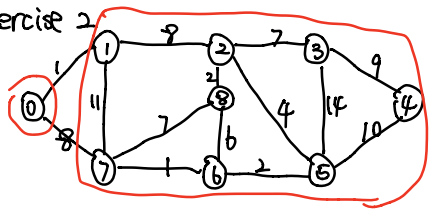
Exercise 2

(i)

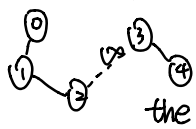


node	0	1	2	3	4	5	6	7	8
selected	T	F	F	F	F	F	F	F	F
minDist	-	inf	inf	inf	inf	inf	inf	inf	inf

for prim algorithm, we need two list, one of them is named selected, which save whether the node has been visited, the other is minDist, if the node has been visited, the minDist is - . We start with any of the node, every time we first update minDist, then choose the node with minimal minDist to be selected.

Now if the graph is not connected. we denote the vertex $V_1, V_2, \cdots V_n$, and we add edge between $V_i, V_{i+1}, i \in n-1$, if there's already a edge then continue. and we set the cost as ∞. Therefore, we can guarantee that the graph is connected.

Now we first create an empty list, res = [], and we choose arbitrary node as the starting node. assume it is 0.



every time add a node we append it in res we finally will get [0,1,2,3,4], but the cost between 2 and 3 is infinite, so we split the res to be [0,1,2], [3,4],

(ii) if T_1 and T_2 are distinct minimum spanning tree, then consider the edge of minimum weight among all the edges that are contained in exactly one of T_1 or T_2, assume this edge appears only in T_1, and we can call it e_1.
Then $T_2 \cup \{e_1\}$ must contain a cycle, and one of the edges of this edges of this cycle call it e_2, is not in T_1.
Since e_2 is a edge different from e_1 and is contained in exactly one of T_1 or T_2, it must be that $w(e_1) < w(e_2)$. Note that $T = T_2 \cup \{e_1\} \setminus \{e_2\}$ is a spanning tree. The total weight of T is smaller than the total weight of T_2, but this is a contradiction, since we have supposed that T_2 is a minimum spanning tree.

(iii) minimum cost spanning set of edges is not necessarily a tree. Because if the cost are all negative and we have a spanning tree, if we add more edges the total cost is lower. So cycle is possible

if all costs are positive, then it must be a tree. Because if the spanning set contain cycle, then remove one edge make total cost to be lower, so there can't be cycle, then it must be a tree