

2(b)

The accuracy is 96 percent.

3(a)

Code, error is 0.01625, or 1.625 percent.

3(b)

'httpaddr'

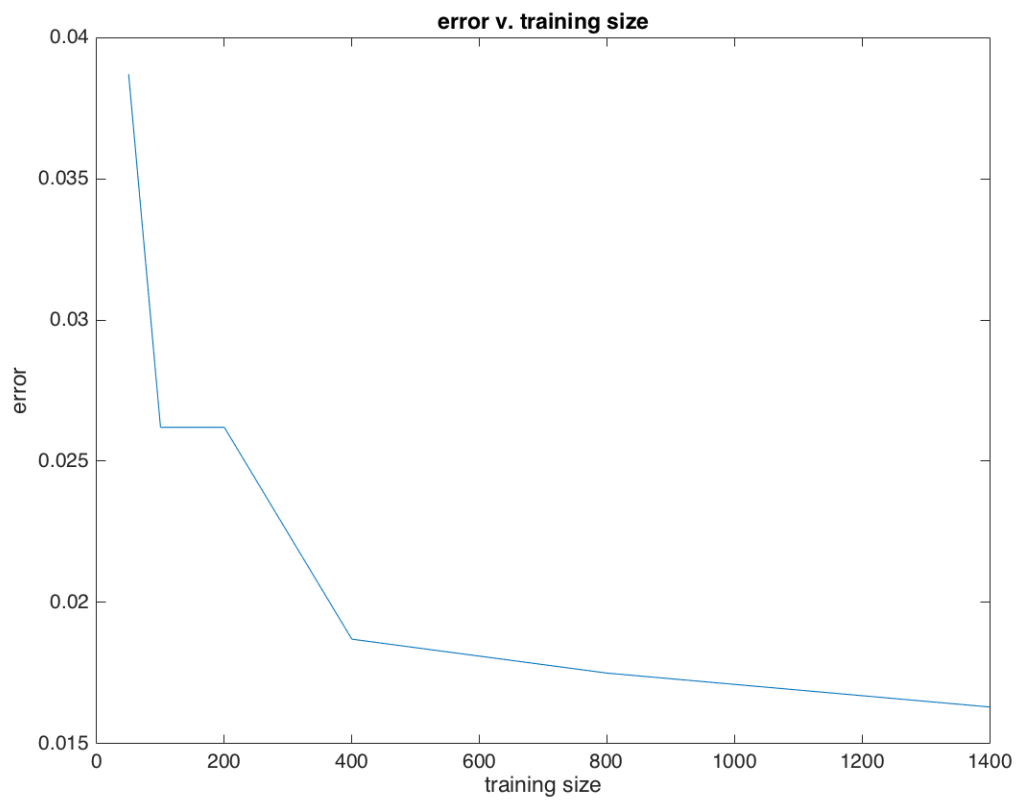
'spam'

'unsubscribe'

'ebai'

'valet'

3(c)



EECS 445 HW2

(a)

$$p(t) = \phi^t (1-\phi)^{1-t}$$

$$P(x|t=0) = \frac{1}{(2\pi)^{\frac{1}{2}} |\Sigma|^{1/2}} \exp\left(-\frac{1}{2} (x-\mu_0)^T \Sigma^{-1} (x-\mu_0)\right)$$

$$P(x|t=1) = \frac{1}{(2\pi)^{\frac{1}{2}} |\Sigma|^{1/2}} \exp\left(-\frac{1}{2} (x-\mu_1)^T \Sigma^{-1} (x-\mu_1)\right)$$

Want to show

$$P(t=1|x) = \frac{1}{1 + \exp(-w^T x)}$$

Use log odds \Rightarrow

$$\begin{aligned} \log\left(\frac{P(C_1|x)}{P(C_2|x)}\right) &= \log \frac{P(C_1|x)}{1 - P(C_1|x)} \\ &= \log \frac{\exp\left(-\frac{1}{2} (x-\mu_0)^T \Sigma^{-1} (x-\mu_0)\right)}{\exp\left(-\frac{1}{2} (x-\mu_1)^T \Sigma^{-1} (x-\mu_1)\right)} + \log \frac{P(C_1)}{P(C_2)} \end{aligned}$$

$$= -\frac{1}{2} (x-\mu_0)^T \Sigma^{-1} (x-\mu_0) - \left\{ -\frac{1}{2} (x-\mu_1)^T \Sigma^{-1} (x-\mu_1) \right\} + \log \frac{P(C_1)}{P(C_2)}$$

$$= (\mu_0 - \mu_1)^T \Sigma^{-1} x - \frac{1}{2} \mu_0^T \Sigma^{-1} \mu_0 + \frac{1}{2} \mu_1^T \Sigma^{-1} \mu_1 + \log \frac{P(C_1)}{P(C_2)}$$

$$= (\Sigma^{-1} (\mu_0 - \mu_1))^T x + w_0, \text{ where}$$

$$w_0 = -\frac{1}{2} \mu_0^T \Sigma^{-1} \mu_0 + \frac{1}{2} \mu_1^T \Sigma^{-1} \mu_1 + \log \frac{P(C_1)}{P(C_2)}$$

To get into form we want, construct $W = \begin{bmatrix} w_0 \\ \Sigma^{-1} (\mu_0 - \mu_1)^T \end{bmatrix}$ and

1(a)
cont

$x' = \begin{bmatrix} 1 \\ x \end{bmatrix}$, s.t. our solution is of the form

$$P(C_1 | x) = \frac{1}{1 + \exp(-w^T x')}, \text{ where } w = \begin{bmatrix} w_0 \\ \Sigma^{-1}(M_0 - M_1)^T \end{bmatrix} \text{ and}$$

$$x' = \begin{bmatrix} 1 \\ x \end{bmatrix}.$$

1 (b) First step is to write $l(w)$

$$l(w) = \log \prod_{i=1}^N P(x^{(i)} | t^{(i)}) P(t^{(i)})$$

$$= \sum_{i=1}^N \log \{P(x^{(i)} | t^{(i)}) P(t^{(i)})\}, \text{ split using indicators.}$$

$$= \sum_{i=1}^N \log \{P(t^{(i)}) P(x^{(i)} | t=1)^{1\{t=1\}} P(x^{(i)} | t=0)^{1\{t=0\}}\}$$

$$= \sum_{i=1}^N \log(P(t^{(i)})) + \log(P(x^{(i)} | t=1)^{1\{t=1\}} + \log(P(x^{(i)} | t=0)^{1\{t=0\}}\}$$

$$= \sum_{i=1}^N t^{(i)} \log(\phi) + (1-t^{(i)}) \log(1-\phi) + 1\{t=1\} \log(P(x^{(i)} | t=1)) + 1\{t=0\} \log(P(x^{(i)} | t=0))$$

We will use this l in our subsequent derivatives \Rightarrow

$$\left[\frac{\partial l}{\partial w} \right] = W$$

$$b(i) \quad \nabla_{\phi} \ell(w) = \sum_{i=1}^N t^{(i)} \left(\frac{1}{\phi} \right) + (1-t^{(i)}) \left(\frac{-1}{1-\phi} \right) \\ = \sum_{i=1}^N \frac{t^{(i)}(1-\phi) - \phi(1-t^{(i)})}{\phi(1-\phi)} = \sum_{i=1}^N \frac{t^{(i)} - \phi}{\phi(1-\phi)}$$

Set to 0, want numerator = 0.

$$\sum_{i=1}^N t^{(i)} - \phi = 0 \Rightarrow \sum_{i=1}^N t^{(i)} - N\phi = 0 \Rightarrow \phi = \frac{1}{N} \sum_{i=1}^N 1\{t^{(i)}=1\} \quad \boxtimes$$

$$(ii) \quad \nabla_{\mu_0} \ell(w) = \nabla_{\mu_0} \sum_{i=1}^N -\frac{1}{2} 1\{t^{(i)}=0\} [(x^{(i)} - \mu_0)^T \Sigma^{-1} (x^{(i)} - \mu_0)]$$

$$= \nabla_{\mu_0} -\frac{1}{2} \sum_{i=1}^N 1\{t^{(i)}=0\} \Sigma^{-1} (x^{(i)} - \mu_0)^T (x^{(i)} - \mu_0)$$

$$= \nabla_{\mu_0} -\frac{1}{2} \sum_{i=1}^N 1\{t^{(i)}=0\} \Sigma^{-1} (x^{(i)} - \mu_0)^2$$

$$= -\frac{1}{2} \sum_{i=1}^N 1\{t^{(i)}=0\} \Sigma^{-1} 2(x^{(i)} - \mu_0) \quad \checkmark$$

$$\Rightarrow \sum_{i=1}^N 1\{t^{(i)}=0\} (x^{(i)} - \mu_0) = 0$$

$$\sum_{i=1}^N 1\{t^{(i)}=0\} x^{(i)} - \sum_{i=1}^N 1\{t^{(i)}=0\} \mu_0 = 0$$

$$\mu_0 = \frac{\sum_{i=1}^N 1\{t^{(i)}=0\} x^{(i)}}{\sum_{i=1}^N 1\{t^{(i)}=0\}} \quad \boxtimes$$

$$\frac{\sum_{i=1}^N 1\{t^{(i)}=0\} x^{(i)}}{\sum_{i=1}^N 1\{t^{(i)}=0\}}$$

$$(iii) \nabla_{\mu_1} \ell(w) = \nabla_{\mu_1} \sum_{i=1}^N -\frac{1}{2} 1\{t^{(i)}=1\} \left[(x^{(i)} - \mu_1)^T \Sigma^{-1} (x^{(i)} - \mu_1) \right]$$

$$= \nabla_{\mu_1} -\frac{1}{2} \sum_{i=1}^N 1\{t^{(i)}=1\} \Sigma^{-1} (x^{(i)} - \mu_1)^2$$

$$= -\frac{1}{2} \sum_{i=1}^N 1\{t^{(i)}=1\} \cancel{\Sigma^{-1}} (x^{(i)} - \mu_1) \cancel{(-1)}$$

$$\Rightarrow \sum_{i=1}^N 1\{t^{(i)}=1\} (x^{(i)} - \mu_1) = 0$$

$$\sum_{i=1}^N 1\{t^{(i)}=1\} (x^{(i)} - \mu_1) = 0 \Rightarrow \sum_{i=1}^N 1\{t^{(i)}=1\} x^{(i)} - \sum_{i=1}^N 1\{t^{(i)}=1\} \mu_1$$

$$\mu_1 = \frac{\sum_{i=1}^N 1\{t^{(i)}=1\} x^{(i)}}{\sum_{i=1}^N 1\{t^{(i)}=1\}}$$

□

$$(iv) \cancel{\nabla_{\mu_1} \ell(w)} \neq \cancel{\sum_{i=1}^N} \cancel{(-1)}$$

Observations: $\Sigma = \sigma^2 \quad |\Sigma|^{\frac{1}{2}} = \sigma$

consolidate indicator variables terms into $t^{(i)}$

$$\nabla_{\Sigma} \ell(w) = \sum_{i=1}^N \nabla \log(p(x^{(i)} | t^{(i)}))$$

$$= \sum_{i=1}^N \nabla \log \left(\frac{1}{(2\pi)^{\frac{1}{2}} \sigma} \right) + \log \left(\exp \left((x - \mu_t)^T \Sigma^{-1} (x - \mu_t) \right) \right)$$

$$= \sum_{i=1}^N \nabla \log \left(\frac{1}{(2\pi)^{\frac{1}{2}}} \right) + \log \left(\frac{1}{\sigma} \right) + \cancel{\frac{1}{2} \Sigma^{-1} (x - \mu_t)^T (x - \mu_t)}$$

$$= \cancel{\sum_{i=1}^N} \cancel{\frac{1}{\sigma}} \cancel{+ (x - \mu_t)^T} \cancel{\frac{1}{\sigma^3}}$$

Continued \Rightarrow

$$\sum_{i=1}^N (-\Sigma + (x^{(i)} - \mu)^T (x^{(i)} - \mu)^T) = 0$$

$$\Rightarrow -N\Sigma + \sum_{i=1}^N (x^{(i)} - \mu)^T (x^{(i)} - \mu)^T = 0$$

$$\Sigma = \frac{1}{N} \sum_{i=1}^N (x^{(i)} - \mu)(x^{(i)} - \mu)^T$$

$$0 = (\mu - \mu) \sum_{i=1}^N (x^{(i)} - \mu) = 0$$

$$\sum_{i=1}^N (x^{(i)} - \mu) = 0 \Rightarrow \sum_{i=1}^N x^{(i)} - N\mu = 0 \Rightarrow \sum_{i=1}^N x^{(i)} = N\mu$$

$$\mu = \frac{1}{N} \sum_{i=1}^N x^{(i)}$$

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Operational: $\Sigma = \frac{1}{N} \sum_{i=1}^N (x^{(i)} - \mu)(x^{(i)} - \mu)^T$

$$\Delta^2 g(\mu) = \frac{1}{N} \sum_{i=1}^N \Delta^2 g(x^{(i)})$$

$$\left(\frac{1}{N} \sum_{i=1}^N \Delta^2 g(x^{(i)}) \right) = \frac{1}{N} \sum_{i=1}^N \Delta^2 g(x^{(i)})$$

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2(a) Wts

$$\nabla_{W_m} \ell(w) = \sum_{i=1}^N \phi(x^{(i)}) [1\{t^{(i)}=m\} - p(t^{(i)}=m|x^{(i)})]$$

Start derivation.

$$\nabla_{W_m} \ell = \sum_{i=1}^N \sum_{m=1}^k \phi(x^{(i)}) 1\{t^{(i)}=m\} - \nabla_{W_m} 1\{t^{(i)}=m\} \log \left(\sum_{j=1}^k \exp(w_j^T \phi(x^{(i)})) \right)$$

$$= \sum_{i=1}^N \sum_{m=1}^k \phi(x^{(i)}) 1\{t^{(i)}=m\} - \nabla_{W_m} (1\{t^{(i)}=m\}) \log \sum_{j=1}^k \exp(w_j^T \phi(x^{(i)}))$$

$$\downarrow$$

$$u = \sum_{j=1}^k \exp(w_j^T \phi(x^{(i)})), \quad \nabla_{W_j} (\log \sum_{j=1}^k \exp(w_j^T \phi(x^{(i)})))$$

$$= \nabla_{W_j} \log(u)$$

$$\Rightarrow \frac{1}{u} \nabla_{W_j} u = \frac{1}{\sum_{j=1}^k \exp(w_j^T \phi(x^{(i)}))}$$

$$\Rightarrow \nabla_{W_j} u = \nabla_{W_j} \left(\sum_{k=1}^k \exp(w_k^T \phi(x^{(i)})) \right)$$

$$= \exp(w_j^T \phi(x)) \cdot \phi(x)$$

$$\Rightarrow \sum_{i=1}^N \sum_{k=1}^k \phi(x^{(i)}) 1\{t^{(i)}=m\} - \phi(x^{(i)}) \left(\frac{\exp(w_m^T \phi(x^{(i)}))}{\sum_{j=1}^k \exp(w_j^T \phi(x^{(i)}))} \right)$$

$$= \sum_{i=1}^N \sum_{k=1}^k \phi(x^{(i)}) (1\{t^{(i)}=m\} - p(t^{(i)}=m|x^{(i)}))$$

We remove k-iteration when we only consider w_m , so

$$\nabla_{W_m} \ell(w) = \sum_{i=1}^N \phi(x^{(i)}) [1\{t^{(i)}=m\} - p(t^{(i)}=m|x^{(i)})]$$

Q