1.

Stars emit luminous energy as blackbodies, and we can estimate the wavelength λ_{max} at which their emission peaks using Wien's Displacement law:

$$\lambda_{\mathrm{max}} = \frac{3 \times 10^{-3}\,\mathrm{m\,K}}{T_\star},$$

with $T_\star$ the temperature of the star.

Plugging in $T_\star = 3000\,\mathrm{K}$ gives

$$\lambda_{\mathrm{max}} = \frac{3 \times 10^{-3}\,\mathrm{m\,K}}{3000\,\mathrm{K}} = 1 \times 10^{-6}\,\mathrm{m} = 1\,\mu\mathrm{m}.$$

In words, the star's emission peaks at $1\,\mu\mathrm{m}$, so a filter centered on that wavelength is likely to return a larger number of photons and better signal-to-noise than filters at other wavelengths.

2.

Recall the relationship between stellar flux F and magnitude M:

$$M = -2.5\log_{10}(F/F_0),$$

where F_0 is some reference flux for an object with $M = 0$.

Thus, we can solve for F as

$$F = F_0 \times 10^{-M/2.5}.$$

Plugging in our numbers:

$$F = \left(1.2 \times 10^{-9}\ \mathrm{W\ m^{-2}}\right) \times 10^{-6.741/2.5} = 2.4 \times 10^{-12}\ \mathrm{W\ m^{-2}}.$$

3.

Energy E for a photon of wavelength λ is

$$E = hc/\lambda,$$

with h Planck's constant and c the speed of light.

Plugging in $\lambda = 1\,\mu\mathrm{m} = 10^{-6}\,\mathrm{m}$:

$$E = \frac{\left(6.6 \times 10^{-34}\,\mathrm{J\,s}\right)\left(3 \times 10^{8}\,\mathrm{m\,s^{-1}}\right)}{\left(10^{-6}\,\mathrm{m}\right)} = 2 \times 10^{-19}\,\mathrm{J}.$$

Therefore, the energy flux $F = 2.4 \times 10^{-12}\,\mathrm{W\,m^{-2}}$ translates into a number flux $F/E = 1.2 \times 10^{7}\,\mathrm{m^{-2}\,s^{-1}}$.

4.

An aperture $D = 20\,\mathrm{m}$ gives a telescope area A:

$$A = \pi(D/2)^2 = \pi\left(20 \times 10^{-2}\,\mathrm{m}/2\right)^2 = 0.03\,\mathrm{m}^2.$$

If our photon flux is $f = 1.2 \times 10^{7}\,\mathrm{m^{-2}\,s^{-1}}$, then in time t 1 second, a telescope of that size will collect

$$N = tAf = (1\,\mathrm{s})\left(0.03\,\mathrm{m}^2\right)\left(1.2 \times 10^{7}\,\mathrm{m^{-2}\,s^{-1}}\right) = 360000\,\mathrm{photons}.$$

According to Poisson statistics, the relative precision for such a measurement is

$$p = \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{360000}} = 0.2\%.$$

5.

The time since the last observation t is given by

$$t = \Delta/\mu = \frac{6234\,\mathrm{as}}{10.39000\,\mathrm{as\ yr}^{-1}} = 600\,\mathrm{yr},$$

meaning the year is 1999 + 600 = 2599! But what about the uncertainty?

The uncertainty σ_t on the time t is given by

$$\begin{aligned}\sigma_t &= \sqrt{\left(\frac{\partial t}{\partial \Delta}\right)^2 \sigma_\Delta^2 + \left(\frac{\partial t}{\partial \mu}\right)^2 \sigma_\mu^2} \\ &= \sqrt{\left(\frac{1}{\mu}\right)^2 \sigma_\Delta^2 + \left(\frac{\Delta}{\mu^2}\right)^2 \sigma_\mu^2}\end{aligned}$$

$$\sigma_t = \sqrt{\left(\frac{1}{10.39000\,\mathrm{as\ yr}^{-1}}\right)^2 (0.03\,\mathrm{as})^2 + \left(\frac{6234.00\,\mathrm{as}}{10.39000\,\mathrm{as\ yr}^{-1}}\right)^2 \left(5\times 10^{-5}\,\mathrm{as\ yr}^{-1}\right)^2} = 0.03\,\mathrm{yr},$$

which is very small.

So the year is $(2599 \pm 0.03)\,\mathrm{yr}$, and Morpheus is very wrong.