Artificial Intelligence

First-order Logic

Logic vị từ

Outline

- Why FOL?
- Syntax and semantics of FOL
- Using FOL
- Wumpus world in FOL
- Knowledge engineering in FOL

Pros and cons of propositional logic

- ☺ Propositional logic is declarative
- ☺ Propositional logic allows partial/disjunctive/negated information
  - (unlike most data structures and databases)
- ☺ Propositional logic is compositional:
  - meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$
- ☺ Meaning in propositional logic is context-independent
  - (unlike natural language, where meaning depends on context)
- ☹ Propositional logic has very limited expressive power
  - (unlike natural language)
  - E.g., cannot say "pits cause breezes in adjacent squares“
    - except by writing one sentence for each square

First-order logic

- Whereas propositional logic assumes the world contains facts,
- first-order logic (like natural language) assumes the world contains
  - Objects: people, houses, numbers, colors, baseball games, wars, …
  - Relations: red, round, prime, brother of, bigger than, part of, comes between, …
  - Functions: father of, best friend, one more than, plus, …

Syntax of FOL: Basic elements

• Constants	KingJohn, 2, NUS,...
• Predicates	Brother, >,...
• Functions	Sqrt, LeftLegOf,...
• Variables	x, y, a, b,...
• Connectives	$\neg$, $\Rightarrow$, $\wedge$, $\vee$, $\Leftrightarrow$
• Equality	=
• Quantifiers	$\forall$, $\exists$

Atomic sentences

Atomic sentence = *predicate* ($term_1$,...,$term_n$)

or $term_1 = term_2$

Term = *function* ($term_1$,...,$term_n$)

or *constant* or *variable*

- E.g., *Brother(KingJohn,RichardTheLionheart)*
- *> (Length(LeftLegOf(Richard)),Length(LeftLegOf(KingJohn)))*

Complex sentences

- Complex sentences are made from atomic sentences using connectives

  $\neg S$, $S_1 \wedge S_2$, $S_1 \vee S_2$, $S_1 \Rightarrow S_2$, $S_1 \Leftrightarrow S_2$,

E.g. *Sibling(KingJohn,Richard)* $\Rightarrow$ *Sibling(Richard,KingJohn)*

  $>(1,2) \vee \leq(1,2)$

  $>(1,2) \wedge \neg >(1,2)$

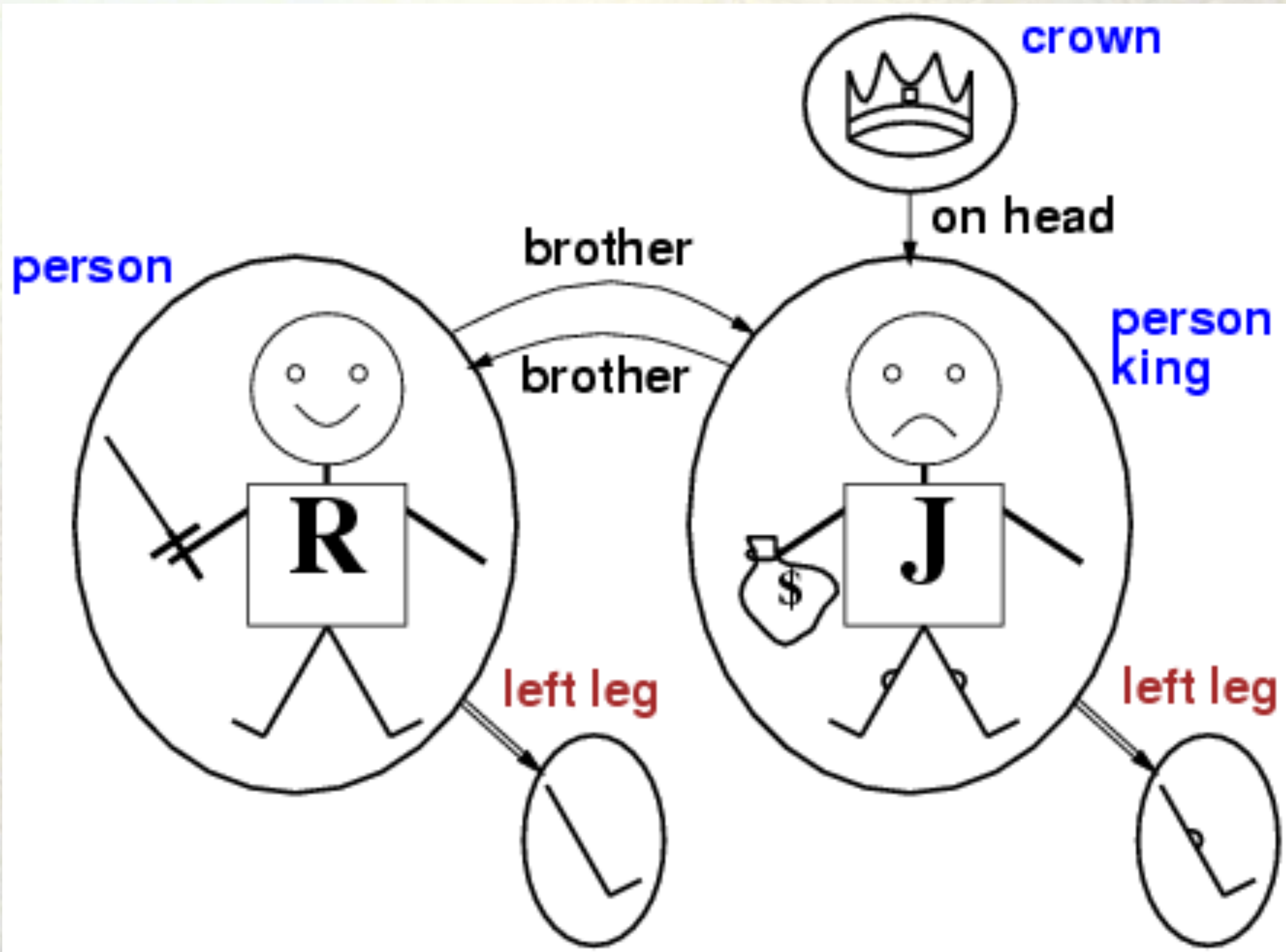
Truth in first-order logic

- Sentences are true with respect to a model and an interpretation
- Model contains objects (domain elements) and relations among them
- Interpretation specifies referents for

constant symbols	→	objects
predicate symbols	→	relations
function symbols	→	functional relations

- An atomic sentence $predicate(term_1,...,term_n)$ is true
  iff the objects referred to by $term_1,...,term_n$
  are in the relation referred to by *predicate*

Models for FOL: Example



Universal quantification

- ∀*<variables>* *<sentence>*

Everyone at Coltech is smart:
$\forall x\ At(x,Coltech) \Rightarrow Smart(x)$

- $\forall x$ *P* is true in a model *m* iff *P* is true with *x* being each possible object in the model

- Roughly speaking, equivalent to the conjunction of instantiations of *P*

$$
\begin{aligned}
& At(KingJohn,Coltech) \Rightarrow Smart(KingJohn) \\
\land\ & At(Richard,Coltech) \Rightarrow Smart(Richard) \\
\land\ & At(A,Coltech) \Rightarrow Smart(A) \\
\land\ & \ldots
\end{aligned}
$$

A common mistake to avoid

- Typically, $\Rightarrow$ is the main connective with $\forall$
- Common mistake: using $\wedge$ as the main connective with $\forall$:

  $\forall x\ At(x,Coltech) \wedge Smart(x)$

  means "Everyone is at Coltech and everyone is smart"

Existential quantification

- ∃*<variables>* *<sentence>*
- Someone at Coltech is smart:
  ∃*x* At(x,Coltech) ∧ Smart(x)

- ∃*x* *P* is true in a model *m* iff *P* is true with *x* being some possible object in the model

- Roughly speaking, equivalent to the disjunction of instantiations of *P*

  At(KingJohn,Coltech) ∧ Smart(KingJohn)
  ∨ At(Richard,Coltech) ∧ Smart(Richard)
  ∨ At(A,Coltech) ∧ Smart(A)
  ∨ ...

Another common mistake to avoid

- Typically, $\wedge$ is the main connective with $\exists$
- Common mistake: using $\Rightarrow$ as the main connective with $\exists$:

$$\exists x\ At(x,Coltech) \Rightarrow Smart(x)$$

is true if there is anyone who is not at Coltech!

Properties of quantifiers

- $\forall x \forall y$ is the same as $\forall y \forall x$
- $\exists x \exists y$ is the same as $\exists y \exists x$
- $\exists x \forall y$ is not the same as $\forall y \exists x$
- $\exists x \forall y$ Loves(x,y)
  - "There is a person who loves everyone in the world"
- $\forall y \exists x$ Loves(x,y)
  - "Everyone in the world is loved by at least one person"
- Quantifier duality: each can be expressed using the other
- $\forall x$ Likes(x,IceCream) $\neg \exists x \neg$Likes(x,IceCream)
- $\exists x$ Likes(x,Broccoli) $\neg \forall x \neg$Likes(x,Broccoli)

Equality

- $term_1 = term_2$ is true under a given interpretation if and only if $term_1$ and $term_2$ refer to the same object

- E.g., definition of *Sibling* in terms of *Parent*:

  $\forall x,y\ Sibling(x,y) \Leftrightarrow [\neg(x = y) \wedge \exists m,f\ \neg(m = f) \wedge Parent(m,x) \wedge Parent(f,x) \wedge Parent(m,y) \wedge Parent(f,y)]$

Using FOL

The kinship domain:

- Brothers are siblings

  $\forall x,y\ \mathit{Brother}(x,y) \Leftrightarrow \mathit{Sibling}(x,y)$

- One's mother is one's female parent

  $\forall m,c\ \mathit{Mother}(c) = m \Leftrightarrow (\mathit{Female}(m) \wedge \mathit{Parent}(m,c))$

- "Sibling" is symmetric

  $\forall x,y\ \mathit{Sibling}(x,y) \Leftrightarrow \mathit{Sibling}(y,x)$

Interacting with FOL KBs

- Suppose a wumpus-world agent is using an FOL KB and perceives a smell and a breeze (but no glitter) at *t=5*:

  Tell(KB,Percept([Smell,Breeze,None],5))
  Ask(KB,∃a BestAction(a,5))
- I.e., does the KB entail some best action at *t=5*?
- Answer: *Yes*, {*a/Shoot*} ← substitution (binding list)

- Given a sentence *S* and a substitution σ,
- *S*σ denotes the result of plugging σ into *S*; e.g.,
  S = Smarter(x,y)
  σ = {x/Hillary,y/Bill}
  *S*σ = Smarter(Hillary,Bill)

- Ask(KB,S) returns some/all σ such that KB ⊨ σ

Knowledge base for the wumpus world

- Perception
  - $\forall t,s,b$ Percept([s,b,Glitter],t) $\Rightarrow$ Glitter(t)
- Reflex
  - $\forall t$ Glitter(t) $\Rightarrow$ BestAction(Grab,t)

Deducing hidden properties

- $\forall x,y,a,b$ *Adjacent*$([x,y],[a,b]) \Leftrightarrow$
  $[a,b] \in \{[x+1,y], [x-1,y],[x,y+1],[x,y-1]\}$

Properties of squares:

- $\forall s,t$ *At*$(\text{Agent},s,t) \wedge \text{Breeze}(t) \Rightarrow \text{Breezy}(s)$

Squares are breezy near a pit:

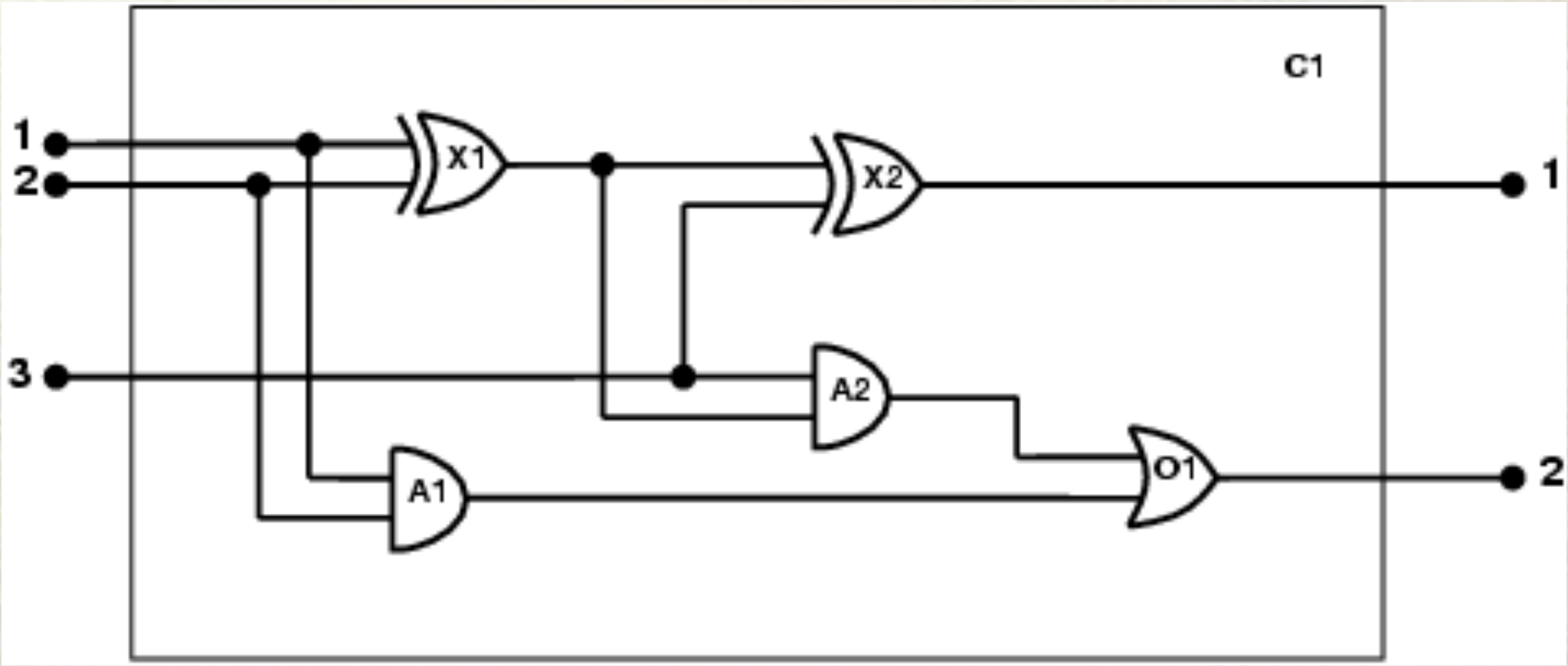
- Diagnostic rule---infer cause from effect
  $\forall s\ \text{Breezy}(s) \Rightarrow \exists r\ \text{Adjacent}(r,s) \wedge \text{Pit}(r)$
- Causal rule---infer effect from cause
  $\forall r\ \text{Pit}(r) \Rightarrow [\forall s\ \text{Adjacent}(r,s) \Rightarrow \text{Breezy}(s)]$

Knowledge engineering in FOL

1. Identify the task
2. Assemble the relevant knowledge
3. Decide on a vocabulary of predicates, functions, and constants
4. Encode general knowledge about the domain
5. Encode a description of the specific problem instance
6. Pose queries to the inference procedure and get answers
7. Debug the knowledge base

The electronic circuits domain

One-bit full adder



The electronic circuits domain

1. Identify the task
   - Does the circuit actually add properly? (circuit verification)
2. Assemble the relevant knowledge
   - Composed of wires and gates; Types of gates (AND, OR, XOR, NOT)
   - Irrelevant: size, shape, color, cost of gates
3. Decide on a vocabulary
   - Alternatives:

     $Type(X_1) = XOR$

     $Type(X_1, XOR)$

     $XOR(X_1)$

The electronic circuits domain

4. Encode general knowledge of the domain
   - $\forall t_1, t_2$ Connected$(t_1, t_2) \Rightarrow$ Signal(t_1) = Signal(t_2)
   - $\forall t$ Signal(t) = 1 $\vee$ Signal(t) = 0
   - $1 \neq 0$
   - $\forall t_1, t_2$ Connected$(t_1, t_2) \Rightarrow$ Connected(t_2, t_1)
   - $\forall g$ Type(g) = OR $\Rightarrow$ Signal(Out(1,g)) = 1 $\Leftrightarrow$ $\exists n$ Signal(In(n,g)) = 1
   - $\forall g$ Type(g) = AND $\Rightarrow$ Signal(Out(1,g)) = 0 $\Leftrightarrow$ $\exists n$ Signal(In(n,g)) = 0
   - $\forall g$ Type(g) = XOR $\Rightarrow$ Signal(Out(1,g)) = 1 $\Leftrightarrow$ Signal(In(1,g)) $\neq$ Signal(In(2,g))
   - $\forall g$ Type(g) = NOT $\Rightarrow$ Signal(Out(1,g)) $\neq$ Signal(In(1,g))

The electronic circuits domain

5. Encode the specific problem instance

   $Type(X_1) = XOR$ $\quad$ $Type(X_2) = XOR$

   $Type(A_1) = AND$ $\quad$ $Type(A_2) = AND$

   $Type(O_1) = OR$

   $Connected(Out(1,X_1),In(1,X_2))$
   $Connected(In(1,C_1),In(1,X_1))$

   $Connected(Out(1,X_1),In(2,A_2))$
   $Connected(In(1,C_1),In(1,A_1))$

   $Connected(Out(1,A_2),In(1,O_1))$
   $Connected(In(2,C_1),In(2,X_1))$

   $Connected(Out(1,A_1),In(2,O_1))$
   $Connected(In(2,C_1),In(2,A_1))$

   $Connected(Out(1,X_2),Out(1,C_1))$
   $Connected(In(3,C_1),In(2,X_2))$

   $Connected(Out(1,O_1),Out(2,C_1))$
   $Connected(In(3,C_1),In(1,A_2))$

The electronic circuits domain

6. Pose queries to the inference procedure

What are the possible sets of values of all the terminals for the adder circuit?

$\exists i_1,i_2,i_3,o_1,o_2$ Signal(In(1,C_1)) = i_1 $\wedge$ Signal(In(2,C_1)) = i_2 $\wedge$ Signal(In(3,C_1)) = i_3 $\wedge$ Signal(Out(1,C_1)) = o_1 $\wedge$ Signal(Out(2,C_1)) = o_2

7. Debug the knowledge base

May have omitted assertions like $1 \neq 0$

Summary

- First-order logic:
  - objects and relations are semantic primitives
  - syntax: constants, functions, predicates, equality, quantifiers
- Increased expressive power: sufficient to define wumpus world

References

- Artificial Intelligence A modern Approach. Chapter 8