Cipher Cracking Solution Write-up

1. There are low hanging fruits for this challenge. You can brute force by trying all possible values for k and f(x) when difficulty level D is low.
2. Analytical solution for f(x):

Consider P(T=1), in this scenario C1 is the max, and C0 is the second largest. The probability for C1 being max is ½. Afterwards, C0 must be the second largest. So P(T=1) = ½.

Consider P(T=2), in this scenario C2 is the max, and C0 is the second largest. The probability for C2 being max is 1/3 (i.e. choosing C2 among C0, C1, and C2), and probability for C0 being the second largest is ½ (i.e. choosing C0 among C0 and C1). So overall P(T=2) = 1/3 * 1/2 = 1/6.

So on and so forth for P(T=3), P(T=4)…

Consider the conditional probability P(T<=x), it is equal to 1 – P(T>x).

Now, P(T>x) implies that C0 is the max of C0, C1, C2… Cx. The probability of choosing C0 to be the max among the sequence is 1/(x+1).

So P(T<=x) = 1 – 1/(x+1)

Putting all together, the expected value of T given T<=x is:

$$f(x) = \frac{1 \times \frac{1}{2} + 2 \times \frac{1}{6} + \cdots + x \cdot \frac{1}{x+1} \cdot \frac{1}{x}}{1 - \frac{1}{x+1}} = (H_{x+1} - 1)\left(1 + \frac{1}{x}\right)$$

Where H_{x+1} is the harmonic number for x+1.

You can use the approximation formula for harmonic number (can be found on Wikipedia) since the challenge only requires 3 decimal places.