Investigating Piecewise Smooth Hybrid Systems

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HYBRID SYSTEMS

These are systems described partly by differential equations and partly by maps: a *hybrid* of continuous time and discrete time systems.

Examples:

- A bell.
- A typewriter.
- Walking motion.

Hybrid Systems: Mathematical Definition

definition

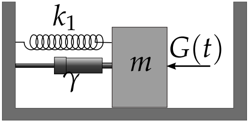
A system described by a set of ODE's and a set of **reset maps**:

$$\dot{x} = F_i(x, \mu), \quad \forall x \in S_i \tag{1}$$
$$x \mapsto R_{ij}(x, \mu), \quad \forall x \in \Sigma_{ij} = \bar{S}_i \cup \bar{S}_j \tag{2}$$

is called a piecewise smooth hybrid system if all the R_i's, F_i's as well as the associated flows φ_i's are smooth in both x and the parameter μ in the appropriate regimes.

EXAMPLE: OSCILLATOR WITH HARD IMPACTS

Figure: Hard impacting oscillator



$$m\ddot{x} = -\gamma\dot{x} - k_1 x + G(t) \qquad \text{for } x < \sigma \qquad (3)$$

$$(x, v) \mapsto (x, -rv) \qquad \text{for } x = \sigma \qquad (4)$$

r is the coefficient of restitution, which is 1 for perfectly elastic collisions.

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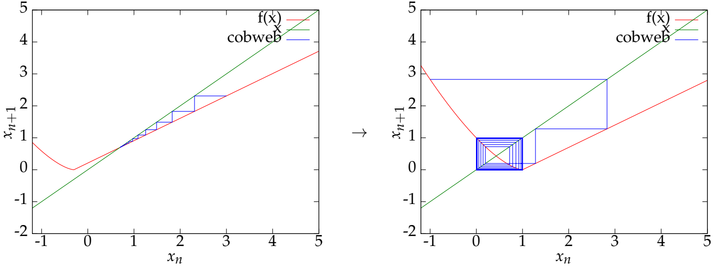
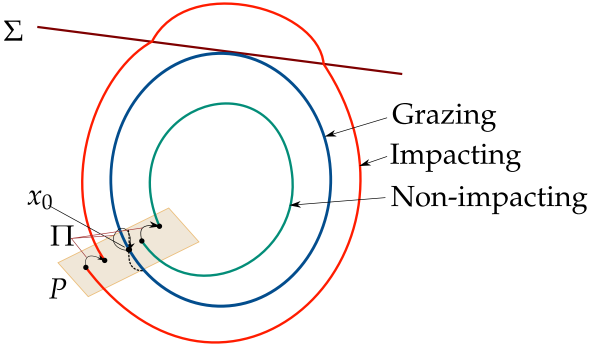


Grazing bifurcation of limit cycles

Figure: Grazing orbit



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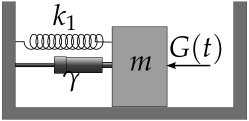
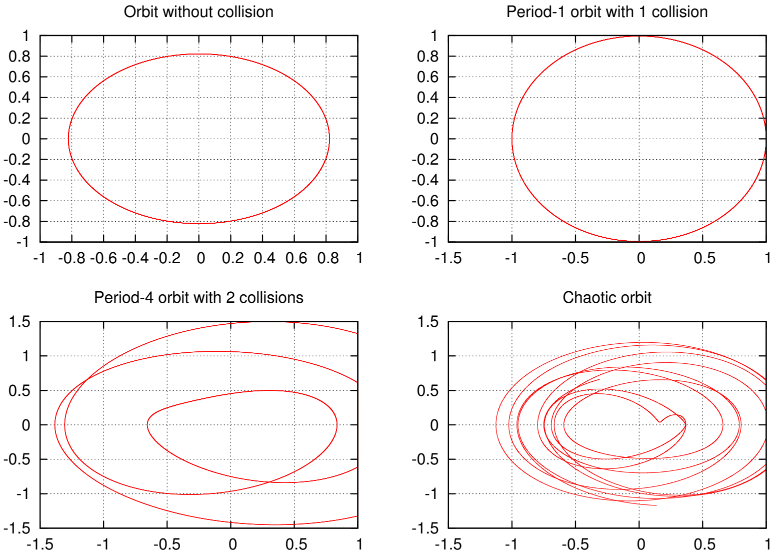
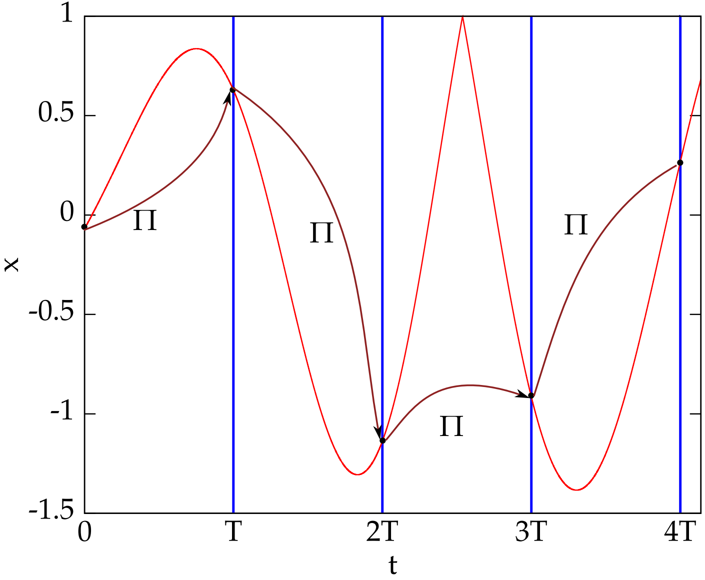


$$
\begin{aligned}
m\ddot{x} &= -\gamma\dot{x} - \omega_0^2 x + F\cos\omega t && \text{for } x < \sigma && (5)\\
(x, v) &\mapsto (x, -rv) && \text{for } x = \sigma && (6)
\end{aligned}
$$

A few possible trajectories



STROBOSCOPIC POINCARÉ MAP



Kundu and Banerjee [?] investigated the grazing bifurcations in this system by deriving an approximate analytical expression for the stroboscopic Poincaré map near grazing.

Their results:

- Unless $n = \frac{2\omega_g}{\omega_{forcing}} \in \mathbb{N}$, chaos [1] immediately follows grazing of the steady state orbit.
- If $n \in \mathbb{N}$, no chaos after grazing.

Experimental data:

- Chaos vanishes not only *at* $n \in \mathbb{N}$, but at small neighbourhoods around each $n \in \mathbb{N}$

[1] $\omega_g = \sqrt{\omega_0^2 - \frac{\gamma^2}{4}}$