<u>Observing Project Questions</u> Josh Gambill

1. The photometric data points you analyzed have uncertainties σ associated with them. How could you estimate the per-point uncertainty? Using your method, what is that uncertainty? How would you expect anomalously outlying data points to affect your estimate? What are some techniques you could use to mitigate the effects of outliers on your estimate?

   Answer:
   One way to estimate the per-point uncertainty is take the chi squared:

$$\chi^2 = \sum_{i=0}^{N-1} \left(\frac{y_i - y(x_i)}{\sigma_i}\right)^2$$

   And force $\chi^2 = 1$, in which our uncertainty can be re-written as:

$$\sigma = \sqrt{\langle y_i^2 \rangle - \langle y_i \rangle^2}$$

   However, outliers in the data set, especially extreme outliers, will cause values such as the standard deviation and the mean to shift dramatically and skew per-point uncertainty. In order to combat this, we can rely on the Median Absolute Deviation (MAD) rather than the mean.

   Defined as:

$$MAD(y_i) = median(|y_i - median(y_i)|)$$

   And our Standard Deviation using the Median Absolute Deviation:

$$\sigma = 1.4826 * MAD$$

   From the analysis:

$$MAD = 5.061$$

$$\sigma = 1.4826 * 5.061$$

$$\sigma = 7.503$$

2. Imagine you switched planets to observe a star that was 2.5 magnitudes brighter than your first target. Assuming Poisson uncertainties, how would you expect σ to change and by how much?

   Answer:

If the star we switch to has a magnitude 2.5 times brighter than the first target, this means that the magnitude of the second star m_2 will be equal to:

$$m_2 = m_1 - 2.5$$

This is true since the magnitudes are logarithmic and as the number grows larger, the dimmer the object appears in the sky.

Since our magnitude is less, and in turn the star is brighter, this means that our detectors will receive a larger number of photons. With more detected photons comes a smaller uncertainty.

Using the equation of magnitude differences from chapter 1 of the textbook:

$$\Delta m = m_1 - m_2 = -2.5\, log_{10}\left(\frac{F_1}{F_2}\right)$$

We can determine that the flux ratios of the two stars is:

$$\frac{F_1}{F_2} = 10^{-0.4(m_1 - m_2)}$$

Plugging in:

$$m_1 - m_2 = 2.5$$

We can determine that:

$$\frac{F_1}{F_2} = 0.1$$

Or

$$\frac{F_2}{F_1} = 10$$

Meaning that the flux of star 2 is ten times the flux of the first.

Using this flux ratio, we can determine the uncertainty ratio as well.

The number of photons is determined by the equation:

$$N = FA\tau$$

Where F is the flux, A is the aperture area, and τ is the exposure time. In our experiment, both the aperture and the exposure time do not change, just the flux. So, if we compare our photon counts $N_1\ and\ N_2$, the ratio reduces to:

$$\frac{N_2}{N_1} = \frac{F_2}{F_1} = 10$$

Using the Poisson definition of uncertainty:

$$\sigma = \sqrt{N}$$

So, the ratio of σ_2 and σ_1 can be expressed as:

$$\frac{\sigma_2}{\sigma_1} = \sqrt{\frac{N_2}{N_1}} = \sqrt{10} \approx 3$$

From this, we can determine that our Signal to Noise ratio between the two stars will decrease by a factor of about 3.

3. Of course, the photometric uncertainty σ will impact your results, specifically the results you get for the estimate of each transit time, t_c. The uncertainty on t_c depends on the system parameters according to the following equation:

$$\sigma_{t_c} = \sqrt{\frac{\tau}{2\Gamma}} \frac{\sigma}{\delta}$$

where τ is related to the ingress or egress duration, Γ is the sampling rate for your data (probably once every 2 minutes), σ is the per-point photometric uncertainty, and δ is the transit depth (how big the planet is compared to the star).

How would your uncertainty on the transit time change if you doubled the photometric uncertainty? How would it change if you doubled the transit depth (made the planet bigger compared to the star)? You can, of course, use the equation to make these estimates, but also explain qualitatively *why* you would expect that behavior? In words, why does the transit timing uncertainty go up or down as you change the photometric uncertainty and the transit depth?

Answer:
From a purely mathematical standpoint, if our photometric uncertainty σ becomes 2σ, without any of the other variables changing, then σ_{t_c} will double as well.

The same logic can apply to transit depth as well. If δ becomes 2δ, and none of the other variables change, then σ_{t_c} will be halved.

From a physical standpoint, we can reason as to why these changes in uncertainty and depth have these effects on σ_{t_c}.

In the case of σ doubling, this could be caused by something such as less photons being detected, this would in turn also decrease σ_{t_c}.

As for the doubling of δ, which is the depth of the transit, we can reason that if the depth doubles, that means that the transit will be easier to pick out from the background star and noise. A clearer transit would in turn decrease σ_{t_c}.

4. Now look at the period value reported on the Exoplanet Archive. We want to know whether your result is consistent (to within uncertainties) with their value. Look at your period value P_{yours} and their period value P_{theirs}, along with the corresponding uncertainties (σ_{yours} and σ_{theirs}, respectively). We want to consider the function $f = P_{yours} - P_{theirs}$, calculate the corresponding uncertainty for that function, and figure out whether the function might be equal to zero to within uncertainties. Consult Chapter 2 in Chromey to refresh your memory about how to propagate uncertainties.

   If your result does not agree with the Archive's, what are some possible reasons? Look at your transit model and compare it to the data. Does it look like a good fit to all the transits?

   Answer:

   From Exoplanet Archive:

$$P_{theirs} = 1.48224686\ days$$
$$\sigma_{theirs} = \pm 6.12\ e - 6\ days$$

   From my analysis:

$$P_{mine} = 1.48225\ days$$
$$\sigma_{mine} = \pm 3.24083\ e - 6\ days$$

   Difference Function:

$$f = |P_{mine} - P_{theirs}|$$

$$f = |1.48225 - 1.48224686\ |$$

$$f = 3.14e - 6\ days$$

   Uncertainty of the Difference Function:

$$\sigma_f^2 = \sum \left(\frac{\partial f}{\partial P}\right)^2 \sigma_P^2$$

$$= \left(\frac{\partial F}{\partial P_{mine}}\right)^2 \sigma_{P_{mine}}^2 + \left(\frac{\partial F}{\partial P_{theirs}}\right)^2 \sigma_{P_{theirs}}^2$$

$$\sigma_f^2 = \sigma_{P_{mine}}^2 + \sigma_{P_{theirs}}^2$$

$$\sigma_f = \sqrt{\sigma^2_{P_{mine}} + \sigma^2_{P_{theirs}}}$$

$$\sigma_f = \sqrt{(3.24083\ e - 6)^2 + (6.12\ e - 6)^2}$$

$$\sigma_f = 6.93\ e - 6\ days$$

Difference function with uncertainty:

$$f = 0.0044 \pm 2.709\ e - 5\ days$$

Determine if the values are consistent:

$$\frac{|f|}{\sigma_f} = \frac{3.14e - 6\ days}{6.93\ e - 6\ days}$$

$$\frac{|f|}{\sigma_f} = 0.453$$

Since:

$$\frac{|f|}{\sigma_f} \leq 3$$

We can conclude that the values are consistent, and my value agrees with the archive's value.

5. Using your period *P* and T_0 value (called "ephemeris_fit_params[1]" in your python notebook), you will estimate the next time that your planet could be observed in transit.

   First, you'll need to figure when your planet will next be visible. One way to check this is to use Stellarium (https://stellarium-web.org/). Most of your targets are in the web version, but a few (WASP-10, HAT-P-19, and Qatar-1) seem not to be. For those, you'll have to download and install Stellarium.

   In Stellarium, run time forward from today and check when your object will next be visible at night. Record that date and convert it to Julian date using this online calculator - https://www.aavso.org/jd-calculator. Don't worry about getting the exact instant the planet is visible at night; just get close.

   Next, you'll need to calculate the times in the future when your object will transit. You can calculate the transit time t_c for the *E*th orbit using this equation: $t_c = T_0 + P\,E$. The first thing you'll need to do is to convert your T_0 value from your fit into Julian date. The fit value you get is in Julian date - 2457000, so start by adding 2457000 to your T_0. Then determine the number of orbits you'll have to wait until t_c is greater than the date you estimated from Stellarium. That should give you the minimum orbit number *E* for when your object is both visible and transiting. Record the next date when your object could be observed transiting and include all your arithmetic (neatly written) as part of your answer to this question.

Answer:

According to Stellarium, TrES-5 is currently always visible in the sky, so the next time will be sunset tomorrow. Or 8:45 pm on April 29.

This date translated to Julian Days:

$$Date\ of\ next\ visiblity: 2460429.86\, JD$$

Transit time:

$$t_c = T_0 + P\, E$$

From my analysis:

$$T_0 = 0.546958\, days$$

Converted to Julian Days:

$$T_0 = 0.546958\ \ days + 2457000\, JD$$

$$T_0 = 2457000.55\, JD$$

Estimate E from transit time equation:

$$t_c = T_0 + P\ E$$

$$E \approx \frac{t_c - T_0}{P}$$

$$E \approx \frac{2460529.86\ JD - 2457000.55\ JD}{1.48225\ \frac{days}{orbit}}$$

$$E \approx 2381.05\ orbits$$

Calculate the next transit time from our estimated E:

$$t_c = T_0 + P\ E$$

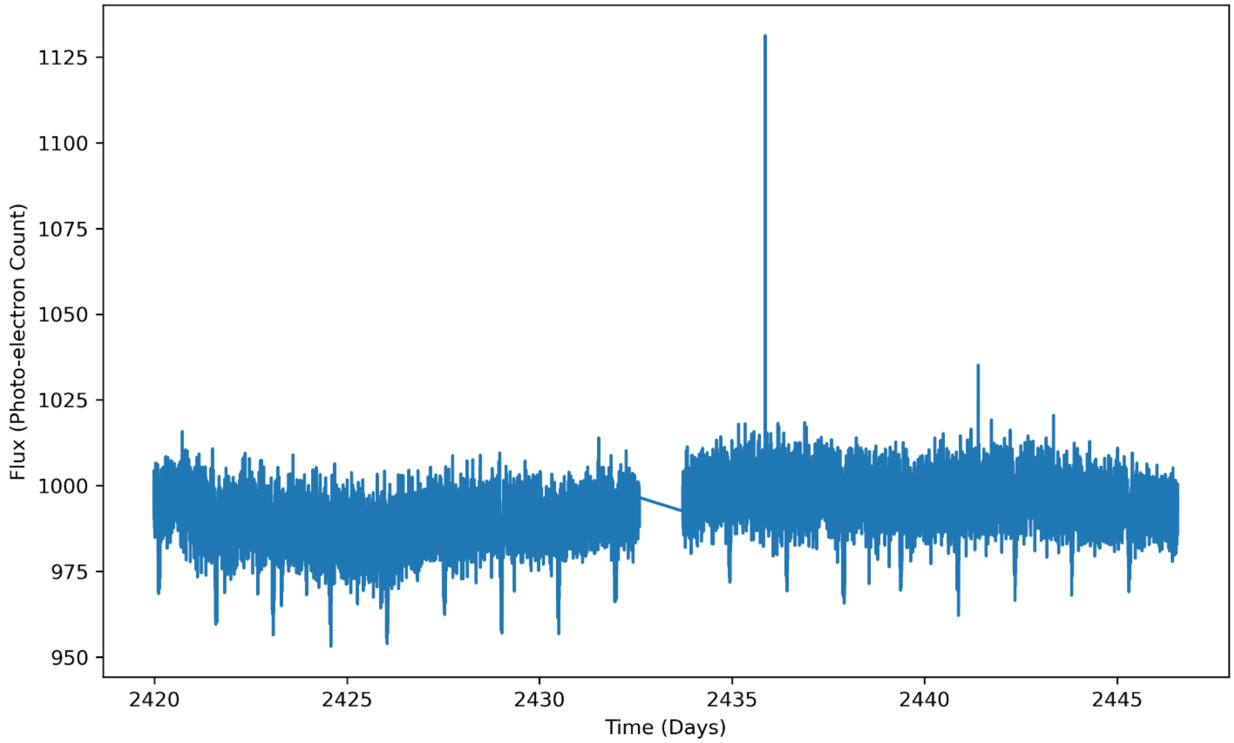
$$t_c = 2457000.55\ JD + (1.48225\ \frac{days}{orbits})\ (2381.05\ orbits)$$

$$t_c = 2460529.86\ JD$$

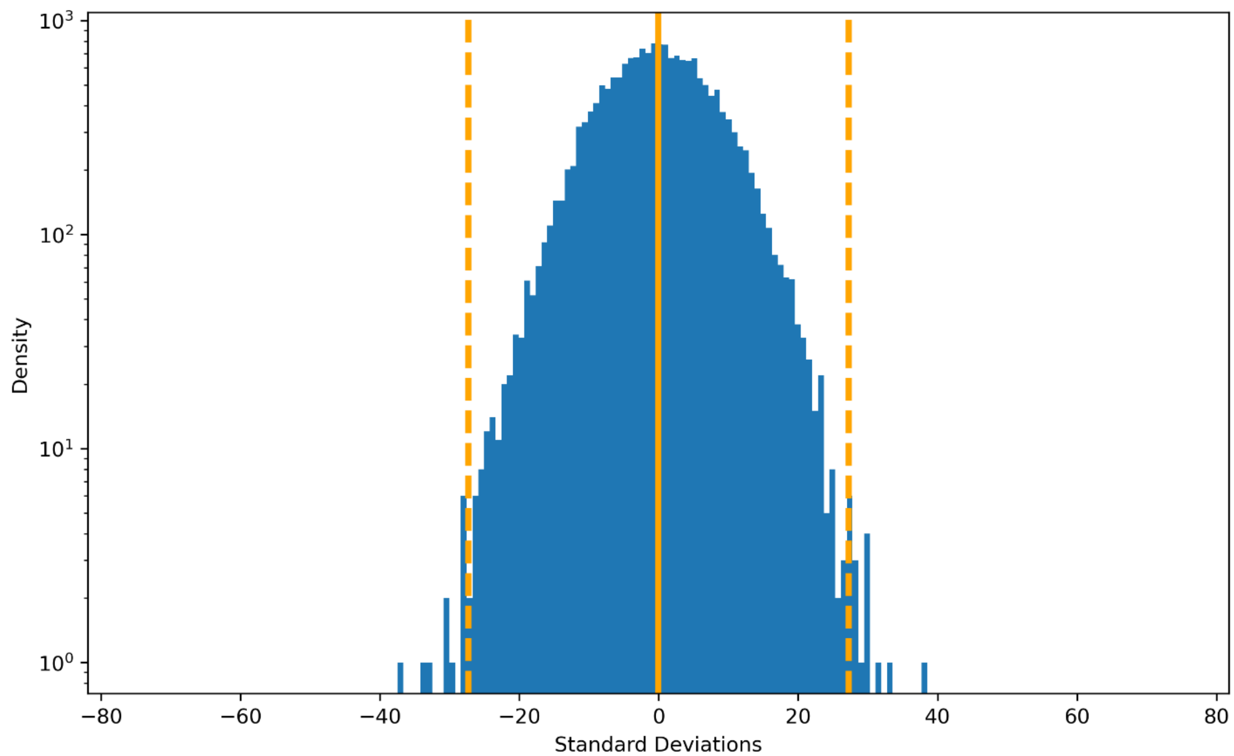
The next transit time of TrES-5 will be August 7th at 8:38 pm

Observing Project Figures for TrES-5b:

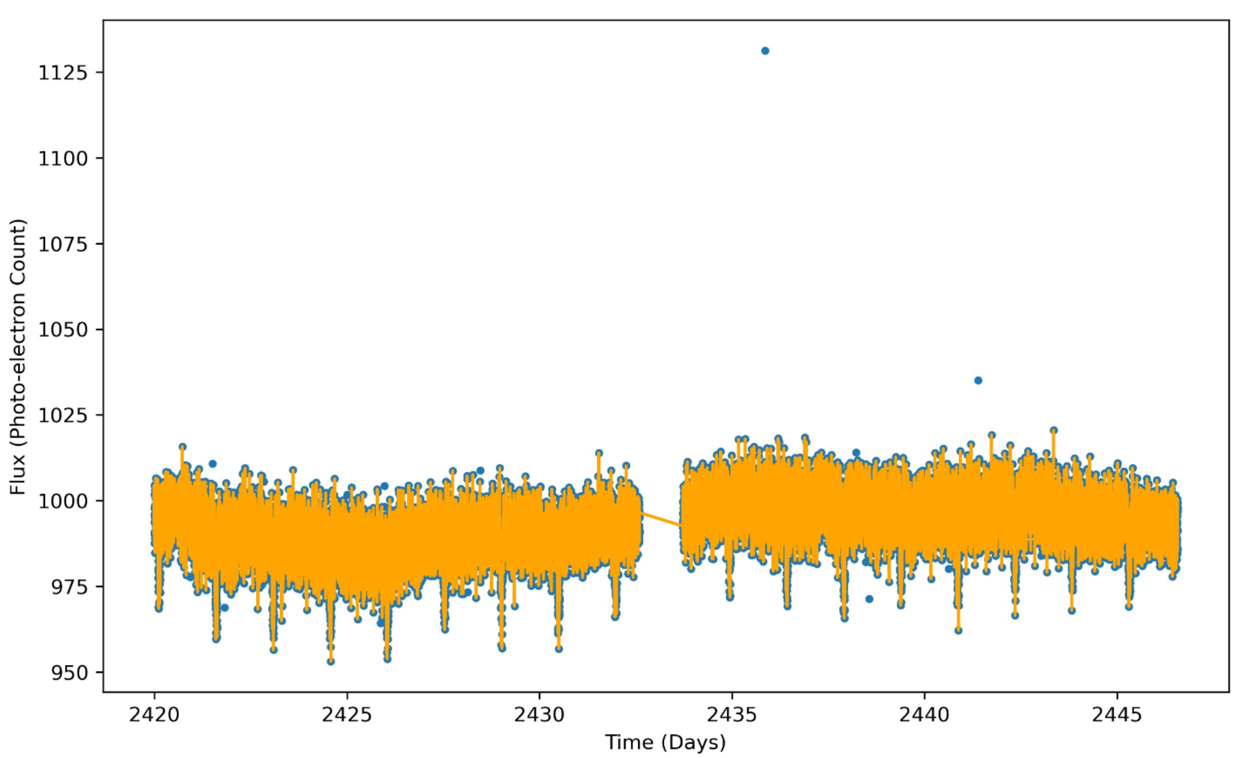
TrES-5b_Flux over time



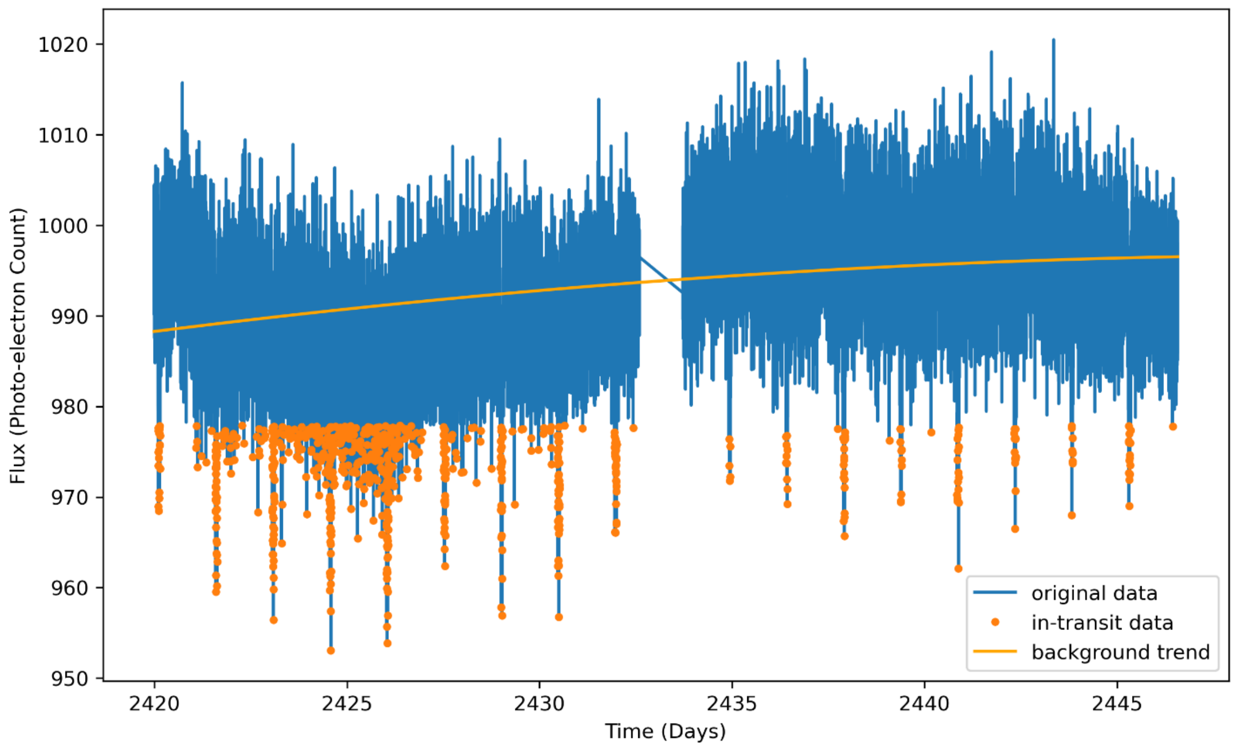
TrES-5b Density of Standard Deviations



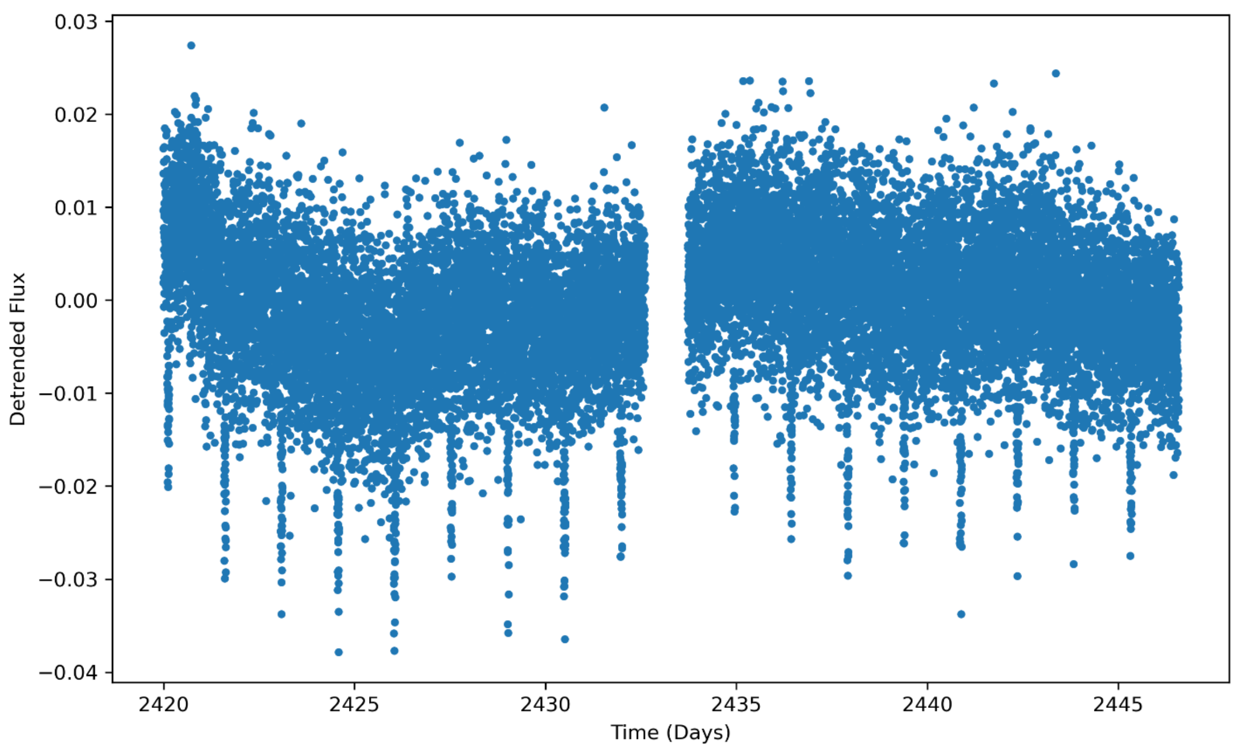
TrES-5b Outlier Mask



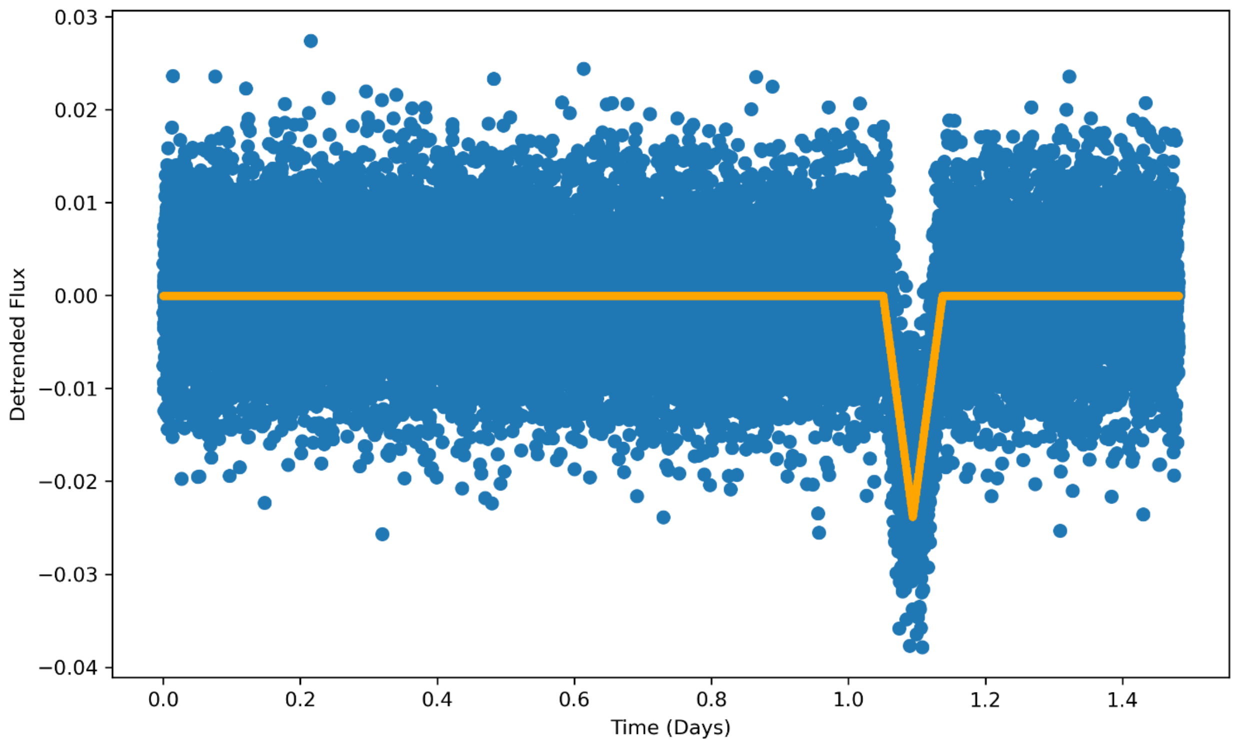
TrES-5b Polyfit



TrES-5b Outlier Filtered



TrES-5b Second Folded Fit



TrES-5b Transit Fit