# Planets in Roche Lobe Contact Due to Orbital Torques and Tidal Heating

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## 1 Introduction

In interacting binary systems, there are two reasons for why mass transfer might occur: Roche lobe overflow or stellar wind accretion. Roche lobe overflow is caused by one body filling its Roche lobe by increasing its radius, or through a decrease in binary separation. Stellar wind accretion is mass ejection in the form of a wind that can be captured gravitationally by the companion. In both cases, mass leaves the Roche lobe through the Lagrange point with the lowest gravitational potential, L1, between the two bodies.

In this paper we consider the fate of gas giant planets on circular orbits which are pushed into Roche lobe contact with their parent star. We assume that the evolution of the system is driven by inward torques or by tidal heating in the planet. The torques remove angular momentum by either tidal torques raised in the star or from interactions with the protoplanetary disk.

We model irradiated gas giants without cores on circular orbits around sunlike stars. We assumed the young planets were located at an orbital separation just outside of Roche lobe contact and then modeled the evolution for ten billion years. The coupled equations for the structure and orbit of the system were integrated numerically using the MESA stellar structure code (Paxton, et al. 2011). MESA is a stellar evolution module comprised of several open source libraries for computational stellar astrophysics.

It has been suggested that if a gas giant planet were to fill its Roche lobe before plunging into its star, two possible outcomes could occur: the mass transfer could be unstable, in which case the planet would be tidally destroyed and consumed by the star, or the mass transfer could be stable, and the planet would gradually transfer the entirety of its mass to its star (Metzger, Giannios, & Spiegel 2012). In both scenarios, the planet has no chance for survival. In this paper we suggest that it is possible for a planet to engage in mass transfer for a few billion years, lose almost 90% mass, but then back away from its parent star and survive. The surviving low mass planets orbit at low periods, between 1 and 3 days.

In §2 of this paper, we derive the equations for the evolution of the planet as it moves into Roche lobe contact and once it engages in mass transfer. We also analyze the possibility of the planet to reverse the direction of motion after mass loss has begun. In §3, we show and discuss the outcomes of our models created with MESA. We briefly summarize and explain the relevance of our findings in §4.

# 2 Methods

## 2.1 Roche Lobe Geometry

We took the assumption that $M_\star \gg M_p$ in order to simplify the equation for the Roche lobe radius around the planet to be (Paczynski 1971)

$$r_L \approx 0.462a\left(\frac{M_p}{M_\star}\right)^{\frac{1}{3}} \tag{1}$$

Using Kepler's third law and the assumption of Roche lobe contact, $R_p = r_L$, one can use this radius to relate the orbital period and orbital separation at which a planet is in contact to the planet's mass and radius and the star's mass

$$P_{orb} = 0.38\ day\left(\frac{M_p}{M_J}\right)^{-\frac{1}{2}}\left(\frac{R_p}{R_J}\right)^{\frac{3}{2}} \tag{2}$$

$$a = 2.21\ R_\odot\left(\frac{R_p}{R_J}\right)\left(\frac{M_J}{M_p}\frac{M_\star}{M_\odot}\right)^{\frac{1}{3}} \tag{3}$$

where $M_J$ and $R_J$ are the mass and radius of Jupiter and $M_\odot$ is the mass of the sun.

## 2.2 Evolution due to Orbital Torques

The evolution of the system is driven by torques caused by tidal interactions with the star and tidal heating in the planet. The planet rises a tide in the star from which the resulting friction causes the star to increase its rate of rotation. In order to conserve the total angular momentum, the orbital angular momentum must decrease, driving the planet towards the star. While the planet is outside of Roche lobe contact, its mass is constant and the orbit moves inward towards the star. Once the planet reaches Roche lobe contact, mass transfer begins at a rapid rate. Differentiating the orbital angular momentum while assuming $M_\star \gg M_p$ and, therefore, treating the stellar mass as constant, gives an equation for change in the semi-major axis

$$J \approx M_p\sqrt{GM_\star a} \tag{4}$$

$$\frac{\dot{a}}{a} = \frac{2\dot{J}}{J} - \frac{2\dot{M}_p}{M_p} \tag{5}$$

From equation (5), one can see the direct relationship between $\dot{M}_p$ and $\dot{a}$. If mass transfer occurs and $\dot{J} > \dot{M}_p$, the planet will continue to move in towards the star. If this occurs for the entire evolution of the system, the planet will run into the star and be destroyed. If the mass loss rate of the planet becomes larger than the change in angular momentum the planet will back away from the star and survive.

Combining the differentiated angular momentum equation with Darwin's theory of tides gives

$$\frac{\dot{J}}{J} = -\frac{9}{4}\left(\frac{G}{M_\star}\right)^{\frac{1}{2}} \frac{R_\star^5 M_p}{Q'_\star} a^{-\frac{13}{2}} \tag{6}$$

(In MESA, there is a factor of $\frac{9}{2}$ used instead of $\frac{9}{4}$)
Where $Q'_\star$ is the tidal dissipation factor, or amount of friction in the planet. For smaller $Q'_\star$ values, the planet dissipates energy less efficiently, moving the planet inwards quicker and reaching contact sooner. Jupiter has a $Q'_\star$ in the range of $10^4 - 10^5$.

Differentiating the Roche lobe equation (1) and combining it with equation (5) yields an equation for how the Roche lobe behaves during evolution

$$\frac{\dot{r_L}}{r_L} = \frac{2\dot{J}}{J} + \frac{-2\dot{M}_p}{M_p}\left(\frac{5}{6} - \frac{M_p}{M_\star}\right) \tag{7}$$

The radius of the Roche lobe around the planet increases proportionally with mass loss so long as $\frac{M_p}{M_\star} \ll \frac{5}{6}$. Since $a \propto J$, the Roche lobe will decrease proportionally to the semi-major axis.

In order to produce a mass loss equation for the planet, let us consider an analytic example case where $R_p \approx R_p(M_p, S)$, where S is the central entropy of the nearly fully convective planet. Then

$$\frac{\dot{R}_p}{R_p} = \alpha\frac{\dot{M}_p}{M_p} + \beta\frac{\dot{S}}{S} \tag{8}$$

where

$$\alpha = \frac{\partial ln R_p}{\partial ln M_p} |_S \tag{9}$$

$$\beta = \frac{\partial ln R_p}{\partial ln S} |_{M_p} \tag{10}$$

Differentiating the Roche lobe equation and placing the planet in contact such that $R_p = r_L$ yields

$$\frac{\dot{R}_p}{R_p} = \frac{\dot{a}}{a} + \frac{\dot{M}_p}{3M_p} \tag{11}$$

Using equations (5), (8), and (11) to solve for $\dot{M}_p$ and $\dot{a}$ we obtain

$$\frac{\dot{M}_p}{M_p} = \left(\frac{1}{\alpha+\frac{5}{3}}\right)\left(2\frac{\dot{J}}{J} - \beta\frac{\dot{S}}{S}\right) \tag{12}$$

$$\frac{\dot{a}}{a} = \left(\frac{2}{\alpha+\frac{5}{3}}\right)\left((\alpha - 1/3)\frac{\dot{J}}{J} + \beta\frac{\dot{S}}{S}\right) \tag{13}$$

### 2.3 Adiabatic Evolution and Cooling

For the adiabatic case, entropy is constant ($\dot{S} = 0$) and the inward torques make $\dot{J} < 0$ giving

$$\frac{\dot{M}_p}{M_p} = -\left(\frac{2}{\alpha+\frac{5}{3}}\right)\left|\frac{\dot{J}}{J}\right| \tag{14}$$

$$\frac{\dot{a}}{a} = -\left(\alpha - \frac{1}{3}\right)\left(\frac{2}{\alpha+\frac{5}{3}}\right)\left|\frac{\dot{J}}{J}\right| \tag{15}$$

In the generic case, $\alpha < \frac{1}{3}$, indicating that the planet backs away from the star. Since $\alpha$ is alway positive, the mass continues to decrease as the orbital separation increases. In adiabats, $R \backsim M^{-\frac{1}{3}}$, so the planet's radius will expand as it backs away from the star. The Roche lobe will also increase during this expansion in accordance to the direct relationship between the Roche lobe radius and orbital separation.

Alternatively, if we set $\dot{J} = 0$,

$$\frac{\dot{M}_p}{M_p} = -\left(\frac{1}{\alpha+5/3}\right)\beta\frac{\dot{S}}{S} \tag{16}$$

By definition, $\beta$ is always positive, indicating that there is heating, $\dot{S} > 0$, as the planet is losing mass in Roche lobe contact. The associated motion is

$$\frac{\dot{a}}{a} = \left(\frac{2}{\alpha+5/3}\right)\beta\frac{\dot{S}}{S} \tag{17}$$

meaning the heated planet planet moves out as it loses mass, independent of $\alpha$, so long as $\alpha > -\frac{5}{3}$.

By analyzing both inward torques and heating independently, we have found that both variables cause the planet to move outwards as it loses mass. In the case that there is simultaneously inward torques ($\dot{J} < 0$) while the planet is cooling ($\dot{S} < 0$), there is a critical cooling rate, $\beta\frac{\dot{S}}{S} = -(\alpha - 1/3)\frac{\dot{J}}{J}$, above which the planet will move inward rather than outward. There is another critical cooling rate above which mass loss will be turned off: $\beta\frac{\dot{S}}{S} < 2\frac{\dot{J}}{J}$. Planets will switch from moving out to moving in when cooling becomes important and will turn off mass loss when increasing cooling even more.

In summary, during the evolution of the system, the planet's radius can either expand or contract, depending on the relative mass loss and cooling times such that

$$\frac{\dot{R}_p}{R_p} = \alpha\frac{\dot{M}_p}{M_p} + \beta\frac{\dot{S}}{S} \tag{18}$$

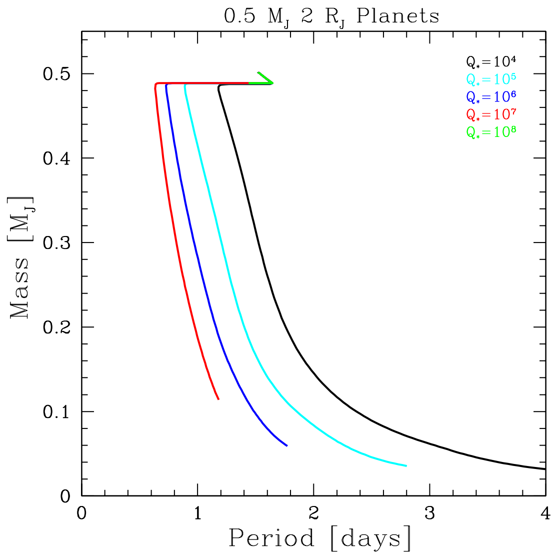
where $\frac{M_p}{\dot{M}_p}$ is the mass loss time, $-\frac{\dot{S}}{S}$ is the cooling time, and $\alpha$ is an index that relates the expansion or contraction rate of the planet to its mass loss rate and $\beta$ to its cooling rate. Rappaport, Joss, and Webbink (1981) found that for highly compact binaries, the structure of the system could be accurately approximated by an n=3/2 polytrope. With this finding, $\alpha = -\frac{1}{3}$. When $-\frac{\dot{S}}{S} \ll \frac{M_p}{\dot{M}_p}$ the planet will cool and shrink with $\alpha \backsimeq 1$. When $\frac{M_p}{\dot{M}_p} \ll \frac{\dot{S}}{S}$, the entropy is constant, and the radius will expand adiabatically with $\beta \backsimeq -\frac{1}{3}$.

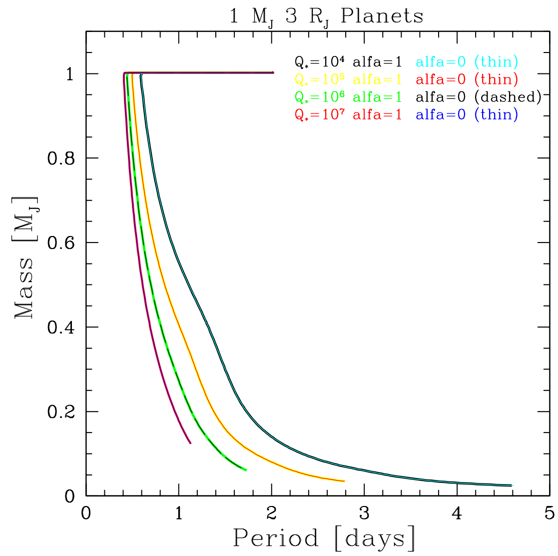
## 3 Modeling the Evolutions with MESA

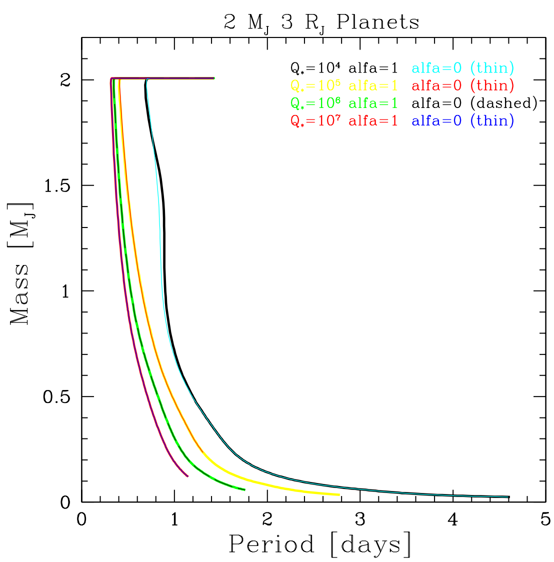
Utilizing the stellar evolution code MESA, we modeled the evolution of irradiated planets without cores on circular orbits around sunlike stars. We used the binary Roche lobe overflow portion of the code in order to evolve the systems and find out where young planets stopped when driven in towards their star by torques and to see how much mass was lost. The parameters we altered in each simulation were: the mass of the planet, the radius of the planet, the initial orbital separation, $Q_\star$ (the tidal dissipation factor), and $\alpha$ (the index that relates the expansion or contraction rate of the planet to its mass loss rate). We found that setting the initial orbital separation either at Roche lobe contact or just outside of contact had no significant effect on the evolution and therefore only include in the following sections the plots produced with the planets initially just out of Roche lobe contact.

In the plots, alfa = $\alpha$. In the $2M_J$ and $3M_J$ cases, we show models with $\alpha = 0$ and $\alpha = 1$. In contrast to previous results, we find that the planets display the same outcome in both scenarios rather than diverge from one another.

## 3.1 Mass Period Relation




Planets start at an orbital period set in MESA to be just out of Roche lobe contact. Planets move inwards towards the star through orbital decay, decreasing the period, and beginning mass loss once reaching its Roche lobe. The planets start to increase the orbital separation a small amount while losing a significant portion of its mass and then back away far enough to terminate the mass loss before losing all of its mass. The lowest $Q_\star$ values move in the fastest and reach contact before the higher values as expected due to its inverse relationship with $\dot{J}$.

Larger mass planets come into Roche Lobe contact at shorter periods due to the fact that the L1 Lagrange point resides where the densities for both the star and planet are the same. Setting

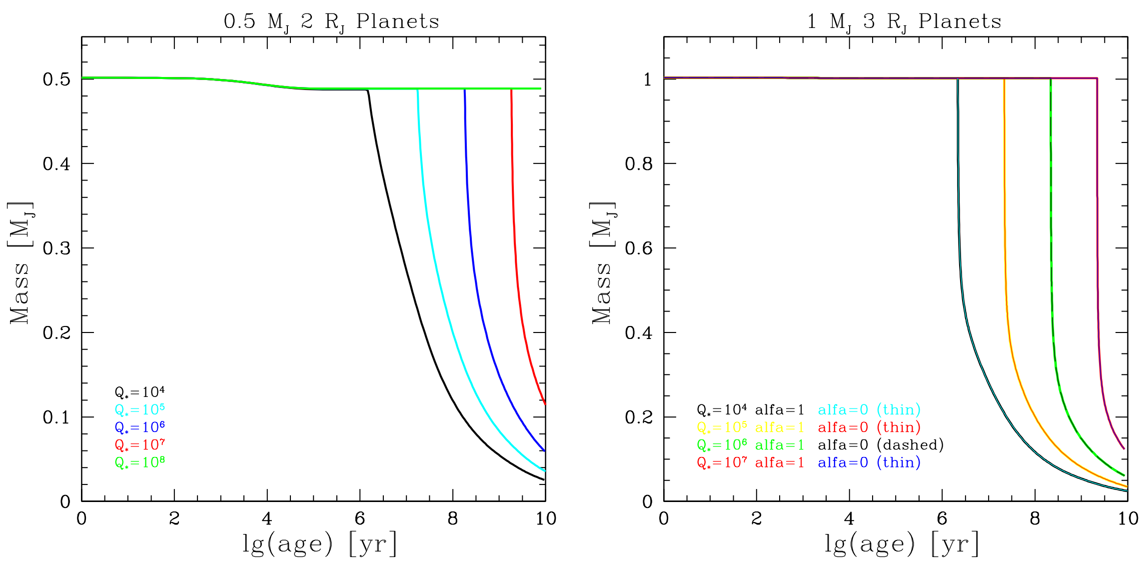
$$\frac{M_\star}{a^3} = \frac{M_p}{R_p^3} \tag{19}$$

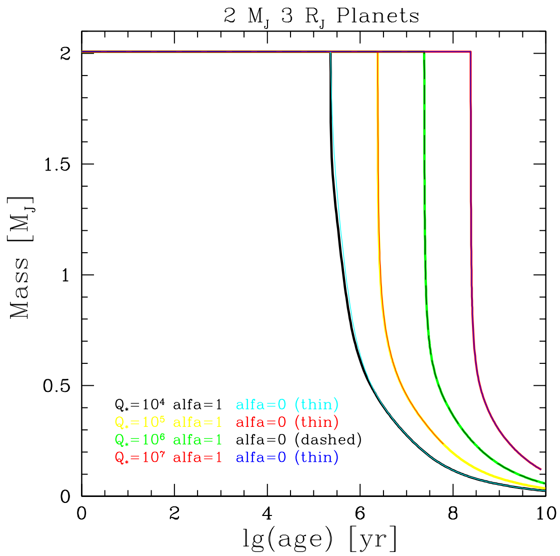
and solving for a

$$a = R_p \left( \frac{M_\star}{M_p} \right)^{\frac{1}{3}} \tag{20}$$

we see that the orbital separation has an inverse relationship with the mass of the planet.

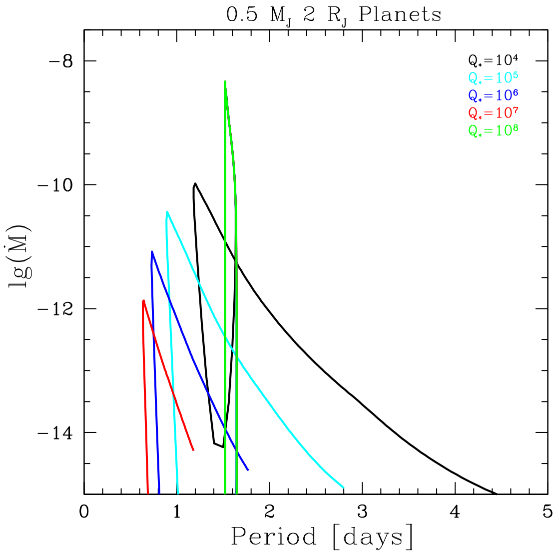
## 3.2 Mass Age Relation

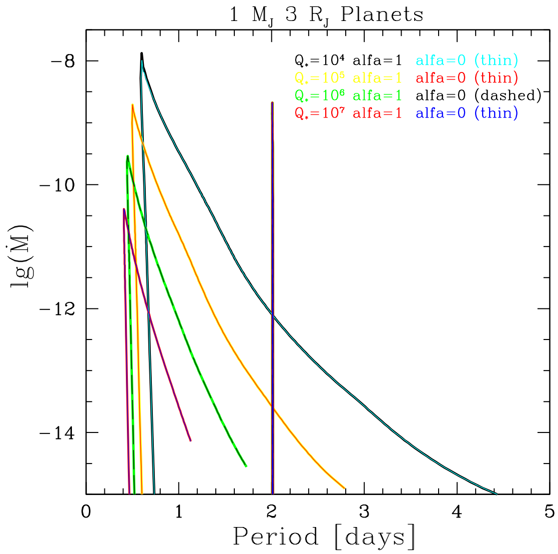


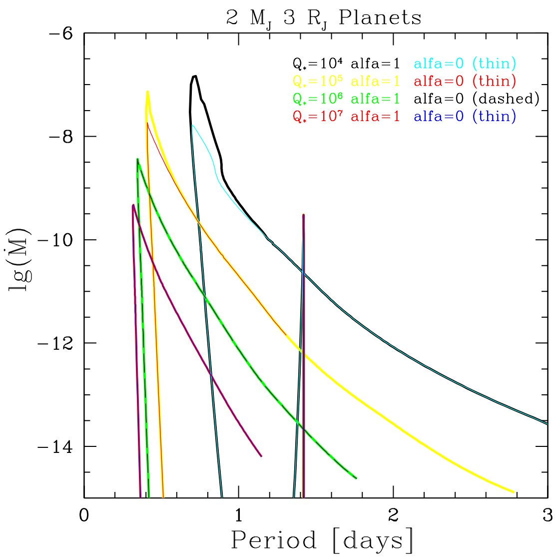



Looking at mass as a function of time, we see that the planets reach Roche lobe contact between one million years and one billion years depending on the tidal dissipation factor. The planets with the lowest $Q_\star$ values make contact the fastest due to the more rapid orbital decay. The higher $Q_\star$ values result in beginning mass loss much later in their evolution in addition to losing a smaller fraction of their total mass. In all cases there is very rapid mass loss initially after reaching the Roche lobe, followed by a tapering off of mass loss rates as the planets lose the majority of their mass.

### 3.3 $\lg(\dot{M})$ Period Relation



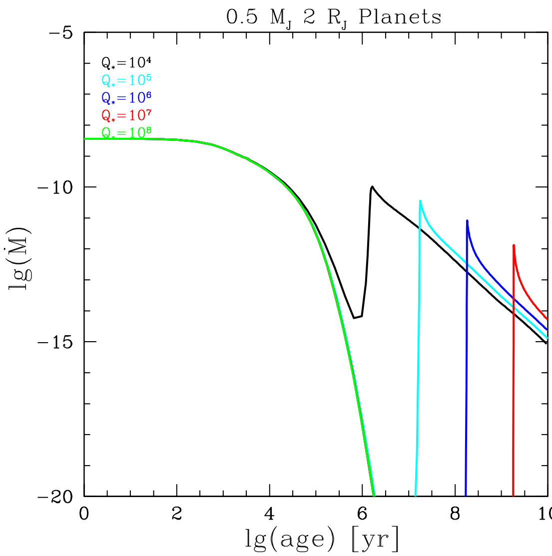



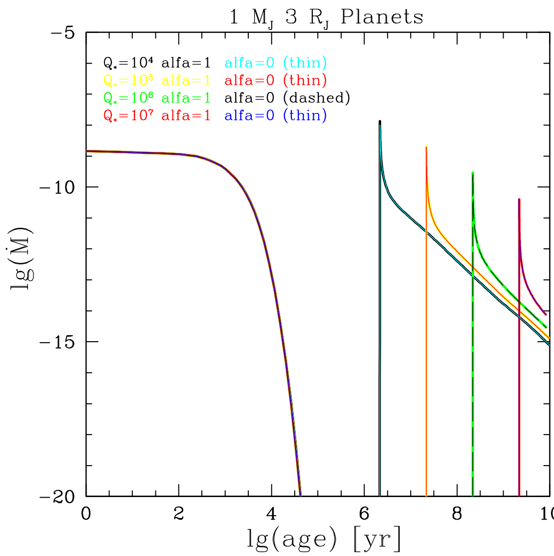



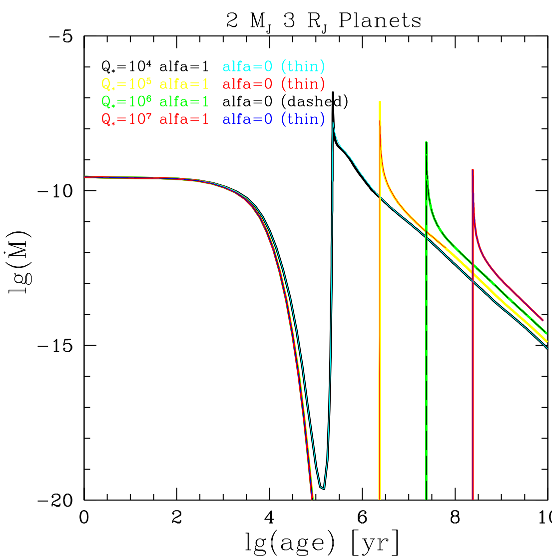

($\dot{M}$ is in units of solar masses per year)

These plots illustrate the planets starting at their critical orbital separation and then moving in to very short periods before losing mass. The lower $Q_\star$ values result in higher peak mass loss rates and occur at longer periods than the higher $Q_\star$ values. This is due to the quicker orbital dissipation causing the planets to reach contact earlier and therefore at a longer period. The models show a slowing of mass loss rates as the planets regress from their stars, hinting that the planets may not be entirely destroyed by the event.

### 3.4 $lg(\dot{M})$ Age Relation

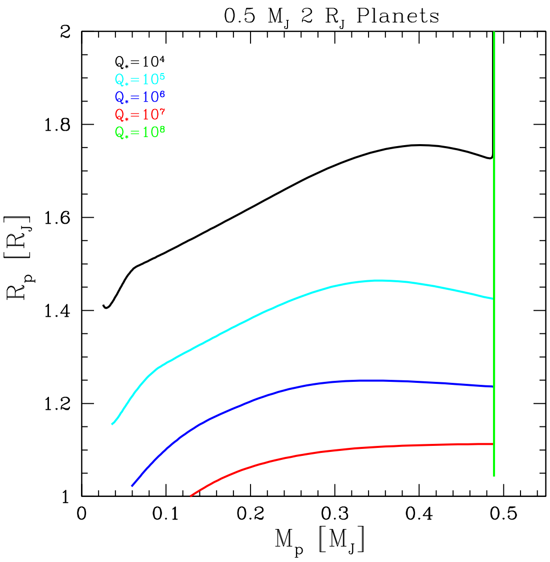


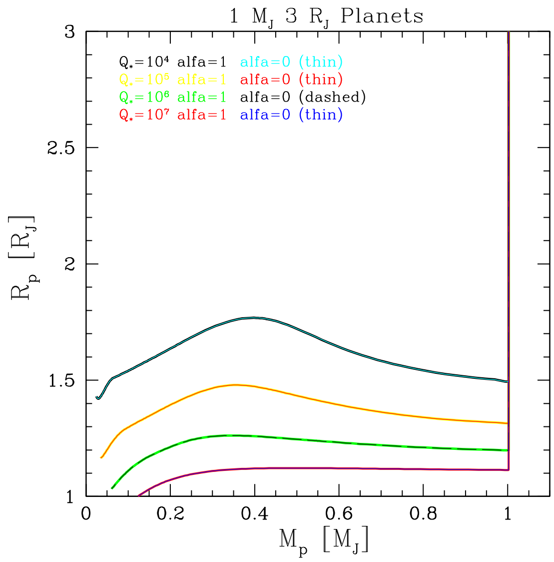



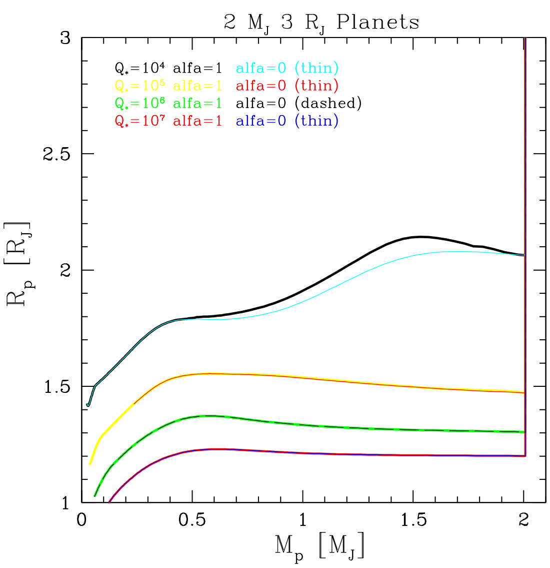




As illustrated in the above plots of mass loss rates against time, orbital decay brings the planets close to the stars until they reach the Roche lobe and mass loss begins at high rates almost instantaneously after contact. Mass loss rates peak when the planet reaches the Roche lobe (ranging from after a few million to a few billion years) and then decrease, but still lose mass for the remainder of the evolution. The lower $Q_\star$ cases peak higher and begin mass loss sooner due to the inverse relationship between $\dot{J}_{orb}$ and $Q_\star$.

## 3.5 Radius Mass Relation



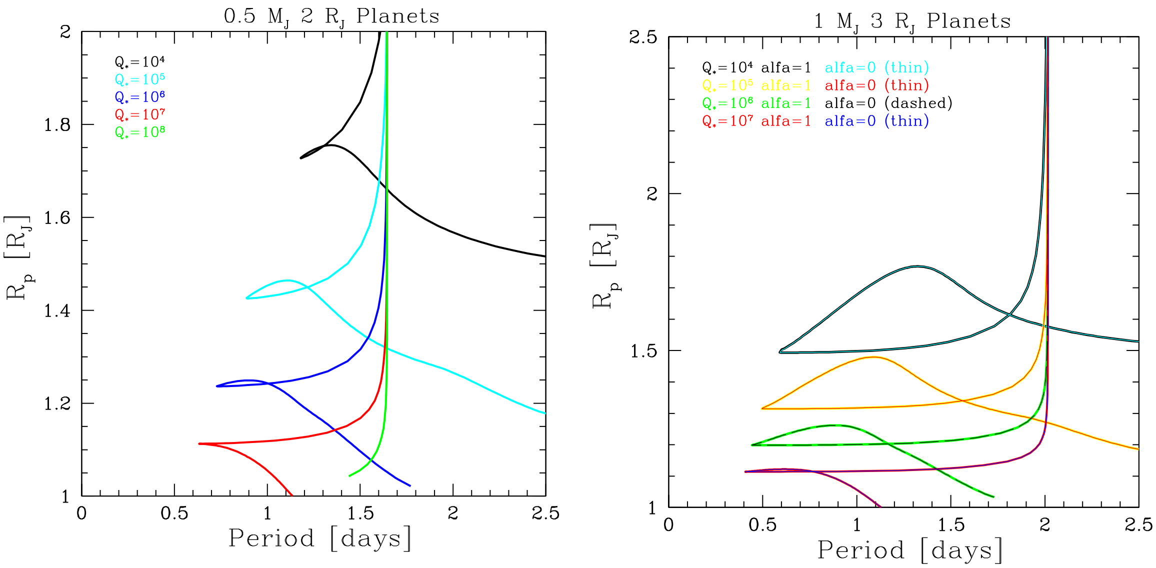

The radius versus mass plots show the planet cooling, causing the radii to shrink, and then the planets beginning to lose mass at different radii. For the higher $Q_\star$ values, the planets shrink by more than half as they move in towards their star and approach the Roche lobe. The lowest $Q_\star$ cases start losing mass at larger radii because the $\dot{J}$ is larger and they come into contact quicker. Once mass loss turns on, the radii expand slightly due to adiabatic activity caused by the cooling time becoming much longer than the mass loss time. After a while, the cooling times once again become comparable and then shorter causing the planets to shrink again, all the while still losing mass.

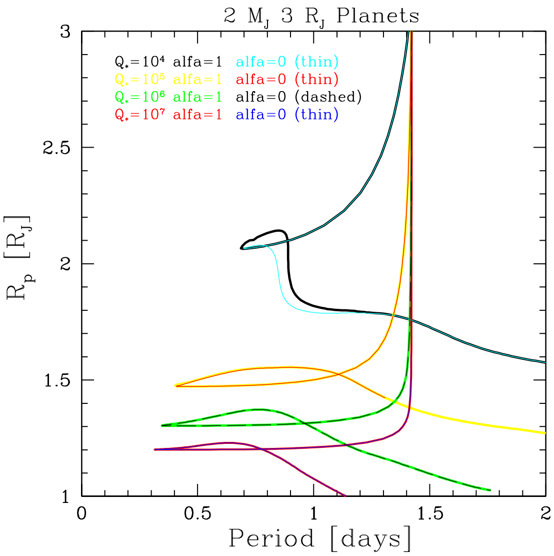
The shape of the curves simulate the $T = 0$ mass-radius relation: at small masses, $(< 1M_J)$, $R \propto M^{\frac{1}{3}}$ as the pressure is constant, and at larger radii, $(> 1M_J)$, $R \propto M^{\frac{-1}{3}}$ as it is supported by degeneracy pressure. In an adiabat,

$$S = const. + \frac{k_b}{\mu m_p} ln\left(\frac{P}{\rho^{\frac{5}{3}}}\right) \tag{21}$$

S and $\frac{k_b}{\mu m_p}$ are constant, so $\frac{P}{\rho^{\frac{5}{3}}}$ must be as well. This is the same as degeneracy pressure, so there is the same mass-radius proportionality of $R \propto M^{-\frac{1}{3}}$.

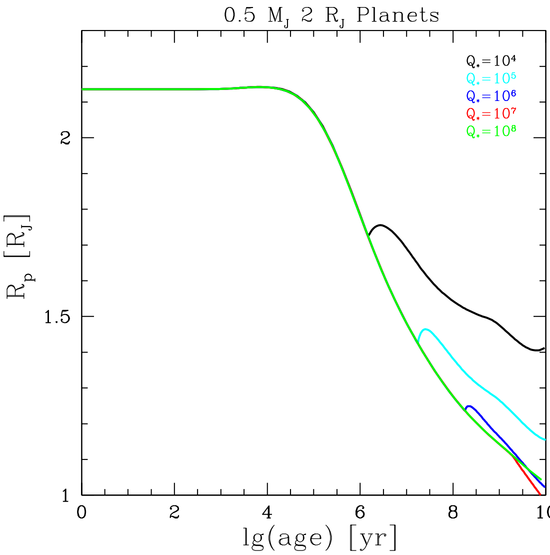
## 3.6 Radius Period Relation

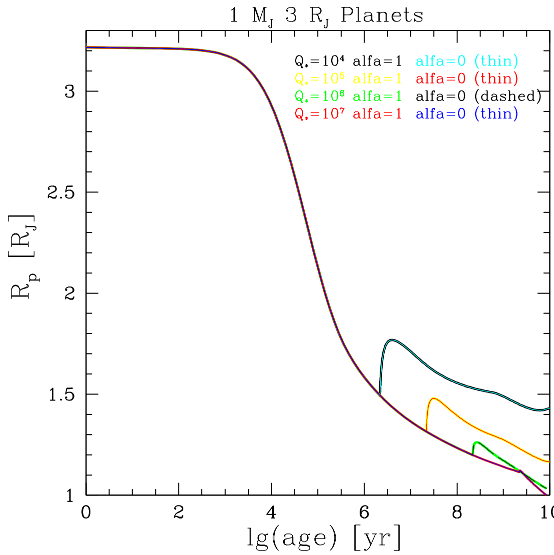


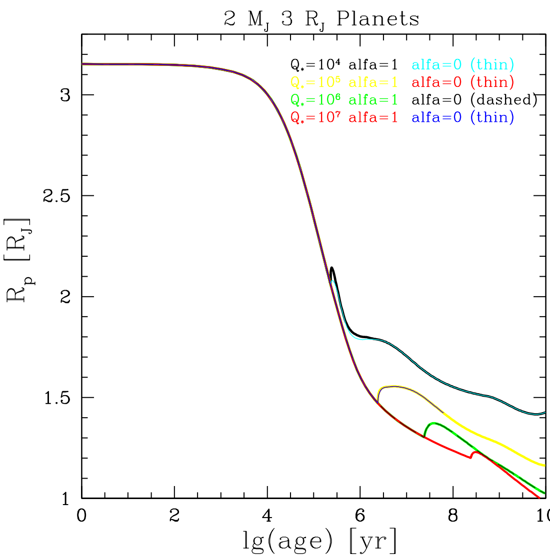



Analyzing the radius-period relations, the planets begin at an orbital period such that they are just out of Roche lobe contact. They then rapidly cool and shrink and start to move in towards the star through orbital decay. As the planets are migrating towards the star, the mass loss time and cooling times are comparable and the radii remain fairly constant. Once the planet reaches the L1 Lagrange point of the Roche lobe and mass loss initiates, the mass loss time becomes shorter than the cooling time and the planet begins to increase its radius through adiabatic motion. The cooling time once again becomes comparable (at the peak of the loop) and then shorter causing the radius to shrink again. In the highest $Q_\star$ case, the radius does not increase very much after making contact and initiating mass loss. This is because the mass loss and cooling times remained comparable for the period of time when $\frac{M}{\dot{M}} < \frac{S}{\dot{S}}$ until the cooling time once again dominated and the radius shrank.

### 3.7 Radius Age Relation




The planets start as large, puffy planets with no time for cooling during the first thousand years. Cooling sets in and the planets shrink for another few thousand years until the mass loss time becomes shorter than the cooling time. This adiabatic activity causes the radii to increase for a very short period until the cooling time become shorter and the planets shrink for the remaining time. The lower $Q_\star$ cases begin adiabatically increasing their radii at an earlier time than the later cases as a result of the inverse dependence on $Q_\star$ with $\dot{J}$.

### 3.8 Cooling Time vs. Mass Loss Time

The following plots illustrate how different factors affect the relationship between the cooling timescale, $\tau_{cool}$, and the mass loss timescale, $\tau_{mass}$. The variables altered are the mass of the planet, the radius of the planet, and the tidal dissipation factor, $Q_\star$. The y-axes are $lg(\tau)$ and are in units of years.

For the initial portion of the evolution of the system, the planet is neither shrinking nor expanding as the cooling timescale and mass loss timescale are comparable. After a few thousand years, the mass loss timescale becomes much larger than the cooling timescale as the planet is cooling, shrinking, and migrating inwards towards its star as driven by the torques between the two bodies. During this period, the cooling timescale is much shorter than the mass loss timescale and the planet is decreasing with an index of $\alpha \simeq 1$ (as seen in equation (18)). If the mass loss timescale is shorter than the cooling time, then the entropy will be constant in time, indicating an adiabatic evolution and the planet will expand with an index of $\alpha \simeq -\frac{1}{3}$. As the planet expands, it backs away from the star until cooling once again becomes dominant and the planet shrinks for the remainder of the evolution.

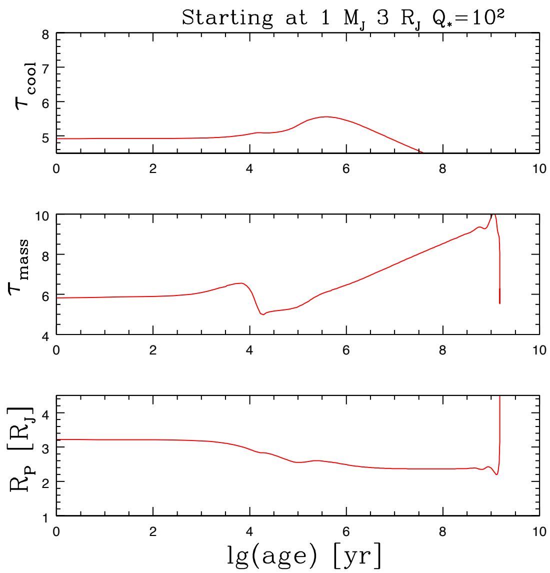
When comparing the effects of mass on $\tau_{mass}$ and $\tau_{cool}$, we find that for higher masses, $\tau_{mass}$ become dominant over $\tau_{cool}$ at an earlier time. Lower masses have a larger variance between the shortest mass loss timescale and longest cooling timescale, resulting in a greater adiabatic increase of the radius of the planet. This is seen more clearly in the plots in section 3.7.

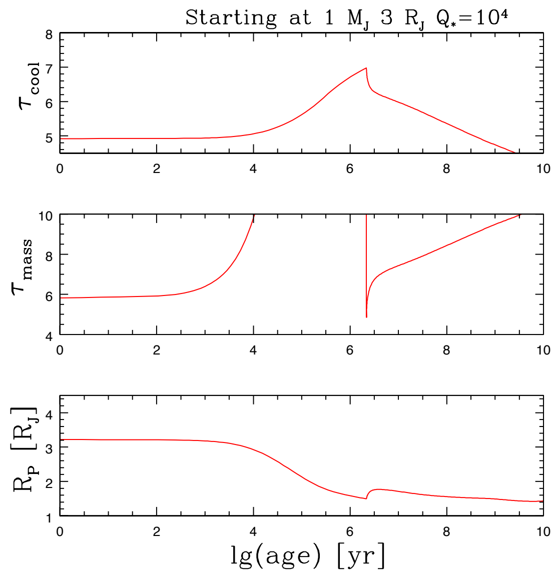
Holding the mass constant, but varying the radii, we see that the mass loss timescale becomes much shorter than the cooling timescale at an earlier time for lower radii. Larger radii have a larger variance between the shortest $\tau_{mass}$ and longest $\tau_{cool}$, producing larger adiabatic increases.

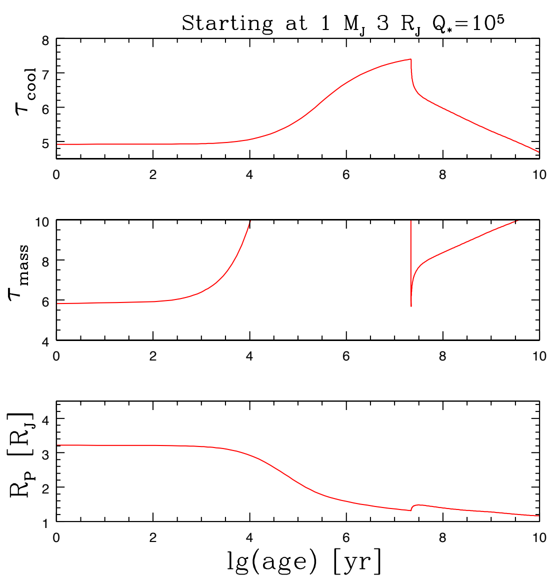
For low mass, small radii planets, there is very little adiabatic activity. The mass loss and cooling timescales are comparable for the majority of the evolution, with cooling being slightly more dominant for the total duration. For higher $Q_\star$ values, ($Q_\star > 10^6$), the mass loss time becomes very large very early on, while the cooling time remains relatively short. In these cases, $\tau_{mass}$ never becomes shorter than $\tau_{cool}$, so the planet continues to shrink for the entirety of the evolution.

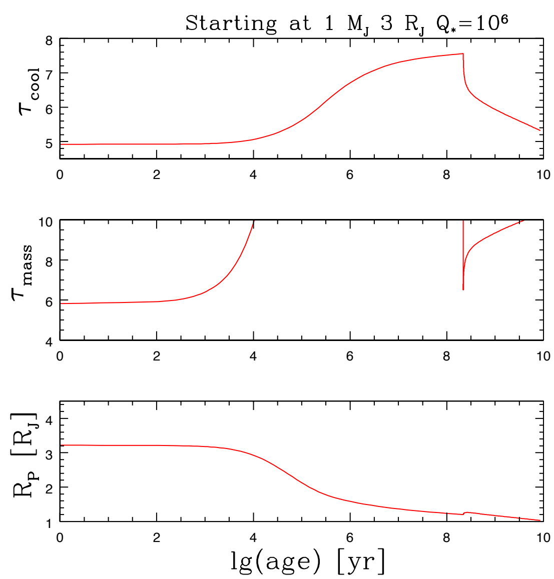
The tidal dissipation factor regulates how quickly the planet moves in towards its star. For lower $Q_\star$ values, the planet moves in and reaches Roche lobe contact faster, therefore starting to lose mass earlier. We see this occurring in the plots below when $\tau_{mass}$ has the sudden drop to its minimum time. Cooling time is dominant for longer periods in the models with higher $Q_\star$ values and they produce smaller adiabatic increases in the radii. For lower $Q_\star$ values, $\tau_{mass} \ll \tau_{cool}$ earlier and at a larger peak differences, causing the planet to increase its radius to a larger size. This is seen in section 3.6.

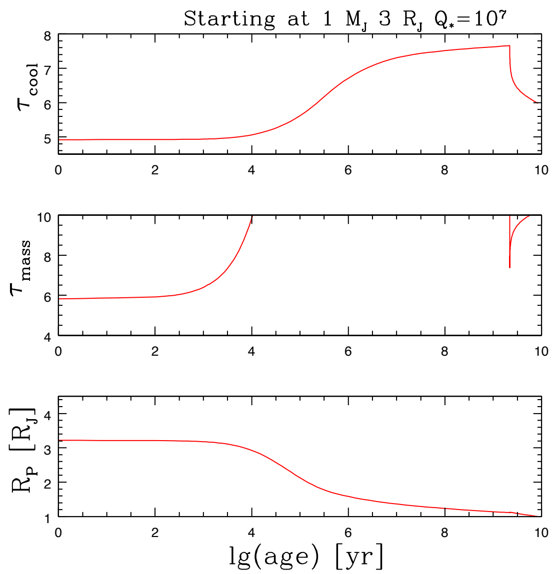
### 3.8.1 Planets Starting at 1 $M_J$ 3 $R_J$

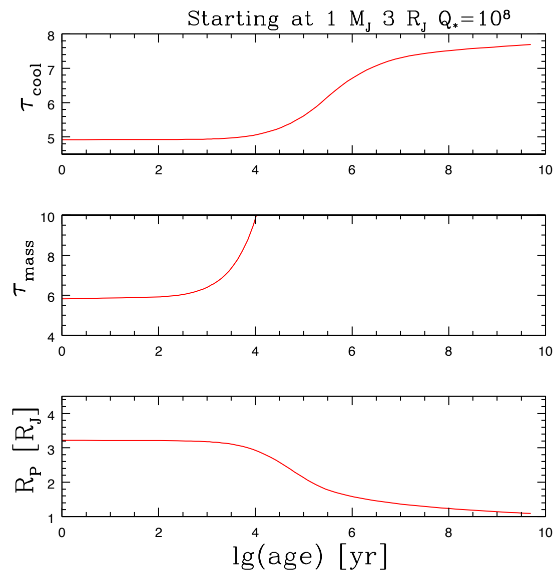


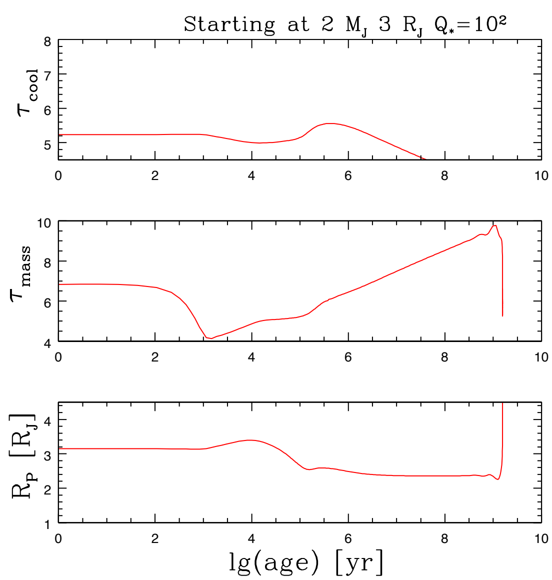



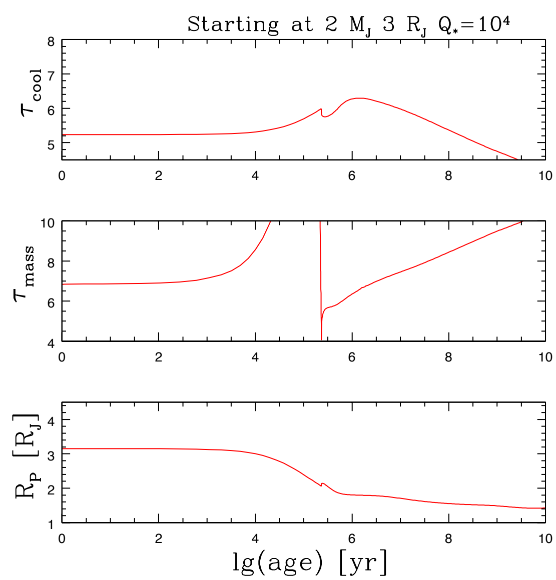



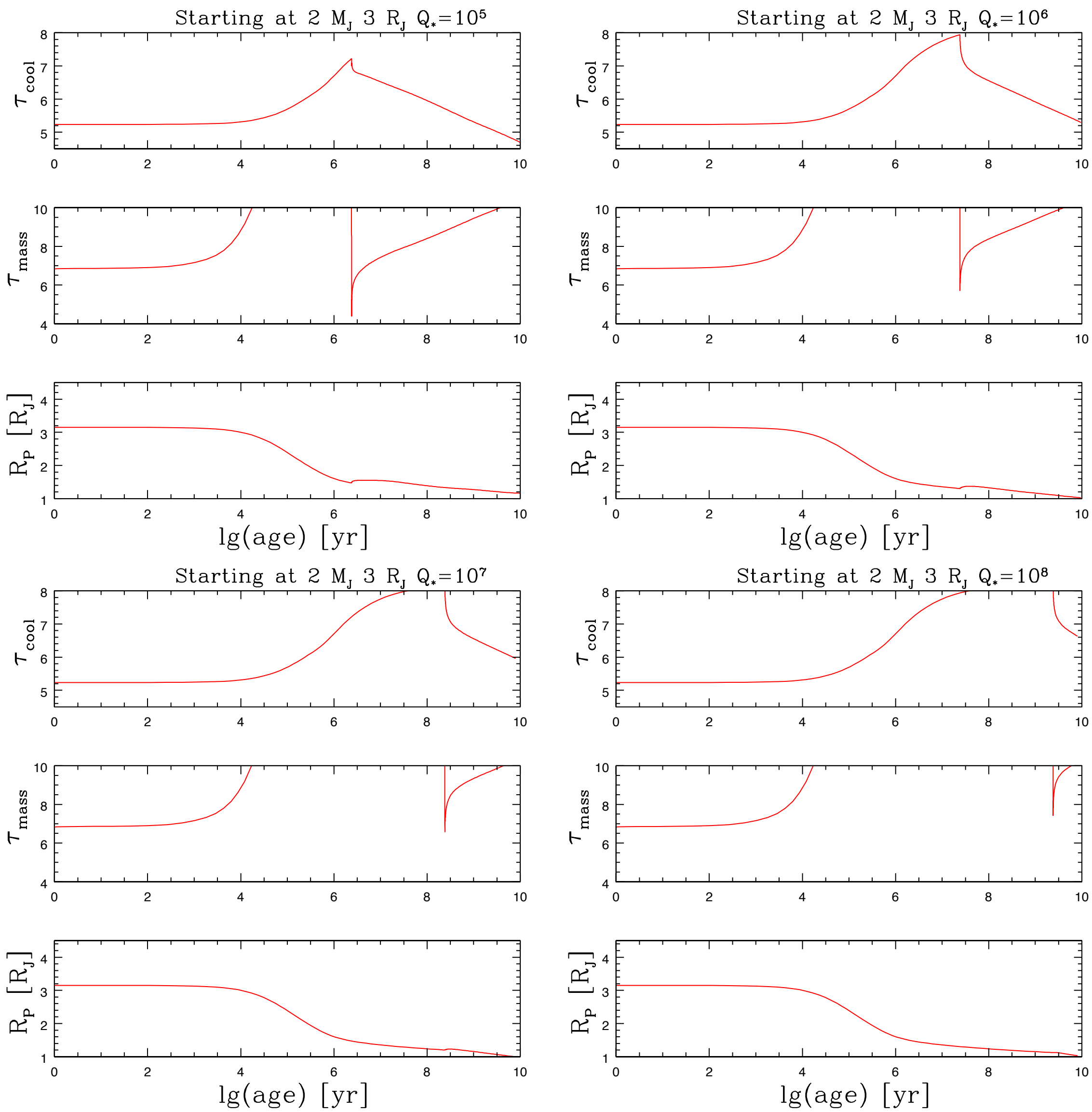


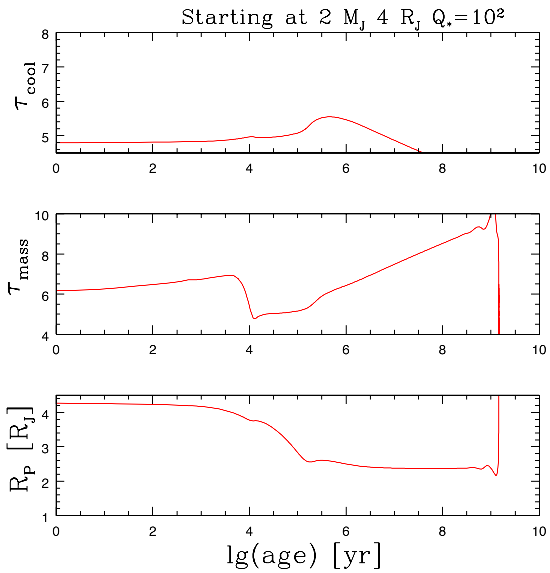

### 3.8.2 Planets Starting at 2 $M_J$ 3 $R_J$

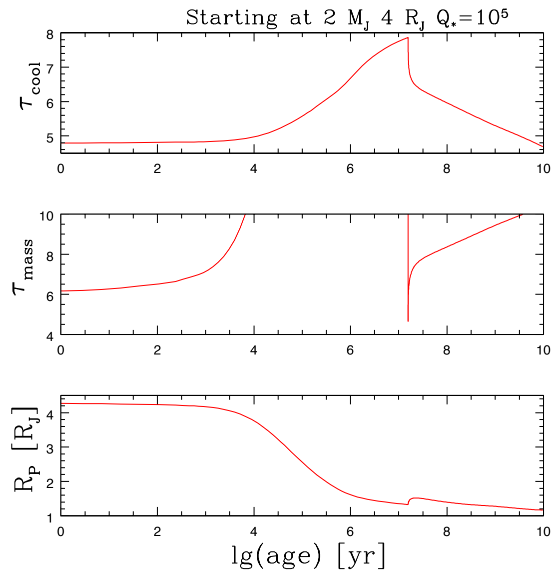


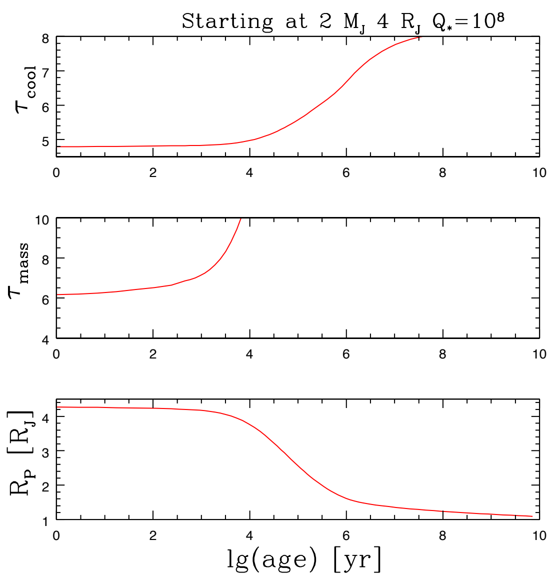




Starting at 2 $M_J$ 3 $R_J$ $Q_*=10^5$
Starting at 2 $M_J$ 3 $R_J$ $Q_*=10^6$
Starting at 2 $M_J$ 3 $R_J$ $Q_*=10^7$
Starting at 2 $M_J$ 3 $R_J$ $Q_*=10^8$
$\tau_{cool}$
$\tau_{mass}$
$R_P$ [$R_J$]
lg(age) [yr]

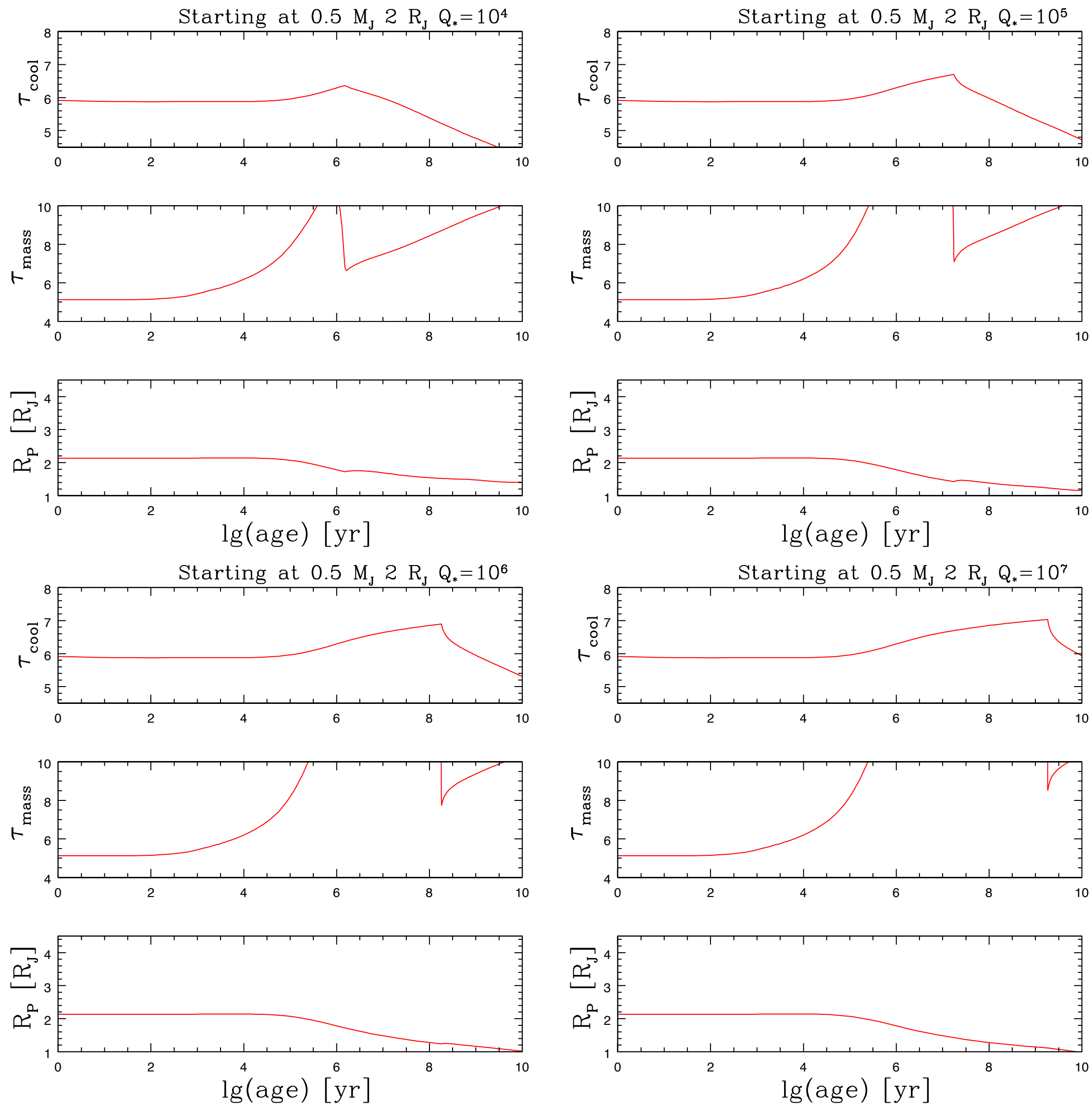
### 3.8.3 Planets Starting at 2 $M_J$ 4 $R_J$

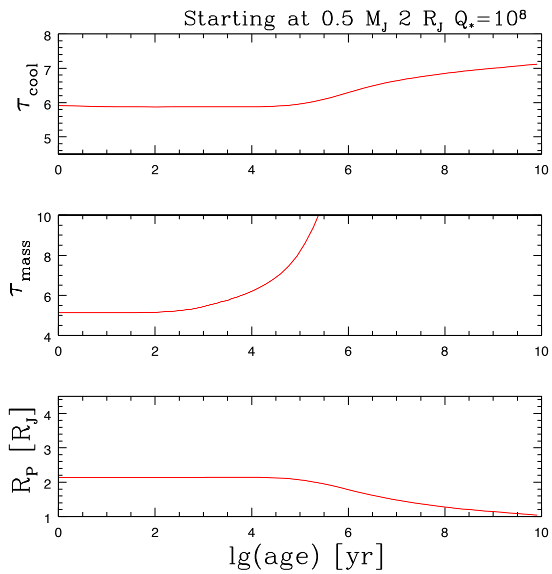


### 3.8.4 Planets Starting at 0.5 $M_J$ 2 $R_J$






# 4 Discussion and Conclusion

We have modeled irradiated gas giants under various conditions in order to determine where young planets stopped when driven in towards their star and to see how much mass was lost. We have found that when placed at a critical orbital separation immediately out of Roche lobe contact and allowed to evolve with both inward torques and heating pushing the planet towards it star, the planets would migrate to a period of approximately half a day and then reverse direction and continue to back away. This contradicts the findings of Metzger, et al. (2012) who suggest that these low density planets engaging in stable mass transfer would alternate between being in and out of contact until the planet lost all of its mass to its star. We found that regardless of mass, radius, $\alpha$, or $Q_\star$, the planets survived after losing nearly 90% of their mass. Metzger, et al. (2012) also suggest that varying $\alpha$, the index that relates the expansion or contraction rate of the planet to its mass loss rate, would directly alter the fate of the planet. Our models presented that $\alpha$ had little effect on the evolution of the planet. Identical results were achieved with both $\alpha = 0, 1$ with the exception of the 2 $M_J$ 3 $R_J$ planets with $Q_\star = 2$ in which they still tracked each other. This result indicates that the survival rate of planets in Roche lobe contact is more likely than previously predicted.

In §2.3, we found that the timescale at which the planets back away from their star and lose mass is set by the orbital torque. We also found that significant tidal heating, which causes the radius of the planet to expand, forces the planet to increase its orbital separation. These are the driving forces for the survival of the planets.

Some planets currently filling a large percentage of their Roche lobe may have been in Roche lobe contact in the early stages of their evolution. If Roche lobe overflow is a common occurrence, we suggest that a population of very short period ($\lesssim 2$ $days$), low mass ($\lesssim 0.2$ $M_J$) remnant planets may exist due to the cooling of a planet within its Roche lobe radius. Such planets may be detectable with Kepler.

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