PHYS305 - Spring 2023 - Exam 1

After meeting up with a ragtag team of mysterious hackers and swallowing a curious pill, you awaken to find yourself in a pod of warm goo, surrounded by millions of other pods containing sleeping humans. You have been rescued from the Matrix, a mental prison constructed by malevolent artificial intelligences in the not-too-distant future. You learn that humans have lost track of what year it is, and you have been rescued because of your training as a professional astronomer in hopes that you can contrive an astronomical means of determining the year so the humans can figure out how long computers have imprisoned the human race. General Morpheus and others in his crew believe that it is about the year 2199. Searching ancient stellar catalogs in the last human city of Zion, you hit upon a way: you will telescopically measure the current position of Barnard's Star in the sky and compare it with the position last recorded in 1999. Since the ancient catalogs record the star's proper motion, you should figure out how long it's been since 1999.

In the following calculations, you will figure out how long the computers have captured humanity. You are allowed to use your textbook, class notes, previous homeworks and solutions, and your calculator. You are NOT allowed to use the internet or to work together.

1. To conduct your observations, you are given several astronomical filters that allow you to conduct observations in different wavelengths to optimize detection of Barnard's Star. Your ancient stellar catalog says that Barnard's Star has a temperature $T \approx 3000\,\mathrm{K}$. Explain why you might choose to use the filter centered at a wavelength $\lambda = 1.0\,\mu\mathrm{m}$.

2. To prepare for your observations, you observe the star Vega in your chosen filter, a star which is much brighter than Barnard's star: Vega has an apparent magnitude in your filter of $m \approx 0$, while Barnard's star has $m = 6.741$. You determine that Vega has a flux of 1.2×10^{-9} W m^{-2}. Show that the flux from Barnard's Star is 2.4×10^{-12} W m^{-2}.

3. Taking the wavelength of your observed photons to be exactly $\lambda = 1\,\mu\mathrm{m}$, what is the energy of each photon? Using this energy, show that the flux of photons from Barnard's star is $1.2 \times 10^{7}\,\mathrm{m}^{-2}\,\mathrm{s}^{-1}$.

4. The humans manage to scrounge up a telescope with a diameter $D = 20\,\mathrm{cm}$ for you to conduct your observations. Show that, using this scope, you can achieve a relative uncertainty on the flux from Barnard's Star of about 0.2% by observing it for about 1 second.

5. With such a precise photometric measurement, you determine the displacement Δ of Barnard's Star relative to its position in 1999 to be $\Delta = (6234.00 \pm 0.03)$ arcsecs. The ancient stellar catalog you have indicates that Barnard's Star exhibits a proper motion $\mu = \left(10.39000 \pm 5 \times 10^{-5}\right)$ arcsecs year^{-1}. Assuming the time since 1999 Δt is given by $\Delta t = \Delta/\mu$, estimate what year it actually is and give your uncertainty, using the uncertainties on Δ and μ. Is General Morpheus right?