

Equitable distribution of bikeshare stations: An optimization approach

Xiaodong Qian ^{a,*}, Miguel Jaller ^{b,c}, Giovanni Circella ^{a,d}

^a 3 Revolutions Future Mobility Program, Institute of Transportation Studies, University of California at Davis, 1606 Tilia Street, Suite 100, Davis, CA 95616, USA

^b Department of Civil Engineering and Environmental Engineering, University of California, Davis, One Shields Avenue, Ghausi Hall, 3143, Davis, CA 95616, USA

^c Sustainable Freight Research Center, Institute of Transportation Studies, University of California, Davis, 1605 Tilia Street, Suite 100, Davis, CA 95616, USA

^d School of Civil and Environmental Engineering, Georgia Institute of Technology, 790 Atlantic Drive, Atlanta, GA 30332, USA



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ABSTRACT

Bikeshare systems have attracted increased research interest ranging from bikeshare planning analyses to operational improvement studies (e.g., rebalancing, or station optimization). However, the interaction between bikeshare station spatial distribution and actual bikeshare activities when addressing equity issues has not been thoroughly considered. Moreover, there is a paucity of research helping governments develop incentive programs for equitable bikeshare services. To fill this research gap, we develop a model to estimate the potential demand (i.e., bikeshare trip production and attraction) and its distribution, and evaluate performance over a set of objectives (e.g., maximization of annual revenue, accessibility improvements) to find the most equitable distribution of stations. We build a genetic algorithm to solve this multi-objective optimization. The study uses the Divvy bikeshare system in Chicago as a case study, and compares the solutions of the model with the system's expansion (new stations added) in 2016, which targeted disadvantaged areas. When selecting accessibility as the main objective, the results indicate the need to provide more stations in disadvantaged areas and those results overlap with the system's expansion in 2016. On the contrary, the goal of revenue maximization results in a smaller network of stations and fewer accessibility improvements, especially in disadvantaged communities. A sensitivity analysis uncovers the greatest obstacle (i.e., station cost) to adding more stations in disadvantaged areas. More importantly, a Pareto frontier of this multi-objective optimization supports several policy suggestions for incentivizing private bikeshare companies to target more disadvantaged populations. Our results show the importance of considering accessibility and other equity constraints in developing a more inclusive, equitable and sustainable transportation system, and we provide several planning suggestions.

1. Introduction

Bikeshare is becoming more common around the world and there are over 800 cities offering bikeshare service, to date. Bikeshare can bring important social benefits, e.g., reducing air pollution and relieving traffic congestion (Fishman, 2016). For individuals, bikeshare systems can help improve physical health, remove the maintenance burden of bicycle ownership, eliminate the risk of bike theft and vandalism, and increase accessibility (i.e., the capability to access different mobility services and/or to reach desired destinations/opportunities) (Buck, 2013; Qian and Niemeier, 2019; Qian and Jaller, 2021). There are two main types of bikesharing systems, dock-based (or station-based) and dockless (or free-floating). In the US, there were over 40 million dock-based bikeshare trips in 2019, which is 17% more than in 2018 (NACTO, 2020), and 96 million dock-less (system) trips, more than

double those from dock-based bikeshare systems (NACTO, 2020).

Despite the benefits, increased popularity, and system expansion of various bikeshare systems, disadvantaged populations are not highly representative of current bikeshare users' profiles (McNeil et al., 2017). There are multiple types of barriers, such as access barriers (Qian and Jaller, 2020), financial resource barriers (e.g., lack of credit card) (Fishman et al., 2012), cultural barriers (Bernatchez et al., 2015; Nina Hoe, 2015; Stewart et al., 2013), and safety concerns (Christie et al., 2011; Griffin et al., 2008; McNeil et al., 2017), among others. Among all the aforementioned barriers, access barriers stand out. Currently, nearly half of the bikeshare systems in the United States are dock-based (NACTO, 2019), which means that users need to return a bike to a specific bikeshare station. Recent research shows that dock-based bikeshare systems may not be providing equitable access, with a station location bias towards specific areas and demographics (Qian and

* Corresponding author.

E-mail addresses: xdqian@ucdavis.edu (X. Qian), mjaller@ucdavis.edu (M. Jaller), gcircella@ucdavis.edu (G. Circella).

Niemeier, 2019). Consequently, a significant access barrier exists for disadvantaged users to enjoy bikeshare services (Bernatchez et al., 2015). Thus, removing this barrier may play an important role in mitigating equity issues in current dock-based bikeshare systems. More recently, a study in San Francisco, CA, showed that while dockless systems can provide increased accessibility to disadvantaged communities as there are less constraints due to the location of stations, additional system characteristics such as bike rebalancing, which determines the number and location of repositioned bikes, still represent some barriers for the equitable access to these services (Qian et al., 2020).

Today, in practice, there are no widely accepted guidelines for designing or evaluating bikeshare station networks (locations, numbers, and capacities), especially in terms of serving disadvantaged communities. Previous research have approached this problem by considering potential demand, bicycle facility and accessibility (Conrow et al., 2018; Hasan, 2016; Caggiani et al., 2020; Beairsto et al., 2021). However, the interaction between station siting plans and trip activities (e.g., trip demand generation and distribution) have not been thoroughly explored to show the trip-level accessibility benefits from bikeshare for disadvantaged communities. To fill this gap, this paper introduces a multi-objective optimization model for bikeshare station locations, which combines both bikeshare trip activities and equity considerations. The model considers the maximization of annual revenue and accessibility improvements to select the optimal number and location of stations. In particular, the model estimates bikeshare trip productions and attractions using trip generation models (Qian and Jaller, 2020) and distributes them in the study area through a destination choice model (Qian and Jaller, 2021). The study uses Chicago's Divvy bikeshare system as the case study and evaluates the station network design under various considerations. In addition, we use Divvy's system expansion in 2016 to compare and validate the results of the optimization under the different objectives (i.e., revenue and accessibility).

This paper is organized as follows. Section 2 reviews the bikeshare location modeling literature. After summarizing the current research gap, the study describes the case study and the optimization model in Section 3. Section 4 discusses the empirical analyses, compares the current station network with suggested locations resulting from the optimization, conducts a sensitivity analysis, and presents a Pareto frontier for this multi-objective optimization. Finally, in the discussion and concluding sections, we present suggestions for how bikeshare stations could be located to service more disadvantaged populations.

2. Literature review

Most of the current studies on bikeshare location decisions apply geographic information systems (GIS) and spatial optimization methods (Wuerzer et al., 2012; Rybarczyk and Changshan, 2010; J. Wang et al., 2016; Croci and Rossi, 2014). Some of these studies focus on the planning and implementation of new systems. For example, Wuerzer et al. (2012) fed GIS-based information into a location optimization model for bikeshare stations in Boise, Idaho. In their model, they considered multiple factors that affect bikeshare demand generation, including population density, employment density, higher education, transit, bike path infrastructure, and the location of other recreational activities. Moreover, they included a budget constraint in their model. After considering different scenarios for bike usage, they provide a suggestion for the total number of bikes and station locations considering capital expenditure constraints. Similarly, Rybarczyk and Changshan (2010) used GIS tools to conduct both network analysis (visualizing spatial distribution) and neighborhood analysis (i.e., Moran's I calculation) for bikeshare planning in Milwaukee (WI, USA).

Other studies have focused on the evaluation of existing bikeshare systems. For example, J. Wang et al. (2016) analyzed public bike rental services in Taiwan, and identified hot spots of bike rack (station) deficits through a spatial-temporal analysis of historical bikeshare activities. The study used trail location theory to suggest ideal rental station

locations. The research showed the benefits of GIS and spatial-temporal analyses for bikeshare planning, management, and operations. Another example is the study of the "BikeMi" system in Milan to identify bikeshare location problems (Croci and Rossi, 2014). The analysis uncovered the fact that metro and train stations, universities, museums, cinemas, and restricted traffic areas in the proximity of bikeshare stations significantly increase bikeshare usage. The work concluded that careful consideration of the surrounding environment is important for the optimal siting of bikeshare stations.

In terms of developing an inclusive bikeshare systems, several studies evaluated current bikeshare station siting through spatial analyses or station optimization from the equity lens. There are studies developing an equity index to judge the spatial distribution of bikeshare stations, e.g., an economic hardship category by Smith et al. (2015), a deprivation quintile by Winters and Hosford (2018), and an accessibility spatial index by Qian and Niemeier (2019). For station optimization, Conrow et al. (2018) applied a location coverage model to maximize the coverage of potential users and bicycle network. Their optimization model can provide a set of station locations, which evenly cover the service area instead of only targeting urban centers. Hasan (2016) applied a linear optimization model to maximize the accessibility indices of the Chicago bikeshare system. The author suggested a redistribution plan to maximize the accessibility of the bikeshare system. However, the accessibility indices only measured the counts of bike share stations within a specific range, which do not include other important opportunities, e.g., jobs. Another study by Beairsto et al. (2021) examined the equitable spatial distribution of bikeshare stations in Glasgow, Scotland through optimizing the product of a demand score and an accessibility score. Nevertheless, the optimization process did not combine the accessibility score with real bikeshare travel activities since not all bikeshare trips generated from a station generate the same accessibility. Other than using accessibility as an equity index, Caggiani et al. (2020) balanced the level of service and system cost when deciding the number of bike stations, the spatial distribution, and the capacity of all stations. The level of service is defined as the number of available bicycles and walking distance to stations.

When comparing current bikeshare station locations, there are multiple criteria to evaluate, thus, a number of researchers have conducted multiple-objective analyses. Jahanshahi et al. (2019) applied multiple-criteria decision making (MCDM) to rank current bikeshare stations and utilized Jenks natural breaks classification method (JENKS) to classify all available stations. Based on their results, 51 stations are unsatisfactory among all 128 stations in Meshad, which is the second-most populous city in Iran. Additionally, they applied the methodology to examine 22 planned stations and provided suggestions on all of these potential locations. Their research provided insights into current station siting and plans. Similarly, Bryant Jr (2013) applied this type of model and provided suggestions based on the multi-objective optimization results. García-Palomares et al. (2012) applied a GIS-based allocation modeling to optimize bikeshare station locations based on minimizing impedance and maximizing coverage. They recommended the second objective, which is more suitable in terms of efficiently covering potential demand. More importantly, after deciding the optimal locations, they evaluated each proposed location regarding accessibility to the different travel activities and discarded those providing limited access. This multi-objective analysis approach has also been applied in other transportation infrastructure planning (C.-H. Wang and Chen, 2020; Zhu and Zhu, 2019; Zhao et al., 2019).

The main feature of the aforementioned analyses is the prediction of potential bicycle populations or bikeshare demand and the suggestion of optimal bikeshare locations. However, there is a lack of research incorporating trip distribution or destination choice into the optimization model. Lack of destination choices in the model limits its ability to fully assess the impact of proposed bikeshare station locations, especially when the objective is to measure accessibility and opportunities to address equity problems faced by current (and potential) bikeshare

users. Additionally, trip distance and trip time distributions will affect the revenue a system can generate, which is critical for the service providers. Therefore, knowing changes in trip distribution based on the location and quantity of stations can help estimate the benefits of the bikeshare system.

This paper contributes to the literature by developing a multi-objective optimization model that: 1) can consider different objectives (e.g., maximizing revenue and improving accessibility); 2) can simulate the interaction between bikeshare station siting and bikeshare activities, e.g., estimating the demand at each station based on the (spatial concentration) location and socioeconomic characteristics of the served population and distributing the trips among all stations based on trip characteristics, and the attractiveness of the locations (to estimate the revenue from such trips). Finally, by interpreting the results of this model, the authors provide quantitative suggestions for local governments to support or incentivize private bikeshare companies so that they expand services to more disadvantaged communities.

3. Case study and methodology

This study uses Chicago as a case study. In 2013, the Chicago Department of Transportation (CDOT) launched the Divvy bikeshare system and contracted with Motivate to purchase, install, and manage the system (Motivate International, 2017a). Divvy obtained start-up federal funding, aimed at reducing traffic congestion and improving air quality. This bikeshare system provides two main kinds of membership: annual pass and day-pass, both of which enjoy a 30-min free ride per trip. The charge framework for both users is listed in Table 1. As we can see, the annual membership fee (\$99 per year) can be a significant amount of budget for some users.

In July of 2015, the Divvy system introduced the “Divvy for Everyone (D4E)” program, which provides affordable membership fees to qualifying residents (Motivate International, 2017b). Thus, Chicago is an ideal case study city to analyze its station location planning strategy. To find the optimal location of bikeshare stations, the study first identifies a set of candidate locations and then evaluates the performance of the system under different objectives, using a discontinuous (and non-differentiable) nonlinear optimization model to provide planning suggestions. The model and solution approach are discussed next.

3.1. Candidate locations

This study focuses on disadvantaged populations, thus the study team first identified whether a station location is in a disadvantaged area following the buffer analysis process described in Qian and Jaller (2020). Specifically, a 400-m buffer was created for every station location and demographic information was compiled for every buffer. Then, the income level and race percentage of all populations within the buffers were used for identification (Table 2).

In 2015, the system had 475 bikeshare stations of which 20.8% were located in disadvantaged areas (Fig. 1). Between 2013 and 2015, the system targeted urban areas and communities with larger cyclist populations. In 2016, the system built 107 additional stations, almost 62% of which are in disadvantaged communities, after the city noticed that not enough bikeshare stations were serving low-income or minority populations. Most of the new stations in 2016, intended to address

Table 1
Charge framework (in dollars) for Divvy in 2016.

Trip duration	Annual member	Day-pass user
Base charge	99 per year	9.95 per day
0–30 min	0	0
31–60 min	1.5	2
61–90 min	4.5	6
91 and more mins	6 per 30 min	8 per 30 min

Table 2
Criteria for disadvantaged communities.

Category	Data	Value
Disadvantaged communities	Income	<\$50,000 per year
	Percentage of white race (low)	<Mean ¹ –0.5×Sd ² (<41.64%)
Other areas	Income	Everything else
	Percentage of white race	

Note: 1. “Mean” is the mean of the percentage of white race;

2. “Sd” stands for “Standard deviation.”

equity issues, are located in the west and south of Chicago, where there are more disadvantaged communities (Fig. 1).

To evaluate the system with respect to its ability to provide improvements in accessibility to disadvantaged communities, the team quantified the system performance using the station network in 2015 as the base case, and considered the 107 stations added in 2016 and another set of 150 locations as candidate locations (a total of 257 potential locations) to conduct the analyses (Fig. 1). To generate this set of 150 locations, this study first identified a number of intersections based on the road network (in addition to the locations added to the system in 2016) at the periphery of the service region in 2015, i.e., approximately 5000 m to the boundary of the service area in 2015. The reason why the candidate locations do not cover the whole area of Chicago is that an isolated rural service area will significantly increase operating costs, which is not feasible. Another reason is that bikeshare is mainly for first/last mile or short-distance trips, so it is unlikely that people will cycle all the way between the city center and rural areas. Then, the study created a unique ID for every intersection and randomly selected a set of 150 locations. During the random selection, the study assigned more weights on locations closer to the city center and meanwhile made sure that the proportion of disadvantaged locations is near 50% (Table 3).

Fig. 1 and Table 3 show the locations of the stations and the number of stations in disadvantaged communities. In total, the analyses considered 257 candidate locations (including 107 stations in 2016 and the other 150 random locations) with about half (56%) of those in disadvantaged areas (Table 3). The red dots in Fig. 1 represent the locations of Divvy’s 2016 expansion, and the blue dots are the potential locations except the stations added in 2016. The black circles show whether a location is in disadvantaged areas or not.

3.2. Bikeshare station optimization problem

The network of all station locations will influence trip generation/atraction and trip distribution, and finally, it will affect the benefits generated by the system. This is because recent research identified spatial proximity between stations to be a significant factor in the production and attraction of bikeshare trips from a particular station (Campbell et al., 2016; Qian and Jaller, 2020). For example (see Fig. 2), a station (blue star) with more nearby stations may generate (yellow buffer) and attract (blue buffer) more bikeshare demand. Thus, the optimization model considers the trip generation and trip distribution for every possible solution of bikeshare siting (the second and third column in Fig. 2). After estimating trip distribution, the model quantifies accessibility improvements and revenue generated by the whole system under one potential siting plan and evaluates the performance under different objectives. The optimal station network will be based on the value of different objectives (Fig. 2). All the variables used in this multi-objective optimization problem are listed in Table 4. The following section will introduce the details of every step in Fig. 2.

3.2.1. Estimating trip productions and attractions

After selecting the set of candidate locations, the model estimates bikeshare trip generation (e.g., productions and attractions) for every location. There are multiple studies quantifying bikeshare demand,

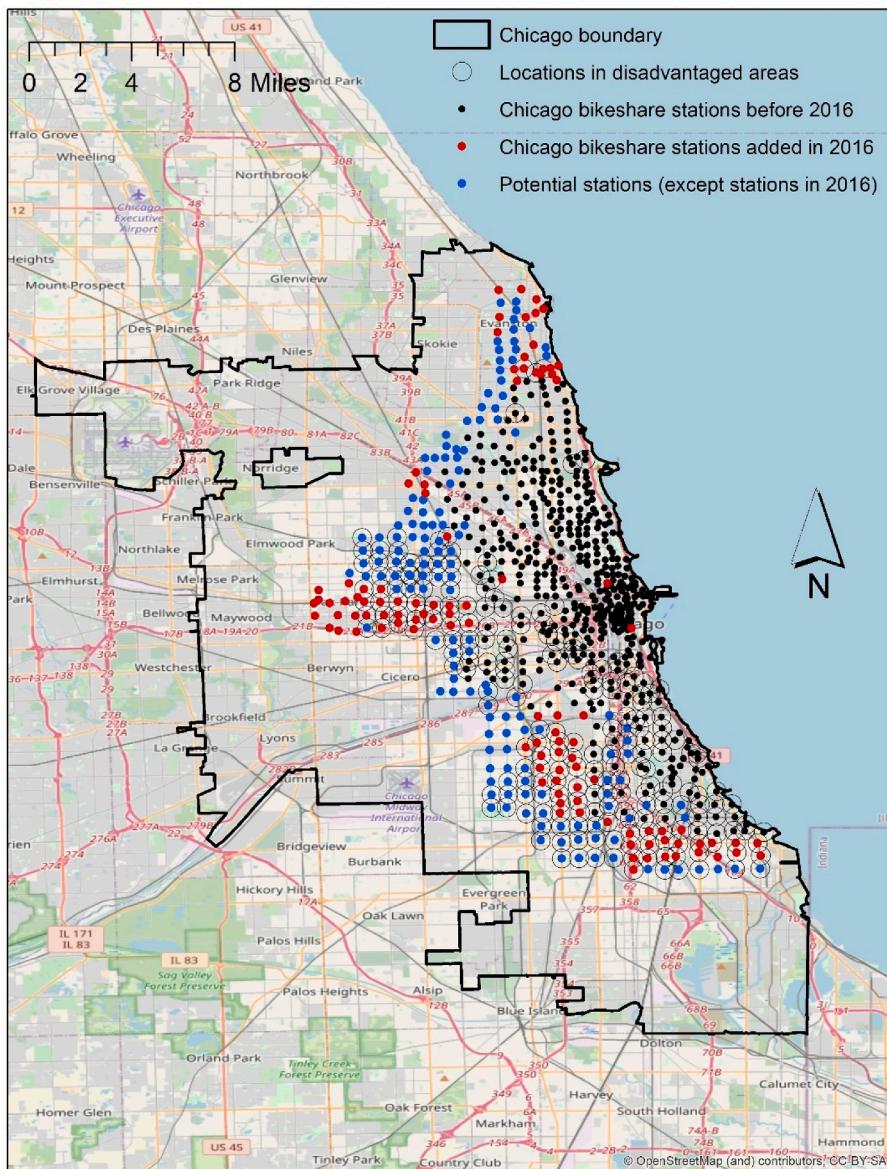


Fig. 1. Locations of existing and candidate bikeshare stations in Chicago.

Table 3
Existing and candidate bikeshare stations in Chicago.

Type	Disadvantaged areas	Other areas	Total	Percentage of stations located in disadvantaged areas
2015	99	376	475	20.8%
2016	66	41	107	61.7%
Other potential	78	72	150	52%
Total	144	113	257	56%

however, this study builds on recent research that developed generation models for Divvy (Qian and Jaller, 2020). The study estimated bikeshare trip production (O_i) and attractions (D_j) as negative binomial regression models considering demographic and other geographic information. The model suggests that the variables affecting the generation of trips include labor force, employment rate, bike path density, park areas, number of nearby bikeshare stations, and area type (disadvantaged or not) around a 400-m buffer. In this trip demand model, there is one parameter (area types: Disadvantaged communities or not) to represent the demographic information including median income and percentage of minority populations. There are other factors (e.g., populations and

number of intersections) that may influence bikeshare trip demand. However, those factors (e.g., populations) are not included in the final model due to correlation and model overall performance based on the Akaike information criterion (AIC) index. For the details of developing this trip demand model, please refer to the research by Qian and Jaller, 2020.

In this study, the authors gathered the same type of information surrounding (buffer) every candidate location. Although most of the variables are independent for each location's buffer, the trip generation model found that the number of other stations within 500 m affects the number of trips generated at the station. Consequently, the evaluation of

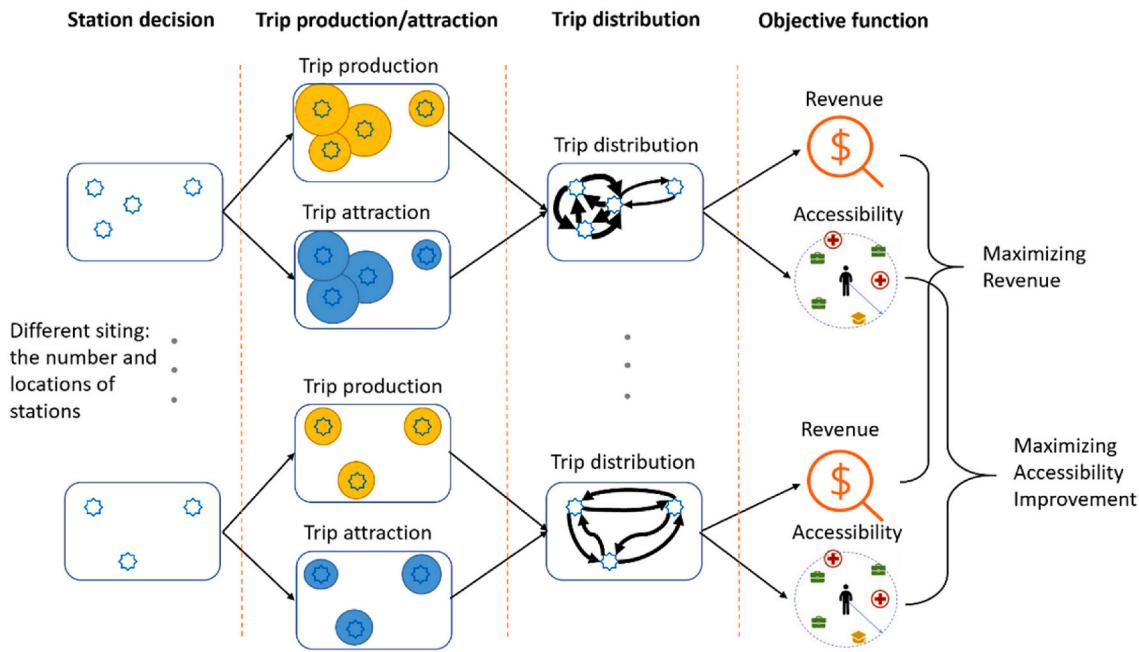


Fig. 2. General schematic for the optimization process.

the different stations' network requires estimating potential trip productions and attractions, each time a new solution is tested. Moreover, the system's performance is dependent not only on the number of stations, but also on specifically which stations are provided. This is because different generations will impact the estimated distribution of such trips. Table 5 shows the trip production and attraction models used here.

The total trips generated are then split into trips made by annual members and day-pass users. The model uses data from the Divvy system during 2016–2017 to generate constant ratios for trip split. Fig. 3 shows the spatial distribution of the ratios of trips by annual members and the histogram of the ratios. Most of those stations have 80% of bikeshare trips from annual members. Thus, annual members generate approximately 80% of all trips.

$$O_{i_annual} = O_i \times 0.8 \quad (1)$$

$$D_{j_annual} = D_j \times 0.8 \quad (2)$$

$$O_{i_day} = O_i \times 0.2 \quad (3)$$

$$D_{j_day} = D_j \times 0.2 \quad (4)$$

3.2.2. Trip distribution

After estimating the trip generation, the model distributes the trips following a competing destination model based on a gravity model, which brings together the important features of both destination station and bikeshare trips. The reader is referred to Qian and Jaller (2021) for additional details about the destination choice model. Overall, solving the entropy model by maximizing travel benefits for the system while minimizing travel decay functions leads to the following functional forms for the number of trips:

$$T_{ij} = A_i O_i B_j D_j (S_{ij})^\rho e^{-\beta C_{ij}} \quad (5)$$

where,

$$A_i = \frac{1}{\sum_j B_j D_j (S_{ij})^\rho e^{-\beta C_{ij}}} \quad (6)$$

$$B_j = \frac{1}{\sum_i A_i O_i (S_{ij})^\rho e^{-\beta C_{ij}}} \quad (7)$$

In Eq. (5), T_{ij} is the total number of trips between origin i and destination j , and ρ represents the influence of attractiveness on trip distribution for each OD pair, β is the travel decay parameter, which estimates traveling distance sensitivity, and, C_{ij} is the network travel distance from location i to j on bicycle estimated by Google API. S_{ij} measures how many opportunities (e.g., jobs, groceries, schools and hospitals) a user can get access to by cycling from location i to j , which is defined as accessibility improvements and estimated by:

$$S_{ij} = \sum_{k=1}^w |\Delta Opp_k| \times Weight_k \quad (8)$$

where for opportunity k , ΔOpp_k measures the opportunity difference between origin i and destination j , and $Weight_k$ is the weight associated with each opportunity (Qian and Niemeier, 2019). The models use a set of ρ and β values for different OD types (i.e., ρ_{ij} and β_{ij}) depending on whether or not the origin or destination are at a disadvantaged community (Qian and Jaller, 2021). Moreover, considering trips for annual members, and day-pass users, Eq. 5 transforms into:

$$T_{ij_annual} = A_{i_annual} O_{i_annual} B_{j_annual} D_{j_annual} (S_{ij})^{\rho_{ij_annual}} e^{-\beta_{ij_annual} C_{ij}} \quad (9)$$

$$T_{ij_day} = A_{i_day} O_{i_day} B_{j_day} D_{j_day} (S_{ij})^{\rho_{ij_day}} e^{-\beta_{ij_day} C_{ij}} \quad (10)$$

ρ_{ij_annual} and β_{ij_annual} are the parameters for annual members of OD pair ij ; and ρ_{ij_day} and β_{ij_day} are the parameters for day-pass users of OD pair ij .

3.2.3. Optimal bikeshare station network

This optimization problem integrates trip generation and distribution, and seeks to identify the optimal bikeshare station network by specific objectives. In this study, the coverage of disadvantaged populations is not considered as an explicit objective; instead, considering accessibility should lead to system that provides coverage where needed, as opposed to finding the network that provides access, which may not necessarily be equitable under budget constraints. As discussed in literature review, some studies have applied accessibility as an index

Table 4

Notation.

Variable	Description
Decision variable	
a_m	Whether a station is placed in location m
Trip production/atraction variables	
O_i	Total number of trips generated at location i
O_{i_annual}	Total number of trips generated at location i by annual members
O_{i_day}	Total number of trips generated at location i by day-pass users
D_j	Total number of trips attracted to location j
D_{j_annual}	Total number of trips attracted to location j by annual members
D_{j_day}	Total number of trips attracted to location j by day-pass users
LF	Labor force: the total number of labor force within 400-m buffers
Emp_rate	Employment rate: employed population divided by the total population of workforce within 400-m buffers
BN_des	Bicycle network density: the total length of bike paths divided by the area of a buffer within 400-m buffers
S_{500m}	Stations within 500 m: number of bikeshare stations within 500 m cycling distance
Pec_young	Percentage of population aged between 20 and 34 years old (%) within 400-m buffers
$Transit$	Number of transit (bus/railway) stations within 400-m buffers
$Park_area$	The total areas of parks within 400-m buffers
$Type$	Whether a station is in disadvantaged communities
Trip distribution variables	
T_{ij_annual}	The total number of trips from location i to location j by annual members
T_{ij_day}	The total number of trips from location i to location j by day-pass users
A_i	The proportionality factor associated with location i
B_j	The proportionality factor associated with location j
S_{ij}	The accessibility improvement by traveling from location i to location j
C_{ij}	The travel distance from location i to location j by bicycles
ρ_{ij_annual}	The parameter of accessibility influences associated with OD ij for annual members
β_{ij_annual}	Travel decay parameter associated with OD ij for annual members
ρ_{ij_day}	The parameter of accessibility influences associated with OD ij for day-pass users
β_{ij_day}	Travel decay parameter associated with OD ij for day-pass users
Objective function variables	
Acc	The total accessibility improvement by the whole bikeshare system
S_{ij}	Same as S_{ij} in Trip distribution
Rev	The total net revenue generated by the whole bikeshare system
$Cost_{ij_annual}$	The cost of a single trip from location i to location j by annual members
$Cost_{ij_day}$	The cost of a single trip from location i to location j by day-pass users
M_Cost_{annual}	The marginal cost for annual members per trip, which is calculated by dividing \$99 annual membership fee by average annual trip number
M_Cost_{day}	The marginal cost for day-pass users, which is calculated by dividing day-pass fee (\$9.95) by average day trip number
$Cost_{station}$	The cost to build a physical station
$Cost_{bike}$	The cost to purchase a bike
Constraints	
M	The candidate location set (257 locations)
N	The maximum number of station proposed (107 stations)

when measuring equity (Hasan, 2016; Beirsto et al., 2021). In addition, previous research shows that bikeshare systems can bring significant accessibility improvements (defined in Eq. 8) for disadvantaged communities (Qian and Niemeier, 2019). Thus, one of the objectives seeks to maximize accessibility (Acc) by all bikeshare trips, and the optimization function could be formulated as:

$$\text{Max } \{Acc\} = \sum_{ij} S_{ij} \times (T_{ij_annual} + T_{ij_day}) \quad (11)$$

In addition to accessibility improvements, we also consider annual revenue (Rev). In this study, the annual revenue does not include fixed operating or other administrative costs because they are assumed fixed,

Table 5

Bikeshare trip production and attraction models (Qian and Jaller, 2020).

Variables	Trip production (O_i)	Trip attraction (D_j)
	Coefficient	Coefficient
Constant	-8.315×10^{-2}	-3.528×10^{-1}
Labor force	5.455×10^{-5}	5.110×10^{-5}
Employment rate	7.107×10^{-2}	7.471×10^{-2}
Bike path density	5.212×10^{-3}	5.122×10^{-3}
Park areas	4.473×10^{-6}	4.415×10^{-6}
Stations within 500 m	8.750×10^{-2}	7.594×10^{-2}
Percentage of young population	3.298×10^{-2}	3.200×10^{-2}
Number of transit stops	2.045×10^{-2}	2.217×10^{-2}
Disadvantaged communities	-3.082×10^{-1}	-2.948×10^{-1}

and the information is not publicly available. The second objective, maximizing annual revenues, is as follows:

$$\begin{aligned} \text{Max } \{Rev\} = & \sum_{ij} (Cost_{ij_annual} \times T_{ij_annual} + Cost_{ij_day} \times T_{ij_day}) \\ & + \sum_{ij} (M_Cost_{annual} \times T_{ij_annual} + M_Cost_{day} \times T_{ij_day}) \\ & - Cost_{station} \times \sum_m a_m - 10 \times Cost_{bike} \times \sum_m a_m \end{aligned} \quad (12)$$

The system is subjected to the following set of constraints:

$$\sum_m a_m \leq N \quad (13)$$

$$a_m = (1, 0) \text{ for all } m \in M \quad (14)$$

M is the set of the candidate locations; and a_m is a binary variable indicating if a bikeshare station is sited at location m , i.e., placing (value = 1) a bikeshare station at candidate locations or not (value = 0). N is the maximum number of bikeshare stations to locate. For the purpose of the analyses, N is set to 107, which is the number of new stations added to the system in 2016; while, N could be any number of stations based on budgetary constraints, having this bound allows comparing the results and trade-offs between the various objectives (i.e., revenue or accessibility maximization). We do not set the total number of suggested stations strictly equal to 107, because it would force such solution that may be sub-optimal. As discussed later, the empirical results found that extreme solutions (i.e., only driven by either of the objectives required sitting less than this bound). Moreover, $Cost_{ij_annual}$ is the trip cost for annual members, while $Cost_{ij_day}$ is the trip cost for day-pass users. The trip cost is estimated based on the charges and fees framework provided by the Divvy system in 2016 (Motivate International, 2017a) (the current framework is different). Since there is no available information about the exact trip times, this study uses an estimated bikeshare trip time from the Google API for each OD pair. An analysis of bikeshare trip time data from 2016 revealed that annual members tend to spend 20% more time than the shortest travel time, while day-pass users spend twice as much time as the shortest travel time, on average. Therefore, the numerical experiments here consider these factors when calculating $Cost_{ij_annual}$ and $Cost_{ij_day}$ by timing the shortest travel time with the corresponding ratios (1.2 for annual members, and 2 for day-pass users).

In addition to the cost for every single trip, bikeshare users also need to pay "membership" fees (i.e., annual membership fee, or day-pass fee). However, these membership fees may need to be distributed among all trips a user makes in one year, which are referred to as marginal trip costs in this study, to be able to estimate comparative trip-based charges. Based on the statistics provided by Nyakuengama (2018), Pratt (2017), and Freund (2018), the authors estimated the marginal trip cost for annual members (\$1.2) and day-pass users (\$4.0). The $Cost_{station}$ and $Cost_{bike}$ are the expenses to build new bikeshare stations (\$40,000) and add bikes (\$1000). The ratio between the number of newly added bikes and the number of newly added stations is fixed (10:1). Besides, this optimization problem is based on the time unit of years. Thus, the daily

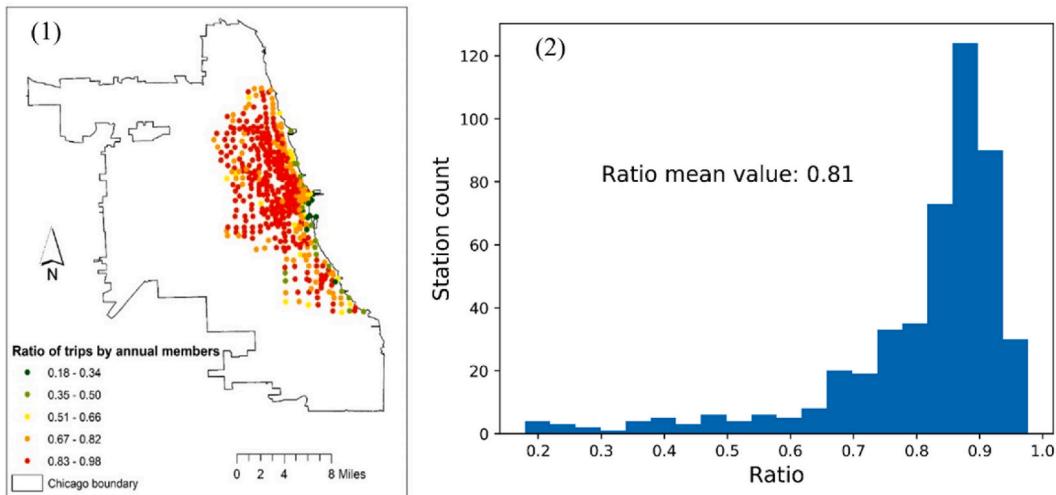


Fig. 3. (1) Spatial distribution and (2) histogram of the ratios of trips by annual members.

operation cost (e.g., bike rebalance or service frequency) is not considered in our model.

3.2.4. Solution approach

The optimization model is a non-linear non-differentiable integer problem because every candidate station location is modeled through a binary variable, and the estimation of trip generation (negative binomial regression) and distribution (Eqs. (9) and (10)) results in non-linear functions. Moreover, as explained before, the generation of trips is affected by the number of stations within a distance buffer of a specific station, thus each potential network design (station siting plan) affects every solution. To solve this complex problem, the authors developed a genetic algorithm (GA), which is an optimization method inspired by the process of natural selection. This approach has been widely used in operations research (Konak et al., 2006; Srinivas and Deb, 1994; Grefenstette, 2013; Fonseca and Fleming, 1993). Therefore, this section only introduces the general GA process (Fig. 4). In the GA, there are essential terms: populations (all station siting plans in our case) and chromosome, which is a set of parameters that define a proposed solution to an optimization problem (one possible siting plan). This study defines a chromosome as the binary sequence representing whether a

station is open (value of 1) at location m , or not (value of 0). For example, a chromosome (P1 in Fig. 4), indicates that stations at locations 1 and 3 (counting from the top) are open, where those at locations 2 and 4 (counting from the top) are closed. A fitness function representing the objectives in this case evaluates each chromosome. The algorithm selects individuals (chromosomes) from the current population based on their fitness to be parents and uses them to produce the children for the next generation. The GA repeatedly modifies these chromosomes through crossover and mutation processes at each iteration. Over successive generations, the population “evolves” towards an optimal solution. Fig. 4 also shows how these crossover and mutation operations correspond to the changes of station siting plans in the model. For example, the pair of siting figures under the mutation operation indicates that there are no potential bikeshare stations in these two gray locations, like a mutation in genes.

The multi-objective optimization model results in a Pareto frontier (of optimal solutions) considering both objectives, which show the trade-offs between accessibility improvements and revenue generated. This type of analysis is common in multi-objective optimization problems (Jaller et al., 2019). Specifically, the Pareto frontier provides insights and practical planning suggestions for bikeshare station siting

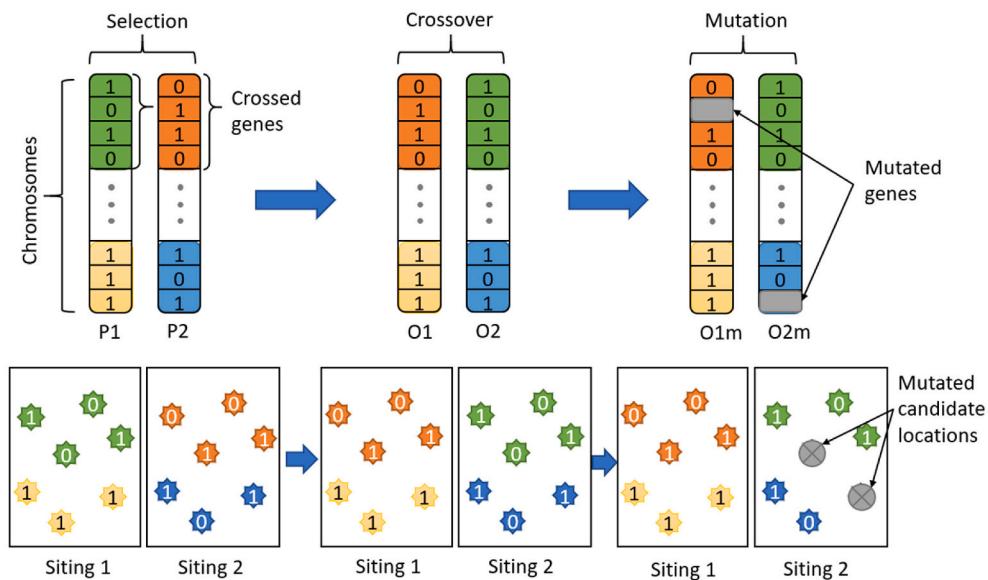


Fig. 4. Illustration of the genetic algorithm in our analysis.

based on how important each objective is for the decision-maker.

The optimization problem was developed in MATLAB R2020a and was solved through the MATLAB genetic algorithm solver on a Window computer with 2.60 GHz Intel Core i7 and 32 GB RAM allocation. The solution process took on average about 3 h, with solutions based on maximizing accessibility (or multi-objective optimization) requiring twice as much time (around 4 h) than those driven by revenue maximization. All the results of this optimization problem are shown in the following section.

4. Empirical results

4.1. Single-objective optimization

As a first step to gain insights into the problem, this section focuses on the evaluation of the optimization model under each objective independently. In Fig. 5, the red line represents the optimization convergence line, while the blue line shows the number of new suggested stations (generation is equal to iteration in the GA optimization process). For the objective of maximizing revenue, the optimal value of revenue generated keeps increasing, but the number of suggested stations is decreasing. However, under maximizing accessibility improvements, the trends of both optimal values and the number of new stations rise as the generation number increases. The difference may result from the fact that the cost of adding a new bikeshare station is a substantial financial burden for bikeshare companies, as reflected in the results. Building more stations will bring more accessibility improvements; however, the siting of those new stations is an important consideration since there is a maximum number of new stations to add. The fluctuation of the optimal number of newly added stations (blue line) in the right panel in Fig. 5 shows this decision process of the GA.

Table 6 shows the details of the two optimization results. Under revenue maximization, the model suggests not building any more bikeshare stations out of the 257 candidate locations. The empirical results show that the system under maximizing revenue could generate approximately \$6,864,611 in annual revenue, which is much more than the estimated revenue (\$1,912,074) under the 2016 scenario (base-case scenario) and what would be realized under the maximizing accessibility scenario (\$2,037,644). However, the station siting suggested by the maximizing revenue objective generates the least accessibility improvements among the three scenarios. This result is consistent with the system's experience, and could be explained by two main factors. First, the system is dense, which decreases travel times between stations (shorter than 30 min for many pairs of stations), which could induce users to split a long trip into multiple short trips to take advantage of the

Table 6
Statistics for optimization results.

Scenario	# of stations suggested/built	Annual revenue (million \$)	Accessibility improvement (E+11)	Total ridership
1. 2016 network	107	1.912	3.197	3,561,023
2. Maximizing revenue	0	6.865 (259.01%)	3.150 (-1.45%)	3,411,170 (-4.21%)
3. Maximizing accessibility	105	2.038 (6.57%)	3.230 (1.04%)	3,576,608 (0.44%)

Note: the numbers in "O" represent the change comparing with the 2016 network scenario.

unlimited 30-min free ride provision. Another reason is that while it would cost more to build more stations in suburban areas, the ridership in those areas may not generate enough revenue to support the expansion. In fact, the system reported that the 2016 expansion decreased Divvy's revenue because of lower ridership in disadvantaged areas (Pratt, 2017). The experience and results are consistent. Under this objective, bikeshare companies may not have a clear motivation to build more stations in disadvantaged areas, if doing so negatively affects their revenue.

The annual revenue generated by maximizing revenue will serve as a baseline for later sensitivity analyses. Note the proportions of revenue generated by different sources are: direct trip cost by annual members (2.0%); marginal trip cost by annual members (46.3%); direct trip cost by day-pass users (13.2%); marginal trip cost by day-pass users (38.5%). The revenue generated by the annual membership fee takes most of the total revenue. This will be reflected in the following sensitivity analysis, and will be explained further.

Under the maximizing accessibility objective, the model suggests a total of 105 stations, which is almost equal to the number of stations added in 2016. Both accessibility improvements and total ridership in the scenario suggested by the model are more than those achieved in the 2016 scenario. Moreover, the estimated annual revenue is 6.57% greater than that associated with the station expansion in 2016 (base case). Under the assumptions of this model, and the set of potential station locations, the model finds a station siting plan with a slight increase in accessibility and total ridership.

However, the maximizing revenue scenario does show a significant increase in revenue (2.5 times more), while accessibility and ridership only decrease by 1.45% and 4.21%, respectively. Careful consideration of these patterns is critical when trying to design an equitable system that improves accessibility. Specifically, the revenue-driven solution will not generate approximately 165,438 (3,576,608 - 3,411,170)

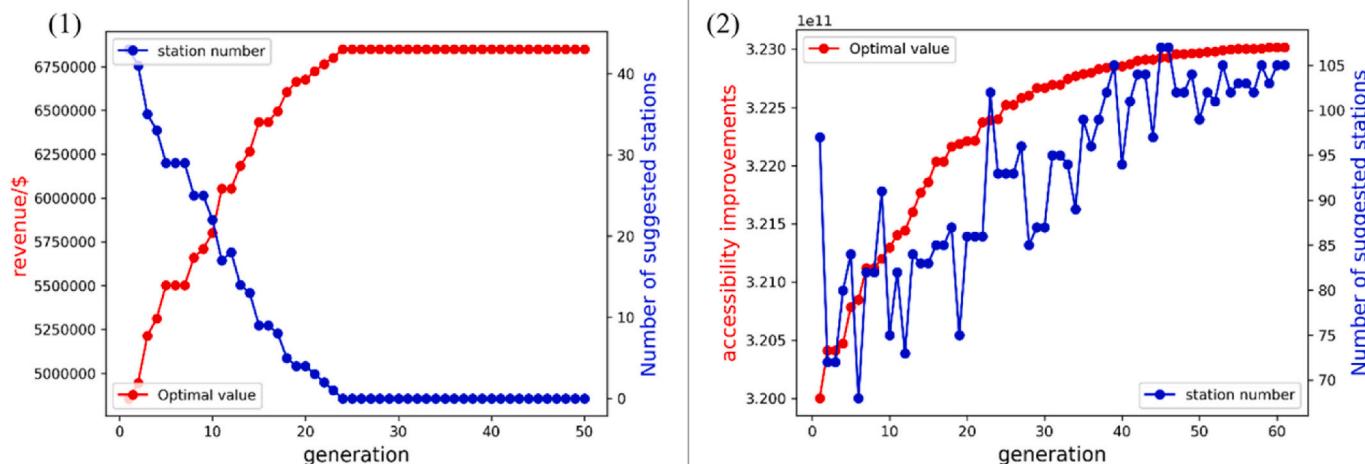


Fig. 5. GA optimization results for maximizing (1) revenue and (2) accessibility improvement.

potential annual trips to and from disadvantaged communities. Again, the objective of the optimization was not to find a system that provided total coverage but to compare an equitable siting decision on the basis of stations to provide improved accessibility.

Fig. 6 shows the optimal locations of stations under the different objectives. The black dots represent the stations built before 2016; the left panel shows (in red dots) the system's new stations in 2016 (base case); the right panel shows the solution for the system that maximizes accessibility. The result under maximizing annual revenue suggests not building new stations (thus, the solution is for only those stations available before 2016). In terms of spatial distribution, the suggested accessibility maximizing locations have a considerable overlap with the 2016 expansion, especially in the west and south of Chicago. As discussed before, the Divvy system targeted more disadvantaged communities in 2016 (left panel in **Fig. 6**). This overlap indicates that designing the system considering accessibility improvements could help address equity access issues by providing more stations in disadvantaged areas even though the accessibility improvements come at a price.

4.2. Sensitivity analysis

To better understand how the results of this multi-objective optimization problem will change according to the variation of the parameters, the authors conducted a sensitivity analysis. Since the objective of maximizing revenue tends not to cover more disadvantaged areas, we want to know how to change this trend by tuning these parameters in the annual revenue objective: marginal cost per trip for both annual members and day-pass users, station cost and bike cost. The implication of increasing/decreasing those marginal costs corresponds to the increase/decrease in annual membership fee or day-pass fee.

First, the analyses examine the sensitivity for every parameter. The top-left, top-right, and bottom-right panels in **Fig. 7** show that the increase in the marginal costs by annual members and day-pass users, which is similar to bike cost, has no influence in the decision of opening more stations, whether or not in disadvantaged areas. The most significant influence comes from the physical station cost. For example, a 60% or more reduction of station costs leads to a larger bikeshare station network. The maximum number of new stations will reach seven when there is no cost to building a new station. The number does not reach to

107 (which was set as the maximum number of added stations in the optimization problem) because bike costs would still be an issue with new stations. However, the percentage increase in revenue compared to the base case (the annual revenue generated by maximizing revenue) by reducing station cost is limited compared to increasing the marginal cost for both annual members and day-pass users. The reason is that the revenue from the membership fee of annual members and day-pass-users takes most of the total revenue.

Fig. 8 shows the exact locations of the added stations when the station cost reduces. These added stations are more likely to be in the city center at first. After the stations in the city center are taken, bikeshare companies will consider extending their systems in the periphery of a city urban area, but still not in disadvantaged areas.

Since the cost of building a new station sets such an economic barrier to expanding the system, especially in disadvantaged areas, the authors modeled a scenario with no station cost to evaluate the influence of the other parameters. The left and middle panels in **Fig. 9** show that as the marginal cost per trip increases, the optimal value (revenue) increases almost linearly. The reason for the almost linear increase is that the marginal cost for both annual members and day-pass users represents the majority of the total revenue generated. With the increase in the optimal revenue, more bikeshare stations are considered. However, almost none of them are in disadvantaged areas.

Even though the total revenue generated by reducing bike costs cannot match that achieved by increasing marginal cost, the number of added stations and stations in disadvantaged areas increases significantly (the right panel in **Fig. 9**). When the bike cost is reduced by 70%, the optimal solution maximizing revenue adds 107 stations (the boundary of maximum station number in this optimization), which is equal to the actual number of stations added in 2016. Among the 107 added stations, approximately 20 stations are in disadvantaged areas, which is less than half of the number (51) that is achieved by solely maximizing accessibility improvements. Moreover, **Fig. 10** shows the spatial distribution of added stations when there is no cost associated with building stations and purchasing bikes. Most of the added stations are in the north and west of Chicago, where there is a limited number of potential stations in disadvantaged areas. Comparing the right panels in **Fig. 6** and **Fig. 10** shows that there is a clear difference in the spatial distribution patterns under the two objectives. Even when bikeshare

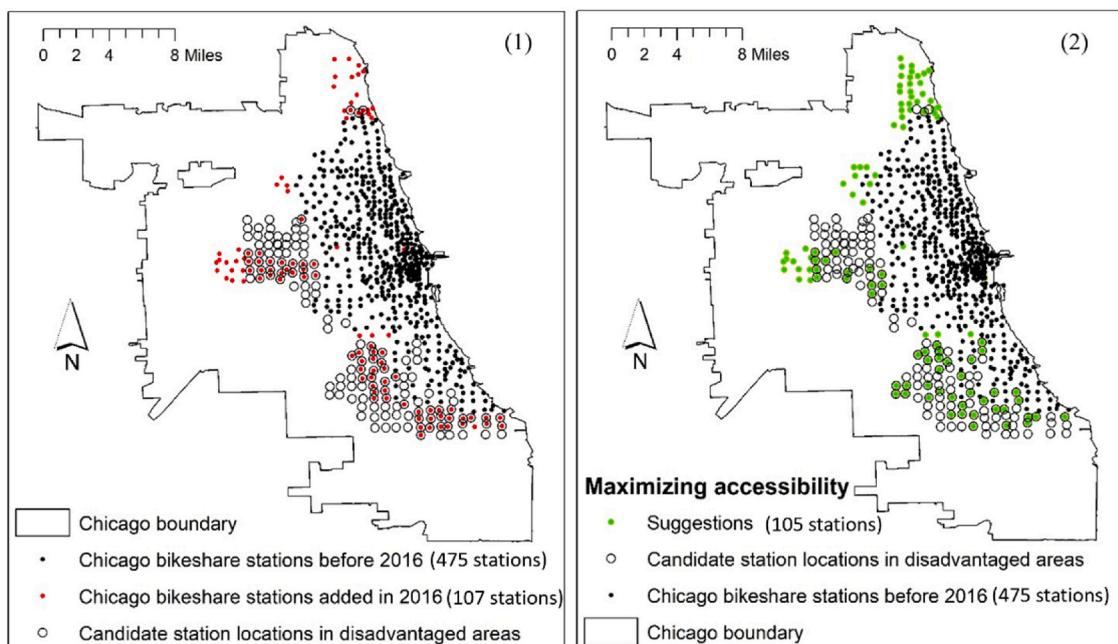


Fig. 6. (1) 2016 Network; (2) optimization results of maximizing accessibility improvements.

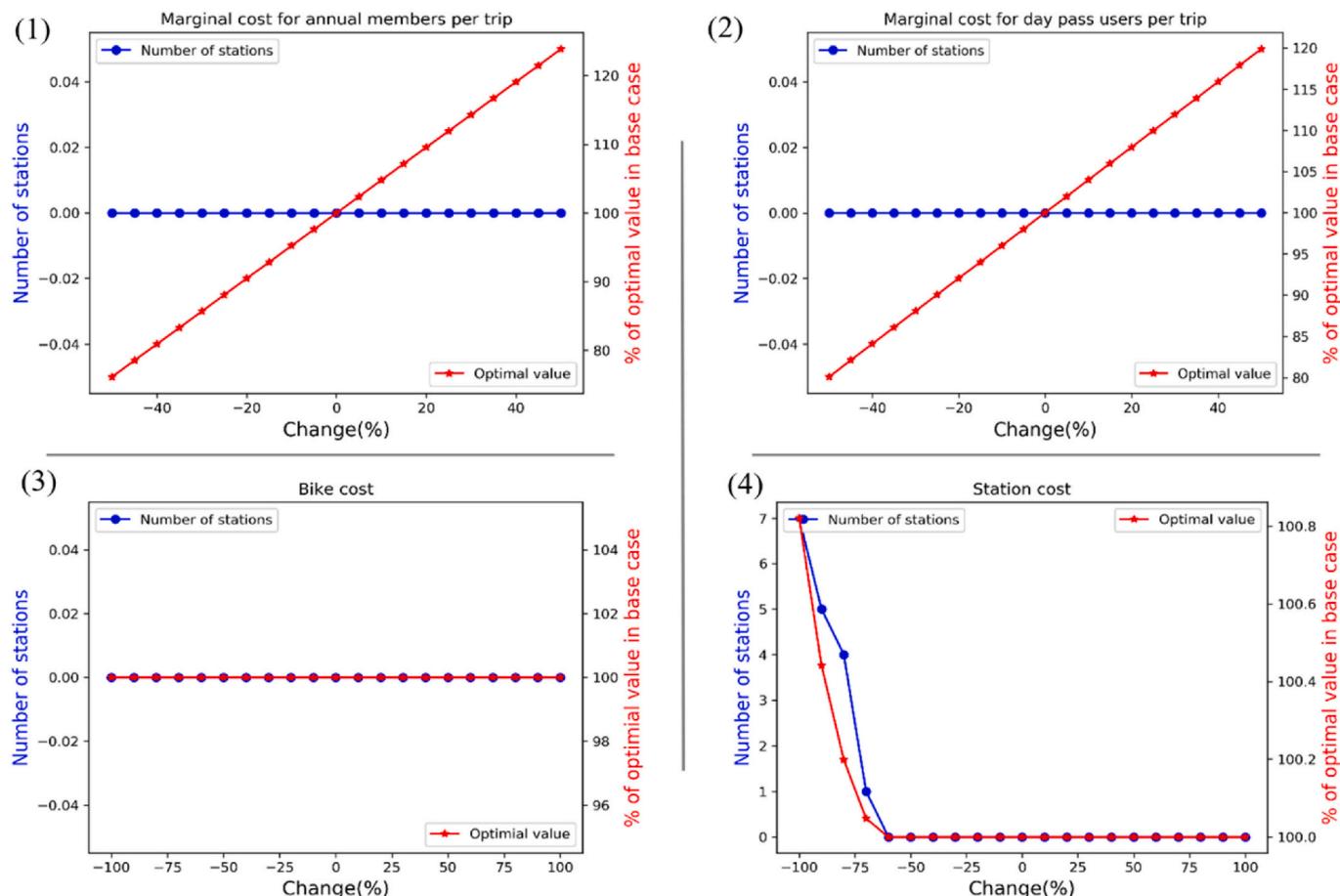


Fig. 7. Sensitivity analysis for various cost estimates.

companies do not need to worry about station and bike costs, the objective of maximizing revenue still results in fewer stations in disadvantaged areas, which may be caused by less bikeshare demand there, as noticed by Qian and Jaller (2020).

4.3. Multi-objective optimization

This section discusses the findings from the multi-objective optimization. Note in this section all of the parameters (e.g., stations and bikes cost) use the value before the sensitivity analysis. Fig. 11 shows the Pareto frontier; the horizontal axis represents the objective of maximizing accessibility improvements, while the vertical axis stands for the objective of maximizing total revenue. To show the trade-off between the two objectives, the Figure uses the absolute objective values expressed as the percentages with respect to the maximum values achieved by the single optimizations. The optimal boundary of the Pareto frontier is convex. The maximization of accessibility improvements and annual revenue cannot be achieved at the same time, as reflected in the previous single-objective analyses. As expected from the single-objective analyses, more stations in disadvantaged areas are built when shifting the objective towards maximizing accessibility, at the expense of annual revenue.

The Pareto frontier has a significant value in economics, as it measures the marginal rate of substitution between different resources. In this case, the two resources refer to annual revenue and accessibility improvements. Examining the marginal rate of substitution from revenue to accessibility improvements, the trend of the substitution rate keeps increasing with more accessibility improvements (Fig. 12). The absolute value of substitution rates may fluctuate because of different settings (parameter values in our model) in the optimization. However,

the trend of the substitution rate should be increasing, which indicates more and more sacrifice in revenue to achieve significant accessibility improvements.

In this case, a city bikeshare planner can first calculate the current accessibility improvement level by bikeshare system and then apply this Pareto frontier curve to estimate how much annual revenue will be sacrificed to achieve a specific objective of accessibility improvements. After that, they can provide the same amount of incentive (equal to the annual revenue sacrifice) to bikeshare operators and instruct them on how to spend these incentives. Based on our sensitivity analysis, these incentives should preferentially cover station and bike cost in disadvantaged areas.

5. Discussion

5.1. Insights from this optimization model

Before 2016, the Divvy bikeshare systems covered most of the central areas in Chicago, where there is more bikeshare demand (Fig. 1). Adding more stations in the suburban areas will not generate enough bikeshare trips and consequent revenue to justify the expense. However, more accessible bikeshare stations will bring significant accessibility improvements, as proved by Qian and Niemeier (2019). A private company may be reluctant to sacrifice revenue to expand a bikeshare system into disadvantaged areas. Thus, local governments need to offer incentives for private companies to overcome these financial constraints to provide more equitable bikeshare services and to integrate accessibility improvements as one of their planning and overall system goals. In general, this optimization analysis can help address current equity issues in bikeshare development from three directions.

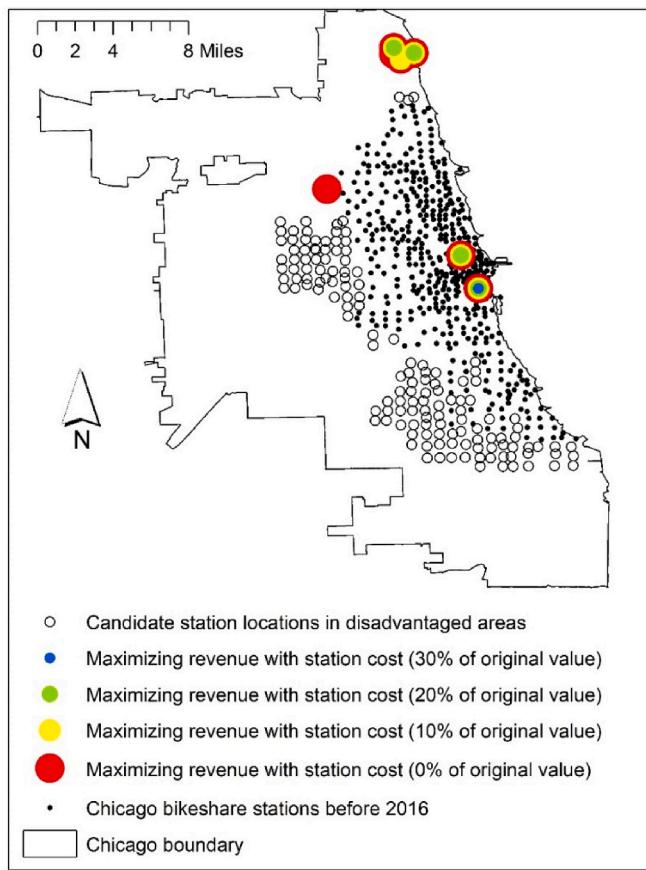


Fig. 8. Locations of added stations when the cost of building stations reduces.

First, this optimization model integrates the whole process of bike-share trip activities (including trip generation and distribution) and the outcomes (including accessibility improvements and revenue) of the

whole system, which can be closer to reality. In addition, the model can provide a guide for both local governments and bikeshare companies on how to decide where to locate new bikeshare stations to target more disadvantaged populations and to increase accessibility at the same time. As that the empirical results indicate, maximizing accessibility can

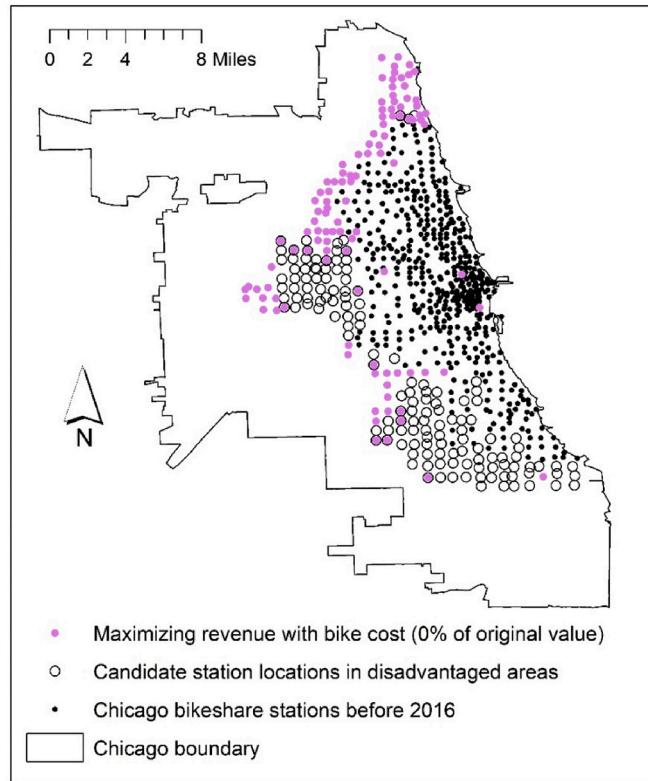


Fig. 10. Locations of added stations when the costs of building stations and bike reduce.

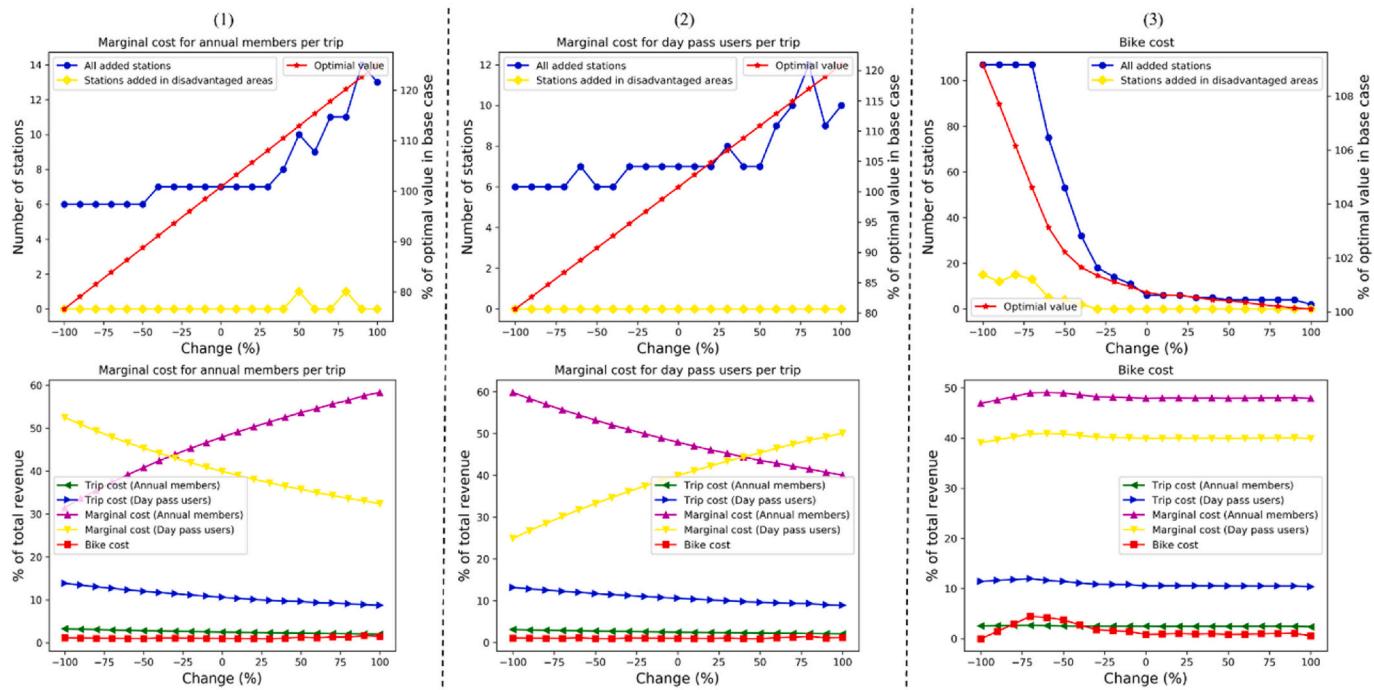


Fig. 9. Sensitivity analysis with no station cost.

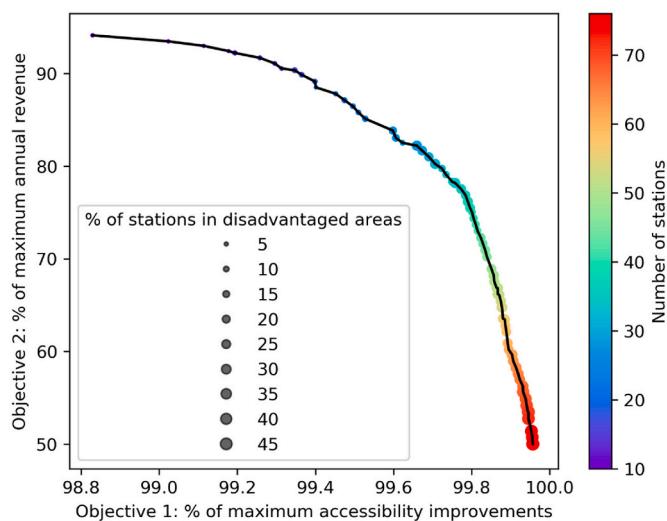


Fig. 11. The Pareto frontier of the multi-objective results.

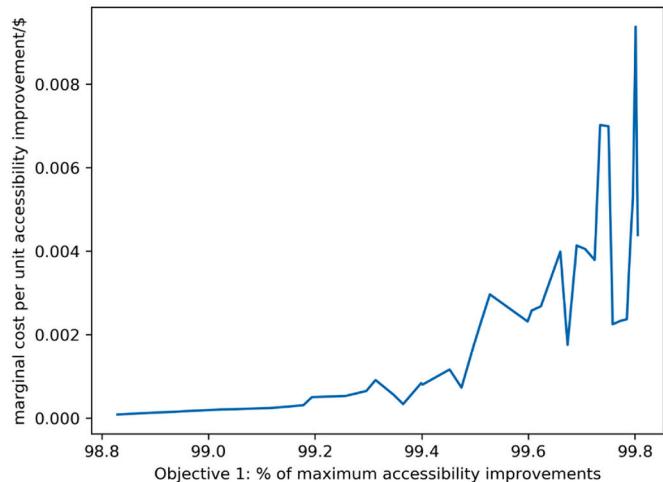


Fig. 12. The marginal cost for accessibility improvements.

shift bikeshare extension plans to target more disadvantaged areas and improve social benefits for them.

Second, the sensitivity analyses indicate where the financial incentive should be prioritized. The cost of building a new bikeshare station is the greatest barrier to adding more stations, especially in disadvantaged areas. Even though reimbursing all costs for building new stations may be infeasible in some situations, local governments could provide more incentives, e.g., using government owned curbside spaces in return for placing stations in disadvantaged areas, sharing advertising revenue, or providing rebates for stations, to promote private companies expanding their services in more disadvantaged areas. A report by Kodransky and Lewenstein (2014) has discussed similar incentives to remove barriers for low-income people to use shared mobility. Moreover, reducing bike costs could further help locate more stations in disadvantaged areas. However, increasing membership fees could hamper people's willingness and ability to join bikeshare programs, even though it can generate more revenue for the bikeshare system. Particularly for disadvantaged areas, bikeshare systems need to provide affordable membership plans, e.g., D4E in Divvy bikeshare (Motivate International, 2017b). Local governments could reimburse companies for the gap between the standard annual membership and the discounted membership fee. Thus, companies will have the motivation to add more stations, and more users from disadvantaged areas will be able to enjoy this mobility option.

Last but not least, based on the previous optimization results, the multi-objective analysis provides more insights into the results from an economic perspective, as well as policy suggestions regarding planning bikeshare for disadvantaged populations. Regarding governments' incentives, the model developed in this paper provides quantitative assessments of the financial assistance that local governments could provide to private companies that expand their services into disadvantaged areas. The Pareto frontier can establish a clear relation between incentive and accessibility improvements. For example, at an early stage, a local government could provide a lower level of incentive to require a certain level of accessibility improvements. As bikeshare systems become larger, an aggressive objective of accessibility improvements will require more financial support. The process of improving accessibility and removing access barriers is an incremental process.

5.2. Research limitation

Although the methodology proposed sheds light on the potential impacts of design considerations, there are some technical and data limitations in the approach. First, bikeshare travel patterns (from different users) are not fully understood, because of the lack of disaggregate travel data. Lacking trip travel time data, the analyses used an estimated single travel time or trip distance for an OD pair from an aggregator such as Google, instead of trip length/time distribution functions. However, the work implements correction factors from a comparison between total trip length/times and those estimated by Google. Future work could try to explicitly simulate travel times between OD pairs. Second, the optimization model does not consider, at the disaggregated level, some of the system operations and management costs (e.g., rebalancing) of bikeshare stations. And third, while the model estimates a general accessibility measure, future work could further develop a measure that considers the social benefit of accessibility for different segments of society. Despite these limitations, the model highlights the trade-offs between the private and social objectives, and identifies the key factors to prioritize when seeking a more equitable transportation system.

6. Conclusions and future work

This study explores the design of bikeshare station networks that address equity issues considering accessibility, while also considering economic sustainability. To compare network designs under two contrasting objectives (i.e., maximizing revenue and accessibility improvements), the work proposes an optimization model integrating the interaction between station siting plans and trip activities. The results highlight the need to develop decision support systems to help design and implement equitable and sustainable new mobility systems. As expected, to provide accessibility improvements to underserved communities, the network requires a larger number of stations, with most of them in disadvantaged areas. On the other hand, it is important to have an economically viable system. Consequently, local governments should cooperate with bikeshare system operators to find a balance between equity and economic feasibility and to find the mechanisms to address the needs of the operators and the community at large. This cooperation could be in the form of funding support for station building, bike purchases, and affordable membership plans. This is of particular importance when defining integrated transportation systems with competing or overlapping services.

Since this research mainly addresses the dock-based bikeshare systems, the next plan would be exploring emerging dockless bikeshare systems. The research team will study how to design an equitable operation system including bike rebalancing to maximize the overall benefits, especially for traditional underserved communities.

Declaration of competing interest

None.

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