



Bicycle sharing system design with capacity allocations

Dilay Çelebi*, Aslı Yörüşün, Hanife Işık

Istanbul Technical University, Department of Management Engineering, Macka, Turkey



ARTICLE INFO

Article history:

Received 26 September 2017

Revised 24 May 2018

Accepted 25 May 2018

Available online 14 June 2018

Keywords:

Bicycle sharing system

Facility location

Capacity allocation

Service level

ABSTRACT

This study presents an integrated approach for the design of a Bicycle Sharing System (BSS) by jointly considering location decisions and capacity allocation. An important distinction of this approach is the **definition of service levels, measured by the amount of unsatisfied demand both for bicycle pick-ups and returns**. The method combines a set-covering model to assign location demands to stations with a queuing model to measure the related service levels. The key quality of this approach is its capacity in addressing the issues related to uncertainties in bicycle pick-up and return demand in BSS network design decisions. Results of the implementation of a BSS design for Istanbul Technical University's Ayazağa Campus show that our approach provides a balanced BSS network by equalizing the mean demand and return rates, which will decrease the need for relocation efforts once the system is put to use.

© 2018 Elsevier Ltd. All rights reserved.

1. Introduction

Bicycle Sharing Systems are ideal for short distance point-to-point trips providing users the ability to pick up a bicycle at any self-serve sharing point and return it to any other point located within the system service area (Zhang et al., 2015). They are convenient in providing an alternative or complementary form of transport to cover short journeys, particularly in traffic restricted areas, such as parks, university campuses, and historic city centers. The fact that bicycles are a cheap and green form of transport, benefits of a Bicycle Sharing System (BSS) include flexible mobility, emission reductions, increased physical activity, reduced congestion and fuel use, individual financial savings and support for multi-modal transport connections (Shaheen et al., 2010).

The evidence from different cities and countries suggests that bicycle sharing can improve the experience, accessibility and affordability of personal travel, through greater transport choice, reduced journey times and reduced mobility costs (Ricci, 2015). Nevertheless, an efficient design of a BSS is vital to prevent critical issues that may lead to shortcomings. For example, despite the high rate of bicycle usage in Beijing, some BSS companies declared bankruptcy and were closed; others closed dozens of bicycle stations to reduce the operational costs in 2009 (Liu et al., 2012). Similarly, in Melbourne the public BSS have had disappointing usage rates despite the significant increase in bicycle use as transportation in Australia over recent years (Fishman et al., 2012). To prevent failures and to minimize the negative impacts of obstacles and uncertainty, a BSS design should address the key success factors for an efficient running of the system, such as future demand, the distribution of bicycle stations, capacity management and the charging mechanism. The key motivating factor for bicycle sharing use is the convenience that users can have access to the service whenever and wherever needed, and that the service is not interrupted because of bicycle shortages or unavailability of empty slots for returns.

* Corresponding author.

E-mail address: celebid@itu.edu.tr (D. Çelebi).

Alvarez-Valdes et al. (2016) defines three phases in the design and operation of a bicycle sharing system. They suggest that the number of stations and their locations have to be decided in the first stage, the number of bicycles in the system has to be determined in the second stage, and a bicycle repositioning system has to be adopted for moving bicycles from stations with an excess to stations with a shortage in the final stage. Nevertheless, the first two steps are highly interdependent. The proximity of residence to docking stations is strongly correlated with frequency of use – accordingly, demand for both bicycle rentals and returns at any station is determined by the convenience of its location and range of its coverage. On the other hand, each bicycle station must carry enough capacity to increase the probability that users can find a bicycle or a docking station when needed.

The model proposed in this paper addresses a station based BSS offering spontaneous one-way trips with a flat subscription fee. Reservation of bicycles is not possible. We developed a mathematical model and a solution method for determining the number and location of the required stations of such a BSS by taking into account the service levels. We also present an application of the proposed model for planning a BSS for Istanbul Technical University's Ayazağa Campus. The remainder of the paper is organized as follows: Section 2 reviews the different research approaches on BSS design and discusses how they relate to the objective of this work. Section 3 presents the model and the method for determining optimal station locations and capacities. Section 4 explains the application of the proposed model, including estimation of bicycle sharing demand and then translating it into transport flow patterns. Section 5 concludes the study and outlines perspectives for further research.

2. Review of BSS models

A BSS is defined as a system offering short-term urban rental bicycles available from a network of unattended locations in public spaces. Users arrive at rental stations, utilize the bicycle for some period of time, and then return it to the station of their choice (George and Xia, 2011). Three (and a half) generations of bicycle sharing systems can be identified (Vogel and Mattfeld, 2010): The first generation originates in Amsterdam in 1965. The initiative was called “White Bicycles” where bicycles in circulation were provided free to be used for one trip and then left unlocked for someone else to use. Second-generation BSS emerged in Copenhagen, under the name “Bycykler København”. The system introduced the coin-deposit model to deter theft and to encourage bicycle returns. Yet, the program still had an issue with theft due to the anonymity of the users. The third generation replaced coin-access with smart card access. It was first launched in Rennes as *àVelo* la carte. It also started the restricted usage time scheme, generally providing 30 min of bicycle use for free. The next generation (3+) of bicycle sharing systems was smartened with real-time availability and GPS tracking. These systems signal the appearance of flexible, clean docking stations, touchscreen kiosks, additional bicycle re-balancing technologies, as well as the integration of a unique individual card allowing a user to make use of both bicycles and public transportation. Currently, there are around 1000 cities equipped with BSSs around the world (Wikipedia, 2017).

BSSs have been receiving growing attention from researchers and policy makers for achieving more sustainable urban transport (Lovelace et al., 2011). Over the past decade, bicycle sharing has become more common, consequently a good inventory of research has been developed for the analysis of BSSs in towns and cities around the world. Some examples are Dublin (Murphy, 2010), Madrid (García-Palomares et al., 2012), Beijing (Liu et al., 2012), Montreal (Bachand-Marleau et al., 2012), Brisbane (Fishman et al., 2012), Helsinki (Jäppinen et al., 2013), Paris (Nair et al., 2013), Milan (Crocì and Rossi, 2014), Coimbra (Frade and Ribeiro, 2015), Palma de Mallorca (Alvarez-Valdes et al., 2016), Taipei City (Yan et al., 2017), Switzerland (Audikana et al., 2017), and New York (Campbell and Brakewood, 2017).

Many of these studies provide a quantitative analysis of existing BSSs, examine empirical evidence of the usage patterns, and analyse the characteristics of these systems through quantitative metrics. Jäppinen et al. (2013) studied the impacts of a BSS on public transport travel times in Greater Helsinki and concluded that a BSS should be viewed as part of public transport rather than a separate cycling scheme. García-Palomares et al. (2012) identified and critically interpreted the available evidence on bicycle sharing to date, on both impacts and processes of implementation and operation. Crocì and Rossi (2014) analysed the case of Milan to assess the factors that influence the use of bicycle sharing stations and examined the different effects of proximity and visibility of bicycle sharing stations of those factors. Zhang et al. (2015) explored the characteristics and commonalities between particular bicycle sharing systems in urban areas in an empirical study in China. Fishman et al. (2013) provided a critical examination of the growing body of literature in an overview of bicycle share programs. An extended review of the enabling conditions for the occurrence and transferability of beneficial impacts of a BSS can be found in Ricci (2015).

One stream of literature focuses on strategic design and the logistics of BSSs regarding the capacity and locations of bicycle rental stations to optimize a measure or a combination of measures for system performance (Boyacı et al., 2015). These studies aim at determining the number and location of stations, total fleet size, and/or the structure of the network of bicycle paths that should be developed to connect the bicycle stations. George and Xia (2011) developed a closed queueing network model for determining optimal fleet size through the use of a profit-based optimization problem. Nair and Miller-Hooks (2016) used an equilibrium network design model to determine locations of a given number of bicycle sharing stations that maximizes the flow potential along bicycle sharing links. Reijnders (2016) used spatial data to identify alternative locations in the target city and used simulation techniques to determine how attractive those areas are for station placement. Yan et al. (2017) applied a time-space network technique to formulate four planning models for leisure-oriented public bicycle rental systems under deterministic and stochastic demands for locating stations, fleet allocation and bicycle

routing. [García-Palomares et al. \(2012\)](#) used a GIS-based method to calculate the spatial distribution of the potential demand for trips, to locate stations using location/allocation models, to determine station capacity and to define the characteristics of the demand for stations.

Another major stream is associated with the relocation of bicycles in a BSS. The problem arises from demand asymmetries which cause an imbalanced accumulation of bicycles at stations. Different strategies and models have been proposed in the literature to cope with this problem. [Kabra et al. \(2016\)](#) studied the impact of demand asymmetry, travel time between stations, uncertainty of demand, the number of docking points, station locations, and bicycle availability on the use of the bicycle share system. They also showed how demand asymmetry affects decisions about the number of bicycles/docking points and the frequency with which bicycles should be reallocated from full stations to empty ones. [Vogel and Mattfeld \(2010\)](#) presented a system dynamics model to assess the effect of dynamic repositioning efforts on service levels. [Nair and Miller-Hooks \(2011\)](#) modeled the stochastic nature of the demand in a vehicle-sharing system. They proposed a chance-constrained, mixed-integer program to generate fleet redistribution plans that correct imbalances. This model was later employed by [Nair et al. \(2013\)](#) with an extension of a framework to characterize the system that integrates system configuration, current state, and offered level-of-service. [Contardo et al. \(2012\)](#) formulated the problem as an arc-flow model on a space-time network and presented an approach utilizing Dantzig-Wolfe and Benders decomposition to solve the problem. [Shu et al. \(2013\)](#) proposed a stochastic network flow model to examine the effectiveness of periodic redistribution of bicycles in the network to support greater flow, and the impact on the number of docks needed. They introduced proportionality constraints to estimate the flow of bicycles within the network and the number of trips supported, given an initial allocation of bicycles at each station. [Raviv and Kolka \(2013\)](#) introduced a user dissatisfaction function to assess the quality of the relocation service and to determine the initial inventory of the station that minimizes the dissatisfaction function. [Frade and Ribeiro \(2015\)](#) added a constraint for the budget in the model which aims at maximizing the benefits of the system by satisfying the demand. [Forma et al. \(2015\)](#) proposed an algorithm by first clustering the stations according to geographic and inventory considerations, and then routing vehicles through the clusters while inventory decisions are made for each individual station separately. [Li et al. \(2016\)](#) dealt with a static bicycle repositioning problem in which multiple types of bicycles are considered. They formulated a mixed integer model and proposed an evolutionary algorithm as a solution. In a recent study, [Zhang et al. \(2017\)](#) developed a novel non-linear time-space network flow model to integrate user dissatisfaction estimations with bicycle repositioning and vehicle routing.

[Alvarez-Valdes et al. \(2016\)](#) used simulation techniques to address the problem of bicycle repositioning in two stages. They first estimated the unsatisfied demand at each station for a given time period in the future and for each possible number of bicycles at the beginning of that period. In the second stage, they used these estimations to guide redistribution algorithms. [Fricker and Gast \(2016\)](#) used a stochastic model and a fluid approximation to investigate the influence of station capacities on the performance of BSSs. They provided analytical expressions for the stationary performance of large-scale systems and quantified the influence of the fleet size that is optimal in terms of minimizing the proportion of problematic stations. [Schuijbroek et al. \(2017\)](#) proposed a new heuristic for determining service level requirements at each bicycle sharing station and designing optimal vehicle routes to rebalance the inventory simultaneously. Their approach includes considering service level feasibility constraints and approximate routing costs.

Only a small number of studies, however, seek to integrate the operational decisions of the above work with strategic decisions. [Martinez et al. \(2012\)](#) developed a mixed integer linear programming model and a heuristic algorithm to design a bicycle sharing network for Lisbon. They handled uncertainty in demand with an elasticity rate and used average travel times as estimates of trip durations. Their model represents each bicycle/user movement on the system separately. [Frade and Ribeiro \(2015\)](#) proposed a similar model to determine the optimal location of the bicycle stations, the fleet size, the capacity of the stations and the number of bicycles in each station and presents an application through a case study in Coimbra. They used station capacities to define an upper and lower limit on the number of available bicycles but did not consider the uncertainties in demand. [Lin and Yang \(2011\)](#) addressed strategic planning of public bicycle sharing systems with service level considerations using a deterministic mathematical model. They presented an integer non-linear program to determine the optimal location of docking stations and the network structure of bicycle paths connected between the stations. In an effort to develop a method for practical situations, [Lin et al. \(2013\)](#) formulated the problem as a hub location inventory model. Service quality in the system is represented by the availability rate for sharing bicycles at each station, determined by a safety stock, together with the total amount of demand covered within a specified distance, determined by the desired distance coverage. To the authors knowledge, all previous work incorporating capacity decisions into network design, dealt with the uncertainty in travel demand and trip duration either by using deterministic estimates or employing an imprecise approximation of service level measures.

In this work, we propose a model that provides an integrated view, which **jointly considers location decisions and capacity allocation to determine the optimal configuration of a bicycle sharing system**. An important distinction of this model is the definition of service level measured by the amount of unsatisfied demand both for bicycle pick-ups and returns. We therefore **combined a set-covering model to assign the location demand to stations with a queuing model to measure the relevant service levels**. The key quality of our approach is its capacity in addressing the issues related to uncertainties in bicycle pick-up and return demand in BSS network design decisions.

3. System overview

Prior to introducing the system structure and presenting the formulation of the model, the notation and symbols are listed below:

Subscripts and sets	
S	set of alternative stations, $j, k \in S$
P	set of demand points, $i, m \in P$
Parameters	
V_{im}	rate of flow from point i to point m
λ_i	mean bicycle pick up rate at point i
μ_i	mean bicycle return rate at point i
V_i^+	bicycle pick up demand at point i ; $V_i^+ \sim \text{Poisson}(\lambda_i)$
V_i^-	bicycle return demand at point i ; $V_i^- \sim \text{Poisson}(\mu_i)$
A_j	set of demand points within the reach of station j
C	total number of bicycles in the system
N	total number of stations
Variables	
X_j	set of demand points covered by station j
C_j	capacity of station j
P_j	set of demand points available to be assigned to station j
$\mathcal{P}(P_j)$	power set of P_j ; $X_j \in \mathcal{P}(P_j)$
λ_j	mean bicycle pick up rate at station j
μ_j	mean bicycle return rate at station j
ρ_j	traffic intensity at station j ; $\rho_j = \frac{\lambda_j}{\mu_j}$
$\pi_j(0)$	steady state probability of having no bicycles at station j
$\pi_j(C_j)$	steady state probability of having no empty slots at station j
L_j	expected inventory level at station j
$\beta_j(X_j, C_j)$	expected number of unsatisfied pick up and return demand in station j

We consider a network of bicycle stations where the set of stations in the system is denoted by S and indexed by $j \in S$ and $k \in S$. We assume that a station covers a predetermined range with an area in which the origin and destination points of trips are located. Let X_j be the set of points covered by station j . That is, if passengers want to travel from point $i \in P$ to point $m \in P$, they pick the closest station j to the point of origin i to pick up the bicycle, and return it to the closest station k to destination point m . We refer to V_{im} as the routing matrix, which measures the rate of flow from point i to point m . **We assume passengers arrive at station j according to a Poisson process with rate λ_j .** Passengers return bicycles to any station $k \in S$ following a Poisson distribution with a mean μ_k . Fig. 1 is an illustration of how passengers move from point i to point m within the network.

We also assume that if a bicycle is not available for a user upon arrival, the potential user leaves without service. Similarly, if there are insufficient parking spaces at a station, the potential user leaves to find another parking space for the bicycle. Though this assumption can be somewhat restrictive, in reality, given that some users may prefer to wait until another user returns or picks up a bicycle, this is a widely accepted assumption in literature (Vogel and Mattfeld, 2010; George and Xia, 2011; Schuijbroek et al., 2017) as the fraction of such users can be assumed negligible. For each station $j \in S$, we denote C_j as the capacity (number of docking units). We assume that the users prefer to use the closest stations to their origin and destination to access the system. Accordingly, demand at point i is covered if a station is located within a given distance. We denote A_j as the set of demand locations within the range of maximum walking distance from station j .

For a given number of stations, the location and capacity allocation model involves (1) the determination of station locations, (2) the allocation of demand points to stations, (3) and the determination of the number of bicycle slots required to satisfy the service level constraints. Given that the universal set P is the set of all points, and the feasible set $P_j \subseteq P$ the set of points available to be assigned to station j , then, $\mathcal{P}(P_j)$ is the power set of P_j , referring to the set of all feasible assignments to station j . We refer to a particular set $X_j \in \mathcal{P}(P_j)$ as an instance of a set of feasible assignments, including all points assigned to station j .

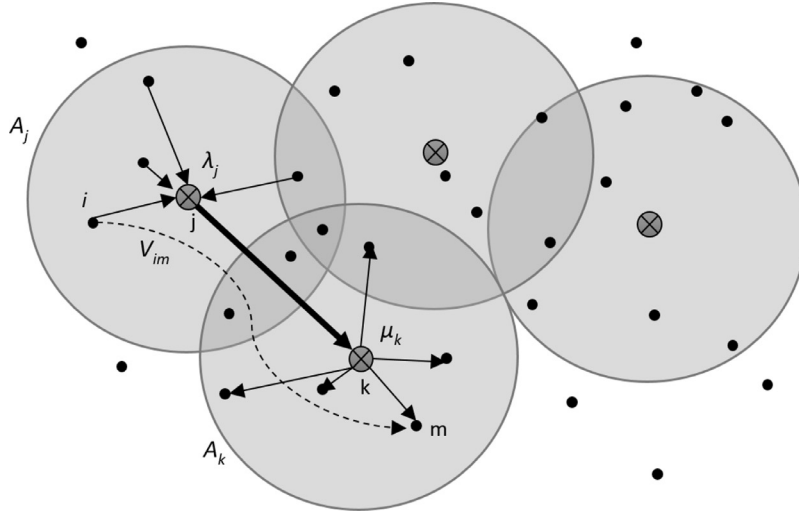


Fig. 1. A representation of system network.

3.1. Formulation of service level measures

We model the capacity assignment problem using the Markov decision process. We assume that **bicycle demand (V_i^+) and th bicycle return rate (V_i^-) at point i are discrete random variables having Poisson distribution** with parameters λ_i and μ_i respectively; and V_i^+ and V_i^- are independent. Then, at station $j \in S$, bicycle demand is distributed as: $\sim \text{Poisson}(\sum_{i \in X_j} \lambda_i)$ and bicycle return is distributed as: $\sim \text{Poisson}(\sum_{i \in X_j} \mu_i)$. Therefore, we consider each station j with C_j bicycles available as a $M/M/1/C_j$ queue (Kendall and Hill, 1953) with an independent external user arrival rate λ_j , service rate μ_j , and a total capacity of $c = C_j$ bicycles. The number of customers in the queue represents the number of bicycles available at the station. For $M/M/1/C_j$ queue, closed-form expressions for steady state probabilities are given by:

$$\pi_j(0) = \begin{cases} \frac{1-\rho_j}{1-\rho_j^{C_j+1}}, & \text{if } \rho_j \neq 1 \\ \frac{1}{C_j+1}, & \text{if } \rho_j = 1 \end{cases} \quad (1)$$

and

$$\pi_j(C_j) = \begin{cases} \rho_j^{C_j} \pi_j(0), & \text{if } \rho_j \neq 1 \\ \frac{1}{C_j+1}, & \text{if } \rho_j = 1 \end{cases} \quad (2)$$

In Eqs. (1) and (2), $\pi_j(0)$ and $\pi_j(C_j)$ are the steady state probabilities of having no bicycles and no empty slots at station j , respectively. Then, traffic intensity in station j is given by $\rho_j = \lambda_j / \mu_j$.

Therefore, total expected number of unsatisfied bicycle pick up and return demand in station j can be formulated as:

$$\beta_j(X_j, C_j) = \pi_j(0)\lambda_j + \pi_j(C_j)\mu_j \quad (3)$$

3.2. Assignment and allocation model

To determine the assignment of demand points $i \in P$ to stations $j \in S$, we obtain a vector X_j and a scalar C_j , for all j , to minimize the total unsatisfied demand

$$\min \sum_{j \in S} \beta_j(X_j, C_j) \quad (4)$$

subject to following set of constraints:

$$\rho_j = \frac{\sum_{i \in X_j} \lambda_i}{\sum_{i \in X_j} \mu_i}, \quad \forall j \in S \quad (5)$$

$$\sum_{j \in S} C_j \leq C \quad \forall j \in S \quad (6)$$

$$X_j \subset A_j \quad \forall j \in S \quad (7)$$

$$\cup X_j = P. \quad (8)$$

Eq. (5) formulates the traffic intensity of station j as a function of the total bicycle rental and return rates at station j , calculated by:

$$\lambda_i = \sum_{m \in A_i} V_{im}, \quad \forall i \in P \quad (9)$$

and

$$\mu_i = \sum_{m \in A_i} V_{mi}, \quad \forall i \in P \quad (10)$$

Constraint (6) enforces an upper limit on the number of total bicycles in the system. Constraints (7) and (8) ensure that each point is assigned to at least one station within the range of allowed walking distance (A_j).

We can show that at a steady state, the expected number of bicycles present at any given station j is given by:

$$L_j = \begin{cases} \frac{\rho_j[1-(C_j+1)\rho_j^{C_j} + C_j\rho_j^{C_j+1}]}{(1-\rho_j^{C_j+1})(1-\rho_j)}, & \text{if } \rho_j \neq 1 \\ \frac{C_j}{2}, & \text{if } \rho_j = 1. \end{cases} \quad (11)$$

L_j shows the expected inventory levels at the stations when the system is in use. Minimizing Eq. (4) will increase the probability of finding at least one bicycle or one docking place available at each station. As such, the objective function of the model also implies a minimization of cases where the relocation of a bicycle is required.

3.3. A dynamic formulation for capacity allocation model

To solve the problem by dynamic programming, define $f_j(P_j, C)$ to be the minimum service loss that can be obtained under given constraints (5)–(8) when the feasible set of points that can be assigned to station j is P_j , the total number of stations is N , and the total number of available bicycles is C . The dynamic programming recursion can be written in the following form:

$$f_N(P_N, C) = \min_{X_N \in \mathcal{P}(P_N) C_N \leq C} \beta_N(X_N, C_N), \quad \text{for } j = N, \quad (12)$$

$$f_j(P_j, C) = \min_{X_j \in \mathcal{P}(P_j) C_j \leq C} \{\beta_j(X_j, C_j) + f_{j+1}(P_j \setminus X_j, C - C_j)\}, \quad \text{for } j \leq N. \quad (13)$$

Proof. Let $f_j^*(P_j, C)$ is the optimal solution to the problem at stage j . We may assume that for a station j there exists an optimal assignment $\{X_j^*, C_j^*\} = \arg \min f_j(P_j, C)$. For any demand point i , we have three cases:

Case 1: $i \in X_k : k = 1, 2, \dots, j-1$. The optimal solution requires point i to be assigned to station $k: k < j$, and $i \notin X_j^*, i \notin P_j$. Then,

$$\begin{aligned} f_j(P_j, C) &= \min_{X_j^* \in \mathcal{P}(P_j) C_j^* \leq C} \{\beta_j(X_j^* \setminus i, C_j^*) + f_{j+1}((P_j \setminus i) \setminus (X_j^* \setminus i), C - C_j^*)\} \\ &= \min_{X_j^* \in \mathcal{P}(P_j) C_j^* \leq C} \{\beta_j(X_j^*, C_j^*) + f_{j+1}(P_j \setminus X_j^*, C - C_j^*)\} \\ &= f_j^*(P_j, C) \end{aligned}$$

Case 2: $i \in X_k : k = j+1, j+2, \dots, N$. The optimal solution requires point i to be assigned to station $k: k > j$, and $i \notin X_j^* & i \in P_j$. Then,

$$\begin{aligned} f_j(P_j, C) &= \min_{\substack{X_j^* \in \mathcal{P}(P_j) \\ C_j^* \leq C}} \{\beta_j(X_j^* \setminus i, C_j^*) + f_{j+1}((P_j \cup i) \setminus (X_j^* \setminus i), C - C_j^*)\} \\ &= \min_{\substack{X_j^* \in \mathcal{P}(P_j) \\ C_j^* \leq C}} \{\beta_j(X_j^*, C_j^*) + f_{j+1}(P_j \setminus X_j^*, C - C_j^*)\} \\ &= f_j^*(P_j, C) \end{aligned}$$

Case 3: $i \in X_k : k = j$. Then the optimal solution requires point i to be assigned to station j , then $i \in X_j^*$ and $i \notin P_j$. Then,

$$f_j(P_j, C) = \min_{X_j^* \in \mathcal{P}(P_j) C_j^* \leq C} \{\beta_j(X_j^* \cup i, C_j^*) + f_{j+1}((P_j \setminus i) \setminus (X_j^* \cup i), C - C_j^*)\}$$



9

According to [Midgley \(2011\)](#), slopes within the range of 4% and 8% can be a limiting constraint for the implementation of a bicycle sharing system. We conducted a spatial examination of slopes in the area and height profiles show that despite the high level of elevation differences between a number of points within the campus, the maximum altitude difference is moderate. The slopes of the two most inclined paths vary between 2% and 8%. With the lengths of both slopes being less than 300 m, together with the high percentage of daily trips of less than 2 km and restricted vehicle traffic on campus, ITU Ayazağa is a potentially suitable site for a bicycle sharing system.

Table 1
Demographics of survey respondents.

Gender	Female	41.4%
Bike usage frequency	Everyday	6.3%
	3–4 times/week	2.6%
	1–2 times/week	6.6%
	1–2 times/month	10.2%
	1–2 times/year	43.0%
Occupation	Never	27.8%
	Undergraduate student	68.2%
	Academic staff	22.8%
	Graduate student	5.8%
Bike ownership	Admin. staff & other	3.1%
	Yes	28.6%

4.1. Predicting bike sharing demand

Two important determinants of BSS demand are geographical convenience and price (Ricci, 2015). Empirical evidence suggests that bicycle sharing programs may attract increased membership by high geographical convenience in combination with docking stations being sufficiently close to frequented points of origin and destinations (Fishman et al., 2013). Similarly, higher prices of the service may lessen the potential demand while it may at the same time be an incentive to add additional capacity. Similar to other public transportation modes, BSS demand is also time-dependent (Nair et al., 2013). In general, demand is higher and even more condensed during the morning and evening times while it is sparse in the remaining times of the day. Obtaining a good prediction of trip demand in BSS design is highly challenging, even after the implementation of the system, as the number of trips a BSS experiences daily is typically inaccessible and published metrics are typically incomparable (de Chardon and Caruso, 2015).

To capture the demand characteristics related to these factors, we conducted a survey to predict the travel behavior of potential system users. The survey consisted of questions regarding general travel behavior, bicycle usage habits, travel routes on campus, bicycle ownership, and usage potential of a BSS. The questionnaire was posted on-line through social media accounts functioning as a focus group of campus inhabitants. Anyone could access the web page and fill out the questionnaire which was available on-line for approximately three weeks. After the incomplete and inconsistent observations were removed, a final sample of 381 respondents was obtained. The number of male respondents was slightly higher in the sample, and accounted for 59% of the respondents. The majority of respondents are students, mainly undergraduates. Table 1 summarizes respondents demographics.

According to the survey, almost 29% of respondents own a bicycle, but only less than 7% of the bicycle owners use a bicycle to travel from one point to another on campus. Around 28% of respondents never use a bicycle for daily trips, even though around 10% of them own a bicycle for personal use. The majority of daily trips are made by walking (72%), and the bicycle is of very low importance. A number of shuttle bus services are provided for transportation within the campus, but again due to the low frequency of the buses, only 7% of daily trips on campus are taken in shuttle buses.

To capture the differences in bicycle demand related to time, we identified four different time slots to collect travel demand information. We only considered the weekdays and the active hours of intensive bicycle usage. We also generated five scenarios based on the pricing schemes and produced demand predictions accordingly. We developed a travel network of 48 nodes by identifying points of demand on campus where trips could start and end. As represented with blue marks in Fig. 2, demand points are spread all around the campus.

We employed the following notations and symbols:

s	Index for sampling group
p	Index for price level
t	Index for time slot
$n_{im}(s, t)$	Number of trips between points i and m for group s at time t
$\alpha_s(p)$	Extension factor of demand for group s at price level p
m_s	Sample size of group s
M_s	Population size of group s
$\gamma_s(p)$	The percentage of users in group s who are willing to pay a price of p or less

We used the survey results to estimate V_{im} , as a function of the time slot t and price level p :

$$V_{im}(p, t) = \sum_s \frac{1}{\alpha_s(p)} n_{im}(s, t) \quad (14)$$

Table 2
Price sensitivity distribution of the sample($\gamma_s(p)$).

Maximum price	Academic staff (%)	Undergraduate students (%)	Graduate students (%)	Admin. staff & others (%)
10	82.8	95.8	100.0	91.7
20	43.7	41.9	45.5	16.7
30	14.9	13.5	18.2	8.3
40	3.4	1.9	0	0
50	1.1	0.4	0	0

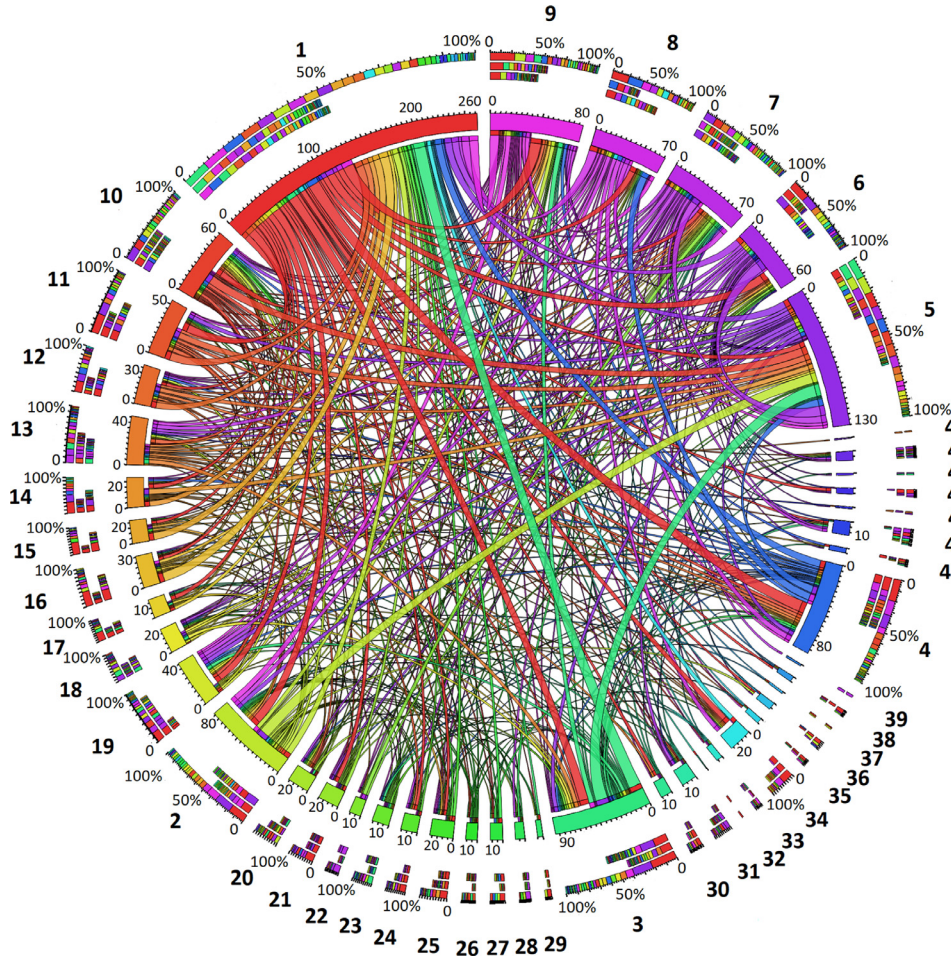


Fig. 3. Daily travel patterns of respondents. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

where $n_{im}(s, t)$ is the number of trips between points i and m at time t , resulting from the data collected from sample group s . We used $1/\alpha_s(p)$ as the extension factor of demand which is calculated as follows:

$$\alpha_s(p) = \frac{m_s}{M_s} \cdot \gamma_s(p). \quad (15)$$

In Eq. (15) m_s is the number of respondents of group s in the sample, M_s is the number of individuals of group s in the population, and $\gamma_s(p)$ is the percentage of users in group s who are willing to pay a price of p or less to use the system. For our sample of 381 respondents, the $\gamma_s(p)$ values are given in Table 2

Using Eq. (15), we predicted the flow patterns among all point pairs in the network and created a matrix of origin-destination pairs. To provide an insight into inter-point flows, daily travel patterns of respondents are plotted as a circular diagram as illustrated in Fig. 3. The demand locations are represented along the circumference of the figure. In this figure, each demand point is represented by a different color. The width of each arc represents the flow potential between the locations connected by that arc. The color of the arc determines the direction of flow, each arc is presented in the color of its origin point. Wider arcs occur when the flow potential is greater. The inner circle shows the total bicycle demand (both

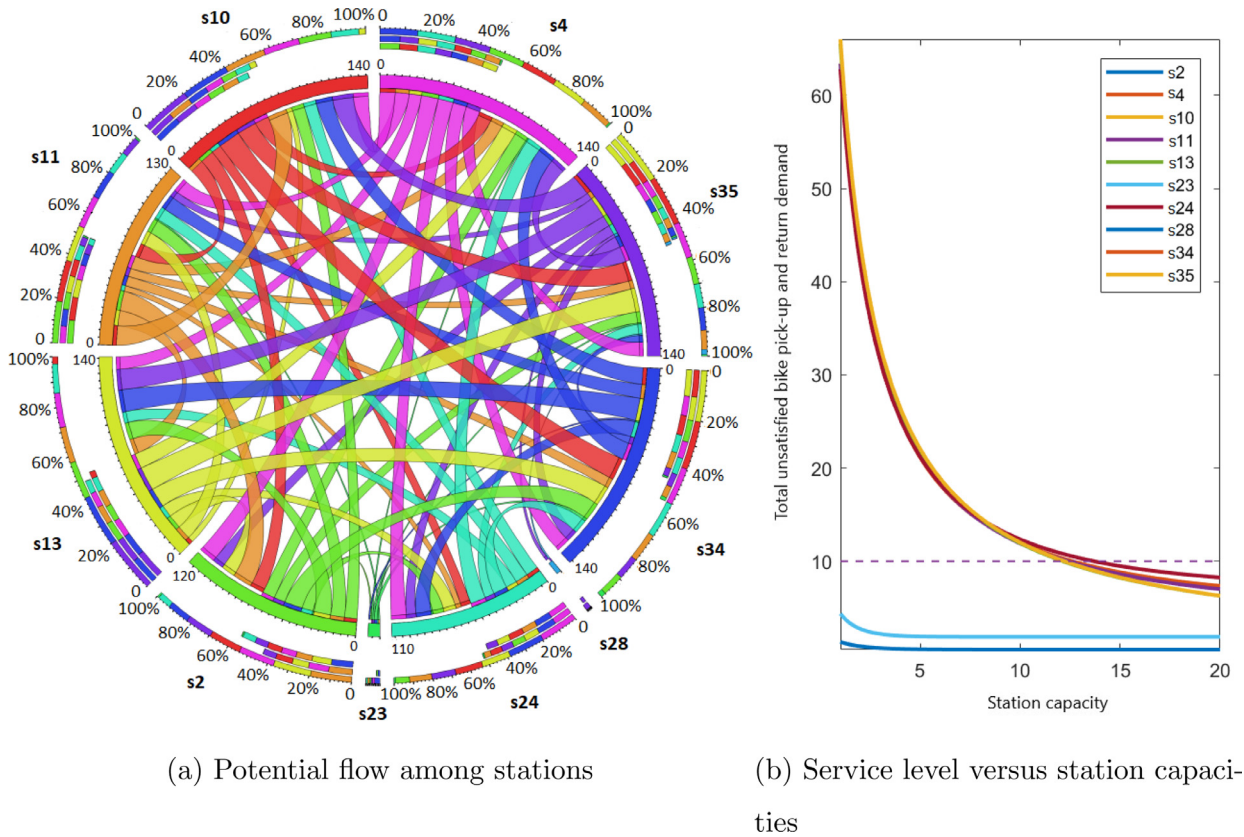


Fig. 4. Test results for the 10 station system. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

incoming and outgoing) at that point. The outer circle shows the percentage share of the points in terms of traffic flow from the given point. For example, around eight percent of the traffic from and to point 1 (shown in red color) comprises the traffic flow from point 3 (shown in light green color).

Fig. 3 shows that there are a number of popular locations, as the average number of trips originated or ended at those points are significantly higher than the average. On the other hand, there are more than 15 centroids where the flow rate is less than 1% of the total demand. Accordingly, we observed considerable asymmetries in flow patterns in the system.

We have considered all demand points as docking station alternatives and obtained 48 location alternatives for stations. Studies show that people within 250m of a station are over twice as likely to become users of the bicycle sharing system than those living further away (Fuller et al., 2011). We are, therefore, able to safely assume that a point of demand i is covered if a station is located within the 250m range.

4.2. Findings

The solution to the model was obtained by solving Eqs. (12) and (13) on Matlab. As a base case scenario, we identified 10 stations at the zero price level. The optimal station locations for this problem are shown in red in Fig. 2 and the average flow of bicycles among those stations is illustrated in Fig. 4. The color and arc representation in this figure is the same as Fig. 3, with a single difference: points are replaced by stations.

In Fig. 4a, the stations are represented along the circumference. Stations with the highest rates of bicycle demand are s4, s13, s34, and s35. This is an interesting result as these stations, particularly s34 and s35, are not among the locations with the highest predicted demand based on the current daily travel patterns on campus. Furthermore, there is almost no identified traffic in location s35, either inflow or outflow. Yet, s35 benefits from proximity to a number of locations with high bicycle demand, such as s7, s15, s16, and s34. The width of each arc represents the flow potential between the stations connected by that arc. Since wider arcs between two stations indicate greater flow potential, the flow potential for s10 to s34 and s13 to s35 are the largest. However, the differences among the magnitudes of arcs are still low, and there is no significant flow which dominates the others.

On the other hand, some locations with the highest demand, for example s1, are not included in the selected list of stations; instead, the demand at location s1 is allocated to three stations: s4, s10, and s14. This provides a key insight into

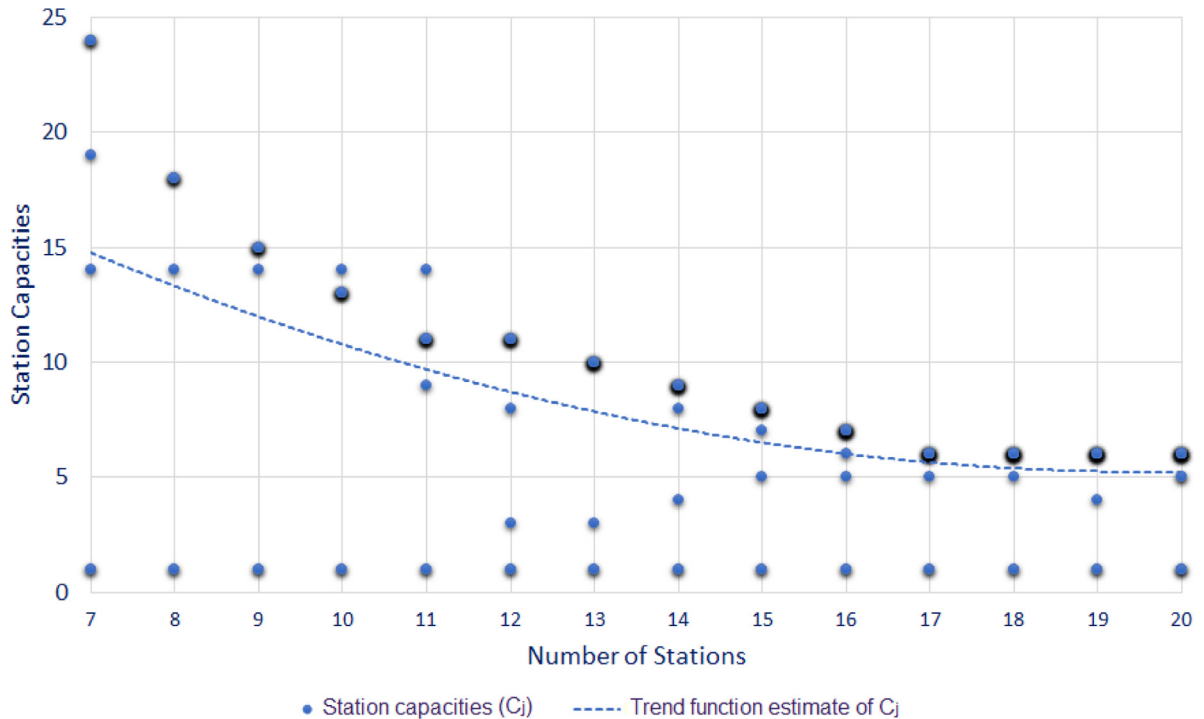


Fig. 5. Allocation of bicycles/docking points to stations.

the configuration of bicycle sharing stations that stations are not necessarily located directly at the points with the highest demand. In fact, allocating the demand at the most popular points to a number of nearby stations provides a more balanced system and decreases the asymmetries in flow potential from one station to another.

Another observation is that all stations are similar in capacity. The only exceptions are s23 and s29, which are geographically restricted. Except for these two, the hourly outflow and inflow rate for all stations are in the range of 55 to 80 bicycles per hour. Equilibrating both the inward and outward flow at stations and the flow of bicycles among the stations generates a balanced system which decreases the minimum number of bicycles required to achieve the desired availability levels. A minimization of Eq. (4) can be achieved when the mean pick-up and return demands are equal for an individual station. It follows then that the model also provides a solution which approximates the expected bicycle inventories at the stations to half of the station capacity (Eq. (11)). Such an allocation will decrease the probability of encountering availability problems and situations requiring the relocation of bicycles.

Fig. 4b illustrates the impact of service constraints – the number of unsatisfied bicycle pick-up and return demand – on station capacities for the 10 station system. Due to the low demand at stations s23 and s28, even a small number of bicycles are sufficient to achieve the desired service levels. On the other hand, for stations s24 and s4, the total number of bicycles (docking stations) should be at least 18 so that the desired level of satisfied bicycle pick-up and return demand is maintained.

We further studied various problem instances with different network configurations. The first set was developed by varying the total number of stations in order to analyse its impact on location configuration and capacity allocation. The findings of this experiments are summarized in Fig. 5 which shows how the variation of station capacities change with the number of stations. The sizes of the points on the graph correspond to the number of stations at the corresponding capacity – the larger the size of the point, the greater the number of stations. For example, when the total number of stations are 20, three of these have capacities of one, two of them have capacities of five and the rest have capacities of six. For the same service level ($\beta \leq 15$), both the mean and standard deviation of station capacities decrease as the number of available stations increase. The figure also shows diminishing returns on the number of stations, which means the rate of decrease is higher for a smaller set of stations, and the capacity values converge after a given number of stations (in our instance, after 17). This finding is in line with García-Palomares et al. (2012) that as the number of stations increase, there is a smaller degree of improvement in the proportion of the population catered for and in the accessibility to stations.

Bearing in mind that most bicycle sharing systems are funded by the government or supported by advertising and sponsorship contracts (Frade and Ribeiro, 2015), the relationship between revenue and demand covered can therefore be very useful when it comes to determining the price of the service. Thus, the second analysis investigated how varying the system service fee will affect bicycle usage rates. Based on the results of Section 4.1, we identified four different price levels and

Table 3
Price sensitivity distribution of the sample($\gamma_s(p)$).

Price	ΣC_j	π_0	π_{C_j}	$\tilde{\beta}_j$	Revenue
0	130	0.119	0.160	9.29	0
10	130	0.120	0.161	9.15	11,891
20	52	0.229	0.268	7.90	10,221
30	15	0.482	0.518	7.47	5,219

determined the relationship between revenue, total station capacity and service level using several scenarios. The variables and the results are presented in Table 3.

It is worth noting that almost identical results were obtained by introducing a low monthly subscription fee. Increasing the price level from 0 to 10, resulted in the total number of bicycles remaining constant and the average probabilities of having no bicycles and no empty slots only slightly increasing. On the other hand, increasing the price level from 10 to 20 more than halved the total demand, with the total number of bicycles required decreasing by 60%. However, the no-bicycle and no-slot probabilities doubled despite the decrease in unsatisfied demand. Lower demand is compensated by higher prices, while the decline in revenue remains small. For a price level of 30, the demand declined significantly, and subsequently the revenue halved. At this price level, the demand is so low that on average one bicycle per one station is sufficient to fulfil the unsatisfied demand.

5. Conclusions

The methodology described in this paper makes it possible to determine the best locations for bicycle sharing stations within an area in relation to the **estimated demand within the area they cover**. We proposed an integrated optimization system that simultaneously determines the number of stations and capacities that minimizes unsatisfied demand. The proposed model contributes to existing literature by using queuing theory to estimate unsatisfied demand and by incorporating dynamic calculations of bicycle pick-up and return rates with respect to station-point allocation. The method outlined here can be directly used to identify optimal station locations and capacities so as to maximize service levels with respect to investment constraints. The model can also be helpful to analyse the impacts of different configurations of an already planned Bicycle Sharing System.

Our findings can provide decision makers with beneficial insights when designing a bicycle sharing system. For example, we observed that creating a balanced allocation of demand across stations gives the best results when the total capacity of the system is fixed. Though this is an expected result, further study is required for a mathematical proof of validity in different cases. Similarly, the comparison of different pricing scenarios can help decision makers to choose the best solution for their particular situation. In fact, it is not reasonable to implement a BSS if there is insufficient demand for its use, so the relationship between expected revenue and service fees must be included in the analysis. It is quite possible that increasing the price level can decrease initial investment and operational costs while keeping revenue and service levels the same, as observed in our case.

There are two kinds of limitations to this study. The first limitation is related to our approach. We did not include any cost considerations in our analysis because we wanted to eliminate any misleading results that may be caused by differences in cost structures. However, most BSS investment decisions require a detailed analysis of various cost factors such as the fixed cost of stations, purchasing, maintenance and relocation. On the other hand, our model can easily be extended to cover relevant costs of the system either by including budget constraints in the model or by using cost terms in the objective function and setting the service level measures as constraints. Another limitation of our approach is in its extent. Even though our model implicitly decreases the need for relocation operations by definition of the objective function, the problem of relocation is beyond the the scope of this study. In a future investigation, this present study can be extended by integrating relocation and routing models into the analysis to observe the impact on the findings.

The second limitation of our research relates to its scope. The non-linear form of the model and dynamic programming based solution algorithm increases the complexity of the problem, making it more difficult to solve when the problem size increases. This may be a limitation in terms of the application of this model to a large scale BSS in big cities. To overcome this problem, in a further study, we plan to focus on developing a heuristic algorithm based on the findings related to balanced allocation in this study. Another limitation arises in our method of demand estimation. We estimated the demand using a simple approximation method based on user segmentation information collected by a survey which was conducted prior to the design of the system. However, demand is subject to change as a function of service quality, changing passenger habits, weather conditions, and bicycle availability after the implementation of such a system. Hence some future work is planned to obtain a more accurate prediction of demand in relation to other factors, including the impact of relocation of bicycles on service levels, for demand is not only the main determinant but also a major indicator of success for BSSs.

Acknowledgments

The authors would like to thank Philip Erasmus for his diligent proofreading of the manuscript.

References

- Alvarez-Valdes, R., Belenguer, J.M., Benavent, E., Bermudez, J.D., Muñoz, F., Vercher, E., Verdejo, F., 2016. Optimizing the level of service quality of a bike sharing system. *Omega* 62, 163–175.
- Audikana, A., Ravalet, E., Baranger, V., Kaufmann, V., 2017. Implementing bikesharing systems in small cities: evidence from the swiss experience. *Transp. Policy* 55, 18–28.
- Bachand-Marleau, J., Lee, B., El-Geneidy, A., 2012. Better understanding of factors influencing likelihood of using shared bicycle systems and frequency of use. *Transp. Res. Rec.: J. Transp. Res. Board* 2314, 66–71.
- Baltes, M., 1996. Factors influencing nondiscretionary work trips by bicycle determined from 1990 us census metropolitan statistical area data. *Transp. Res. Rec.: J. Transp. Res. Board* 1538, 96–101.
- Bauman, A., Crane, M., Drayton, B.A., Titze, S., 2017. The unrealised potential of bike share schemes to influence population physical activity levels – a narrative review. *Prev. Med.* 103, S7–S14.
- Boyaci, B., Zografos, K.G., Geroliminis, N., 2015. An optimization framework for the development of efficient one-way car-sharing systems. *Eur. J. Oper. Res.* 240 (3), 718–733.
- Campbell, K.B., Brakewood, C., 2017. Sharing riders: how bikesharing impacts bus ridership in New York City. *Transp. Res. Part A: Policy Pract.* 100, 264–282.
- de Chardon, C.M., Caruso, G., 2015. Estimating bike-share trips using station level data. *Transp. Res. Part B: Methodol.* 78, 260–279.
- Contardo, C., Morency, C., Rousseau, L.-M., 2012. Balancing a Dynamic Public Bike-sharing System, 4. Cirrelet, Montreal.
- Croci, E., Rossi, D., 2014. Optimizing the Position of Bike Sharing Stations. The Milan Case.
- Fishman, E., Washington, S., Haworth, N., 2012. Barriers and facilitators to public bicycle scheme use: a qualitative approach. *Transp. Res. Part F: Traffic Psychol. Behav.* 15 (6), 686–698.
- Fishman, E., Washington, S., Haworth, N., 2013. Bike share: a synthesis of the literature. *Transp. Rev.* 33 (2), 148–165.
- Forma, I.A., Raviv, T., Tzur, M., 2015. A 3-step math heuristic for the static repositioning problem in bike-sharing systems. *Transp. Res. Part B: Methodol.* 71, 230–247.
- Frade, I., Ribeiro, A., 2014. Bicycle sharing systems demand. *Procedia – Soc. Behav. Sci.* 111, 518–527.
- Frade, I., Ribeiro, A., 2015. Bike-sharing stations: a maximal covering location approach. *Transp. Res. Part A: Policy Pract.* 82, 216–227.
- Fricker, C., Gast, N., 2016. Incentives and redistribution in homogeneous bike-sharing systems with stations of finite capacity. *EURO J. Transp. Logist.* 5 (3), 261–291.
- Fuller, D., Gauvin, L., Kestens, Y., Daniel, M., Fournier, M., Morency, P., Drouin, L., 2011. Use of a new public bicycle share program in Montreal, Canada. *Am. J. Prev. Med.* 41 (1), 80–83.
- García-Palomares, J.C., Gutiérrez, J., Latorre, M., 2012. Optimizing the location of stations in bike-sharing programs: a gis approach. *Appl. Geogr.* 35 (1), 235–246.
- George, D.K., Xia, C.H., 2011. Fleet-sizing and service availability for a vehicle rental system via closed queueing networks. *Eur. J. Oper. Res.* 211 (1), 198–207.
- Handy, S.L., Xing, Y., Buehler, T.J., 2010. Factors associated with bicycle ownership and use: a study of six small us cities. *Transportation* 37 (6), 967–985.
- Jäppinen, S., Toivonen, T., Salonen, M., 2013. Modelling the potential effect of shared bicycles on public transport travel times in greater Helsinki: an open data approach. *Appl. Geogr.* 43, 13–24.
- Kabra, A., Belavina, E., Girotra, K., 2016. Bike-share systems. In: *Environmentally Responsible Supply Chains*. Springer, pp. 127–142.
- Kendall, M.G., Hill, A.B., 1953. The analysis of economic time-series-part I: prices. *J. R. Stat. Soc. Ser. A (Gen.)* 116 (1), 11–34.
- Li, Y., Szeto, W., Long, J., Shui, C., 2016. A multiple type bike repositioning problem. *Transp. Res. Part B: Methodol.* 90, 263–278.
- Lin, J.-R., Yang, T.-H., 2011. Strategic design of public bicycle sharing systems with service level constraints. *Transp. Res. Part E: Logist. Transp. Rev.* 47 (2), 284–294.
- Lin, J.-R., Yang, T.-H., Chang, Y.-C., 2013. A hub location inventory model for bicycle sharing system design: formulation and solution. *Comput. Ind. Eng.* 65 (1), 77–86.
- Liu, Z., Jia, X., Cheng, W., 2012. Solving the last mile problem: ensure the success of public bicycle system in beijing. *Procedia – Soc. Behav. Sci.* 43, 73–78.
- Lovelace, R., Beck, S., Watson, M., Wild, A., 2011. Assessing the energy implications of replacing car trips with bicycle trips in Sheffield, UK. *Energy Policy* 39 (4), 2075–2087.
- Martinez, L.M., Caetano, L., Eiró, T., Cruz, F., 2012. An optimisation algorithm to establish the location of stations of a mixed fleet biking system: an application to the city of lisbon. *Procedia – Soc. Behav. Sci.* 54, 513–524.
- Midgley, P., 2011. Bicycle-sharing Schemes: Enhancing Sustainable Mobility in Urban Areas. United Nations, Department of Economic and Social Affairs, pp. 1–12.
- Murphy, H., 2010. Dublin Bikes: An Investigation in the Context of Multimodal Transport. Dublin Institute of Technology, Dublin, Ireland. M.Sc. in Sustainable Development
- Nair, R., Miller-Hooks, E., 2011. Fleet management for vehicle sharing operations. *Transp. Sci.* 45 (4), 524–540.
- Nair, R., Miller-Hooks, E., 2016. Equilibrium design of bicycle sharing systems: the case of washington dc. *EURO J. Transp. Logist.* 5 (3), 321–344.
- Nair, R., Miller-Hooks, E., Hampshire, R.C., Bušić, A., 2013. Large-scale vehicle sharing systems: analysis of vélib. *Int. J. Sustain. Transp.* 7 (1), 85–106.
- Raviv, T., Kolka, O., 2013. Optimal inventory management of a bike-sharing station. *IIE Trans.* 45 (10), 1077–1093.
- Reijsbergen, D., 2016. Probabilistic modelling of station locations in bicycle-sharing systems. In: *Proceedings of Federation of International Conferences on Software Technologies: Applications and Foundations*. Springer, pp. 83–97.
- Ricci, M., 2015. Bike sharing: a review of evidence on impacts and processes of implementation and operation. *Res. Transp. Bus. Manag.* 15, 28–38.
- Schuijbroek, J., Hampshire, R.C., Van Hoeve, W.-J., 2017. Inventory rebalancing and vehicle routing in bike sharing systems. *Eur. J. Oper. Res.* 257 (3), 992–1004.
- Shaheen, S., Guzman, S., Zhang, H., 2010. Bikesharing in Europe, the Americas, and Asia: past, present, and future. *Transp. Res. Rec.: J. Transp. Res. Board* 2143, 159–167.
- Shu, J., Chou, M.C., Liu, Q., Teo, C.-P., Wang, I.-L., 2013. Models for effective deployment and redistribution of bicycles within public bicycle-sharing systems. *Oper. Res.* 61 (6), 1346–1359.
- Vogel, P., Mattfeld, D.C., 2010. Modeling of repositioning activities in bike-sharing systems. In: *Proceedings of World Conference on Transport Research (WCTR)*.
- Wikipedia, 2017. Bicycle-sharing System. [Online; accessed 25-September-2017].
- Yan, S., Lin, J.-R., Chen, Y.-C., Xie, F.-R., 2017. Rental bike location and allocation under stochastic demands. *Comput. Ind. Eng.* 107, 1–11.
- Zhang, D., Yu, C., Desai, J., Lau, H., Srivathsan, S., 2017. A time-space network flow approach to dynamic repositioning in bicycle sharing systems. *Transp. Res. Part B: Methodol.* 103, 188–207. *Green Urban Transportation*
- Zhang, L., Zhang, J., Duan, Z.-y., Bryde, D., 2015. Sustainable bike-sharing systems: characteristics and commonalities across cases in urban China. *J. Clean. Prod.* 97, 124–133.