ITP20001/ECE 20010 Data Structures

Chapter 5

- *introduction*
- *tree, binary tree, binary search tree*
- *heaps data structure*
  - *complete binary tree*
  - *priority queues (Chapter 9)*
  - *binary heap and min-heap*
  - *max-heap demo*
  - *max-heap implementation*
  - ***heap sort (Chapter 7)***


Hyphaene Compressa - Doum Palm
© Shlomit Pinter

Basic plan for in-place sort

- **1st Pass:** Create max-heap with all **N** keys.
- **2nd Pass:**

Basic plan for in-place sort

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										N
S	O	R	T	E	X	A	M	P	L	E
1	2	3	4	5	6	7	8	9	10	11

Basic plan for in-place sort

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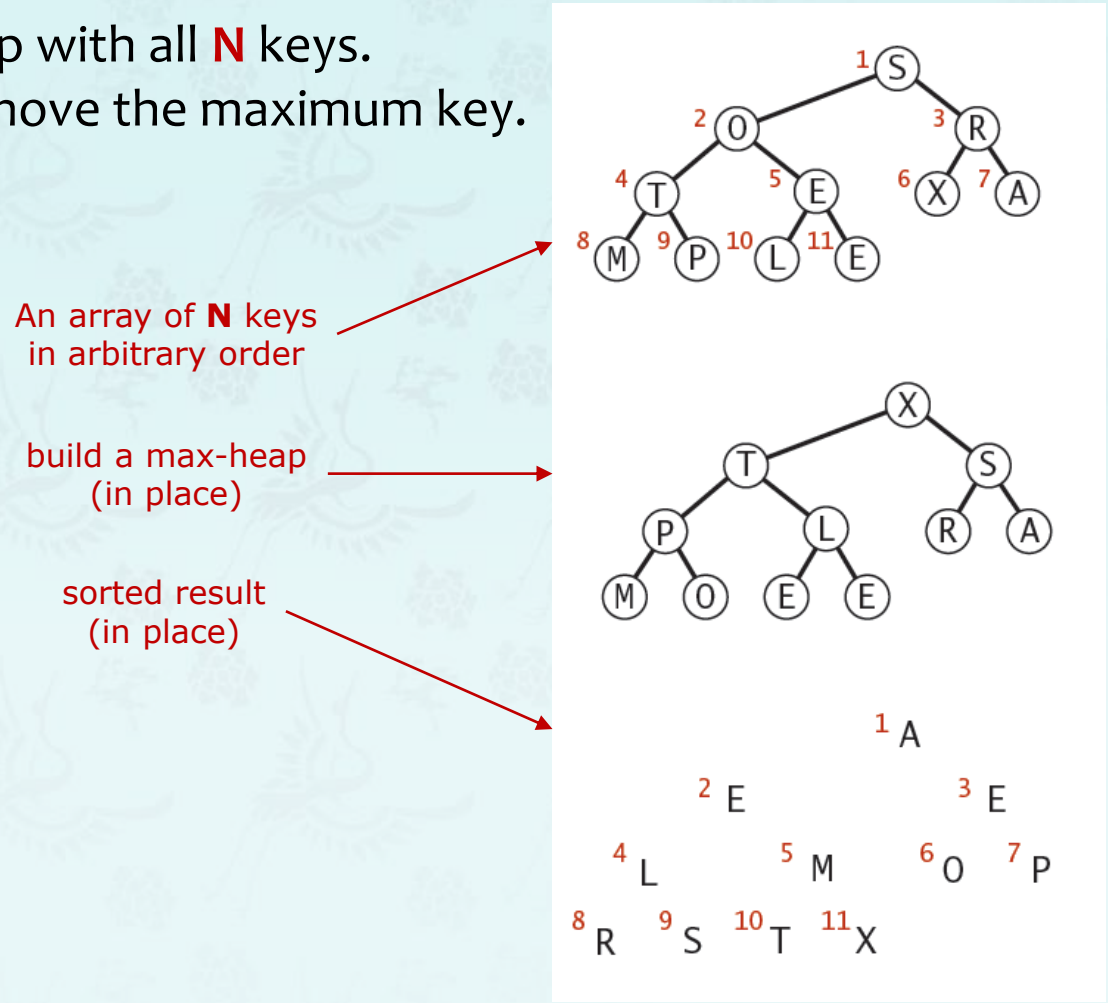


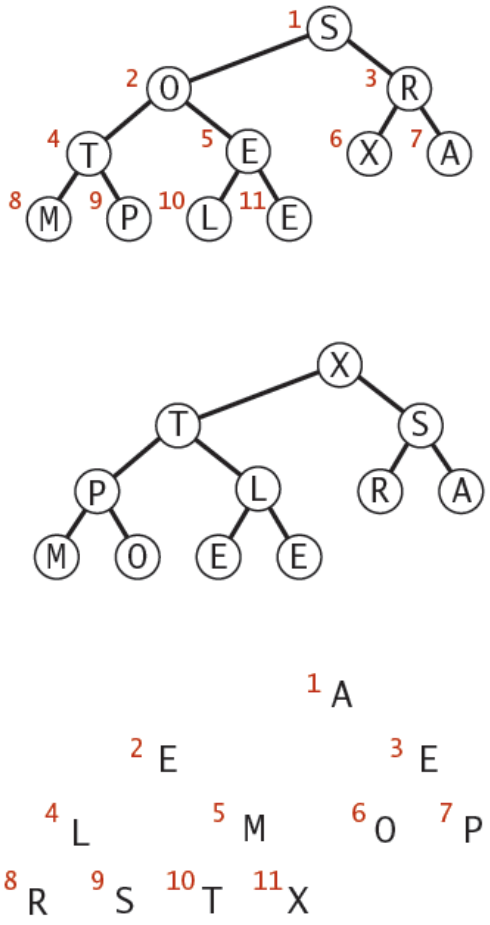
Basic plan for in-place sort

- **1st Pass:** Create max-heap with all **N** keys.
- **2nd Pass:** Repeatedly remove the maximum key.

An array of **N** keys in arbitrary order

build a max-heap (in place)

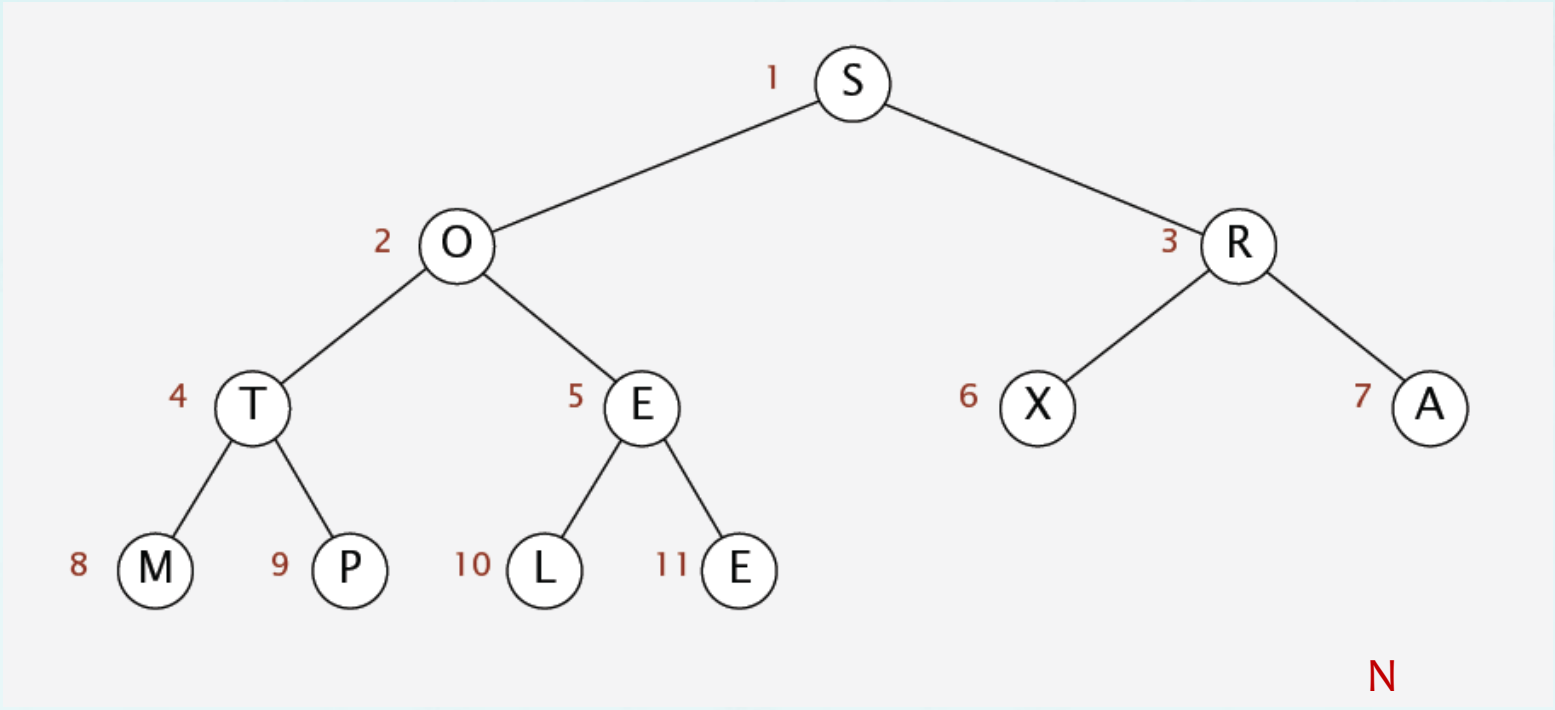
sorted result (in place)



A	E	E	L	N	O	P	R	S	T	X
1	2	3	4	5	6	7	8	9	10	11

- **1st Pass: Heap construction(heapify)**
  Build max heap using bottom-up method.
  (we assume array entries are indexed from 1 to N.)

array in arbitrary order



N

S	O	R	T	E	X	A	M	P	L	E
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- **1st Pass: Heap construction(heapify)**
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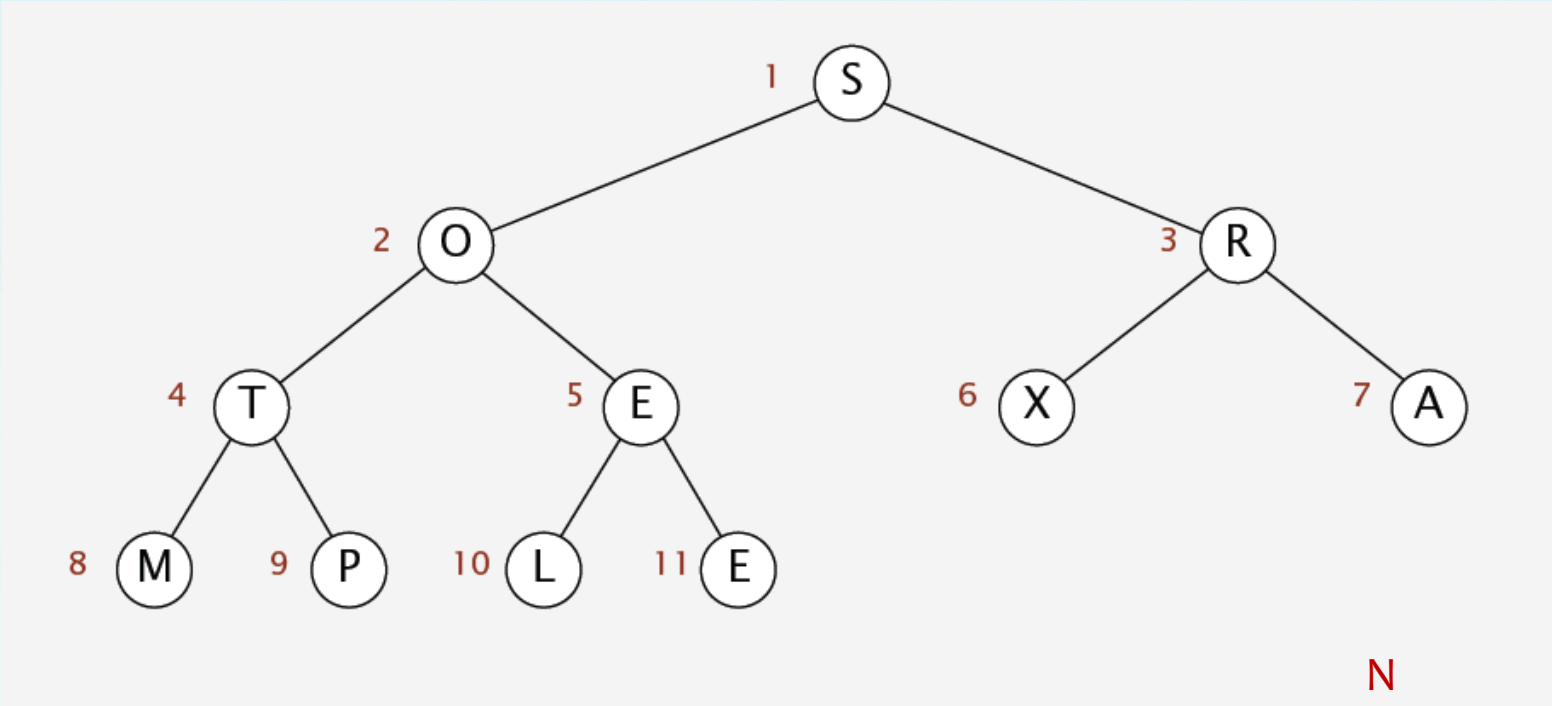
Where should we start from?

array in arbitrary order



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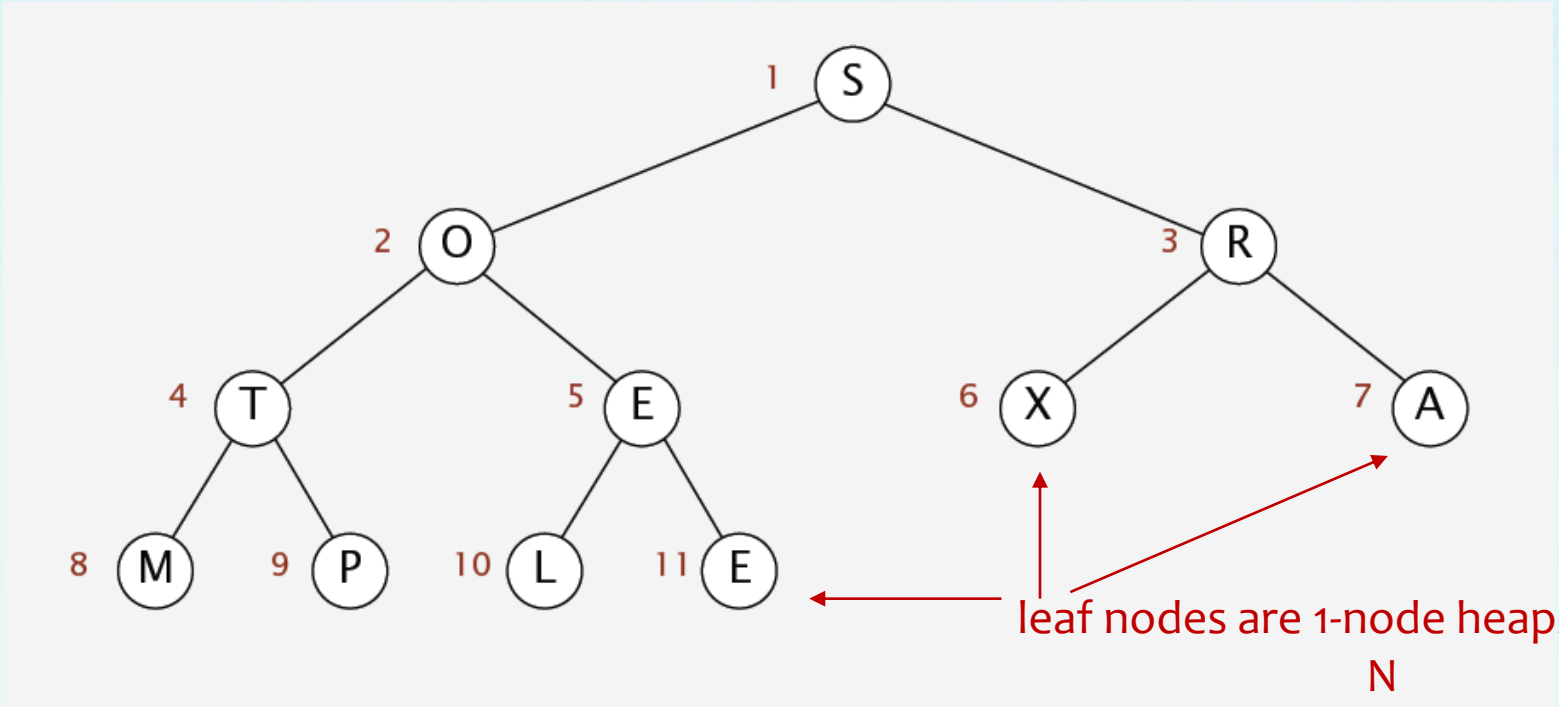

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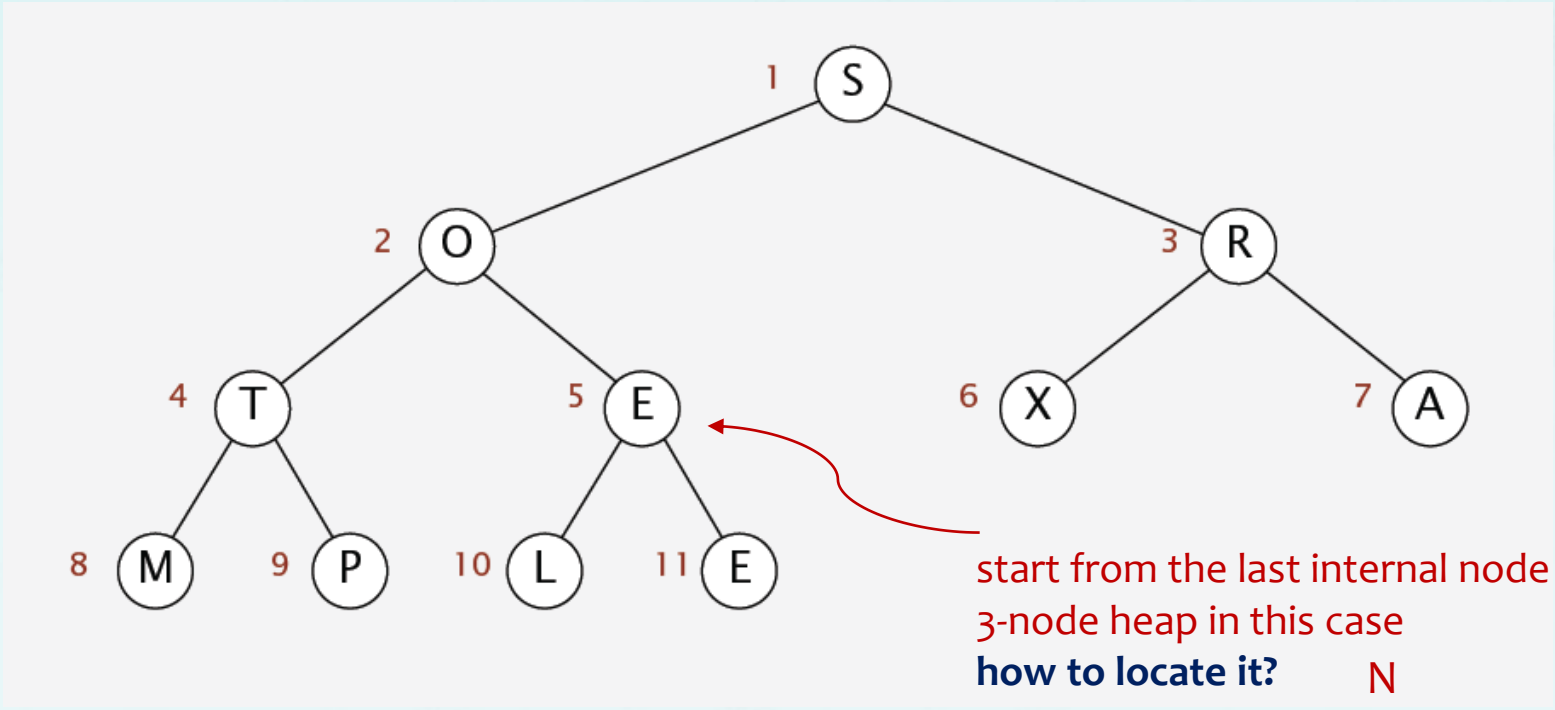

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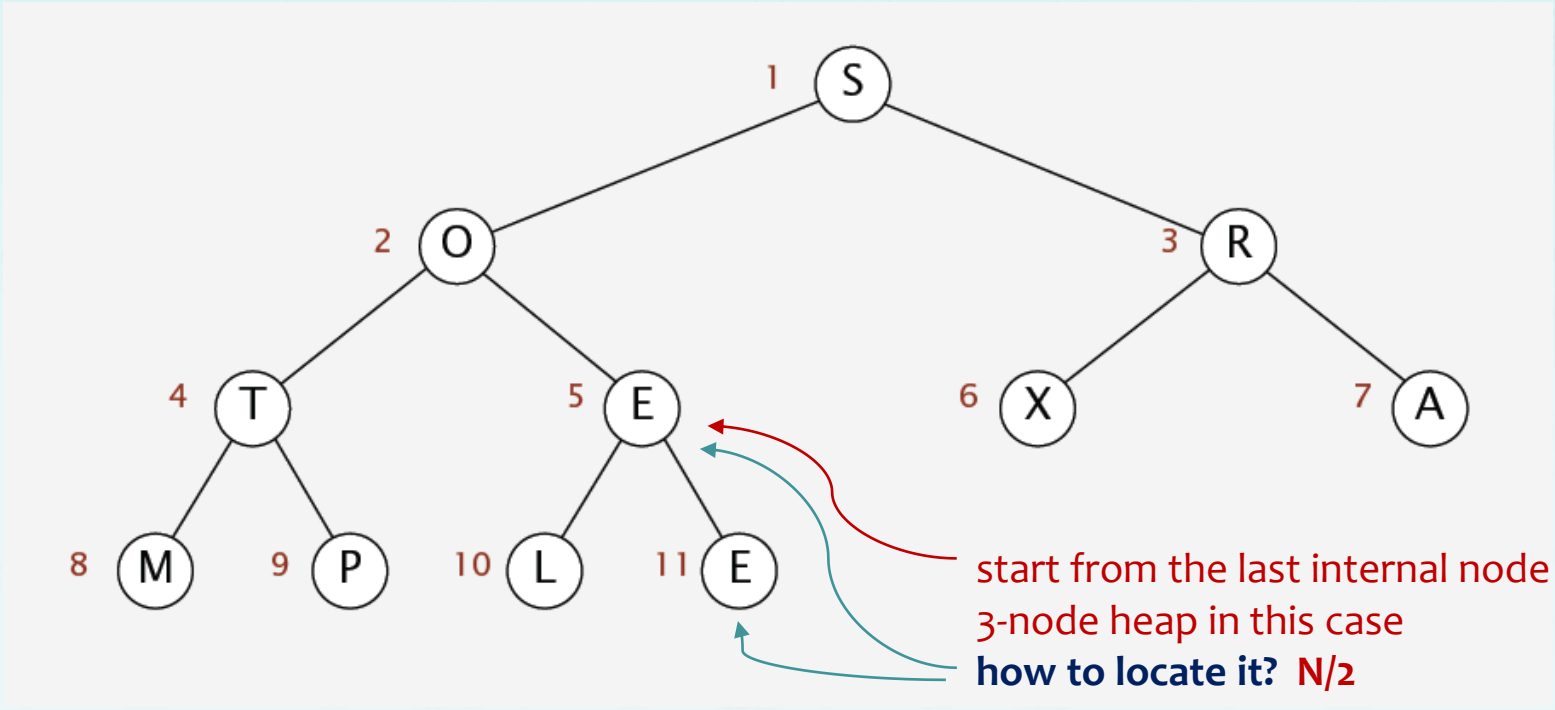
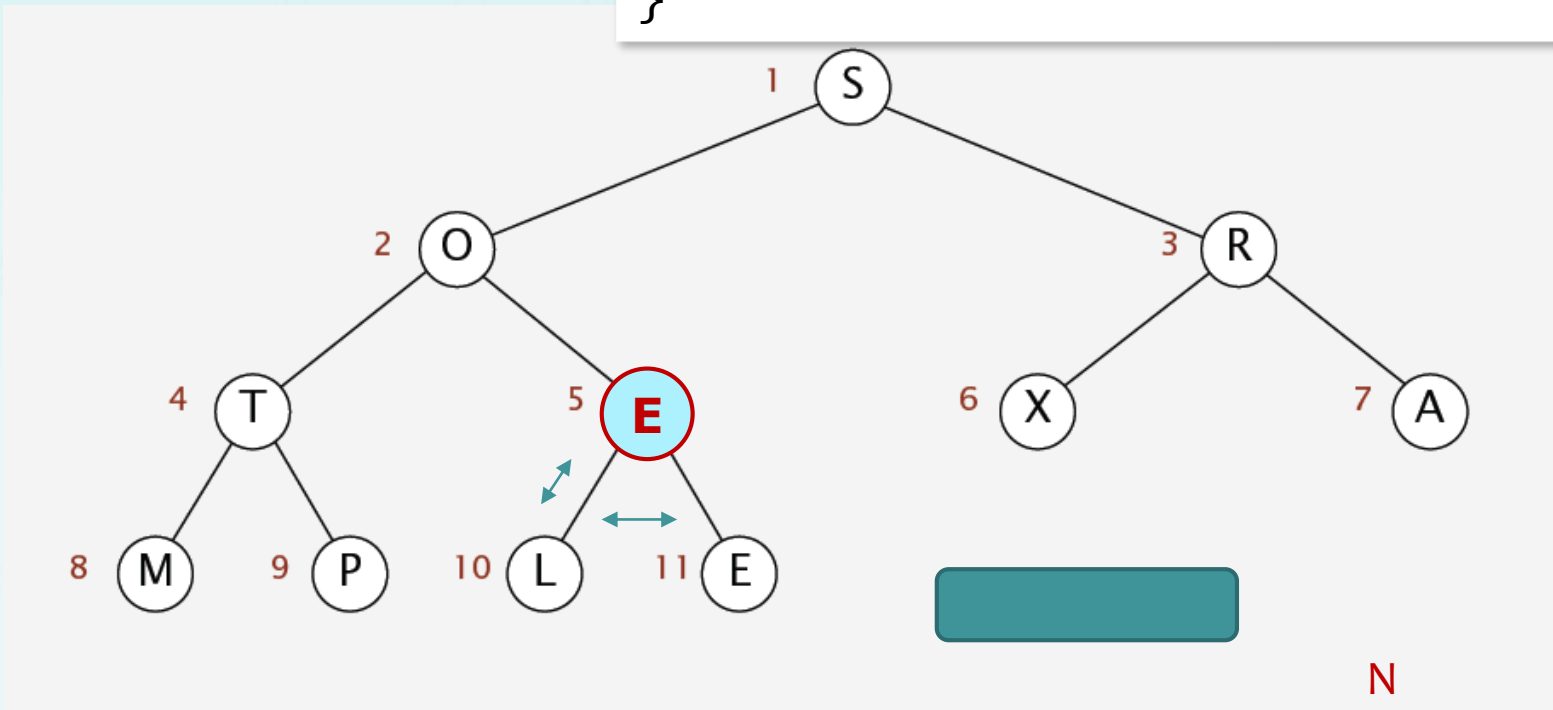


S	O	R	T	E	X	A	M	P	L	E
1	2	3	4	5	6	7	8	9	10	11

Chapter 7.6 Heap sort

- **1st Pass: Heap construction(**
  Build max heap using bottor
  (we assume array entries are indexe

```
void sink(heap h, int k) {
   while (2 * k <= h->N) {
      int j = 2 * k;
      if (j < h->N && less(h, j, j + 1)) j++;
      if (!less(h, k, j)) break;
      swap(h, k, j);
      k = j;
   }
}
```



S	O	R	T	E	X	A	M	P	L	E
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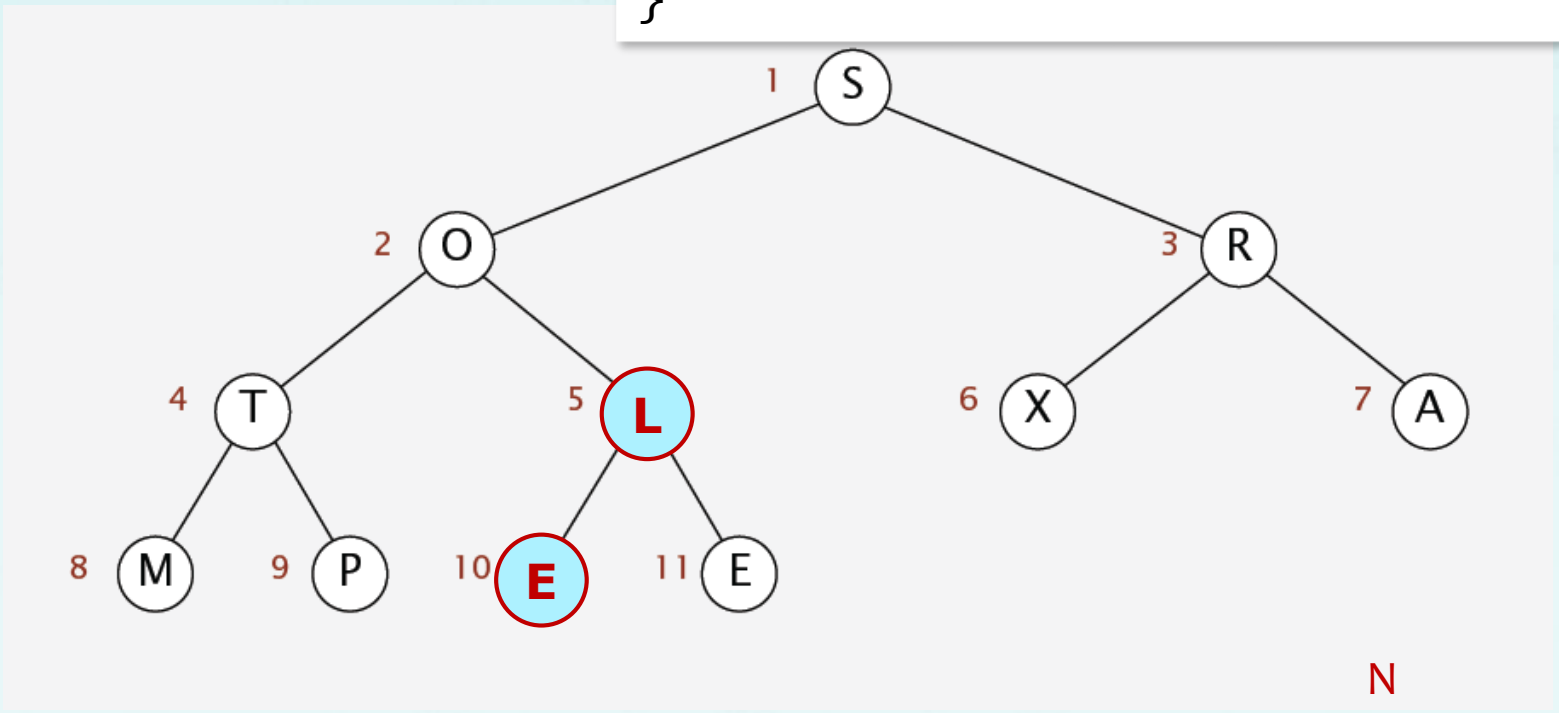

S	O	R	T	L	X	A	M	P	E	E
1	2	3	4	5	6	7	8	9	10	11

- **1st Pass: Heap construction(heapify)**
  Build max heap using bottom-up method.
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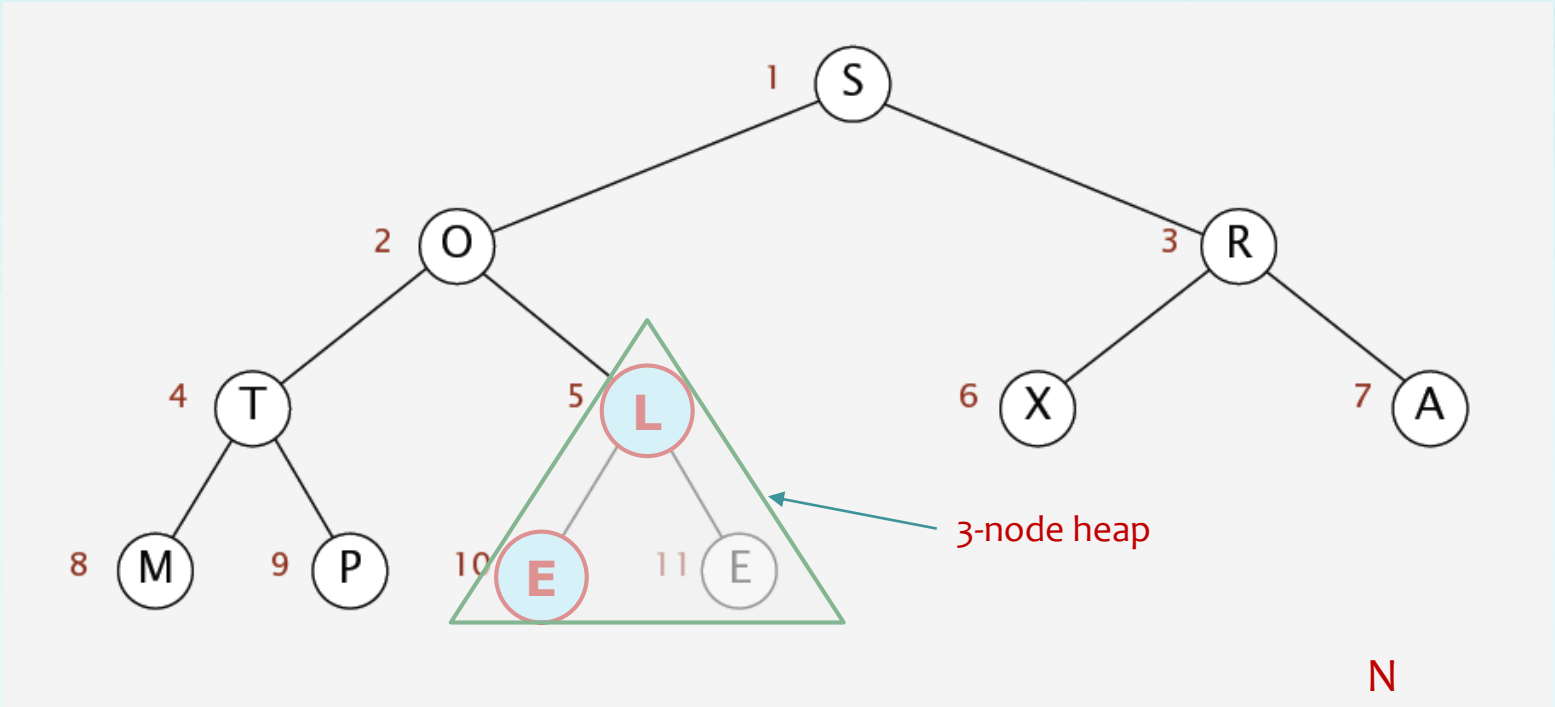
sink 5



N

S	O	R	T	**L**	X	A	M	P	**E**	E
1	2	3	4	5	6	7	8	9	10	11

Chapter 7.6 Heap sort

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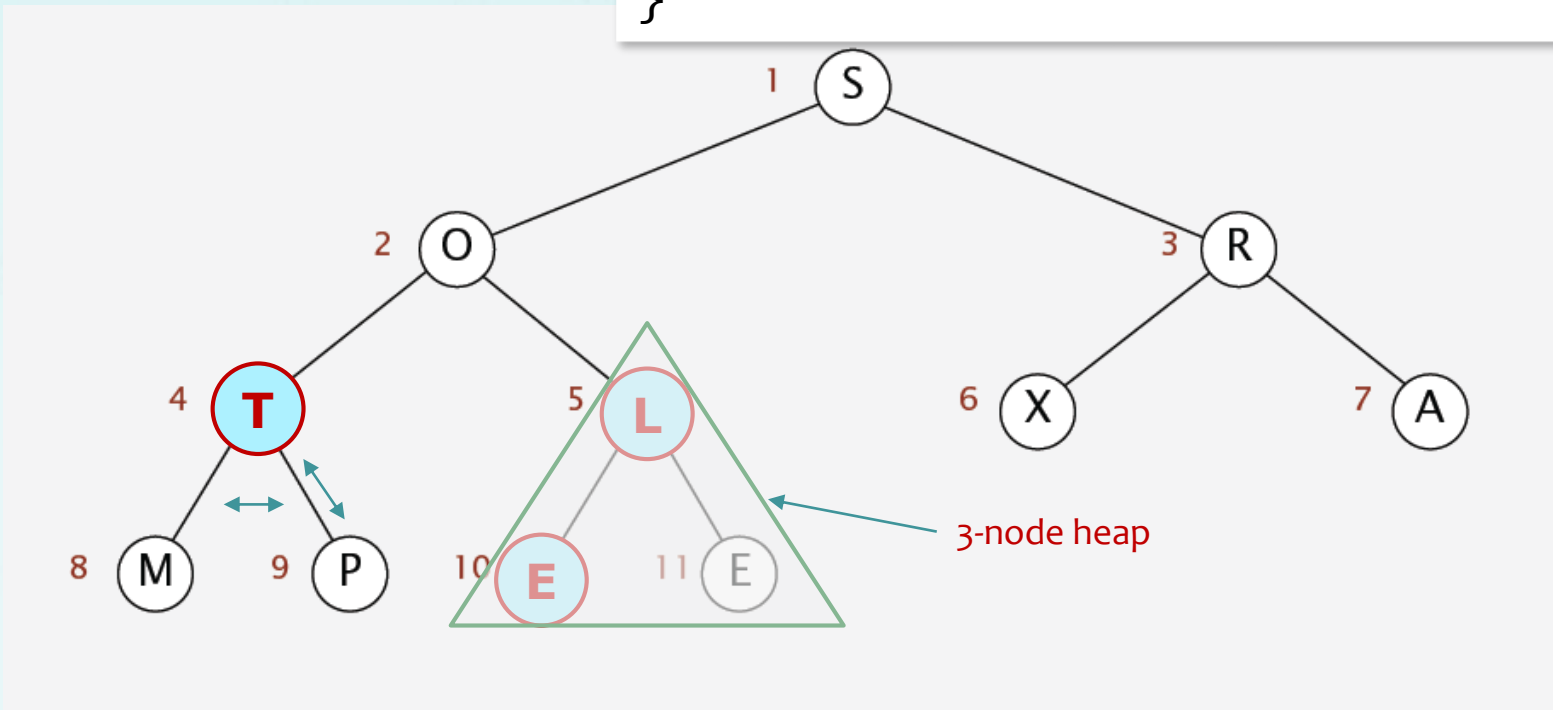

S	O	R	T	L	X	A	M	P	E	E
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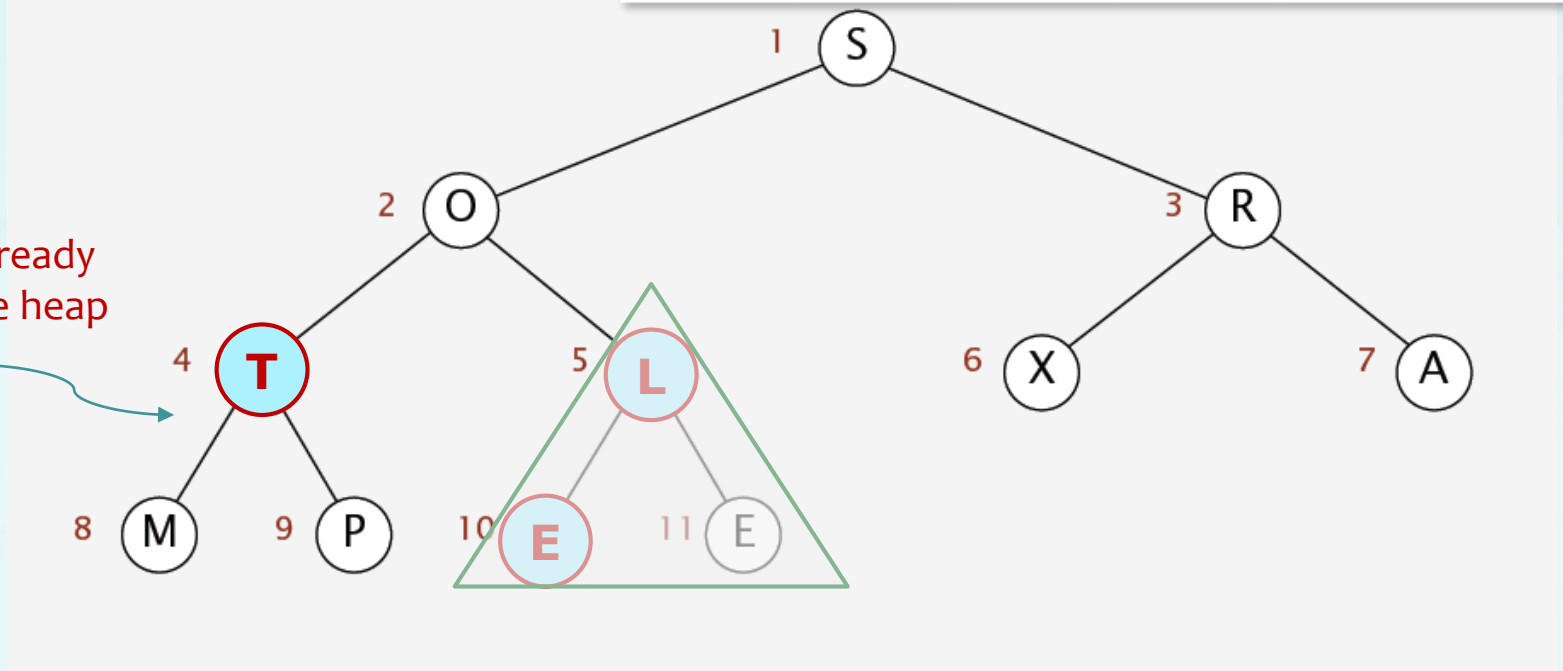
      k = j;
   }
}
```

sink 4

It's already
3-node heap



S	O	R	T	L	X	A	M	P	E	E
1	2	3	4	5	6	7	8	9	10	11

- **1st Pass: Heap construction(heapify)**
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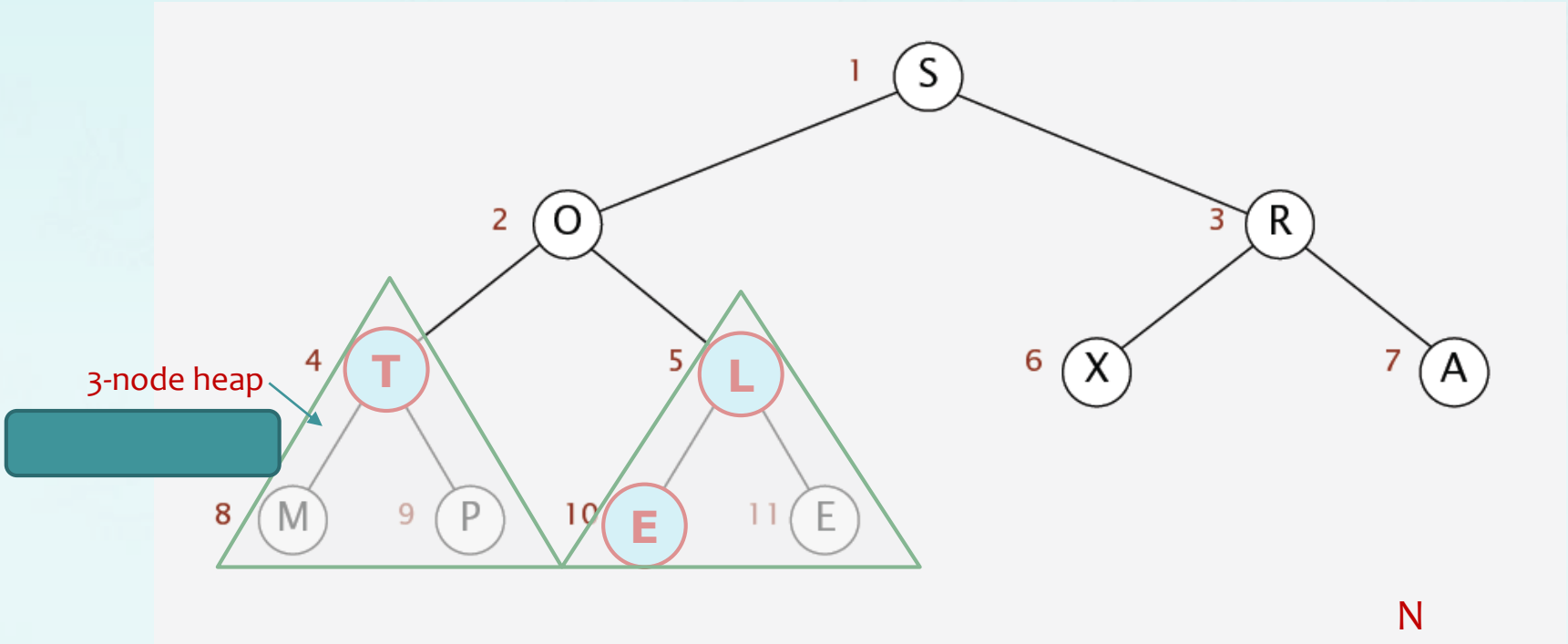
sink 4



S	O	R	**T**	**L**	X	A	M	P	**E**	E
1	2	3	4	5	6	7	8	9	10	11

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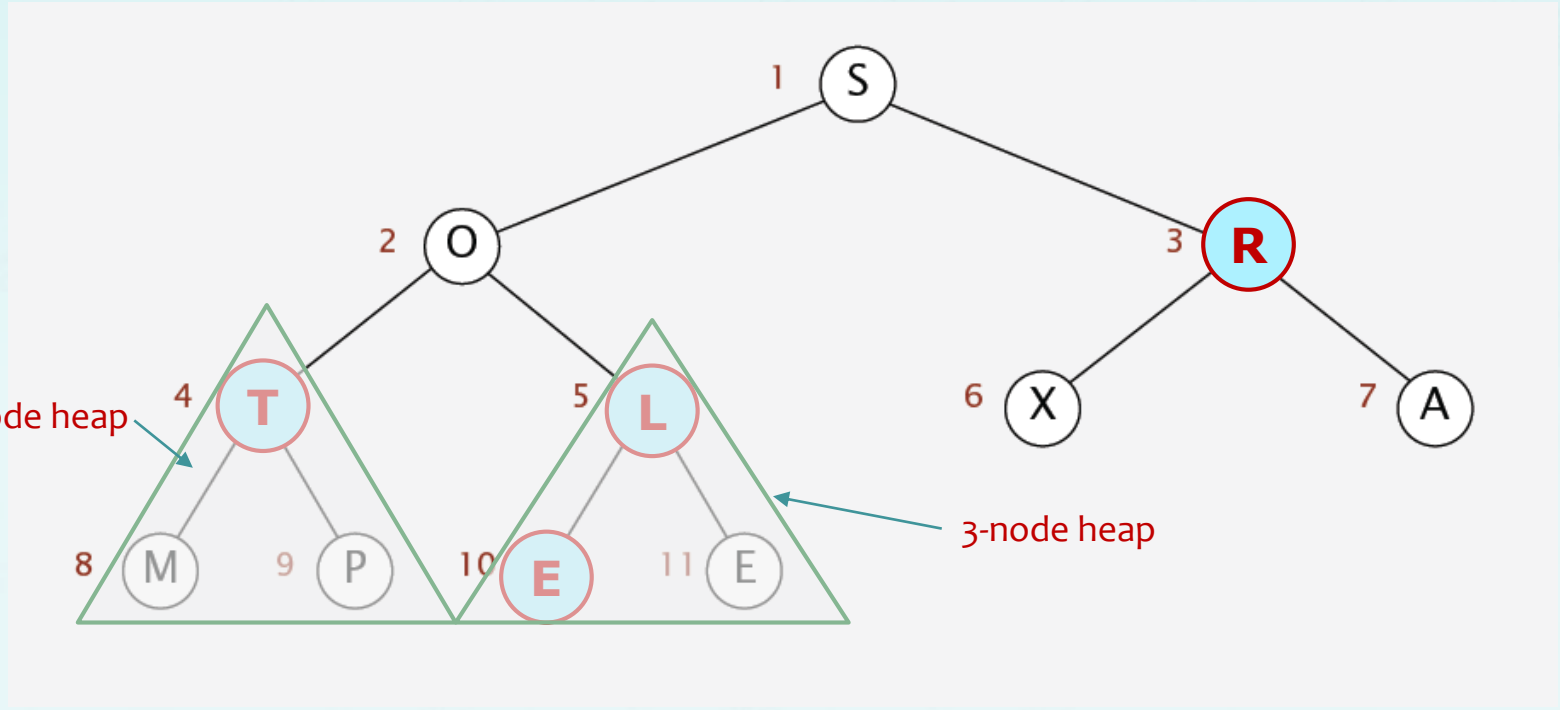
sink 3



S	O	R	**T**	**L**	X	A	M	P	**E**	E
1	2	3	4	5	6	7	8	9	10	11

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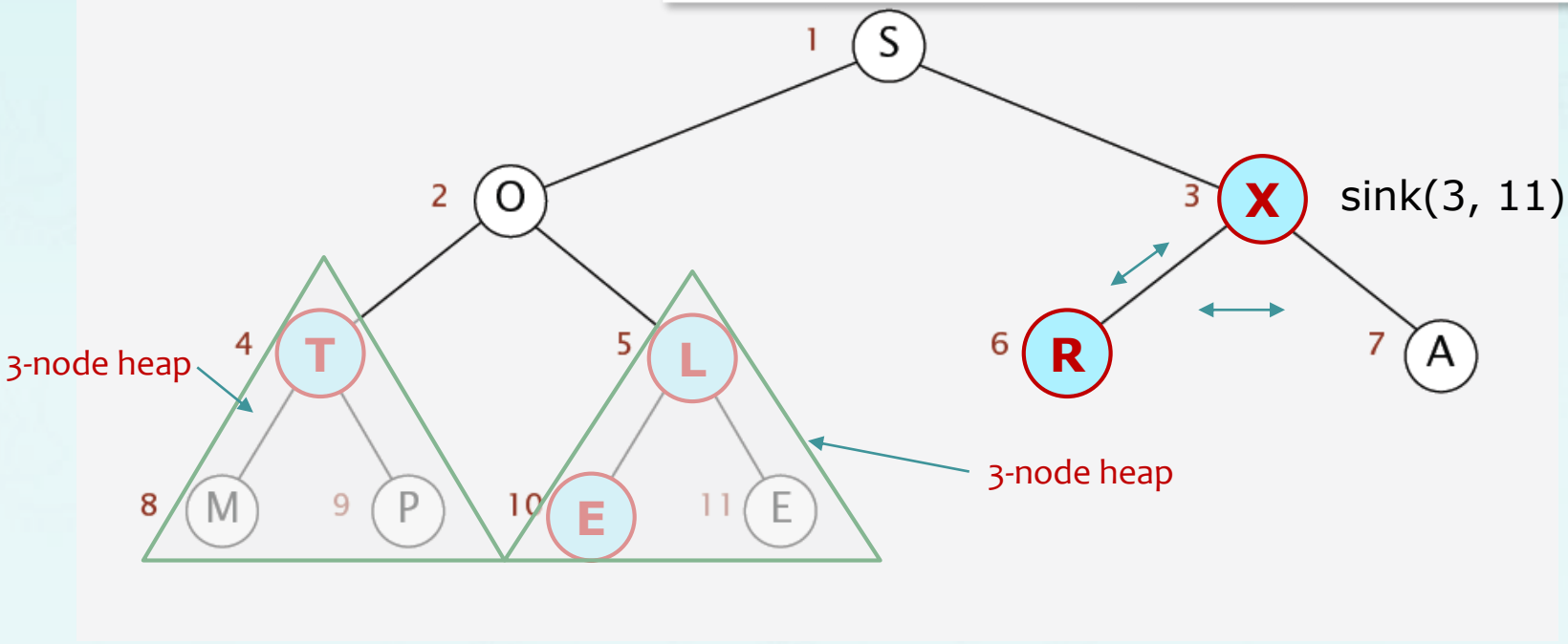
sink 3



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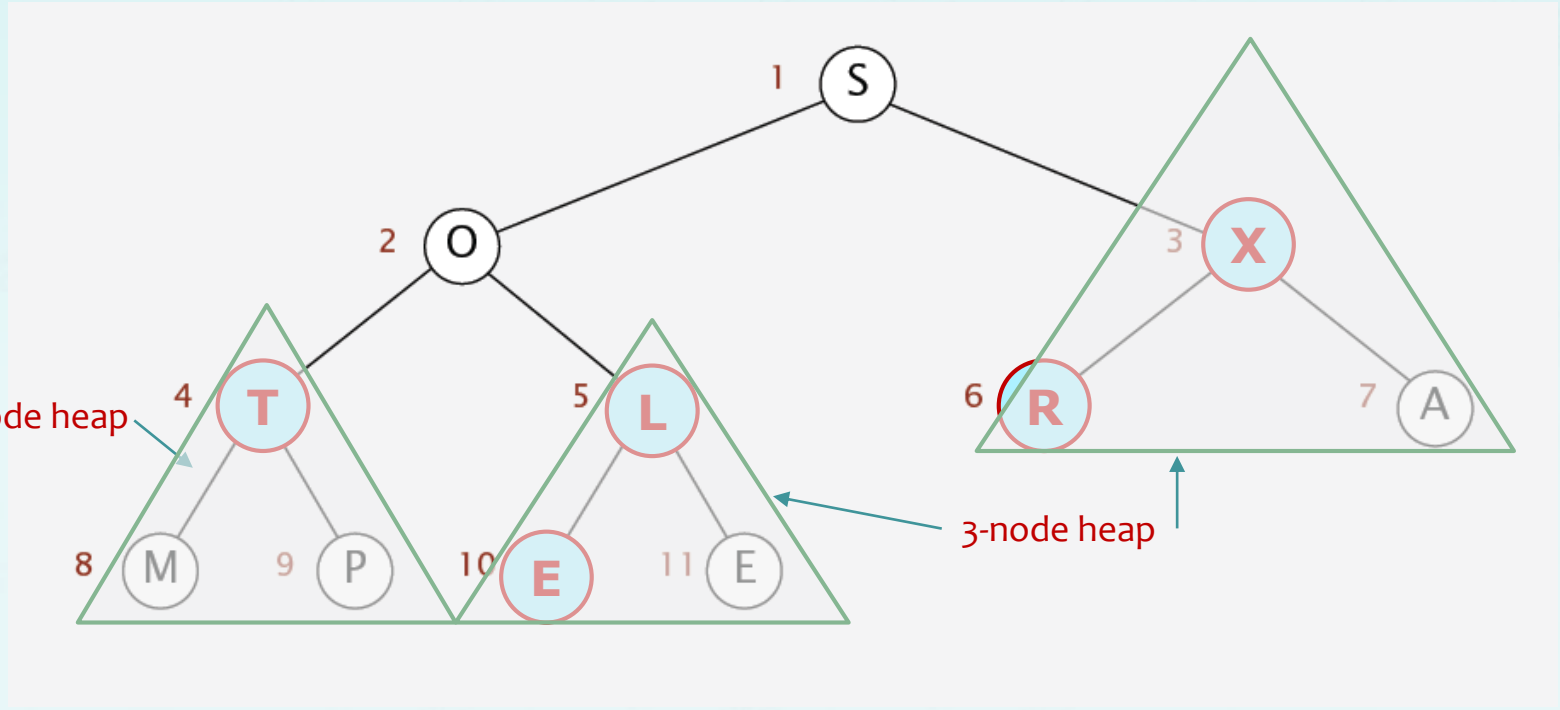

S	O	X	T	L	R	A	M	P	E	E
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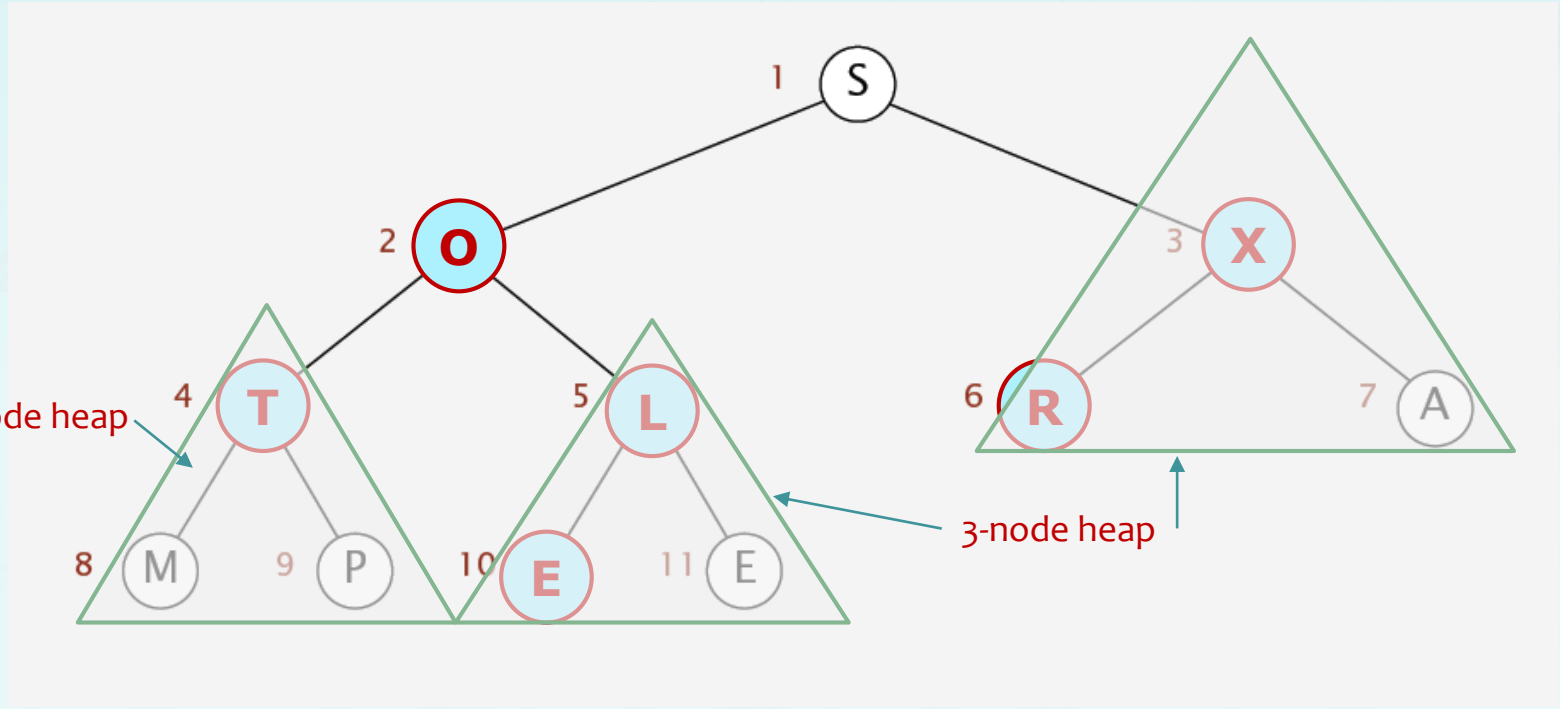
sink 2



S	O	X	T	L	R	A	M	P	E	E
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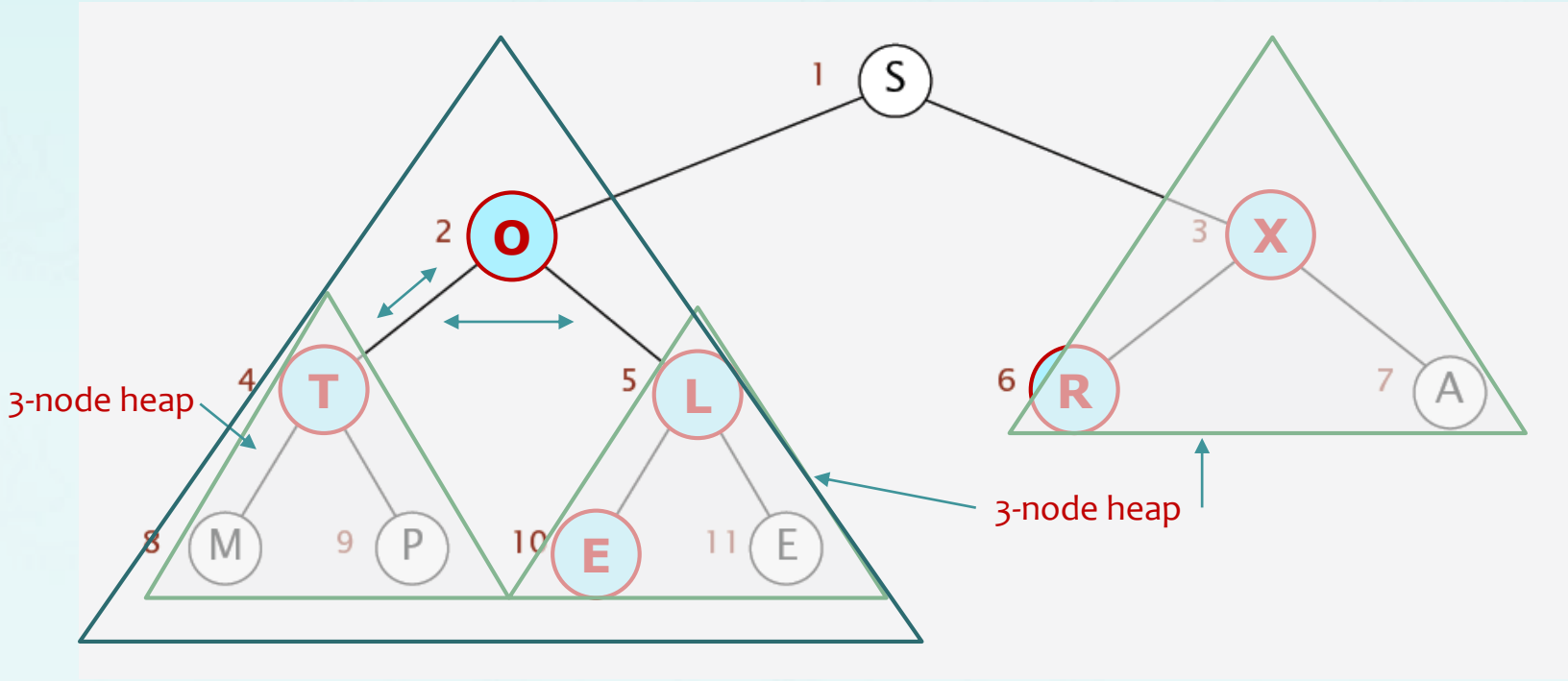

S	O	X	T	L	R	A	M	P	E	E
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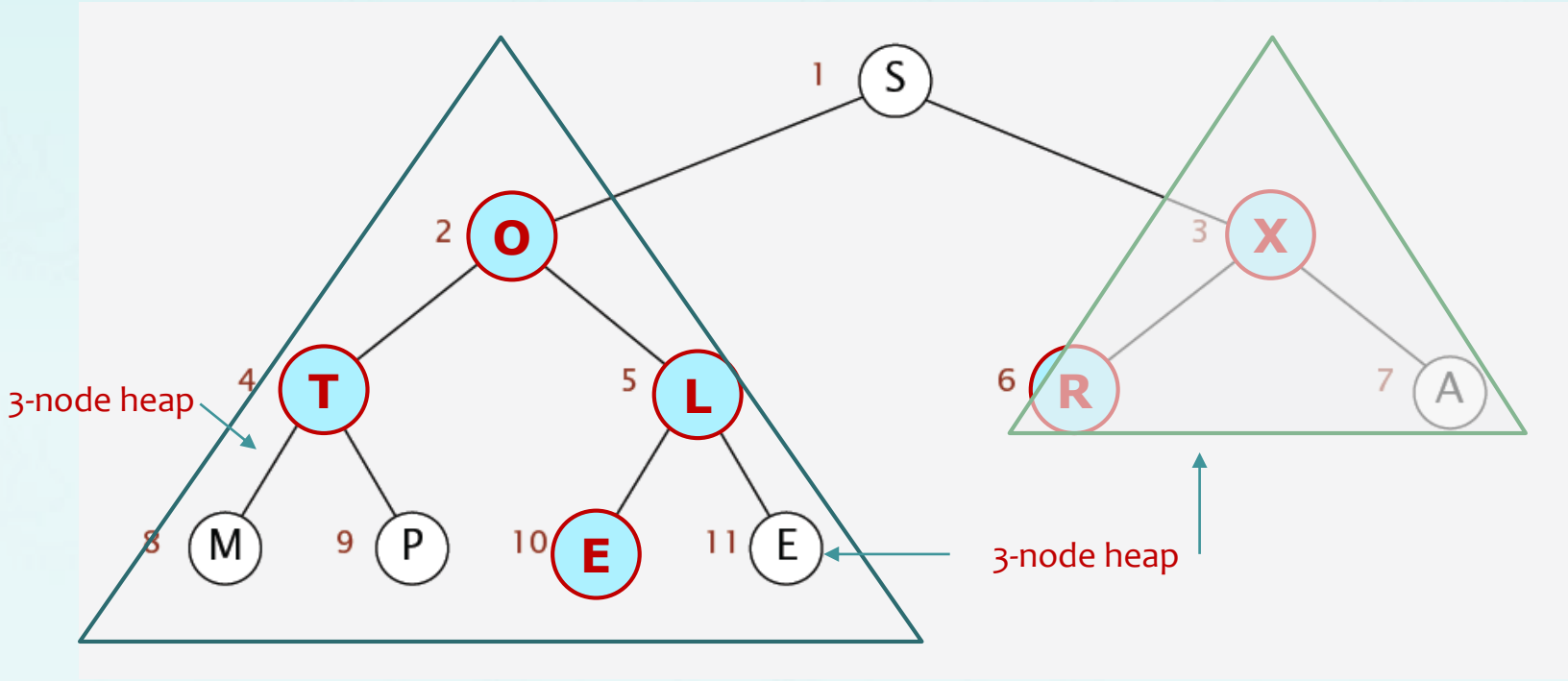

S	O	X	T	L	R	A	M	P	E	E
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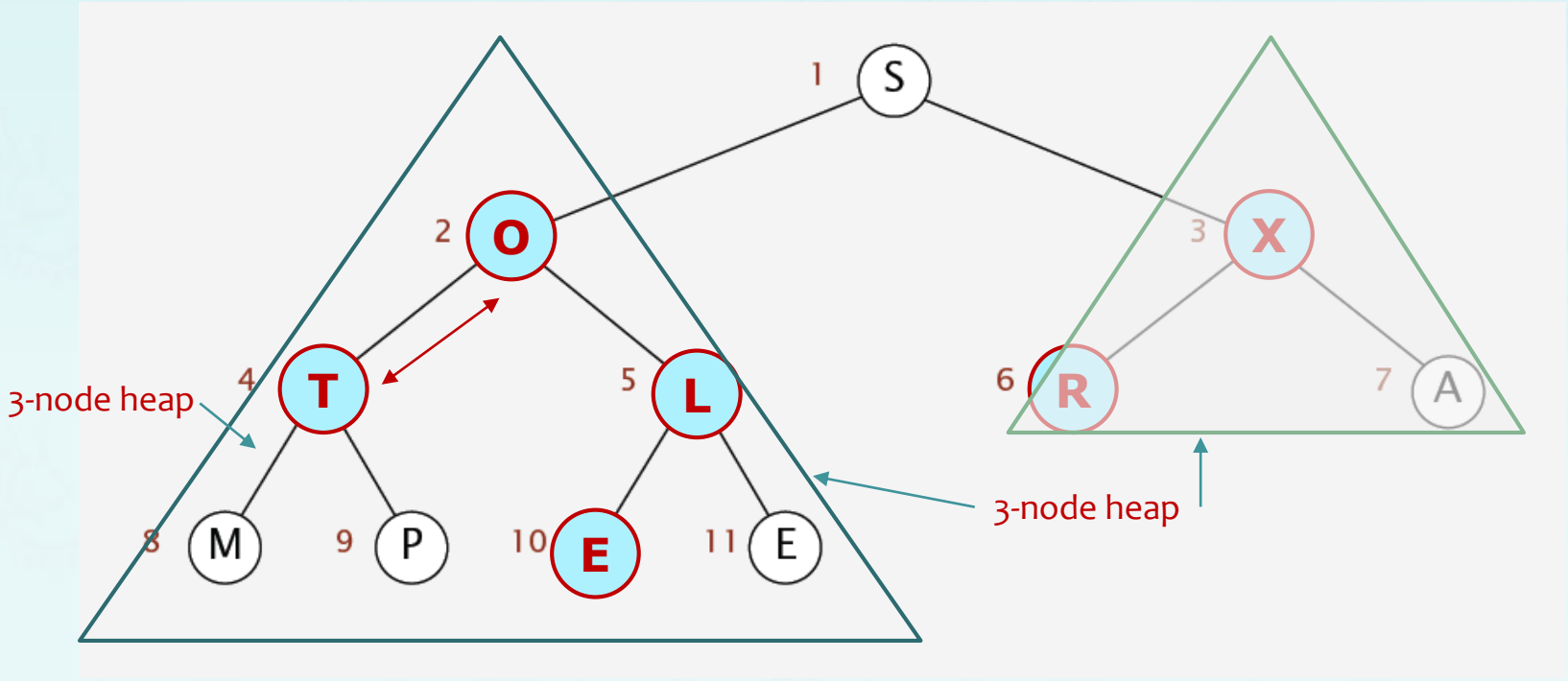

S	O	X	T	L	R	A	M	P	E	E
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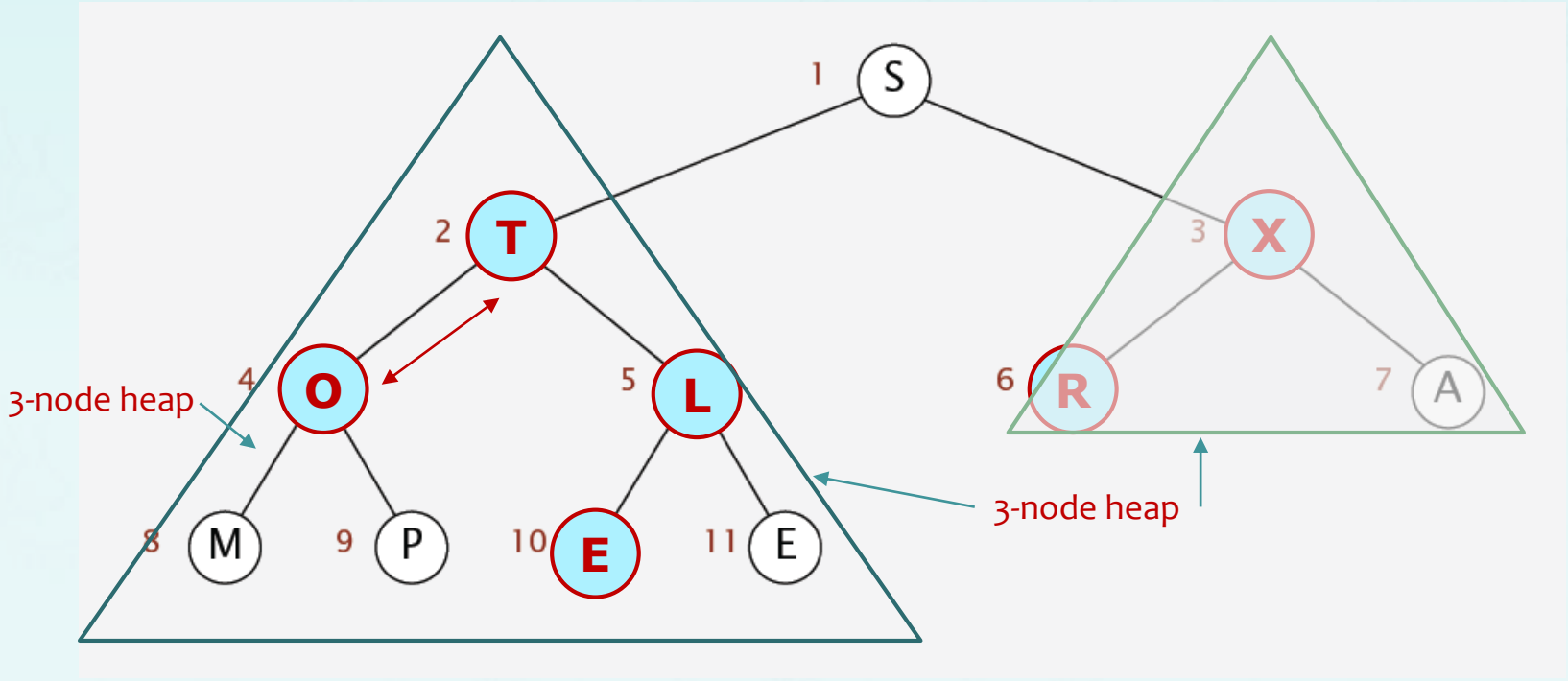

S	T	X	O	L	R	A	M	P	E	E
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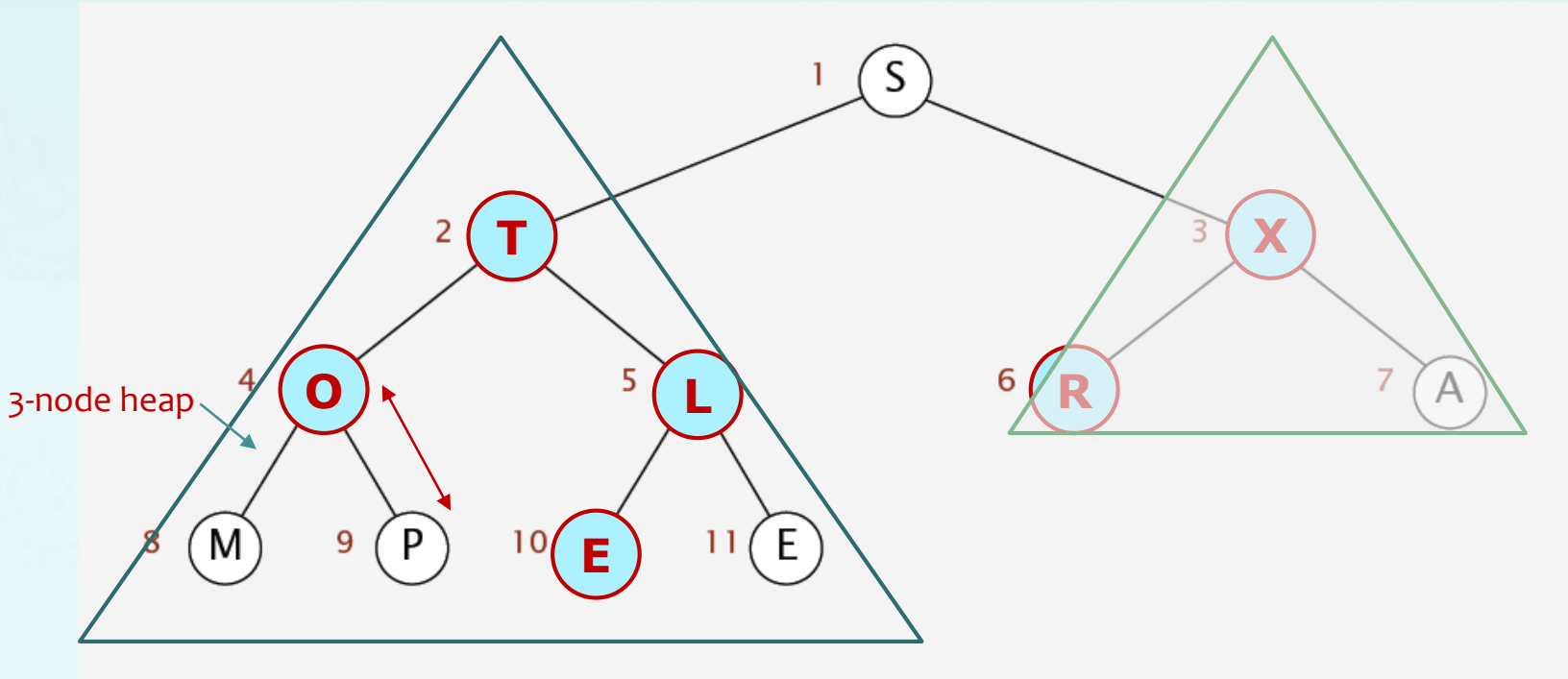

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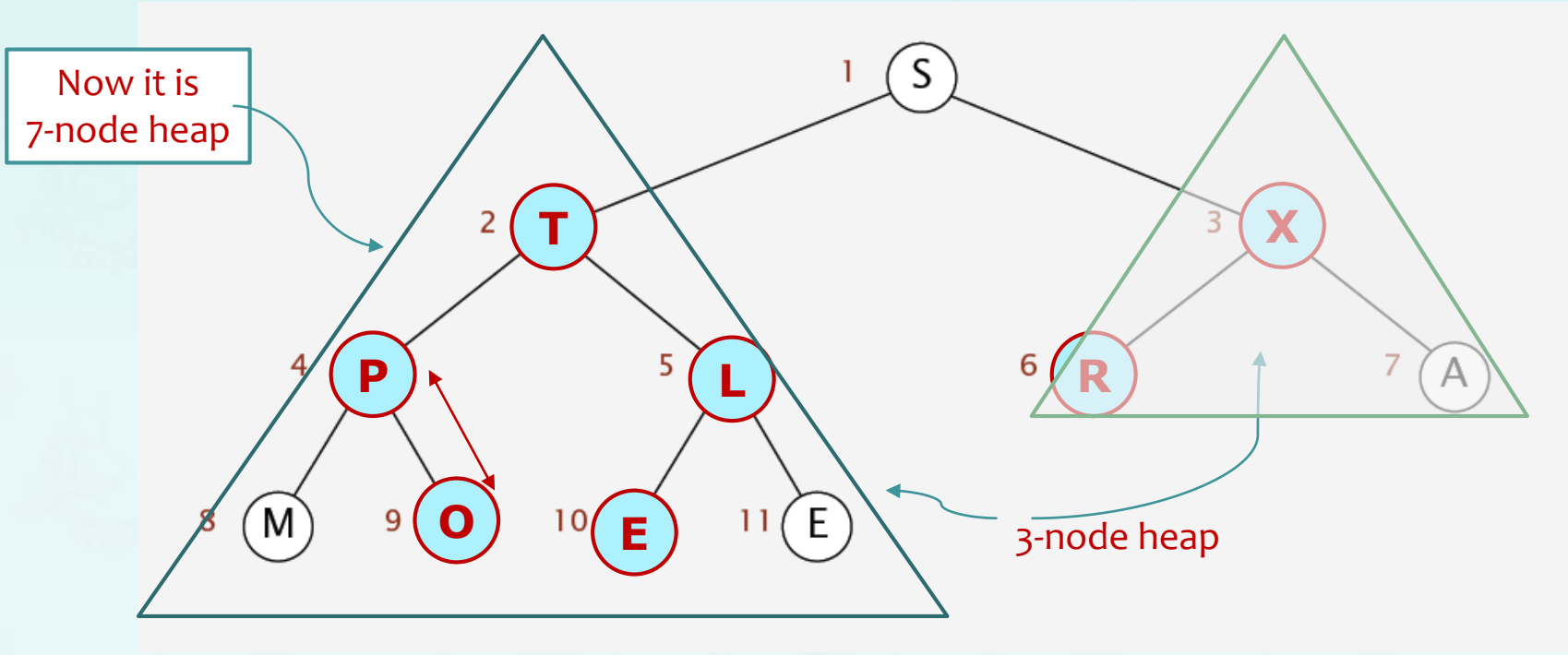

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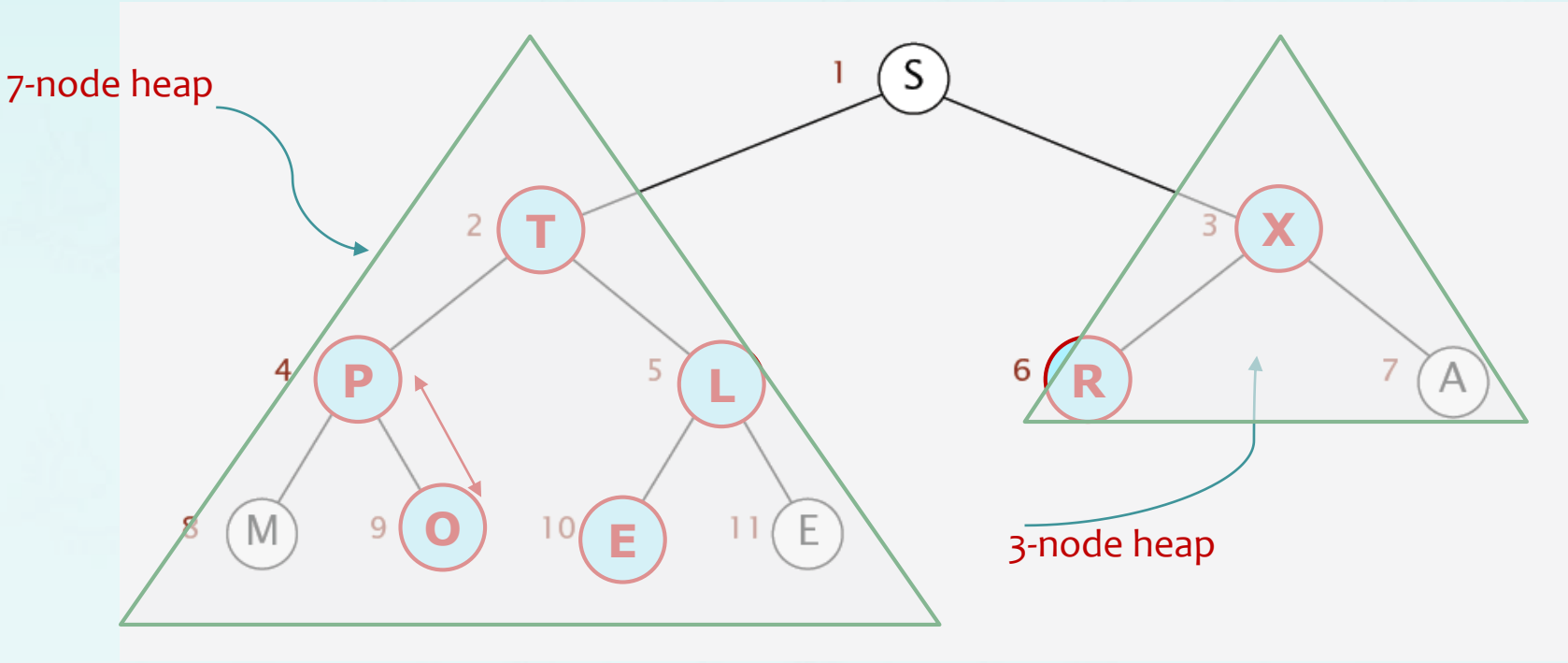

S	T	X	P	L	R	A	M	O	E	E
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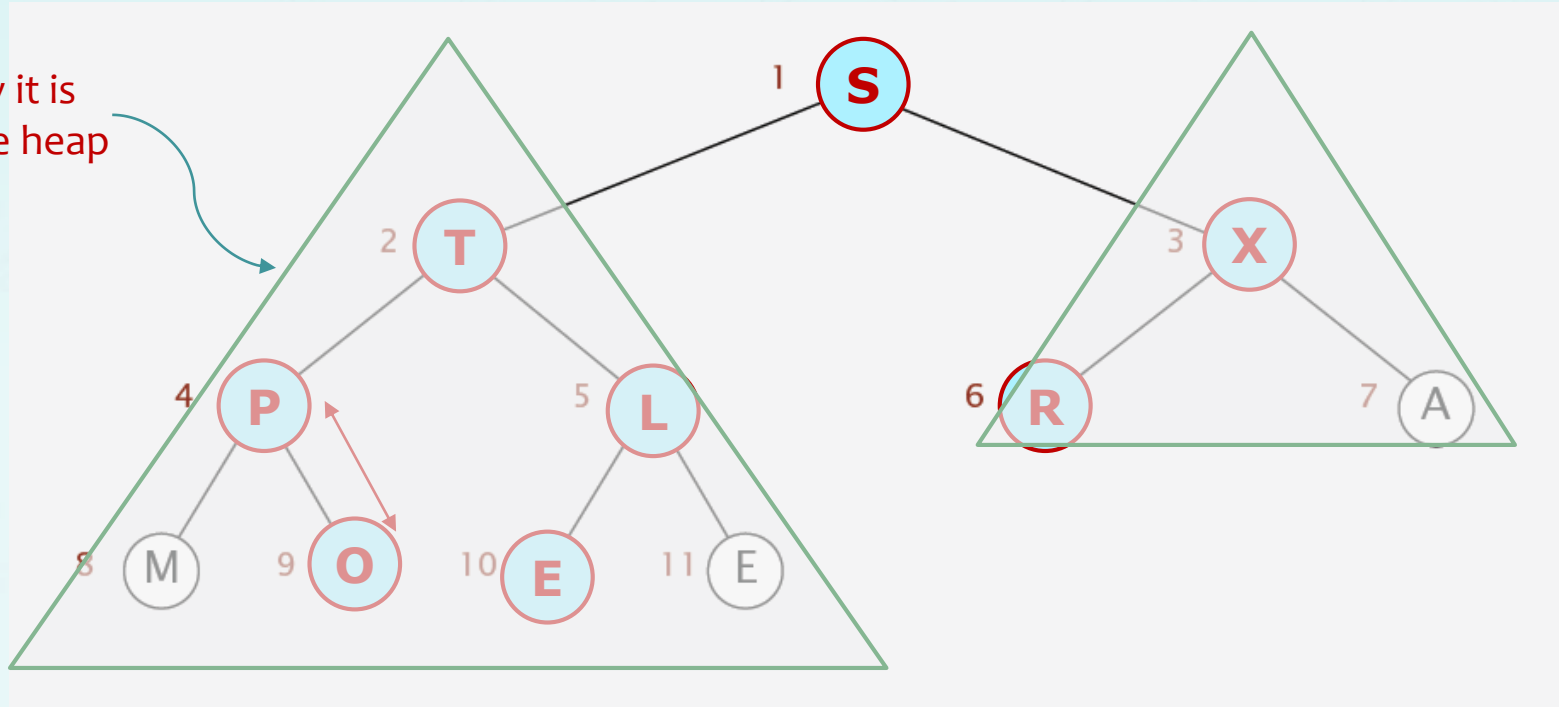
sink 1

sink(1, 11)

Now it is 7-node heap



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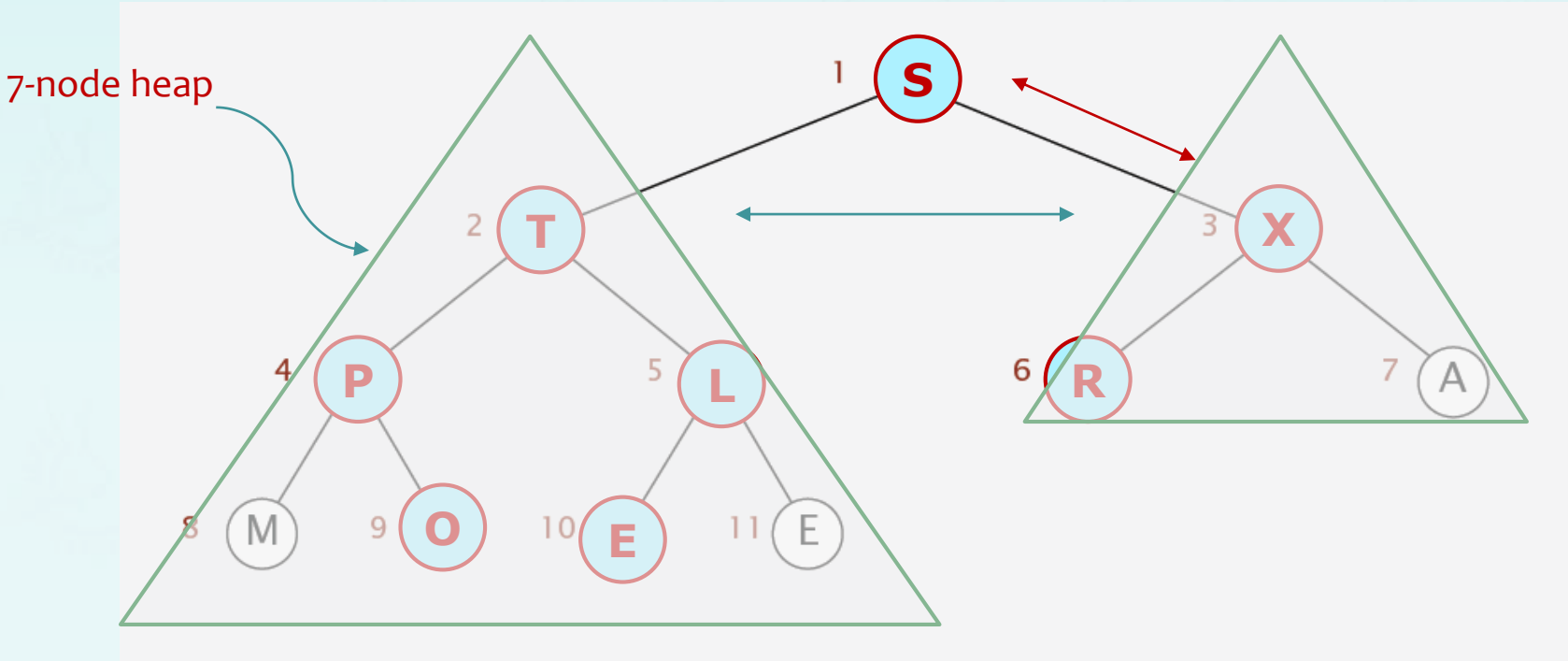

S	T	X	P	L	R	A	M	O	E	E
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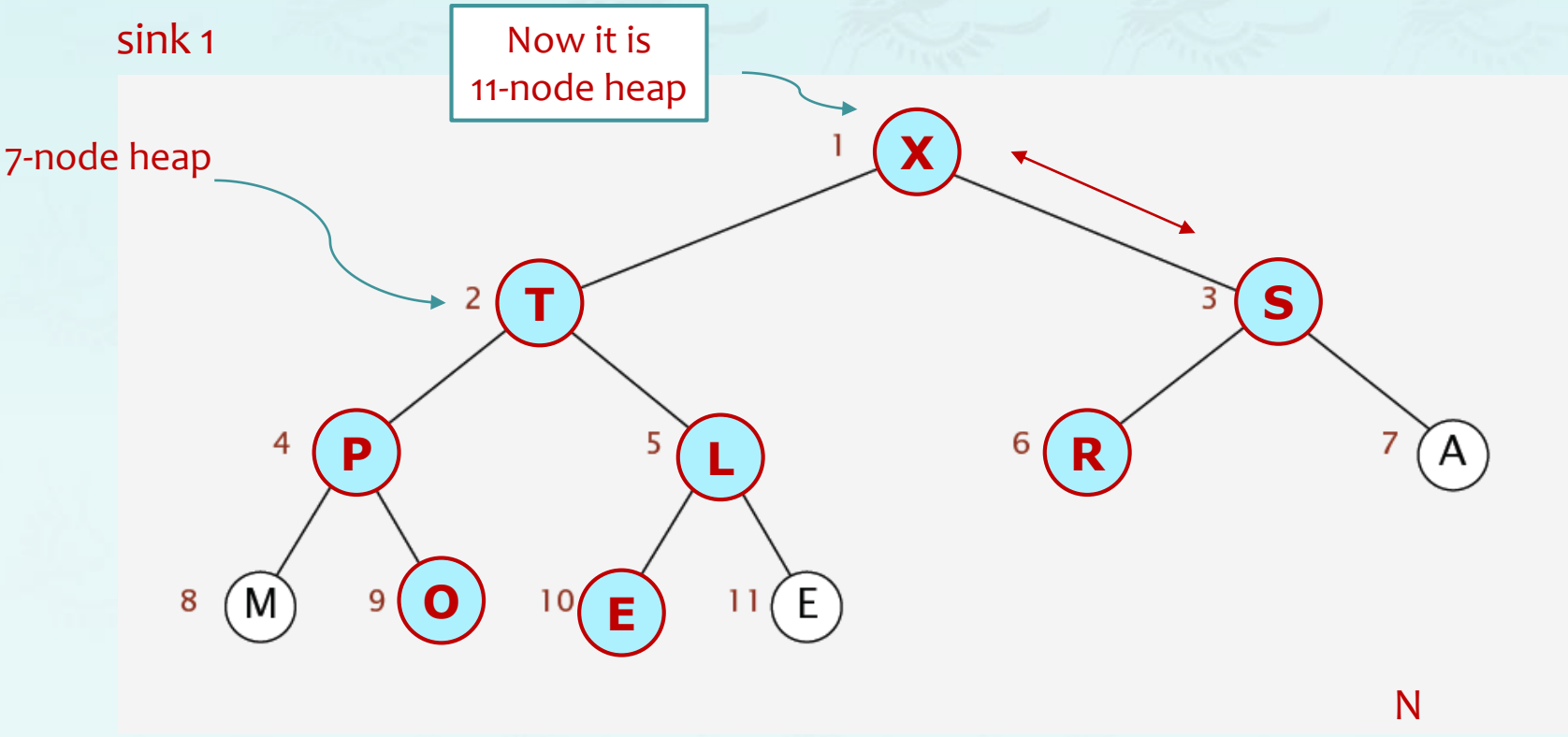

S	T	X	P	L	R	A	M	O	E	E
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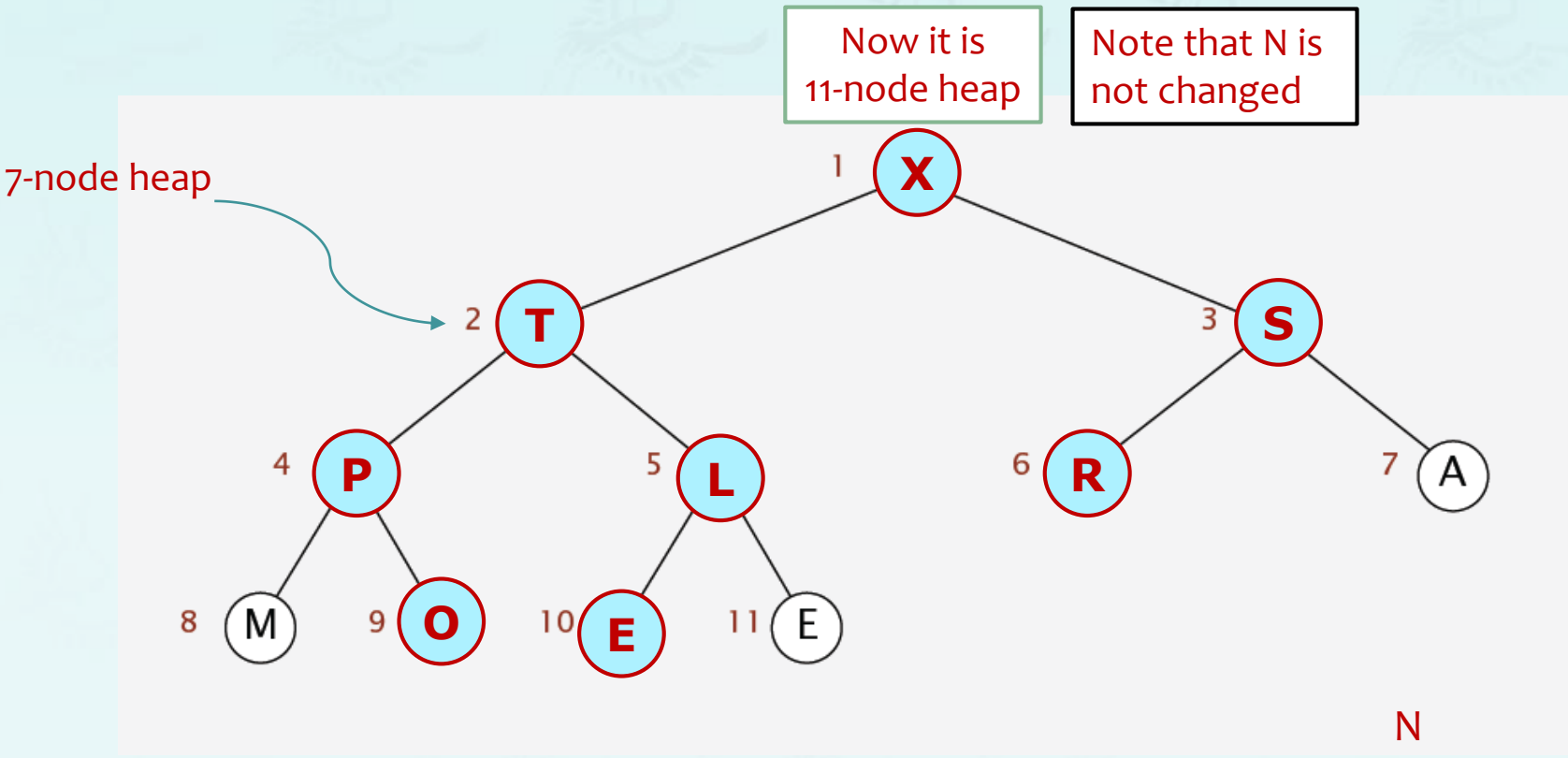

X	T	S	P	L	R	A	M	O	E	E
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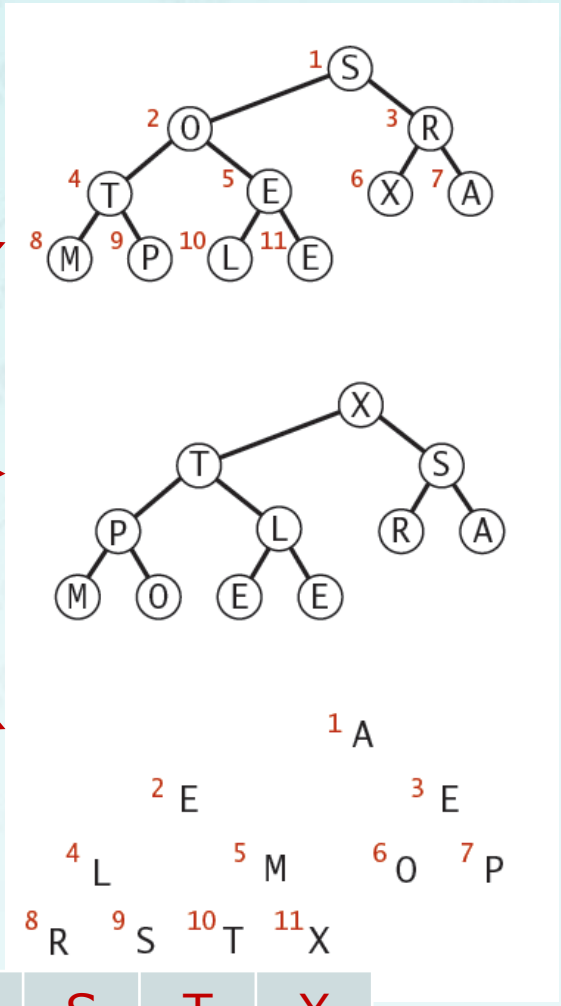
Basic plan for in-place sort

- **1st Pass**: Create max-heap with all **N** keys.
- **2nd Pass: Repeatedly remove the maximum key.**



A	E	E	L	N	O	P	R	S	T	X
1	2	3	4	5	6	7	8	9	10	11

- **2nd Pass:**
  - Remove the maximum
  - Leave them in array, in

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void sink(heap h, int k) {
   while (2 * k <= h->N) {
      int j = 2 * k;
      if (j < h->N && less(h, j, j + 1)) j++;
      if (!less(h, k, j)) break;
      swap(h, k, j);
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}
```
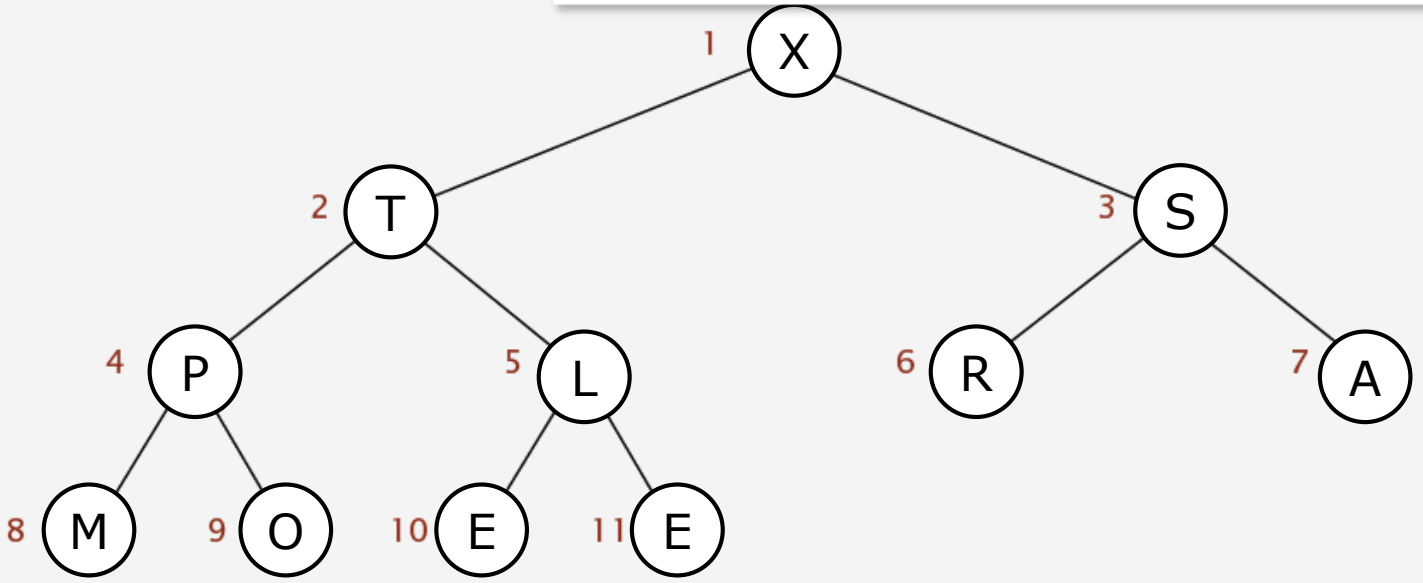
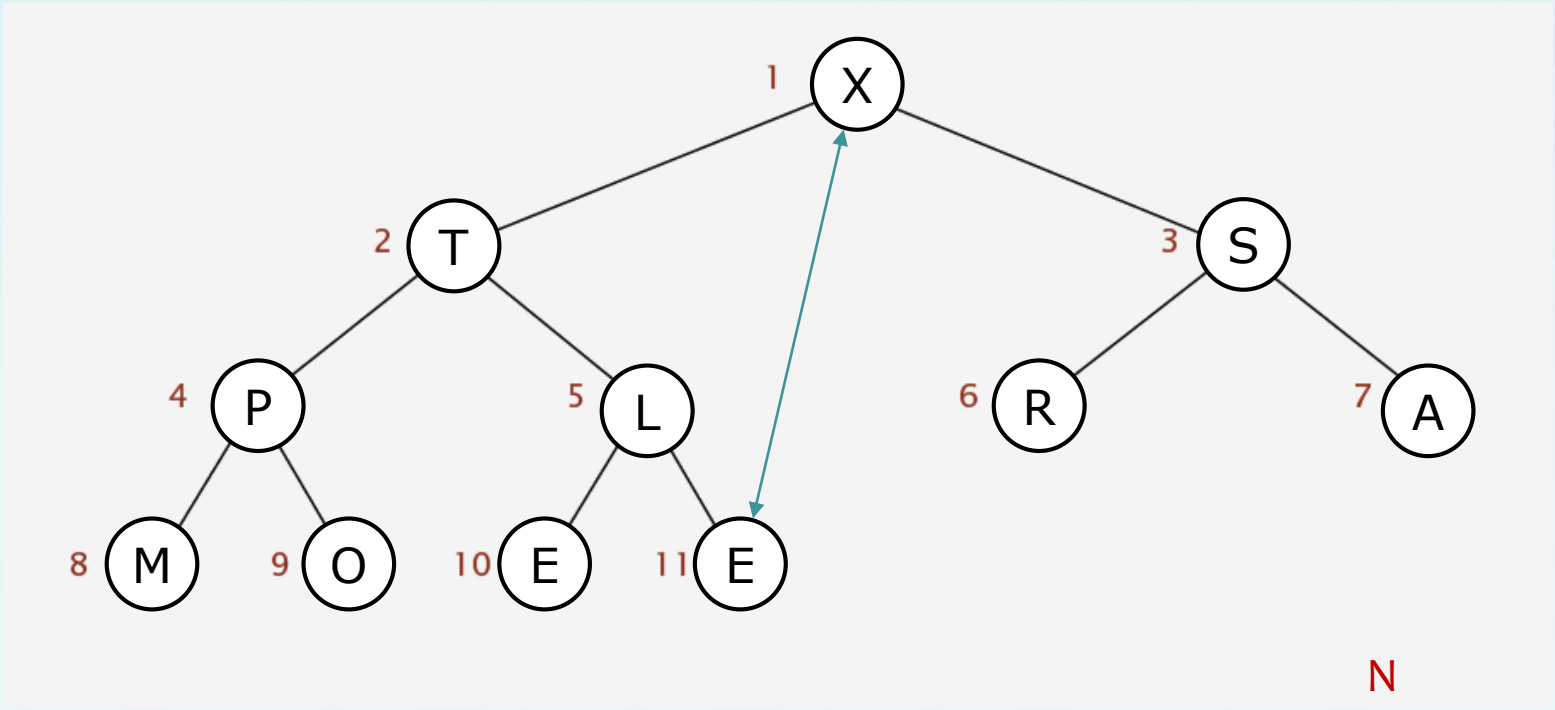



N

X	T	S	P	L	R	A	M	O	E	E
1	2	3	4	5	6	7	8	9	10	11

- **2nd Pass:**
  - Remove the maximum, one at a time.
  - Leave them in array, instead of nulling out

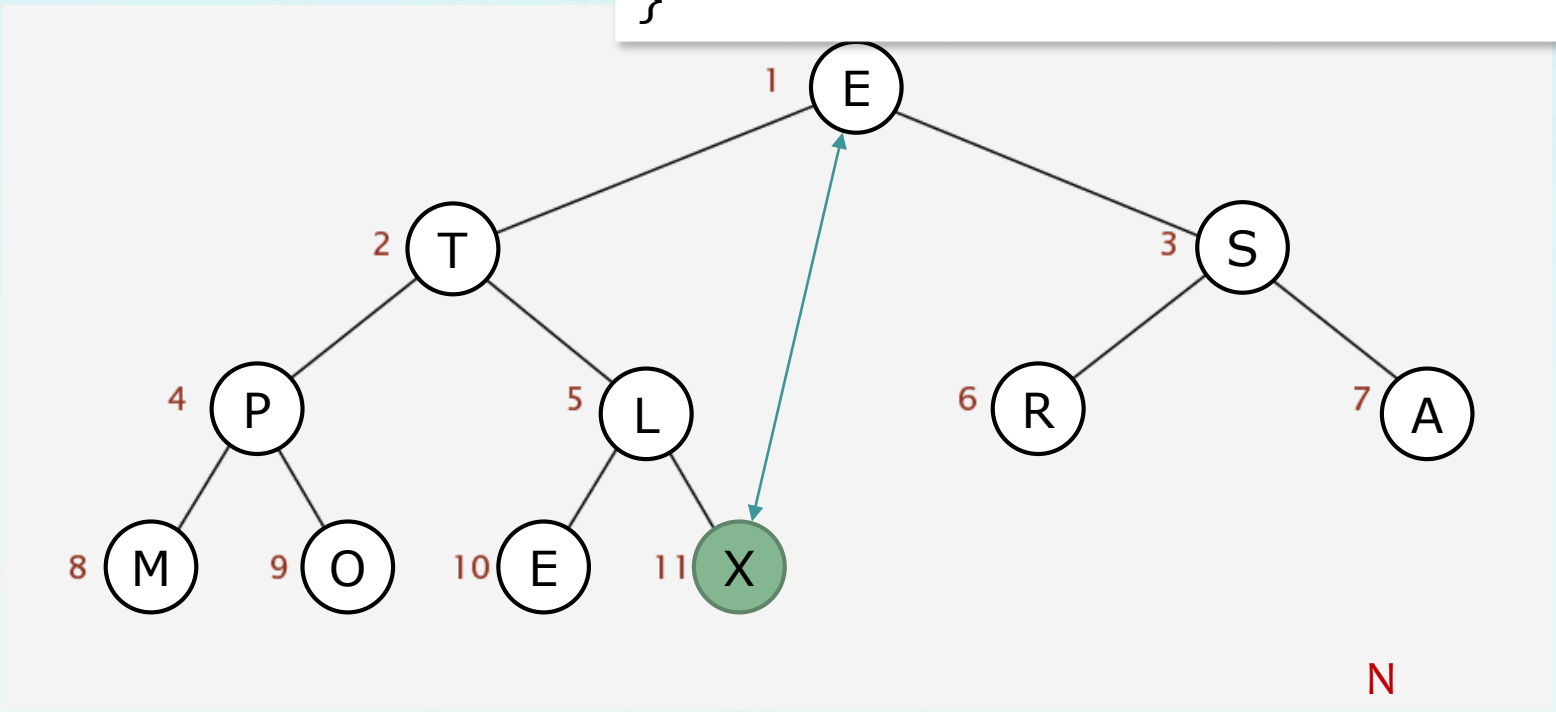


	X	T	S	P	L	R	A	M	O	E	E
	1	2	3	4	5	6	7	8	9	10	11

N

- **2nd Pass:**
  - Remove the maximum
  - Leave them in array, in

```
void sink(heap h, int k) {
   while (2 * k <= h->N) {
      int j = 2 * k;
      if (j < h->N && less(h, j, j + 1)) j++;
      if (!less(h, k, j)) break;
      swap(h, k, j);
      k = j;
   }
}
```

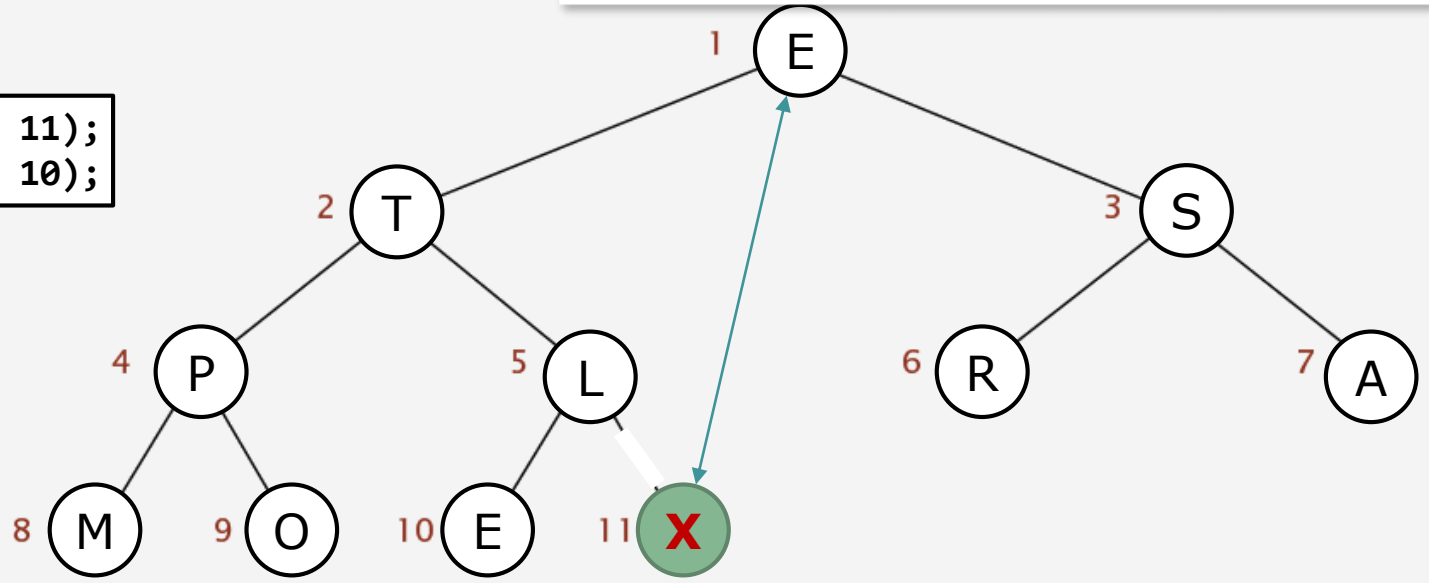


X	T	S	P	L	R	A	M	O	E	E
1	2	3	4	5	6	7	8	9	10	11

- 2nd Pass:
  - Remove the maximum
  - Leave them in array, in

```
void sink(heap h, int k) {
   while (2 * k <= h->N) {
      int j = 2 * k;
      if (j < h->N && less(h, j, j + 1)) j++;
      if (!less(h, k, j)) break;
      swap(h, k, j);
      k = j;
   }
}
```

```
swap(1, 11);
sink(1, 10);
```

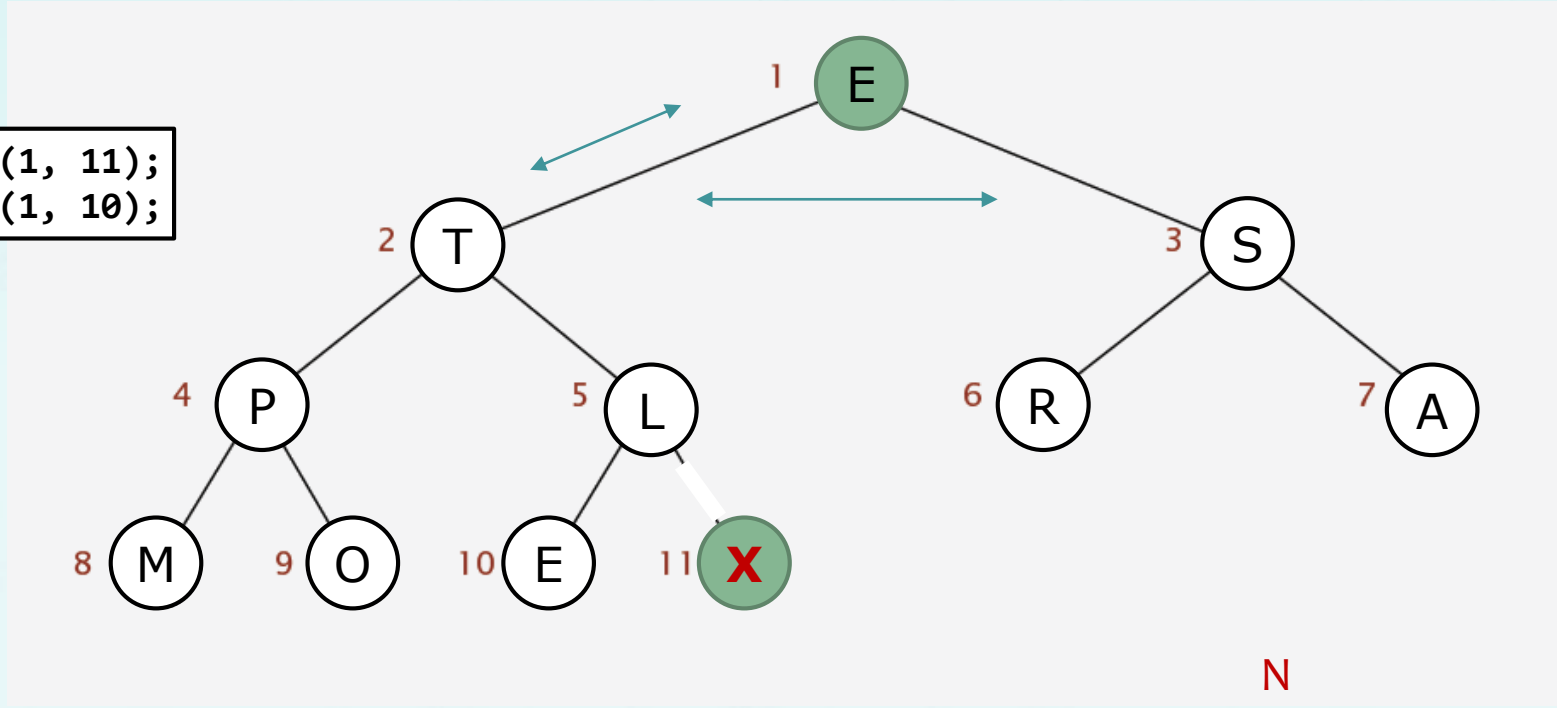


N

E	T	S	P	L	R	A	M	O	E	X
1	2	3	4	5	6	7	8	9	10	11

- **2nd Pass:**
  - Remove the maximum, one at a time.
  - Leave them in array, instead of nulling out

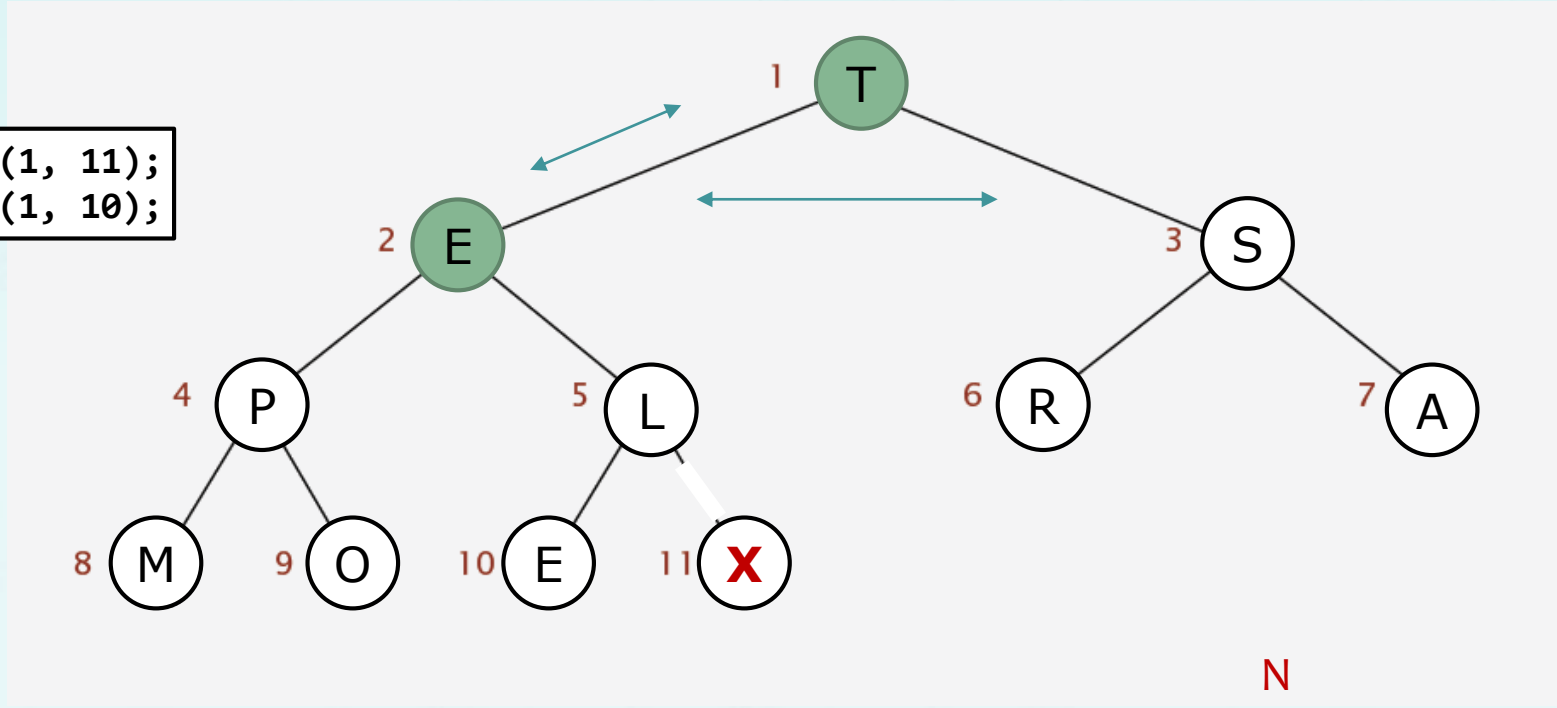
```
swap(1, 11);
sink(1, 10);
```



									N	
E	T	S	P	L	R	A	M	O	E	X
1	2	3	4	5	6	7	8	9	10	11

- **2nd Pass:**
  - Remove the maximum, one at a time.
  - Leave them in array, instead of nulling out

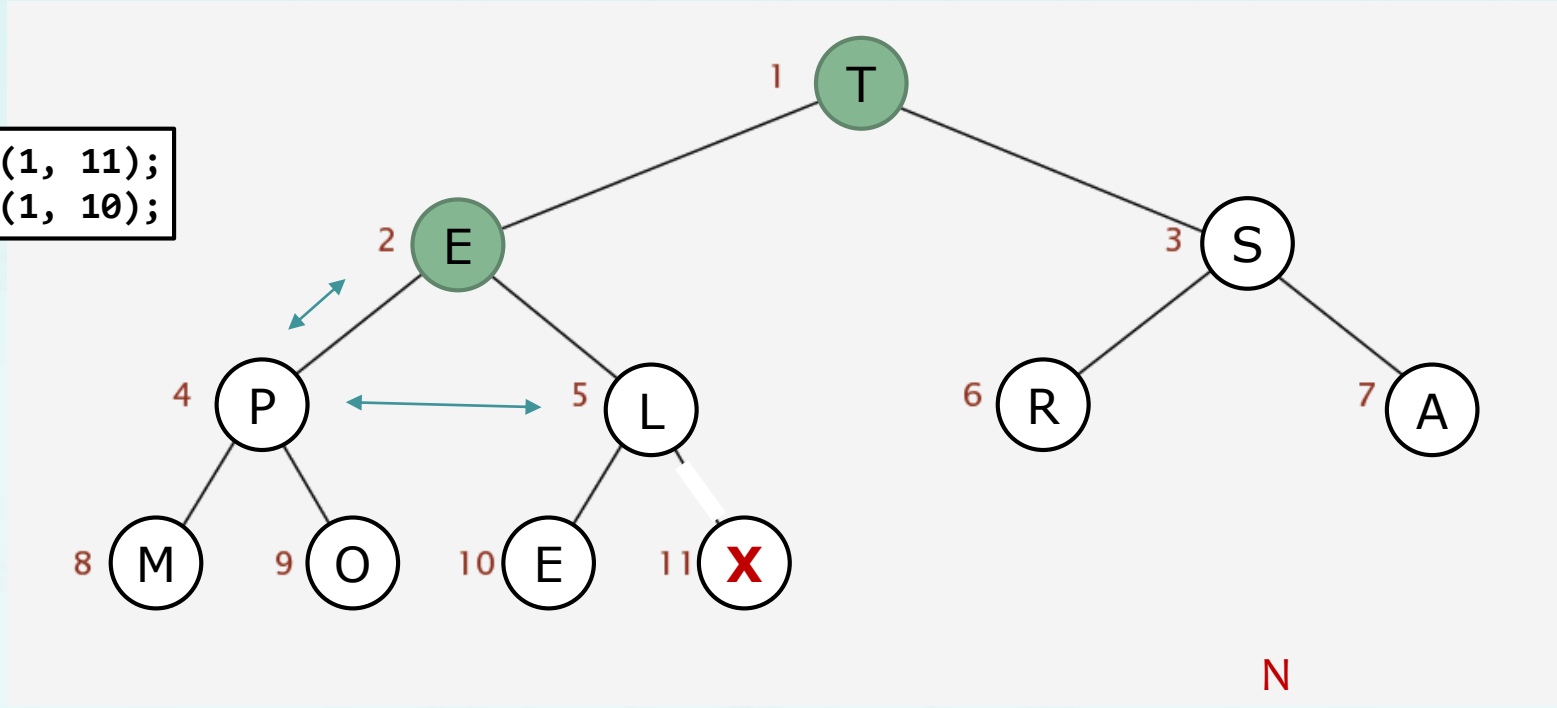
```
swap(1, 11);
sink(1, 10);
```



									N	
E	T	S	P	L	R	A	M	O	E	X
1	2	3	4	5	6	7	8	9	10	11

- **2nd Pass:**
  - Remove the maximum, one at a time.
  - Leave them in array, instead of nulling out

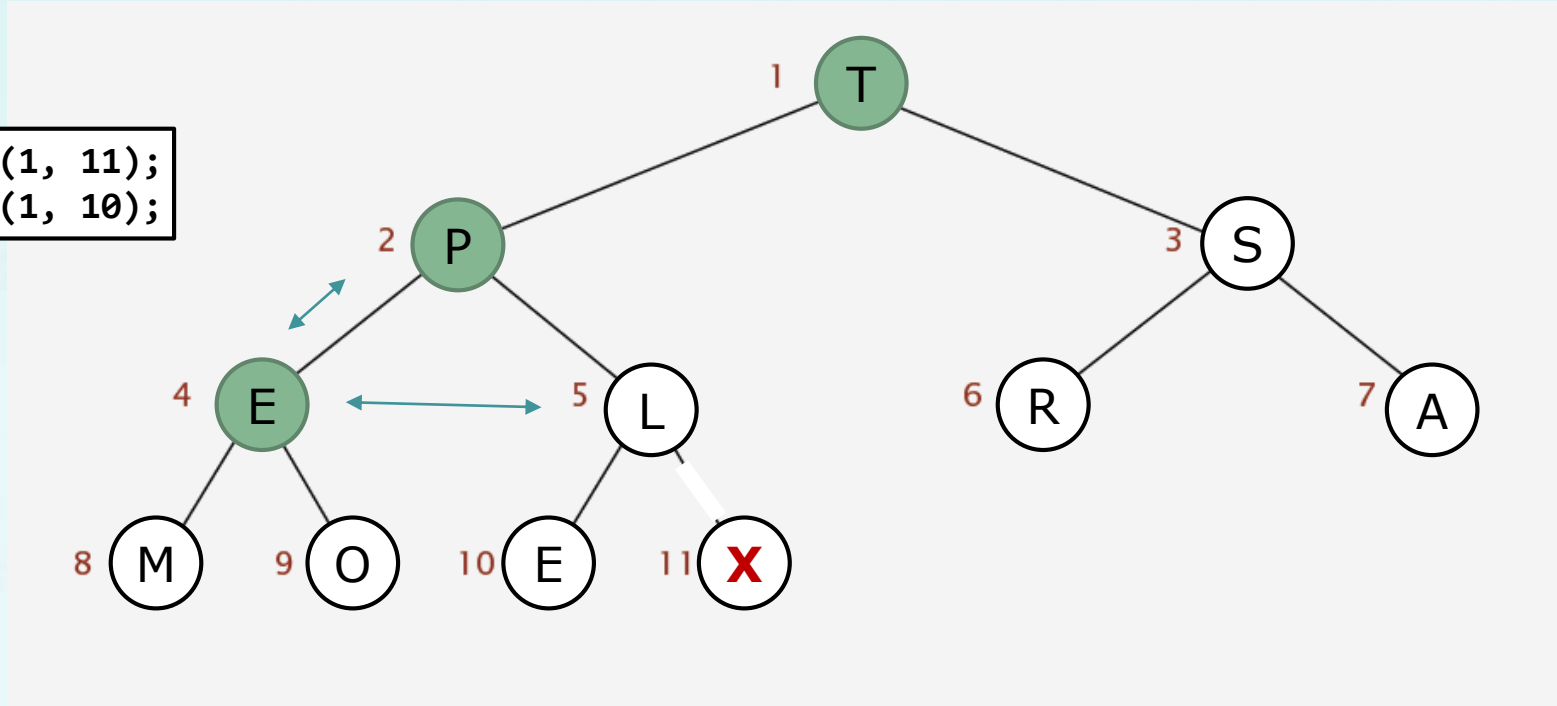
```
swap(1, 11);
sink(1, 10);
```



									N	
E	T	S	P	L	R	A	M	O	E	X
1	2	3	4	5	6	7	8	9	10	11

- **2nd Pass:**
  - Remove the maximum, one at a time.
  - Leave them in array, instead of nulling out

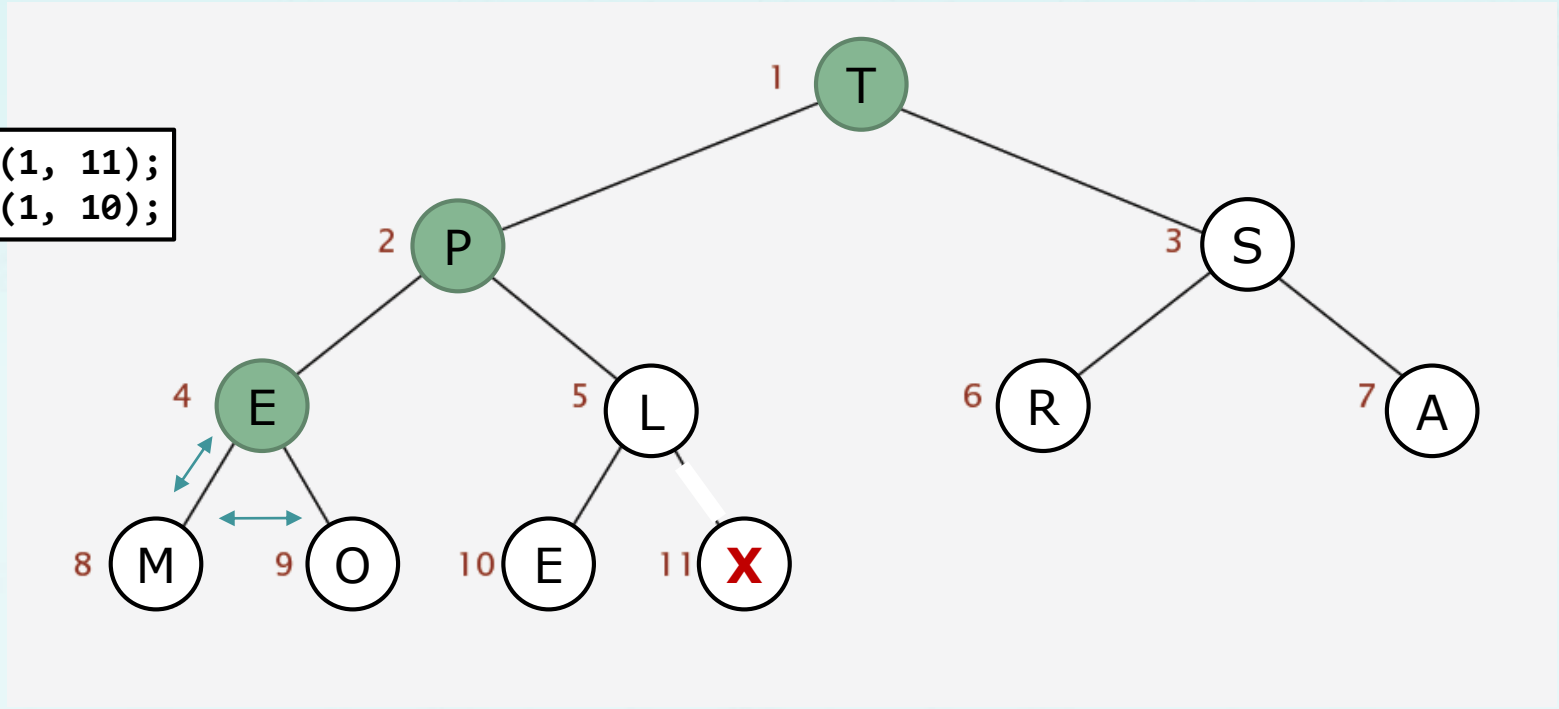
```
swap(1, 11);
sink(1, 10);
```



E	T	S	P	L	R	A	M	O	E	X
1	2	3	4	5	6	7	8	9	10	11

- **2nd Pass:**
  - Remove the maximum, one at a time.
  - Leave them in array, instead of nulling out

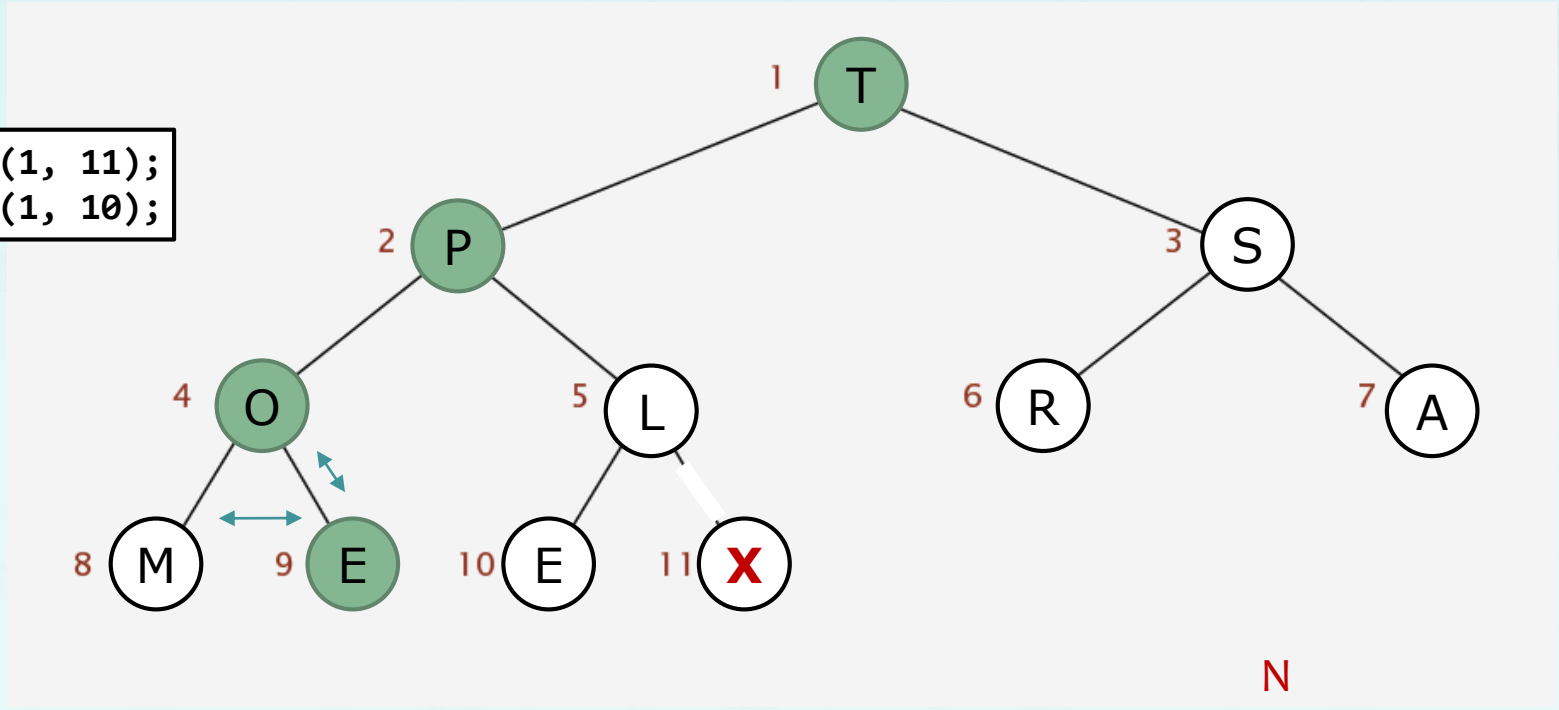
```
swap(1, 11);
sink(1, 10);
```



E	T	S	P	L	R	A	M	O	E	X
1	2	3	4	5	6	7	8	9	10	11

- **2nd Pass:**
  - Remove the maximum, one at a time.
  - Leave them in array, instead of nulling out

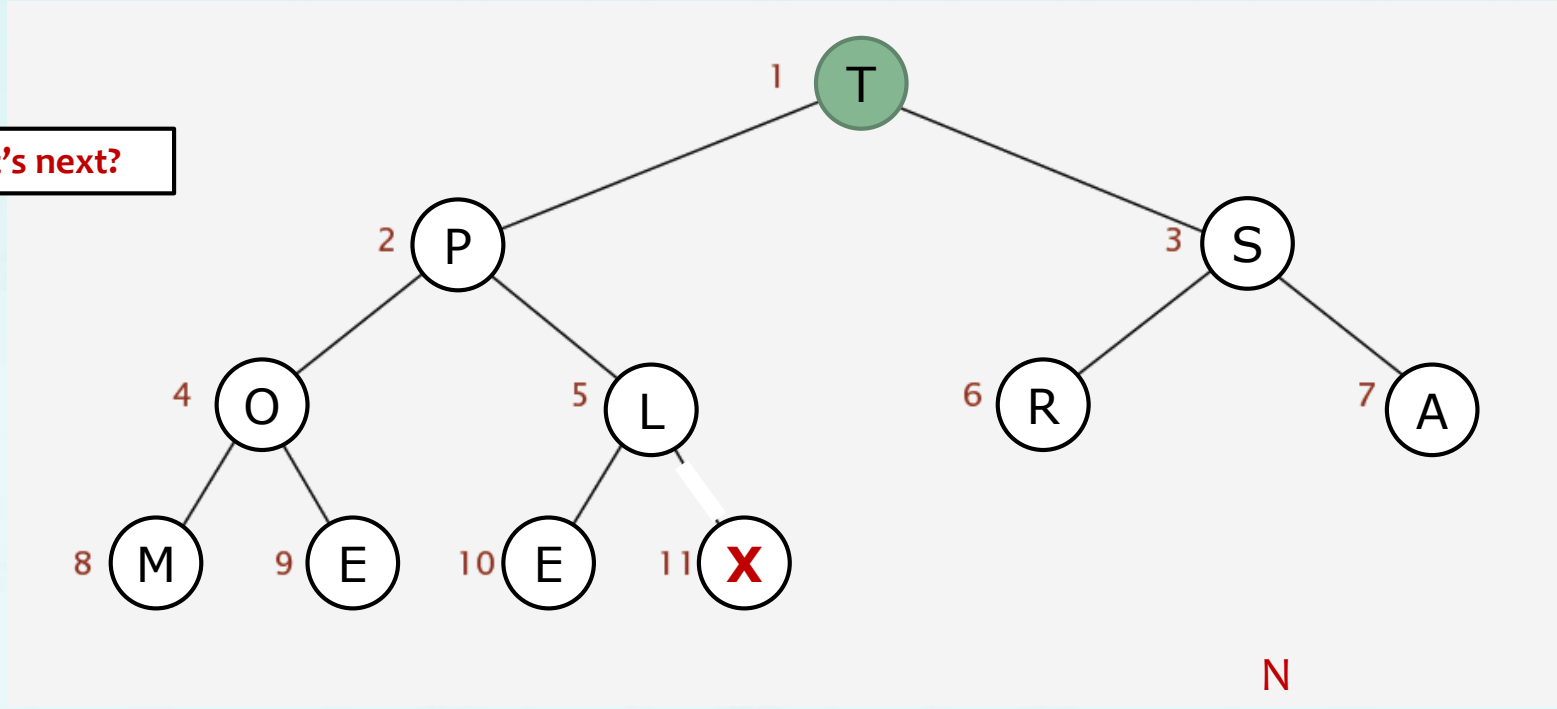
```
swap(1, 11);
sink(1, 10);
```



									N	
T	P	S	O	L	R	A	M	E	E	X
1	2	3	4	5	6	7	8	9	10	11

- **2nd Pass:**
  - Remove the maximum, one at a time.
  - Leave them in array, instead of nulling out

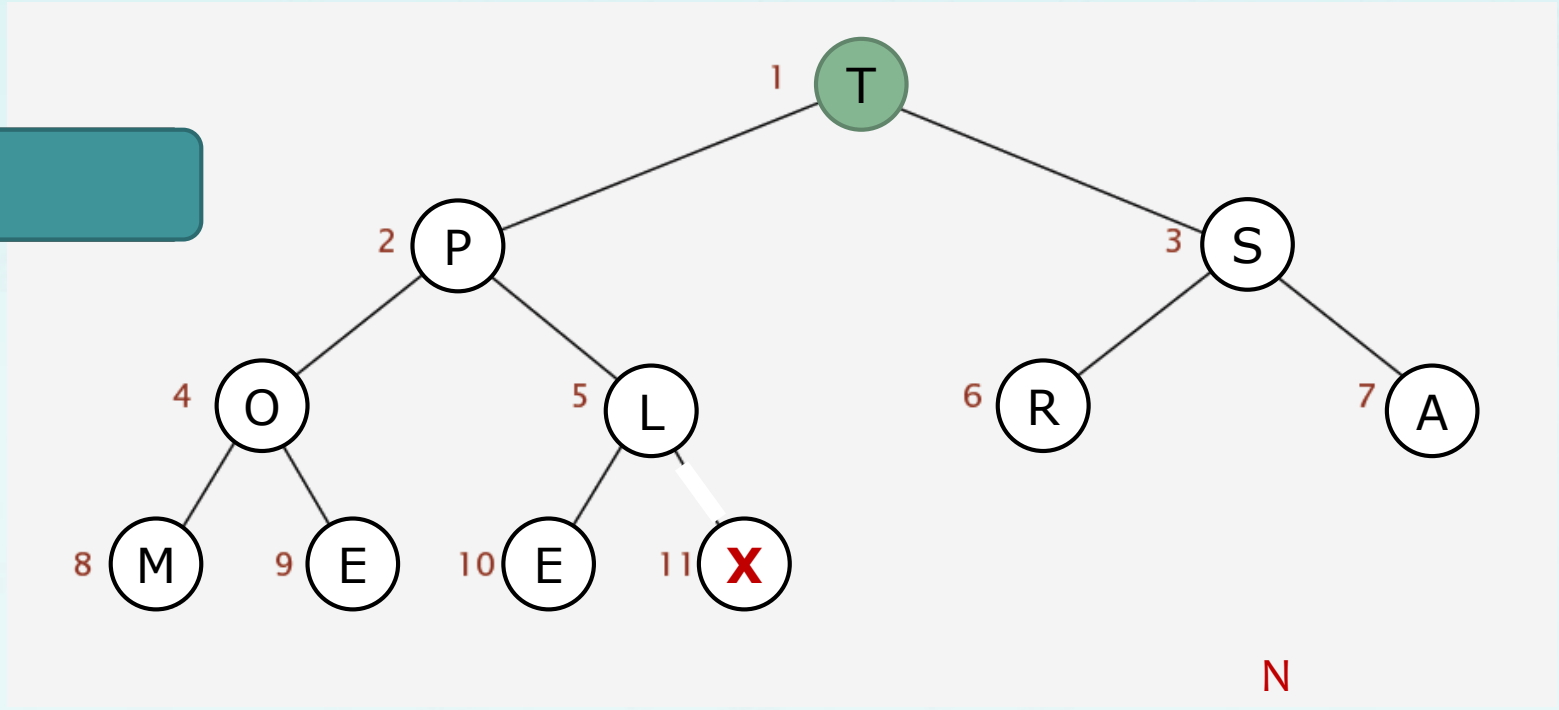
What's next?



N

T	P	S	O	L	R	A	M	E	E	X
1	2	3	4	5	6	7	8	9	10	11

- **2nd Pass:**
  - Remove the maximum, one at a time.
  - Leave them in array, instead of nulling out

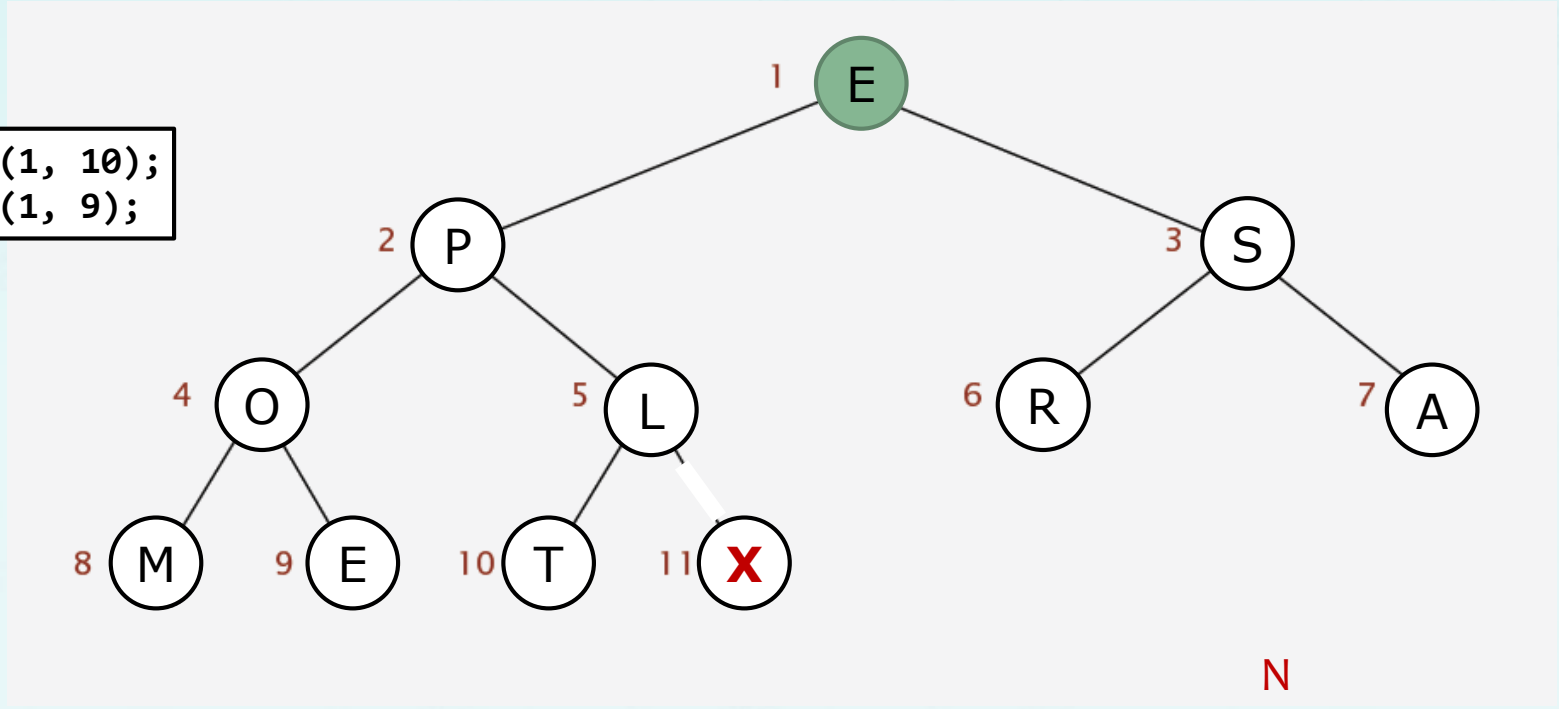


N

T	P	S	O	L	R	A	M	E	E	X
1	2	3	4	5	6	7	8	9	10	11

- **2nd Pass:**
  - Remove the maximum, one at a time.
  - Leave them in array, instead of nulling out

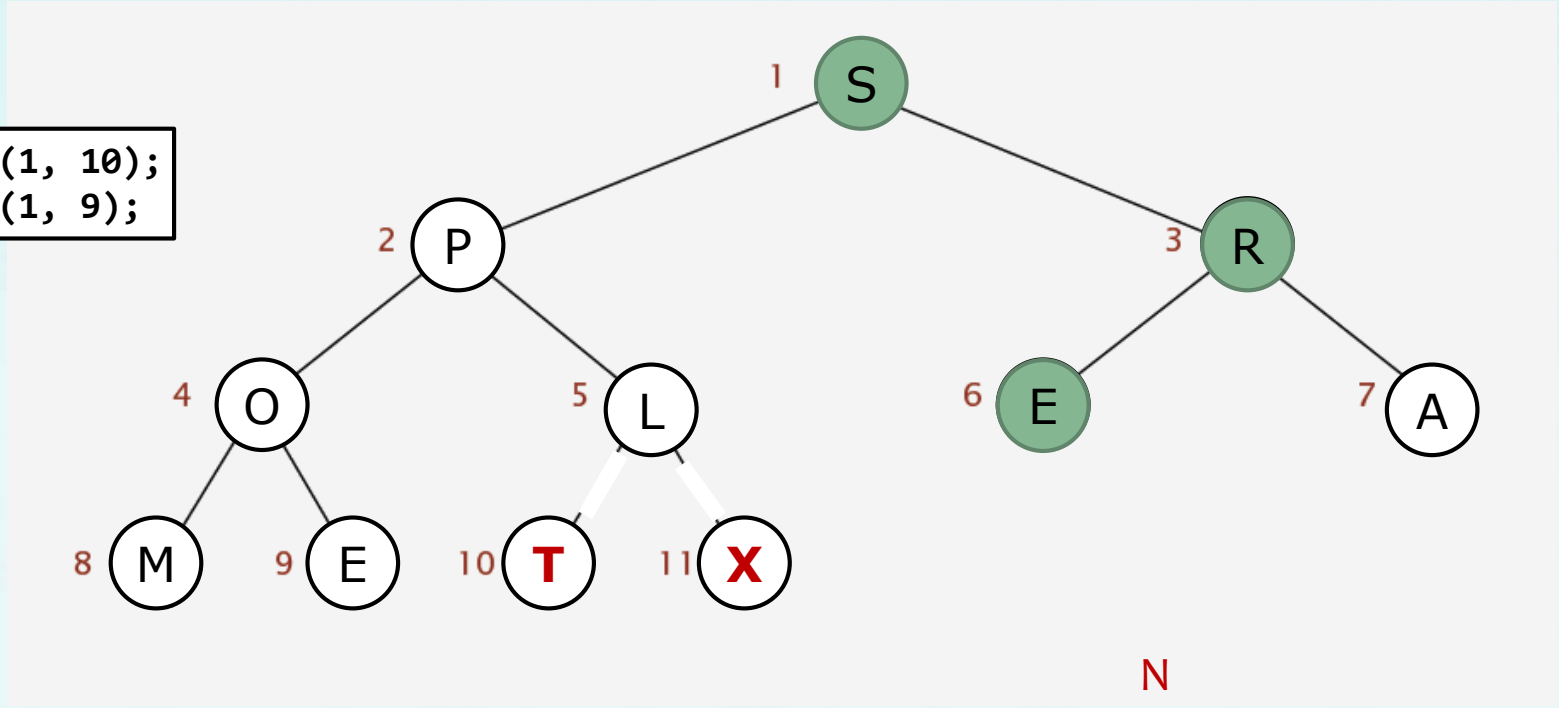
```
swap(1, 10);
sink(1, 9);
```



									N	
T	P	S	O	L	R	A	M	E	E	X
1	2	3	4	5	6	7	8	9	10	11

- **2nd Pass:**
  - Remove the maximum, one at a time.
  - Leave them in array, instead of nulling out

```
swap(1, 10);
sink(1, 9);
```

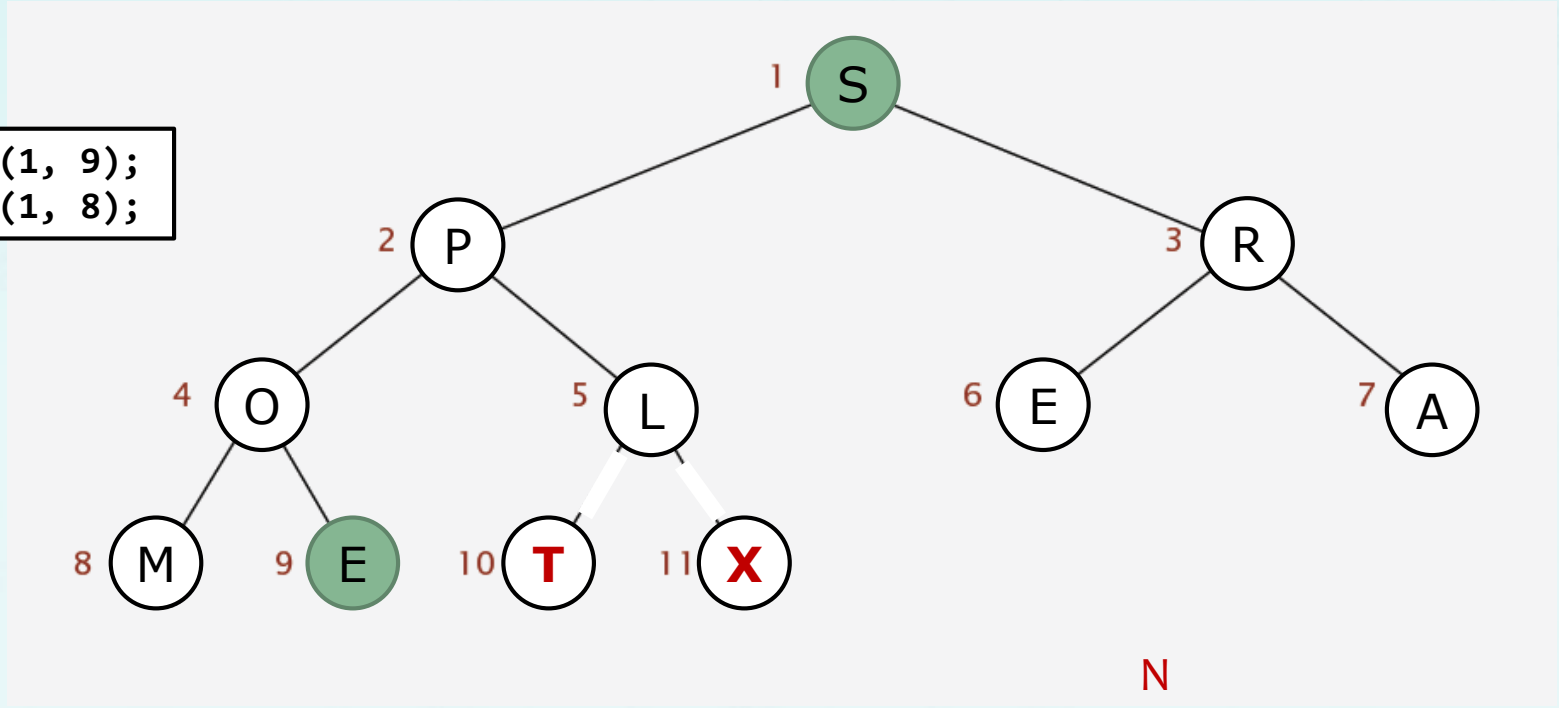


N

T	P	S	O	L	R	A	M	E	T	X
1	2	3	4	5	6	7	8	9	10	11

- **2nd Pass:**
  - Remove the maximum, one at a time.
  - Leave them in array, instead of nulling out

```
swap(1, 9);
sink(1, 8);
```

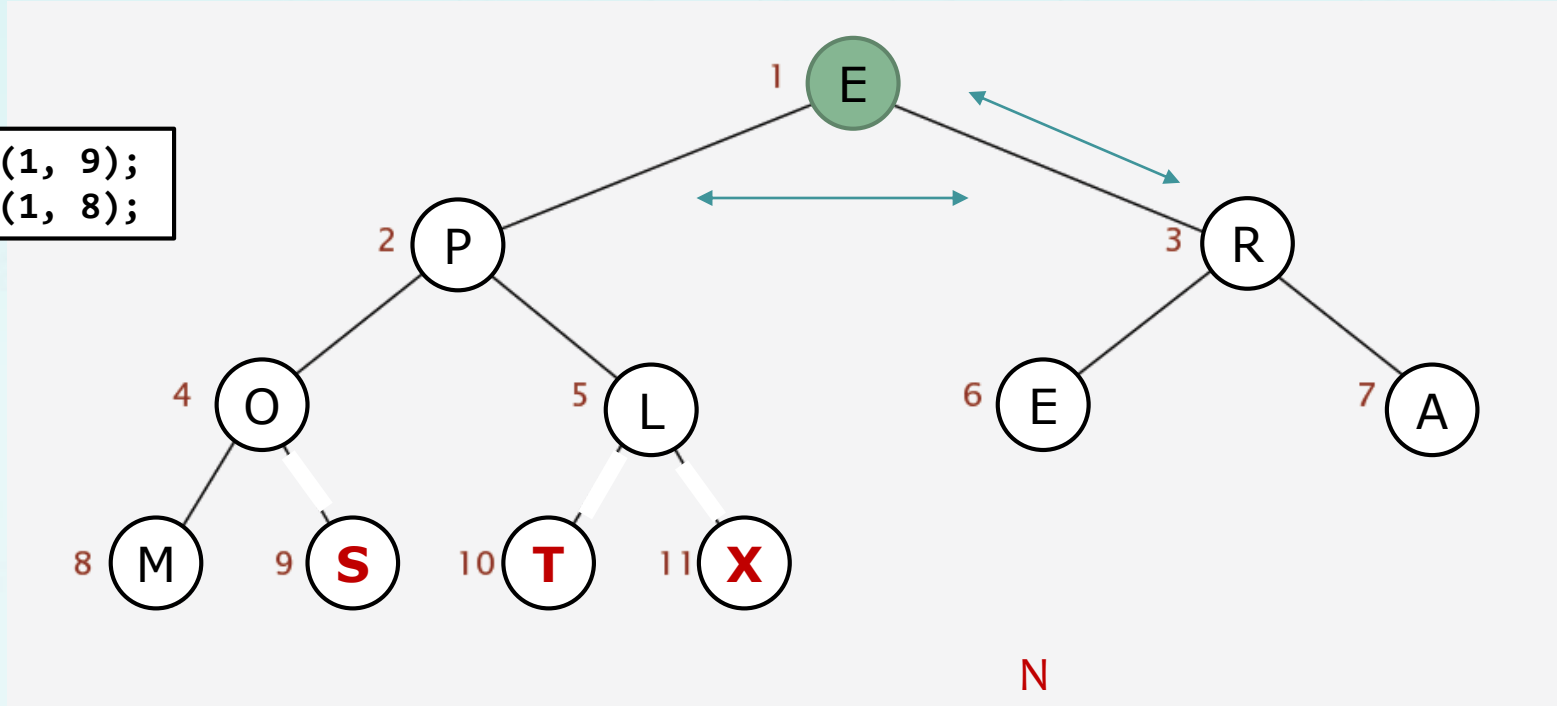


N

T	P	S	O	L	R	A	M	E	T	X
1	2	3	4	5	6	7	8	9	10	11

- **2nd Pass:**
  - Remove the maximum, one at a time.
  - Leave them in array, instead of nulling out

```
swap(1, 9);
sink(1, 8);
```

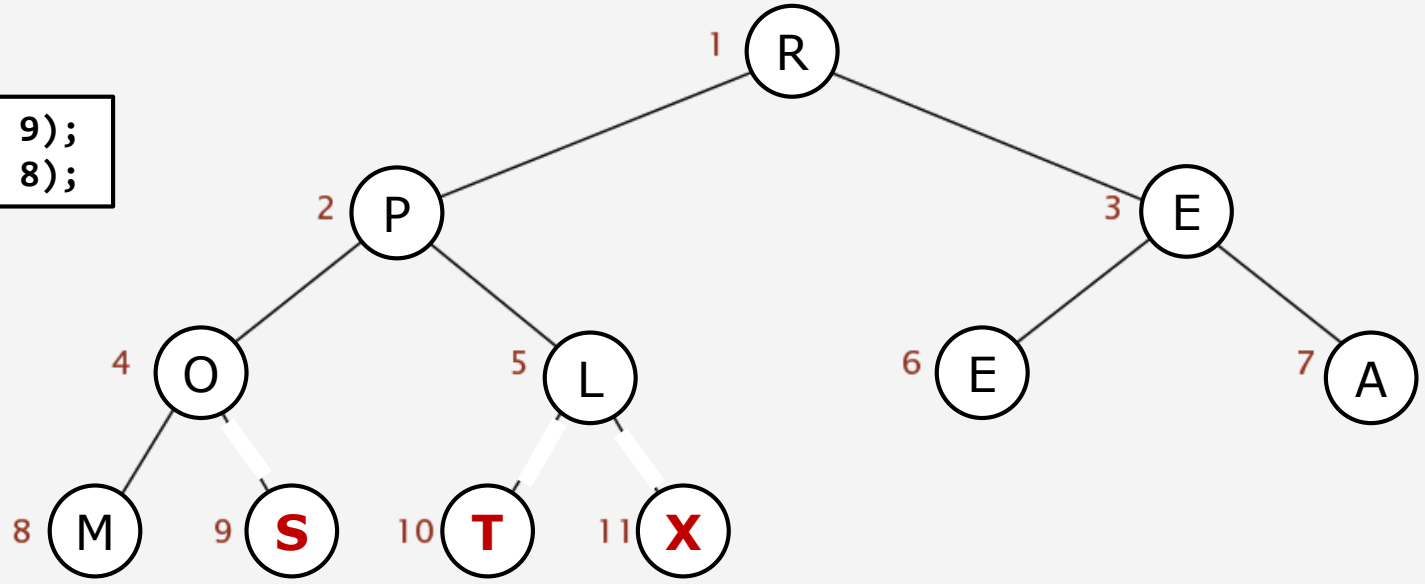


N

E	P	S	O	L	R	A	M	S	T	X
1	2	3	4	5	6	7	8	9	10	11

- **2nd Pass:**
  - Remove the maximum, one at a time.
  - Leave them in array, instead of nulling out

```
swap(1, 9);
sink(1, 8);
```



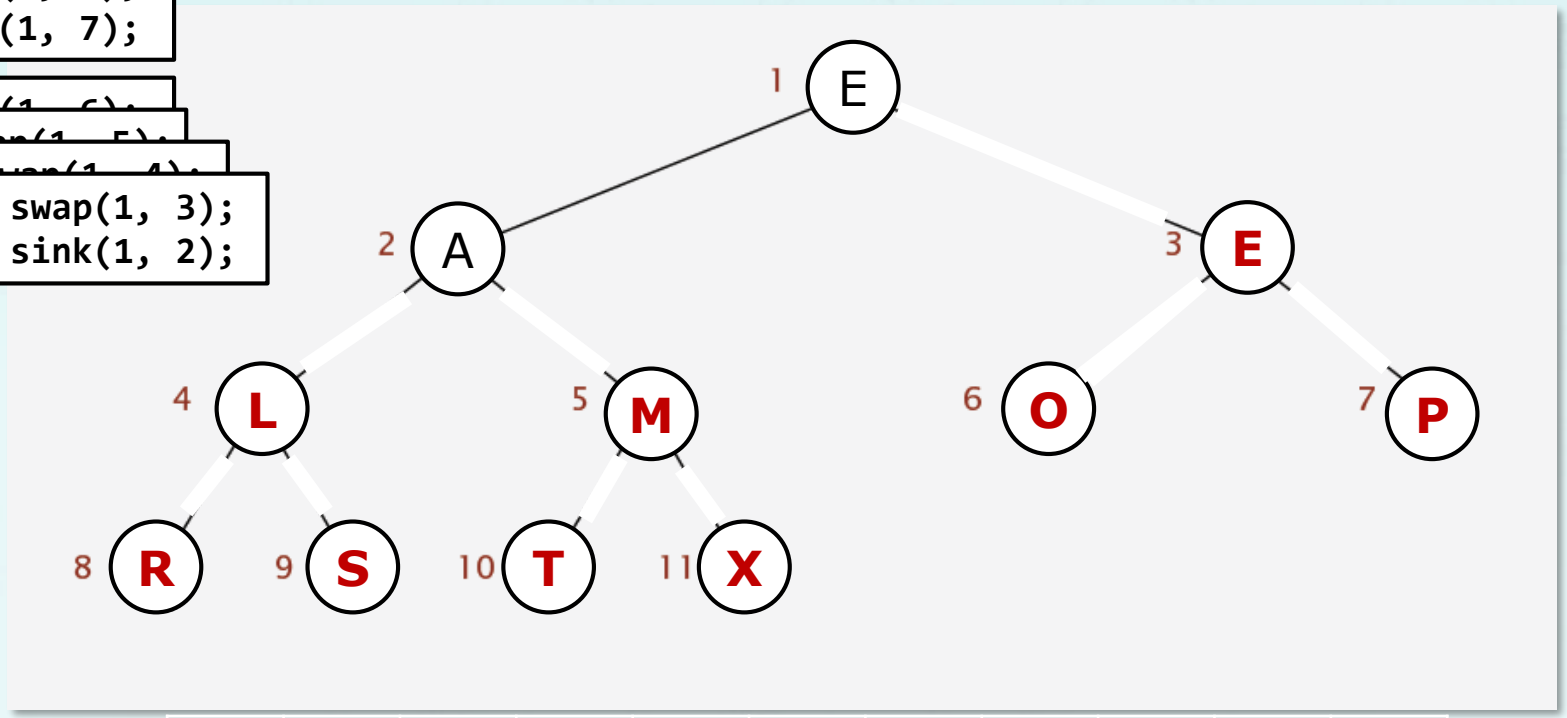
N

R	P	E	O	L	R	A	M	S	T	X
1	2	3	4	5	6	7	8	9	10	11

- **2nd Pass:**
  - Remove the maximum, one at a time.
  - Leave them in array, instead of nulling out

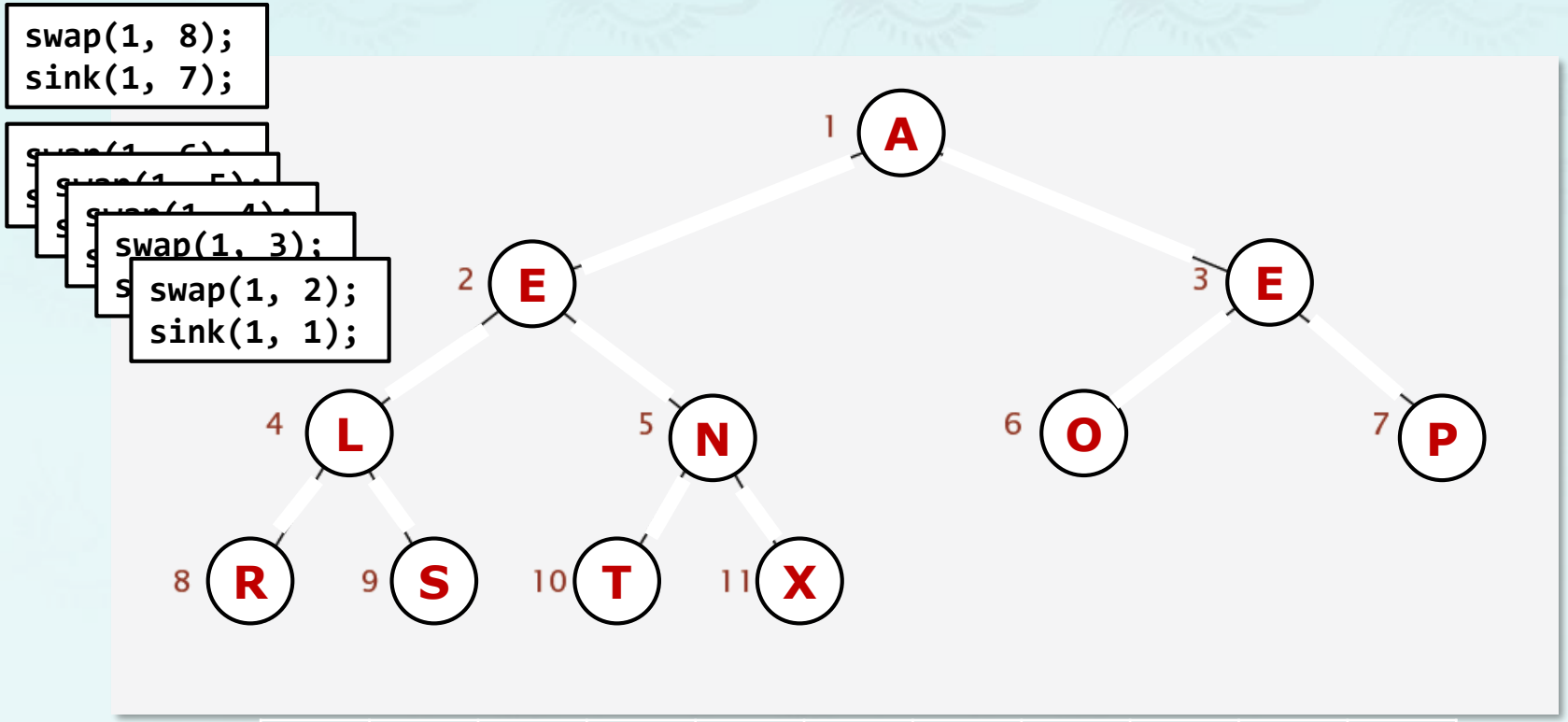
```
swap(1, 8);
sink(1, 7);
```

```
swap(1, 3);
sink(1, 2);
```



E	A	E	L	M	O	P	R	S	T	X
1	2	3	4	5	6	7	8	9	10	11

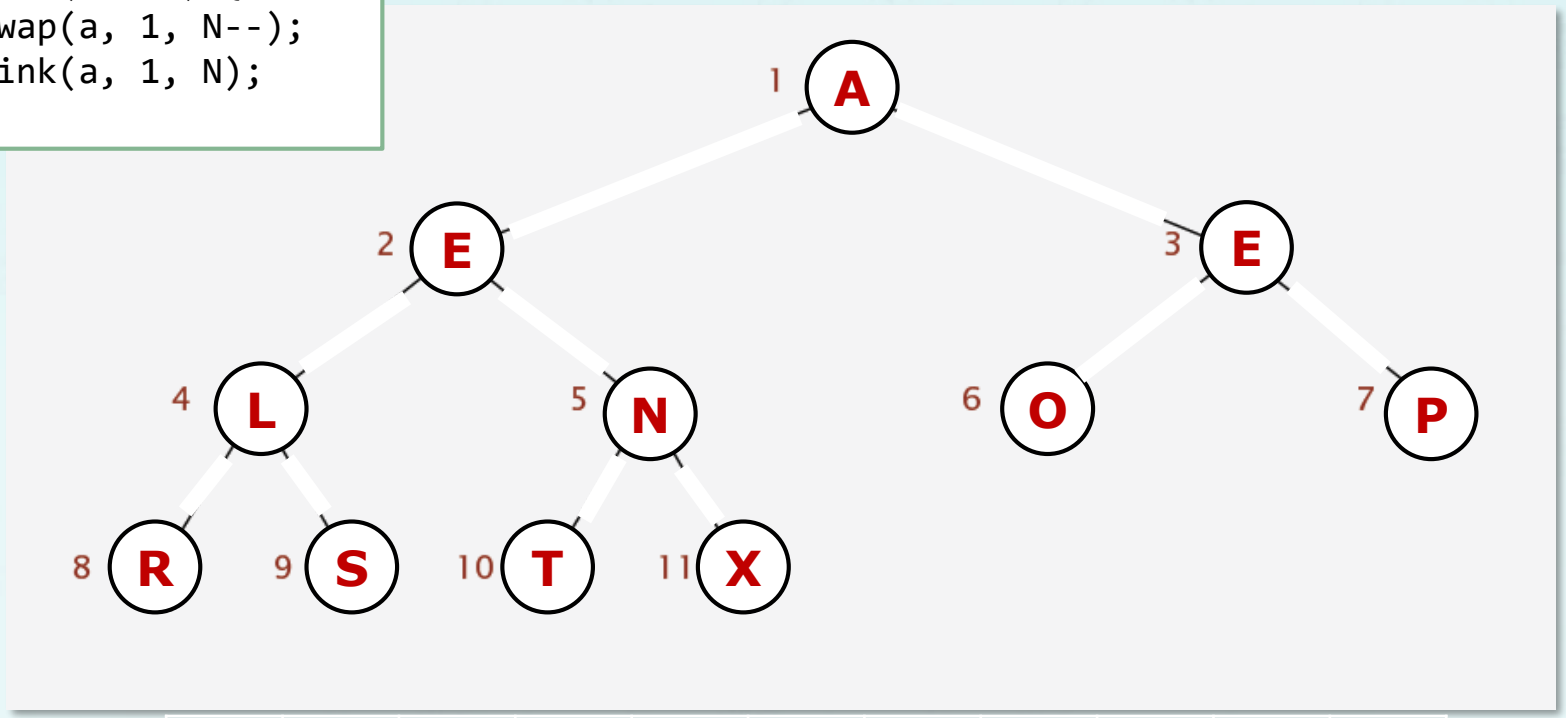
- 2nd Pass:
  - Remove the maximum, one at a time.
  - Leave them in array, instead of nulling out



A	E	E	L	N	O	P	R	S	T	X
1	2	3	4	5	6	7	8	9	10	11

- 2nd Pass:
  - Remove the maximum, one at a time.
  - Leave them in array, instead of nulling out

```
while (N > 1) {
  swap(a, 1, N--);
  sink(a, 1, N);
}
```



A	E	E	L	N	O	P	R	S	T	X
1	2	3	4	5	6	7	8	9	10	11

Chapter 7.6 Heap sort tracing

```
Enter a word to sort: SORTEXAMPLE
     Unsorted: S O R T E X A M P L E
   N=11  k=5: S O R T L X A M P E E
   N=11  k=4: S O R T L X A M P E E
   N=11  k=3: S O X T L R A M P E E
   N=11  k=2: S T X P L R A M O E E
   N=11  k=1: X T S P L R A M O E E
 Heap ordered: X T S P L R A M O E E
```

printed in main()

printed in main()

1st path printed in sink()

printed in heapSort()

Chapter 7.6 Heap sort tracing

```
Enter a word to sort: SORTEXAMPLE
     Unsorted: S O R T E X A M P L E
    N=11  k=5: S O R T L X A M P E E
    N=11  k=4: S O R T L X A M P E E
    N=11  k=3: S O X T L R A M P E E
    N=11  k=2: S T X P L R A M O E E
    N=11  k=1: X T S P L R A M O E E
  Heap ordered: X T S P L R A M O E E
    N=10  k=1: T P S O L R A M E E
    N= 9  k=1: S P R O L E A M E
    N= 8  k=1: R P E O L E A M
    N= 7  k=1: P O E M L E A
    N= 6  k=1: O M E A L E
    N= 5  k=1: M L E A E
    N= 4  k=1: L E E A
    N= 3  k=1: E A E
    N= 2  k=1: E A
    N= 1  k=1: A
       Sorted: A E E L M O P R S T X
```

printed in main()

printed in main()

1st path printed in sink()

printed in heapSort()

2nd path printed in sink()

printed in main()

Chapter 7.6 Heap sort tracing

```
Enter a word to sort: SORTEXAMPLE
     Unsorted: S O R T E X A M P L E
    N=11  k=5: S O R T L X A M P E E
    N=11  k=4: S O R T L X A M P E E
    N=11  k=3: S O X T L R A M P E E
    N=11  k=2: S T X P L R A M O E E
    N=11  k=1: X T S P L R A M O E E
 Heap ordered: X T S P L R A M O E E
    N=10  k=1: T P S O L R A M E E
    N= 9  k=1: S P R O L E A M E
    N= 8  k=1: R P E O L E A M
    N= 7  k=1: P O E M L E A
    N= 6  k=1: O M E A L E
    N= 5  k=1: M L E A E
    N= 4  k=1: L E E A
    N= 3  k=1: E A E
    N= 2  k=1: E A
    N= 1  k=1: A
       Sorted: A E E L M O P R S T X
```

printed in main()

printed in main()

1st path printed in sink()

printed in heapSort()

2nd path printed in sink()

printed in main()

- **NOTE: This implementation does not sort the first element in the array.**
- **NOTE: N=?? k=? .... lines are outputs at the end of each sink()**

ITP20001/ECE 20010 Data Structures

Chapter 5

- *introduction*
- *tree, binary tree, binary search tree*
- *heaps data structure*
  - *complete binary tree*
  - *priority queues (Chapter 9)*
  - *binary heap and min-heap*
  - *max-heap demo*
  - *max-heap implementation*
  - ***heap sort (Chapter 7)***