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Station Site Optimization in Bike Sharing Systems

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***Abstract*—Bike sharing systems, aiming at providing the missing links in the public transportation systems, are becoming popular in urban cities. In an ideal bike sharing network, the station locations are usually selected in a way that there are balanced pick-ups and drop-offs among stations. This can help avoid expensive re-balancing operations and maintain high user satisfaction. However, it is a challenging task to develop such an efficient bike sharing system with appropriate station locations. Indeed, the bike station demand is influenced by multiple factors of surrounding environment and complex public transportation networks. Limited efforts have been made to develop demand-and-balance prediction models for bike sharing systems by considering all these factors. To this end, in this paper, we propose a bike sharing network optimization approach by considering multiple influential factors. The goal is to enhance the quality and efficiency of the bike sharing service by selecting the right station locations. Along this line, we first extract fine-grained discriminative features from human mobility data, point of interests (POI), as well as station network structures. Then, prediction models based on Artificial Neural Networks (ANN) are developed for predicting station demand and balance. In addition, based on the learned patterns of station demand and balance, a genetic algorithm based optimization model is built to choose a set of stations from a large number of candidates in a way such that the station usage is maximized and the number of unbalanced stations is minimized. Finally, the extensive experimental results on the NYC CitiBike sharing system show the advantages of our approach for optimizing the station site allocation in terms of the bike usage as well as the required re-balancing efforts.**

I. Introduction

Recent years have witnessed a worldwide prevalence and popularity of public bike sharing system [1, 2]. Bike sharing system is a short-term bicycle rental service with many automatic rental stations scattered over an urban city that provides bicycles for inner-city transportation from one established bike station to another. With the exploding growth, public bike sharing system has rapidly emerged as an innovative and sustainable transportation option with various benefits. On the one hand, the bike sharing systems offer an environment friendly solution for the "first-and-last mile" connection for both short and long distance destinations as well as for bridging the gap between existing transportation modes such as subways and rail systems. On the other hand, from the perspective of customers, bicycling is an affordable, convenient, healthy and sometimes even more efficient than vehicles or other public transportation in congested urban cities by avoiding the traffic hold-ups and detours.

To offer immediate and convenient access, a network of bike docking stations are positioned throughout an urban area. However, developing an efficient bike sharing system with proper station locations is a challenging task. To construct a successful bike sharing network, we must consider the station locations in the bike sharing network and their relationship with trip demand and balance [3–5]. Specifically, there are two major challenges for bike station site selections. First, bike sharing system is an undirected network that the performance (i.e., bicycle demand) of one station highly depends on its connection to other stations and its surrounding human activities. The multi-factor effects of surrounding environment and station network structure make it difficult to predict station demand. Second, the demand distribution is unbalanced both geographically and temporally. It is costly to dispatch bikes from full stations to empty stations for re-balancing, and the efficiency of the station usage is reduced during the unavailable period. The empty or full stations also make customers inconvenient to pick-up or drop-off.

Recently, a number of researches on bike sharing systems analysis have been conducted from different aspects. Most of the studies have focused on the historic development of bicycle sharing system [1], promotion strategies [6], bicycle temporal and geographical usage patterns analysis [7], station demand related factors [8] and re-balancing bicycles among established bike stations [9, 10]. However, there are relatively few studies quantitatively addressing the relationship between the multiple influential factors and the station demand or its geographically imbalance distribution.

To solve the aforementioned challenges, in this paper, we first extract insightful features from human mobility data, POIs and bike station network structures. Next, we propose an Artificial Neural Network based prediction model for station demand and balance prediction according to the features extracted. Then an optimization problem aiming at maximizing station demand and minimizing the number of unbalanced stations is addressed and solved using a genetic algorithm. The performance of our prediction model and optimization strategy is comprehensively evaluated on real world bike sharing system data generated by NYC CitiBike System and the experimental results demonstrated the effectiveness and efficiency of our proposed method.

II. Problem Formulation

In this section, we first introduce some preliminaries used throughout this paper, and then formally define the problem of bike station network optimization.

A. Preliminaries

1) Station Network: The bike station network is represented by a direct graph $G = (S, E)$. With each station $s \in S$ as a node, the edges in E are directed connections of bike stations $e_{ij} = (s_i, s_j) \in E$. Each node and edge have several attributes. For example, $e_{ij}.f$ represents the commuting

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frequency of a pick-up at station s_i and a drop-off at station s_j.

Since need-based customers will choose the station closest to their current locations or final destinations, we partition the bike station in service area using a Voronoi-based gridding method [11], from which the map is partitioned into regions based on walking distance to bike stations. Each grid is centered by one bike station and the points within one region is closest to its center. As a result, pick-up/drop-off points for taxi trips and POIs are mapped to the nearest bike station.

Definition 1 (Voronoi Region)**.** Let X be a space coordinate endowed with a walking distance wd extracted from Google Maps Distance Matrix API. The Voronoi region R_{s_i} associated with station s_i is the set of all points in X whose distance to s_i is no greater than their distance to other stations: $R_{s_i} = \{x \in X | wd(x, s_i) \leq wd(x, s_j), \forall j \neq i\}$.

The NYC CitiBike in service area (Manhattan island below 61st street and western Brooklyn) is partitioned into Voronoi Regions centered by each CitiBike Station (see Figure 1(a)).

2) Station Demand: The station demand is defined as the average pick-up frequency/hour when this station is available. Station availability means the station is in service and there are bikes available for pick-up. Station pick-up unavailability is usually due to maintaining and empty dock. We do not consider the station demand during its unavailable period.

Definition 2 (Station Demand)**.** Let $s_i.f(T)$ and $s_i.a(T)$ represent the daily pick-up frequency and station in service time duration (hour) in day T. The station demand (SD_i) is defined as: $SD_i = \frac{1}{T}\sum_T \frac{s_i.f(T)}{s_i.a(T)}$.

The station demand distribution of stations of NYC CitiBike sharing system is presented in Figure 1(b) as an example. In Figure 1(b), each dot represents a current in service bike station in NYC with its size representing its bike demand defined by Defintion 2.

3) Station Balance: Due to unbalanced bike demand distribution, bikes from full stations are dispatched by truck to empty stations, which greatly increases the operation cost of bike sharing systems and affects customers' conveniences. We investigate the station imbalance problem by first introducing the concept of station net pick-up/drop-off frequency from the daily transaction records. Let $\{s_i.pd(t_j)|j = 0, 1...\}$ represents the pick-up/drop-off events of station s_i at time t_j, where $s_i.pd(t_j) = 1$ for pick-up record and $s_i.pd(t_j) = -1$ for drop-off record. The net pick-up (net drop-off) $s_i.np$ ($s_i.nd$) is defined as the contiguous subarray of series $\{s_i.pd(t)\}$ whose values have the largest positive sum (smallest negative sum).

In our study, if the average net pick-up or net drop-off of station exceeds a threshold γ (decided by the tolerance of a station vacancy rate), we discriminate this station as unbalanced. For the situation of NYC CitiBike system, γ equals to the average dock numbers of the CitiBike stations.

Definition 3 (Station Balance)**.** Let $(s_i.np(T_j), s_i.nd(T_j))$, $j = 1, ..., n$ represents the net-pick/drop frequency of station s_i from day T_1 to T_n, the station balance is identified as a binary variable discriminated by a delta function $\delta(x) = 1$ if x is TRUE and 0 otherwise:
$SB_i = \delta(\frac{1}{n}\sum_{j=1}^{n} s_i.np(T_j) \geq \gamma \; or \; \frac{1}{n}\sum_{j=1}^{n} s_i.nd(T_j) \geq \gamma)$

For the example of NYC CitiBike sharing system, the station net pick-up/drop-offs and their balance patterns are presented in Figure 1(c) and Figure 1(d) respectively, with the size of each dot representing the value of net pick-up/drop-off and unbalanced station highlighted in red.

B. Problem Formulation

The bike station network optimization problem for bike sharing systems can be separated into two stages: station demand and balance prediction; station network optimization.

1) Station Demand and Balance Prediction: Given a set of bike station locations and their surrounding features (**F**), the problem of station demand and balance prediction is to predict the station demand defined in Definition 2 and to identify if the station is unbalanced according to Definition 3. In our study, we feed multi-factor features extracted from human mobilities, POIs and station network structures into prediction models based on neural network $NN_{SD}(s_i; \mathbf{F})$ for station demand prediction and neural network $NN_{SB}(s_i; \mathbf{F})$ for station balance prediction.

2) Station Network Optimization: Given the well trained neural network prediction models NN_{SD} and NN_{SB} from stage 1 and a set of bike station location candidates SC of size $|SC| = m$, the problem of station network optimization is to find an optimal subset OC of the location candidates SC such that the total demands from all chosen stations are maximized while the number of unbalanced stations are minimized. Formally, our objective function for station network optimization is defined as follows:

$$\max \mathcal{F}(\mathbf{y}) = \sum_{i=1}^{m} y_i(\frac{1}{n}NN_{SD}(s_i) - \lambda NN_{SB}(s_i)) \quad (1)$$

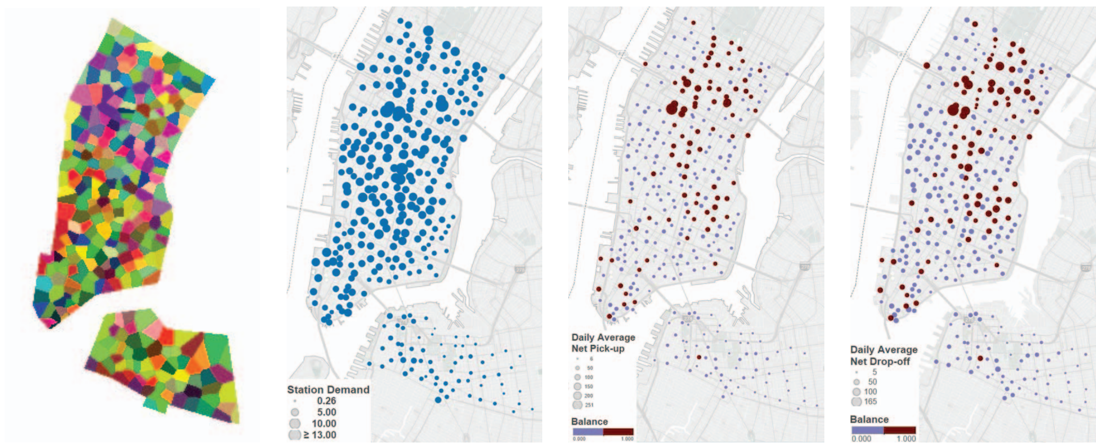
$$s.t. \quad \sum_{i=1}^{k} y_i = n_1 \quad (2)$$

$$\sum_{i=k+1}^{m} y_i = n_2 \quad (3)$$

$$\|s_i - s_j\| \geq y_i y_j d \quad \forall i \neq j \quad (4)$$

$$y_i \in \{0, 1\} \quad j = 1, 2, ..., m \quad (5)$$

where $\mathbf{y} = \{y_1, y_2, ..., y_m\}$ is a binary variable vector. y_i=1 indicates location candidate s_i is chosen to be an optimal station site location, otherwise, $y_i = 0$. "$\|a - b\|$" is spherical earth (ignoring ellipsoidal effects) distance calculation according to the coordinates of two points. λ is a penalty parameter representing the additional cost and demand losing for unbalanced stations. Constrain (2) and Constrain (3) specify the limits of total number of stations in different areas respectively and the total number of stations n is pre-determined. Constrain (4) specifies the minimum distance between any optimal stations. Different from other optimization problems which treat station candidates independently, the station demand and balance are non-functionally decided by the chosen stations indicated by indicator vector **y**. For the same selected candidate s_i, a different network will have different Voronoi Regions and different network structures which will affect the station demand and balance pattern for station s_i.



(a) Station Voronoi Region (b) Demand Distribution (c) Net-pick-up Distribution (d) Net-drop-off Distribution

Figure 1. NYC CitiBike stations in service area Voronoi Region partition, bike demand distribution and balance distribution.

III. Feature Extraction

In this section, we introduce 10 fine-grained features extracted from station network, bicycle trajectories, taxi trajectories and POIs for station demand balance prediction.

A. Transportation Related Features

Public bicycles are widely used for short-term distance traveling and transportation missing link connection. It is very common that people will take bikes to nearby locations with more convenient accesses to other long-distance transportation like subways, taxis, etc. Thus, we extract the walking distance from each bike station to its nearest parking lot (PL), the walking distance to the nearest subway entrance (SE), the taxi pick-up densities (TP) and the number of faster bicycle routes (FR) as our transportation related features. Taxi pick-up density mapped to station s_i is the number of taxi pick-up in Voronoi Regin R_{s_i} divided by the region size: $s_i.TP = \sum_{k_t} \delta(TP_{k_t} \in R_{s_i})/|R_{s_i}|$. Because of traffic jams and vehicle detours, bicycling is faster than vehicles in some areas. For the same origins and destinations, people are more willing to take bikes if it is faster, cheaper and more convenient than vehicles. By tracking bicycles and taxies as speed sensors, we are able to define the feature of number of faster bicycle routes as follows: Let $e_{ij}.vt, e_{ij}.vb$ represents the average transportation time of taxis and bicycles from station s_i to station s_j. The feature number of faster bicycle route is defined as the number of edges taking a bicycle is faster than a taxi: $s_i.FR = \sum_{j \neq i} \delta(e_{ij}.vb - e_{ij}.vt > 0)$.

B. POIs Features

POIs provide us various information about the city from different aspects. The density of POIs is an indicator of human crowd intensity. A high population density means a high probability of bicycle demand. In terms of station balance, the stations near some POI caterogies, like schools and restaurants, are more likely to have a large net pick-up/drop-off during after-school time period and dining time. In this study, we use the densities 4 major categories of POIs surrounding each bicycle station. The POIs are from Yelp dataset including 4569 entertainments (PO_EN), 9303 restaurants (PO_R), 11031 shopping centers (PO_S) and 2238 educations (PO_E).

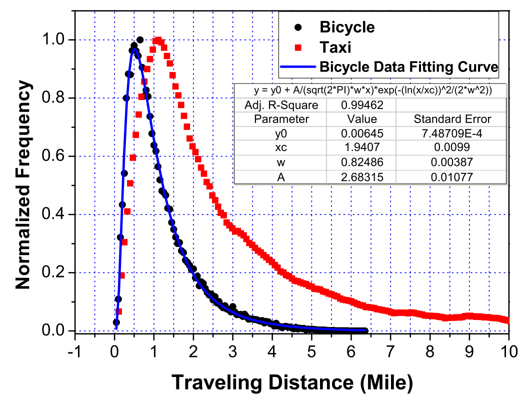


Figure 2. Distance preference comparisons between bicycles and taxis

C. Station Network Profile

Station Scale. The station scale is represented by the total number of docks (ND). Although the pick-up frequency is restricted by the station scale since a small station is more likely to be empty, the station demand from our definition is not restricted by this situation because the empty time period is not counted. In addition, because of the bike sharing re-balance system, the stations with size smaller than the threshold γ defined in Definition 3 can still have a net pick-up/drop-off larger than γ and our station balance definition is not restricted.

Nearby Station Score. Bicycle users will have a different traveling distance preference compared to vehicle users. From the historical traveling distance records of bicycles and taxis, we calculate the statistic frequencies of trips in different trip distance intervals and represent the frequencies as distance preference. The difference of the transportation distance preference between taxis and bicycles are presented in Figure 2. From Figure 2, we can see that people usually take bicycles for 0.5-1.5 mile distance transportation while most people take taxis for long distance destinations where the bicycles can hardly reach. Mathematically, the normalized pick-up frequency versus station distance forms a log-normal distribution (see the blue fitting line in Figure 2). Therefore, given the locations of two stations $s_i.c, s_j.c$ associated with their distance

$x \equiv \|s_i.c - s_j.c\|$, we can estimate the users' preference of taking bicycles from s_i to s_j, which is defined by a single nearby station score ($SNSS_{ij} = y_0 + \frac{A}{\sqrt{2\pi}wx} exp(-\frac{(ln(x/x_c))^2}{2w^2})$). y_0, A, w, x_c are fitting parameters (see fitting results in inserted table of Figure 2). The feature of nearby station score is then defined as $NSS_i = \sum_{j \neq i}^{n} SNSS_{ij}$. From the definition of NSS, we can see that a station should not be located too close or too far away from other stations and the station demand should be positively correlated to the NSS.

IV. Methodology

A. Prediction Model

We propose an artificial neural network (ANN) to predict the station demand and station balance based on the features extracted. The comparative advantage of ANN over most conventional prediction models is that it can implicity detect complex nonlinear relationships between the features from different domains and the targets without any prior assumptions about the underlying data generating process [12]. The details of the specification and estimation of our M-layer ANN model is summarized below.

Layer Input. The net input to unit i in layer $k+1$ is the linear combinations of the outputs α^k in layer k. The network input α^0 is the feature vector normalized within [0,1] ranges by mapping $x = \frac{x - x_{min}}{x_{max} - x_{min}}$.

Layer Output. The output of unit i in layer $k+1$ is mapped from l^{k+1} using a sigmoid activation function $a^{k+1}(i) = \frac{1}{1+e^{-l^{k+1}}}$. The output layer is a linear layer for regression problem of station demand prediction and the final output a^M is t_{sd} (continue variable). For station balance prediction, a threshold output layer is trained and the final output a^M is binary variable t_{sb}.

Training Algorithm. Our training task is to learn the associations between the inputs and the outputs of our training set which aims at minimizing the prediction error: $V = \frac{1}{2}\sum_{i=1}^{nt}(t_i - a_i^M)^2$. The Levenberg-Marquardt algorithm [13] is applied for parameter training. Moreover, a testing set is used for monitoring validation error and overfitting control without affecting training parameters during the training process.

B. Optimization Model

The station network optimization problem is to find a binary indicator vector **y** that maximizes our objective function (equation (1)). We first simulate $k = 1702$ and $m - k = 634$ locations as candidates in Manhattan and Brooklyn areas. Among which, we select $n_1 = 252$ optimal stations from Manhattan and $n_2 = 68$ optimal stations from Brooklyn. The candidates are simulated with equally distanced interval which cover the NYC CitiBike in service area and the docks number of each candidate is simulated to be 35 (the average number of docks of current bike system).

A genetic algorithm (GA) can be understood as a probabilistic search algorithm which is applicable to our combinational optimization problem [14]. In our case, each possible solution (an optimal station network) represented by our indicator vector **y** is identified by a chromosome with each element y_i representing one piece of gene. The process for solving the bike station network optimization problem starts by randomly initializing 1000 individuals as the first generation, which are transformed to the next generation through the designed tournament selection [15], recombination and mutation [16]. The termination criteria is setup by identifying if best objective is varying within 0.2% for 5 continuous generations.

In the tournament selection process of our study, 3 individuals are selected randomly from the large population and the selected individuals compete against each other. The individual with the highest value of objective function among the three is selected as one of the next generation population. This procedure is repeated 100 times and 100 individuals are selected for genetic operation of recombination and mutation to generate next generation. In recombination process, a multiple points crossover specified by a binary vector $S = (s_1, s_2, ..., s_m)$ is applied to determine the genes inherited from the two parents. In general, the crossover point marker S can be arbitrarily decided. However, we limit the structure of S to guarantee the constrain (2) and (3) in our optimization problem. Mutation is applied to explore newly possible offsprings for diversified generation. Two pieces of gene of offsprings from crossover are randomly selected to have 2 genes mutated.

V. Experiment

To validate the efficiency and effectiveness of our proposed method, extensive experiments are performed on real world NYC CitiBike trajectory data of 320 stations in Manhattan and Brooklyn area (see Figure 1(b)). The stations are randomly split into 80% (256) for training and 20% (64) for validation. Their demand and balance information are extracted from the CitiBike system historical data as our ground truth.

A. Data Description

Citibike Transactions. Citibike transactions are generated by NYC Bike Sharing System which is public available from Citibike official website. 11.3 million transactions are extracted from July 2013 to November 2014 with winter session from December 2013 to March 2014 excluded because the demand for bicycle during the winter was very low. This data set contains the following information: station id, bicycle pick-up station, pick-up time, drop-off station and drop-off time.

Taxi GPS Transactions. Taxi GPS transaction dataset is generated by taxis in New York City in August 2013 which is public available. 11.3 million taxi transactions are collected with each record containing the information of trip distance, taxi pick-up coordinate, taxi pick-up time, taxi drop-off coordinate and taxi drop-off time.

B. Feature Analysis

1) Correlation Analysis: We first perform a correlation analysis investigating the correlation relationship between our targets (station demand and station balance) and the features extracted from real world data (see Figure 3). The Pearson correlation coefficient is applied for station demand and features. For the correlation of station balance and features, we use Point-Biserial correlation. From Figure 3 we can see that all features are correlated to the targets we investigate, compared to a simulated random noise feature (RN). Moreover, the features of distance to subway entrance and parking lot are

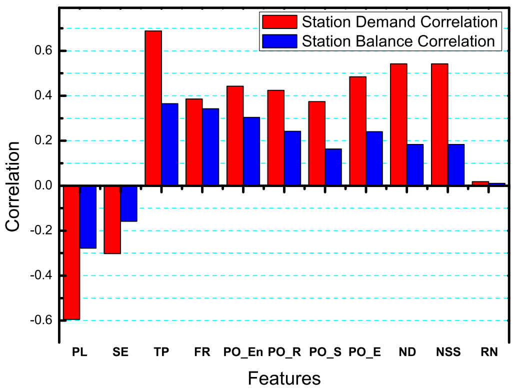
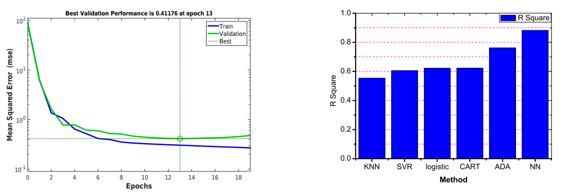


Figure 3. Correlation of features and station demand & balance patterns



(a) Training progress of ANN (b) Performance comparison

Figure 4. Station demand prediction training progress and performance

negative correlated, which indicate the bike station is treated as an transportation missing link connection.

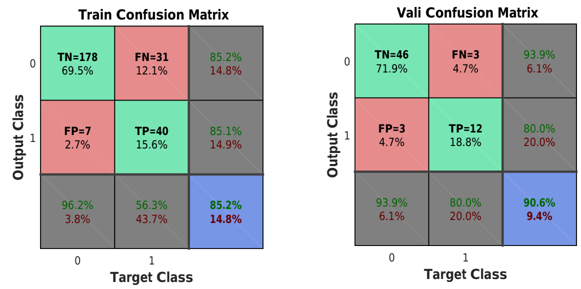
C. Station Demand Prediction

Evaluation Metrics. To show the effectiveness of our proposed method for station demand prediction, we use the coefficient of determination for the prediction error measurement.

Training Progress. Figure 4(a) shows how fast the ANN converges using Levenberg-Marquardt algorithm. Although the training error continues to decrease, the optimized ANN is chosen at epoch 13 of minimum validation error.

Baseline Algorithm. We evaluate the effectiveness of our model for station demand prediction with a set of baselines, including (1)K-Nearest Neighbor; (2)Logistic Regression (3)SVR with RBF kernal; (4)CART; and (5) Adaboost Decision Tree Regression. All baseline algorithms are trained on the same training data set and their performance are compared using the same validation set as our Regression Neural Network. All the baselines are implemented by a python machine learning library named Scikit-Learn [17].

Overall Performance. The overall performance comparison of different methods is summarized in Figure 4(b). Our proposed Neural Network achieves an R^2 of 0.88168, which obviously outperforms the baseline algorithms with a significant margin. Among the 5 baseline algorithms, only AdaBoosted decision tree can achieve a relatively high R^2 of 0.76152. The algorithms of KNN (0.55322) logistic regression (0.62134), Suport Vector Regressor (0.60479) and CART (0.62261) are not able to predict station demand based on the features extracted.



(a) Training Confusion Matrix (b) Validation Confusion Matrix

Figure 5. Confusion Matrix of ANN training and validation outputs

D. Station Balance Prediction

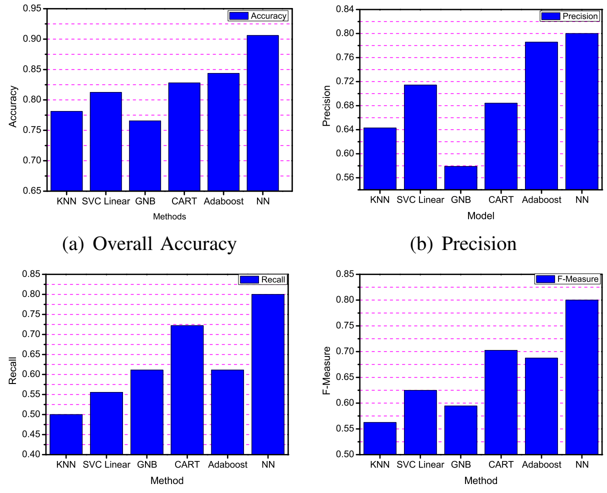
Evaluation Metrics. The classification performance of the optimized artificial neural networks for station balance prediction is evaluated using evaluation metrics including overall accuracy, precision, recall and F-measure.

Baseline Algorithm. We evaluate the effectiveness of our station balance prediction model with a set of baselines: (1)K-Nearest Neighbor Classifier (KNN); (2)SVC with linear kernal; (3)Gaussian Naive Bayes (GNB) classifier; (4)CART and (5)Adaboost Decision Tree Classifier.

Overall Performance. The training and the validation performances of artificial neural network based station balance prediction are presented by two confusion matrixes in Figure 5. Our proposed prediction model can achieve an accuracy of 85.2% for the 256 stations (185 balanced and 71 unbalanced stations) in training set and the validation accuracy reaches 90.6% for the rest 64 stations (49 balanced and 15 unbalanced stations). The overall performance comparison of different methods is summarized in Figure 6. As can be seen from Figure 6(a), our proposed method achieves the highest prediction accuracy compared to the 5 most commonly used classification algorithms. The overall validation accuracy of AdaBoost is above 84% and the Gaussian Naive Bayes has the lowest accuracy of 76.6%. Moreover, from Figure 6(b), 6(c) and 6(d), our method outperforms other baseline algorithms in terms of precision, recall and F-measure.

E. Station Network Optimization

Based on our prediction models, a bike sharing network optimization is conducted to find 252 optimal stations from 1720 station candidates in Manhattan area and 68 optimal stations from 967 station candidates in Brooklyn. Figure 7(a) shows the progress of searching best station network. It can be seen, the optimization converges at 109th generation with the best objective of 3.42323, significantly higher than the current station network that has the same number of stations but obtains a much lower objective of 2.71. The optimum stations achieve a high objective from two respects: the average station demand is 3.98323 compared to current stations with an average demand of 3.57092; the number of unbalanced stations decreases from 86 to 56 (see Figure 7(b)).The distribution of optimum stations is presented in Figure 7(c) with the potential station demands are represented by the dot sizes and the red dots indicate unbalanced stations.



(a) Overall Accuracy

(b) Precision

(c) Recall

(d) F-Measure

Figure 6. Performance comparison of models for staion balance prediction

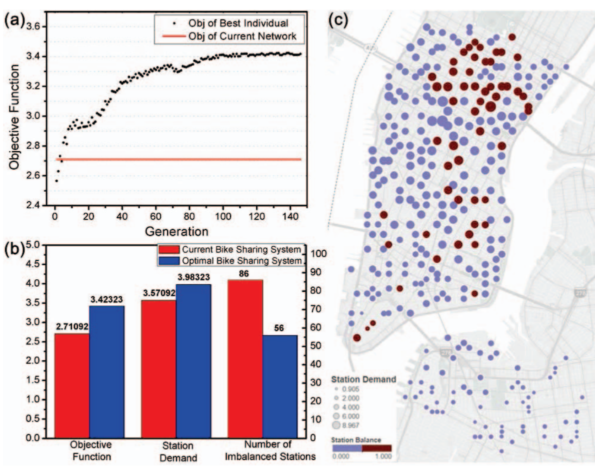


Figure 7. Distribution and statistics of optimum bike stations

VI. CONCLUSION

In this paper, we developed a comprehensive bike station network optimization approach by selecting bike station locations with high demand and balanced pick-ups/drop-offs . To the best of our knowledge, this paper is the first attempt to integrate multiple factors from human mobilities, surrounding POIs and station network structures for station demand prediction and balance evaluation in bike sharing systems. Specifically, artificial neural network based prediction models was developed to build the complex nonlinear relationships between the features extracted from different factors and the patterns of station demand and balance. Evaluated by bike sharing system data generated by NYC CitiBike System, our proposed model manifested the best prediction performance among other state of the art algorithms. Moreover, an genetic algorithm based optimization strategy aiming at maximizing station network demand as well as minimizing number of unbalanced stations was conducted by selecting optimal station locations from a large set of station locations.

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