1->3.27
2->4.85
3->4.100
4->5.20
5->8.40
6->7.17
7->7.25
8->7.30

3.27 $E[X] = \sum_{j=1}^{\infty} j\, P[X=j] = \sum_{j=1}^{\infty} j \frac{c}{j^2} = c \sum_{j=1}^{\infty} \frac{1}{j} = \infty$

mean does not exist.

$E[X^2] = \sum_{j=1}^{\infty} j^2 \frac{c}{j^2} = c \sum_{j=1}^{\infty} 1 = \infty$

none of the moments exist

This pmf decays sufficiently fast that probabilities add to 1, but too slowly for moments to exist.

4.85 X : Gaussian, $Y = aX + b$, a linear combination of X.
Y is also Gaussian

$$E[Y] = aE[X] + b = am + b = m'$$

$$Var[Y] = a^2 Var[X] = a^2\alpha^2 = \alpha'^2$$

$$a = \alpha'/\alpha, \;\; b = m' - am = m' - m\alpha'/\alpha$$

4.100

~~5.82~~ $Y = X/n$

$$E[Y] = E[X]/n = np/n = p$$

$$VAR[Y] = VAR[X]/n^2 = npq/n^2 = pq/n, \; q = 1-p$$

$$P[\{|Y-p|\} > a] \leq \frac{\sigma^2}{a^2} = \frac{pq}{na^2}$$

$$\text{as } n \to \infty \;\; P[\{|Y-p|\} > a] \to 0 \text{ for any fixed } a > 0$$

5.20 (b) $F_X(x) = F_{XY}(x, \infty)$

$$= \begin{cases} 1 - 1/x^2 & , x > 1 \\ 0 & , \text{otherwise} \end{cases}$$

$F_Y(y) = F_{XY}(\infty, y)$

$$= \begin{cases} 1 - 1/y^2 & , y > 1 \\ 0 & , \text{otherwise} \end{cases}$$

(c) $P\{X < 3, Y \leq 5\}$

$= F_{XY}(3, 5)$

$= (1 - \frac{1}{9})(1 - \frac{1}{25})$

$= \frac{64}{75}$ //

$P\{X > 4, Y > 3\}$

$= 1 - F_{XY}(4, \infty) - F_{XY}(\infty, 3) + F_{XY}(4, 3)$

$= 1 - (1 - \frac{1}{16}) - (1 - \frac{1}{9}) + (1 - \frac{1}{16})(1 - \frac{1}{9})$

$= 1 - \frac{15}{16} - \frac{8}{9} + \frac{5}{6}$

$= \frac{1}{144}$ //

8.40

~~**5.32**~~ The width of the confidence interval given by Eqn. 8.52 ~~5.37~~ is

$$\left(M_n + \frac{z_{\alpha/2}\sigma}{\sqrt{n}}\right) - \left(M_n - \frac{z_{\alpha/2}\sigma}{\sqrt{n}}\right) = \frac{2z_{\alpha/2}\sigma}{\sqrt{n}}$$

a) For 95% confidence intervals $z_{\alpha/2} = 1.96$, so $(\sigma = 1)$

$$\text{width of interval} = \frac{2(1.96)}{\sqrt{n}} = \begin{cases} 1.96 & n = 4 \\ 0.98 & n = 16 \\ 0.29 & n = 100 \end{cases}$$

b) For 99% confidence intervals $z_{\alpha/2} = 2.576$ so

$$\text{width of interval} = \frac{2(2.576)}{\sqrt{n}} = \begin{cases} 2.576 & n = 4 \\ 1.288 & n = 16 \\ 0.515 & n = 100 \end{cases}$$

7.17

$$\begin{aligned}
M_{\cancel{100}\,20} &= \frac{1}{20\cancel{100}}(X_1 + \dots + X_{100}) = \frac{1}{20\cancel{100}} S_{\cancel{100}\,20} \\
\mu &= \mathcal{E}[X] = \frac{1 + 2 + \dots + 6}{6} = 3.5 \\
\sigma_X^2 &= \frac{1}{\cancel{7}\,6}(1 + s^2 + 3^2 + 4^2 + 5^2 + 6^2) - (3.5)^2 = 2.91667
\end{aligned}$$

$$\begin{aligned}
P[\overset{60}{\cancel{300}} < S_{100} < \overset{80}{\cancel{400}}] &= \left[3 < \overset{M_{100}}{\frac{S_{\cancel{100}\,20}}{\cancel{100}\,20}} < 4\right] \\
&= P[-.5 < M_{\cancel{100}}^{20} - 3.5 < .5] \\
&= P[|M_{\cancel{100}}^{20} - \cancel{3}5| < .5] \\
&\geq 1 - \frac{2.92}{20\cancel{100}\left(\frac{1}{2}\right)^2} = 0.416
\end{aligned}$$

7.25

~~5.25~~ $\mathcal{E}[X_i] = \frac{1}{\lambda} = 36 \qquad VAR(X_i) = \frac{1}{\lambda^2} = 36^2$

$$S = X_1 + \ldots + X_{16} \qquad \mathcal{E}[S] = 16(36) \qquad VAR(S) = 16(36)^2$$

$$\begin{aligned} P[S < 600] &= P\left[\frac{S - 16(36)}{4(36)} < \frac{600 - 16(36)}{4(36)}\right] \\ &\cong 1 - Q\left(\frac{1}{6}\right) = 0.5692 \end{aligned}$$

7.30

~~5.30~~ Total error is

$$S_{\cancel{100}\,64} = X_1 + X_2 + \ldots + X_{\cancel{100}\,64}$$

where X_i uniform is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

$$\mathcal{E}[X_i] = 0 \qquad VAR[X_i] = \frac{1}{12}$$

$$P[S_{\cancel{100}\,64} > \cancel{6}\,4] = P\left[\frac{S_{100}}{\sqrt{\frac{\cancel{100}\,64}{12}}} > \frac{4}{\sqrt{\frac{\cancel{100}\,64}{12}}}\right] \approx Q(\cancel{2.078}\ 1.7321) = \cancel{1.79}\ 4.16(10^{-2})$$