NAV Simulator

Input : FORCES and MOMENTS

$$\tau = [X \quad Y \quad Z \quad K \quad M \quad N]^T$$

Output: ACCELERATION $\rightarrow$ VELOCITY $\rightarrow$ POSITION

Dynamic Equation : $M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\eta) = \tau$

Output $= \dot{\nu}$ (acceleration) : $\boxed{\dot{\nu} = M^{-1}\left[\tau - C(\nu)\nu + D(\nu)\nu + g(\eta)\right]}$

ν – linear and angular velocity

$$\nu = [\overbrace{u \quad v \quad w}^{\nu_1} \quad \overbrace{p \quad q \quad r}^{\nu_2}]^T$$

1) $M = M_{RB} + M_A$

rigid body inertia — added mass – hydrodynamic effect

2) $C(\nu) = C_{RB}(\nu) + C_A(\nu)$

dynamic Coriolis and Centripetal terms — Hydrodynamic Coriolis and centripetal terms

3) $D(\nu)$ – hydrodynamic damping

4) $g(\eta)$ – restoring forces and moments

$$M_A = -\operatorname{diag}\{X_{\dot{u}}, Y_{\dot{v}}, Z_{\dot{w}}, K_{\dot{p}}, M_{\dot{q}}, N_{\dot{r}}\}$$

~~Assume~~

$$\cancel{A_{22} = \pi\rho a^2}$$

$$\cancel{A_{33} =}$$

$$X_{\dot{u}} \simeq -0.1\,m$$

$$Y_{\dot{u}} \simeq -\pi\rho R^2 L$$

$$Z_{\dot{u}} \simeq -\pi\rho R^2 L$$

$$K_{\dot{p}} \simeq -\frac{\pi\rho R^5}{4} \quad \text{or} \quad 0$$

$$M_{\dot{q}} \simeq -\frac{\pi\rho L^3 R^2}{12}$$

$$N_{\dot{r}} \simeq -\frac{\pi\rho L^3 R^2}{12}$$

2) $C(v) = C_{RB}(v) + C_A(v)$

$$C_{RB}(v) = \begin{bmatrix} mS(v_2) & -mS(S(v_2)r_G) \\ -mS(S(v_2)r_G) & mS(S(v_1)r_G) - S(J_0 v_2) \end{bmatrix}$$

1) $M = M_{RB} + M_A$

$$M_{RB} = \begin{bmatrix} m I_{3\times3} & -m S(r_G) \\ m S(r_G) & I_o \end{bmatrix}$$

$$M_{RB} = \begin{bmatrix} m & 0 & 0 & 0 & m z_G & -m y_G \\ 0 & m & 0 & -m z_G & 0 & m x_G \\ 0 & 0 & m & m y_G & -m x_G & 0 \\ 0 & -m z_G & m y_G & I_x & -\cancel{I_{xy}}\,0 & -\cancel{I_{xz}}\,0 \\ m z_G & 0 & -m x_G & -\cancel{I_{yx}}\,0 & I_y & -\cancel{I_{yz}}\,0 \\ -m y_G & m x_G & 0 & -\cancel{I_{zx}}\,0 & -\cancel{I_{zy}}\,0 & I_z \end{bmatrix}$$

$$M_A = - \begin{bmatrix} X_{\dot u} & X_{\dot v} & X_{\dot w} & X_{\dot p} & X_{\dot q} & X_{\dot r} \\ Y_{\dot u} & Y_{\dot v} & Y_{\dot w} & Y_{\dot p} & Y_{\dot q} & Y_{\dot r} \\ Z_{\dot u} & Z_{\dot v} & Z_{\dot w} & Z_{\dot p} & Z_{\dot q} & Z_{\dot r} \\ K_{\dot u} & K_{\dot v} & K_{\dot w} & K_{\dot p} & K_{\dot q} & K_{\dot r} \\ M_{\dot u} & M_{\dot v} & M_{\dot w} & M_{\dot p} & M_{\dot q} & M_{\dot r} \\ N_{\dot u} & N_{\dot v} & N_{\dot w} & N_{\dot p} & N_{\dot q} & N_{\dot r} \end{bmatrix}$$

If the values are not given based on experiments they can be computed but only for the principal diagonal $\Longrightarrow$

$$C_{RB} = \begin{bmatrix} 0 & 0 & 0 & m(y_G q + z_G r) & -m(x_G q - w) & -m(x_G r + v) \\ 0 & 0 & 0 & -m(y_G p + w) & m(z_G r + x_G p) & -m(y_G r - u) \\ 0 & 0 & 0 & -m(z_G p - v) & -m(z_G q + u) & m(x_G p + y_G q) \\ -m(y_G q + z_G r) & m(y_G p + w) & m(z_G p - v) & 0 & -J_{yz} q - J_{xz} p + J_z r & J_{yz} r + J_{xy} p - J_y q \\ m(x_G q - w) & -m(z_G r + x_G p) & m(z_G q + u) & J_{yz} q + J_{xz} p - J_z r & 0 & -J_{xz} r - J_{xy} q - J_x p \\ m(x_G r + v) & m(y_G r - u) & -m(x_G p + y_G q) & -J_{yz} r - J_{xy} p + J_y p & J_{xz} r + J_{xy} q - J_x p & 0 \end{bmatrix}$$

$$C_A(v) = \begin{bmatrix} 0 & 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & a_3 & -a_1 \\ 0 & 0 & 0 & -a_2 & 0 \\ 0 & -a_3 & a_2 & 0 & b_2 \\ a_3 & 0 & -a_1 & b_3 & -b_1 \\ -a_2 & a_1 & 0 & -b_2 & 0 \end{bmatrix}$$

where

$$a_1 = X_{\dot u} u + X_{\dot v} v + X_{\dot w} w + X_{\dot p} p + X_{\dot q} q + X_{\dot r} r$$

$$a_2 = X_{\dot u} u + Y_{\dot v} v + Y_{\dot w} w + Y_{\dot p} p + Y_{\dot q} q + Y_{\dot r} r$$

$$a_3 = X_{\dot w} w + Y_{\dot w} v + Z_{\dot w} w + Z_{\dot p} p + Z_{\dot q} q + Z_{\dot r} r$$

$$b_1 = X_{\dot p} u + Y_{\dot p} v + Z_{\dot p} w + K_{\dot p} p + K_{\dot q} q + K_{\dot r} r$$

$$b_2 = X_{\dot q} u + Y_{\dot q} v + Z_{\dot q} w + K_{\dot q} p + M_{\dot q} q + M_{\dot r} r$$

$$b_3 = X_{\dot r} u + Y_{\dot r} v + Z_{\dot r} w + K_{\dot r} p + M_{\dot r} q + N_{\dot r} r$$

$D(v)$ = only linear and quadratic damping
(when modelled → vehicle moving in 6DOF at slow speed)

linear drag

$$D(v) = -\,\mathrm{diag}\{X_u, Y_v, Z_w, K_p, M_q, N_r\} -$$
$$-\,\mathrm{diag}\{X_{u|u|}|u|, Y_{v|v|}|v|, Z_{w|w|}|w|, K_{p|p|}|p|, M_{q|q|}|q|, N_{r|r|}|r|\}$$

quadratic drag

Terms defined based on

$$f(U) = -\frac{1}{2}\rho\, C_D(R_n)\, A_{section}\, |U| U$$

$$R_n = \frac{U \cdot D}{\nu}$$

D → length of body / Diameter

ν → kinematic viscosity coeff ($\nu = 1.56 \cdot 10^{-6}$ for salt water)

↓

Graph ⇒ $C_D(R_n)$

$$
g(\eta) = \begin{bmatrix} (W-B)\sin\theta \\ -(W-B)\cos\theta\sin\phi \\ -(W-B)\cos\theta\cos\phi \\ -(y_G W - y_B B)\cos\theta\cos\phi + (z_G W - z_B B)\cdot \\ \cdot\cos\theta\sin\phi \\ (z_G W - z_B B)\sin\theta + (z_G W - x_B B)\cos\theta\cos\phi \\ -(x_G W - x_B B)\cos\theta\sin\phi - (y_G W - y_B B)\sin\theta \end{bmatrix}
$$

where $W = m \cdot g$

$B = \rho_W V_{disp}\, g$

- mass of AUV [kg]

$r_G = [x_G \ y_G \ z_G]^T$ - position of center of gravity [m]

$S(r_G)$ - skew - symmetric matrix

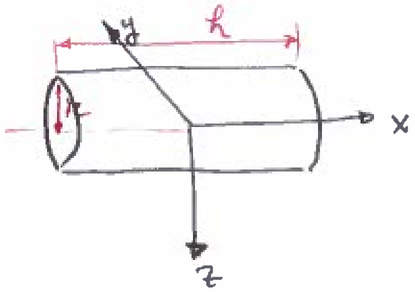
$$S(r_G) = \begin{bmatrix} 0 & z_G & -y_G \\ -z_G & 0 & x_G \\ y_G & x_G & 0 \end{bmatrix}$$

J_o - inertia tensor w.r.t. origin of AUV [mass · length^2]

$$J_o = \begin{bmatrix} J_x & -J_{xy} & -J_{xz} \\ -J_{yx} & J_y & -J_{yz} \\ -J_{zx} & -J_{zy} & J_z \end{bmatrix}$$

For simplicity $\Rightarrow$ $J_o = \begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix}$

Wikipedia :



$$J_x = \frac{m r^2}{2}$$

$$J_y = J_z = \frac{1}{12} m \left(3r^2 + h^2\right)$$