Coupled Oscillations in Predator-Prey Systems

Debsankha Manik   Arobendo Mondal

April 19, 2010

Outline

Predator-Prey Systems

- Predator-prey system is a biological system where two species interact: one is a predator and the other is its prey.
- Due to the predator-prey relationship between the two species, the dynamics of the two populations become closely interrelated.
- Some of the interesting features arising out of this interrelation are:

Predator-Prey Systems

- Predator-prey system is a biological system where two species interact: one is a predator and the other is its prey.
- Due to the predator-prey relationship between the two species, the dynamics of the two populations become closely interrelated.
- Some of the interesting features arising out of this interrelation are:

Predator-Prey Systems

- Predator-prey system is a biological system where two species interact: one is a predator and the other is its prey.
- Due to the predator-prey relationship between the two species, the dynamics of the two populations become closely interrelated.
- Some of the interesting features arising out of this interrelation are:

Predator-Prey Systems

- If the predation rate is too high, the prey population die out exponentially and the predator population follows suit.
- If the predation rate is too low compared to the reproduction rate of the prey, then the prey population grows exponentially.
- For certain cases, both the populations reach equillibrium values.
- And most interestingly, sometimes the two populations undergo **coupled oscillations**.

Lotka-Volterra Model

The model says that the prey population (denoted by x) and the predator population (denoted by y) will evolve in time according to these two first-order, non-linear, coupled differential equations:

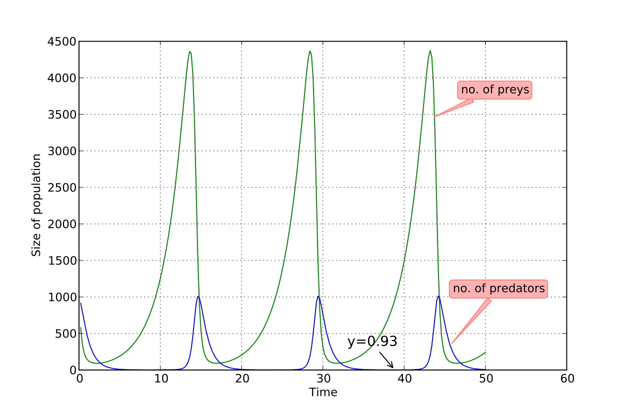
$$\frac{dx}{dt} = x(\alpha - \beta y) \tag{1}$$

$$\frac{dy}{dt} = -y(\gamma - \delta \beta x) \tag{2}$$

where $\alpha, \beta, \gamma, \delta$ are parameters representing the interaction of the two species.

Flaws of The Lotka-Volterra Model

The values α=0.08, β=0.0005, γ=0.2 and δ=0.4 yields this result:



Our Approach

- It is totally absurd to talk about a differential increase in a population.
- Besides, it is difficult to get to the physical meaning of those differential equations.
- So we decided to go for a simple rule-based stochastic model and see how the population develops under our set of rules.

The Set of Rules

1. The mice (prey) reproduce with a constant time interval between successive reproductions.
2. During a time interval $\triangle t$, the number of mice eaten by each cat (predator) is: the total no. of mice$\times \triangle t$ $\times$ a constant (Let's call it catch_rate).
3. After eating a mouse, the mass of the cat increases by: the mass of the mouse killed$\times$a constant factor (Let's call it efficiency).
4. A cat reproduces only after it's mass reaches a particular limit.
5. The number of cats dying in a certain time interval is proportional to $tan^{-1}(k \times \textit{the number of cats}) \times$ a constant factor (Let's call it extinction_rate).

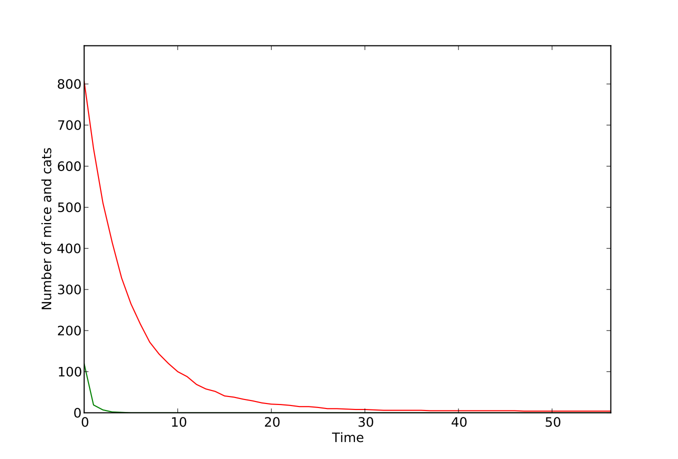
The Parameters

So our model employs four basic parameters:

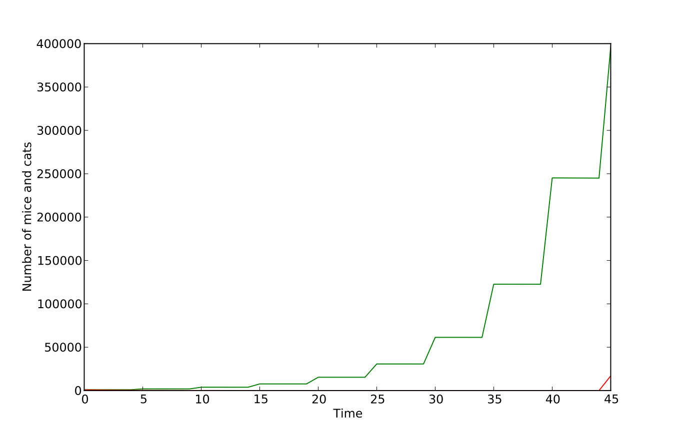
- The reproduction rate of the mice (α to 1/(the gap between two successive reproductions)).
- The catch_rate defined in rule no. 2 (α to the rate at which a cat catches mice).
- The efficiency defined in rule no. 3 (α to the fraction of the mass assimilated when a cat eats a mouse)
- The extinction_rate defined in rule no. 4 (α to the rate at which the cat population diminishes)

Therefore, depending on the values of these four parameters, the dynamics of our system should change.

Extinction of Both The Species

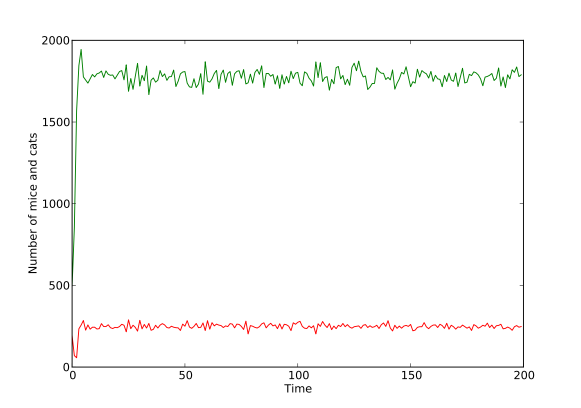


Exponential Growth

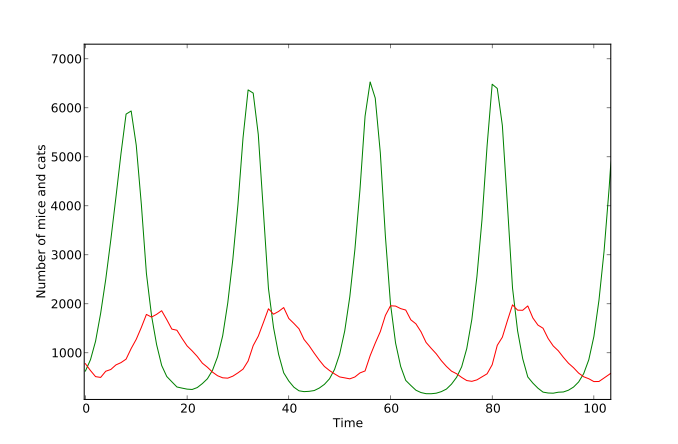


Equillibrium State

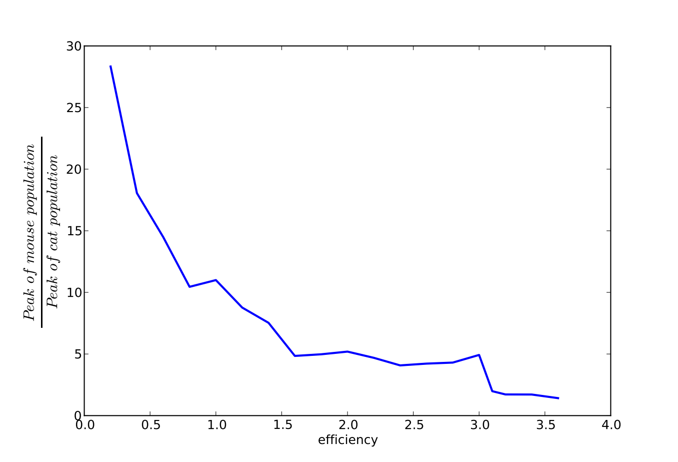
Apart from small-scale chance fluctuations, the population sizes don't change appreciably:



Lag of Predator Population



Effect of Changing Efficiency



Effect of Changing Reproduction Rate

Conclusions

In this project, we proposed a stochastic rule-based model to understand the population dynamics in a predator-prey system. By means of a set of simple rules, we managed to simulate the four major type of population characteristics:

1. Exponential dieout of both the species.
2. Exponential boom in the prey population.
3. Stability of both the populations.
4. Steady-state oscillation in both the populations.

Conclusions

Most importantly, this being a rule-based model, it is very straightforward to interpret the patterns observed in terms of physical parameters. Since this model takes into account the discrete nature of the population, we avoided the atto-fox problem, which is characteristic of the differential equation based models like the lotka-volterra model.

Future Plans

In our model, we assumed an infinite food supply for the mice (prey). We wanted to introduce competition among the prey population but couldn't do it due to lack of time. We want to do so in a future project. We would expect the cooperation of our biology department during that future endeavour.

Acknowledgement

Thanks to:

- Sudipta Tung
- Prof. Jayasri Das Sarma
- Prof. Mohit Prasad