PHYS305 - Spring 2023 - Exam 2

Galileo was the first known astronomer to point a telescope heavenwards, an act that allowed him to make many seminal discoveries in astronomy. In Galileo's time, it was common to encode a scientific discovery as an anagram and then to distribute the scrambled message among other scientists, only revealing the true meaning when you finally had time to completely confirm and publish the discovery. In 1610, the year after he first began telescopic observations, Galileo sent an anagram recording a discovery to another famous astronomer, Johannes Kepler, in the German states. In his feverish desire to work out Galileo's discovery, Kepler tried to unscramble the message and decided it meant Galileo had discovered two moons orbiting Mars. Mars does, indeed, have two very small moons, Phobos and Deimos, orbiting very close to Mars.

For this exam, you will try to work out whether Galileo could have observed the moons of Mars using his telescope. You are allowed to use your textbook, class notes, previous homeworks and solutions, and your calculator. You are NOT allowed to use the internet or to work together.

(If you want to hear how this story ends, visit this url - `https://www.thisamericanlife.org/475/send-a-message/prologue` - after completing the exam, of course).

1. Figure 1 below illustrates the components of Galileo's scope. The objective consists of a converging lens with a spherical surface on both sides which bends rays toward the lens's optical axis, and the eyepiece (or ocular) consist of a diverging lens with a spherical surface on both sides which bends rays away from the lens's optical axis. The optical axis for each lens is shown as a dashed line. Complete the optical diagram for the objective, showing how ray 1 passes through the lens. Then, using the rays shown for each lens, show where the focal points are. Be precise. The rays should show the correct relationships to the focal points. It will probably help to use a straight-edge.

2. Figure 2 below shows how the lenses were arranged in Galileo's scope. The distance between the lenses $d = |f_1| - |f_2|$ where $|f_i|$ is the absolute value of the focal length for lens i. (Remember, a diverging lens has negative focal length.) Mark the focal points for each lens and show how ray 2 propagates through the telescope. Again, be precise. The rays should show the correct relationships to the focal points. It will probably help to use a straight-edge.

3. One benefit of Galileo's scope is that, by adjusting f_2 and d together, he could increase the magnification. By increasing magnification M, you increase the apparent angle between two objects in the field of view as $\theta' = M\theta$, where θ is the angle between the two objects as seen in the sky and θ' is the apparent angle as seen through the telescope. Show that, if $d = |f_1| - |f_2|$, the magnification M be made arbitrarily large by adjusting $d \to |f_1|$. The focal lengths for Galileo's scope were $f_1 = 1330\,\text{mm}$ and $f_2 = -94\,\text{mm}$. Show that, with these values, Galileo's scope provided a magnification $M = 14$.

4. In order for Galileo to have seen both moons, he needed to be able to differentiate the two moons from Mars. If his telescope had too small an angular resolution, it would have been impossible to differentiate the moons from Mars. Galileo's scope had a focal ratio $R = 28$. Assuming Galileo observed at a wavelength $\lambda = 380\,\text{nm}$ (just about the bluest wavelength a human can see) and diffraction-limited seeing (much better than Galileo would actually have had), calculate the minimum angular resolution achievable by Galileo's scope.

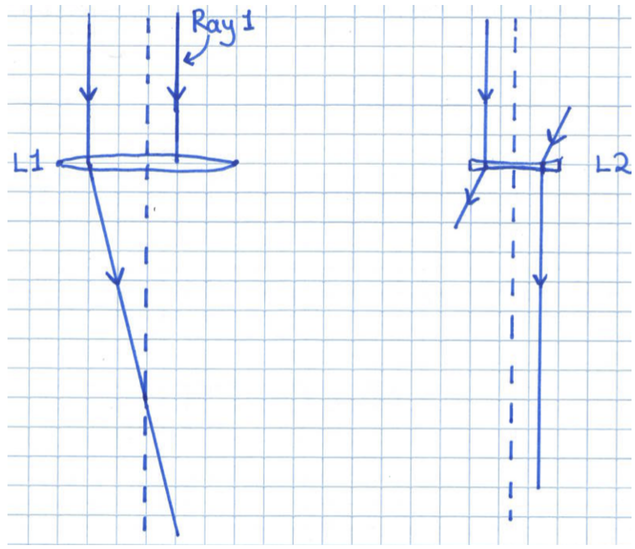

Figure 1: Lens 1 (L1) and Lens 2 (L2) with optical axes shown as dashed lines.

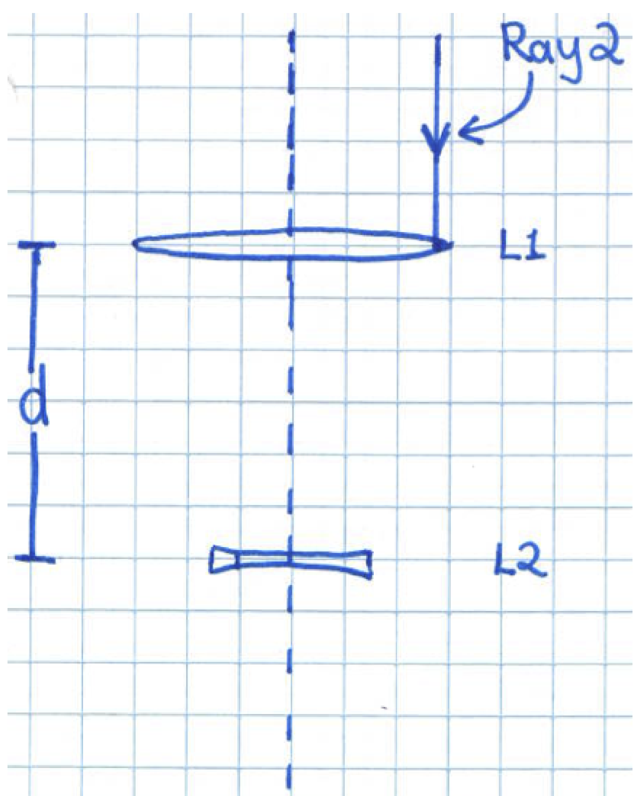

Figure 2: Optical diagram of Galileo's telescope, with lenses shown to the exact same scale (relative to the grid in the diagram) as in Figure 1.

5. In October 1610, Mars (and its moons) were about 66 million kilometers away from Earth. The moon nearest to Mars, Phobos, orbits about 6000 km above Mars' surface, while Deimos orbits about 20000 km above Mars' surface. Incorporating the magnification from above, determine whether Galileo's scope have had sufficient angular resolution to differentiate the moons from Mars.

6. The last requirement for Galileo to have seen the moons is that they were sufficiently bright. The brightest moon, Phobos, has an apparent magnitude of 11.8. The darkest object you can see with your naked eye is about magnitude 6. Using a telescope allows your eye to detect dimmer objects by increasing the total flux your eye receives, lowering the minimum flux your eye can detect by a factor of $(D_{\text{eye}}/D_{\text{telescope}})^2$, where $D_{\text{telescope}}$ is the telescope diameter and D_{eye} is your pupil diameter (4 mm). First, convert magnitude 6 to a (relative) flux. Then, using the scaling above, calculate the new minimum (relative) flux Galileo could have seen with his scope, assuming $D_{\text{telescope}} = 50\,\text{mm}$. (Your fluxes will be relative to the unspecified flux for a magnitude 0 object. As long as you leave them relative to the unspecified flux, you should be fine.) Finally, determine the maximum magnitude Galileo could have detected. Would he have been able to see the moons of Mars?