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An equality-based model for bike-sharing stations location in bicycle-public transport multimodal mobility

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ABSTRACT

Bike-sharing systems can be implemented to complete the coverage of public transport networks which could be insufficient to serve an entire urban area. Some methodologies that maximise coverage or accessibility are suggested in the literature. In this paper, we propose a bike-sharing stations location model that includes not only these issues but also equality aspects. The model aims to minimise inequalities in bicycle-public transport mobility among observed population groups trying to maintain specified levels of accessibility and coverage at the same time. We evaluated the performance of the model on a test network and carried out a sensitivity analysis according to the available budget. The results showed that maximising accessibility or coverage alone, without considering equality, may lead to an unequal distribution of accessibility among the population, producing discrimination between different groups. The outcomes of the application revealed the significance of the model in evaluating equality in the network design phase for achieving not only a satisfactory bike-sharing system and public transport multimodal accessibility of each zone but also a high equality measure among the considered population groups. Budget availability also played an essential role since a minimum budget value is needed to achieve higher levels of equality. The proposed approach could serve transport and public authorities as a decision support system in planning future investment as well as promoting multimodal mobility because it links bike-sharing stations with stops/stations of the public transport lines networks.

1. Introduction

In the last decades, the promotion of Bike-Sharing Systems (BSSs) and the concept of "sustainable cities" in urban areas have been wildly encouraged through various policies, initiatives and directives. BSS embraces full advantages for urban mobility through emission mitigation, users' health and lifestyle improvement, reduction of traffic congestions, traffic systems improvement, integration with public and multimodal transport, etc. (Zhang and Zhang, 2018). The support of public authorities as well as the endorsement of safety standards for bike lanes, the required implementation time and the infrastructure costs for bike network design, are one of the primary considerations for implementing a BSS network. In particular, the variations in implementation and infrastructure costs are due to different factors, depending on the bike schemes (with or without the stations), considering that the docking points and station locations have the highest cost rate (Mooney et al., 2019). In addition to costs, other factors that can be considered in the planning phase of a BSS are those affecting the users' satisfaction degree and the bike-sharing usage (Guo et al,

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2017, Medard de Chardon et al., 2017), which can be identified according to predicting models (Frade and Ribeiro, 2014; Zhang et al., 2016). Among these factors we can find, for example, household bike ownership, BSS users' perception and bike-sharing station locations (Guo et al, 2017).

BSSs programs are nowadays spread all over the world (Meddin and DeMaio, 2020). Many of them, operating worldwide, are investigated in the literature (Parkes et al., 2013; Etienne and Latifa, 2014; Hu et al., 2019; Fishman, 2019). China is the leading country with the most extensive BSSs programs (O'Brien, 2020) and with a high number of shared cycle users, especially in East China (Zhou et al., 2020). As of April 2020, the city of Suzhou with the Wujiang District has more than 33,000 bikes allocated over 2500 stations. The other larger BSSs are still found mainly in China in the cities of Weifang, Hángzhōu and Putian City. In Europe, BSSs are operating in many cities (i.e. Paris, London, Barcelona, Berlin, Milan, etc.). The largest one is active in Paris, with over 15,000 bikes and around 1400 stations (O'Brien, 2020). Bike-share use in the U.S. is also dramatically growing. In 2017, 35 million trips were recorded with 25% more than in 2016. With 12,000 bikes and 750 stations, New York's BSS is the largest system in North America (NYCDOT, 2018).

The case studies of BSSs in the various cities in China investigate the opportunities, the environmental contribution and influence of such systems in urban areas (Zhang et al., 2015; Liu et al., 2012). Otero et al., (2018) provide a case study in Europe that examine the advantages of replacing car trips with bike ones; the bike substitution in twelve considered BSSs results in a 5.17% reduction of annual deaths. Also, the paper underlines that replacement of all car trips would lead to 73.25% reduction of injuries and savings of 18 million euros per year.

The cycling infrastructure of BSSs enhances the concept of "green transport" in urban areas and increases the efficiency of public transport (Mobike, 2017). Mobike, with the support of Tsinghua University's China, presents a study of BSSs in China that analyses the influence of cycling systems in an urban area and its influence on the inhabitants, energy-saving, public transport and the environment. The increase in bike trips results in a 55% decrease in car trips (with an emission reduction of 540,000 tons of CO_2 and energy savings of 29 million tons of oil), and it improves connectivity with bus and subways stations. Additionally, for a suitable "green" BSS, would be appropriate to find the right balance between the total number of systems bikes, operator-based bikes relocation (due to one-way trips), and demand satisfaction (Shui and Szeto, 2018; Luo et al., 2020). The bike and rebalancing vehicles fleet size, as well as the relocation operations, can be optimised, for example, minimising greenhouse gas emissions considering the Life Cycle Assessment of the system as proposed in the work of Luo et al. (2020). In the literature are proposed several strategies and models for optimising relocation operations (Legros, 2019; Zhang et al., 2019; Caggiani et al., 2018; Pal and Zhang, 2017). Among these models, the minimisation of the local polluting emissions of a given BSS with fossil-fuelled rebalancing vehicles is also presented (Wang and Szeto, 2018). For a comprehensive review of relocation problems and other issues related to the design and management of BSSs, see Shui and Szeto (2020).

Thus, BSSs have been showing contribution regarding environmental, health and financial point of view, but if not properly designed, in their spatial and non-spatial properties, different groups may experience unequal access to them as it could happen for any type of transport systems (Martens, 2016). In recent years, more considerable attention is paid to the design of transport systems based on equity or equality concepts. While the idea of "equity" generally refers to fairness or justice (Pereira et al., 2017), the "equality" concepts concern the allocation of goods or services, such as accessibility, regardless of moral judgement (van Wee and Geurs, 2011).

However, in the literature, the promotion of equality and multimodal accessibility related to network design of BSSs is not investigated enough. In particular, the location problem of BSSs is usually based on various indicators (characteristics of bike users, bike stations, accessibility of bike stations, performance of bike lane) but not equality concepts (Zhang, 2011; Loidl et al., 2019).

With this work, we propose to plan and design a Bike-Sharing Stations Location (BSSL) in an urban context considering equality principles and the connection with public transport. In particular, this paper presents a multimodal BSS and public transport accessibility measure and a novel network design optimisation for BSSL based on it. The application of the proposed model primarily ensures equal accessibility among different groups and throughout the studied urban area. Moreover, the spatial distribution of BSS stations resulting from the model tends to promote connectivity between the BSS and the considered public transport systems, because it links bike-sharing stations with stops/stations of the public transport lines networks. Furthermore, the proposed model could serve transport and public authorities as a decision support system in planning future investment for public transport systems related to bike-sharing infrastructures.

The paper is organised as follows. In Section 2, we present a literature review related to the problem that we consider. Section 3 is dedicated to the problem description, including the introduction of the Theil index, the definition of accessibility formulation for calculating equality and the proposed network design model for BSSL. In Section 4, we test the model with a numerical application and provide a sensitivity analysis of the obtained results. Section 5 is devoted to the conclusion and contribution of the proposed model.

2. Literature review

In this section, we present some literature works to describe the framework in which our proposed model is positioned. In Subsection 2.1, the works related to BSSs network design are shown first. Then follows a description of the papers that connect the design of these systems with public transport networks. The subsection ends with the models on BSSs or facilities network design based on equality issues. Since our work is focused on an equality index, in Subsection 2.2, the literature presenting the use of the two most suggested equality measure indices is described.

2.1. Bike-Sharing Systems and network design

According to the literature, the network design problem of bike-sharing systems can be solved by setting three main categories of objective functions: maximising coverage or demand, maximising accessibility expressed in terms of travel times and minimising systems costs/maximising profits. For instance, the work of García-Palomares et al. (2012) belongs to the first category. They develop a GIS-based method and a location-allocation model to determine the demand and the optimal location of bike-sharing stations in the city of Madrid. Additionally, Frade and Ribeiro (2015) propose a demand coverage maximisation method, to find optimal station locations and number of bikes in every station. More recently, Li et al. (2020) propose a new formulation of the maximal covering location problem, which aims to maximise customers' demand. Saharidis et al. (2014) and Çelebi et al. (2018) define the locations of bike-sharing stations minimising the unmet users' demand. Similarly, Caggiani et al. (2019a) propose to enhance a bike-sharing system defining the optimal number of parking lots minimising the unsatisfied demand. As far as concerns the second category Chen and Sun (2015) propose a mathematical model to minimise the total travel time considering cyclists preference regarding the pick-up and drop off stations. Fang et al. (2019) develop a bike stations location problem with the objective of maximising service quality. The authors measure the service quality as a function of each user travel time. Instead of the total time, the distance to be travelled, as suggested by Guo et al. (2014), can be minimised. A common constraint of most of these cited works is the investment budget required to implement the system. As regards the minimisation of the total costs, Lin and Yang (2011) and Romero et al. (2012) propose a model that also optimises the locations of bike paths and maximises the number of system users, respectively. In addition to the costs related to the implementation and operation of BSS, such as bikes relocation costs, those for unsatisfied demand can also be considered (Lin et al., 2013; Soriguera and Jiménez, 2020). On the other hand, instead of minimising costs, maximising profits or revenue is also proposed (Sayarshad et al., 2012; Martinez et al., 2012).

Several papers in the literature introduce the connection between cycling infrastructure (bike paths, bike racks and bike-sharing stations) and public transport. These works show the central role of the bike infrastructure locations in the context of multimodality. In particular, Brand et al. (2017) propose a model for multimodal transport networks considering bus, walking and cycling transport mode. They investigate the implementation of different bus services on their access and egress modes and analyse the effects of integrating these services with other transport modalities. The study is implemented in the Amstelland-Meerlanden (The City Region of Amsterdam - The Netherlands) and the results show the preference of cycling on the access side due to the higher diversity of transport modes. In contrast, on the egress side, this is not the case. However, this could be improved with the integration of bus and bike networks through the installation of the parking bike facilities on the access side to the bus network and providing bike-sharing opportunities on the egress side. Li et al. (2013) propose a multimodal network equilibrium model for investigating the effect of BSS in a multi-road network including auto and buses, considering accessibility to the bike rental stations and emissions rates from motorised vehicles. Ma et al. (2019) investigate the relationship and impact of free-floating BSSs on bus ridership in Chengdu (China). One of the results shows the increase of daily bus ridership on weekdays and the reduction of daily bus ridership on weekends, suggesting that user's demand affects the relationship between BSSs and public transport. Zhao et al. (2019) provide a case study in Nanjing that shows that demographic, built environment and transport-infrastructure indicators, have a significant influence on shared bikes reallocation, especially regarding the places and attractions with high utility and demand patterns. Liu et al. (2015) investigate the public transport system optimisation considering transport and public bike station network design, from a practical and governmental point of view. They propose a novel two-stage integer programming model, tested on Sioux-Falls and Anaheim network, that shows the improvement of the travel time costs and traffic issues. Instead, Lin and Yang (2011) propose the integration of BSSs and public transport system regarding the infrastructure, location of bike stations and travel paths considering the interests of users.

However, in literature, most of the papers that connect the design of BSSs with other public transport modes do not include issues related to social equity or equality among users of these systems. Only a few papers address these issues linked to BSSs network design. For example, Conrow et al. (2018) propose an optimisation approach related to GIS and spatial covering for equitable bike-sharing station siting. The model chooses the number of needed bikes and the optimal station locations. Most of the authors investigate equity/equality principles not in the design phase of a BSS but for the evaluation of the current state through several case studies mainly related to spatial and demographic accessibility (Chen and Yang, 2004; Behbahani et al., 2019a). For example, Winters et al. (2019) analyse the equity patterns of the groups of the bike share members in the city of Vancouver, referred to as "super-users". The results reveal that the higher population of super-users is related to young men with lower income and fewer opportunities for travelling with different transport modes. Instead, Mooney et al. (2019) investigate spatial equity of dockless BSSs, considering that these systems could have larger spatial accessibility since they do not require investment in docking stations infrastructure. They aggregate the location measures into neighbourhoods and obtain that neighbourhoods with higher income and accessibility to community resources tend to have more bikes.

Equity or equality principles are also applied in the literature to more general issues of network design and facilities location. For example, these papers, similarly to what is shown for BSSs, propose the maximisation of accessibility. Among these works, recently, Mousazadeh et al., (2018) propose a facility health care location-based model that use equity for maximising the accessibility and utility of health care services. Behbahani et al., (2019b) use John's Rawl's theory of justice for designing road network to maximise the accessibility of lower socio-economic groups of the population. The comparison of John's Rawl's theory of justice with classical methods resulted in reducing low accessibility by 30%. Zhang and Waller (2019) develop a multi-objective transport network design model that aims to minimise total travel time and optimise equity related to the travel time and energy consumption. Some authors, in particular, use properly defined equity/equality indices in the design phase or to evaluate the state of networks or transport systems. For example, Cavallaro et al. (2020) investigate the spatial and social equity for High-Speed Railway (HSR), through social equity railway indices. These indices evaluate the changes in travel time, ticket price and the number of connections in Italy. The results of the north-western part of Italy indicate the decrease of direct connections and a reduction of the travel time, followed by an increase in the ticket price. On the other hand, the results of these performances in Turin and Genova show the opposite outcome and this implies the territorial inequalities caused by HSR.

Our proposed model is based on an inequality index defined explicitly for the multimodal BSS and public transport case. In the following subsection, we describe the most used equality indices in the literature.

2.2. Equality measure indices

In literature, the measure of equalities is often accessed through the weighted coefficients of variation, the Gini index or the Theil index (Gini, 1912; Theil, 1967). Gini index is a measure of inequalities that uses the Lorenz curve and compares a population's distribution income to the egalitarian line (Arnold and Sarabia, 2018). Gini index is widely applied for evaluating the transport infrastructure and systems inequality (Pritchard et al., 2019; Mollanejad and Zhang, 2014; Camporeale et al., 2019; Gallo, 2020). In particular, in this paper, we considered the Theil index for measuring equality.

The Theil index, proposed by Henri Theil, was primarily introduced in economic as a measure of the level of inequalities among the different groups and individuals with unique properties (Conceição and Ferreira, 2000). The concept of inequalities is a matter for every society since it arises the question of opportunities, possibilities and rights for every individual in the population. For example, in most cases, the countries with high-income diversity among people, seen as "income inequalities", have lower economic growth respect to the others. Instead, if every individual has the same or similar opportunities for economic prosperity, this diversity is less. Therefore, the equity among the population is higher and this contributes to the wealth-being of overall society (McKay, 2002).

In the field of transport, the Theil index, as well as the Gini index, is mainly used to find the equality value related to already existing transport infrastructures (Delafontaine et al., 2011; Efthymiou et al., 2014; Hamidi et al., 2019). For example, Chen and Haynes (2017) investigate the influence of high-speed rail on the economic disparity, where the accessibility regarding the development of high-speed rail has been measured through weighted average travel time, potential accessibility and daily accessibility. They adjusted the Theil index to measure intra-regional and inter-regional economic disparity. Only a few studies use these indicators in the network design phase (Santos et al., 2008; Feng and Zhang, 2012; Camporeale et al., 2019; Caggiani et al., 2017a; Caggiani et al., 2017b).

Considering that previous studies scarcely investigate the equality concept in the multimodal and public network design phase, in this study, we introduce it in the design model. Furthermore, one of the primary causes of interruptions and half-realised projects in the network design phase is due to the limited resources and budget availability leading to network inaccessibility, fewer opportunities and transport inequalities for some users among the population groups, especially in urban zones. Therefore, we introduce the equality concept, based on the Theil index, for transport accessibility in the multimodal and public network design phase to create opportunities for all individuals to reach destinations using a combination of transport modes. In particular, the objective of the paper is the achievement, under a limited available budget, of equitable BSS through the minimisation of inequalities in multimodal transport accessibility (bike and public transport in the urban zones) among different population groups. In that way, all population groups could have equal opportunities to reach the desired destination as better explained in the next section.

3. Problem description

In this section, we describe the model that we developed regarding BSSL network design. In Subsection 3.1, we introduce the concept of equality based on the Theil index. In Subsection 3.2, our novel accessibility index and the Theil between index for BSS-public transport framework is presented. Then in Subsection 3.2. we propose our formulation of the BSSL network design model, based on the suggested indices.

3.1. Assessing transport equality through the Theil index

Our model is based on the Theil index to minimise the inequalities between observed population groups. The general formulation of Theil index T is given as follows (Eq. (1)):

$$T = \frac{1}{P_{tot}} \sum_{j=1}^{m} \frac{y_j}{\bar{y}}\, ln\left(\frac{y_j}{\bar{y}}\right) \tag{1}$$

where the value of inequality is based on the total population P_{tot} made by m individuals j, on the variable y_j (income of an individual j) and the variable $\bar{y}$ (average per capita income in the study area).

The measure of inequality in Theil index is composed of two parts (see Eq. (2)), Theil within and Theil between, where the first part points out the variations among the individuals of the same group, while the second part quantifies the inequality between different groups (Bellu and Liberati, 2006; Caggiani et al., 2019b):

$$T = \sum_{i=1}^{g} \sum\nolimits_{j}^{m_i} \left(\frac{1}{P_{tot}} \frac{y_{ij}}{y_i}\right) ln\left(\frac{y_{ij}}{y_i}\right) + \sum_{i=1}^{g} \left(\frac{P_i}{P_{tot}} \frac{y_i}{\bar{y}}\right) ln\left(\frac{y_i}{\bar{y}}\right) \tag{2}$$

where we have g groups i, and each is constituted by m_i individuals; P_i is the number of people belonging to the group i, y_i is the average per capita income of the group and y_{ij} is the income of a single individual.

3.2. Proposed accessibility and inequality indices

We use the Theil between index in the suggested objective function, described as follows. Since the aim of our model is to minimise the difference in transport accessibility among population groups, instead of y_j we used an accessibility measure, defined in this paper, called Multimodal BSS-Public Transport Accessibility (MBPTA).

This proposed accessibility measure can be applied to any urban public transport system that operates on fixed stops and routes such as bus, metro or tram. Indeed, when the characteristics of the services/lines of the analysed public transport are known (such as travel times and stop locations), the MBPTA can be calculated considering the travel times between the origins and destinations, as described in detail below.

To specify MBPTA, the network must be divided into n zones (districts). Each zone is associated with a centroid and the possibility of using one or combined modes of transport.

The value of MBPTA, denoted by $A_o(\mathbf{S})$, from origin zone o to all other n zones, is measured as follows (Eq. (3)):

$$A_o(\mathbf{S}) = \frac{\sum_{d=1}^{n} \frac{t_{min}^{od}}{t_{mu\_min}^{od}(\mathbf{S})}}{n-1}, \quad d \neq o \tag{3}$$

where:

- t_{min}^{od} is the minimum travel time between origin and destination zone considering the shortest path and the speed of the fastest transport mode among those considered (i.e. bus, metro, etc.). This value is calculated whether or not there is a public transport system that connects the origin o to the destination d along the shortest path;
- $t_{mu\_min}^{od}$ is the minimum travel time associated with all possible combinations of transport modes between origin zone o and destination zone d. In particular, this value depends on the set of all bike station locations $\mathbf{S}$. However, if o-d pair is not associated with any multimodal transport combination, this value is equal to infinity.

For example, if we consider the simple case of an area consisting of three zones (n = 3) with a bus line connecting the centroids 1 and 3 and a bike-sharing system with a station in each one of the three centroids, $A_1(\mathbf{S})$ would be calculated with its following components:

- t_{min}^{1-2} that is the time the bus would take (supposed that it is the fastest mode between bike and bus) to go from 1 to 2 along the shortest path (it is a hypothetical value since zone 1 is not connected to zone 2 by bus).
- t_{min}^{1-3} that is the time the bus takes (would take) to go from 1 to 3 if its route is (is not) along the shortest path between 1 and 3.
- $t_{mu\_min}^{1-2}(\mathbf{S})$ that is the minimum time between two possible cases: a) the time to cycle along the shortest path between 1 and 2; b) the time to go by bus along its real route between 1 and 3 added to the time for cycling 3 to 2.
- $t_{mu\_min}^{1-3}(\mathbf{S})$ that is the minimum time between two possible cases: a) the time to cycle along the shortest path between 1 and 3; b) the time to go by bus along its real route between 1 and 3.

These values, when considering the bus mode, include the average waiting times at the stop. If we had chosen another public transport system, instead of bus lines, the MBPTA calculation procedure would not have changed. The only difference would have been in setting the appropriate travel and waiting times at stops/stations of the considered public transport.

According to the given formulation (Eq. (3)), the value of $A_o(\mathbf{S})$ can be greater than zero and lower than or equal to one. The lower values of $A_o(\mathbf{S})$ indicate that the lowest multimodal travel times, from observed zone o to the other zones, are much higher than their absolute minimum values. Instead, the higher values of $A_o(\mathbf{S})$ indicate higher MBPTA from the zone o to the other zones, according to the defined scenarios of transport modes. In other words, this accessibility measure shows, on average for an origin o, how quickly we can reach all the destinations, compared to the hypothetical fastest case. The closer this measure is to one, the faster the destinations will be accessible from the origin o.

Considering $A_o(\mathbf{S})$, we can calculate the average MBPTA of each group i (Eq. (4)) and the average MBPTA of the total population (Eq. (5)), where P_i^o is the number of people belonging to the group i and the zone o.

$$A_i(\mathbf{S}) = \frac{\sum_{o=1}^{n} P_i^o \cdot A_o(\mathbf{S})}{\sum_{o=1}^{n} P_i^o} \tag{4}$$

$$\bar{A}(\mathbf{S}) = \frac{\sum_{i=1}^{g} \sum_{o=1}^{n} P_i^o \cdot A_o(\mathbf{S})}{P_{tot}} \tag{5}$$

Replacing (4) and (5) in the expression of Theil between, we obtain the following objective function:

$$T_b(\mathbf{S}) = \sum_{i=1}^{g} \left(\frac{P_i}{P_{tot}} \frac{A_i(\mathbf{S})}{\bar{A}(\mathbf{S})} \right) ln\left(\frac{A_i(\mathbf{S})}{\bar{A}(\mathbf{S})} \right) \tag{6}$$

Similarly to what is pointed out in the literature, the primary variable contained in this proposed inequality index relates to accessibility/travel times. It depends on the position of the bike-sharing stations **S**, as specified by equations (3), (4) and (5). This adjusted formulation of Theil between assesses the value of multimodal BSS-public transport accessibility inequality between g groups. These groups, in general, can refer to two main and opposite categories of the population, seen as the “advantaged” and “disadvantaged” people according to the level of income or education, the age, the state of employment, etc. $T_b(\mathbf{S})$ can take on positive or negative values, where the lower values indicate the less diversity between population groups. According to the equality principle, the ideal bike stations location would require that all individuals in the observed groups have equal transport opportunities. Therefore, our goal is to minimise $T_b(\mathbf{S})$, as described in the following subsection.

3.3. Bike-Sharing stations network design model

As seen in the literature section, the objectives often adopted for bike-sharing systems design are the maximisation of coverage/accessibility or the minimisation of users’ travel times. These objective functions do not take into account equality aspects. One of the essential bike-sharing systems purposes should be to provide a cheaper alternative to the ownership and maintenance of a bike. Actually, the costs associated with a personal bike can be excessive for low-income people (Brown, 2017). Nevertheless, these systems are frequently implemented in higher-income neighbourhoods, thus restricting the use by those who need them most (McNeil et al., 2017; ITDP, 2018), creating more significant discrimination between different groups of the same population.

For these reasons, we developed a network design model for BSSL to select them among all possible solutions according to the previously defined equality principles between observed groups, under a limited available budget. The model investigates the accessibility inequalities among individuals in the population associated with each group i, for each zone of the network. In particular, the goal of the network design model proposed here is the minimisation of Theil between according to the MBPTA, ensuring specific levels of accessibility and coverage at the same time. Therefore, the proposed problem for BSSL network design model is given as follows:

$$min T_b(\mathbf{S}) = \sum_{i=1}^{g}\left(\frac{P_i}{P_{tot}}\frac{A_i(\mathbf{S})}{\bar{A}(\mathbf{S})}\right) ln\left(\frac{A_i(\mathbf{S})}{\bar{A}(\mathbf{S})}\right) \tag{7}$$

s.t.

$$C(\mathbf{S}) = B \tag{8}$$

$$A\bar{C}C(\mathbf{S}) \geq \alpha \quad \text{with} \quad A\bar{C}C(\mathbf{S}) = \frac{\sum_{o=1}^{n} A_o(\mathbf{S})}{n-1} \tag{9}$$

$$C\bar{O}V(\mathbf{S}) \geq c \quad \text{with} \quad C\bar{O}V(\mathbf{S}) = \frac{\sum_{o=1}^{n}\sum_{d=1}^{n} cov_{o,d}(\mathbf{S})}{n(n-1)} \tag{10}$$

$$cov_{o,d}(\mathbf{S}) = \begin{cases} 1 & \text{if } t_{mu\_min}^{od}(\mathbf{S}) \neq \infty \\ 0 & \text{if } t_{mu\_min}^{od}(\mathbf{S}) = \infty \end{cases} \tag{11}$$

$$\mathbf{S} \subseteq \mathbf{H} \tag{12}$$

The decision variable of the problem is the set **S** of chosen locations for the implementation of the bike-sharing stations. The objective function (7) aims to minimise the difference in Multimodal BSS-Public Transport Accessibility (MBPTA) between population groups. The constraint (8) is the budget constraint where the total infrastructure cost for building bike stations $C(\mathbf{S})$ in the bike-sharing system must be equal to available budget B. In the constraint (9), the mean value of $A_o(\mathbf{S})$ (i.e. $A\bar{C}C(\mathbf{S})$) should be higher than a constant value α. This constraint ensures that the minimisation of inequalities does not reduce the average accessibility below a specific value. With constraint (10) we establish that the total coverage of the entire area, $C\bar{O}V(\mathbf{S})$,should be higher than a constant value c. $C\bar{O}V(\mathbf{S})$ depend on $cov_{o,d}(\mathbf{S})$ variable. This variable, constraint (11), is equal to one, if a given pair of zones is connected with one of the predefined transport modes, (i.e. $t_{mu\_min}^{od}(\mathbf{S})$ take a finite value) or is equal to zero if no predefined transport modes connect the two zones. Finally, with constraint (12) we impose that the set of the chosen bike station locations **S** should be selected among set **H** of all the potential locations. This set can be identified according to some basic principles. For example, bike-sharing stations should be easy to find/reach by pedestrians, cyclists and maintenance/relocation vehicles. They should be located near public transport stops and along paths with the high pedestrian flow, low volume of cars and low-speed limits, sited near to bike paths and so on. For a detailed description of the factors affecting the layout, location and sizing of BSSs see, for example, the NACTO (2016) and the ITDP (2018) guides.

The stations sizing is not the subject of this work and the problem (7)-(12) does not consider the origin–destination demand. In a BSS-public transport framework, for proper stations sizing, a joint design that considers as variables the number of docks/bikes of each bike station, as well as the capacities/frequencies of the public transport lines, is required. In this problem, it should be taken into account both bike-sharing and public transport demand (single-mode and multimodal demand). Besides the equality of the station locations, the dimensioning should guarantee equally distributed resources among all users in terms of the number of docks/bikes and public transport frequencies. For this reason, in a multimodal context, a BSS dimensioning equality problem would deserve a separate study. In a future work will be suggested a bi-level model for defining the optimal number of docks/bikes to be assigned to each bike station location of the set **S** obtained through the proposed model.

4. Numerical application

In this section, we apply and test the proposed model on a bus line network in an urban context through a numerical application. However, both the MBPTA accessibility measure (Eq. (3)) with the related Theil between index (Eq. (6)) and the proposed problem (7)-(12), can also be applied to other public transport systems or even to more than one public transport at the same time. Instead of bus lines, for example, metro or tram lines might be considered. The main difference with the following numerical application would consist only in setting the appropriate travel and waiting times at stops/stations of the public transport lines.

The data and the network used in the application are set to represent a possible urban environment configuration. We evaluate the proposed model considering a test network with a generic grid layout, but it can be applied to any urban network structure and public transport lines, regardless of streets configuration. Indeed, all involved variables of the problem (7)-(12) depend only on zone population and on travel time between points. These travel times, useful for calculating the proposed accessibility measure (MBPTA), are based on distances measured along urban streets and not along the straight-line between points (Euclidean distances). They can be determined in the same way both for similar or different layouts compared to the test network (such as for tree, radial or cellular streets structures).

In Subsection 4.1, we define the basic assumptions and the data related to the network and previously defined parameters. In Subsection 4.2, we describe the optimisation process and the obtained results. The evaluation of used parameters as well as the performance/effectiveness of the proposed model is carried out with a sensitivity analysis in Subsection 4.3.

4.1. Basic assumptions and starting data

The proposed network design model for a bike-sharing stations location is tested on a network with 693 nodes and 2616 arcs (Fig. 1). The system is defined on a grid of 3.0 km × 3.6 km composed of streets, three bus lines with bus stops and potential bike stations. We set 200 possible bike station locations (e.g. where a station is spatially feasible) and divided the network into 29 zones with its corresponding origin/destination centroid. Our model can be applied to any number of groups. In this application, we have assumed two ($g = 2$) opponent groups (e.g. low-income people and the rest of the population).

For this application we defined four possible single/combinations modes of transport (Fig. 2) as follows: bus only, bike of BSS only, bike of BSS and bus - case 1 (each mode used for a single trip), bike of BSS and bus - case 2 (bike used for the stretch towards the departure bus stop and the stretch from the arrival bus stop to the destination).

Each combination of transport modes includes walking distance from centroids to bike-sharing/bus stops stations and vice versa. Couples of centroids less than 500 m apart are not considered as we assume that among them, the users move on foot (Pongprasert and Kubota, 2019). For distances between centroids greater than this value, all users take BSS bikes and buses to reach their destinations. Moreover, in this application, we do not take into account the transfer between bus lines.

For a bike-sharing station or a bus stop to be considered accessible, it is necessary to evaluate the willingness of a user to reach it in terms of time or space spent. For this reason, to make the application as realistic as possible and to define the choice set of alternatives, for each mode used for travelling between the centroids, we imposed some distance limitations. In literature, several studies propose the willingness to walk or to cycle distances/times. These distances are not the same for each user and depend on many factors including, for instance, income or age. The survey of Sarker et al. (2019) shows the willingness to walk to a bus stop is between 300 and 600 m for most of the respondents of the inner city area of Munich regardless the public transport mode considered (bus, metro, tram and suburban train). However, they show that people walk farther to reach a train than a bus/tram. According to Kabra et al. (2016), most of the bike-share usage comes from areas within 300 m from the stations. As for the maximum distance that can be covered by bike, 5 km could be considered a threshold (Winters et al., 2010; Dill, 2009). It is also noted that, since the transfer is perceived as an inconvenience, except for metro journeys (Alshalalfah and Shalaby, 2007), users of a transport system would be less willing to walk in the case of transfer (van Soest et al. 2020). However, these works are often related to a specific case study and cannot be generalised. These values should be calibrated on the particular case taking into account that the lower the distances that users have to walk, the higher the accessibility to transport systems and the number of users available to accept these distances. In particular, in our numerical application, we set the following distance thresholds (they must be measured along the arcs of the network and not in a direct line):

- the maximum cycling distance is 5 km;
- the maximum walking distance:
  - from a centroid to a bus stop and vice-versa is 300 m;
  - from a centroid to a bike station and vice-versa is 250 m;
  - between bike station and bus stop is 150 m.

These limitations may imply that some *o-d* pairs (for example zones very distant from each other or not reachable with BSS/bus) are not accessible by the defined transport modes. In other words, it is necessary to use a private car or different transport modes to travel between these zones. Therefore, the potential stations that do not respect these maximum distances are not considered in **H**. In this way, the set **H** consists of a reduced number of potential stations' positions.

Since the accessibility in the model (see Eq. (3)) is calculated considering travel times, we set the average speed for each transport mode equal to 5 km/h for pedestrians, 10 km/h for bikes and 15 km/h for buses. Furthermore, the average user's waiting time at bus stop is set to be 5 min. For calculating the travel time t_{min}^{od} of Eq. (3), we consider the average speed of buses (speed of the fastest

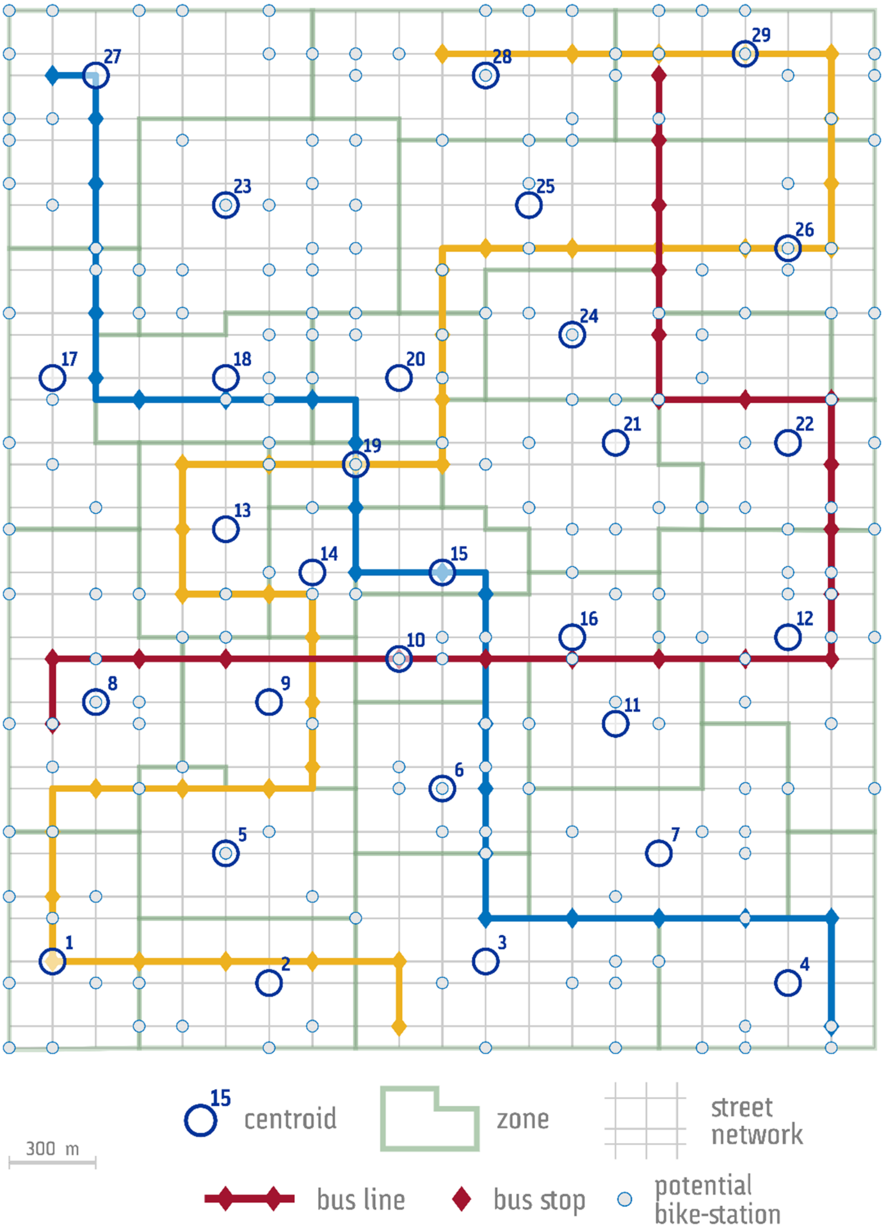

Fig. 1. Transport network tested with numerical application.

transport mode among all alternatives).

The values of parameters described in the proposed model (7)-(12) are presented in Table 1.

In particular, we represent the budget through the number of stations that could be constructed according to available funds, considering an equal average number of docks/racks for each station (the design of the number of docks for each station is not the subject of this work).

4.2. Optimisation and results

The test network has a high number of possible *o-d* combinations and transport mono/multimodal alternatives. The techniques for solving these network sized optimisation problems are mainly selected among heuristic and metaheuristic approaches. Therefore, we used a genetic algorithm for finding an optimal set of chosen bike stations, preserving equality among population groups, according to the available budget. We generate the set of individuals as a "chromosome" with the length equal to the number of possible bike station locations (**H**). Each chromosome individual is seen as a variable taking the value one or zero if the bike station location is chosen or not. The genetic algorithm generates the set of solutions based on the evaluation of fitness function that aims to minimise

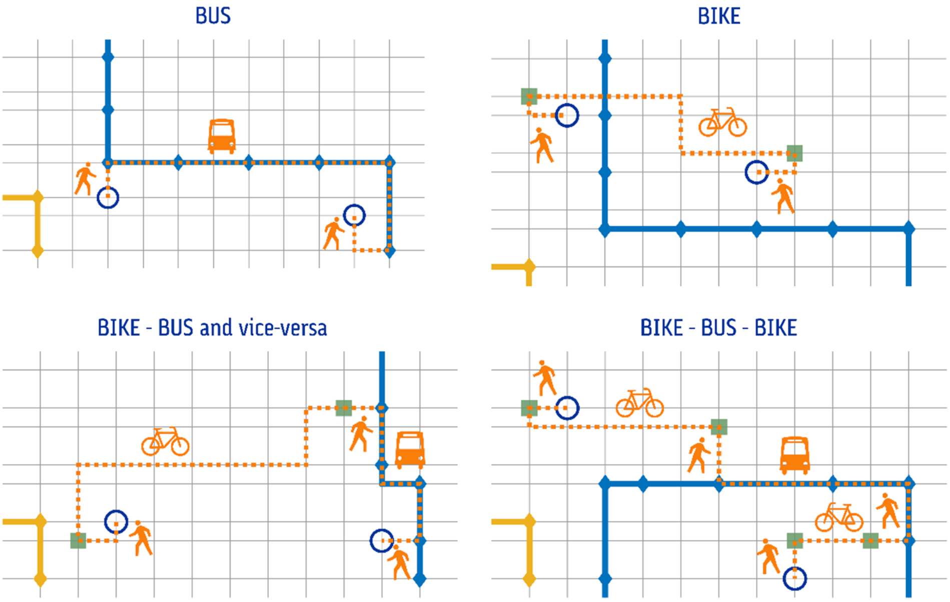

Fig. 2. Defined combinations of transport modes.

Table 1
Model parameters' value.

parameter	*description*	*value*
B	budget (maximum number of stations)	30
α	minimum value of mean accessibility	0.4
c	minimum value of total coverage	0.9

$T_b(\mathbf{S})$ and achieve equal accessibility among the two population groups.

The proposed model was run on a computer with a 64-bit Windows 10 operating system with an Intel® Core™ i7-4790 CPU (3.60 GHz) and 16 GB of RAM. The results were obtained after running 30 optimisations solved in MATLAB® R2017b with an average computation time of around 75 min for each optimisation.

The graphical representation of the best solution found of the numerical application (the set $\mathbf{S}^*$ of the BSS chosen locations) is presented in Fig. 3.

In this figure, the grey squares represent the 30 bike station locations chosen by the proposed model. As we expected, most of them are located near/over a centroid/bus stop due to the maximum distances that can be travelled on foot.

The objective function value of the optimal solution $\mathbf{S}^*$ is $T_b(\mathbf{S}^*)$ = -3.84E-13, with an average value of accessibility equal to $A\bar{C}C(\mathbf{S}^*)$ = 0.414 and a total coverage $C\bar{O}V(\mathbf{S}^*)$ = 0.938. The performance and the value of the solution are satisfactory since the value of the Theil is very low with good values of accessibility and coverage of the examined area.

In fact, as we show in the next subsection, the $T_b(\mathbf{S}^*)$ value achieved is several orders of magnitude smaller than those obtainable through the maximisation of accessibility or coverage (objective functions often proposed in literature).

The statistical parameters, over 30 simulations, regarding the minimum value, maximum value, average value and variance of obtained results are presented in Table 2.

The obtained values of 30 simulations related to the objective function, the average accessibility and coverage have low diversity. In particular, for all simulation, we got lower values of $T_b(\mathbf{S}^*)$ which indicate that our model can achieve high equality among population groups, offering them equal travel opportunities.

4.2.1. Understanding the obtained results

According to Table 2 and the results related to the best solution found, we can observe that the maximum values of $A\bar{C}C(\mathbf{S}^*)$ and $C\bar{O}V(\mathbf{S}^*)$ do not correspond to the lowest value of $T_b(\mathbf{S}^*)$ that is the highest equality among the population groups. To better understand this behaviour, we calculated the maximum value of average accessibility and the maximum value of the total coverage according to the following two problems (13)-(15) and (16)-(19):

maximisation of the average accessibility:

$$max A\bar{C}C(\mathbf{S}) = \frac{\sum_{o=1}^{n} A_o(\mathbf{S})}{n-1} \tag{13}$$

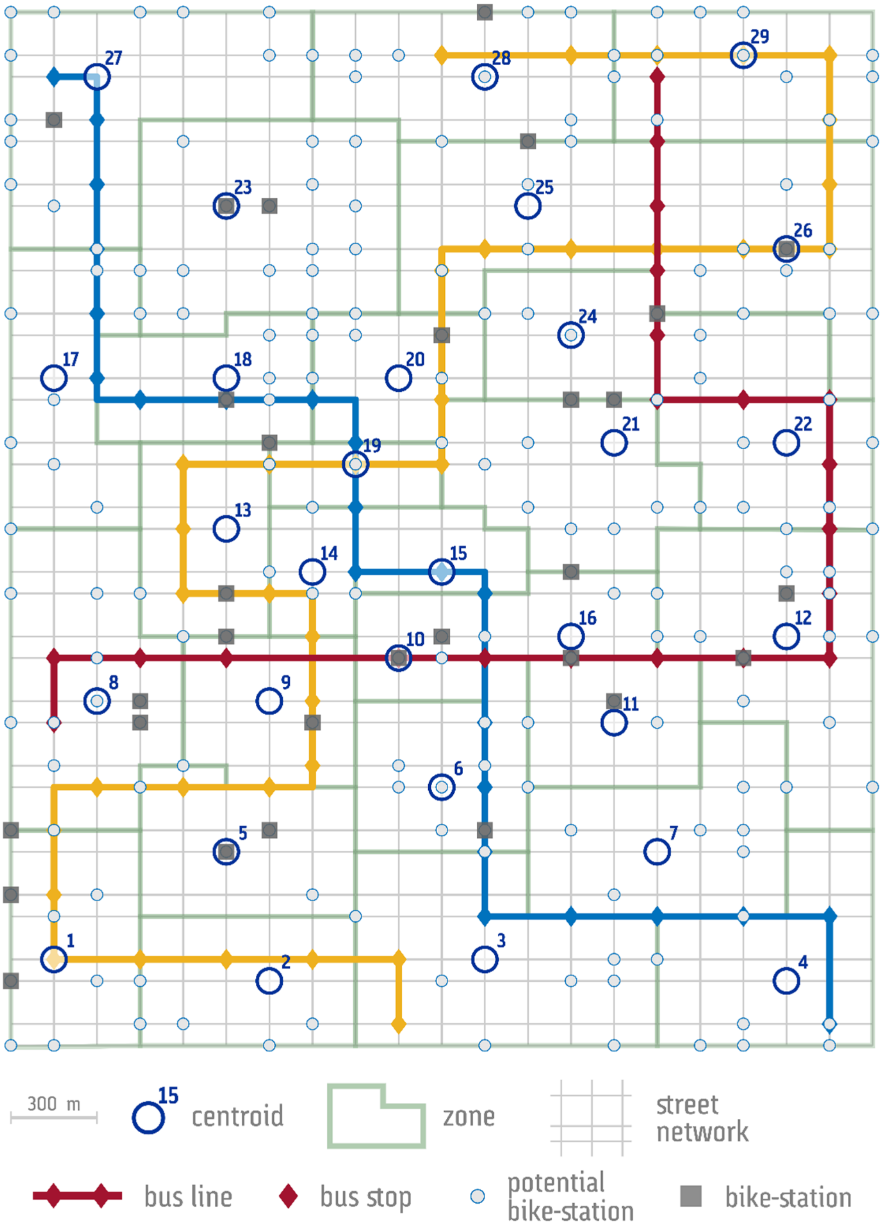

Fig. 3. The best solution found.

Table 2
Statistical parameters.

	min	mean	max	var
$T_b(\mathbf{S}^*)$	−3.84E-13	5.16E-12	7.10E-11	1.67E-22
$A\breve{C}C(\mathbf{S}^*)$	0.403	0.432	0.480	3.50E-04
$C\bar{O}V(\mathbf{S}^*)$	0.915	0.951	0.988	3.15E-04

s.t.

$$c(\mathbf{S}) = B \tag{14}$$

$$\mathbf{S} \subseteq \mathbf{H} \tag{15}$$

maximisation of the total coverage:

Table 3
Statistical parameters of the average accessibility maximisation.

	min	*mean*	max	*var*
$T_b(\mathbf{S}^*)$	1.63E-06	5.05E-05	1.70E-04	2.14E-09
$A\bar{C}C(\mathbf{S}^*)$	0.492	0.515	0.539	9.89E-05
$C\bar{O}V(\mathbf{S}^*)$	0.985	0.997	1.000	2.30E-05

$$maxC\bar{O}V(\mathbf{S}) = \frac{\sum_{o=1}^{n}\sum_{d=1}^{n} cov_{o,d}(\mathbf{S})}{n(n-1)} \tag{16}$$

s.t.

$$c(\mathbf{S}) = B \tag{17}$$

$$cov_{o,d}(\mathbf{S}) = \begin{cases} 1 & \text{if } t_{mu\_min}^{od}(\mathbf{S}) \neq \infty \\ 0 & \text{if } t_{mu\_min}^{od}(\mathbf{S}) = \infty \end{cases} \tag{18}$$

$$\mathbf{S} \subseteq \mathbf{H} \tag{19}$$

The $A\bar{C}C(\mathbf{S})$ and $C\bar{O}V(\mathbf{S})$ values are calculated as presented in (7)-(12). The values of Theil between $T_b(\mathbf{S}^*)$ are calculated according to Eq. (6) starting from the set of chosen locations $\mathbf{S}^*$ belonging to the solution of the problems (13)-(15) and (16)-(19). The results are obtained, also in this case, using a genetic algorithm.

The statistical parameters of obtained results, over 30 simulations, are presented in Table 3 and Table 4.

The objective function of the best solution related to the average accessibility maximisation is equal to $A\bar{C}C(\mathbf{S}^*) = 0.539$ with a total coverage $C\bar{O}V(\mathbf{S}^*) = 1$ and a Theil value $T_b(\mathbf{S}^*) = 1.24\text{E-05}$. The best solution found regarding the second problem (total coverage maximisation) resulted in the value of objective function equal to $C\bar{O}V(\mathbf{S}^*) = 1$ with $A\bar{C}C(\mathbf{S}^*) = 0.452$ and $T_b(\mathbf{S}^*) = 2.42\text{E-06}$. As expected, the accessibility and the coverage values are higher than those obtained with the minimisation of inequalities. Concerning accessibility, the result is in line with the literature. Santos et al. (2008), obtain similar results relative to the road network design with equity objectives. However, we can observe that in these cases, the value of inequality significantly increases. Maximising accessibility or coverage might result in high values of inequality among potential BSS and public transport users. Moreover, this is even more valid if we observe that $A\bar{C}C(\mathbf{S}^*)$ and $C\bar{O}V(\mathbf{S}^*)$ do not change significantly compared to the case in which the Theil value is minimised. Thus, the contribution of our work highlights the relevance of introducing equality in BSSL objective function, especially in the designing phase. We can reach dual benefits expressed through equality achieved among population groups and satisfactory results regarding multimodal accessibility and coverage, appropriately constraining the problem.

4.3. Sensitivity analysis

We carried out a sensitivity analysis related to the available budget in the proposed BSSL problem. In particular, we solved the problem (7)-(12) imposing a total number of stations (B) lower and higher than 30. The main results, $T_b(\mathbf{S}^*)$, $A\bar{C}C(\mathbf{S}^*)$ and $C\bar{O}V(\mathbf{S}^*)$, according to the available budget, are presented in Fig. 4.

When the maximum number of stations equals 10 ($B = 10$), problem (7)-(12) is infeasible. Consequently, 10 stations are not enough to obtain a minimum value of mean accessibility equal to $\alpha = 0.4$ and a minimum value of total coverage of $c = 0.9$. Accordingly, in this case, we set $\alpha = 0.3$ and $c = 0.8$. In all the other cases, the values of α and c are as presented in Table 1.

It is observed that with lower budgets (10 or 20 stations) the $T_b(\mathbf{S}^*)$ results in relatively higher values. The lowest value of $T_b(\mathbf{S}^*)$ is obtained with 40 stations. Regarding MBPTA, this number of stations seems to provide the highest level of equality among population groups and good-quality values of accessibility and coverage.

With 50 stations, we would have expected even lower inequality values with a downward trend. This probably does not occur due to the growing complexity of the problem, for which it is not possible to find an objective function value even lower than the case with 40 stations. However, it should be noted that in any case all the best values found of $T_b(\mathbf{S}^*)$ (regardless of the available budget) are very low, which is between 1.2E-14 and −5.0E-13. The average accessibility and the total coverage are substantially constant irrespective of the available budget, except for the case with 10 stations (for the reasons mentioned above).

Similarly, to the previous subsection, we carried out a sensitivity analysis by solving problems (13)-(15) and (16)-(19) assuming

Table 4
Statistical parameters of the total coverage maximisation.

	min	*mean*	max	*var*
$T_b(\mathbf{S}^*)$	7.70E-07	6.52E-05	4.05E-04	8.28E-09
$A\bar{C}C(\mathbf{S}^*)$	0.432	0.460	0.492	1.74E-04
$C\bar{O}V(\mathbf{S}^*)$	0.990	0.997	1.000	1.46E-05

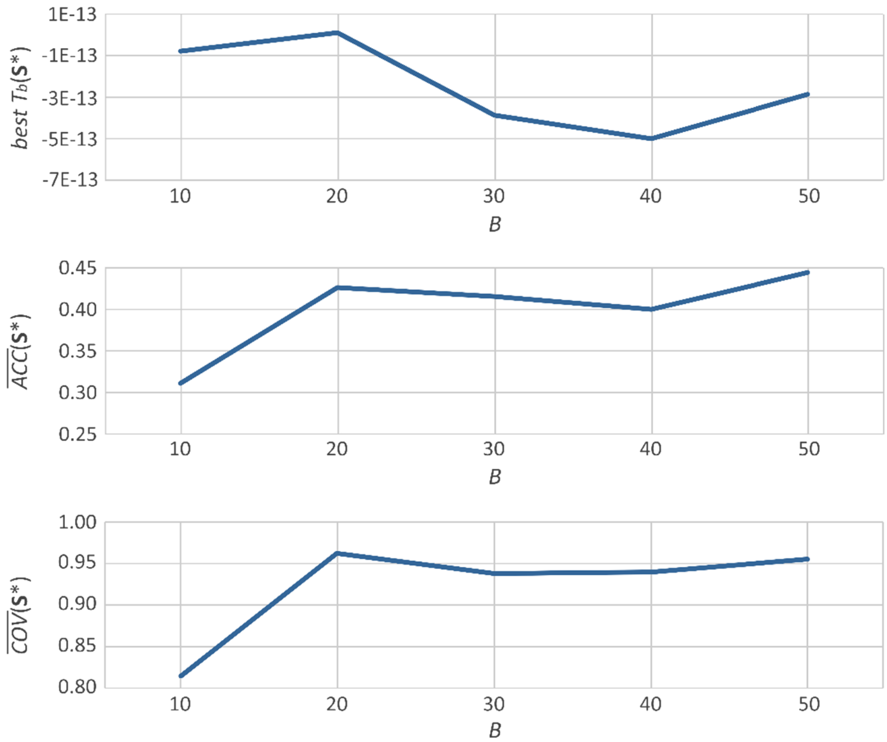

Fig. 4. Main results of the sensitivity analysis.

the available budget values equal to 10, 20, 40 and 50 stations. The summary of the results (for 30 optimisations), including the solutions of problem (7)-(12), is presented in Table 5 and Table 6.

As depicted in Table 5 and Table 6, the maximisation of the average accessibility or the total coverage leads to high inequality among population groups regardless of the budget. In these cases, the $T_b(\mathbf{S}^*)$ values are significantly higher than those obtained with the proposed model (column $minT_b(\mathbf{S})$). However, for all cases, the values of inequality achieved according to a high available budget are relatively lower. Moreover, the values of $A\bar{C}C(\mathbf{S}^*)$ are increasing as the available budget increases.

5. Conclusions

Bike-sharing systems are progressively adopted by municipal administrations to promote sustainable mobility and support existing public transport systems. In particular, BSSs are implemented to expand the transport supply, increasing accessibility and

Table 5
Statistical parameters of the sensitivity analysis - budget lower than 30.

	$B = 10$								
	$minT_b(\mathbf{S})$			$maxA\bar{C}C(\mathbf{S})$			$maxC\bar{O}V(\mathbf{S})$		
	min	max	*best*	min	max	*best*	min	max	*best*
$T_b(\mathbf{S}^*)$	−7.7E-14	2.5E-09	−7.7E-14	7.2E-07	8.5E-04	5.8E-05	6.0E-07	8.5E-04	1.4E-04
$A\bar{C}C(\mathbf{S}^*)$	0.302	0.331	0.312	0.312	0.347	0.347	0.285	0.339	0.317
$C\bar{O}V(\mathbf{S}^*)$	0.805	0.868	0.815	0.766	0.868	0.850	0.803	0.875	0.875
	$B = 20$								
	$minT_b(\mathbf{S})$			$maxA\bar{C}C(\mathbf{S})$			$maxC\bar{O}V(\mathbf{S})$		
	min	max	*best*	min	max	*best*	min	max	*best*
$T_b(\mathbf{S}^*)$	1.2E-14	3.6E-11	1.2E-14	5.3E-08	4.0E-04	1.1E-04	9.3E-09	1.1E-03	1.5E-06
$A\bar{C}C(\mathbf{S}^*)$	0.400	0.454	0.426	0.440	0.477	0.477	0.406	0.451	0.413
$C\bar{O}V(\mathbf{S}^*)$	0.905	0.970	0.963	0.935	0.985	0.963	0.960	1.000	1.000

Table 6
Statistical parameters of the sensitivity analysis – budget higher than 30.

	$B = 40$								
	$minT_b(\mathbf{S})$			$max\bar{ACC}(\mathbf{S})$			$max\bar{COV}(\mathbf{S})$		
	min	max	*best*	min	max	*best*	min	max	*best*
$T_b(\mathbf{S}^*)$	−5.0E-13	4.6E-11	−5.0E-13	6.8E-08	6.0E-05	3.0E-05	2.6E-07	1.6E-04	2.6E-07
$\bar{ACC}(\mathbf{S}^*)$	0.399	0.486	0.399	0.521	0.544	0.544	0.449	0.508	0.461
$\bar{COV}(\mathbf{S}^*)$	0.900	0.993	0.940	0.993	1.000	1.000	0.995	1.000	1.000
	$B = 50$								
	$minT_b(\mathbf{S})$			$max\bar{ACC}(\mathbf{S})$			$max\bar{COV}(\mathbf{S})$		
	min	max	*best*	min	max	*best*	min	max	*best*
$T_b(\mathbf{S}^*)$	−2.8E-13	9.7E-11	−2.8E-13	1.9E-05	5.3E-05	3.1E-05	1.7E-07	1.8E-04	1.7E-07
$\bar{ACC}(\mathbf{S}^*)$	0.405	0.492	0.444	0.537	0.547	0.547	0.464	0.506	0.470
$\bar{COV}(\mathbf{S}^*)$	0.910	0.980	0.955	1.000	1.000	1.000	1.000	1.000	1.000

territory coverage. Therefore, it is crucial to investigate the issues related to the design of BSS systems, especially regarding potential advantages they can offer to the population. In this work, we propose a new model for stations location of a BSS. Differently from the literature, we want to link the location design of a BSS to urban public transport lines considering social equality principles. We use the Theil index to calculate equality, considering as a variable the multimodal accessibility BSS-public transport. The objective of the proposed model is to find, under a limited available budget, the best locations for the stations of a BSS to minimise inequalities between different groups of the population, considering multimodal BSS-public transport accessibility. The results obtained, through the application of our model, show its effectiveness. The locations identified for the BSS stations guarantee a low level of inequality among two different income population groups. Furthermore, we found that the maximisation of BSS-bus accessibility or coverage leads to high values of inequality. It is, therefore, necessary to consider equality in the design of the BSSs if you want to assure equal multimodal BSS-bus accessibility among the population. The proposed approach might be applied not only to bus lines but also to metro, tram or other urban public transport systems on any type of urban street networks. It also could serve transport and public authorities as a decision support system for future investment related to bike-sharing infrastructures. The proposed accessibility measure, named Multimodal BSS-Public Transport Accessibility, can also be applied to design new urban public transport lines or to modify the routes and stops of existing ones. Thus, additional research may relate to the design of public transport urban lines by keeping bike-sharing station locations as constants or variables, to maximise the equality between different groups of the population. Future developments of this work will concern stations sizing, defining the number of docks for each station and the total number of bikes, depending on bike-sharing and public transport demand. An equality-based bi-level programming will be developed, with the proposed model at the first level and a demand assignment model at the second one. This model, in addition to the sizing variables, will also consider public transport trip frequencies, optimising vehicle scheduling starting from the overall origin–destination demand. Moreover, further research could be carried out considering vertical equity indices, to support some disadvantaged groups of the population over others, and other shared vehicle systems in the mobility as a service framework.

CRediT authorship contribution statement

Leonardo Caggiani: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing - original draft, Writing - review & editing, Visualization. **Aleksandra Colovic:** Conceptualization, Methodology, Validation, Formal analysis, Investigation, Writing - original draft, Writing - review & editing. **Michele Ottomanelli:** Conceptualization, Methodology, Validation, Resources, Writing - review & editing, Supervision, Project administration.

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