Assignment 2

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ex.1 see the py file attached

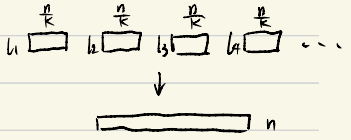
ex.2 see the py file attached

ex.3 see the py file attached

ex.4 (1)

Write the recursion formula: $f(n) = \begin{cases} 0, & \text{if } n<1 \\ \underbrace{f(\frac{n}{k}) + f(\frac{n}{k}) + \cdots + f(\frac{n}{k})}_{k} + g(n) \end{cases}$

for k sorted lists to be merged together:



mergedList := merge(l1, l2, ... lk):

tmp = [None] * n

For all $i \in \{1, 2, \ldots k\}$, choose $x_i \in l_i$, $\forall y \in \{ \text{head}(l_1) \cup \text{head}(l_2) \cup \text{head}(l_3) \cdots \cup \text{head}(l_k) \}$, $x \le y$:

tmp[k] = x

l_i.pop(x)

return tmp

We can obtain the number of comparisons is as follows:

① Best case: clear the list one by one $k \cdot \frac{n}{k} + (k-1) \cdot \frac{n}{k} + \cdots + (k-(k-1)) \frac{n}{k} = \frac{k(k+1)}{2} \cdot \frac{n}{k} = \frac{k+1}{2} \cdot n$ (only two lists left)

② worst case: no lists will be cleared beforehand $k(n-1) + 1 + 2 + \cdots + k = k(n + \frac{k-1}{2})$

So $f(n) = f(\frac{n}{k}) + f(\frac{n}{k}) + \cdots + f(\frac{n}{k}) + k(n + \frac{k-1}{2})$

Substitute n by k^n, we get $d_n = k d_{n-1} + k^{n+1} + \frac{k(k-1)}{2}$, whose characteristic equation is $(x-k)^2(x-1)$

We can solve the solution is $d_n = c_1 \cdot n \cdot k^n + c_2 \cdot k^n + c_3$, convert it back to n,

$f(n) = c_1 \log_k n \cdot n + c_2 \cdot n + c_3$, where $\log_k n = \frac{\log_2 n}{\log_2 k}$

Therefore, we say $f(n) \in O(n \log_2 n)$, the proof is complete.

(2) (3) see the py file attached