Home ▸ New Articles ▸ Privacy Policy and Contact

Search

Lotka-Volterra equation

Lotka-Volterra equation: Encyclopedia - Lotka-Volterra equation

The Lotka-Volterra equations, also known as the predator-prey equations, are a pair of first order, non-linear, differential equations frequently used to describe the dynamics of biological systems in which two species interact, one a predator and one its prey. They were proposed independently by Alfred J. Lotka in 1925 and Vito Volterra in 1926. A classic model using the equations is of the population dynamics of the lynx and the snowshoe hare, popularised due to the extensive data collected on the relative populations of the ...

Including:

Lotka-Volterra equation, Lotka-Volterra equation - Bibliography, Lotka-Volterra equation - Dynamics of the system, Lotka-Volterra equation - Physical meanings of the equations, Lotka-Volterra equation - Population equilibrium, Lotka-Volterra equation - Predators, Lotka-Volterra equation - Prey, Lotka-Volterra equation - Solutions to the equations, Lotka-Volterra equation - Stability of the fixed points, Lotka-Volterra equation - The equations, Lotka-Volterra inter-specific competition equations, Population dynamics

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- Cultural Creatives
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- Dream Interpretation
- Dream interpreter
- Dreams
- Enlightenment
- Essential Oils

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2012 And Mayan Calendar
- Affirmations
- Alternative Health
- Astrology
- Aura
- Ayurveda
- Bentonite Clay
- Buddhism

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Lotka-Volterra equation

The **Lotka-Volterra equations**, also known as the *predator-prey equations*, are a pair of first order, non-linear, differential equations frequently used to describe the dynamics of biological systems in which two species interact, one a predator and one its prey. They were proposed independently by Alfred J. Lotka in 1925 and Vito Volterra in 1926. A classic model using the equations is of the population dynamics of the lynx and the snowshoe hare, popularised due to the extensive data collected on the relative populations of the two species by the Hudson Bay company during the 19th century.

Lotka-Volterra equation - The equations

The usual form of the equations is:

where

- y is the number of some predator (for example, dingoes);
- x is the number of its prey (for example, wallabies);
- t represents the growth of the two populations against time; and
- α, β, γ and δ are parameters representing the interaction of the two species.

Lotka-Volterra inter-specific competition equations, Population dynamics

Lotka-Volterra equation - Physical meanings of the equations

When multiplied out, the equations take a form useful for physical interpretation.

Lotka-Volterra equation - Prey

The prey equation becomes:

The prey are assumed to have an unlimited food supply, and to reproduce exponentially unless subject to predation; this exponential growth is represented in equation above by the term αx. The rate of predation upon the prey is assumed to be proportional to the rate at which the predators and the prey meet; this is represented above by βxy. If either x or y is zero then there can be no predation.

With these two terms the equation above can be interpreted as: the change in the prey's numbers is given by its own growth minus the rate at which it is preyed upon.

Lotka-Volterra equation - Predators

The predator equation becomes:

In this equation, δxy represents the growth of the predator population.

(Note the similarity to the predation rate; however, a different constant is used as the rate at which the predator population grows is not necessarily equal to the rate at which it consumes the prey). γy represents the natural death of the predators; it is an exponential decay.

Hence the equation represents the change in the predator population as the growth of the predator population, minus natural death.

Lotka-Volterra equation - Solutions to the equations

The equations have periodic solutions which do not have a simple expression in terms of the usual trigonometric functions. However, an approximate linearised solution yields a simple harmonic motion with the population of predators leading that of prey by 90°.

Lotka-Volterra equation - Dynamics of the system

In the model system, the predators thrive when there are plentiful prey but, ultimately, outstrip their food supply and decline. As the predator population is low the prey population will increase again. These dynamics continue in a cycle of growth and decline.

Lotka-Volterra equation - Population equilibrium

Population equilibrium occurs in the model when neither of the population levels are changing, i.e. when both of the differential equations are equal to 0.

$x(\alpha - \beta y) = 0 - y(\gamma - \delta x) = 0$
When solved for x and y the above system of equations yields

$\{y = 0, x = 0\}$
and

,
hence there are two equilibria.

The first solution effectively represents the extinction of both species. If both populations are at 0, then they will continue to be so indefinitely. The second solution represents a fixed point at which both populations sustain their current, non-zero numbers, and, in the simplified model, do so indefinitely. The levels of population at which this equilibrium is achieved depends on the chosen values of the parameters, α, β, γ, and δ.

Lotka-Volterra equation - Stability of the fixed points

The stability of the fixed points can be determined by performing a linearization using partial derivatives.

The Jacobian matrix of the predator-prey model is

When evaluated at the steady state of (0,0) the Jacobian matrix J becomes

The eigenvalues of this matrix are

$\lambda_1 = \alpha, \lambda_2 = -\gamma$
In the model α and γ are always greater than zero, and as such the sign of the eigenvalues above will always differ. Hence the fixed point at the origin is a saddle point.

The stability of this fixed point is of importance. If it was stable, non-zero populations might be attracted towards it, and as such the dynamics of the system might lead towards the extinction of both species for many cases of initial population levels. However, as the fixed point at the origin is a saddle point, and hence unstable, we find that the extinction of both species is difficult in the model.

Evaluating J at the second fixed point we get

The eigenvalues of this matrix are

As the eigenvalues are both complex, this fixed point is a focus. The real part is zero in both cases so more specifically it is a centre. This means that the levels of the predator and prey populations cycle, and oscillate around this fixed point.

See also

- Lotka-Volterra inter-specific competition equations
- Population dynamics

Lotka-Volterra equation - Bibliography

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Main Page
for
Lotka-volterra Equation

Index of Articles
related to
Lotka-volterra Equation

« Back