Welcome to Data Structures(ECE20010/ITP20001)

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Data Structures

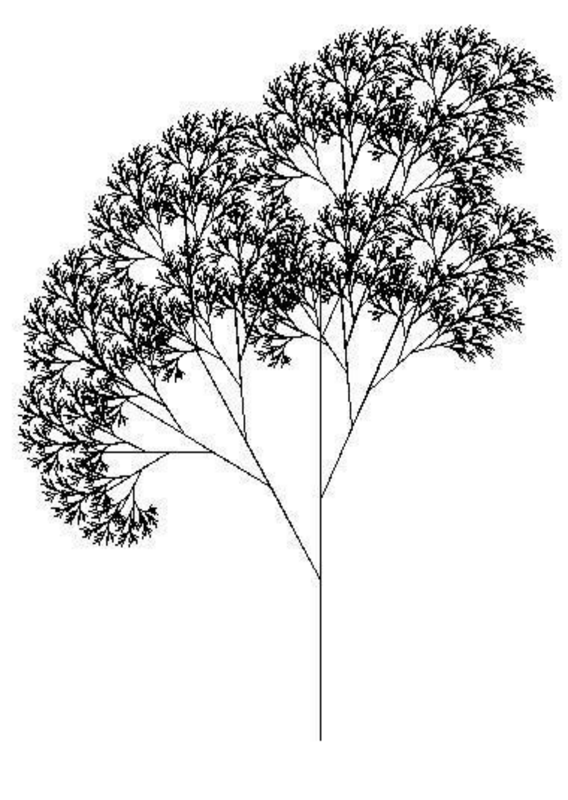
Chapter 1

- ***algorithm specification***
  recursive algorithm
- *data abstraction*
- *performance analysis - time complexity*

1.3 Algorithm specification (p.8)

- Input
- Output
- Definiteness – clear and unambiguous
- Finiteness – it terminates after a finite number of steps
- Effectiveness – it is carried out and feasible

- Ex. **program = algorithms + data structures**
  flowchart is not an algorithm.



Recursion

- When solving a problem using **recursion**, the idea is to transform a big problem into a **smaller**, similar problem.
- Eventually, as this process **repeats itself** and the size of the problem is reduced at each step, we will arrive at a **very small**, **easy-to-solve** problem.

Exercise: With five students, compute 4! using recursion.

1.3 Recursive algorithms

Execution sequence of recursive functions:

Exercise: What is the output of the function (num=0)?

execution sequence

```
void  recursiveFunction(int num) {
      printf("%d\n", num);
      if (num < 4)
            recursiveFunction(num + 1);
}
```

1.3 Recursive algorithms

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```

1	recursiveFunction (0)				
2	printf (0)				
3		recursiveFunction (0+1)			
4		printf (1)			
5			recursiveFunction (1+1)		
6			printf (2)		
7				recursiveFunction (2+1)	
8				printf (3)	
9					recursiveFunction (3+1)
10					printf (4)

1.3 Recursive algorithms

Execution sequence of recursive functions:

Exercise: What is the output of the function (num=0)?

execution sequence

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void  recursiveFunction(int num) {
      if (num < 4)
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}
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1	recursiveFunction (0)				
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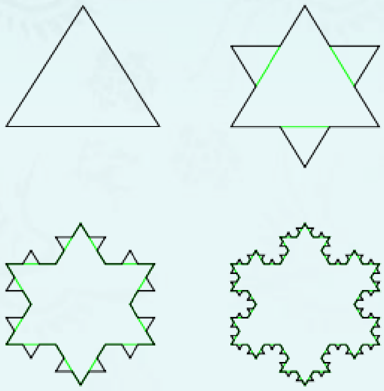
5					recursiveFunction (3+1)
6					printf (4)
7				printf (3)	
8			printf (2)		
9		printf (1)			
10	printf (0)				

1.3 Recursive algorithms

Recursion is a method where the solution to a problem depends on solutions to smaller instances of the same problem (as opposed to iteration).



Four stages in the construction of a **Koch snowflake**. The stages are obtained via a recursive definition.

1.3 Recursive algorithms

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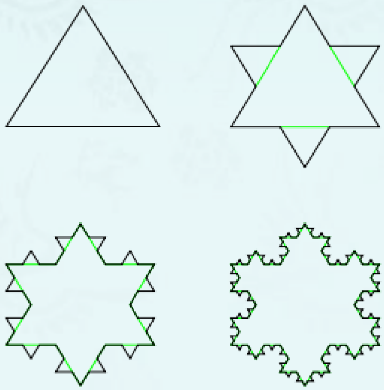
Recursive algorithm is expressed in terms of

1. **base case(s)** for which the solution can be stated **non-recursively**,



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1.3 Recursive algorithms

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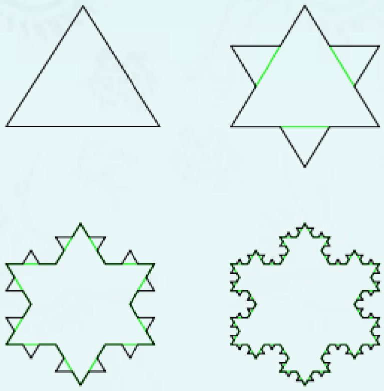
Recursive algorithm is expressed in terms of

1. **base case(s)** for which the solution can be stated **non-recursively**,
2. **recursive case(s)** for which the solution can be expressed in terms of a smaller version of itself.



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1.3 Recursive algorithms

Example: Factorial

$$\text{fact}(n) = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot \text{fact}(n-1) & \text{if } n > 0 \end{cases}$$

1.3 Recursive algorithms

Example: Factorial

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factorial(n)
function factorial **input**: integer *n* such that *n* >= 0 **output:** [*n* × (*n*-1) × (*n*-2) × … × 1] 1. if *n* is 0, **return** 1 2. otherwise, **return** [*n* × factorial(*n*-1)] **end** factorial

1.3 Recursive algorithms

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factorial (n = 4)
$f_4 = 4 * f_3$ $= 4 * (3 * f_2)$ $= 4 * (3 * (2 * f_1))$ $= 4 * (3 * (2 * (1 * f_0)))$ $= 4 * (3 * (2 * (1 * 1)))$ $= 4 * (3 * (2 * 1))$ $= 4 * (3 * 2)$ $= 4 * 6$ $= 24$

1.3 Recursive algorithms

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Exercise: **GCD** recursively with gcd (x=259, y=111) = ?

1.3 Recursive algorithms

Example: GCD (Great common divisor)

$$\gcd(x,y) = \begin{cases} x & \text{if } y = 0 \\ \gcd(y, \text{remainder}(x,y)) & \text{if } y > 0 \end{cases}$$

1.3 Recursive algorithms

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gcd(x, y)
function gcd **input**: integer *x*, *y* such that x >= y, y > 0 **output**: gcd of x and y 1. if y is 0, **return** x 2. otherwise, **return** [*gcd* (y, *x%y*)] **end** gcd

1.3 Recursive algorithms

Example: GCD (Great common divisor)

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gcd(x, y)	gcd (x=259, y=111)
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1.3 Recursive algorithms

Example: GCD (Great common divisor)

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Exercises: gcd(91, 52)

Exercises: Fibonacci, Binomial coefficients(p.14), Akerman’s function(p.17)

1.3 Recursive algorithms

Example: Recursive binary search

It searches a *sorted* array of **int**s for a particular **int**. Let **i** be an array of **int**s sorted from least to greatest. For instance, {-3, -2, 0, 0, 1, 5, 5}. We want to search **the array for the value** "wallly". If we find "wally", we return its array *index*; otherwise, we return FAILURE(-1).
Let's suppose "wally" is 1.

1.3 Recursive algorithms

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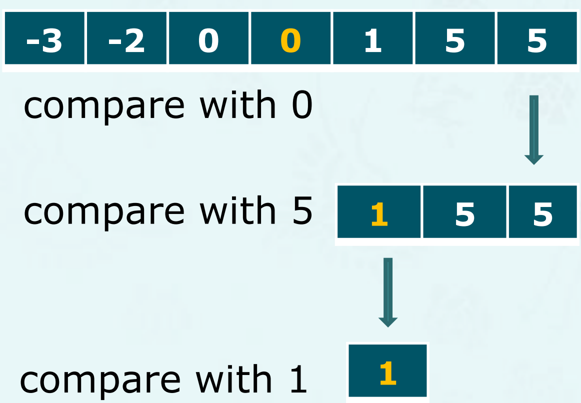

Exercise: Base case(s) & recursive case(s):?

int binarySearch(int list[], int wally, int **left**, int **right**)

1.3 Recursive algorithms

Example: Recursive binary search

Exercise: Base case(s) & recursive case(s):?

```
int binarySearch(int list[], int wally, int left, int right)

if (left > right) return FAILURE;                         // base case

mid = (left + right)/2;
if (wally == list[mid]) return mid;                       // base case
if (wally < list[mid])
    return binarySearch(list, wally, left, mid-1 ); // recursive
else
    return binarySearch(list, wally, mid+1, right); // recursive
```

1.3 Recursive algorithms

Example: Recursive binary search

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```

How long does the binarySearch() take?
In one call to binarySearch(), we eliminate at least half the elements from consideration. Hence, it takes $\log_2 n$ (the base 2 logarithm of n) binarySearch() calls to pare down the possibilities to one. Therefore binarySearch takes time proportional to $\log_2 n$.

1.3 Recursive algorithms

Example: Recursive binary search – revisited

Exercise:

-3	-2	0	0	1	5	5

compare with 0

compare with 5

1	5	5

compare with 1

1

	Stack	Stack	Heap
bSearch()	left[4] right[4] middle[4]	wally[1] list[.]	
bSearch()	left[4] right[6] middle[5]	wally[1] list[.]	
bSearch()	left[0] right[6] middle[3]	wally[1] list[.]	
bSearch()	wally[1]	list[.]	[-3 -2 0 0 1 5 5]
main()		args[.]	args[]

Most operating systems give a program enough stack space for a few thousand stack frames. If you use a recursive procedure to walk through a million-node list, the program will try to create a million stack frames, and **the stack will run out of space**. The result is a run-time error. Refer to p.108, p.111

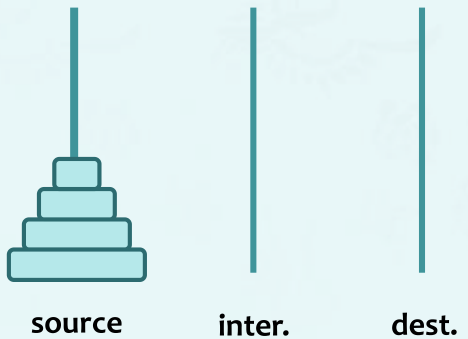
1.3 Recursive algorithms

Example: Tower of Hanoi (Refer to p.17, Ex11)

Given three pegs, one with a set of N disks of increasing size, determine the minimum (optimal) number of steps it takes to move all the disks from their initial position to a single **stack** on another peg *without placing a larger disk on top of a smaller one. Only one disk can be moved at any time.*

Recursive algorithm:

*(1) Move the top **n-1** disks from **source** to **intermediate**.*
*(2) Move the remaining (**largest**) disk from **source** to **destination**.*
*(3) Move the **n-1** disks from **intermediate** to **destination**.*



1.3 Recursive algorithms

Example: Tower of Hanoi

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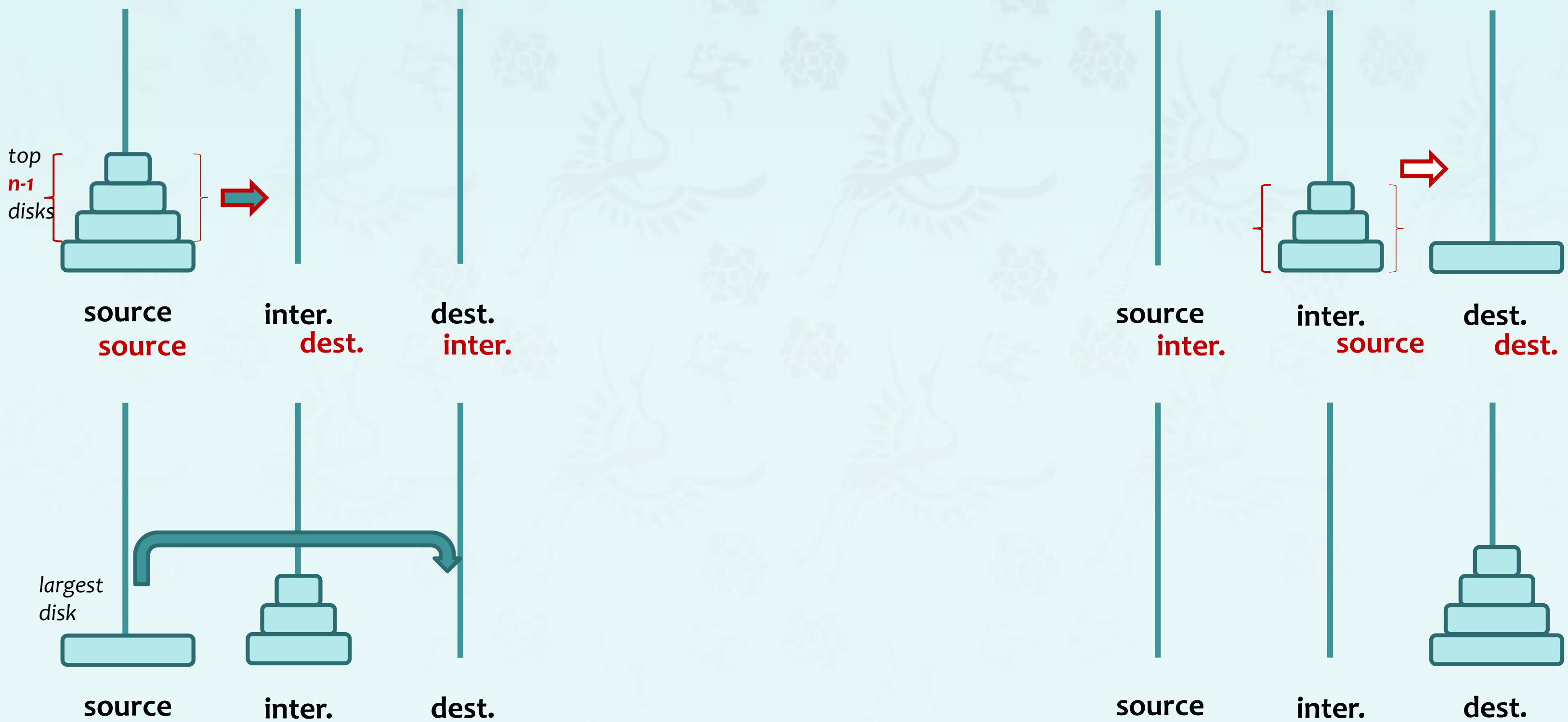

1.3 Recursive algorithms

Example: Tower of Hanoi

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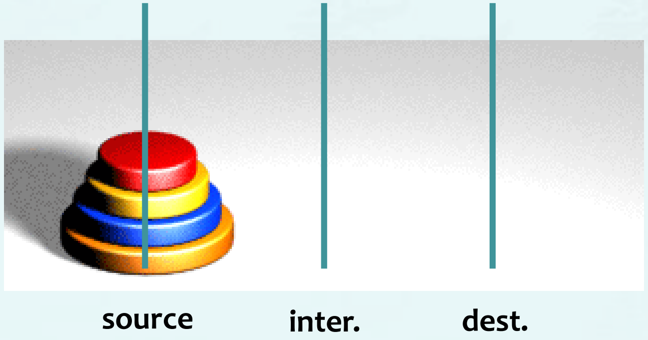


1.3 Recursive algorithms

Exercise: Tower of Hanoi – revisited

Recursive algorithm:
*(1) Move the top **n-1** disks from **source** to **intermediate**.*
*(2) Move the remaining (**largest**) disk from **source** to **destination**.*
*(3) Move the **n-1** disks from **intermediate** to **destination**.*

How do you program this to have the output as shown below?
Disk 1 from A to C
Disk 2 from A to B
Disk 1 from C to B
Disk 3 from A to C
Disk 1 from B to A
Disk 2 from B to C
Disk 1 from A to C

hanoi()

```
void hanoi(int n, char from, char inter, char to) {
    if (n == 1)
        printf ("Disk 1 from %c to %c\n", from, to);
    else {
        hanoi(n – 1  from, to, inter ) ;
        printf("Disk %d from %c to %c\n", n, from, to);
        hanoi(n – 1,  inter, from, to );
    }
}
```

1.3 Recursive algorithms

Exercise: How many moves for n disks in Tower of Hanoi, hanoi(n)?

Recursive algorithm:

*(1) Move the top **n-1** disks from **source** to **intermediate**.*
*(2) Move the remaining (**largest**) disk from **source** to **destination**.*
*(3) Move the **n-1** disks from **intermediate** to **destination**.*

hanoi(*n-1*) move
hanoi(*1*) move
hanoi(*n-1*) move

$$\mathrm{hanoi}(n) = \begin{cases} 1 & \text{if } n = 1 \\ 2 \cdot \mathrm{hanoi}(n-1) + 1 & \text{if } n > 1 \end{cases}$$

Exercise: hanoi(2) =
hanoi(4) =
hanoi(32) =
hanoi(64) = ?

hanoi(n = 4)
hanoi(4) = 2*hanoi(3) + 1 = 2*(2*hanoi(2) + 1) + 1 = 2*(2*(2*hanoi(1) + 1) + 1) + 1 = 2*(2*(2*1 + 1) + 1) + 1 = 2*(2*(3) + 1) + 1 = 2*(7) + 1 = 15

1.3 Recursive algorithms

Q: Is the recursive version usually faster?

A: No -- it's usually slower (due to the overhead of maintaining the stack)

Q: Does the recursive version usually use less memory?

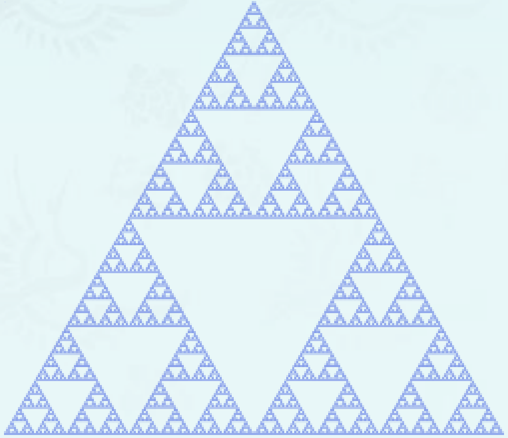
A: No -- it usually uses ***more*** memory (for the stack).

Q: Then ***why*** use recursion?

A: Sometimes it is much simpler to write the recursive version.

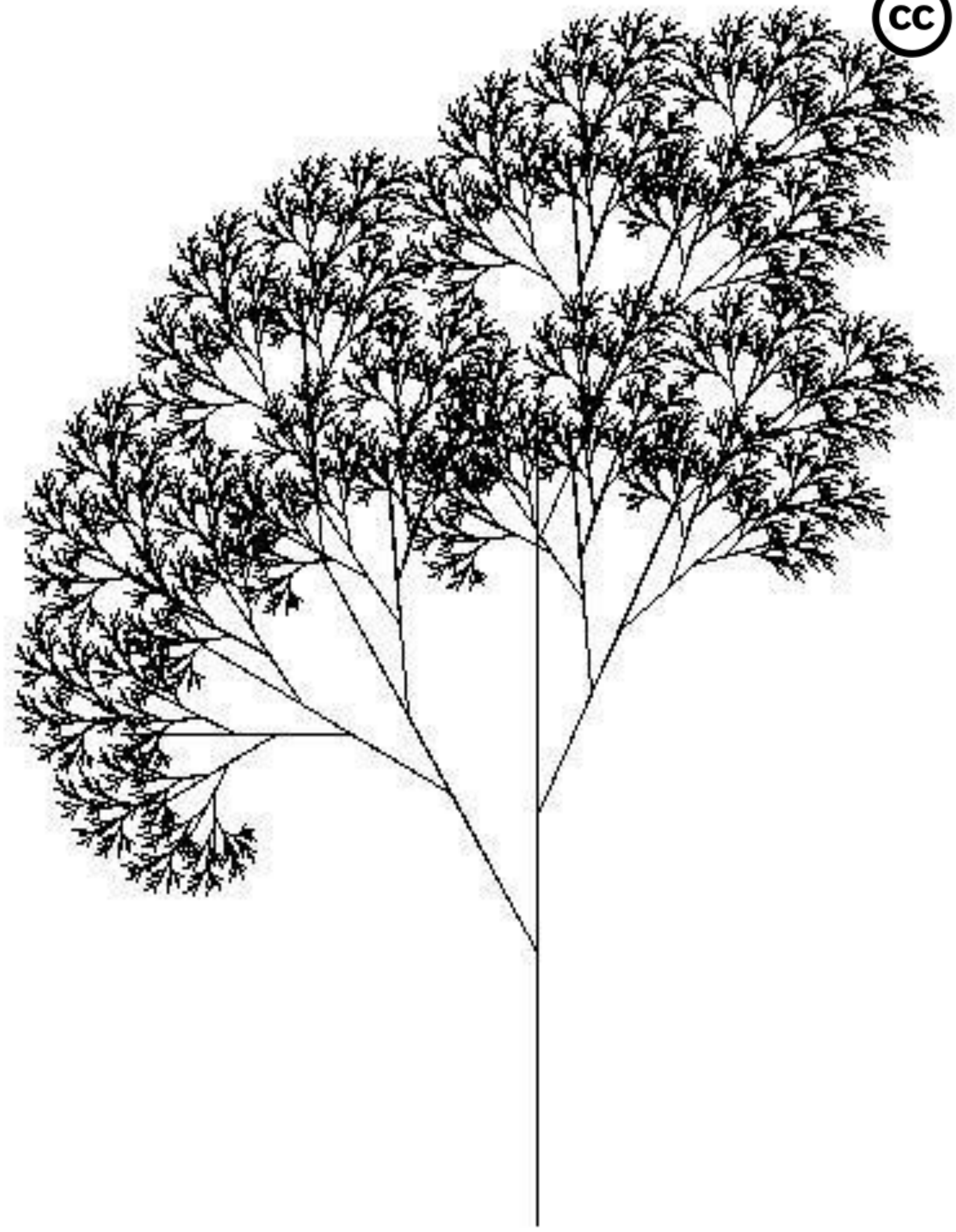
How the function call work? See[System Stack] in p.108.

*Because the recursive version causes an **activation record** to be pushed onto the system stack for every call, it is also more limited than the iterative version (it will fail, with a "stack overflow" error), for large values of N.*



Sierpinski Triangle:
a confined recursion of triangles to form a geometric lattice

Chapter 1 – Basic concepts



Recursion	*see Recursion*
GNU	GNU's not Unix.

ECE 20010 Data Structures

Data Structures

Chapter 1

- *algorithm specification*
  - *recursive algorithm*
  - *problem set 03*
- ***data abstraction***
- *performance analysis - time complexity*