ITP20001/ECE20010 Data Structures

Chapter 5

- *introduction*
- *binary tree*
- *complete binary tree*
  - *max heap, min heap*
  - *Chapter 7 – heap sorting*
  - *Chapter 9 - priority queues*
- ***binary search tree***

Major references:
1. Fundamentals of Data Structures by Horowitz, Sahni, Anderson-Freed,
2. Algorithms 4th edition - Part 1 & Part 2 by Robert Sedgewick and Kevin Wayne
3. Wikipedia and many resources available from internet

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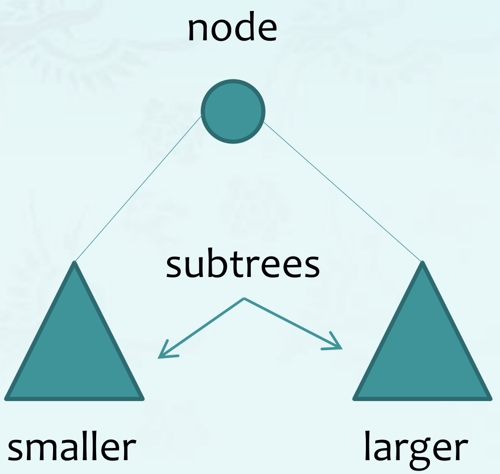
Definition: A binary search tree is a binary tree in symmetric order.

- A binary tree is either
  - empty
  - a key-value pair and two binary trees [neither of which contain that key]

equal keys ruled out

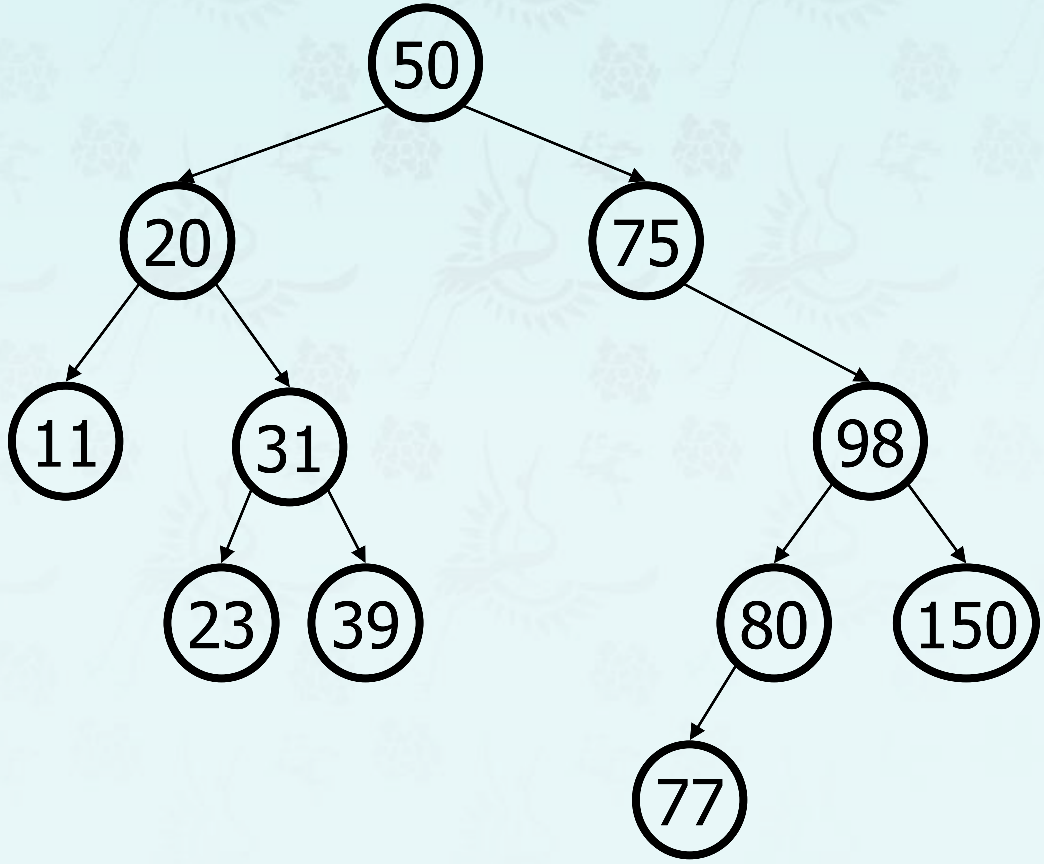
- Symmetric order means that
  - every node has a key
  - every node's key is
    larger than all keys in its left subtree
    smaller than all keys in its right subtree



Operations: Insert

- Q: Draw what a binary search tree would look like if the following values were added to an initially empty tree in this order:

50
20
75
98
80
31
150
39
23
11
77



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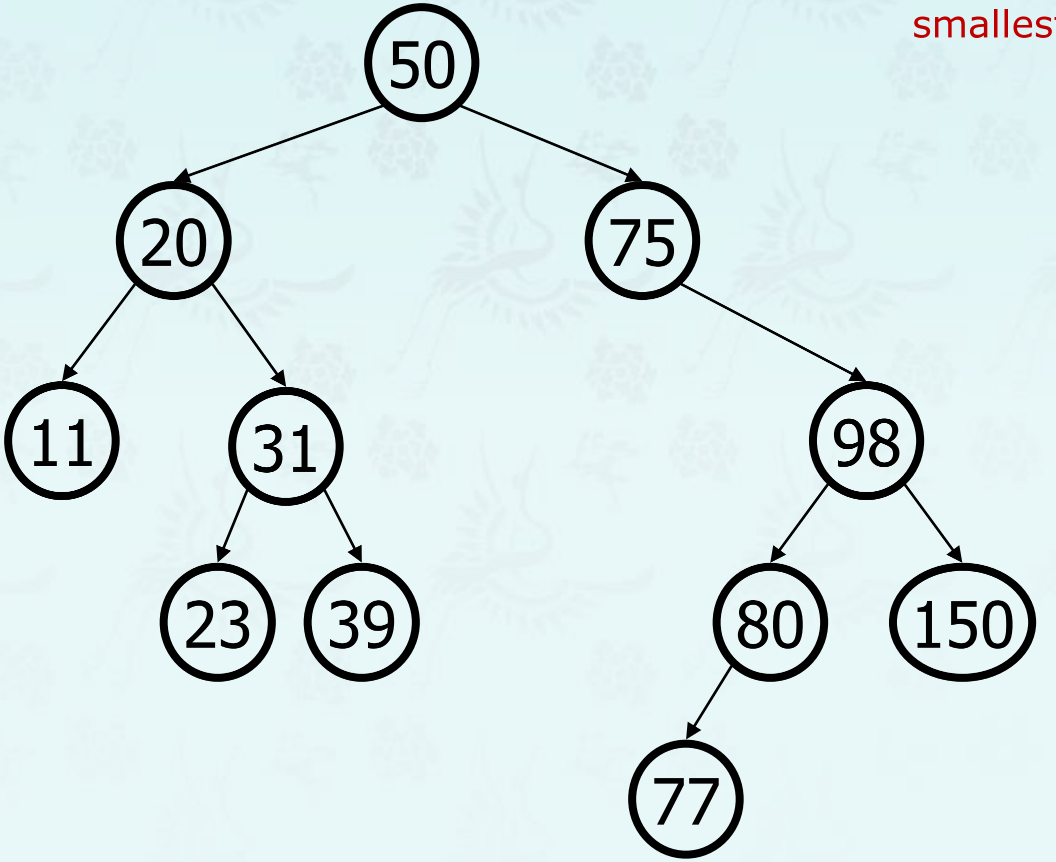

smallest?

Operations: Insert

- Q: Draw what a binary search tree would look like if the following values were added to an initially empty tree in this order:

50
20
75
98
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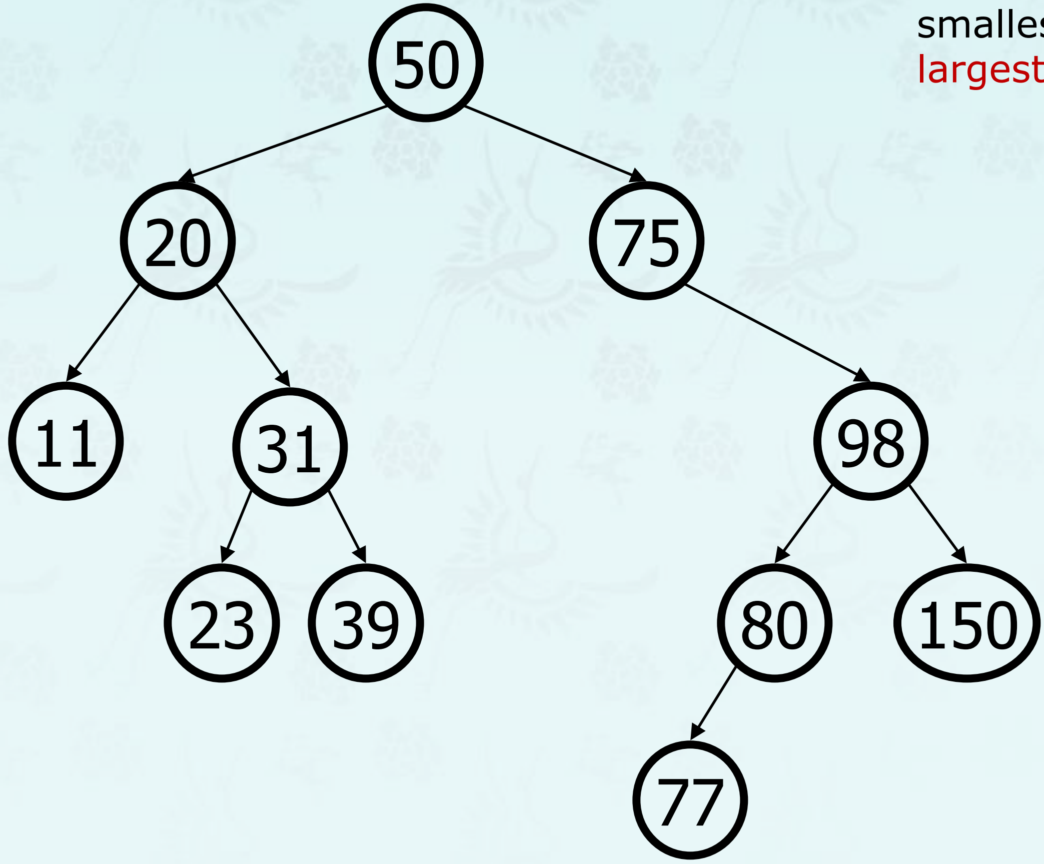
77



smallest?
largest?

Operations: Insert

- Q: Draw what a binary search tree would look like if the following values were added to an initially empty tree in this order:

50
20
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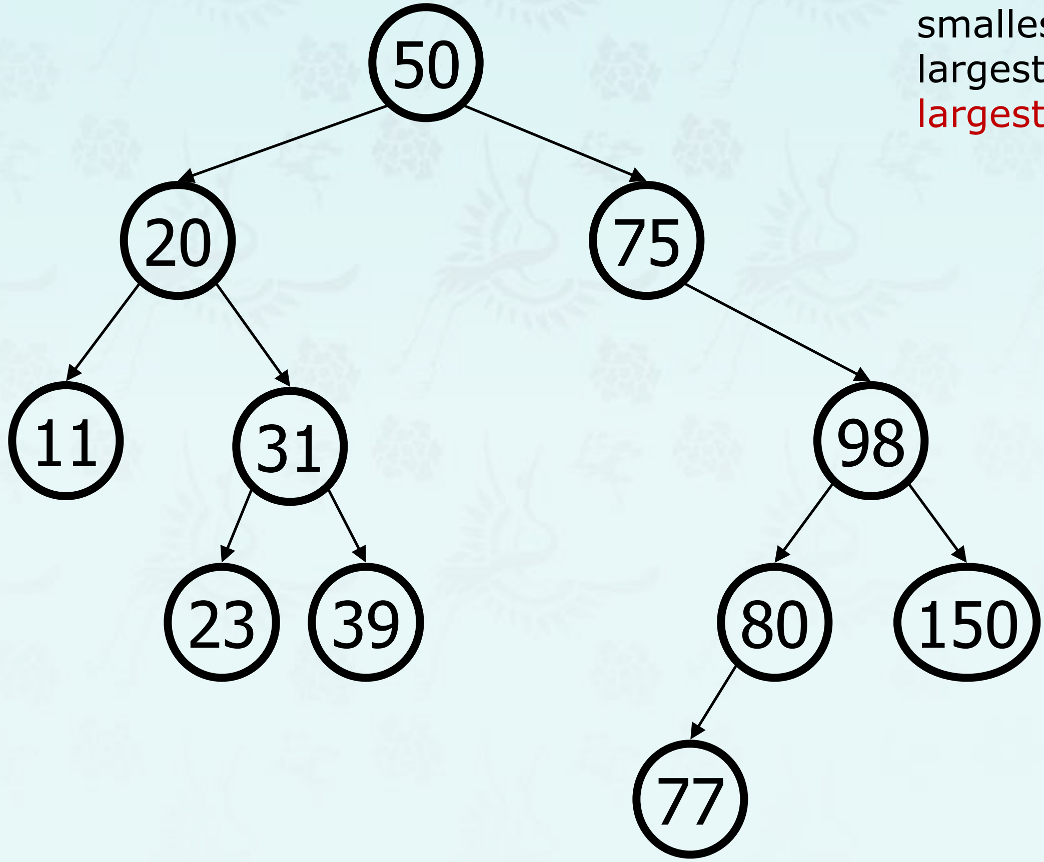
150
39
23
11
77



smallest?
largest?
largest in left?

Operations: Insert

- Q: Draw what a binary search tree would look like if the following values were added to an initially empty tree in this order:

50
20
75
98
80
31
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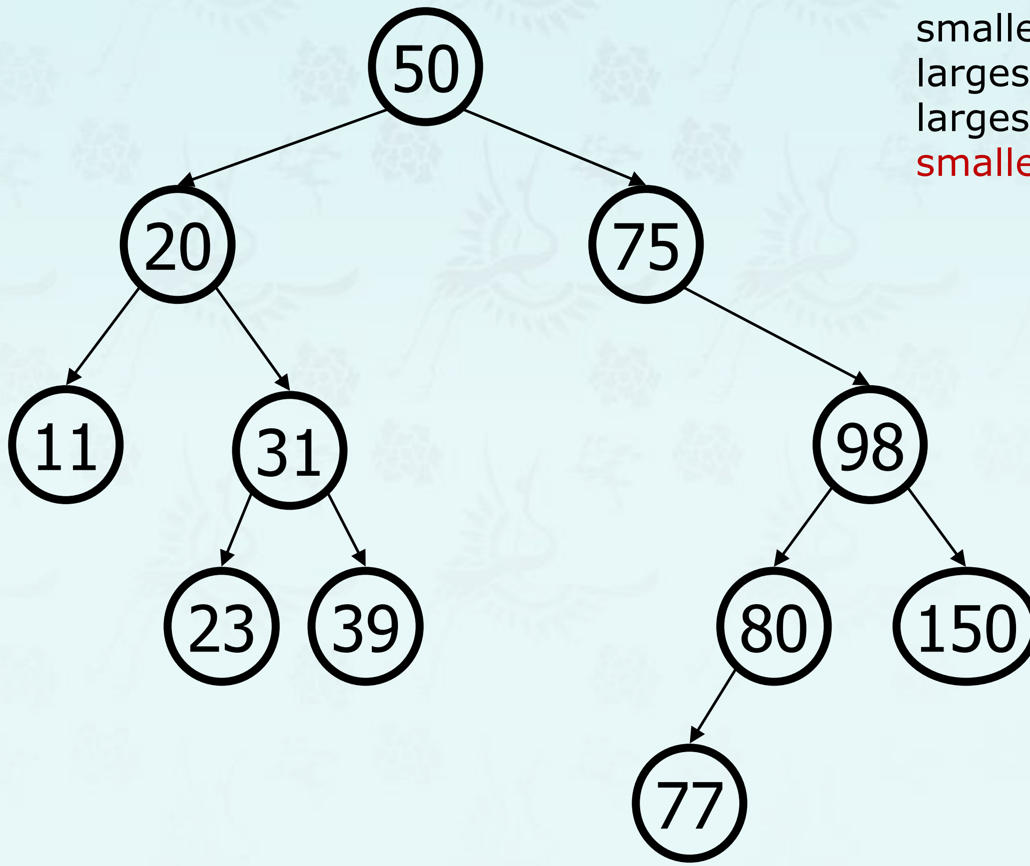
11
77

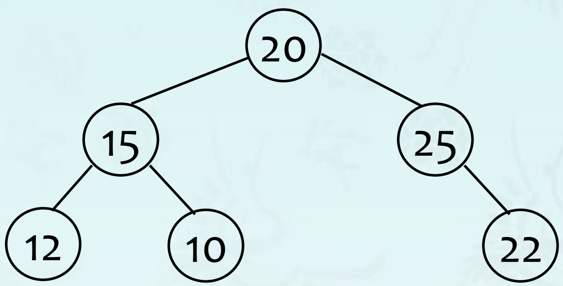


smallest?
largest?
largest in left?
smallest in right?

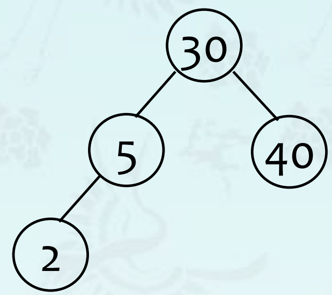
Chapter 5.7 Binary search trees

Definition: A binary search tree is a binary tree in symmetric order.

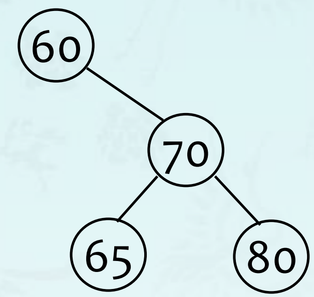
Exercise: Identify non-BST(s) and correct them if not.



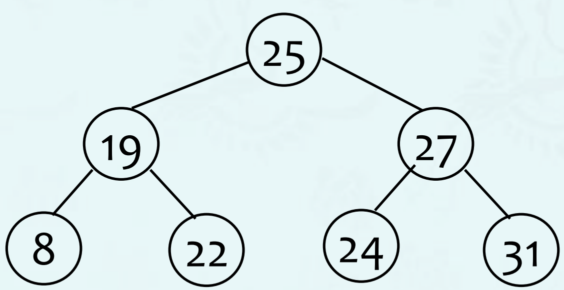
(a)



(b)



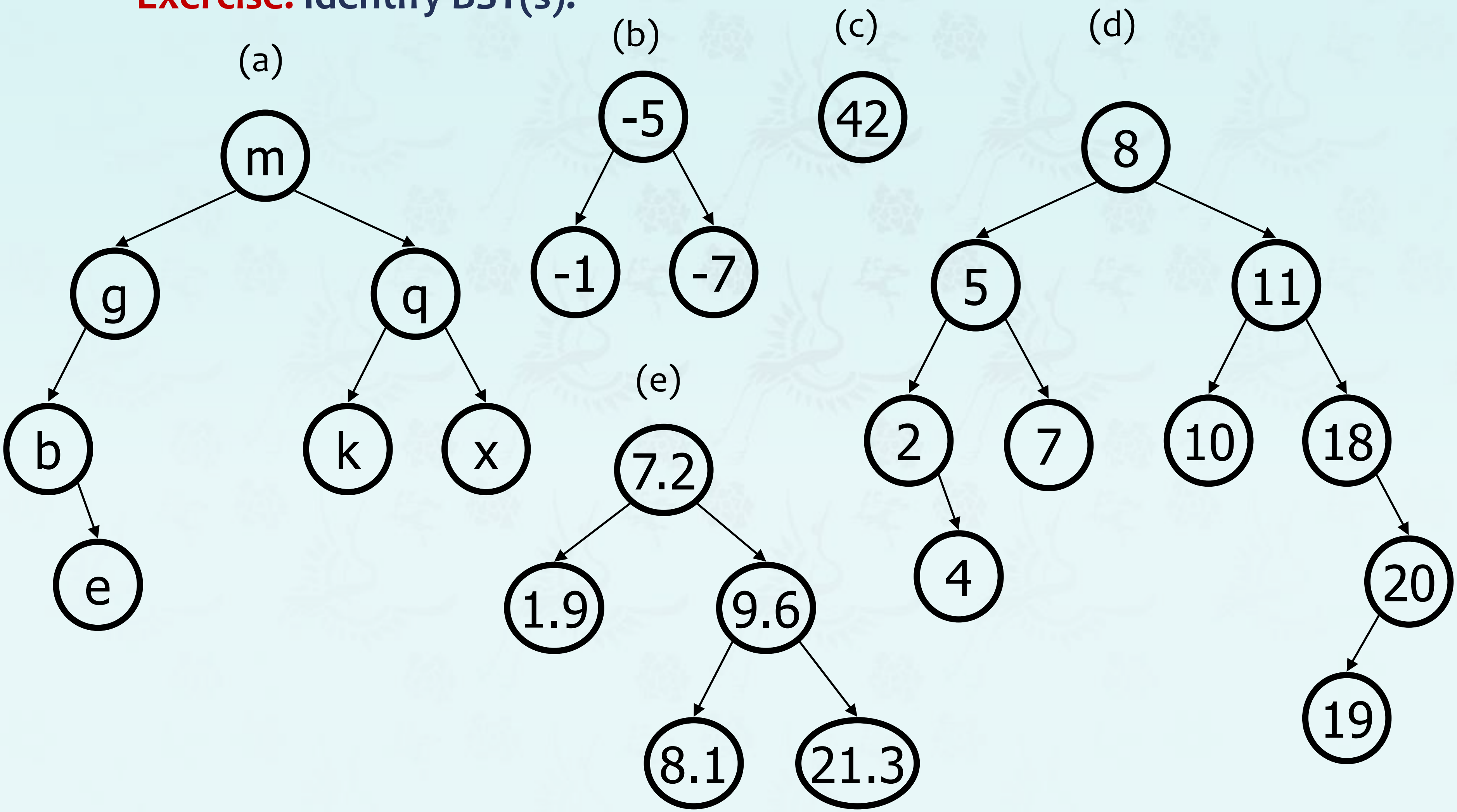
(c)



(d)

Definition: A binary search tree is a binary tree in symmetric order.

Exercise: Identify BST(s).



Node structure:

<table>
<tr><td colspan="2">Key</td></tr>
<tr><td colspan="2">Value</td></tr>
<tr><td>Left</td><td>Right</td></tr>
</table>

Operations:

- **Query – search, min/max, successor, predecessor**
- **Insert**
- **Delete**

Chapter 5.7 Binary search trees

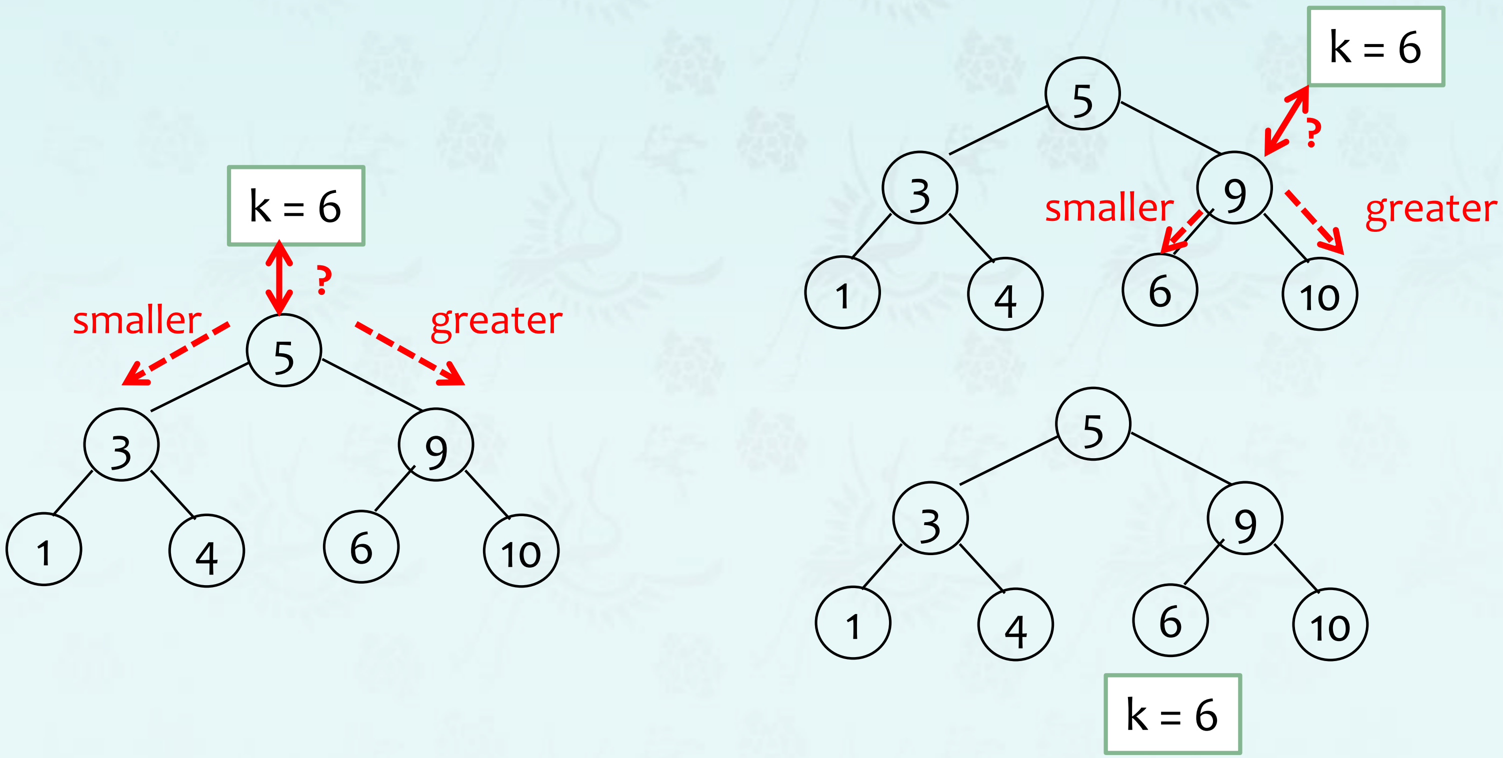
Binary search tree(BST) node structure:

Key key	
pValue value	
pTree left	pTree right

```
typedef int        Key;     // can be replace by different type
typedef char       Value;
typedef char*      pValue;

typedef struct node *pTree;
typedef struct node {
  pTree  left;              // left child
  pTree  right;             // right child
  Key    key;               // sorted by key
  pValue value;             // associated data with key
} node;
```

Operations: Search or "contains"

Search(T, k) – search the BST, T for a key k



❖ Search operation takes time O(h), where h is the height of a BST.

Operations: Search or “contains”

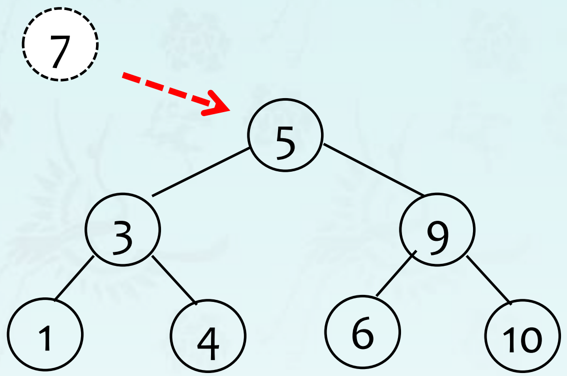
```
// does there exist a key-value pair with given key?
// search a key in binary search tree iteratively

int containsIteration(pTree node, Key key)
{
    if (node == NULL) return false;
    while (node) {
        if (key == node->key) return true;
        if (key  < node->key)
            node = node->left;
        else
            node = node->right;
    }
    return false;
}
```

Operations: Search or "contains"

```
// does there exist a key-value pair with given key?
// search a key in binary search tree recursively

int contains(pTree node, Key key)
{
    if (node == NULL)        return false;

    if (key  == node->key) return true;

    if (key  <  node->key)
        return contains(node->left, key);

    return contains(node->right, key);
}
```

Operations: Insert

- Insert(**T**, k)
  - Insert a node with Key = k into BST **T**
  - Time complexity? O(h)
- **Step 1**:
  if the tree is empty,
  then Root(**T**) = k
- **Step 2**:
  Pretending we are searching for k in BST,
  until we meet a null node
- **Step 3**:
  Insert k



Q: Where is it inserted at?

Operations: Insert

- Insert(**T**, k)
  - Insert a node with Key = k into BST **T**
  - Time complexity? O(h)
- **Step 1**:
  if the tree is empty,
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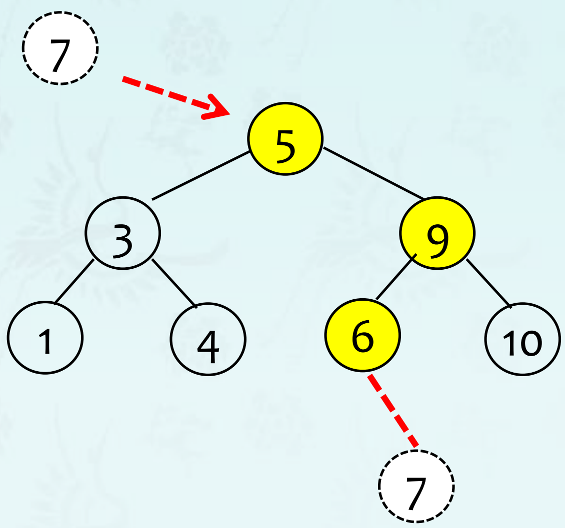
- **Step 3**:
  Insert k



The light nodes are compared with key.

Operations: Insert

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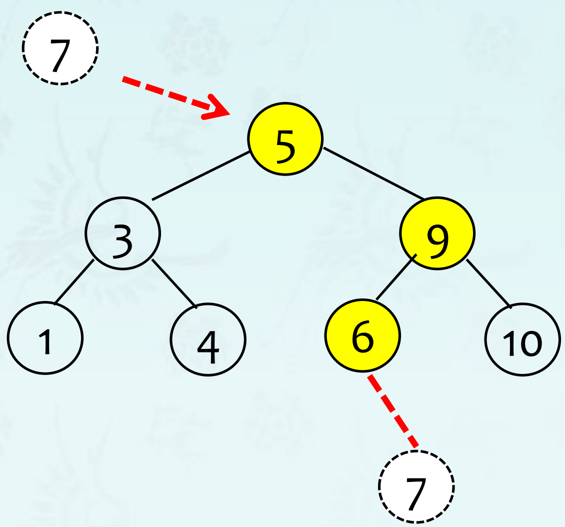
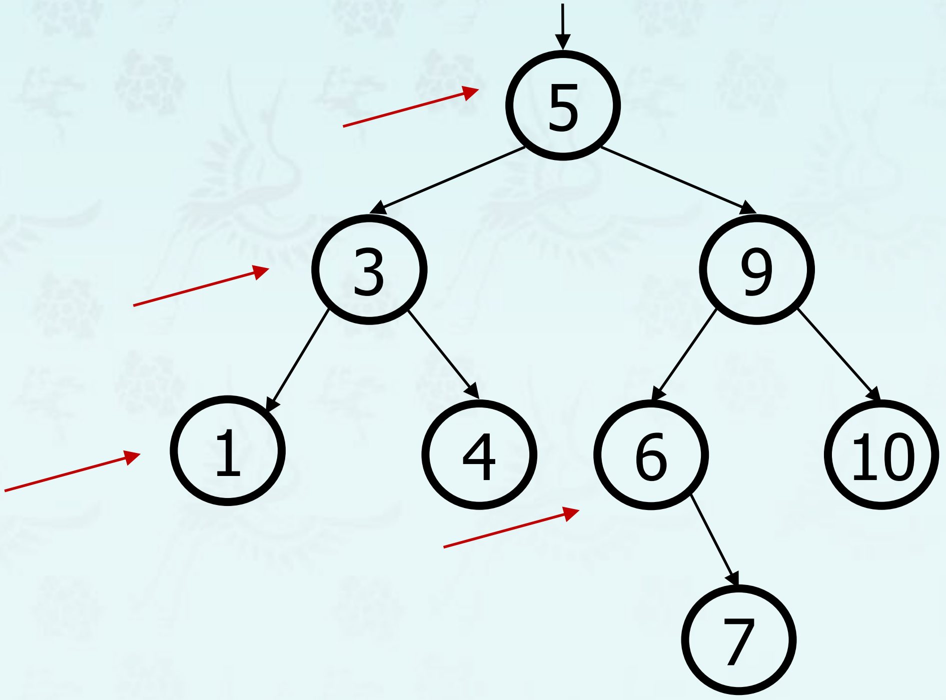


The light nodes are compared with key.

Q: Do you see the difference between the complete binary tree and binary search tree?

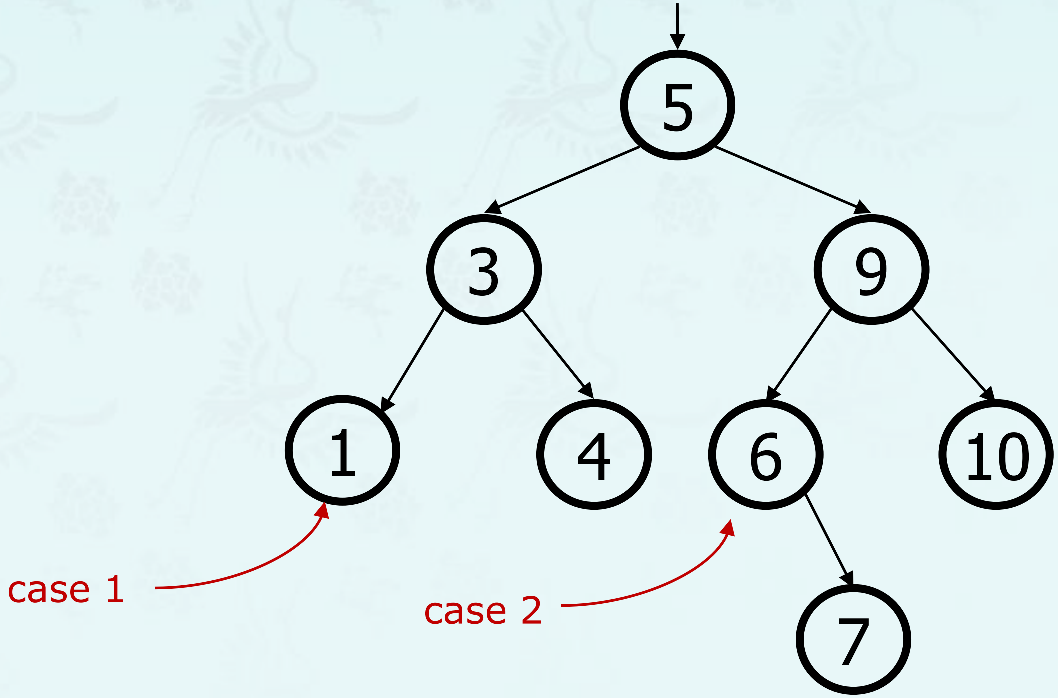
Operations: Delete

- How can we delete a value from a BST in such a way as to maintain proper BST ordering?
  - `delete(1);`
  - `delete(3);`
  - `delete(6);`
  - `delete(5);`



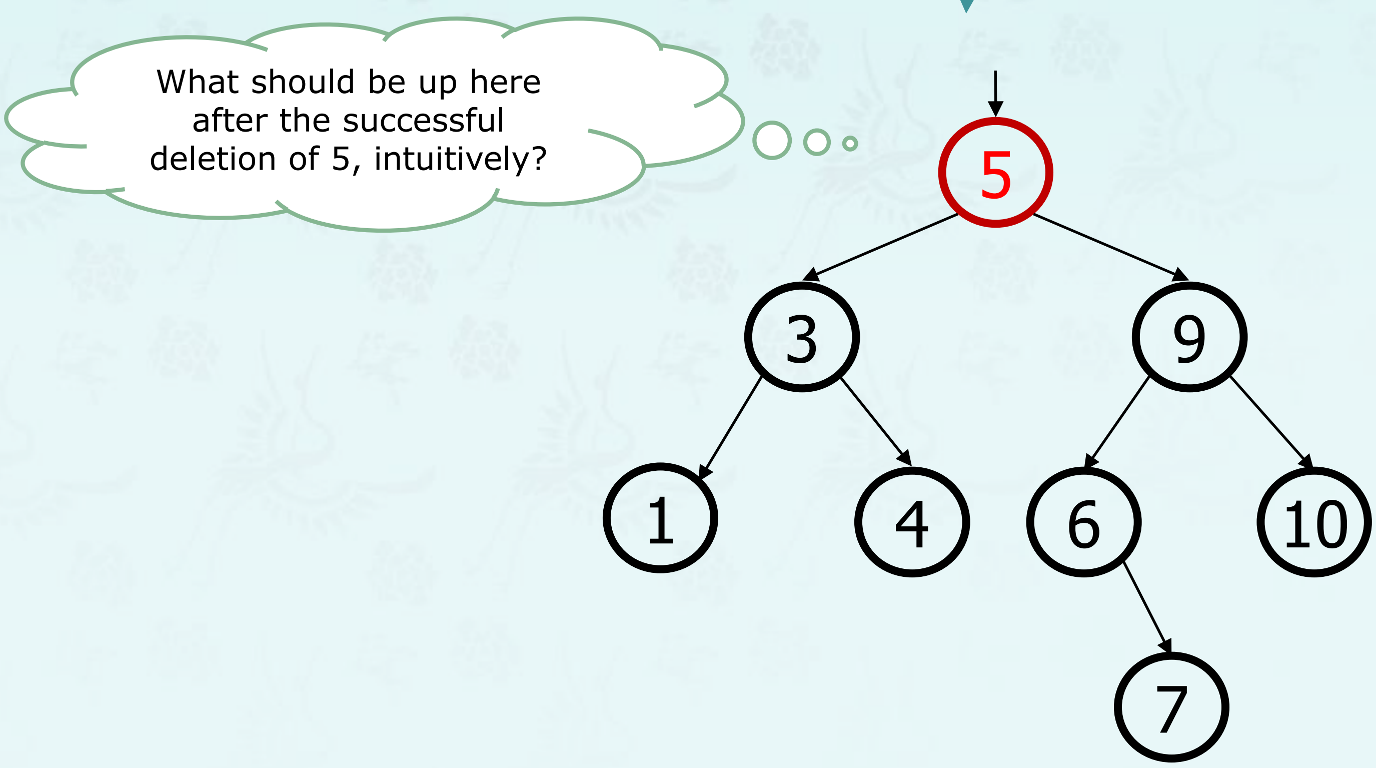
Operations: Delete

- **case 1:** leaf
  - a leaf — replace with NULL
- **case 2:** one child case
  - a node with a left child only — replaced with left child
  - a node with a right child only — replaced with right child
- **case 3:** ?



Operations: Delete

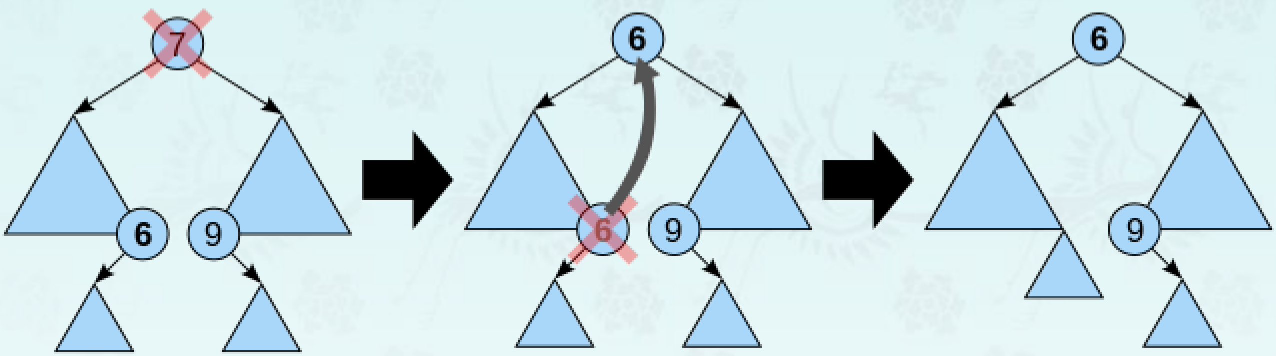
- **case 3**: two children case
  - What can we replace **5** with?



Operations: Delete

- **case 3:** two children case

Where is predecessor or successor of root 7?



1. The rightmost node in the left subtree, the inorder **predecessor 6**, is identified.
2. Its value is copied into the node being deleted.
3. The inorder **predecessor** can then be deleted because it has at most one child.

NOTE: The same method works symmetrically using the inorder **successor** labelled **9**.

Operations: Delete

- **case 3**: two children case

Idea: Replace the deleted node with a value guaranteed to be between the two child subtrees

Options:

- *predecessor* from left subtree: **`findMax( )`**
- *successor* from right subtree: **`findMin( )`**
  - These are the easy cases of predecessor/successor

Now delete the original node containing *successor* or *predecessor*

- It becomes leaf or one child case – easy cases of delete!

Operations: Delete

- **case 3**: two children case

Idea: Replace the deleted node with a value guaranteed to be between the two child subtrees

Options:

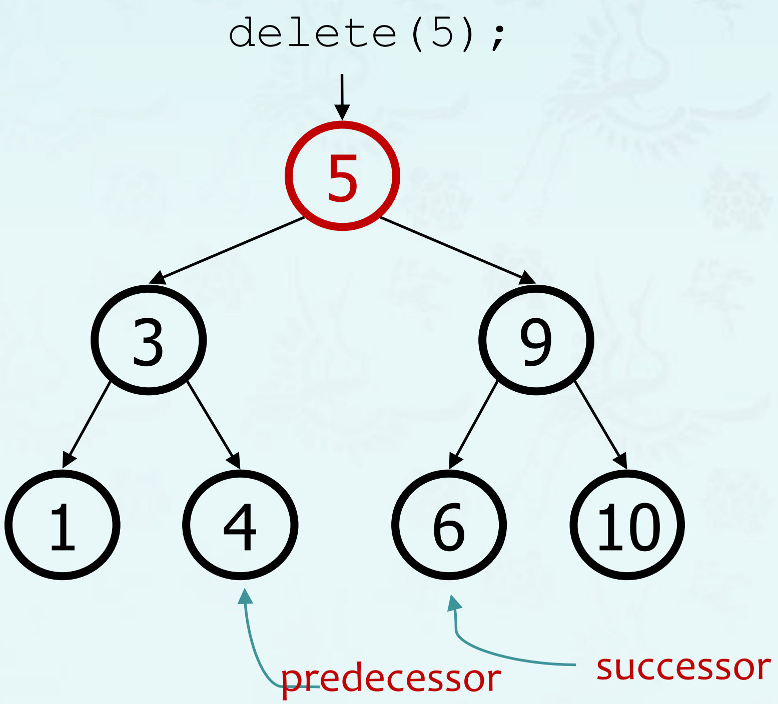
- ***predecessor*** from left subtree: `findMax(node->left )`
- ***successor*** from right subtree: `findMin(node->right)`
  - These are the easy cases of predecessor/successor

Now delete the original node containing *successor* or *predecessor*

- It becomes leaf or one child case – easy cases of delete!

Operations: Delete

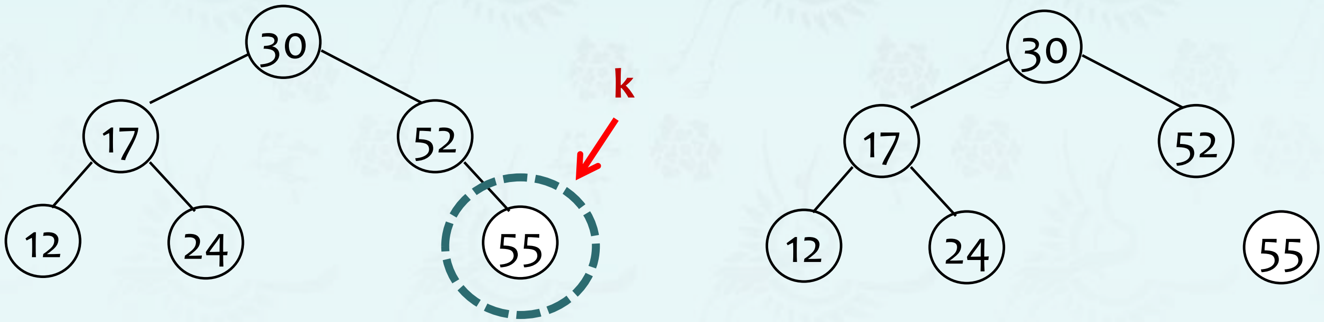
- **case 3**: two children case
  - Replace with min from right or max from left
  - **Where is predecessor or successor of root 5?**



Operations: Delete

- Delete(**T**, k)
    - Delete a node with Key = k into BST **T**
    - Time complexity: O(h)

Case 1: k has no child

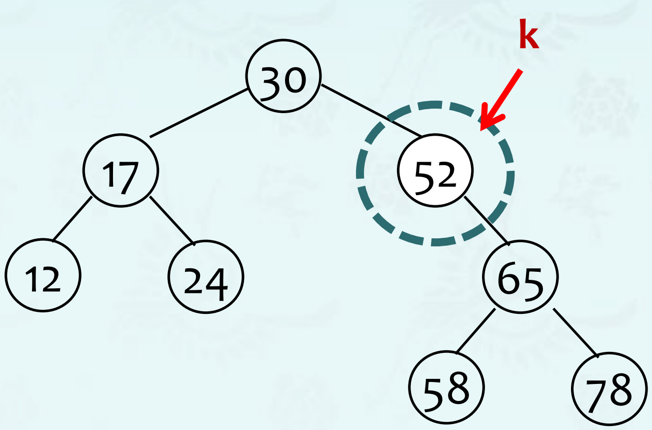


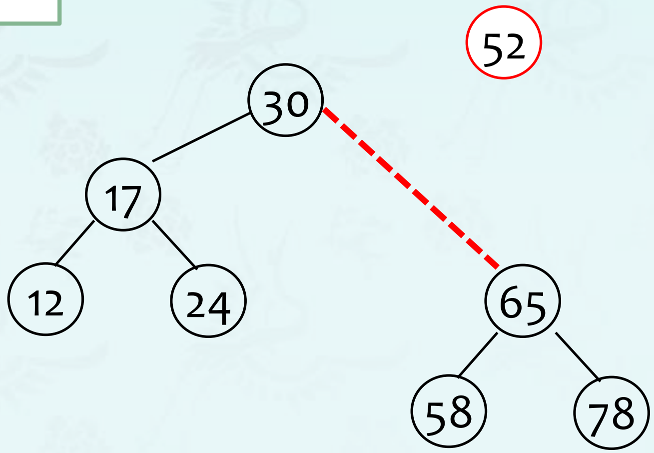
We can simply delete it from the tree

Operations: Delete

- Delete(**T**, k)
  - Delete a node with Key = k into BST **T**
  - Time complexity: O(h)

Case 2: **k has one child**



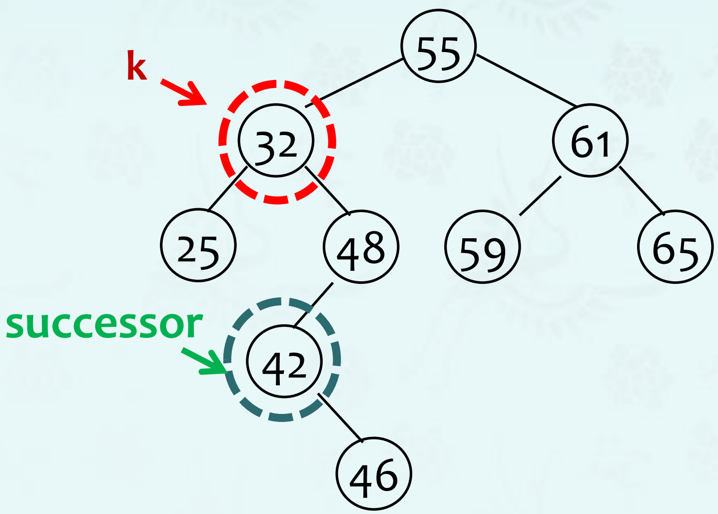


After removing it, connect it's subtree to it's parent node.

Operations: Delete

- Delete(**T**, k)
  - Delete a node with Key = k into BST **T**
  - Time complexity: O(h)

Case 3: k has two children

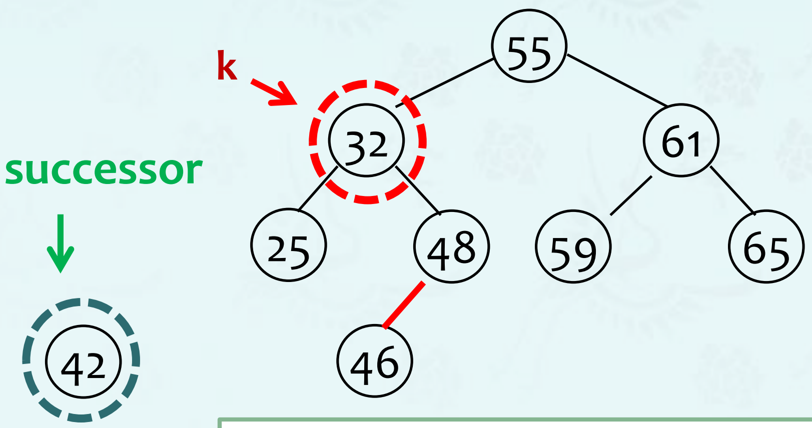


Find it's successor

Operations: Delete

- Delete(**T**, k)
  - Delete a node with Key = k into BST **T**
  - Time complexity: O(h)

Case 2: **k has two children**



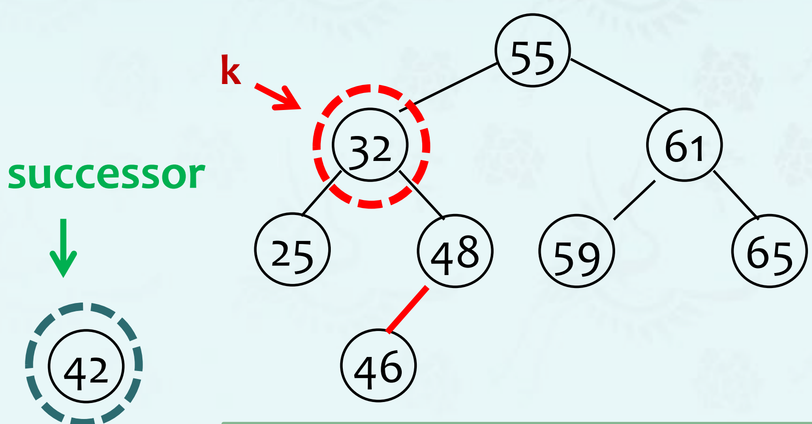
Pull out successor,
and connect the tree with it's child

Q: What if successor has **two** children?

Operations: Delete

- Delete(**T**, k)
  - Delete a node with Key = k into BST **T**
  - Time complexity: O(h)

Case 2: k has two children



Pull out successor,
and connect the tree with it’s child

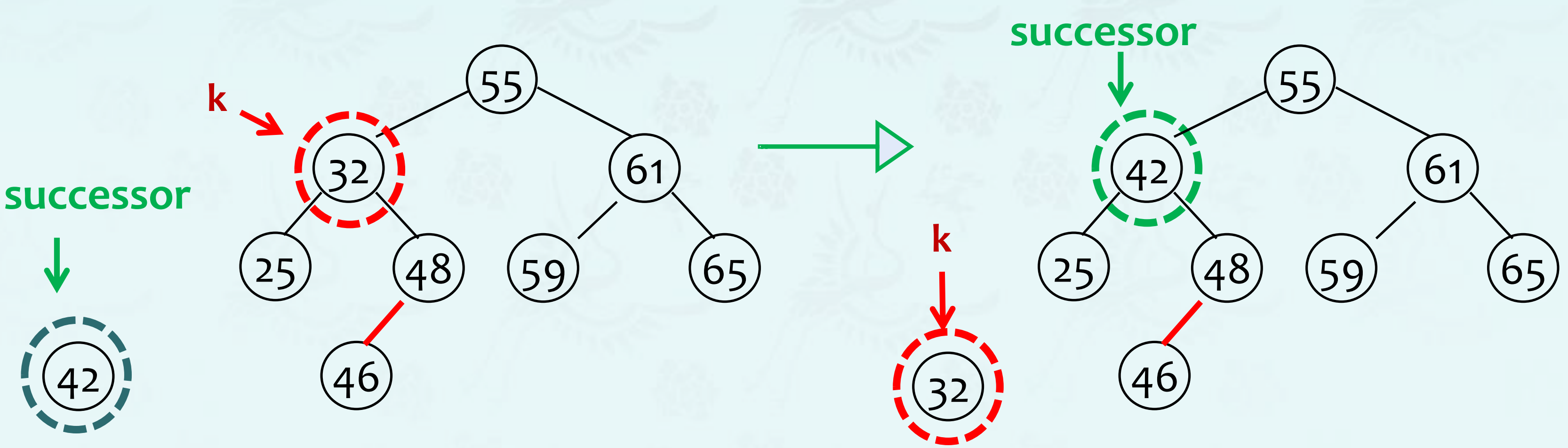
A: Not possible !
Because if it has two nodes, at least one of them is less than it, then in the process of finding successor, we won't pick it !

Q: What if successor has **two** children?

Operations: Delete

- Delete(**T**, k)
    - Delete a node with Key = k into BST **T**
    - Time complexity: O(h)

Case 2: k has two children



Replace the key with it’s successor

More Operations:

- **Query – search, min/max, successor, predecessor**

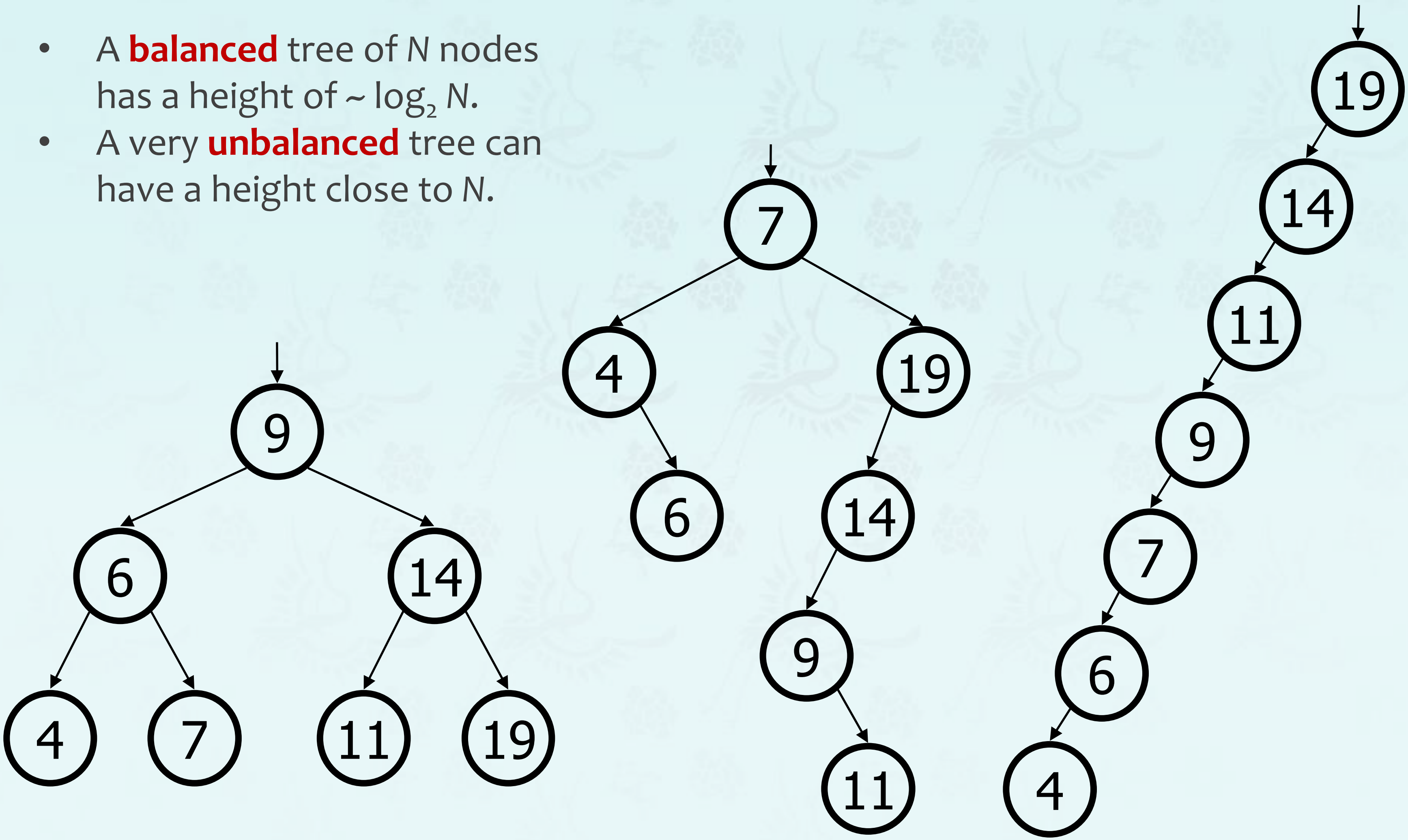
Min/max

- For min, we simply follow the left pointer until we find a null node. Why?
- Similar for Max
- Time complexity: O(h)

❖ Search operation takes time O(h), where h is the height of a BST.

Observations: What do you see in the following BSTs?

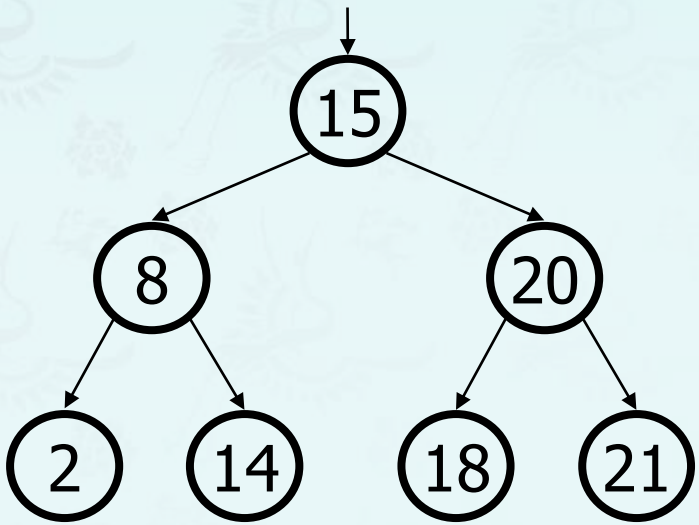
- A **balanced** tree of N nodes has a height of ~ $\log_2 N$.
- A very **unbalanced** tree can have a height close to N.



Observations: What do you see in the following BSTs?

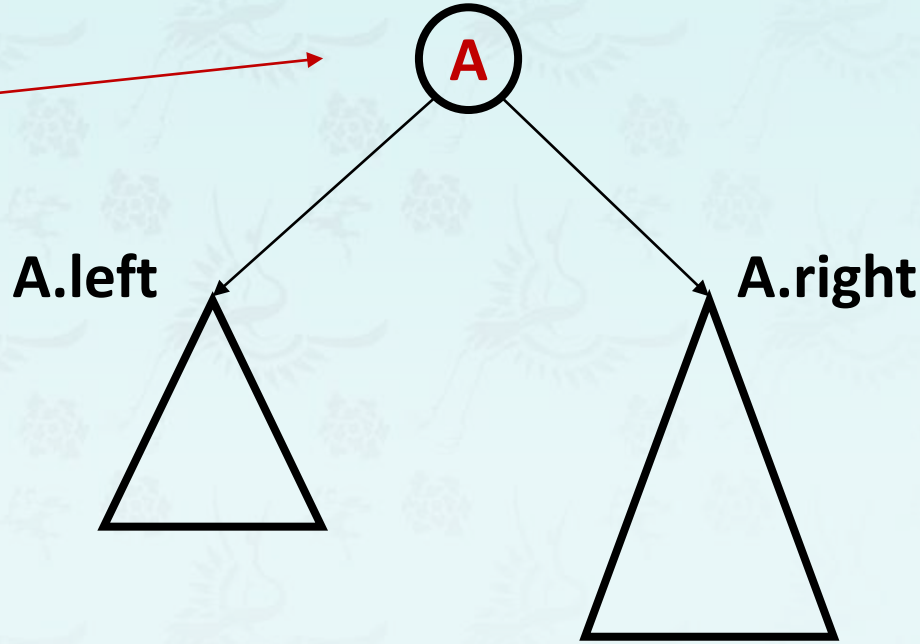
- *Observation:* The shallower the BST the better.
  - Average case height is O(log N)
  - Worst case height is O(N)
  - Simple cases such as adding (1, 2, 3, ..., N), or the opposite order, lead to the worst case scenario: height O(N).

- For binary tree of height h:
  - max # of leaves: 2^{h-1}
  - max # of nodes: $2^h - 1$
  - min # of leaves: 1
  - min # of nodes: h



Q: Calculate tree height.

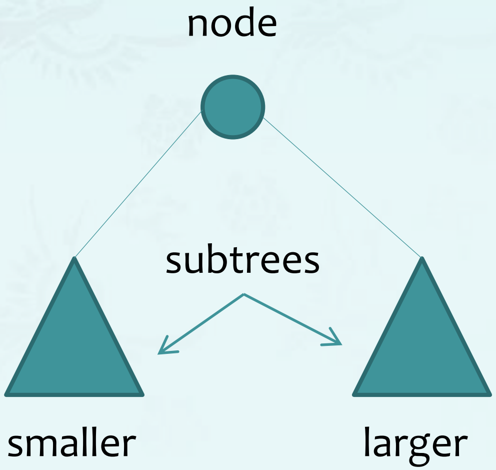
- ***Height*** *is* max number of nodes in path from root to any leaf.
  - height(null) = 0
  - height(a leaf) = ?
  - height(**A**) = ?
- ***Hint***:
  - use recursive.
  - use max(a, b).



- ***A:***
  - height(a leaf) = 1
  - height(A) = 1 + max( )

Conclusion:

- If you have a sorted sequence, and we want to design a data structure for it
- **Array or BST? and why?**



Conclusion:

- If you have a sorted sequence, and we want to design a data structure for it
- **Array or BST? and why?**

Time Complexity	
BST	$O(h)$
Array	$O(\log n)$

Conclusion:

Q. When searching, we're traversing a path (since we're always moving to one of the children); since the length of the longest path is the height h of the binary search tree, then finding an element takes $O(h)$.

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Since $h = \lg n$ (where n is the number of elements), then it's good! – right?

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No, of course, it is wrong! Why?

Conclusion:

Q. When searching, we're traversing a path (since we're always moving to one of the children); since the length of the longest path is the height h of the binary search tree, then finding an element takes $O(h)$.

Since $h = \lg n$ (where n is the number of elements), then it's good! – right?

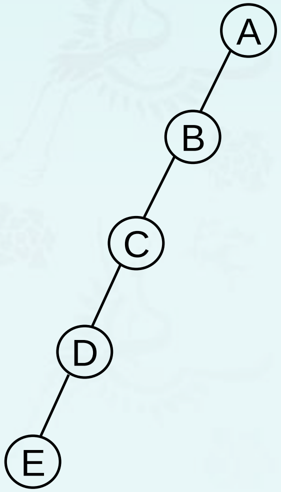
No, of course, it is wrong! Why?

A. The nodes could be arranged in linear sequence in BST, so the $\boldsymbol{height}$ $\boldsymbol{h}$ could be $\boldsymbol{n}$. In worst case, it is $\boldsymbol{O(n)}$ instead of $\boldsymbol{O(h)}$.

Conclusion:

- We already know that n is fixed, but h differs from how we insert those elements!

- So why we still need BST?
  - Easier insertion and deletion
  - And with some optimization, we can avoid the worst case!



$n = h$

a skew binary search tree