Bifurcations in Dynamical systems with singularities

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DEFINITION

A simple piecewise smooth function:

$$f(x) = \begin{cases} g(x) & : x < \mu \\ h(x) & : x > \mu \end{cases}$$

Continuity demands $g(\mu) = h(\mu)$.

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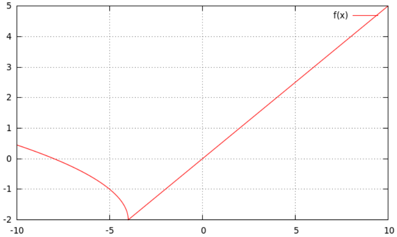
Orger of singularity:
The power of the leading order term in the power series expansion of

$$g(x) - h(x)$$

around $x = \mu$

A system with singularity:

Figure :



A system with singularity:

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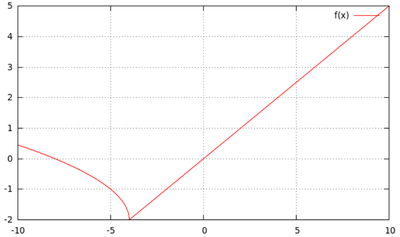


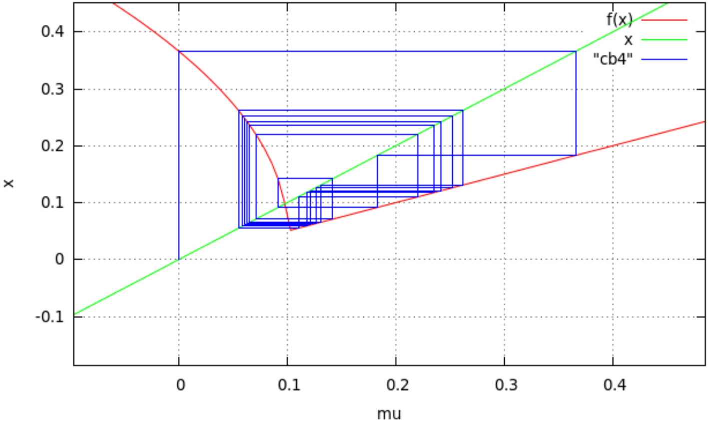
$$f(x) = \begin{cases} \sqrt(\mu - x) - \mu\nu & : x < \mu \\ \nu x & : x > \mu \end{cases}$$

$$g(\mu - x) - h(\mu - x) = \sqrt(x) + O(x)$$

This is square root singularity.

Cobweb Diagrams

Figure : Cobweb Diagram



Attractor must be bounded.

For $\mu < 0$, the stable fixed point is at $x = 0$.
Border collision occurs at $\mu = 0$.
What happens when μ crosses 0 to positive values?

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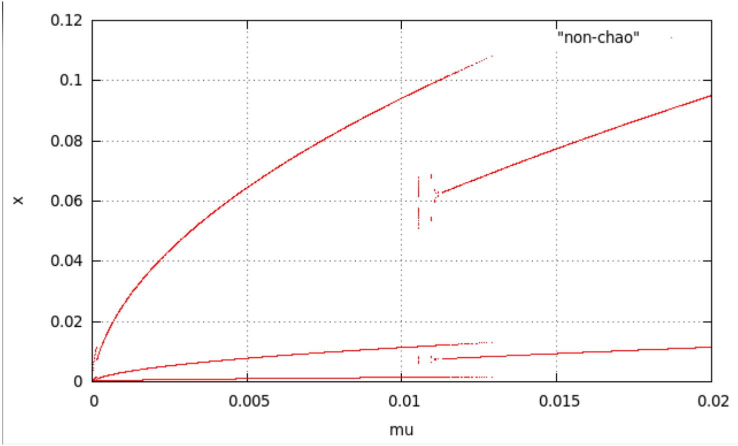
What happens when μ crosses 0 to positive values?

1. $\frac{2}{3} < \nu < 1$: Immediate onset of chaos.
2. $\frac{1}{4} < \nu < \frac{2}{3}$: There is a period-adding cascade as mu decreases, with infinite-period orbits at $\mu = 0$. Moreover, chaotic regions between each $periodm$ and $periodm + 1$ orbits.
3. $0 < \nu < \frac{1}{4}$: Periodic orbits.

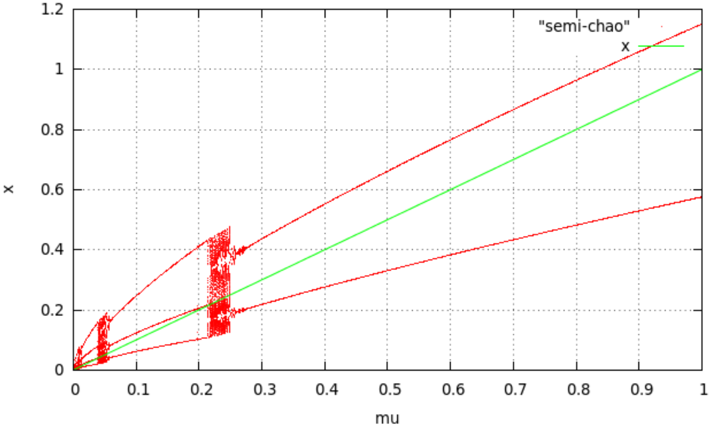
CASE 1: $\nu = 0.12$

Figure :



CASE 2: $\nu = 0.5$

Figure :



CASE 3: $\nu = 0.8$

Figure :