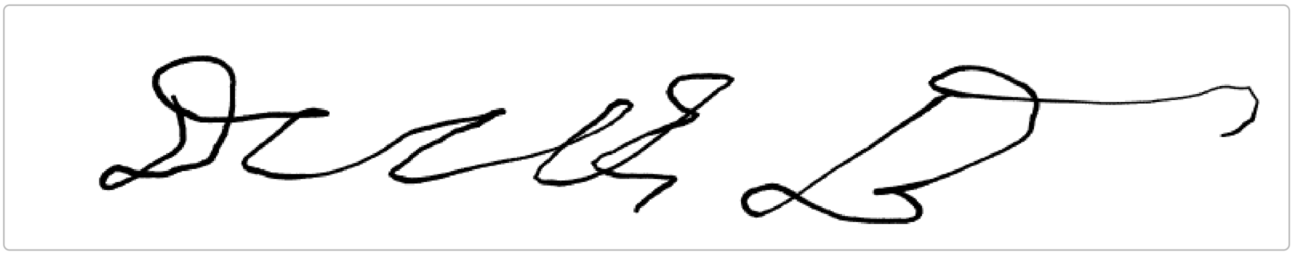
Phone: 6123605935

Email: d.brown13335@gmail.com

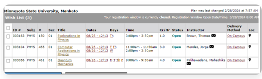
Service Details

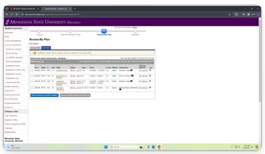
Service Report Number	FL-354564564
Date	2025-07-22
Technician	Derek
Technician Email	d.brown13335@gmail.com
Technician Phone	6123605935
Work Order	Work_Order
Reason For Service	Work_Order
Customer Asset Number	Test
Serial Number	Test
Incident	Test
Work Order Type	test
Start Time	08:00 AM
End Time	05:44 PM
On Site Duration	9h 44m
Functional Location Address	13335 85th Ave N.
Products	Work_Order
Service Task Inspections	Work_Order
Customer Notes	Work_Order
Customer Name	Derek Alexander Brown

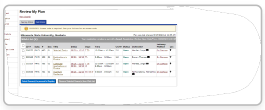
Signature

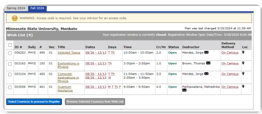


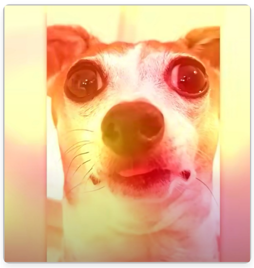
Photos

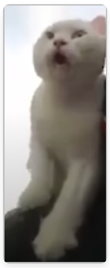


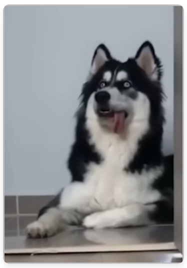


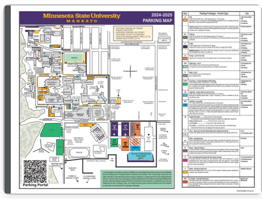


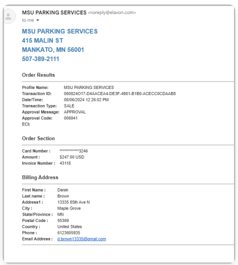












MSU PARKING SERVICES <noreply@elavon.com>
to me

MSU PARKING SERVICES
415 MALIN ST
MANKATO, MN 56001
507-389-2111

Order Results

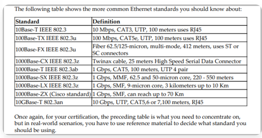
Profile Name: MSU PARKING SERVICES
Transaction ID: 060824O17-D4AACEA4-DE3F-4981-B1B0-ACECC0CDAAB8
Date/Time: 08/06/2024 12:26:02 PM
Transaction Type: SALE
Approval Message: APPROVAL
Approval Code: 006941
ECI:

Order Section

Card Number : ************3246
Amount : $247.00 USD
Invoice Number : 43118

Billing Address

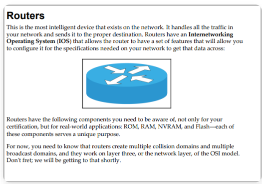
First Name : Derek
Last name : Brown
Address1 : 13335 85th Ave N
City : Maple Grove
State/Province : MN
Postal Code : 55369
Country : United States
Phone : 6123605935
Email Address : d.brown13335@gmail.com



The following table shows the more common Ethernet standards you should know about:

Standard	Definition
10Base-T IEEE 802.3	10 Mbps, CAT3, UTP, 100 meters uses RJ45
100Base-TX IEEE 802.3u	100 Mbps, CAT5e, UTP, 100 meters uses RJ45
100Base-FX IEEE 802.3u	Fiber 62.5/125-micron, multi-mode, 412 meters, uses ST or SC connectors
1000Base-CX IEEE 802.3z	Twinax cable, 25 meters High Speed Serial Data Connector
1000Base-T IEEE 802.3ab	1 Gbps, CAT5, 100 meters, UTP 4 pair
1000Base-SX IEEE 802.3z	1 Gbps, MMF, 62.5 and 50-micron core, 220 - 550 meters
1000Base-LX IEEE 802.3z	1 Gbps, SMF, 9-micron core, 3 kilometers up to 10 Km
1000Base-ZX (Cisco standard)	1 Gbps, SMF, can reach up to 70 Km
10GBase-T 802.3an	10 Gbps, UTP, CAT5,6 or 7,100 meters, RJ45

Once again, for your certification, the preceding table is what you need to concentrate on, but in real-world scenarios, you have to use reference material to decide what standard you should be using.

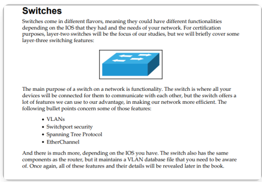


Routers

This is the most intelligent device that exists on the network. It handles all the traffic in your network and sends it to the proper destination. Routers have an **Internetworking Operating System** (**IOS**) that allows the router to have a set of features that will allow you to configure it for the specifications needed on your network to get that data across:

Routers have the following components you need to be aware of, not only for your certification, but for real-world applications: ROM, RAM, NVRAM, and Flash—each of these components serves a unique purpose.

For now, you need to know that routers create multiple collision domains and multiple broadcast domains, and they work on layer three, or the network layer, of the OSI model. Don't fret; we will be getting to that shortly.



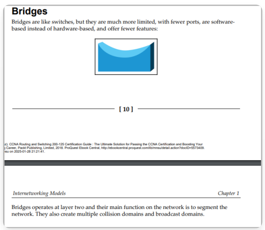
Switches

Switches come in different flavors, meaning they could have different functionalities depending on the IOS that they had and the needs of your network. For certification purposes, layer-two switches will be the focus of our studies, but we will briefly cover some layer-three switching features:

The main purpose of a switch on a network is functionality. The switch is where all your devices will be connected for them to communicate with each other, but the switch offers a lot of features we can use to our advantage, in making our network more efficient. The following bullet points concern some of those features:

- VLANs
- Switchport security
- Spanning Tree Protocol
- EtherChannel

And there is much more, depending on the IOS you have. The switch also has the same components as the router, but it maintains a VLAN database file that you need to be aware of. Once again, all of these features and their details will be revealed later in the book.



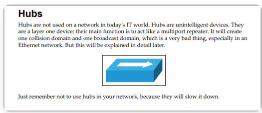
Bridges

Bridges are like switches, but they are much more limited, with fewer ports, are software-based instead of hardware-based, and offer fewer features:

[10]

Internetworking Models — Chapter 1

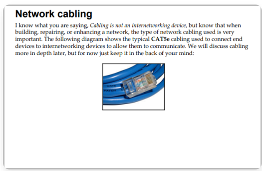
Bridges operates at layer two and their main function on the network is to segment the network. They also create multiple collision domains and broadcast domains.



Hubs

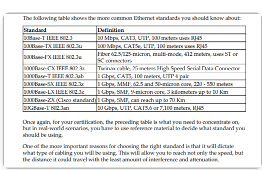
Hubs are not used on a network in today's IT world. Hubs are unintelligent devices. They are a layer one device; their main function is to act like a multiport repeater. It will create one collision domain and one broadcast domain, which is a very bad thing, especially in an Ethernet network. But this will be explained in detail later.

Just remember not to use hubs in your network, because they will slow it down.



Network cabling

I know what you are saying, *Cabling is not an internetworking device*, but know that when building, repairing, or enhancing a network, the type of network cabling used is very important. The following diagram shows the typical **CAT5e** cabling used to connect end devices to internetworking devices to allow them to communicate. We will discuss cabling more in depth later, but for now just keep it in the back of your mind:

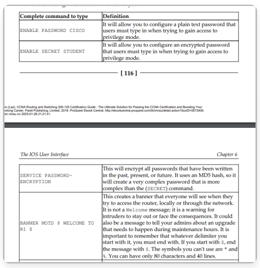


The following table shows the more common Ethernet standards you should know about:

Standard	Definition
10Base-T IEEE 802.3	10 Mbps, CAT3, UTP, 100 meters uses RJ45
100Base-TX IEEE 802.3u	100 Mbps, CAT5e, UTP, 100 meters uses RJ45
100Base-FX IEEE 802.3u	Fiber 62.5/125-micron, multi-mode, 412 meters, uses ST or SC connectors
1000Base-CX IEEE 802.3z	Twinax cable, 25 meters High Speed Serial Data Connector
1000Base-T IEEE 802.3ab	1 Gbps, CAT5, 100 meters, UTP 4 pair
1000Base-SX IEEE 802.3z	1 Gbps, MMF, 62.5 and 50-micron core, 220 - 550 meters
1000Base-LX IEEE 802.3z	1 Gbps, SMF, 9-micron core, 3 kilometers up to 10 Km
1000Base-ZX (Cisco standard)	1 Gbps, SMF, can reach up to 70 Km
10GBase-T 802.3an	10 Gbps, UTP, CAT5,6 or 7,100 meters, RJ45

Once again, for your certification, the preceding table is what you need to concentrate on, but in real-world scenarios, you have to use reference material to decide what standard you should be using.

One of the more important reasons for choosing the right standard is that it will dictate what type of cabling you will be using. This will allow you to reach not only the speed, but the distance it could travel with the least amount of interference and attenuation.

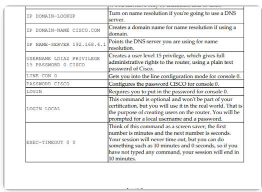


Complete command to type	Definition
`ENABLE PASSWORD CISCO`	It will allow you to configure a plain text password that users must type in when trying to gain access to privilege mode.
`ENABLE SECRET STUDENT`	It will allow you to configure an encrypted password that users must type in when trying to gain access to privilege mode.

[116]

The IOS User Interface — Chapter 6

`SERVICE PASSWORD-ENCRYPTION`	This will encrypt all passwords that have been written in the past, present, or future. It uses an MD5 hash, so it will create a very complex password that is more complex than the (`SECRET`) command.
`BANNER MOTD $ WELCOME TO R1 $`	This creates a banner that everyone will see when they try to access the router, locally or through the network. It is not a `Welcome` message; it is a warning for intruders to stay out or face the consequences. It could also be a message to tell your admins about an upgrade that needs to happen during maintenance hours. It is important to remember that whatever delimiter you start with it, you must end with. If you start with `$`, end the message with `$`. The symbols you can't use are `"` and `%`. You can have only 80 characters and 40 lines.



`IP DOMAIN-LOOKUP`	Turn on name resolution if you're going to use a DNS server.
`IP DOMAIN-NAME CISCO.COM`	Creates a domain name for name resolution if using a domain.
`IP NAME-SERVER 192.168.4.1`	Points the DNS server you are using for name resolution.
`USERNAME LDIAZ PRIVILEGE 15 PASSWORD 0 CISCO`	Creates a user level 15 privilege, which gives full administrative rights to the router, using a plain text password of Cisco.
`LINE CON 0`	Gets you into the line configuration mode for console 0.
`PASSWORD CISCO`	Configures the password CISCO for console 0.
`LOGIN`	Requires you to put in the password for console 0.
`LOGIN LOCAL`	This command is optional and won't be part of your certification, but you will use it in the real world. That is the purpose of creating users on the router. You will be prompted for a local username and a password.
`EXEC-TIMEOUT 0 0`	Think of this command as a screen saver; the first number is minutes and the next number is seconds. Your session will never time out, but you can do something such as 10 minutes and 0 seconds, so if you have not typed any command, your session will end in 10 minutes.

LOGGING SYNCHRONOUS	As you type, the router gives you feedback, and that is great, but it does interrupt your typing. So, if you type this command on whichever line you're configuring on, your typing will not be interrupted by the feedback.
LINE AUX 0	This is just another port you can access your router on, but you must have an external modem, and you must configure the router for this to work.
LINE VTY 0 15	This is to enter your virtual terminal lines; the default is 0 4; if you have more, you could type 0 100, for example, and make passwords and the login local, so individuals would have to type a username. These are your Telnet lines and SSH lines.
EXIT	This command just takes you one step back, meaning you could go from interface mode, to global config, then to privilege mode. You are backing out one step at a time. A shortcut to this is a keyboard command Ctrl + Z, so if you're in sub-interface mode, you could simply type Ctrl + Z and you would go straight to privilege mode.
IP SSH AUTHENTICATION-RETRIES 3	This command is to configure SSH, so you can set how many times you can type in a password before getting locked out.
IP SSH TIME-OUT 120	If you SSH into a router and do not type anything for 120 seconds, your session will time out.
CRYPTO KEY GENERATE RSA	This is the command to generate an RSA key. It's useful to know that RSA stands for Rivest Adi Shamir. The initials of its co-founders.
How many bits in the modulus [512]: 512	This command will tell it how much encryption we are going to use. Keep in mind, the higher the encryption, the more overhead on the router.
TRANSPORT INPUT SSH	Instructs the router. If you are connecting through the network, you could only use SSH.
INT F0/0	This command lets you get into the Fast Ethernet 0/0 interface.
IP ADDRESS 192.168.1.254 255.255.255.0	This allows you to put an IP address on the router.

The IOS User Interface Chapter 4

DESCRIPTION CONNECTION TO LAN	This is a courtesy command; when someone is doing any type of troubleshooting, it would be nice to know what interface is facing what and what type of interface it is. You could specify whatever you think is going to be useful for someone.
NO SHUT	Turns on an interface.
CLOCK RATE 4000000	This is the actual physical speed of that connection. You normally enter this on serial cables and on the DCE or Data Communication Equipment side of the cable.
COPY RUN START	This will copy all your configuration you have in RAM into NVRAM, which is the startup-configuration file or backup. This is the file that is used on boot up.
Destination filename [startup-config]? HIT THE ENTER KEY	This is the router asking you where you want to save the configuration. The default is in-between the brackets, so simply hit the Enter key. This is not so on all routers; for higher routers, you have to save the configuration.

The following table will help you configure your Cisco devices faster; they are called **editing commands**:

Ctrl + A	Moves your cursor to the beginning of the line
Ctrl + E	Moves your cursor to the end of the line
Ctrl + B	Moves back one character
Ctrl + F	Moves forward one word
Ctrl + D	Deletes a single character (when the cursor is in front of that command)
Ctrl + R	Redisplays a line
Ctrl + U	Erases a line
Ctrl + W	Erases a word
Ctrl + Z	Takes you back to privilege mode, no matter what mode you are in in the router
Tab	Finishes typing a command for you
Esc + B	Moves back one word
Esc + F	Moves forward one word
Backspace	Deletes a single character (when cursor is behind that command)

The IOS User Interface Chapter 4

I did a traceroute from R3 to PC1 using the destination IP address of PC1. The traceroute command tells me the next hop address of R2 and then the next hop address of R1 and finally the destination IP of PC1. This is a very helpful troubleshooting tool, because you can pinpoint where the packet dropped or lost connectivity, and further troubleshoot from that point.

While we are here, another tool in our arsenal to look at things in a router or switch is called the **pipe** command. The pipe command is an output modifier (|) using this will allow you to take a more direct approach when looking at configurations on your router. The following would be an example of using the pipe command:

```
Router#sh run | begin interface
Router#sh ip route | include 10.1.1.4
```

In the certification exam, you won't be using it, but in your job, you will. It is essential that you learn how to use the pipe. Imagine this scenario. You type the command to look at the running-config file and hit Enter. It is an enterprise company, there may be thousands of lines that make up your configuration, and you will overwhelm your router and maybe even crash it. At a minimum, you will go through excruciating pain wasting time scrolling through all those lines, not to mention the boss will be very upset.

Using the pipe (|) will allow you to see exactly what you want to see.

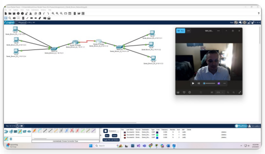
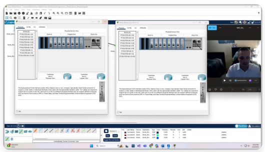
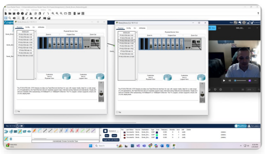
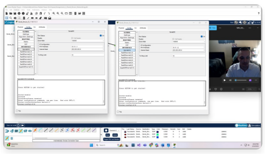
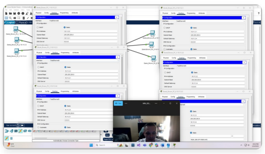
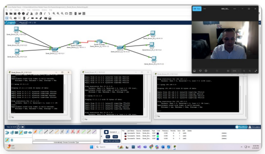
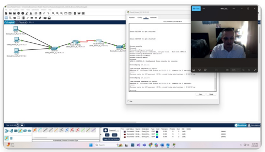
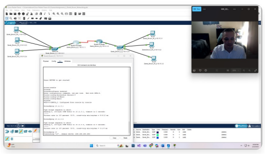
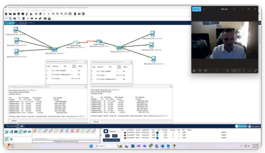
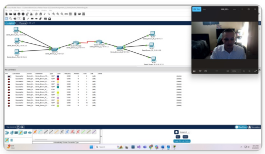
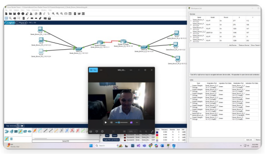
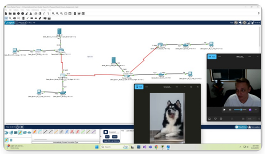
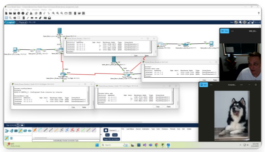
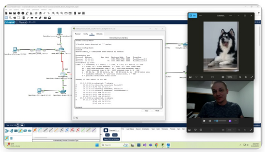
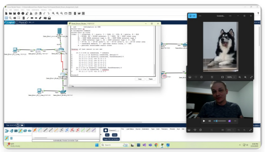
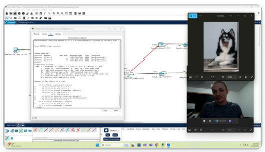
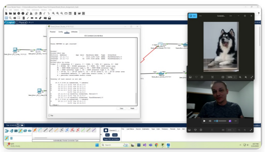
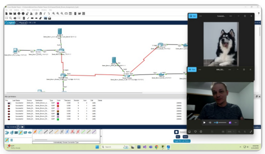
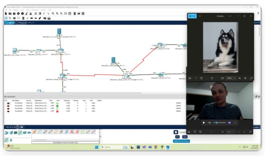
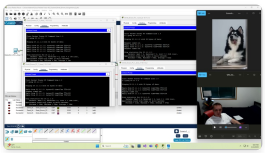
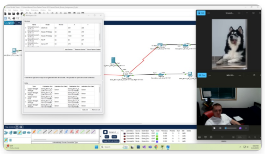
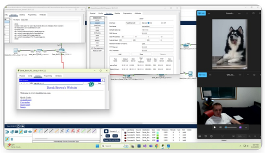
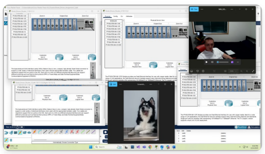
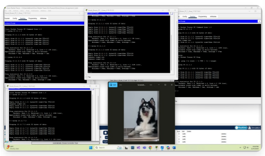
The following table will help you configure your Cisco devices faster; they are called **editing commands**:

Ctrl + A	Moves your cursor to the beginning of the line
Ctrl + E	Moves your cursor to the end of the line
Ctrl + B	Moves back one character
Ctrl + F	Moves forward one word
Ctrl + D	Deletes a single character (when the cursor is in front of that command)
Ctrl + R	Redisplays a line
Ctrl + U	Erases a line
Ctrl + W	Erases a word
Ctrl + Z	Takes you back to privilege mode, no matter what mode you are in in the router
Tab	Finishes typing a command for you
Esc + B	Moves back one word

You can only type the show command in privilege mode, and its purpose is to show us the output of what we want to see. There is a huge list of subcommands you can type, but the following table is what I need you to store in your NVRAM, meaning your brain:

Commands	Purpose
show running-configuration	Will show configuration in RAM
show startup-configuration	Will show configuration in NVRAM
show ip interface brief	Will show a summary of the interface, IP, and status
show ip route	Will show the routing table
show ip protocol	Will show routing protocols configured on the router
show ip dhcp binding	Will show which address you're binding with
show ip nat translation	Will show the translation from private to public address and ports
show access-list	Will show all access lists on the router that are configured

This is just the tip of the iceberg; there are a lot of show commands for you to use, and these are just a few to get you started. We will be going over more show commands in the other chapters. The only downside to the show command is that you must use it in privilege mode, so if you're in router configuration mode, or standard named access-list mode, you need to go back to privilege mode to do a show command. But that is what you must do for certification purposes.

1. Harmonic Wave Characteristics:

Question: For a harmonic wave given by the equation:

$$y(x,t) = 5\cos(2\pi(0.2x - 3t))\,\mathrm{m}$$

Determine the following: (a) Wavelength
(b) Frequency
(c) Propagation constant
(d) Angular frequency
(e) Period
(f) Velocity
(g) Amplitude

2. Phase Velocity:

Question: Use the constant phase condition to determine the velocity of the following waves in terms of the constants A, B, C, and D. Distances are in meters and time in seconds. Verify your results dimensionally. (a) $y(x,t) = A\cos(Bx - Ct + D)$
(b) $y(x,t) = A\sin(Cx + Bt + D)$

3. Displacement of a Harmonic Wave:

Question: A harmonic wave traveling in the positive x-direction has, at $x = 2\,\mathrm{m}$, a displacement of 13 units at $t = 0$ and a displacement of -8 units at $x = 3\,\mathrm{m}$ and $t = 5\,\mathrm{s}$. Write the equation for the wave.

4. Sinusoidal Wave Maximum Displacement:

Question: (a) Show that if the maximum positive displacement of a sinusoidal wave occurs at distance 5 cm from the origin, when its initial phase angle is $\phi_0 = \frac{\pi}{2}$, the wavelength is 10 cm.
(b) Determine the initial phase and sketch the wave when $A = 3\,\mathrm{cm}$, $k = \frac{\pi}{5}\,\mathrm{cm}^{-1}$, and $t = 0$.
(c) What are the appropriate initial phase angles for (b) when a cosine function is used instead?

5. Plane Wave Equation in Three Dimensions:

Question: (a) Write the equation for a sinusoidal plane wave propagating along the x-axis in three dimensions.
(b) Now write the equation for a sinusoidal plane wave propagating along the y-axis.
(c) Write the equation for a sinusoidal plane wave propagating perpendicular to the planes of the wave.

6. Complex Number Phase Shift:

Question: Show that if z is a complex number, the following hold: (a) $\mathrm{Re}(z) = \frac{z+\bar{z}}{2}$
(b) $\mathrm{Im}(z) = \frac{z-\bar{z}}{2i}$
(c) $|z| = \sqrt{z\bar{z}}$
(d) $\arg(z) = \tan^{-1}\left(\frac{\mathrm{Im}(z)}{\mathrm{Re}(z)}\right)$

7. Phase Shift in Complex Form:

Question: Show that a wave function, expressed in complex form, is shifted in phase: (a) by $\frac{\pi}{2}$ when multiplied by i
(b) by π when multiplied by -1

8. Superposition of Waves:

Question: Two waves of the same amplitude, speed, and frequency travel together in the same region of space. The resultant wave may be written as a sum of the individual waves. With the help of complex exponentials, show that:

$$y(x,t) = A\left[\exp(i(kx - \omega t)) + \exp(i(kx + \omega t))\right]$$

9. Solar Energy Flux:

Question: The energy flow to the Earth's surface associated with sunlight is about $1.0\,\mathrm{kW/m^2}$. Find the maximum values of the electric field amplitude E_0 and magnetic field amplitude B_0 for a wave of this power density.

10. Light in Glass:

Question: A light wave is traveling in glass of refractive index $n = 1.50$. If the electric field amplitude of the wave is known to be $100\,\mathrm{V/m}$, find: (a) The wavelength of the light in glass
(b) The speed of light in the glass
(c) The energy flux density in the glass

11. Harmonic Wave Equation in Terms of Constants:

Question: Derive the equation for a harmonic wave traveling in the x-direction. Use the following known variables: amplitude A, wave number $k = \frac{2\pi}{\lambda}$, angular frequency $\omega = 2\pi f$, and velocity v. Write the wave equation in terms of the amplitude, frequency, and wavelength.

12. Cylindrical Wave Amplitude:

Question: Show that, in order to conserve flux, the amplitude of a cylindrical wave must vary inversely with the distance from the source.

13. Doppler Effect in Light:

Question: How does the Doppler effect formula for light change when the source is moving away from the observer at a high velocity? Derive the equation and provide a general solution.

14. Light Polarization and Amplitude:

Question: A light wave is traveling through a polarizer. If the initial amplitude of the electric field is E_0, find the amplitude of the electric field after the light passes through the polarizer at an angle θ with respect to the axis of polarization.

15. Frequency of a Moving Source:

Question: A source of sound is moving towards an observer at 30 m/s. The frequency emitted by the source is 500 Hz. Find the frequency heard by the observer, assuming the speed of sound in air is 343 m/s.

16. Light Wave at the Earth's Surface:

Question: The solar constant is the radiant flux density (irradiance) from the sun at the surface of the Earth's atmosphere and is about $1361\,\mathrm{W/m^2}$. Assume an average wavelength of $700\,\mathrm{nm}$ for the sun's radiation. Find: (a) The amplitude of the electric field E_0 for this irradiance
(b) The number of photons that arrive each second on each square meter of a solar panel.

17. Maximum Electric Field in a Laser Beam:

Question: A laser beam of power $P = 2\,\mathrm{W}$ is concentrated into a circular spot with radius $r = 0.01\,\mathrm{m}$. Find the maximum electric field amplitude E_0 at the center of the spot.

18. Irradiance of Light from a Lamp:

Question: A light from a 220-W lamp spreads uniformly in all directions. Find: (a) The irradiance of the optical electromagnetic waves at a distance of 10 m from the lamp.
(b) The amplitude of the electric field at this distance.

19. Phase Shift of a Complex Exponential:

Question: Show that multiplying a wave function $y(x,t) = Ae^{i(kx-\omega t)}$ by $e^{i\phi}$ results in a phase shift of ϕ.

20. Doppler Shift for a Moving Quasar:

Question: A quasar near the limits of the observed universe shows a wavelength that is 4.80 times the wavelength emitted by the same molecules on Earth. If the Doppler effect is responsible for this shift, what velocity does it determine for the quasar?

1. Harmonic Wave Characteristics:

Given the wave equation:

$$y(x,t) = 5\cos(2\pi(0.2x - 3t))\,\mathrm{m}$$

(a) **Wavelength:**
The wave number k is $2\pi/\lambda$, and from the wave equation, we can identify $k = 2\pi(0.2) = 0.4\pi$.
So, $\lambda = \frac{2\pi}{0.4\pi} = 5\,\mathrm{m}$.

(b) **Frequency:**
The angular frequency $\omega = 2\pi f$, and from the wave equation, we identify $\omega = 6\pi$.
So, $f = \frac{6\pi}{2\pi} = 3\,\mathrm{Hz}$.

(c) **Propagation Constant:**
From the equation, the propagation constant is $k = 0.4\pi\,\mathrm{rad/m}$.

(d) **Angular Frequency:**
The angular frequency is $\omega = 6\pi\,\mathrm{rad/s}$.

(e) **Period:**
The period $T = \frac{1}{f} = \frac{1}{3} = 0.333\,\mathrm{s}$.

(f) **Velocity:**
The velocity $v = \frac{\omega}{k} = \frac{6\pi}{0.4\pi} = 15\,\mathrm{m/s}$.

(g) **Amplitude:**
The amplitude is $A = 5\,\mathrm{m}$.

2. Phase Velocity:

(a) For $y(x,t) = A\cos(Bx - Ct + D)$,
The phase velocity is $v = \frac{C}{B}$.

(b) For $y(x,t) = A\sin(Cx + Bt + D)$,
The phase velocity is $v = \frac{-B}{C}$.

3. Displacement of a Harmonic Wave:

We are given the following conditions:
At $x = 2\,\mathrm{m}$, displacement is 13 units, and at $x = 3\,\mathrm{m}$, displacement is -8 units at $t = 5\,\mathrm{s}$.

The general form of the wave equation is:

$$y(x,t) = A\cos(kx - \omega t + \phi)$$

Using the known values, we can solve for A, k, ω, and ϕ.

4. Sinusoidal Wave Maximum Displacement:

(a) The equation for maximum positive displacement is $A\cos(kx - \omega t + \phi)$. If the maximum displacement is at $x = 5\,\mathrm{cm}$, then we use $\phi = \frac{\pi}{2}$.

(b) To determine the initial phase, we need the form $A\cos(kx - \omega t)$. The parameters $k = \frac{\pi}{5}\,\mathrm{cm}^{-1}$ and $t = 0$.

(c) If we use a cosine function, the initial phase angles would be adjusted based on ϕ in the cosine form.

5. Plane Wave Equation in Three Dimensions:

(a) A plane wave propagating along the x-axis:

$$y(x,t) = A\cos(kx - \omega t)$$

(b) A plane wave propagating along the y-axis:

$$y(y,t) = A\cos(ky - \omega t)$$

(c) A plane wave propagating perpendicular to the planes:

$$y(x,y,z,t) = A\cos(kx + ky + kz - \omega t)$$

6. Complex Number Phase Shift:

(a) $\mathrm{Re}(z) = \frac{z+\bar{z}}{2}$
(b) $\mathrm{Im}(z) = \frac{z-\bar{z}}{2i}$
(c) $|z| = \sqrt{z\bar{z}}$
(d) $\arg(z) = \tan^{-1}\left(\frac{\mathrm{Im}(z)}{\mathrm{Re}(z)}\right)$

7. Phase Shift in Complex Form:

(a) If we multiply a wave function by i, we get a phase shift of $\frac{\pi}{2}$.
(b) If we multiply by -1, we get a phase shift of π.

8. Superposition of Waves:

The sum of two complex exponentials:

$$y(x,t) = A\left[\exp(i(kx-\omega t)) + \exp(i(kx+\omega t))\right]$$

Simplifying:

$$y(x,t) = 2A\cos(kx-\omega t)$$

9. Solar Energy Flux:

(a) The power density $S = 1.0\,\mathrm{kW/m^2}$.
Using the relation $S = \frac{E_0^2}{Z_0}$, where $Z_0 \approx 377\,\Omega$, we solve for E_0.
$E_0 = \sqrt{SZ_0} = \sqrt{1000 \times 377} \approx 19.4\,\mathrm{V/m}$.
(b) The magnetic field amplitude $B_0 = \frac{E_0}{c}$, where $c = 3\times10^8\,\mathrm{m/s}$, so $B_0 \approx 6.47\times10^{-8}\,\mathrm{T}$.

10. Light in Glass:

(a) The wavelength in glass is $\lambda_{\text{glass}} = \frac{\lambda_0}{n} = \frac{700\times10^{-9}}{1.5} = 467\,\mathrm{nm}$.
(b) The speed of light in glass is $v = \frac{c}{n} = \frac{3\times10^8}{1.5} = 2\times10^8\,\mathrm{m/s}$.
(c) The energy flux density is $S = \frac{E_0^2}{Z_0}$.

11. Harmonic Wave Equation in Terms of Constants:

The general form of the wave equation is:

$$y(x,t) = A\cos(kx-\omega t)$$

Here $k = \frac{2\pi}{\lambda}$, and $\omega = 2\pi f$, so the wave equation is:

$$y(x,t) = A\cos\left(\frac{2\pi}{\lambda}x - 2\pi f t\right)$$

12. Cylindrical Wave Amplitude:

To conserve flux, the amplitude of a cylindrical wave must decrease with distance as $A(r) \propto \frac{1}{\sqrt{r}}$, where r is the radial distance from the source.

13. Doppler Effect in Light:

For light, the Doppler shift formula is:

$$\lambda' = \lambda\left(1+\frac{v}{c}\right)$$

where λ' is the observed wavelength, λ is the emitted wavelength, v is the velocity of the source, and c is the speed of light.

14. Light Polarization and Amplitude:

The amplitude of the electric field after passing through a polarizer is:

$$E = E_0\cos(\theta)$$

15. Frequency of a Moving Source:

The frequency heard by the observer is given by the Doppler effect formula:

$$f' = f\left(\frac{c}{c-v_s}\right)$$

Substituting $f = 500\,\mathrm{Hz}$, $v_s = 30\,\mathrm{m/s}$, and $c = 343\,\mathrm{m/s}$, we get:

$$f' = 500 \times \frac{343}{343-30} \approx 552.7\,\mathrm{Hz}$$

16. Light Wave at the Earth's Surface:

(a) The irradiance is $S = 1361\,\mathrm{W/m^2}$.
(b) The amplitude of the electric field $E_0 = \sqrt{SZ_0} = \sqrt{1361\times377} \approx 67.4\,\mathrm{V/m}$.

17. Maximum Electric Field in a Laser Beam:

The irradiance is given by:

$$I = \frac{P}{A}$$

where $P = 2\,\mathrm{W}$ and $A = \pi r^2 = \pi(0.01)^2 = 3.14\times10^{-4}\,\mathrm{m^2}$.
The irradiance $I = \frac{2}{3.14\times10^{-4}} = 6369\,\mathrm{W/m^2}$.
The maximum electric field is $E_0 = \sqrt{IZ_0} \approx \sqrt{6369\times377} \approx 49.3\,\mathrm{V/m}$.

18. Irradiance of Light from a Lamp:

(a) The irradiance at a distance $r = 10\,\mathrm{m}$ is:

$$I = \frac{P}{4\pi r^2}$$

Substituting $P = 220\,\mathrm{W}$:

$$I = \frac{220}{4\pi(10)^2} \approx 0.175\,\mathrm{W/m^2}$$

(b) The amplitude of the electric field is:

$$E_0 = \sqrt{IZ_0} \approx \sqrt{0.175\times377} \approx 4.1\,\mathrm{V/m}$$

19. Phase Shift of a Complex Exponential:

Multiplying by $e^{i\phi}$ results in a phase shift by ϕ:

$$y(x,t) = Ae^{i(kx-\omega t)} \times e^{i\phi} = Ae^{i(kx-\omega t+\phi)}$$

20. Doppler Shift for a Moving Quasar:

Given that the wavelength is 4.80 times the emitted wavelength, the velocity of the quasar is given by the relativistic Doppler shift formula:

$$z = \frac{\lambda'-\lambda}{\lambda} = \frac{v}{c}$$

Substitute $\lambda' = 4.80\lambda$ to find the velocity v.

1. Two plane waves are given by:

$$y_1(x,t) = 5\,\mathrm{cm}\cos(2\pi(2x-3t))$$
$$y_2(x,t) = 7\,\mathrm{cm}\cos(2\pi(2x-3t+\pi))$$

a. Describe the motion of the two waves.
b. At what instant is their superposition everywhere zero?
c. At what point is their superposition always zero?

2. Two harmonic waves are represented as:

$$y_1 = 3\,\mathrm{cm}\cos(2\pi(x-t))$$
$$y_2 = 4\,\mathrm{cm}\cos(2\pi(x-t+\frac{\pi}{2}))$$

a. Show the motion of these waves on a phasor diagram.
b. Determine the mathematical expression for the resultant wave.

3. Find the resultant of the superposition of two harmonic waves with amplitudes of 3 and 4 and phases of $\frac{\pi}{4}$ and $\frac{\pi}{2}$, respectively. Both waves have a period of 2 s.

4. Two waves traveling along the same line are given by:

$$y_1 = 5\,\mathrm{cm}\cos(2\pi(3x-4t))$$
$$y_2 = 7\,\mathrm{cm}\cos(2\pi(3x-4t+\frac{\pi}{3}))$$

Write the form of the resultant wave.

5. Plot and write the equation for the superposition of the following harmonic waves, where the period of each is 3 s:

$$y_1 = 5\,\mathrm{cm}\cos(2\pi\left(\frac{x}{3}-\frac{t}{3}\right))$$
$$y_2 = 6\,\mathrm{cm}\cos(2\pi\left(\frac{x}{3}-\frac{t}{3}+\frac{\pi}{4}\right))$$

6. One hundred antennas are emitting identical waves:

$$y = 3\,\mathrm{cm}\cos(2\pi(x-t))$$

6. One hundred antennas are emitting identical waves:

$$y = 3\,\mathrm{cm}\cos(2\pi(x-t))$$

The waves are brought together at a point. What is the amplitude of the resultant when: a. All waves are in phase (coherent sources).
b. The waves have random phase differences (incoherent sources).

7. Two plane waves of the same frequency and with vibrations in the z-direction are given by:

$$y_1(x,t) = (2\,\mathrm{cm})\cos(2\pi(0.2x-3t+\frac{\pi}{2}))$$
$$y_2(x,t) = (4\,\mathrm{cm})\cos(2\pi(0.2x-3t+\frac{\pi}{4}))$$

Write the resultant wave form expressing their superposition at the point $x = 0, t = 0$.

8. Beginning with the relation between group velocity and phase velocity:

a. Express the relation in terms of refractive index n and wavelength λ.
b. Determine whether the group velocity is greater or less than the phase velocity in a medium having normal dispersion.

9. The dispersive power of glass is defined as the ratio of the change in refractive index with respect to wavelength. Find the approximate group velocity in glass whose dispersive power is 0.2 and refractive index at $\lambda = 500\,\mathrm{nm}$ is $n = 1.5$.

10. The dispersion curve of glass is given by Cauchy's empirical equation. Find the phase and group velocities for light of 500-nm wavelength in a particular glass for which $A = 1.4$ and $B = 2.5\times10^6$.

11. The dielectric constant K of a gas is related to its index of refraction by the relation $n = \sqrt{K}$.

a. Show that the group velocity for waves traveling in the gas may be expressed in terms of the dielectric constant by $v_g = \frac{c}{\sqrt{K}}$.
b. If K varies with frequency, show the dependence of group velocity on frequency.

12. Show that group velocity can be expressed as $v_g = \frac{c}{n} - \frac{1}{n^2}\frac{dn}{d\lambda}$.

b. Find the group velocity for plane waves in a dispersive medium, for which $n = A + B\lambda^2$. Interpret the result.

13. Waves on the ocean have different velocities, depending on their depth. Long-wavelength waves, traveling deep in the ocean, have a speed given by $v = \sqrt{\frac{g}{k}}$, where k is the wave number.

a. Show that the group velocity for long-wavelength waves is equal to their phase velocity.
b. For short-wavelength waves, show that the group velocity is less than the phase velocity.

14. A laser emits a monochromatic beam of wavelength $\lambda = 600\,\mathrm{nm}$, which is reflected normally from a plane mirror receding at a speed of $3\,\mathrm{m/s}$. What is the beat frequency between the incident and reflected light?

15. Standing waves are produced by the superposition of the wave:

$$y = (6\,\mathrm{cm})\sin(2\pi\left(\frac{x}{4}-\frac{t}{3}\right))$$

and its reflection in a medium whose absorption is negligible. Find the amplitude, wavelength, length of one loop, velocity, and period.

16. A medium is disturbed by an oscillation described by:

$$y(x,t) = 8\,\mathrm{cm}\cos(2\pi(x-4t))$$

a. Determine the amplitude, frequency, wavelength, speed, and direction of the component waves.
b. What is the internodal distance?
c. What are the displacement, velocity, and acceleration of a particle in the medium at $x = 0$ and $t = 0$?

17. Express the plane waves of Eqs. (19) and (20) in the complex representation. In this form, show that the superposition of the waves is the standing wave given by Eq. (22).

18. A certain argon-io
Estimate the number
the laser cavity is 1 m

19. A signal is modula
wave has a frequency
frequency modulatinc

18. A certain argon-ion laser can support lasing over a frequency range of 6 GHz. Estimate the number of standing wave modes that might be in the laser output if the laser cavity is 1 m long.

19. A signal is modulated using amplitude modulation (AM) in which the carrier wave has a frequency of $100\,\mathrm{MHz}$. The modulated signal contains a lower-frequency modulating signal with a frequency of $1\,\mathrm{MHz}$.

a. What is the group velocity of the signal?
b. What is the phase velocity of the carrier wave?

20. A wave traveling in a medium has a wavelength of $2\,\mathrm{m}$ and frequency $10\,\mathrm{Hz}$.

a. Find the phase velocity of the wave.
b. If the refractive index of the medium changes with wavelength, what is the group velocity of the wave?

18. A certain argon-ion laser can support lasing over a frequency range of 6 GHz. Estimate the number of standing wave modes that might be in the laser output if the laser cavity is 1 m long.

19. A signal is modulated using amplitude modulation (AM) in which the carrier wave has a frequency of $100\,\mathrm{MHz}$. The modulated signal contains a lower-frequency modulating signal with a frequency of $1\,\mathrm{MHz}$.

a. What is the group velocity of the signal?
b. What is the phase velocity of the carrier wave?

20. A wave traveling in a medium has a wavelength of $2\,\mathrm{m}$ and frequency $10\,\mathrm{Hz}$.

a. Find the phase velocity of the wave.
b. If the refractive index of the medium changes with wavelength, what is the group velocity of the wave?

1. Two plane waves are given by:

$$y_1(x,t) = 5\,\mathrm{cm}\cos(2\pi(2x-3t))$$
$$y_2(x,t) = 7\,\mathrm{cm}\cos(2\pi(2x-3t+\pi))$$

a. Describe the motion of the two waves:

Both waves have the same frequency and wavelength (since they are of the same form, $2\pi(2x-3t)$), so they move in sync but with a phase shift of π.

b. At what instant is their superposition everywhere zero?

Since the waves are in opposite phases (due to the phase shift of π), their superposition is zero when the amplitude of both waves is equal in magnitude and opposite in sign. This happens at any time when the waves' phases align in such a way that they cancel out each other.

The superposition of the two waves:

$$y(x,t) = 5\cos(2\pi(2x-3t)) + 7\cos(2\pi(2x-3t)+\pi)$$

Since $\cos(\theta+\pi) = -\cos(\theta)$, we have:

$$y(x,t) = 5\cos(2\pi(2x-3t)) - 7\cos(2\pi(2x-3t)) = -2\cos(2\pi(2x-3t))$$

The superposition is always nonzero, but the amplitude is always negative.

c. At what point is their superposition always zero?

There is no point where their superposition is always zero, since the waves are out of phase but not of equal amplitude at all points.

2. Two harmonic waves are represented as:

$$y_1 = 3\,\mathrm{cm}\cos(2\pi(x-t))$$
$$y_2 = 4\,\mathrm{cm}\cos(2\pi(x-t+\frac{\pi}{2}))$$

a. Show the motion of these waves on a phasor diagram.

The phasor diagram shows two vectors representing the waves. One vector points at $3\,\mathrm{cm}$ along the real axis, and the second vector points at $4\,\mathrm{cm}$ at a phase angle of $\frac{\pi}{2}$.

b. Determine the mathematical expression for the resultant wave.

The resultant wave $y(x,t)$ can be found by adding the two waves. Since $\cos(\theta+\frac{\pi}{2}) = -\sin(\theta)$, we have:

$$y(x,t) = 3\cos(2\pi(x-t)) + 4\sin(2\pi(x-t))$$

Using the trigonometric identity $R\cos(\theta-\phi) = A\cos(\theta) + B\sin(\theta)$, we find:

$$R = \sqrt{3^2+4^2} = 5\,\mathrm{cm}$$

The phase ϕ is given by $\tan(\phi) = \frac{4}{3}$, so:

$$\phi = \tan^{-1}\left(\frac{4}{3}\right)$$

Therefore, the resultant wave is:

$$y(x,t) = 5\cos\left(2\pi(x-t)-\phi\right)$$

where $\phi = \tan^{-1}\left(\frac{4}{3}\right)$.

3. Find the resultant of the superposition of two harmonic waves with amplitudes of 3 and 4 and phases of $\frac{\pi}{4}$ and $\frac{\pi}{2}$, respectively. Both waves have a period of 2 s.

Using the formula for the resultant of two harmonic waves with different amplitudes and phases, we can calculate the amplitude R and phase ϕ:

$$R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_2-\phi_1)}$$

Here, $A_1 = 3$, $A_2 = 4$, $\phi_1 = \frac{\pi}{4}$, $\phi_2 = \frac{\pi}{2}$:

$$R = \sqrt{3^2+4^2+2\cdot3\cdot4\cos\left(\frac{\pi}{2}-\frac{\pi}{4}\right)}$$

$$R = \sqrt{9+16+24\cos\left(\frac{\pi}{4}\right)} = \sqrt{25+24\cdot\frac{\sqrt{2}}{2}} = \sqrt{25+12\sqrt{2}} \approx 9.06$$

The phase ϕ is:

$$\tan(\phi) = \frac{A_2\sin(\phi_2)+A_1\sin(\phi_1)}{A_2\cos(\phi_2)+A_1\cos(\phi_1)}$$

Using the given values, you can solve for the phase.

4. Two waves traveling along the same line are given by:

$$y_1 = 5\,\mathrm{cm}\cos(2\pi(3x-4t))$$
$$y_2 = 7\,\mathrm{cm}\cos(2\pi(3x-4t+\frac{\pi}{3}))$$

The resultant wave can be found using the same method as before:

$$y(x,t) = 5\cos(2\pi(3x-4t)) + 7\cos(2\pi(3x-4t+\frac{\pi}{3}))$$

Using trigonometric identities:

$$y(x,t) = \sqrt{5^2+7^2+2\cdot5\cdot7\cos\left(\frac{\pi}{3}\right)}\cos(2\pi(3x-4t)+\phi)$$

Calculate the value of the amplitude R and the phase ϕ as done in previous steps.

5. Plot and write the equation for the superposition of the following harmonic waves, where the period of each is 3 s:

The two waves are:

$$y_1 = 5\,\mathrm{cm}\cos(2\pi\left(\frac{x}{3}-\frac{t}{3}\right))$$
$$y_2 = 6\,\mathrm{cm}\cos(2\pi\left(\frac{x}{3}-\frac{t}{3}+\frac{\pi}{4}\right))$$

The resultant wave is:

$$y(x,t) = \sqrt{5^2+6^2+2\cdot5\cdot6\cos\left(\frac{\pi}{4}\right)}\cos\left(2\pi\left(\frac{x}{3}-\frac{t}{3}\right)+\phi\right)$$

Calculate R and ϕ to get the final expression.

6. One hundred antennas are emitting identical waves:

(a) All waves are in phase (coherent sources):
The amplitude is $R = 100\cdot A$ since the waves are in phase.

(b) The waves have random phase differences (incoherent sources):
The amplitude is $R = A\sqrt{100} = 10A$, where A is the amplitude of each individual wave.

7. Two plane waves of the same frequency and with vibrations in the z-direction are given by:

$$y_1(x,t) = (2\,\mathrm{cm})\cos(2\pi(0.2x-3t+\frac{\pi}{2}))$$
$$y_2(x,t) = (4\,\mathrm{cm})\cos(2\pi(0.2x-3t+\frac{\pi}{4}))$$

The resultant wave can be found by applying the superposition principle and using trigonometric identities as described in previous problems.

8. Beginning with the relation between group velocity and phase velocity in the form:

$$v_p = \frac{c}{n}, \quad v_g = \frac{d\omega}{dk}$$

(a) Express the relation in terms of n and k:

We know that:

$$\omega = v_pk = \frac{c}{n}k$$

Now, we differentiate ω with respect to k:

$$v_g = \frac{d\omega}{dk} = \frac{d}{dk}\left(\frac{c}{n}k\right)$$

Since n depends on k (i.e., the refractive index is frequency-dependent in dispersive media), we can apply the chain rule:

$$v_g = \frac{c}{n} - \frac{c}{n^2}\frac{dn}{dk}$$

Thus, the group velocity in terms of n and k is:

$$v_g = \frac{c}{n} - \frac{c}{n^2}\frac{dn}{dk}$$

(b) Determine whether the group velocity is greater or less than the phase velocity in a medium having normal dispersion:

In a medium with normal dispersion, $\frac{dn}{dk} > 0$, meaning that the refractive index increases with frequency. This leads to:

$$v_g < v_p$$

So, in a medium with normal dispersion, the group velocity is less than the phase velocity.

9. The dispersive power of glass is defined as the ratio:

$$\text{Dispersive power} = \frac{1}{\lambda}\left(\frac{d^2n}{d\lambda^2}\right)$$

where C, D, and F refer to the Fraunhofer wavelengths.

Find the approximate group velocity in glass whose dispersive power is given, and for which:

Use the formula for group velocity in a dispersive medium:

$$v_g = \frac{c}{n} - \frac{c}{n^2}\frac{dn}{dk}$$

For glass, we have empirical data for n and $\frac{dn}{d\lambda}$, and $\frac{d^2n}{d\lambda^2}$. Using these, you can estimate v_g.

10. The dispersion curve of glass can be represented approximately by Cauchy's empirical equation:

$$n(\lambda) = A + \frac{B}{\lambda^2}$$

Find the phase and group velocities for light of 500-nm wavelength in a particular glass for which $A = 1.40$ and $B = 2.5\times10^6\,\mathrm{nm}^2$:

1. **Phase velocity:**

$$v_p = \frac{c}{n(\lambda)}$$

For $\lambda = 500\,\mathrm{nm}$, substitute into the Cauchy equation:

$$n(500) = 1.40 + \frac{2.5\times10^6}{500^2}$$

Compute $n(500)$, then use $v_p = \frac{c}{n(\lambda)}$.

2. **Group velocity:** To find v_g, we differentiate $n(\lambda)$ with respect to λ:

$$\frac{dn}{d\lambda} = -2\cdot\frac{B}{\lambda^3}$$

Then, use the formula for group velocity:

$$v_g = \frac{c}{n(\lambda)} - \frac{c}{n(\lambda)^2}\cdot\frac{dn}{d\lambda}$$

11. The dielectric constant K of a gas is related to its index of refraction by the relation:

$$K = n^2$$

(a) Show that the group velocity for waves traveling in the gas may be expressed in terms of the dielectric constant:

Start with the relation for the group velocity v_g in terms of the refractive index:

$$v_g = \frac{c}{n} - \frac{c}{n^2}\frac{dn}{dk}$$

Now use the fact that $n = \sqrt{K}$, so:

$$v_g = \frac{c}{\sqrt{K}} - \frac{c}{K}\frac{d(\sqrt{K})}{dk}$$

(b) If the second term is very small compared to the first, show that:

If the second term is negligible, then:

$$v_g \approx \frac{c}{\sqrt{K}}$$

This means that the group velocity is approximately the speed of light divided by the square root of the dielectric constant.

12. Show that group velocity can be expressed as:

$$v_g = \frac{c}{n}\left(1 - \frac{\lambda}{n}\frac{dn}{d\lambda}\right)$$

This is a general formula for group velocity in a dispersive medium. You can show this by differentiating the expression for ω with respect to k, and using the chain rule for the relationship between n and λ.

(b) Find the group velocity for plane waves in a dispersive medium:

Given a relationship of the form:

$$n = A + B\lambda^2$$

You can differentiate $n(\lambda)$ with respect to λ and substitute into the group velocity formula to find v_g.

13. Waves on the ocean have different velocities depending on their depth:

For **long-wavelength waves**, the speed is approximately:

$$v = \sqrt{gD}$$

where D is the depth of the ocean, and g is the acceleration due to gravity.

For **short-wavelength waves**, the speed is:

$$v = \sqrt{\frac{T}{\rho}}$$

where T is the surface tension, and ρ is the density.

Show that the group velocity for long-wavelength waves is their phase velocity:

For long-wavelength waves, the phase velocity is $v_p = \sqrt{gD}$, and the group velocity is the same, $v_g = \sqrt{gD}$. Therefore, the group velocity equals the phase velocity for long-wavelength waves.

For short-wavelength waves, the phase and group velocities are equal as well.

14. A laser emits a monochromatic beam of wavelength λ, which is reflected normally from a plane mirror receding at a speed v:

The frequency shift due to the Doppler effect is given by:

$$\Delta f = \frac{2v}{\lambda}$$

This is the beat frequency between the incident and reflected light.

15. Standing waves are produced by the superposition of the wave $y = (7\,\mathrm{cm})\sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right)$ and its reflection:

The amplitude of the standing wave is the sum of the individual wave amplitudes:

$$A_{\text{total}} = 2A$$

The wavelength is λ, the velocity is $v = \frac{\lambda}{T}$, and the period is T.

16. A medium is disturbed by an oscillation described by:

(a) Amplitude, frequency, wavelength, speed, and direction of the component waves:

From the given disturbance, you can find the amplitude and wavelength by analyzing the oscillation.

(b) Internodal distance:

For a standing wave, the internodal distance is $\frac{\lambda}{2}$, where λ is the wavelength.

17. Express the plane waves of Eqs. (19) and (20) in complex form:

Express the waves in the complex representation:

$$y_1(x,t) = Ae^{i(kx-\omega t)}$$

This form simplifies the analysis of wave superposition.

18. Estimate the number of standing wave modes in a laser cavity 1 m long with a frequency range of 6 GHz:

The number of modes N is given by:

$$N = \frac{\Delta f L}{c}$$

where Δf is the frequency range, and L is the length of the cavity.

For a 1 m long cavity and a frequency range of 6 GHz, the number of modes is:

$$N = \frac{6\times10^9\times1}{3\times10^8} = 20\ \text{modes}$$

19. Express the plane waves of Eqs. (19) and (20) in the complex representation:

Given the plane waves $y_1(x,t)$ and $y_2(x,t)$, we need to express them in complex form.

General form of a plane wave:

A general plane wave can be written in the complex form as:

$$y(x,t) = Ae^{i(kx-\omega t)}$$

where:

- A is the amplitude of the wave,
- k is the wave number (related to the wavelength λ),
- ω is the angular frequency (related to the period T),
- x and t are the position and time variables, respectively.

Now, let's examine how to represent the given waves.

1. **For wave $y_1(x,t)$:** Assuming that the equation for $y_1(x,t)$ looks like something of the form:

$$y_1(x,t) = A\sin(kx-\omega t)$$

We can represent this in complex form using Euler's formula. Remember that:

$$\sin(kx-\omega t) = \frac{1}{2i}\left(e^{i(kx-\omega t)} - e^{-i(kx-\omega t)}\right)$$

$$\sin(kx-\omega t) = \frac{1}{2i}\left(e^{i(kx-\omega t)} - e^{-i(kx-\omega t)}\right)$$

So, in complex form, the wave can be written as:

$$y_1(x,t) = Ae^{i(kx-\omega t)}$$

This represents the forward traveling wave. For the backward traveling wave, you would use $e^{-i(kx-\omega t)}$.

2. **For wave $y_2(x,t)$:** If the second wave $y_2(x,t)$ is of a similar form, say:

$$y_2(x,t) = B\sin(kx-\omega t+\phi)$$

Again, using Euler's formula:

$$y_2(x,t) = Be^{i(kx-\omega t+\phi)}$$

This can be written as a complex exponential with a phase shift ϕ.

Resultant wave:

When two waves $y_1(x,t)$ and $y_2(x,t)$ are superimposed, their resultant wave in complex form will be the sum of the individual waves:

$$y_{\text{total}}(x,t) = Ae^{i(kx-\omega t)} + Be^{i(kx-\omega t+\phi)}$$

This can then be simplified (if needed) depending on the values of A, B, and the phase shift ϕ.

20. Estimate the number of standing wave modes in a laser cavity 1 meter long with a frequency range of 6 GHz:

To estimate the number of standing wave modes in a laser cavity, we use the formula for the number of modes in a cavity. A laser cavity supports discrete standing wave modes, and the number of modes can be approximated by the following formula:

$$N = \frac{\Delta f\cdot L}{c}$$

where:

- Δf is the frequency range,
- L is the length of the cavity,
- c is the speed of light in vacuum ($c \approx 3\times10^8\,\mathrm{m/s}$).

Given data:

- Frequency range $\Delta f = 6\,\mathrm{GHz} = 6\times10^9\,\mathrm{Hz}$,
- Cavity length $L = 1\,\mathrm{m}$,
- Speed of light $c = 3\times10^8\,\mathrm{m/s}$.

Now, substitute the values into the formula:

$$N = \frac{(6 \times 10^9) \cdot (1)}{3 \times 10^8}$$

$$N = \frac{6 \times 10^9}{3 \times 10^8} = 20$$

Conclusion:

The number of standing wave modes in the laser cavity is approximately **20** modes.

$$E(x,t) = 2E_0 \cos\left(\frac{k_1+k_2}{2}x - \frac{v_1+v_2}{2}t\right) \cos\left(\frac{k_1-k_2}{2}x - \frac{v_1-v_2}{2}t\right)$$

$$f_b = |f_1 - f_2|$$

2. Phase and Group Velocities

- Phase Velocity (Equation 36):
  - The **phase velocity**, v_p, is the speed at which a single wave crest (or individual component of the wave) propagates.
  - It is given by:

$$v_p = \frac{\omega}{k}$$

  where ω is the angular frequency and k is the wave number.
  - For a carrier wave, the phase velocity represents how fast the oscillations of that wave move.
- Group Velocity (Equation 37):
  - The **group velocity**, v_g, is the speed at which the overall shape of the wave (such as the envelope of the modulated wave) propagates.
  - It is given by:

$$v_g = \frac{d\omega}{dk}$$

  where ω is the angular frequency and k is the wave number.
  - The group velocity is important because it determines how fast energy or information is transmitted through the wave.

$$v_g = v_p \pm \frac{dv_p}{dk}$$

Example Problem - Dispersion in Glass

Given:

- The refractive index for visible wavelengths in crown glass is approximated as:

$$n(\lambda) = 1.5255 + \frac{9650}{\lambda^2} \quad (\text{where } \lambda \text{ is in nm})$$

- The problem provides specific wavelengths: 400 nm, 500 nm, and 700 nm.

Solution Steps:

1. Calculate Refractive Index for Each Wavelength:
   - For $\lambda = 400\,\mathrm{nm}$, $\lambda = 500\,\mathrm{nm}$, and $\lambda = 700\,\mathrm{nm}$, calculate the refractive index using the formula.
2. Phase Velocity (Eq. 36):
   - For each wavelength, calculate the **phase velocity** v_p using the relation:

$$v_p = \frac{c}{n}$$

   where $c = 3 \times 10^8\,\mathrm{m/s}$ is the speed of light in a vacuum and n is the refractive index.

Group Velocity (Eq. 37):

- To find the **group velocity**, v_g, for each wavelength, use the relation:

$$v_g = \frac{d\omega}{dk}$$

  where the change in refractive index $dn/d\lambda$ can be used to estimate how the group velocity changes with wavelength.

6. Example Problem: Dispersion in Glass

- Given:
  - A type of crown glass has a refractive index $n(\lambda) = 1.5255 + \frac{9650}{\lambda^2}$, where λ is the wavelength in nanometers (nm).
- Tasks:
  1. **Calculate the refractive index** for $\lambda = 400\,\mathrm{nm}$, $\lambda = 500\,\mathrm{nm}$, and $\lambda = 700\,\mathrm{nm}$.
  2. **Calculate the phase velocity** for light with a central wavelength of $500\,\mathrm{nm}$.
  3. **Calculate the group velocity** for light with a central wavelength of $500\,\mathrm{nm}$.

- Solutions:
  1. Refractive Index Calculation:
     - For $\lambda = 400\,\mathrm{nm}$:

$$n(400) = 1.5255 + \frac{9650}{(400)^2} = 1.5557$$

     - For $\lambda = 500\,\mathrm{nm}$:

$$n(500) = 1.5255 + \frac{9650}{(500)^2} = 1.5448$$

     - For $\lambda = 700\,\mathrm{nm}$:

$$n(700) = 1.5255 + \frac{9650}{(700)^2} = 1.5353$$

  2. Phase Velocity Calculation:
     - The phase velocity for $\lambda = 500\,\mathrm{nm}$ is calculated as:

$$v_p = \frac{c}{n} = \frac{3 \times 10^8\,\mathrm{m/s}}{1.5448} = 1.942 \times 10^8\,\mathrm{m/s}$$

3. Group Velocity Calculation:
   - Using the refractive index, calculate the group velocity v_g for $\lambda = 500\,\mathrm{nm}$.
   - The approximation for group velocity would depend on the wavelength and the refractive index dispersion.

Notecard 2: Superposition of Two Waves

Front: What is the equation for the superposition of two waves?

Back: The resultant wave $E(x,t)$ is given by:

$$E(x,t) = 2E_0 \cos\left(\frac{k_1+k_2}{2}x - \frac{v_1+v_2}{2}t\right) \cos\left(\frac{k_1-k_2}{2}x - \frac{v_1-v_2}{2}t\right)$$

Notecard 4: Beat Phenomenon

Front: What is the phenomenon of beats?

Back: Beats occur when two waves of slightly different frequencies interfere, creating a fluctuating amplitude. The frequency of this fluctuation is the **beat frequency**, which is the difference between the two frequencies:

$$f_b = |f_1 - f_2|$$

Notecard 5: Beat Frequency

Front: How is the beat frequency related to wave frequencies?

Back: The beat frequency is simply the difference in frequencies of the two waves. If f_1 and f_2 are the frequencies, then:

$$f_b = |f_1 - f_2|$$

Notecard 6: Energy Transfer in Beats

Front: How does the beat phenomenon affect energy transfer?

Back: The energy transferred by the traveling pulse exhibits oscillations at the beat frequency, f_b. This causes periodic fluctuations in the intensity (irradiance) of the resultant wave.

Notecard 7: Phase and Group Velocities

Front: What are phase and group velocities?

Back:

- **Phase Velocity** (v_p): The speed at which the high-frequency carrier wave moves.

$$v_p = \frac{\omega}{k}$$

- **Group Velocity** (v_g): The speed at which the modulating envelope (low-frequency wave) moves.

$$v_g = \frac{d\omega}{dk}$$

Notecard 8: Relationship Between Phase and Group Velocities

Front: How are phase and group velocities related in a dispersive medium?

Back: In a **dispersive medium**, the phase velocity (v_p) and group velocity (v_g) are generally not the same. Their relationship is given by:

$$v_g = v_p \pm \frac{dv_p}{dk}$$

In a **nondispersive medium**, $v_p = v_g$.

Notecard 9: Dispersion in a Medium

Front: What is dispersion in a medium?

Back: Dispersion occurs when different frequency components of a wave travel at different speeds through the medium, causing the pulse to spread and distort over time.

Notecard 10: Pulse Propagation in Dispersive Media

Front: How does pulse propagation behave in a dispersive medium?

Back: In a dispersive medium, the different frequency components of a pulse travel at different speeds. As a result, the pulse may spread out and distort as it propagates, depending on the degree of dispersion.

Notecard 11: Relationship Between Wavelength and Velocity

Front: What is the relationship between wavelength, velocity, and frequency in a wave?

Back: The velocity of a wave is related to its wavelength (λ) and frequency (f) by:

$$v = f\lambda$$

Notecard 12: Example of Dispersion in Glass

Front: How is dispersion in crown glass for visible light wavelengths described?

Back: The refractive index $n(\lambda)$ of crown glass is given by:

$$n(\lambda) = 1.5255 + \frac{9650}{\lambda^2}$$

Where λ is the wavelength in nanometers.

Notecard 13: Calculating Refractive Index

Front: How do you calculate the refractive index for different wavelengths of light in crown glass?

Back: For different wavelengths, substitute λ into the formula $n(\lambda) = 1.5255 + \frac{9650}{\lambda^2}$.

Example calculations:

- For $\lambda = 400\,\mathrm{nm}$, $n(400) = 1.5557$
- For $\lambda = 500\,\mathrm{nm}$, $n(500) = 1.5448$
- For $\lambda = 700\,\mathrm{nm}$, $n(700) = 1.5353$

Notecard 14: Phase Velocity in Glass

Front: How do you calculate phase velocity in glass for a given wavelength?

Back: Phase velocity is calculated as:

$$v_p = \frac{c}{n(\lambda)}$$

Where c is the speed of light in a vacuum, and $n(\lambda)$ is the refractive index at the specific wavelength.

Notecard 15: Group Velocity in Glass

Front: How do you calculate the group velocity in glass for a given wavelength?

Back: Group velocity is calculated using the dispersion relation:

$$v_g = \frac{d\omega}{dk}$$

In practical cases, this can be approximated based on the refractive index's dependence on wavelength.

Notecard 16: Effects of Dispersion on Pulse Shape

Front: How does dispersion affect pulse shape?

Back: In a dispersive medium, the different frequency components of a pulse travel at different speeds. This causes the pulse to spread out and distort, leading to changes in the overall shape of the wave.

Notecard 17: Group and Phase Velocity in Quantum Mechanics

Front: What is the significance of group and phase velocities in quantum mechanics?

Back: In quantum mechanics, particles like electrons are represented by localized wave packets, which consist of a range of frequencies. The **group velocity** of the wave packet represents the velocity of the electron, while the **phase velocity** corresponds to the velocity of the individual harmonic components.

Notecard 18: Practical Application: Beats in Music

Front: How are beats used in music for tuning instruments?

Back: Musicians use the phenomenon of beats to tune instruments by listening for the beat frequency. When two tuning forks or guitars are played together, the beat frequency is the difference between their frequencies. Adjusting the tension in the strings eliminates the beat, ensuring both instruments are in tune.

Notecard 19: Doppler Radar and Beats

Front: How are beats used in Doppler radar systems?

Back: In Doppler radar, the beat phenomenon helps measure the difference between the emitted radar wave and the Doppler-shifted return signal. This is used to determine velocity, such as wind speed in weather radar systems.

Notecard 20: Superposition of Many Waves

Front: What happens when many harmonic waves superpose to form a pulse?

Back: When many harmonic waves combine, they form a narrow pulse. The resulting wave can be described by both a **phase velocity** (average velocity of the individual waves) and a **group velocity** (velocity of the overall modulated pulse).

Notecard 1: Superposition of Two Waves (General Form)

Front: What is the mathematical expression for the superposition of two waves?

Back: The resultant wave from the superposition of two waves is given by:

$$E(x,t) = 2E_0 \cos\left(\frac{k_1+k_2}{2}x - \frac{v_1+v_2}{2}t\right) \cos\left(\frac{k_1-k_2}{2}x - \frac{v_1-v_2}{2}t\right)$$

Where:

- $E(x,t)$ is the resultant wave
- E_0 is the amplitude of the individual waves
- k_1, k_2 are the propagation constants
- v_1, v_2 are the wave velocities

Notecard 2: Beat Frequency Formula

Front: How do you calculate the beat frequency?

Back: The beat frequency f_b is the difference in frequencies of the two waves:

$$f_b = |f_1 - f_2|$$

Where f_1 and f_2 are the frequencies of the two component waves.

Notecard 3: Group Velocity Formula

Front: What is the formula for group velocity?

Back: The group velocity v_g is given by:

$$v_g = \frac{d\omega}{dk}$$

Where:

- ω is the angular frequency
- k is the propagation constant

Notecard 4: Phase Velocity Formula

Front: What is the formula for phase velocity?

Back: The phase velocity v_p is given by:

$$v_p = \frac{\omega}{k}$$

Where:

- ω is the angular frequency
- k is the propagation constant

Notecard 5: Relationship Between Phase and Group Velocities

Front: What is the general relationship between phase velocity and group velocity?

Back: In a dispersive medium, phase velocity v_p and group velocity v_g are related by:

$$v_g = v_p \pm \frac{dv_p}{dk}$$

Where $\frac{dv_p}{dk}$ represents the change in phase velocity with respect to the wave number.

Notecard 6: Dispersion Relation

Front: What is the dispersion relation for phase and group velocities in a dispersive medium?

Back: The dispersion relation links phase velocity v_p and group velocity v_g in a dispersive medium:

$$v_g = \frac{d\omega}{dk}$$

In nondispersive media, $v_p = v_g$, meaning the phase and group velocities are equal.

Notecard 7: Formula for Wavelength and Frequency Relation

Front: How is the velocity related to wavelength and frequency in a wave?

Back: The velocity v of a wave is related to its wavelength λ and frequency f by the equation:

$$v = f\lambda$$

Where:

- v is the wave velocity
- f is the frequency
- λ is the wavelength

Notecard 8: Refractive Index and Phase Velocity

Front: How does the refractive index n relate to the phase velocity?

Back: The refractive index n of a material is related to the phase velocity v_p by:

$$v_p = \frac{c}{n}$$

Where:

- c is the speed of light in vacuum
- n is the refractive index of the medium

Notecard 9: Dispersion in Terms of Refractive Index

Front: How is dispersion in a medium expressed in terms of the refractive index?

Back: The dispersion in a medium can be expressed as:

$$n(\lambda) = n_0 + \frac{A}{\lambda^2}$$

Where:

- $n(\lambda)$ is the refractive index as a function of wavelength λ
- n_0 is the refractive index at a reference wavelength
- A is a constant dependent on the material

Notecard 10: Index of Refraction Formula for Crown Glass

Front: What is the formula for the refractive index of crown glass?

Back: For crown glass, the refractive index as a function of wavelength λ (in nanometers) is given by:

$$n(\lambda) = 1.5255 + \frac{9650}{\lambda^2}$$

Where λ is the wavelength in nanometers.

Notecard 11: Phase Velocity in Crown Glass

Front: How do you calculate the phase velocity in crown glass?

Back: The phase velocity v_p in crown glass for a given wavelength λ is calculated as:

$$v_p = \frac{c}{n(\lambda)}$$

Where:

- c is the speed of light in vacuum
- $n(\lambda)$ is the refractive index of crown glass at wavelength λ

Notecard 12: Group Velocity Formula in Dispersive Medium

Front: What is the formula for group velocity in a dispersive medium?

Back: The group velocity v_g is given by:

$$v_g = \frac{d\omega}{dk}$$

Where ω is the angular frequency and k is the wave number (propagation constant). This relation helps calculate how fast the pulse (or modulation) moves.

Notecard 13: Group and Phase Velocities in Nondispersive Medium

Front: What happens to the group and phase velocities in a nondispersive medium?

Back: In a nondispersive medium, the phase velocity v_p and the group velocity v_g are equal:

$$v_p = v_g$$

This means that all components of the wave travel at the same speed.

Notecard 14: Relationship Between Frequency and Wave Number

Front: How is the wave number k related to the frequency f?

Back: The wave number k and frequency f are related by the equation:

$$k = \frac{2\pi}{\lambda}$$

Where λ is the wavelength.

Notecard 15: Relationship Between Angular Frequency and Frequency

Front: How is the angular frequency ω related to the frequency f?

Back: The angular frequency ω is related to the frequency f by:

$$\omega = 2\pi f$$

Where ω is the angular frequency in radians per second and f is the frequency in hertz.

Notecard 16: Group Velocity and Signal Velocity

Front: What is the relationship between group velocity and signal velocity?

Back: The **group velocity** is the velocity at which the signal (or modulation) travels, and it determines the speed of energy transfer in the wave. It is the actual measurable speed for pulses or modulated waves.

Notecard 17: Relation Between Phase and Group Velocities in Dispersive Media

Front: What is the relationship between phase and group velocities in a dispersive medium?

Back: In a dispersive medium, the phase velocity v_p and group velocity v_g are generally different. The group velocity is influenced by the variation of the refractive index with wavelength:

$$v_g = v_p \pm \frac{dv_p}{dk}$$

This reflects how dispersion causes phase and group velocities to differ.

Notecard 1: Harmonic Waves - General Concept

- **Key Concept**: Harmonic waves can be represented by complex exponential functions.
- **Formula**: $y(x,t) = Ae^{i(kx-\omega t)}$
- **Key Practice**: To find the physical wave, take the real or imaginary part of the complex function.
- **Usefulness**: This method simplifies calculations when dealing with sinusoidal waves.

Notecard 2: Plane Waves

- **Definition**: A plane wave represents a disturbance traveling in a specific direction, with planar wavefronts.
- **Equation**: $y(x,t) = A\sin(kx - \omega t)$
- **Wavefronts**: Planes perpendicular to the propagation direction.
- **Physical Example**: Sound wave propagating through air.
- **Approximation**: Plane waves are an approximation when wavefronts appear nearly planar in a region of interest.

Notecard 3: Generalized Plane Wave Equation (3D)

- **General Equation**: $y(r,t) = A\sin(\mathbf{k} \cdot \mathbf{r} - \omega t)$
- **Key Variables**:
  - $\mathbf{k}$ = Wave vector (propagation direction)
  - $\mathbf{r}$ = Position vector in space
  - ω = Angular frequency
  - t = Time
- **Explanation**: This equation generalizes the harmonic wave to any direction in space.

Notecard 4: Wave Equation in 3D

- **Wave Equation**: $\nabla^2 y = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$
- **Laplacian Operator**: Represents the spatial second derivatives, denoted as ∇^2.
- **Physical Meaning**: This equation governs the behavior of waves in three dimensions (both spatial and temporal changes).

Notecard 5: Spherical Waves

- **Definition**: Waves emanating from a point source with spherical wavefronts.
- **Equation**: $y(r,t) = \frac{A}{r}\sin(kr - \omega t)$
- **Key Point**: Amplitude decreases with distance, proportional to $\frac{1}{r}$.
- **Inverse Square Law**: Irradiance (intensity) follows the inverse square law.
- **Application**: Represents sound waves or any disturbance propagating radially outward from a point.

Notecard 6: Cylindrical Waves

- **Definition**: Waves propagating from a line of symmetry with cylindrical wavefronts.
- **Equation**: $y(r,t) = \frac{A}{r}e^{i(kr-\omega t)}$
- **Physical Example**: Waves emerging from a slit illuminated by a plane wave.
- **Approximation**: Valid for large distances but does not exactly satisfy the wave equation for small regions.

Notecard 7: Gaussian Beams

- **Definition**: A type of laser beam with a transverse irradiance profile that is Gaussian.
- **Spot Size**: The minimum transverse size, called the beam waist, occurs where the wavefronts are planar.
- **Beam Divergence**: Smaller beam waists lead to larger divergence angles.
- **Equation**: Gaussian beam solutions approximate the wave equation well in the far field.

Notecard 8: Electromagnetic Waves

- **Key Concept**: Electromagnetic waves consist of oscillating electric and magnetic fields, perpendicular to each other and to the direction of wave propagation.
- **Maxwell's Equations**: Govern the behavior of electromagnetic waves.
- **Wave Equations**:
  - Electric field: $E = E_0 \sin(k \cdot r - \omega t)$
  - Magnetic field: $B = B_0 \sin(k \cdot r - \omega t)$
- **Relationship**: $E = cB$, where c is the speed of light in the medium.

Notecard 9: Wave Propagation Speed

- **Speed of Wave**: $c = \frac{1}{\sqrt{\mu\epsilon}}$ (Electromagnetic Waves in Vacuum)
- **Electromagnetic Waves**: Travel at the speed of light c in a vacuum (around $3 \times 10^8\, m/s$).

Notecard 10: Energy Propagation in Waves

- **Energy Transfer**: Waves carry energy from one location to another.
- **Plane Waves**: Energy is carried uniformly through space.
- **Spherical Waves**: Energy spreads out as the wavefronts grow larger, causing the intensity to decrease as $\frac{1}{r^2}$.

Notecard 11: Wavefronts

- **Definition**: A surface of constant phase.
- **Plane Waves**: Wavefronts are flat planes.
- **Spherical Waves**: Wavefronts are spherical surfaces.
- **Cylindrical Waves**: Wavefronts are cylindrical surfaces.

Notecard 12: Wave Equation for Complex Functions

- **Complex Wave Representation**:
  - $y(r,t) = Ae^{i(k \cdot r - \omega t)}$
  - This form represents the wave, with the physical wave recovered by taking the real or imaginary part.

Notecard 13: Laplacian Operator in Wave Equation

- **Laplacian**: The operator ∇^2 represents the spatial second derivatives.
- **Wave Equation Form**: $\nabla^2 y = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$.

Notecard 1: General Harmonic Waveform

- **Equation**: $y(x,t) = Ae^{i(kx-\omega t)}$
- **Description**: This is the general form of a harmonic wave, where:
  - A = Amplitude of the wave
  - k = Wave number ($k = \frac{2\pi}{\lambda}$)
  - ω = Angular frequency ($\omega = 2\pi f$)
  - t = Time
  - x = Spatial position
- **Physical Wave**: Take the real or imaginary part to recover the physical wave.

Notecard 2: **Plane Wave Equation**

- **Equation**: $y(x,t) = A\sin(kx - \omega t)$
- **Description**: A wave propagating along the x-direction. It is the real part of the complex harmonic wave.
- **Phase**: $kx - \omega t$ represents the phase of the wave.
- **Wavefronts**: Planes perpendicular to the direction of propagation.

Notecard 3: **Three-Dimensional Plane Wave Equation**

- **General Equation**: $y(\mathbf{r},t) = A\sin(\mathbf{k}\cdot\mathbf{r} - \omega t)$
- **Description**: Generalized to three dimensions where:
  - $\mathbf{k}$ = Wave vector, pointing in the direction of propagation
  - $\mathbf{r}$ = Position vector in space
  - ω = Angular frequency
  - t = Time
- **Physical Meaning**: This equation represents a wave propagating in any direction in space.

Notecard 4: **Wave Equation (3D)**

- **Wave Equation**: $\nabla^2 y = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$
- **Laplacian**: ∇^2 is the spatial second derivative operator.
- **Description**: Governs the behavior of waves in three dimensions, where:
  - ∇^2 = Laplacian operator (spatial derivatives)
  - c = Wave propagation speed (e.g., speed of light for electromagnetic waves)
  - y = Wave function
  - t = Time

Notecard 5: **Laplacian Operator (3D)**

- **Laplacian**: $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$
- **Description**: The Laplacian operator calculates the sum of second partial derivatives with respect to the spatial coordinates.
- **Application**: Used in the wave equation to describe how the wave propagates in space.

Notecard 6: **Wave Equation for Spherical Waves**

- **Equation**: $y(r,t) = \frac{A}{r}\sin(kr - \omega t)$
- **Description**: Represents a spherical wave originating from a point source:
  - r = Radial distance from the source
  - A = Amplitude (not the overall wave amplitude, but relative to distance)
  - k = Wave number
  - ω = Angular frequency
- **Inverse Square Law**: Amplitude decreases as $\frac{1}{r}$, and irradiance decreases with $\frac{1}{r^2}$.

Notecard 7: **Wave Equation for Cylindrical Waves**

- **Equation**: $y(r,t) = \frac{A}{r}e^{i(kr - \omega t)}$
- **Description**: Represents cylindrical waves that propagate outward from a line of symmetry.
  - r = Perpendicular distance from the line of symmetry
  - A = Amplitude
  - k = Wave number
  - ω = Angular frequency
- **Limitation**: Cylindrical waves are approximations and don't exactly satisfy the wave equation over small regions.

Notecard 8: **Wave Equation for Gaussian Beams**

- **Equation**: Approximate solutions involving Hermite-Gaussian functions are used to describe laser beams.
- **Spot Size**: The beam's transverse size at the beam waist is often denoted as w_0.
- **Divergence**: The beam spreads as it moves further from the beam waist, with a larger divergence for smaller beam waists.
- **Beam Waist**: The smallest spot size, where the wavefronts are nearly planar.

Notecard 9: **Electromagnetic Wave Equations**

- **Electric Field**: $E = E_0\sin(k\cdot r - \omega t)$
- **Magnetic Field**: $B = B_0\sin(k\cdot r - \omega t)$
- **Relationship Between E and B**: $E = cB$, where c is the speed of light.
- **Wave Propagation**: Both fields oscillate perpendicular to each other and to the direction of wave propagation.

Notecard 10: **Wave Number (k) and Angular Frequency (ω)**

- **Wave Number**: $k = \frac{2\pi}{\lambda}$, where λ is the wavelength.
- **Angular Frequency**: $\omega = 2\pi f$, where f is the frequency.
- **Relationship**: $k\cdot r = \frac{2\pi}{\lambda}\cdot r$ and $\omega\cdot t = 2\pi f\cdot t$.

Notecard 11: **Harmonic Wave Velocity (v)**

- **Wave Velocity**: $v = \frac{\omega}{k}$
- **Description**: The velocity of a harmonic wave depends on the angular frequency (ω) and wave number (k).
- **For Electromagnetic Waves**: $v = c$, the speed of light.

Notecard 12: **Phase Velocity and Group Velocity**

- **Phase Velocity**: $v_p = \frac{\omega}{k}$
- **Group Velocity**: $v_g = \frac{d\omega}{dk}$
- **Description**: The phase velocity represents the velocity of individual wave crests, while the group velocity represents the velocity of the energy or information in a wave packet.

Notecard 13: **Irradiance of Spherical Waves**

- **Irradiance**: $I(r) \propto \frac{1}{r^2}$
- **Explanation**: As a spherical wave propagates outward from a point source, the intensity (irradiance) decreases following the inverse square law.

Notecard 14: **Relationship Between Wave Equations**

- **Wave Equation in Terms of Derivatives:**
  - $\nabla^2 y = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$
  - Describes the wave function y in terms of both spatial and temporal changes.
- **Laplacian Form**: $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

Notecard 15: **Electromagnetic Wave Propagation (Maxwell's Equations)**

- **Electric Field and Magnetic Field**: Both fields vary sinusoidally and are perpendicular to each other.
  - $E = E_0\sin(k\cdot r - \omega t)$
  - $B = B_0\sin(k\cdot r - \omega t)$
- **Relation Between Fields**: $E = cB$, where c is the speed of light.

It follows that $k\lambda = 2\pi$, so that the *propagation constant* k contains information regarding the wavelength.

$$k = \frac{2\pi}{\lambda} \tag{4}$$

Alternatively, if the wave is viewed from a fixed position, as in Figure 2b, it is periodic in time with a repetitive temporal unit called the *period* T. Increasing all t by T, the waveform is exactly reproduced, so that

$$A\sin k[x + v(t + T)] = A\sin[k(x + vt) + 2\pi]$$

or

$$A\sin(kx + kvt + kvT) = A\sin(kx + kvt + 2\pi)$$

Clearly, $kvT = 2\pi$, and we have an expression that relates the period T to the propagation constant k and wave velocity v. The same information is included in the relation

$$v = \nu\lambda \tag{5}$$

where we have used Eq. (4) together with the reciprocal relation between period T and frequency ν

$$\nu = \frac{1}{T} \tag{6}$$

Related descriptions of wave parameters are often used. The combination $\omega = 2\pi\nu$ is called the *angular frequency*, and the reciprocal of the wavelength $\kappa = 1/\lambda$ is called the *wave number*. Note that the propagation constant k is related to the *spatial period* (i.e., the wavelength) of the wave in the same way that the angular frequency ω is related to the temporal period T. Therefore, the propagation constant k is the *spatial frequency* of the wave. With these relationships it is easy to show the equivalence of the following common forms for harmonic waves:

$$y = A\,{}^{\sin}_{\cos}[k(x \pm vt)] \tag{7}$$

$$y = A\,{}^{\sin}_{\cos}\left[2\pi\left(\frac{x}{\lambda} \pm \frac{t}{T}\right)\right] \tag{8}$$

$$y = A\,{}^{\sin}_{\cos}[(kx \pm \omega t)] \tag{9}$$

In any case, the argument of the sine or cosine, which depends on space and time, is called the *phase*, φ. For example, in Eq. (7),

$$\varphi = k(x \pm vt) \tag{10}$$

When x and t change together in such a way that φ is constant, the displacement $y = A\sin\varphi$ is also constant. The condition of constant phase evidently describes the motion of a fixed point on the waveform, which moves with the velocity of the wave. Thus if φ is constant,

$$d\varphi = 0 = k(dx \pm v\,dt)$$

$$\frac{dx}{dt} = \mp v$$

Wave Equations

confirming that v represents the wave velocity, which is in the negative x-direction when $\varphi = k(x + vt)$ and in the positive x-direction when $\varphi = k(x - vt)$.

Notice that the waveforms of Eqs. (7) through (9) that use the sine function all represent waves for which $y = 0$ at position $x = 0$ and time $t = 0$. All of the cosine waveforms in these equations represent waves for which $y = A$ at position $x = 0$ and time $t = 0$. As pointed out previously, both situations could be handled by either the sine or cosine function if an angle of $\pi/2$ is added to the phase. In general, to accommodate any arbitrary initial displacement, some angle φ_0 must be added to the phase. For example, Eq. (7) with the sine function becomes

$$y = A\sin[k(x \pm vt) + \varphi_0]$$

Now suppose our initial boundary conditions are such that $y = y_0$ when $x = 0$ and $t = 0$. Then

$$y = A\sin\varphi_0 = y_0$$

from which the required *initial phase angle* φ_0 can be calculated as

$$\varphi_0 = \sin^{-1}\left(\frac{y_0}{A}\right)$$

The waveforms in Eqs. (7) to (9) can be generalized further to yield any initial displacement, therefore, by the addition of an initial phase angle φ_0 to the phase. In many cases, the precise phase of the wave is not of interest. Then φ_0 can be set equal to zero for simplicity.

4 HARMONIC WAVES AS COMPLEX FUNCTIONS

Using Euler's formula, it is possible to express a harmonic wave as the real (or imaginary) part of the complex function

$$\tilde{y} = Ae^{i(kx - \omega t)} \tag{18}$$

so that

$$y = \mathrm{Re}\,(\tilde{y}) = A\cos(kx - \omega t) \quad \text{or} \quad y = \mathrm{Im}\,(\tilde{y}) = A\sin(kx - \omega t) \tag{19}$$

Note that any equation that involves only terms that are linear in $\tilde{y}$ and its derivatives will also hold for $y = \mathrm{Re}\,(\tilde{y})$ or $y = \mathrm{Im}\,(\tilde{y})$. Many mathematical manipulations can be carried out more simply with exponential functions than with trigonometric functions. As a result, it is common practice to use the complex waveform Eq. (18) to represent a harmonic wave when doing calculations and then to take the real or imaginary part of this complex function to recover the physical wave represented by one of the forms in Eq. (19).

14 A laser emits a monochromatic beam of wavelength λ, which is reflected normally from a plane mirror, receding at a speed v. What is the beat frequency between the incident and reflected light?

13 Waves on the ocean have different velocities, depending on their depth. Long-wavelength waves, traveling deep in the ocean, have a speed given approximately by

$$v_p = \left(\frac{g\lambda}{2\pi}\right)^{1/2}$$

where g is the acceleration of gravity. Short-wavelength waves, corresponding to surface ripples, have a velocity given approximately by

$$v_p = \left(\frac{2\pi T}{\lambda\rho}\right)^{1/2}$$

where ρ is the density and T is the surface tension. Show that the group velocity for long-wavelength waves is 1/2 their phase velocity and the group velocity for short-wavelength waves is 3/2 their phase velocity.

12 a. Show that group velocity can be expressed as

$$v_g = v_p - \lambda\left(\frac{dv_p}{d\lambda}\right)$$

b. Find the group velocity for plane waves in a dispersive medium, for which $v_p = A + B\lambda$, where A and B are constants. Interpret the result.

11 The dielectric constant K of a gas is related to its index of refraction by the relation $K = n^2$.

a. Show that the group velocity for waves traveling in the gas may be expressed in terms of the dielectric constant by

$$v_g = \frac{c}{\sqrt{K}}\left[1 - \frac{\omega}{2K}\frac{dK}{d\omega}\right]$$

where c is the speed of light in vacuum.

b. An empirical relation giving the variation of K with frequency is

$$K = 1 + [A/(\omega_0^2 - \omega^2)]$$

where A and ω_0 are constants for the gas. If the second term is very small compared to the first, show that

$$v_g \cong c\left[\frac{\omega^2 A}{(\omega_0^2 - \omega^2)^2}\right]$$

10 The dispersion curve of glass can be represented approximately by Cauchy's empirical equation, $n = A + B/\lambda^2$. Find the phase and group velocities for light of 500-nm wavelength in a particular glass for which $A = 1.40$ and $B = 2.5 \times 10^6$ Å^2.

8 Beginning with the relation between group velocity and phase velocity in the form

$$v_g = v_p - \lambda(dv_p/d\lambda)$$

(a) express the relation in terms of n and ω and (b) determine whether the group velocity is greater or less than the phase velocity in a medium having a normal dispersion.

6 One hundred antennas are putting out identical waves, given by

$$E = 0.02\cos(\varepsilon - \omega t)\ \mathrm{V/m}$$

The waves are brought together at a point. What is the amplitude of the resultant when (a) all waves are in phase (coherent sources) and (b) the waves have random phase differences?

5 Plot and write the equation of the superposition of the following harmonic waves:

$E_1 = \sin\left(\frac{\pi}{18} - \omega t\right)$, $E_2 = 3\cos\left(\frac{5\pi}{9} - \omega t\right)$, and $E_3 = 2\sin\left(\frac{\pi}{6} - \omega t\right)$, where the period of each is 2 s.

2 a. Show in a phasor diagram the following two harmonic waves:

$$E_1 = 2\cos\omega t \quad \text{and} \quad E_2 = 7\cos\left(\frac{\pi}{4} - \omega t\right)$$

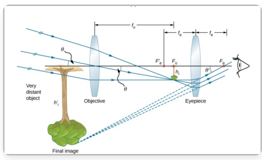
b. Determine the mathematical expression for the resultant wave.

1 Two plane waves are given by

$$E_1 = \frac{5E_0}{[(3/\mathrm{m})x - (4/\mathrm{s})t]^2 + 2} \quad \text{and}$$

$$E_2 = \frac{-5E_0}{[(3/\mathrm{m})x + (4/\mathrm{s})t - 6]^2 + 2}$$

a. Describe the motion of the two waves.
b. At what instant is their superposition everywhere zero?
c. At what point is their superposition always zero?



IPG PHOTONICS