The drift diffusion model of decision formation

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Outline

- The basic drift diffusion model
  - Interrogation paradigm
  - Free response paradigm
- Neural implementation of the DDM
- Value-based drift diffusion model
  - Relation to softmax
  - Differentiating drift vs threshold changes using response times
- An example: what drives random exploration?
- Summary

The drift diffusion model interrogation paradigm

Jury duty … deciding on guilt or innocence

- Evidence is presented sequentially over the course of the trial
- As each piece of evidence is presented your opinion as to guilt or innocence will evolve over time
- This can be represented graphically …

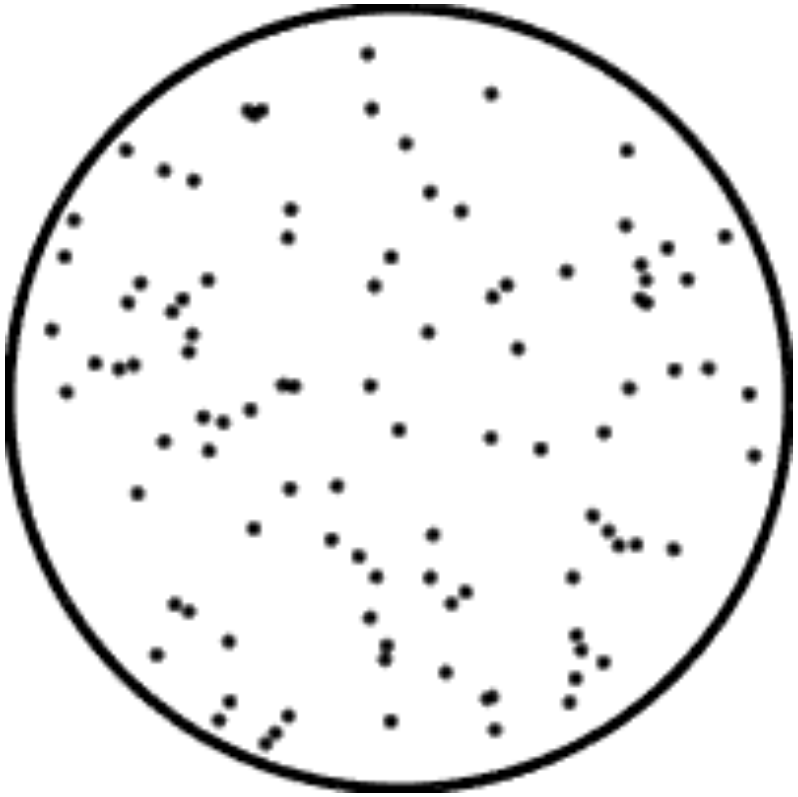
Integrating evidence over time

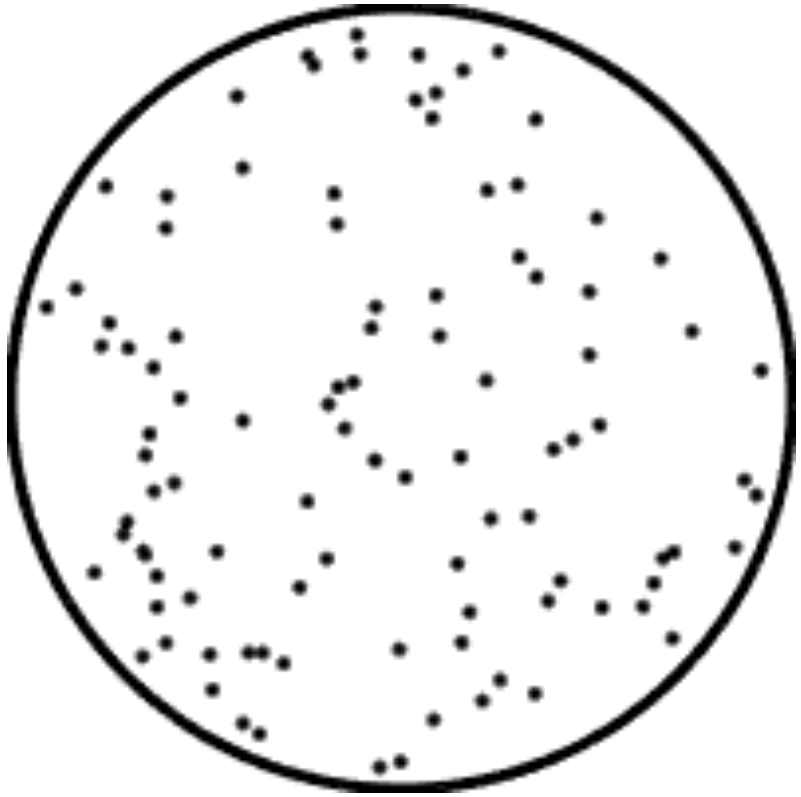
Integrating neural evidence

- The drift diffusion model assumes a similar process of evidence plays out on a much faster time scale to make rapid fire decisions
- The “evidence” is neural firing in favor of one option or another
- The classic example of this is the perceptual decision about a moving dot stimulus …

Moving dots …

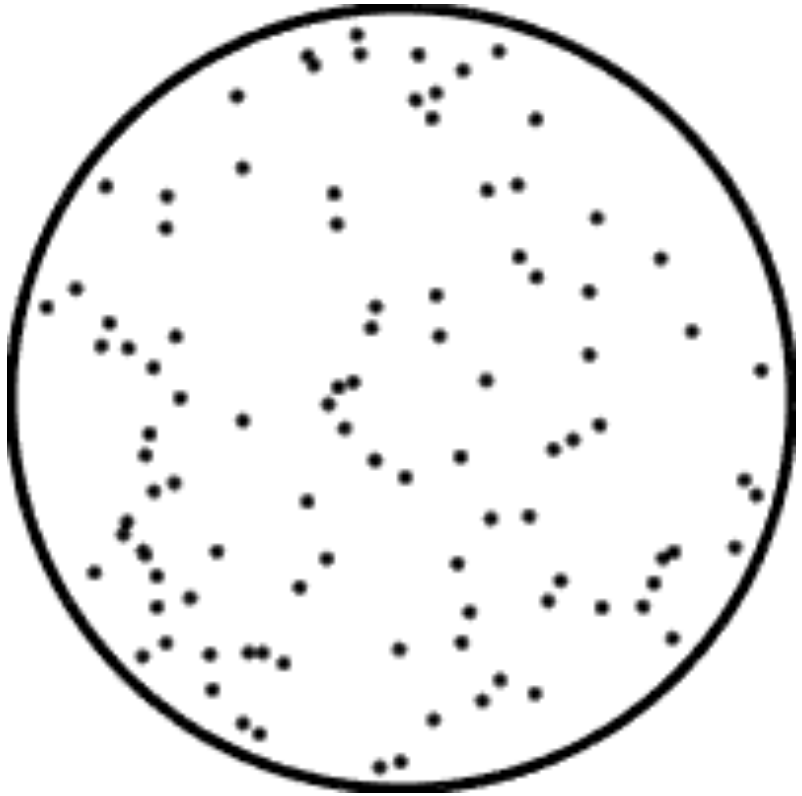
- Which way are most of the the dots moving?



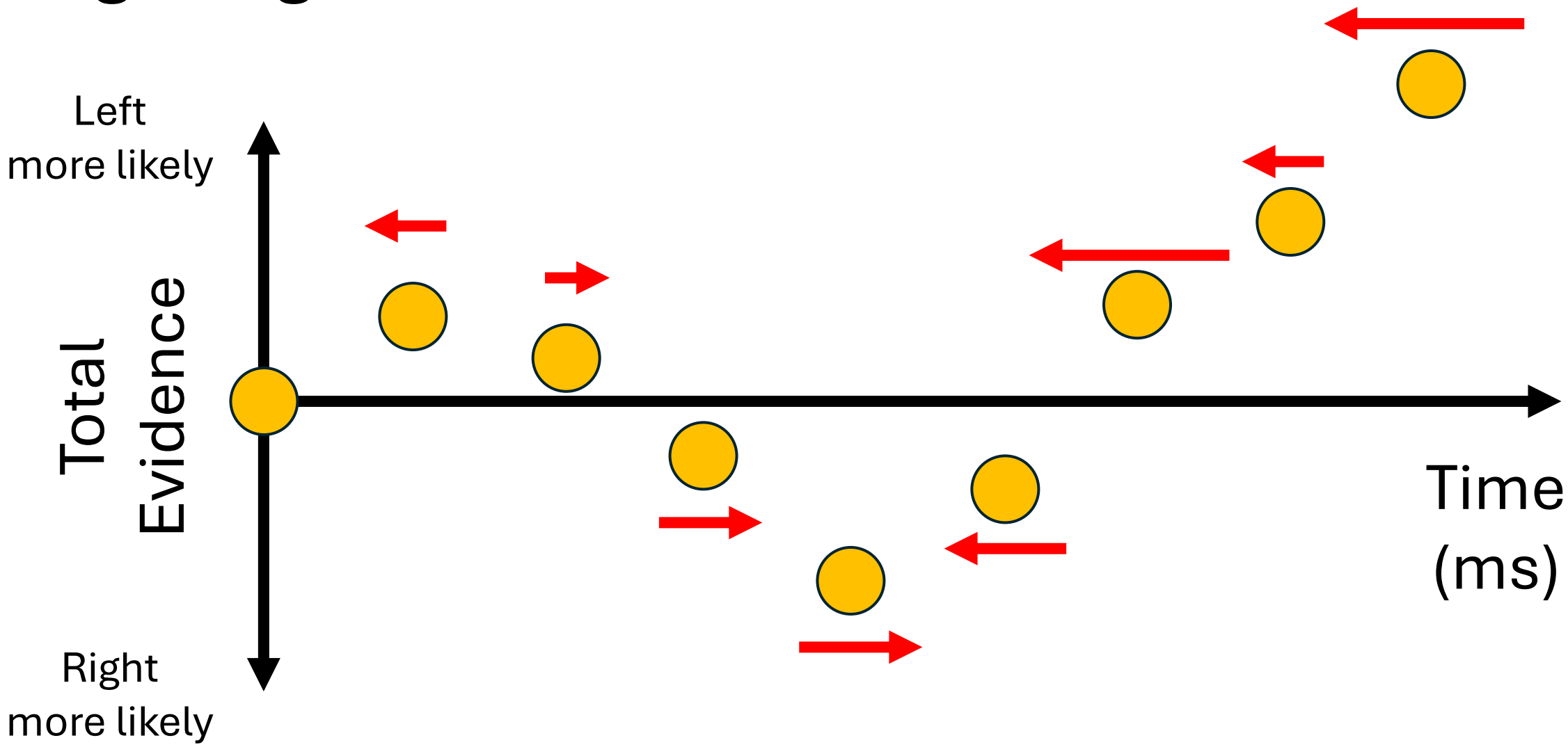


Moving dots …

- Each dot moves randomly left or right and can change from frame to frame of the movie
- More dots move in one direction
- To make the correct decision need to integrate the evidence (left or right) over time

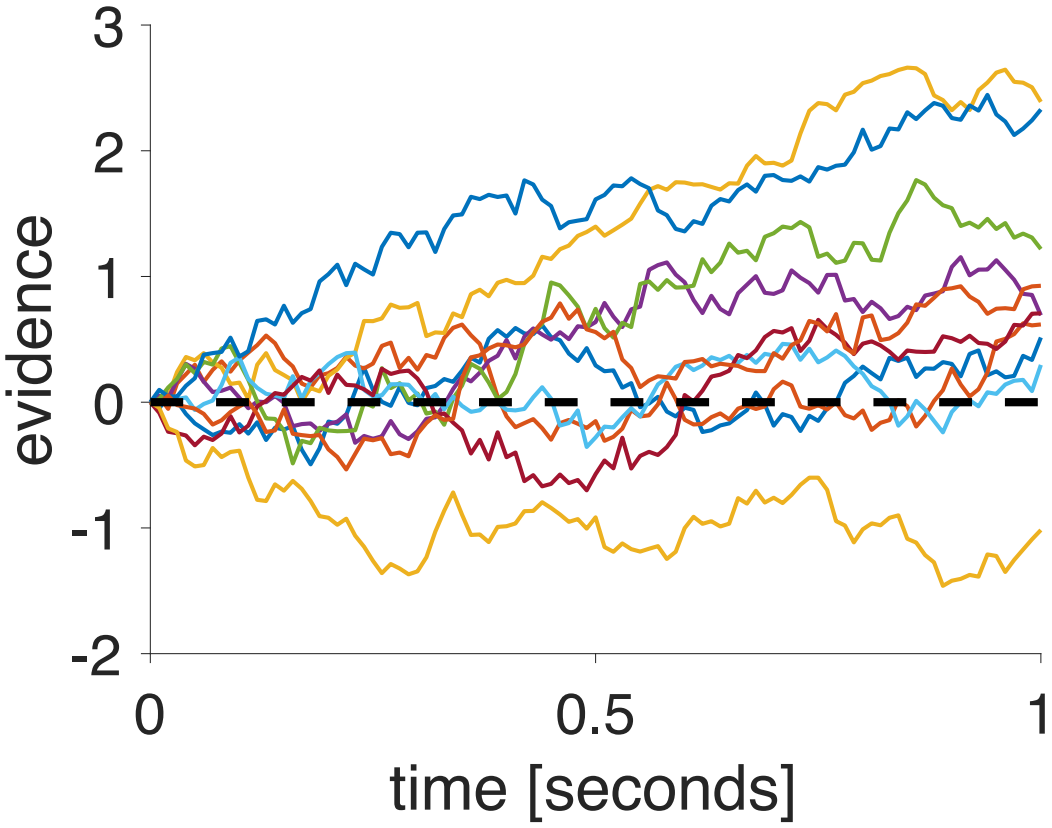


Integrating evidence over time

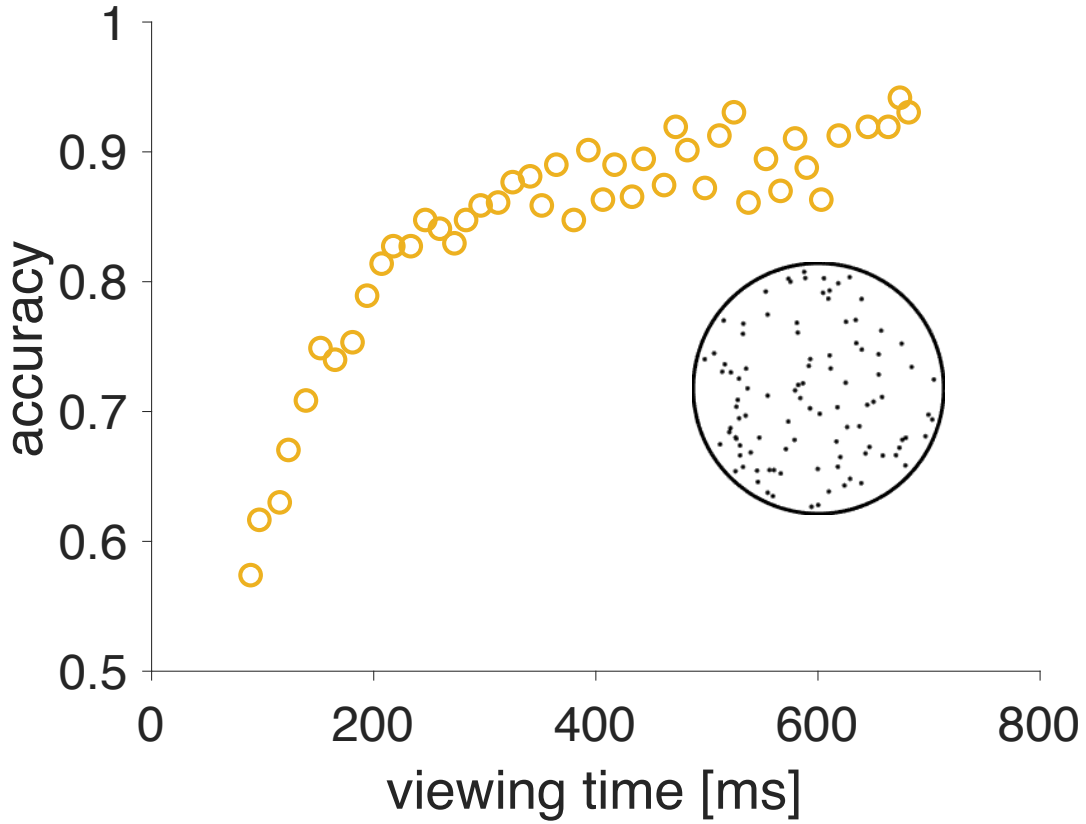


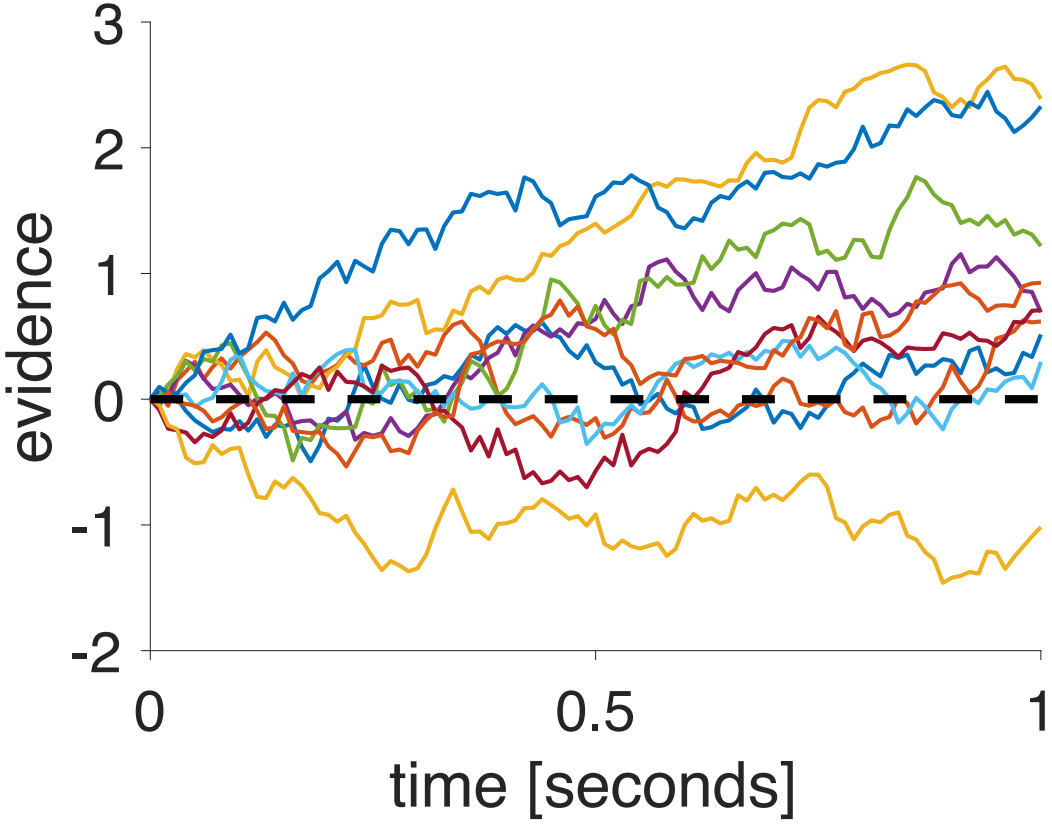
Evidence contains both signal and noise …

- On average, evidence points in the correct direction, **signal**
- But randomness in the stimulus and the brain (e.g. random spiking) causes **noise**
- As a result of noise, the accumulated evidence **drifts** (signal) and **diffuses** (noise) in the correct direction
- Because noise is random, every trial has a different trajectory



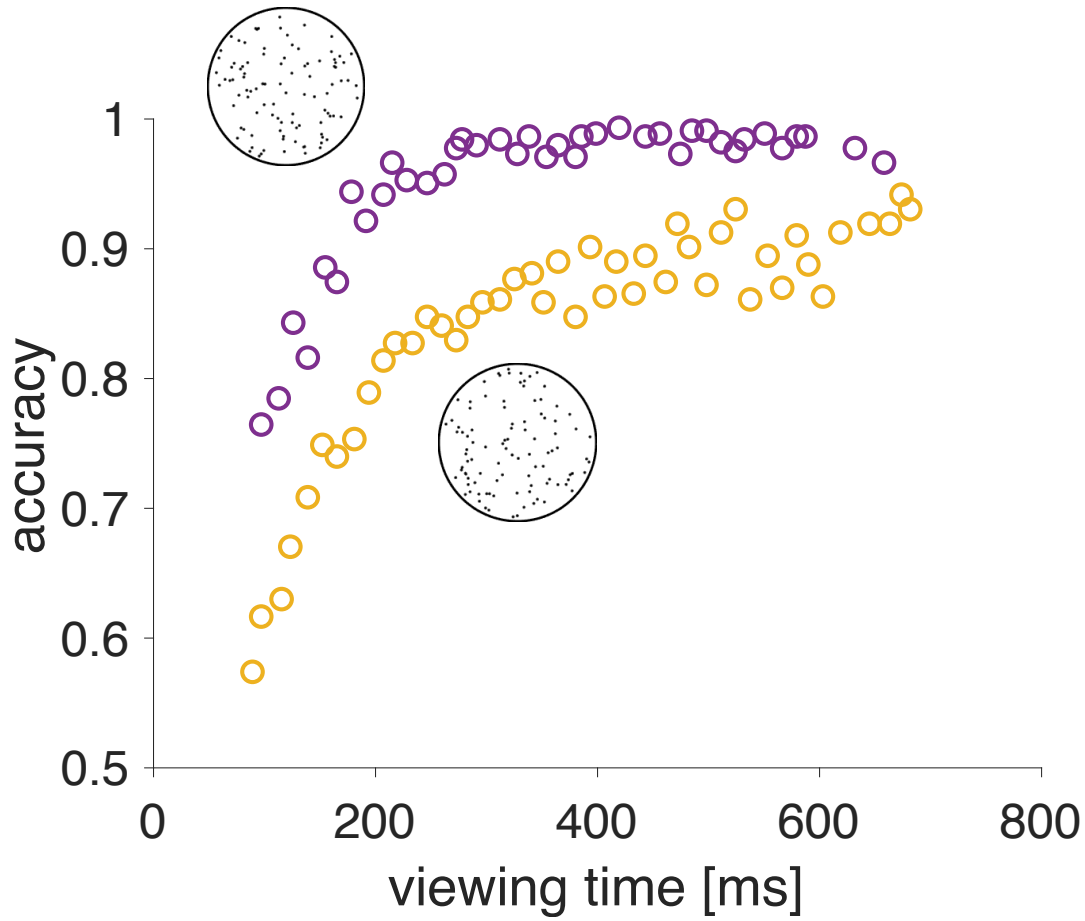
Increasing viewing time increases accuracy

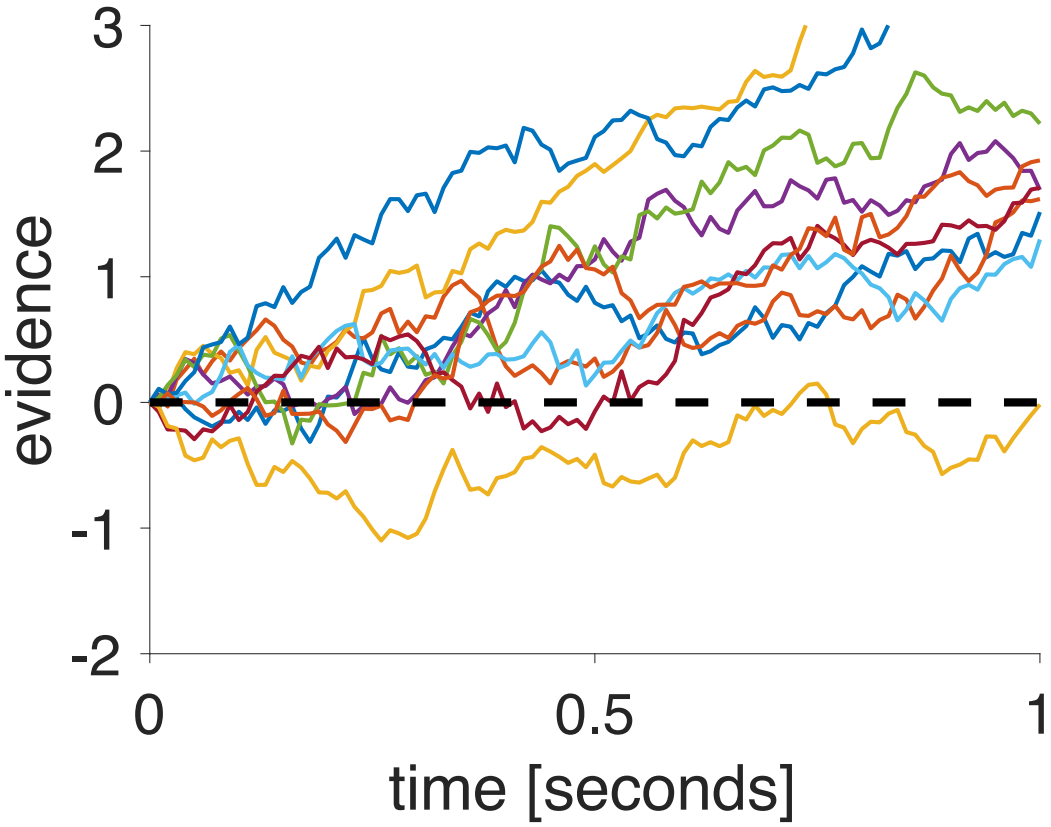




Data from Gold and Shadlen (2004)

Increasing drift rate increases accuracy

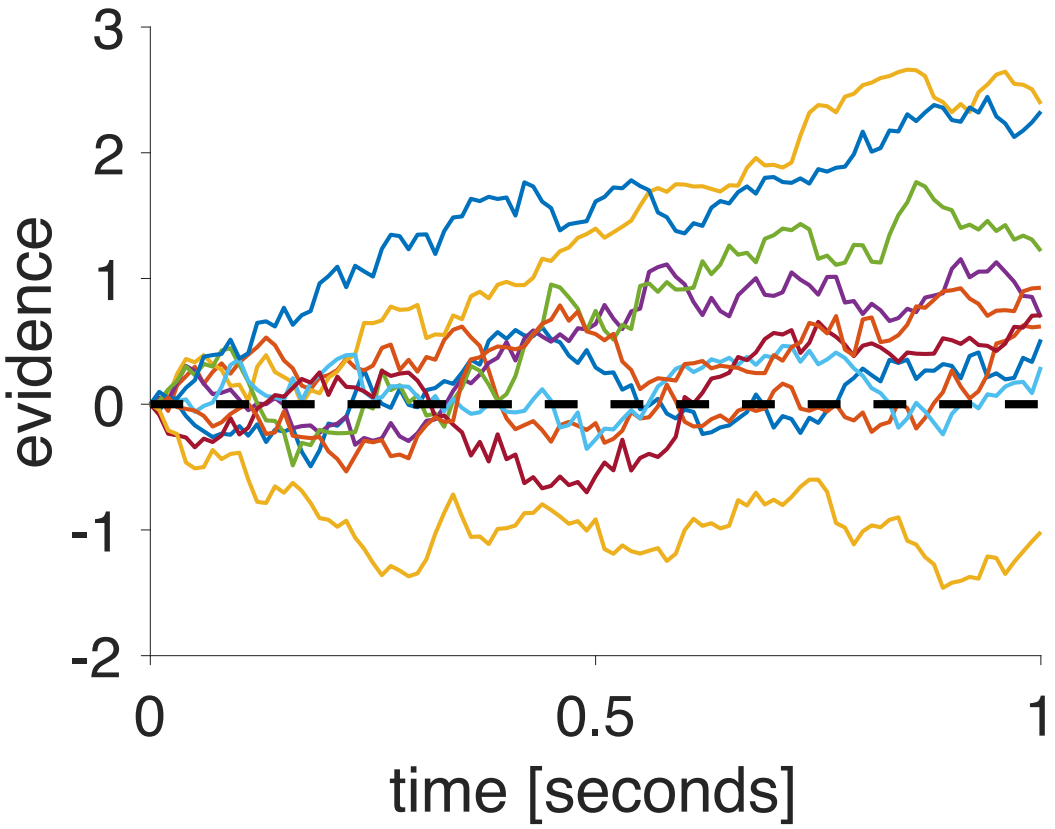




Data from Gold and Shadlen (2004)

DDM in the interrogation paradigm (jury duty)

- Evidence presented sequentially over time
- Evidence is integrated over time
- Decision is made after all evidence is presented
- Accuracy increases with viewing time

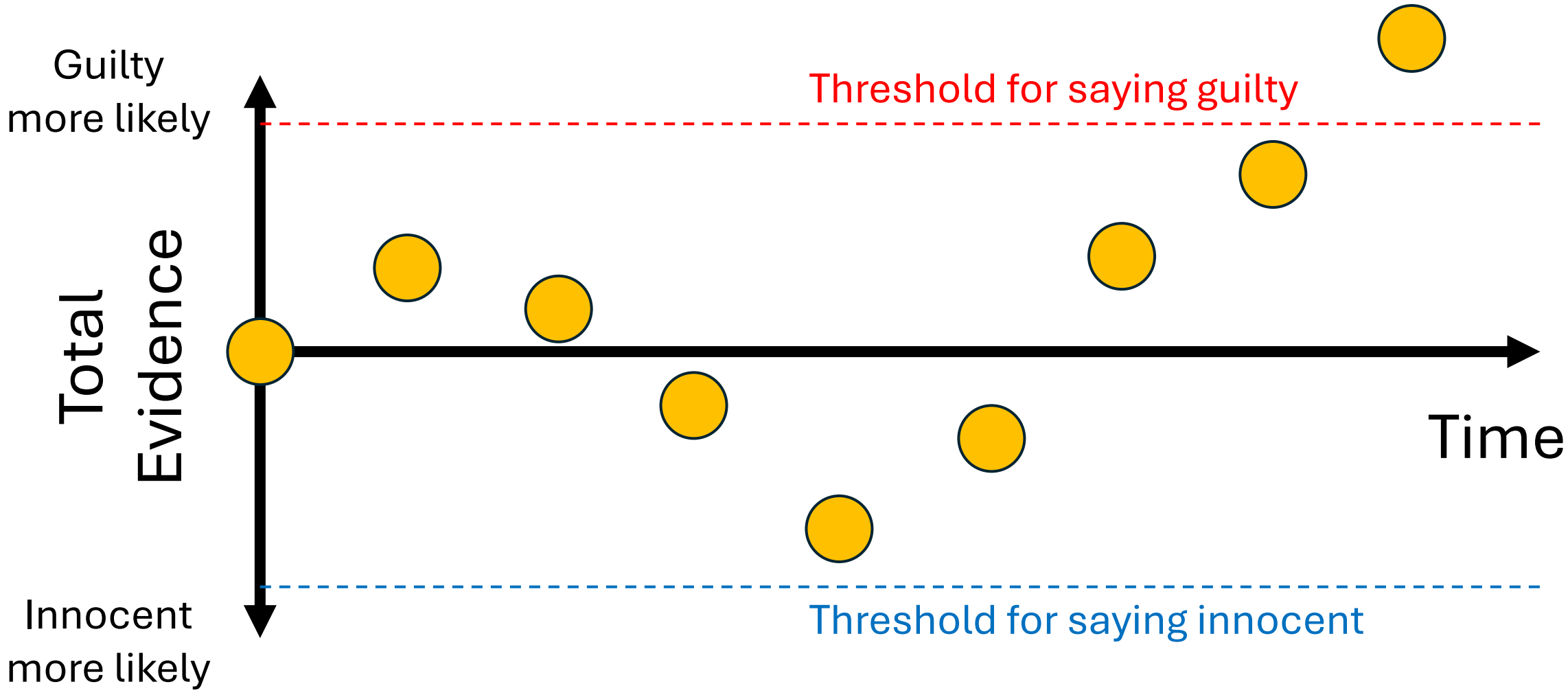


The drift diffusion model
free response paradigm

What if you are the investigator?

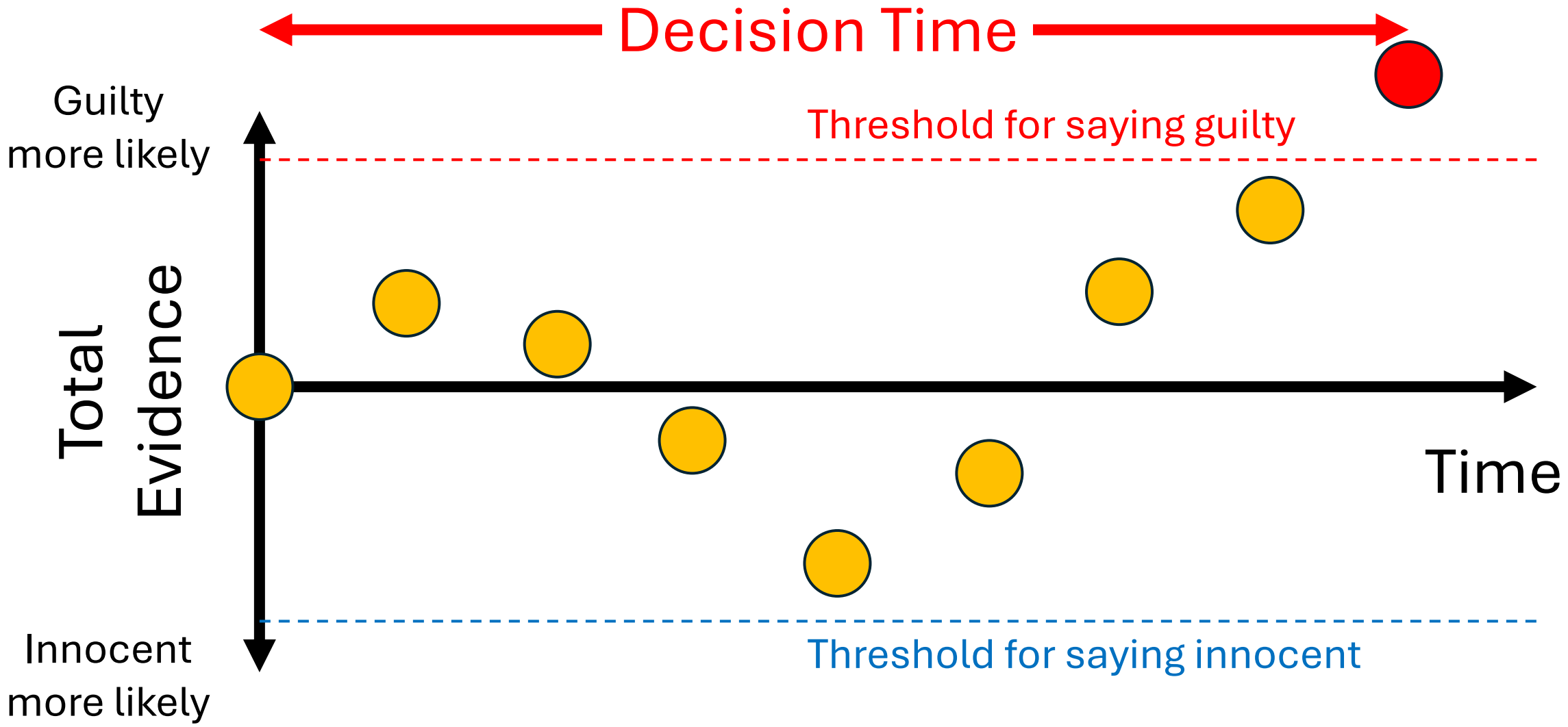
- So far considered case where evidence is presented and decision is made at the end
  - Like jury duty
- What about the case where you also decide when to stop collecting evidence?
  - Like an investigator
- Investigators stop investigating when they have enough evidence to convict, when the total evidence passes a ***threshold***

The decision thresholds ...



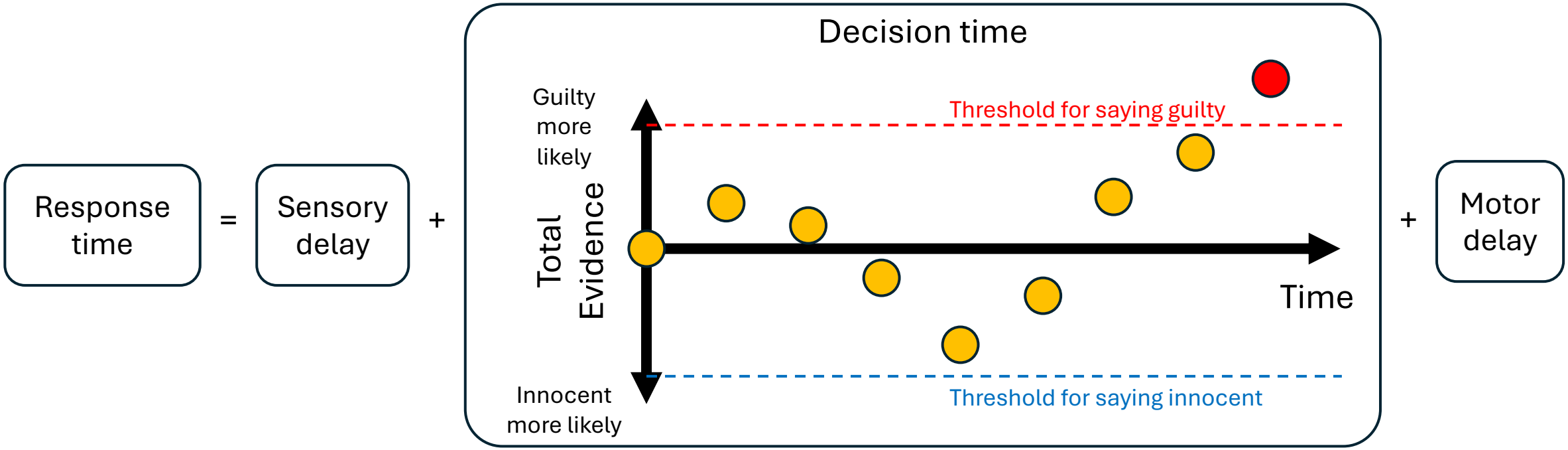
Decision = guilty

The decision thresholds ...



DDM in the free response paradigm

- Models choices AND decision times
- Transform decision times to response times by accounting for delays

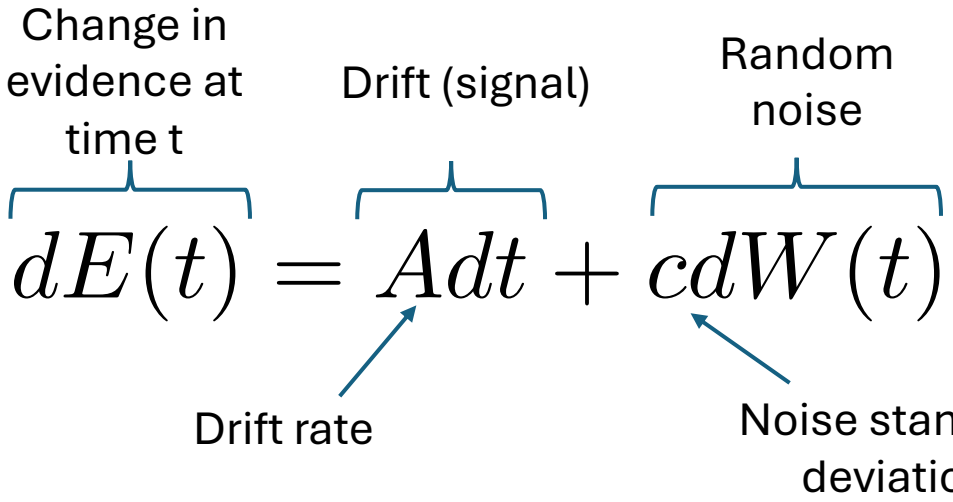


$$RT = T_0 + DT$$

T0 = Non-decision time

Some math …

- A stochastic differential equation for the evidence …



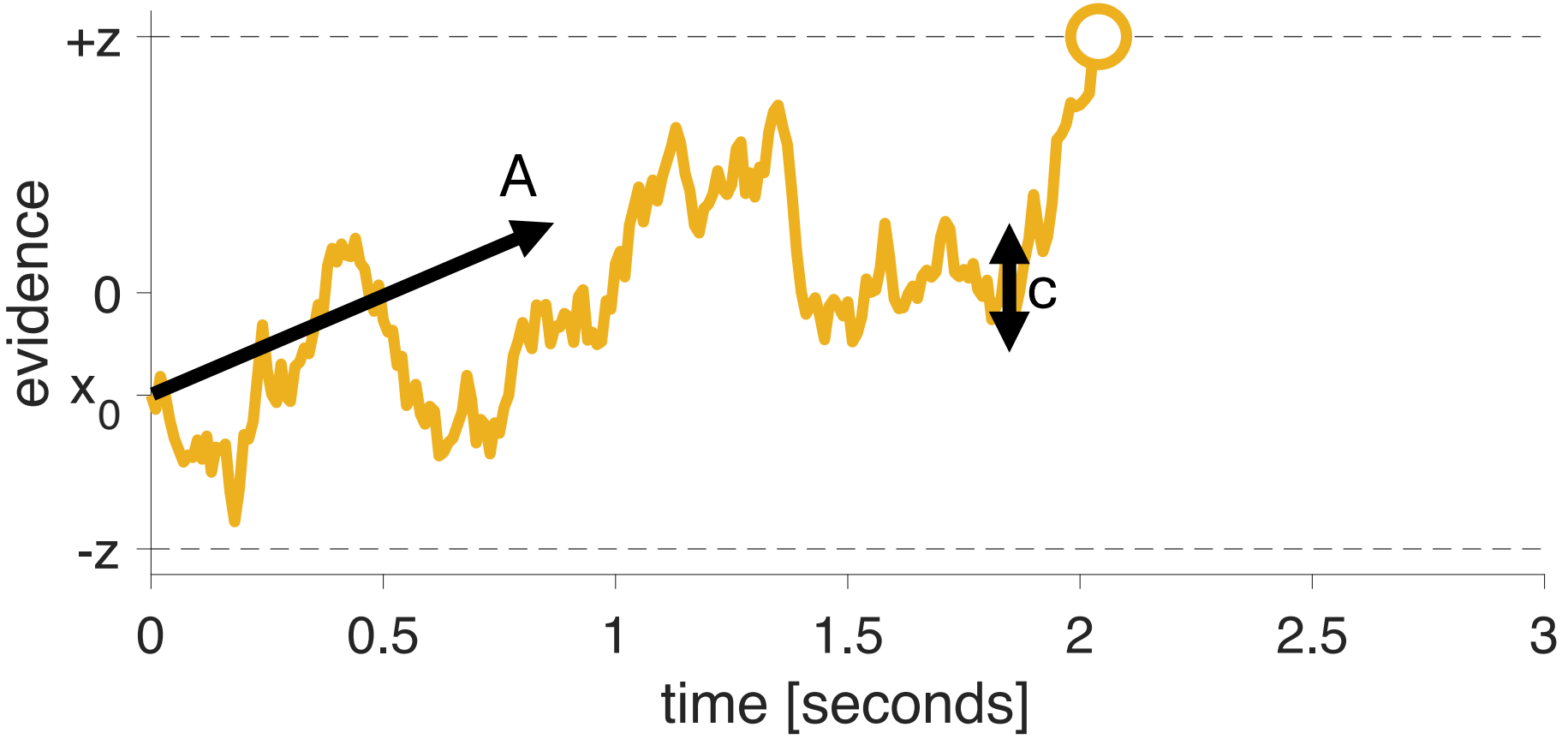
- Evidence is integrated over time, starting at initial bias x_0
- Decision is made when evidence crosses threshold at +z or –z, which gives choice and decision time, DT
- Response time is decision time + non-decision time

$$RT = T_0 + DT$$

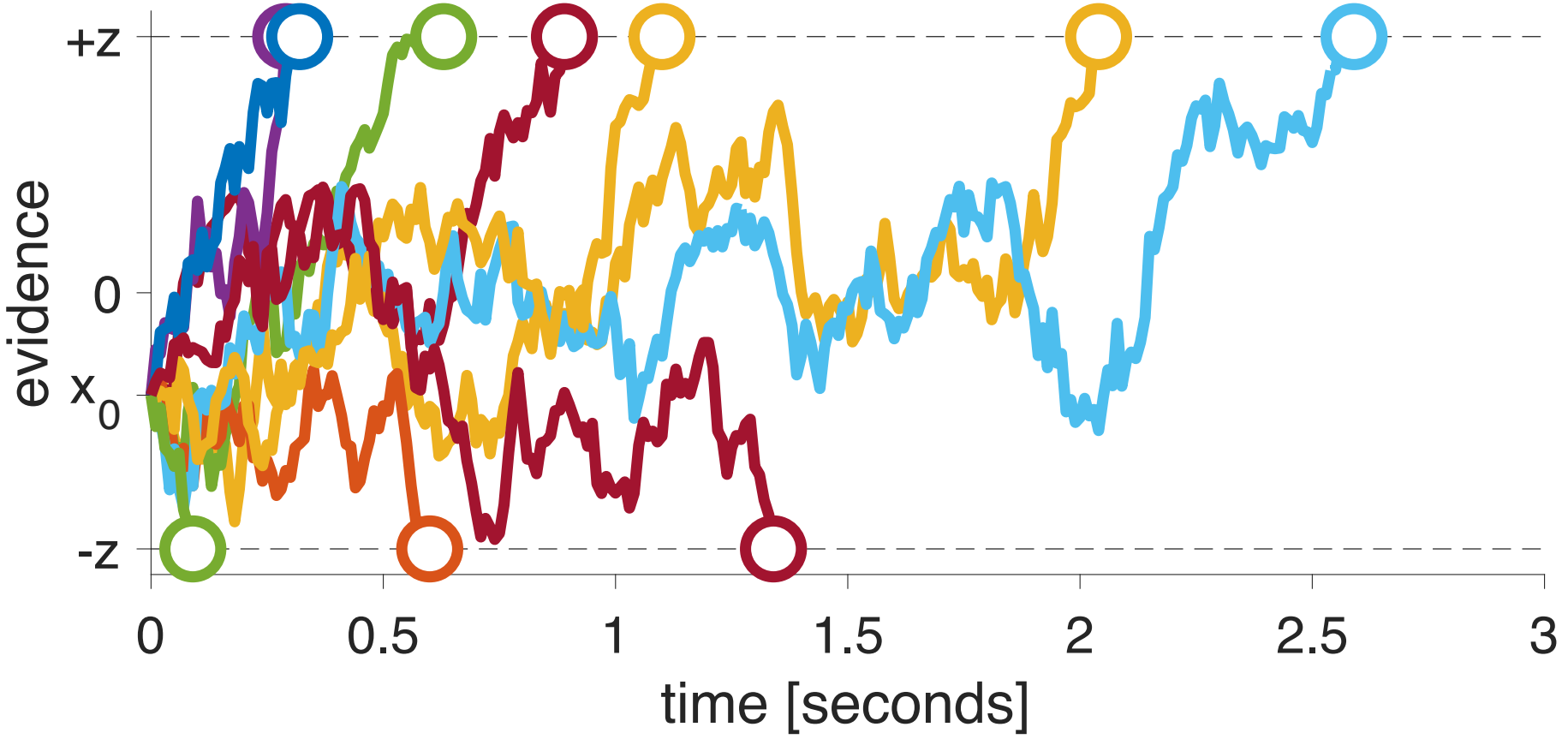
Parameters

- Non-decision time T_0
- Starting point (initial bias) x_0
- Drift rate (signal-to-noise ratio) A
- Threshold z
- Noise standard deviation c
  - In practice set to 1 and interpret drift rate as a signal-to-noise ratio

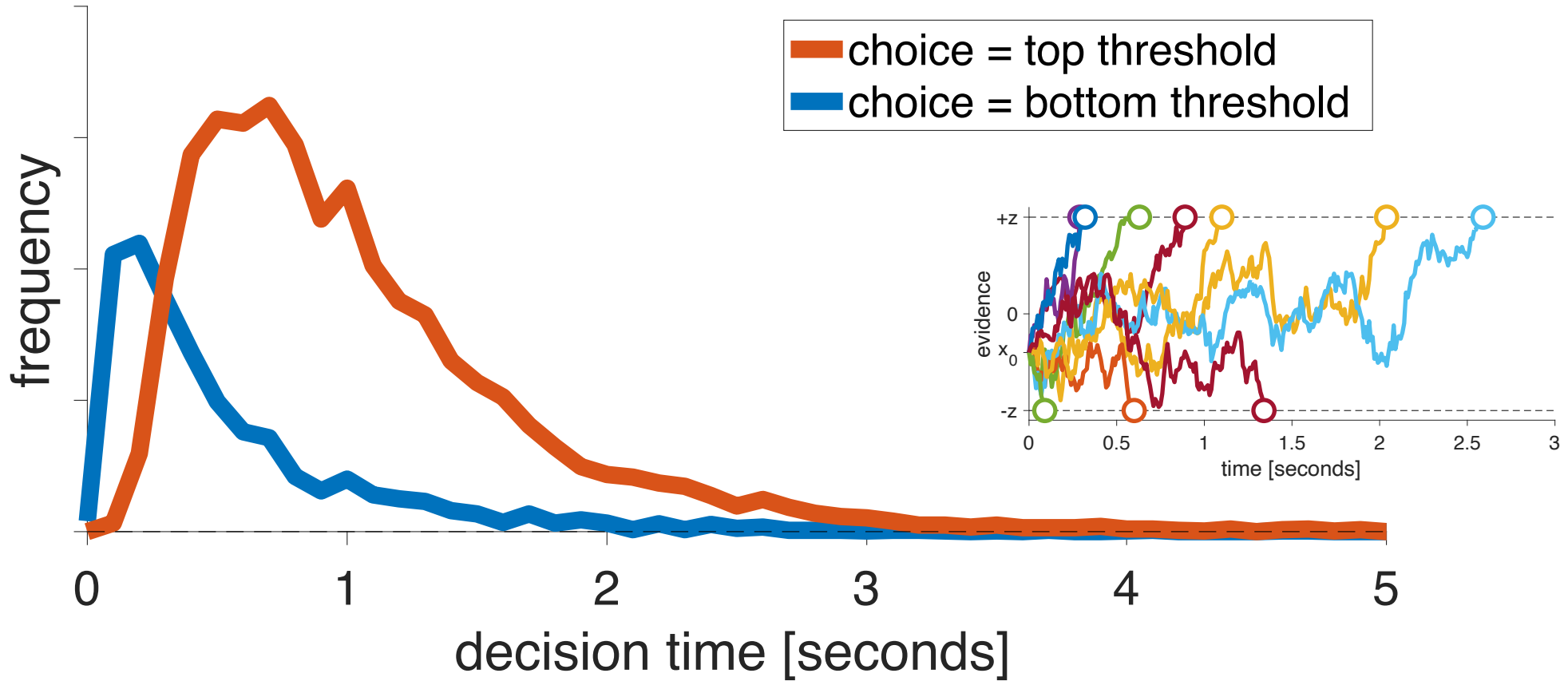
Classic DDM picture ...



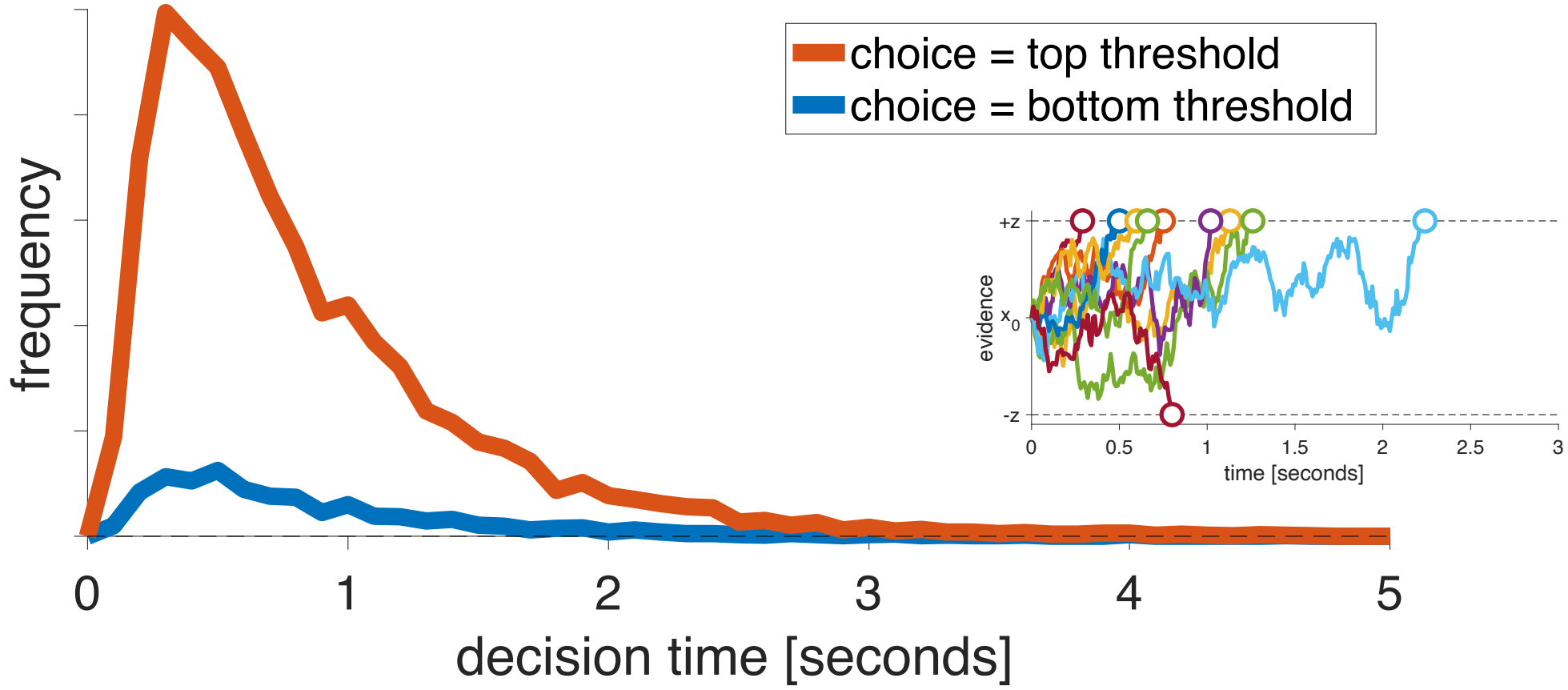
Noise causes variability in choices and decision times



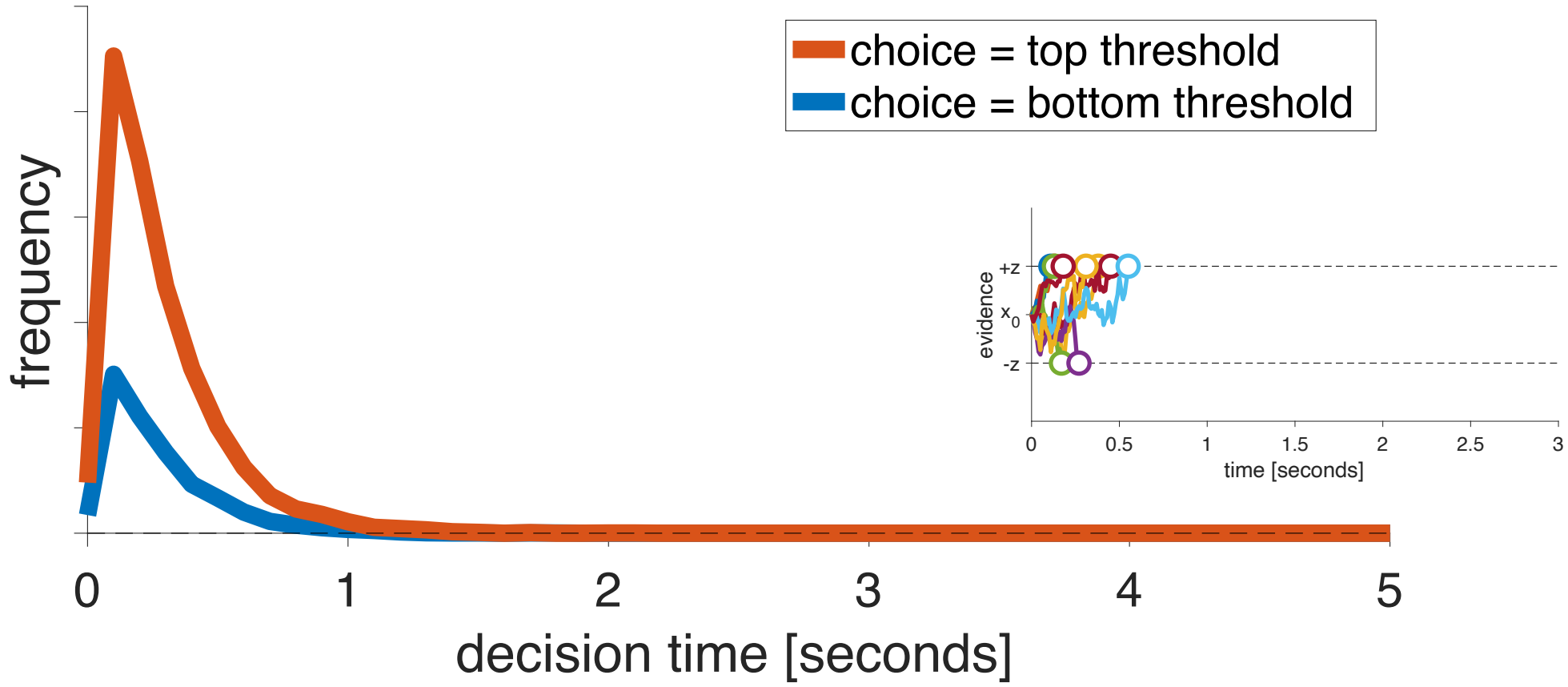
Choice and response time distributions



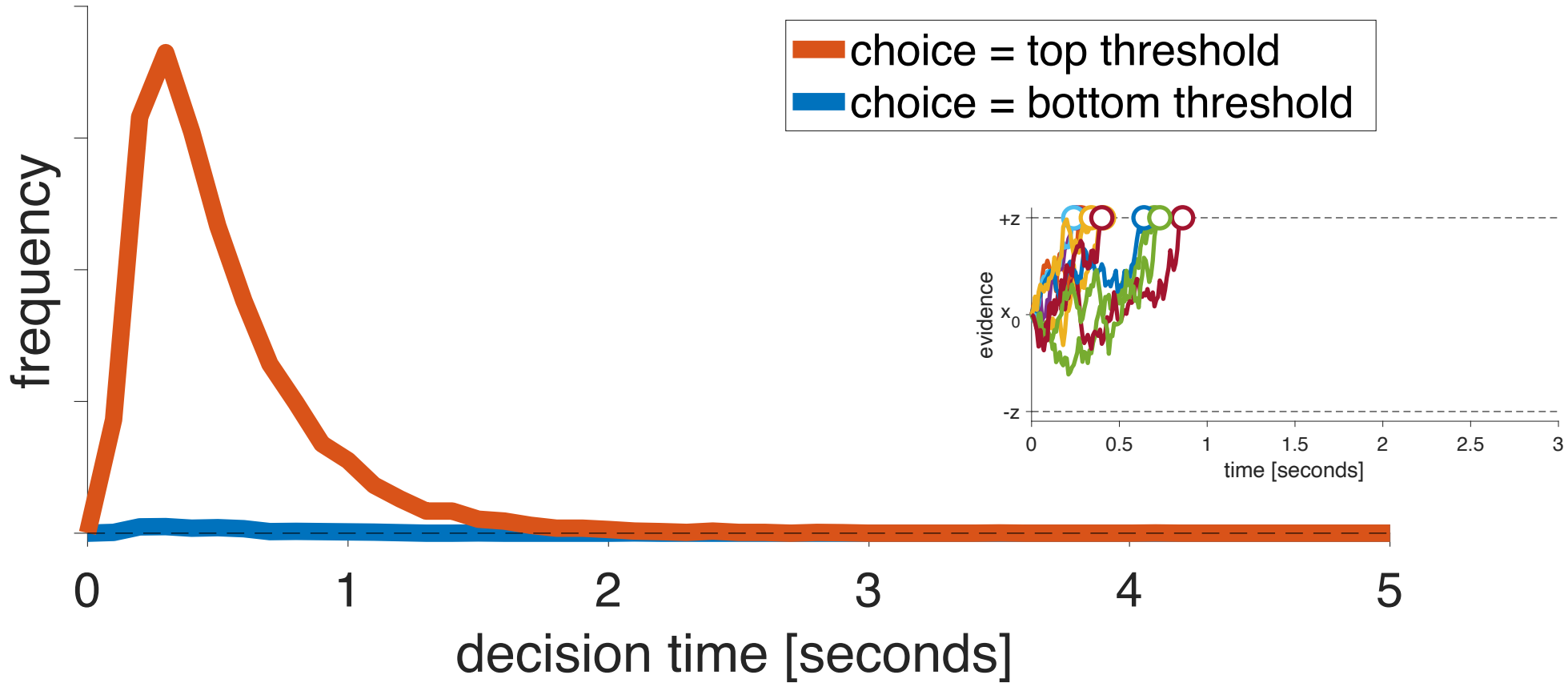
Changing starting point
=> changes type and amount of errors



Lower threshold
=> faster, less accurate



Higher drift
=> faster, more accurate



Analytic expressions for mean accuracy and response time

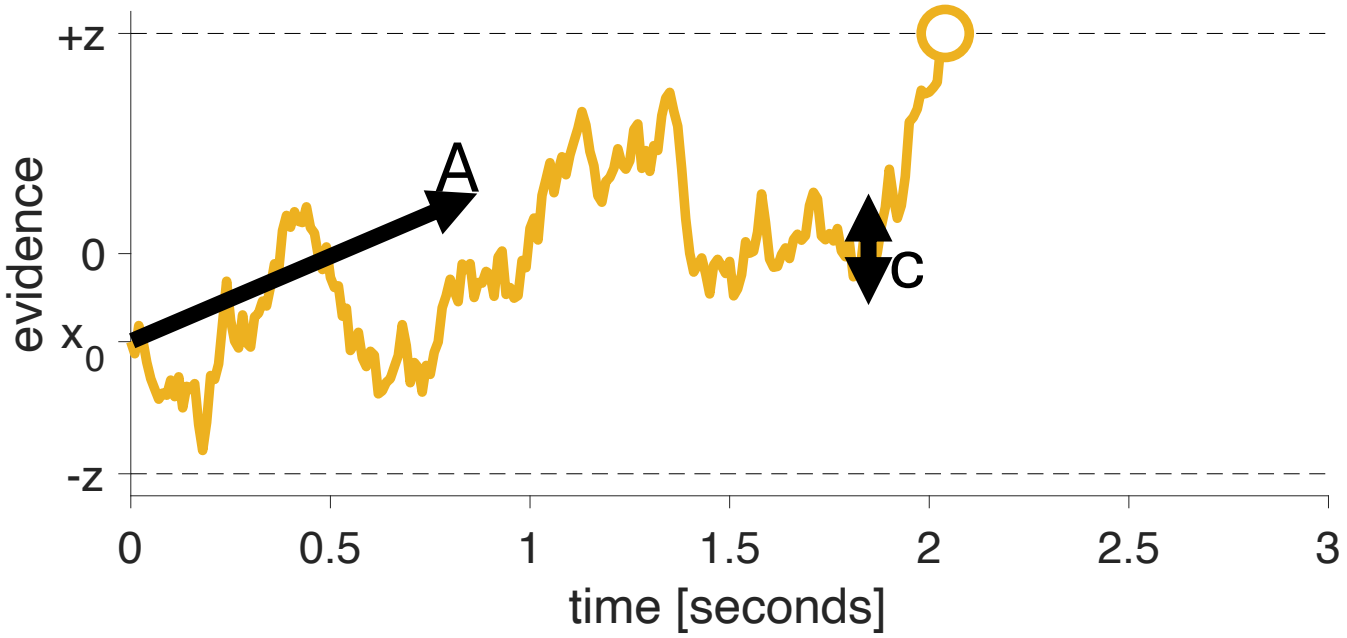
$$p(\text{choice} = \text{left}) = \frac{1}{1 + \exp(2zA)} - \frac{1 - \exp(-2x_0 A)}{\exp(2zA) - \exp(-2zA)}$$

$$RT = T_0 + \frac{z}{A}\tanh(zA) + \frac{z}{A}\frac{2(1 - \exp(-2x_0 A)}{\exp(2zA) - \exp(-2zA)} - \frac{x_0}{A}$$

- Can be handy for visualization of results
- Can also be useful for analysis of reward rate – i.e. reward per unit time
- Also – coming up – equates to softmax equation for value-based DDMs

DDM free response paradigm …

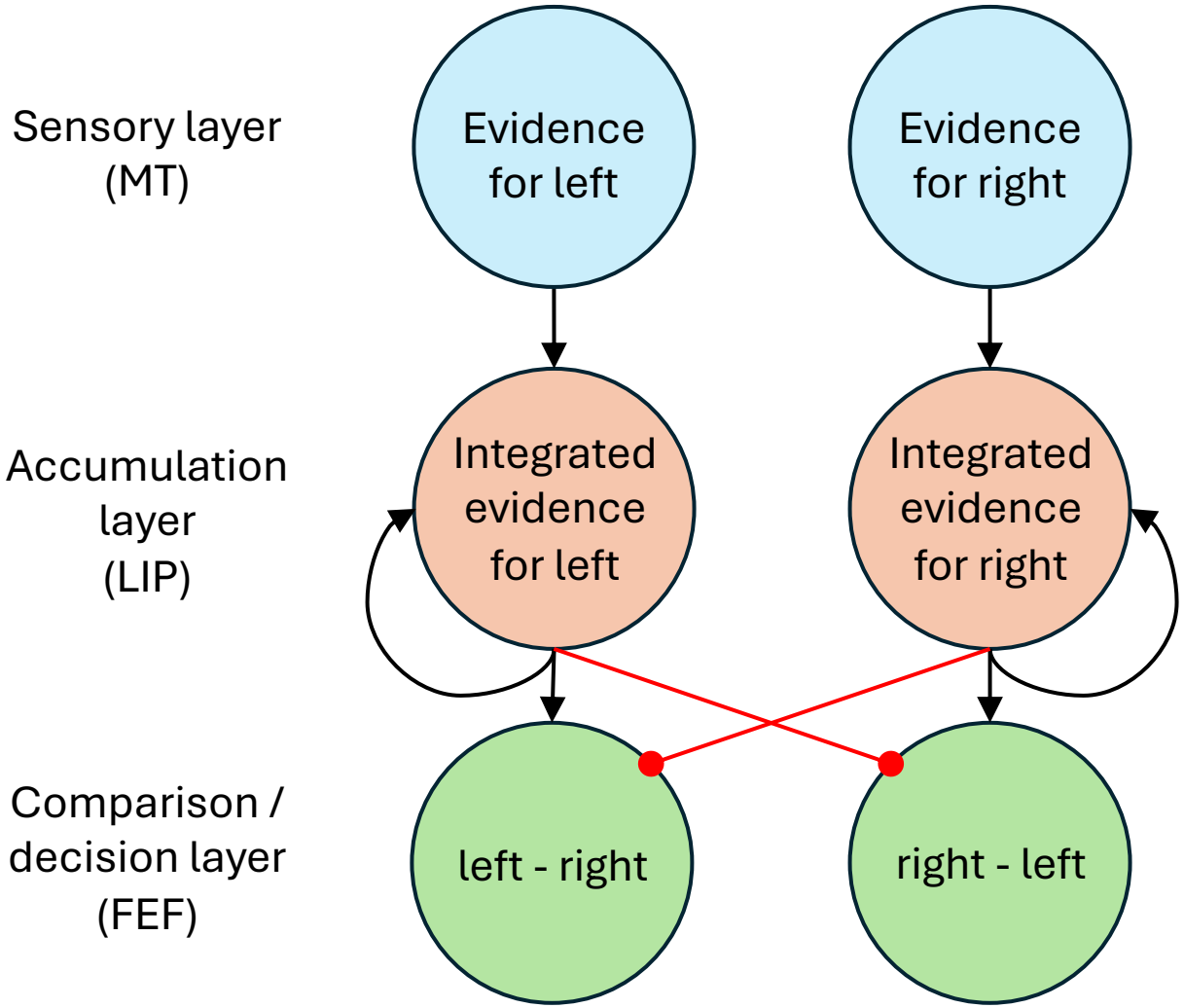
- Integrate evidence to threshold
- Which threshold crossed determines choice
- When threshold crossed determines decision time



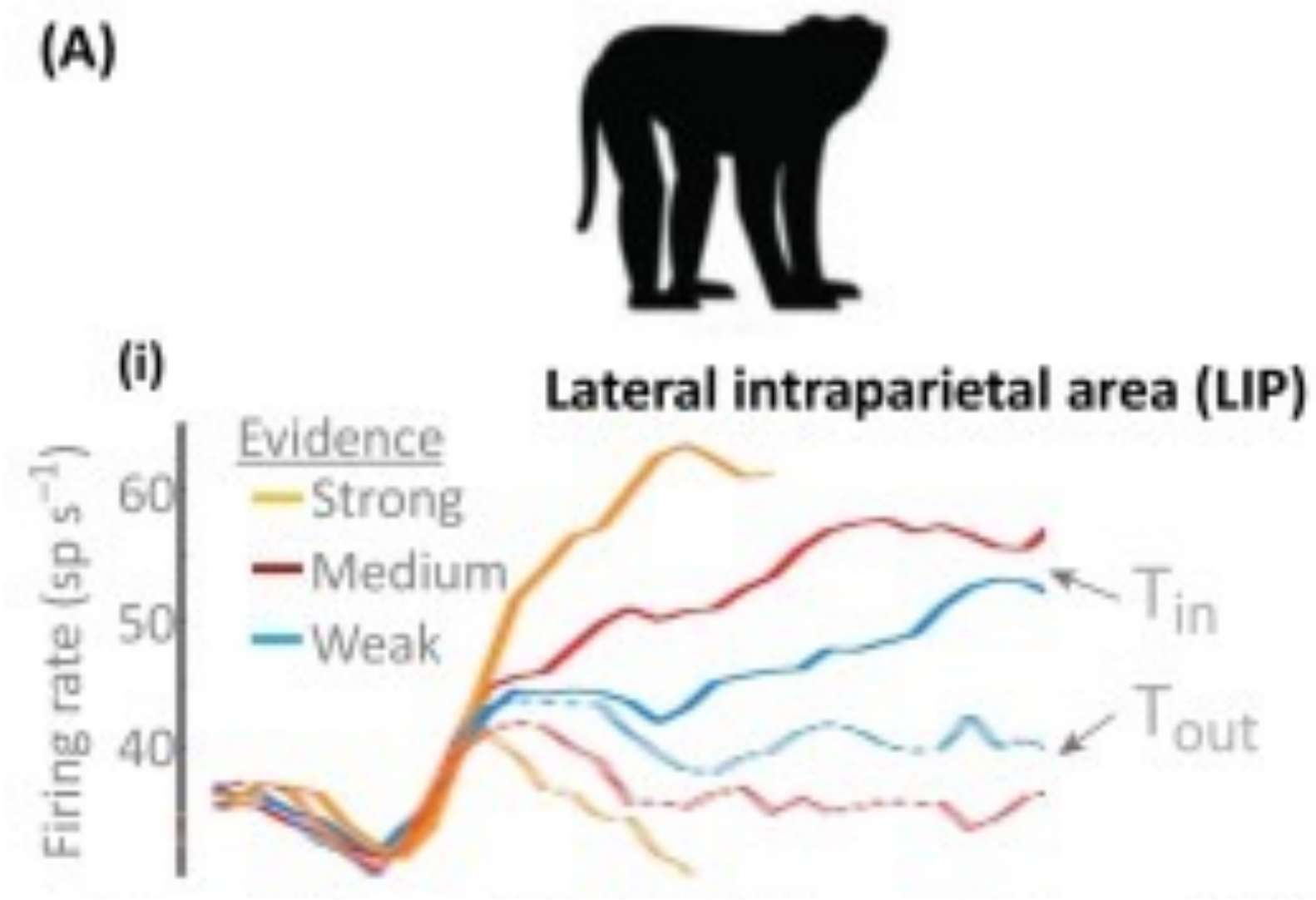
Brief aside …
Neural basis of the DDM

Neural network implementation

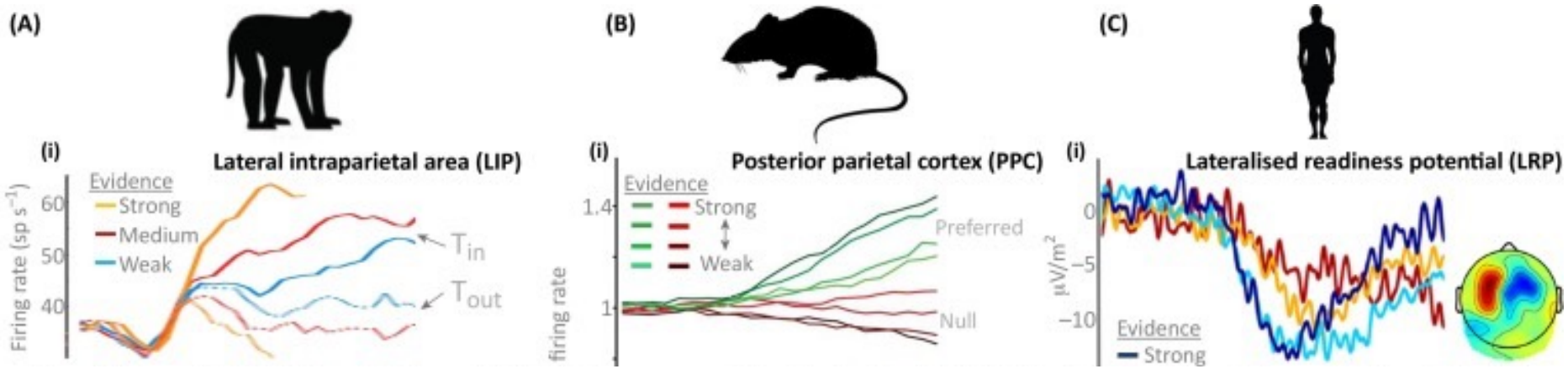
- Different ways of implementing DDM in a neural network
- Most have a similar structure, although underlying mathematics can be different



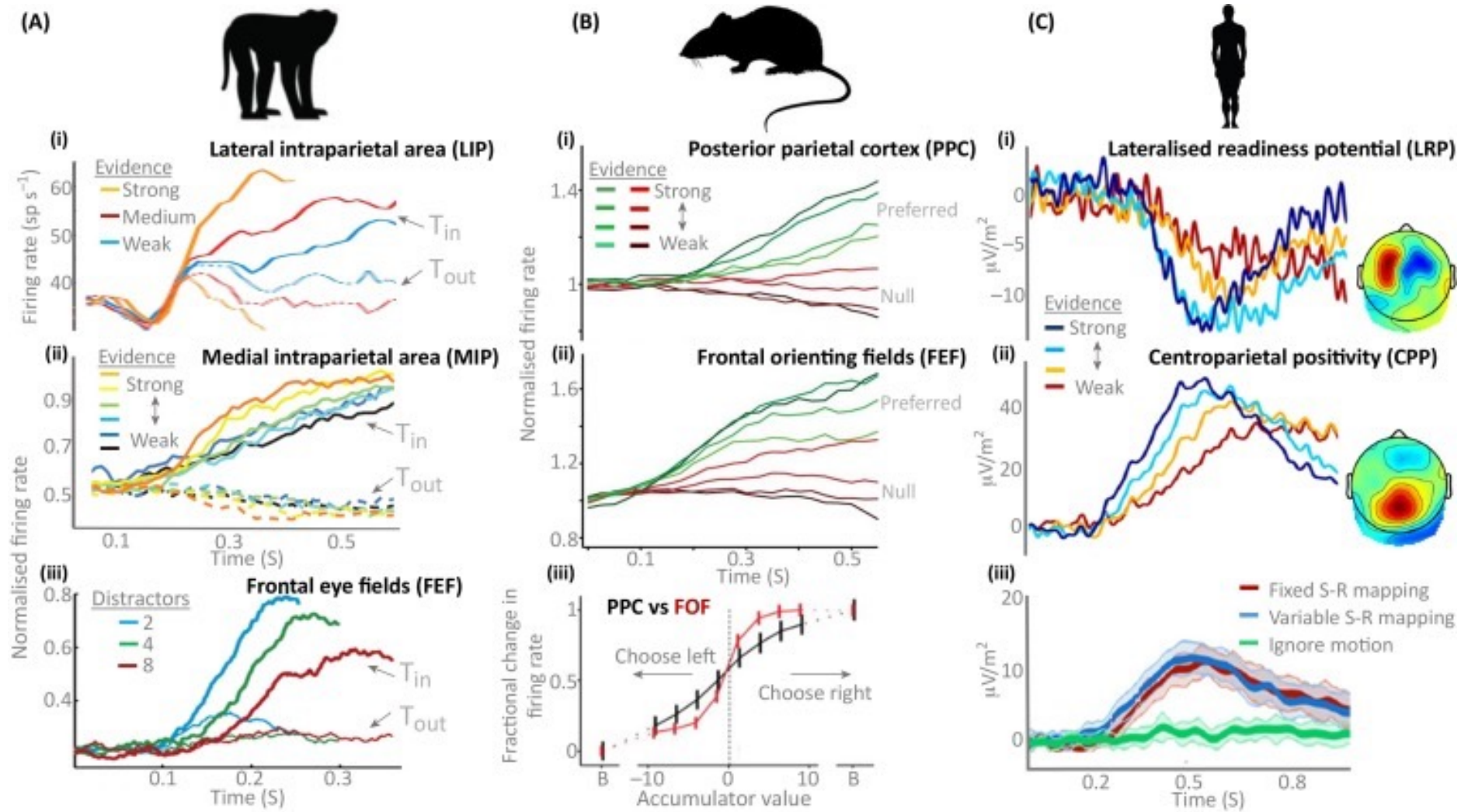
DDMs in the brain ...



DDMs in the brain ...



DDMs in the brain ...



Trends in Neurosciences

Value-based drift diffusion model

Applying the drift diffusion model to value-based decisions

- Value-based decision implies having a value for different options
- Focus on two-choice case
- Choose between option 1 with value V_1 and option 2 with value V_2
- Simple way to connect to DDM is to set the drift rate proportional to the difference in value …

$$A = k(V_2 - V_1) = k\Delta V$$

Value-based DDM – choices

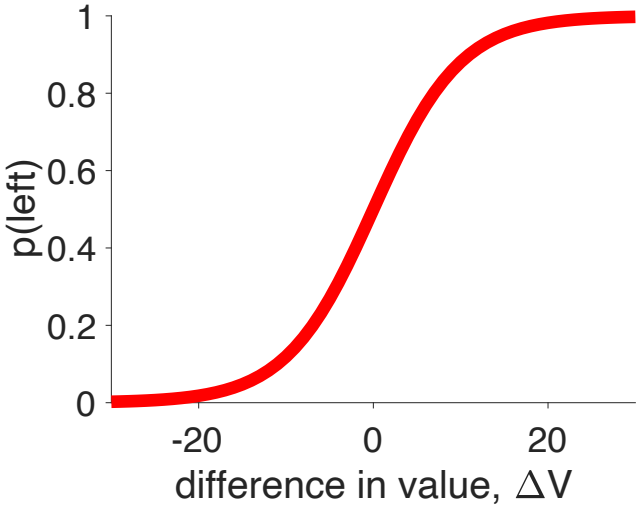
- Choice probabilities …

$$p(\text{choice} = \text{left}) = \frac{1}{1+\exp(2zk\Delta V)} - \frac{1-\exp(-2x_0 k\Delta V)}{\exp(2zk\Delta V)-\exp(-2zk\Delta V)}$$

- Special case when unbiased starting point, $x_0 = 0$

$$p(\text{choice} = \text{left}) = \frac{1}{1+\exp(2zk\Delta V)}$$

- This is the **softmax choice probability!**



Softmax and the value-based DDM

- Compare with softmax equation …

$$p(\text{choice} = \text{left}) = \frac{1}{1 + \exp(2zk\Delta V)} \qquad p(\text{choice} = \text{left}) = \frac{1}{1 + \exp(\beta\Delta V)}$$

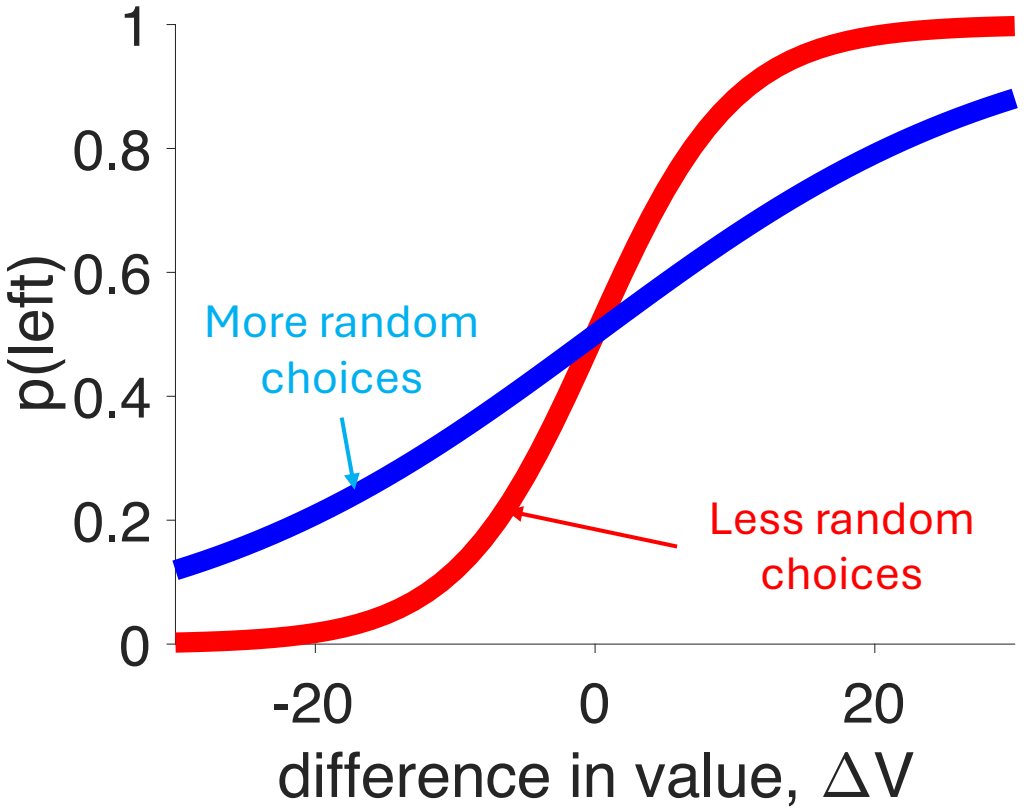
- Inverse temperature parameter of softmax is controlled by TWO DDM parameters

$$\beta = 2zk$$

- Threshold, z, and signal-to-noise ratio, k

Softmax and the value-based DDM

- More random choices can be generated by
  - Lower drift rate (signal-to-noise ratio), k
  - Lower threshold, z
- Different mechanisms – ***cannot be distinguished by choices alone***



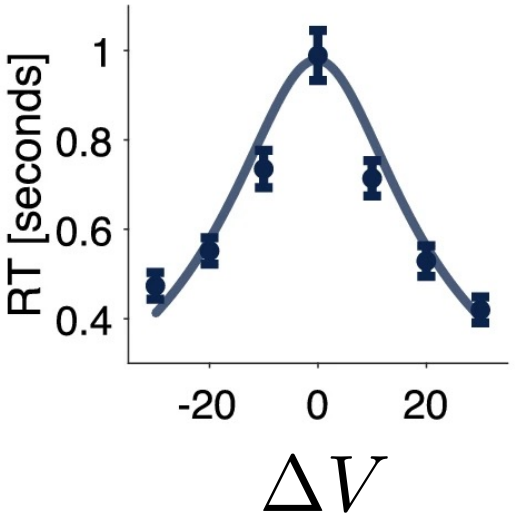
Value-based DDM – response times

- Response times

$$RT = T_0 + \frac{z}{\Delta V}\tanh(z\Delta V) + \frac{z}{\Delta V}\frac{2(1-\exp(-2x_0\Delta V)}{\exp(2z\Delta V)-\exp(-2z\Delta V)} - x_0\Delta V$$

- Special case when unbiased starting point, $x_0 = 0$

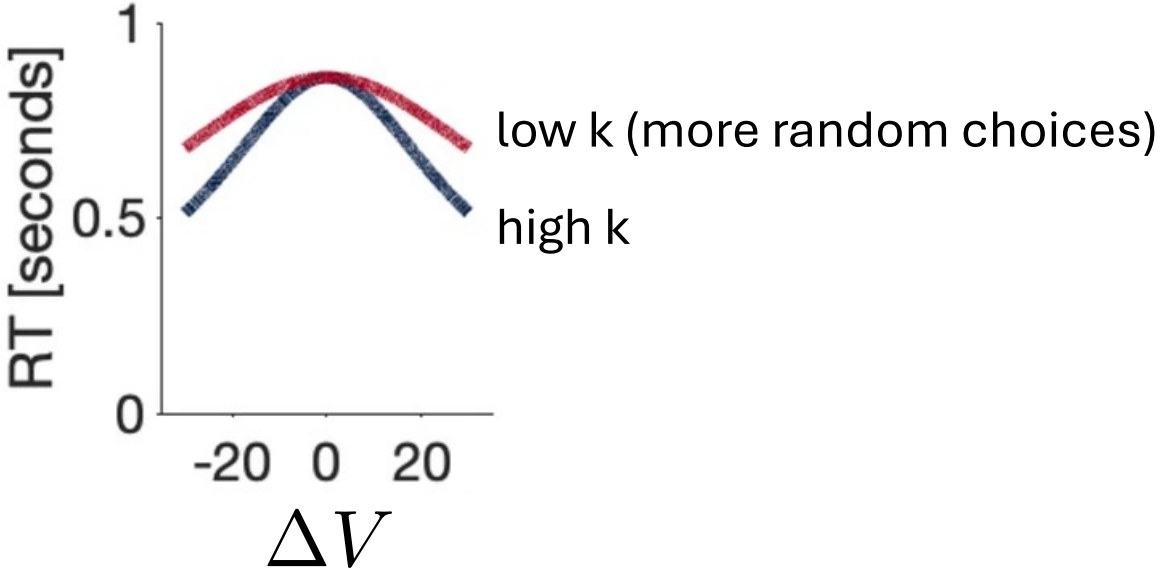
$$RT = T_0 + \frac{z}{\Delta V}\tanh(z\Delta V)$$



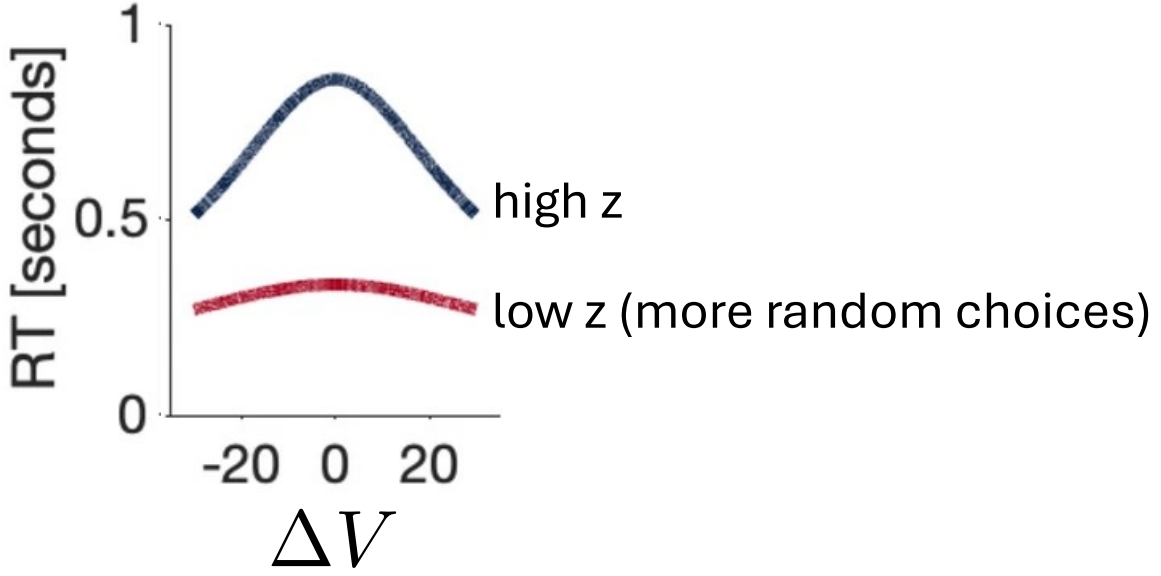
Value-based DDM – response times

- Changes to drift and threshold ratio have ***opposite*** effects on response times …

Change in drift



Change in threshold



Value-based DDM

- Set drift rate proportional to difference in value between options
- Gives softmax choice probabilities when initial condition is 0
- Randomness in choice can be controlled by two difference mechanisms
  - Drift change
  - Threshold change
- These mechanisms can be distinguished using response times

Example: What drives random exploration?

Explore-exploit decisions

STARTERS

Pizza Bianca (White Pizza) ... $5.50
Sprinkled with Parmesan Cheese, Oregano, and Olive Oil.

Mozzarella in Carozza $6.50
Fried Mozzarella with Anchovies, Garlic and Cream Sauce.

Calzone $5.50
Made fresh on the premises.

PASTA

Fettuccine Alfredo $10.95
Egg Noodles in Cream Sauce and Parmesan Cheese.

Seafood Scampi $16.95
Shrimp, scallops, squid sautéed in garlic, olive oil. Served on fresh linguine.

New **Ravioli di Ricotta** $11.95
Pasta Stuffed with Ricotta and Parmesan Cheese. Served with Tomato Sauce.

Linguine Vegetariani $13.00
Sautéed in butter with mushrooms, zucchini and red pepper.

PIZZA

	10"	14"
Cheese and Tomato ..... based on the "Pines of Rome" classic	$11.75	$13.75
Fresh Herb ..... a blend of herbs, topped with an olive oil & chèvre cheese	$14.75	$16.95
New **Thai Chicken** ..... chicken marinated in a spicy peanut-ginger sesame sauce	$15.75	$17.95
Garden Veggie ..... olives, onions, green peppers, & tomatoes	$13.75	$15.95
Greek Pizza ..... mediterranean spiced grilled chicken & mozzarella	$14.75	$16.75
New **The Works** ..... Canadian bacon, salami, pepperoni, sausage, beef, olives, pineapple, mushrooms, onions, green peppers, etc. etc.	$18.95	22.95
Hawaiian BBQ ..... fresh pineapple, BBQ chicken, & smoked gouda	$16.75	$18.95

Explore-exploit decisions

exploit

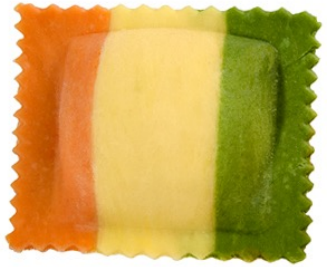
The explore-exploit dilemma

explore

exploit

The Horizon Task

explore



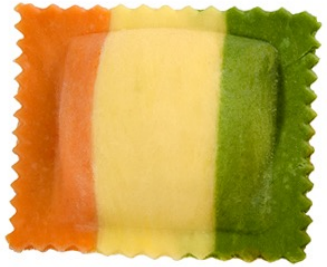
exploit

- Suppose you're leaving town tomorrow
  - Short horizon
- Suppose you're leaving town in a year
  - Long horizon
- The difference in behavior can quantify exploration

Wilson et al 2014

The Horizon Task

explore



exploit

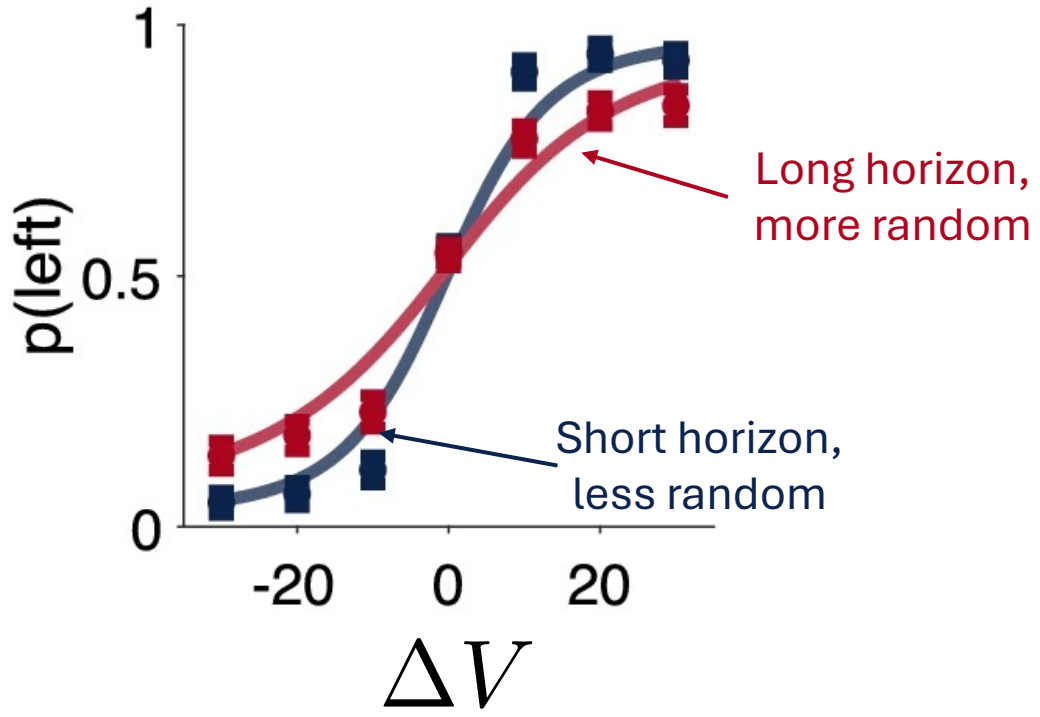
Wilson et al 2014

Two strategies for exploration

- Directed exploration
  - Bias choices towards more informative options
- Random exploration
  - Choose more randomly

Random exploration

- When it is valuable to explore, participants choose more randomly
- The slope of their choice curve is lower
  - The softmax parameter is smaller when it is more valuable to explore

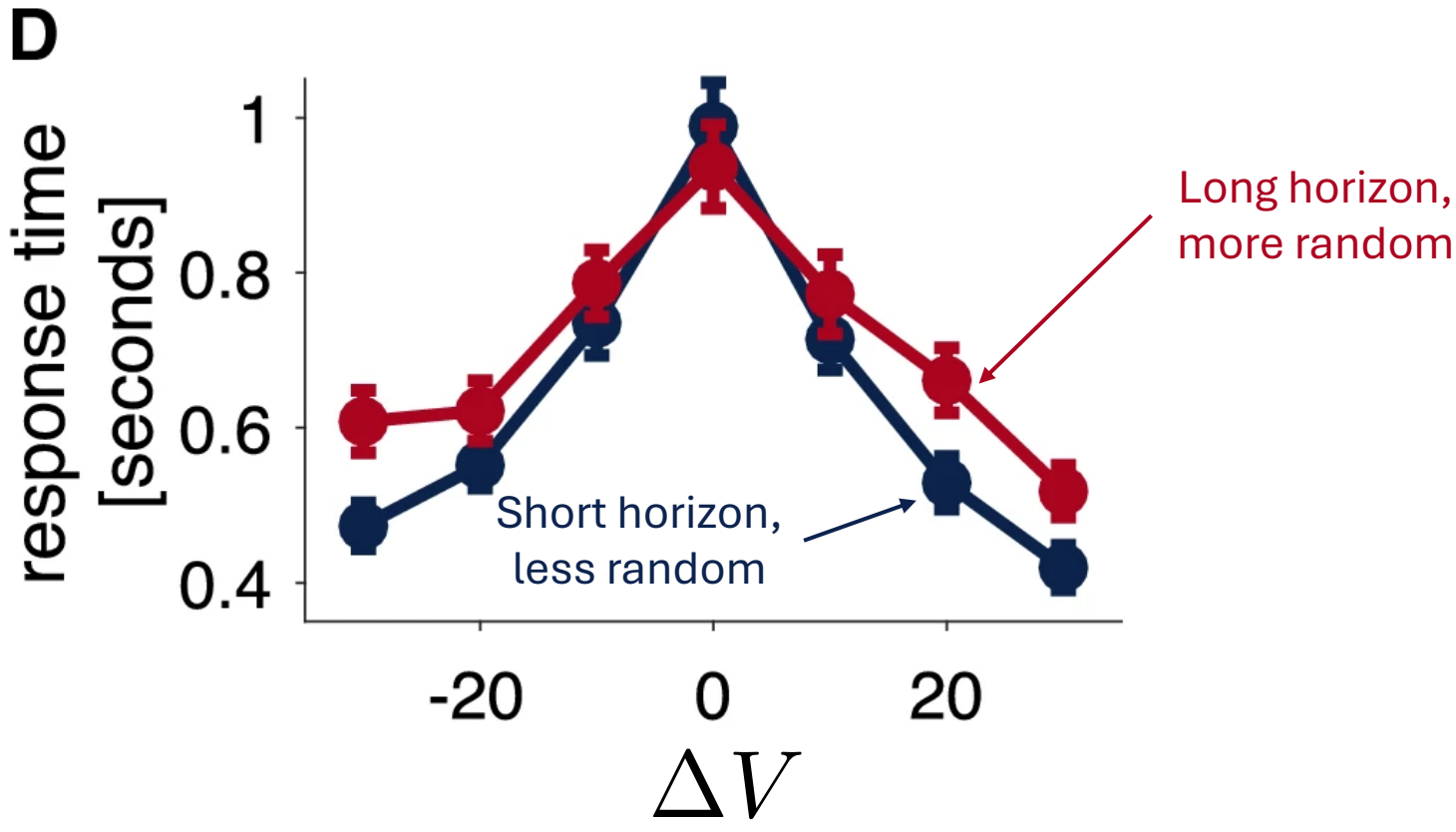


- BUT WHY?

Two mechanisms for random exploration

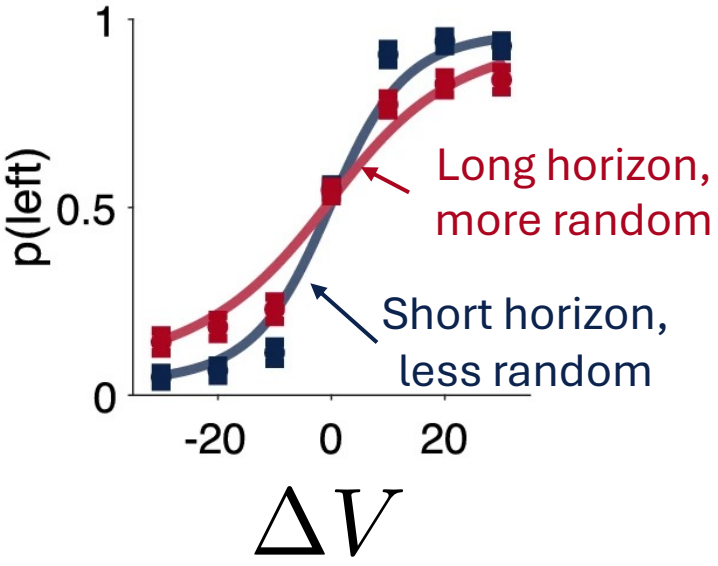
- If decision is made using a drift diffusion process, then randomness can be modulated in two different ways …
  - Reducing threshold
    - increases randomness, speeds up response times
  - Reducing drift rate
    - Increases randomness, slows down response times
- So … do people speed up (=> threshold change) or slow down (=> drift change) when they are randomly exploring?

They slow down => drift rate change (mostly)

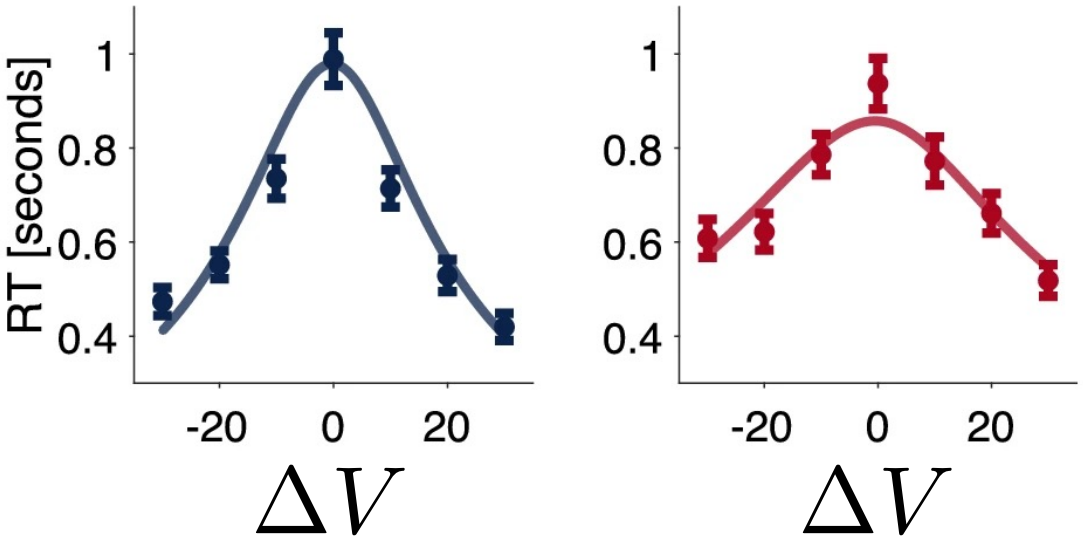


Be more precise by fitting the DDM …

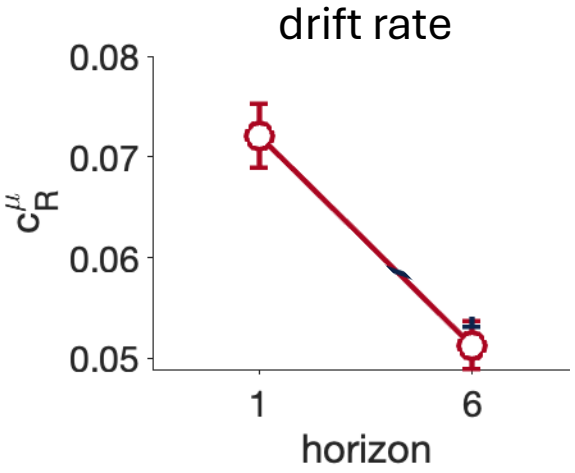
choices



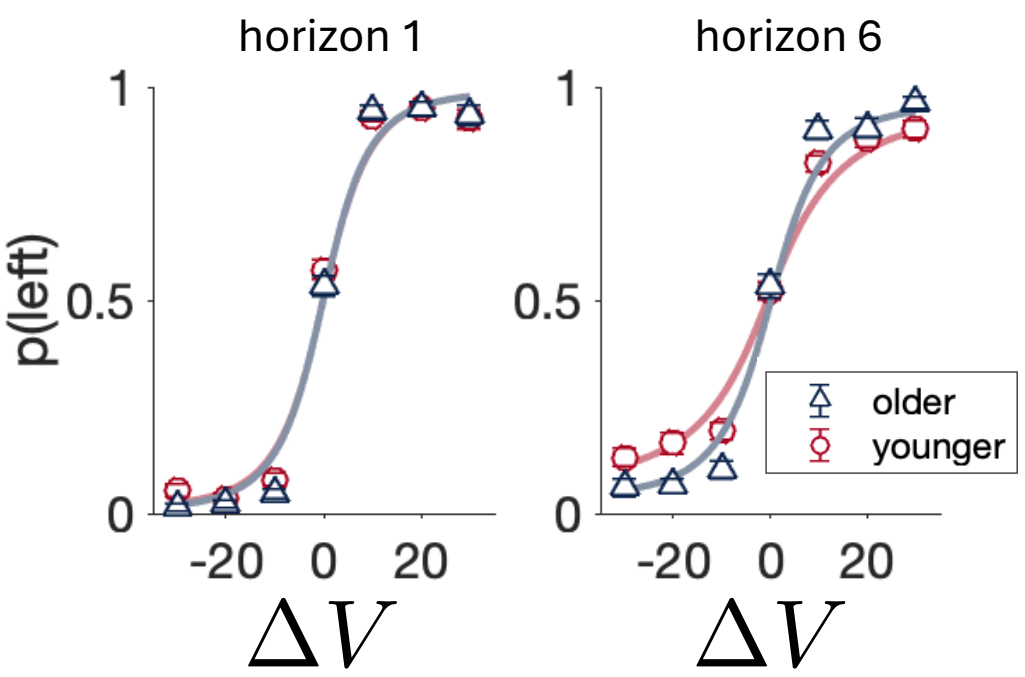
response times

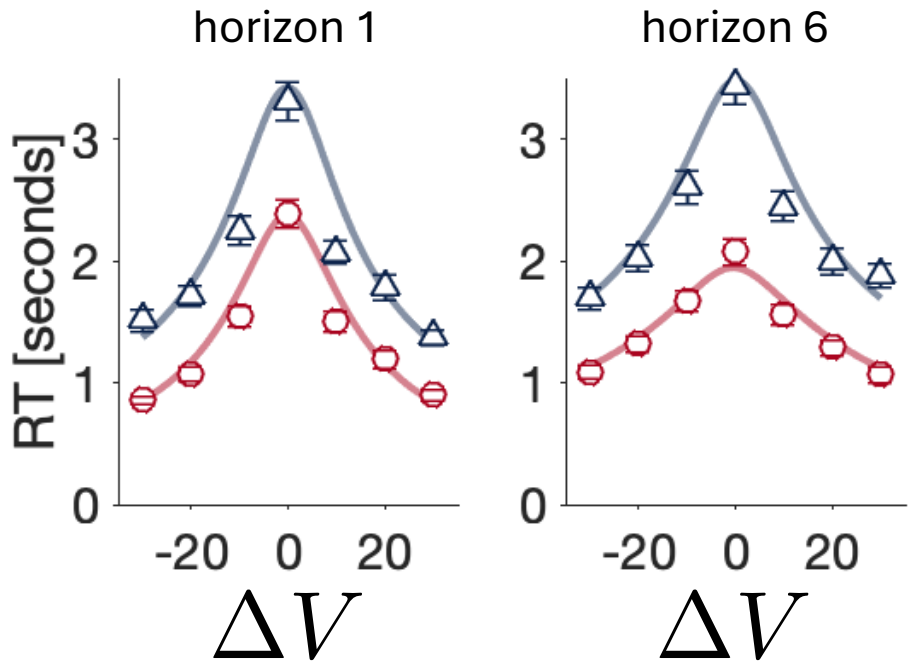


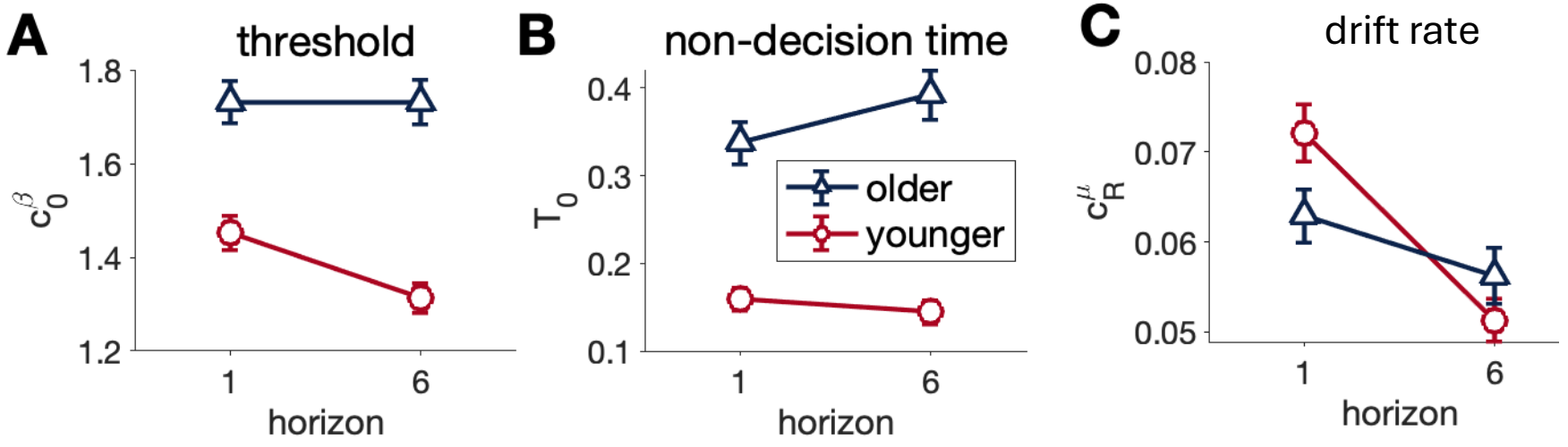
drift rate



What about aging?







DDM and random exploration

- Random exploration is primarily driven by a change in drift rate, not a change in response threshold
  - Suggests a signal-to-noise mechanism
    - But can't distinguish whether it's a change in signal, noise, or both using the DDM, need neural data to go further
- Same pattern is seen in older adults
- But older adults have
  - Higher threshold
  - Longer non-decision time
  - Less modulation of drift rate for exploration
- Amounts to slower responses and less random exploration in older adults

Summary

Drift diffusion model

- Mechanistic model of decision formation
  - Can be connected to neural implementation
- Models choices and response times
- Can be applied to value-based decisions
  - Softmax is a special case
- Allows for better characterization of individual differences and mechanistic answers about the nature of choice variability

Limitations of the DDM

- Main formulation only deals with two-alternative forced choice situations
  - Ignores multi-alternative models
  - Extensions exist
- Assumption that parameters are constant over time
  - Relaxing this constraint can make things a lot more complicated
- Assumption that decision is made quickly in one process

Beware different parameterizations!

- There are different parameterizations of the drift diffusion model
  - The same symbol can mean very different things depending on who's writing the paper!
    - E.g. z for threshold vs z for starting point

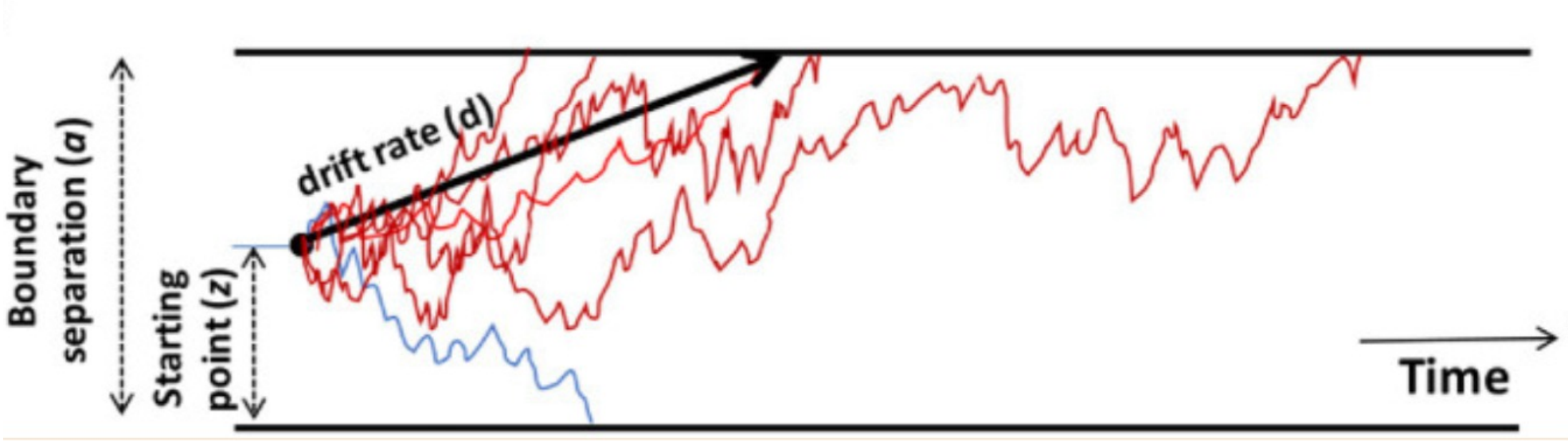


Figure from Myers, C. E., et al (2022).