Chapter 1

Introduction

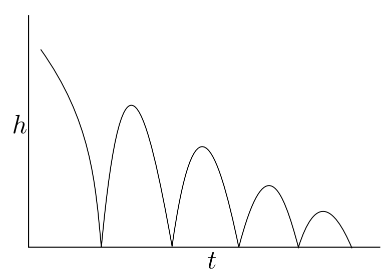
1.1 Overview

Theory of dynamical systems enables us to gain valuable insights into many problems. By using the powerful tools it provides, in many cases we can explain how a system will behave when the actual equations governing its evolution are too difficult to solve analytically. This approach has been proved time and again to be extremely useful in a diverse range of areas: fluid flows, electrical circuits, ecological systems etc.

However, conventional treatments of the subject generally places the demand that the systems be describable in terms of *smooth* functions of the dynamical variables. There are valid reasons for doing so. For a system's stability analysis, the Jacobian plays a role of paramount importance. But its existence cannot be guaranteed everywhere in the phase space for non smooth systems.

Unfortunately, many systems we need to deal with regularly are non smooth: electrical circuits involving switches, systems exhibiting sliding or chattering motion, impacting systems etc. One subclass of these systems is called *piecewise smooth* systems. The equation governing the evolution

Figure 1.1: Trajectory of a bouncing ball : a piecewise smooth system



of these system changes the moment the system co-ordinates cross what is known as a "switching manifold", which is a surface with dimension lower than that of the phase space. But between any two of those switches, the system evolves smoothly.

Chapter 2

Our system