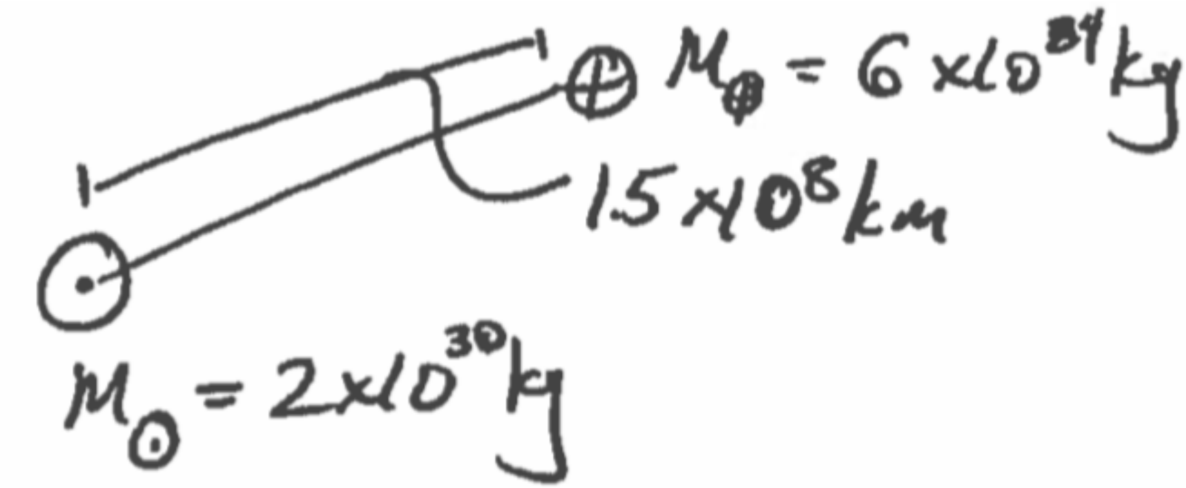
In [2]:
```
%matplotlib inline
%config InlineBackend.figure_format='retina'

from IPython.display import Image
```

3.16

In [2]:
```
Image(filename='Ch. 3, problem 3.16.png', width=600)
```

Out[2]:



Let's place the origin of our coordinate system at the center of the Sun. Then we'll calculate how far from the center of the Sun in the direction toward the Earth the center of mass is, $R_{\rm cm}$.

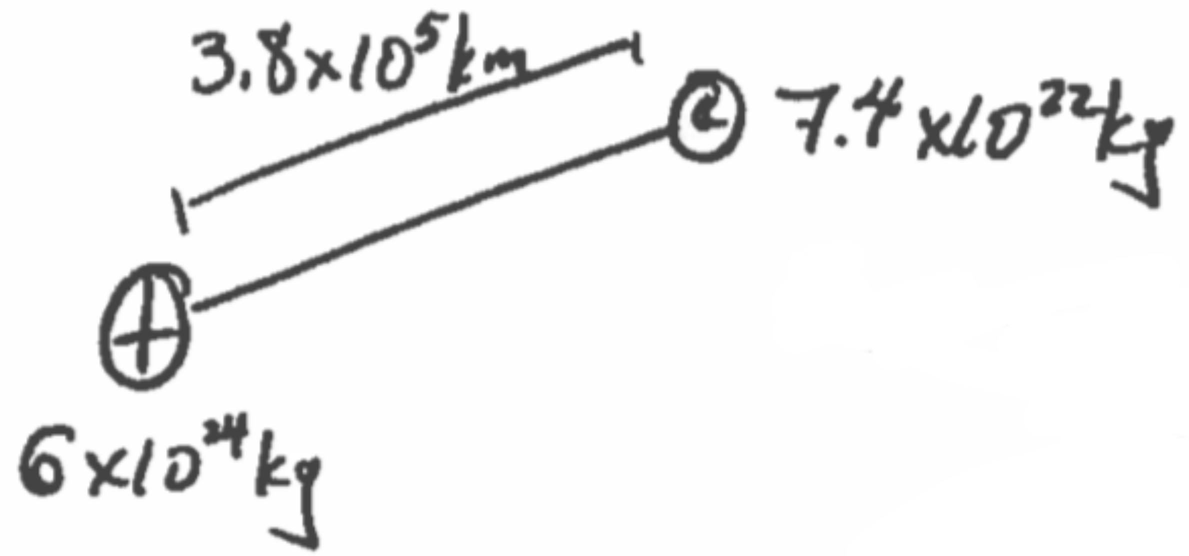
$$R_{\rm cm} = \frac{M_{\rm Sun}\left(0\right) + M_{\rm Earth}\left(1.5\times10^8\ {\rm km}\right)}{M_{\rm Sun} + M_{\rm Earth}} = \left(3\times10^{-6}\right)\left(1.5\times10^8\ {\rm km}\right) = \boxed{450\ {\rm km\ from\ the\ Sun's\ center.}}$$

The radius of the Sun $R_{\rm Sun}$ = 750,000 km, so the center of mass lies inside the Sun.

3.17

In [3]:
```
Image(filename='Ch. 3, problem 3.17.png', width=600)
```

Out[3]:



As measured from the center of the Earth,

$$R_{\mathrm{cm}} = \frac{M_{\mathrm{Moon}} d}{M_{\mathrm{Moon}} + M_{\mathrm{Earth}}} = \frac{\left(7.4 \times 10^{22}\ \mathrm{kg}\right)\left(3.8 \times 10^{5}\ \mathrm{km}\right)}{7.4 \times 10^{22}\ \mathrm{kg} + 6 \times 10^{24}\ \mathrm{kg}} \approx (0.012)\left(3.8 \times 10^{5}\ \mathrm{km}\right) = \boxed{4.6 \times 10^{3}\ \mathrm{km}},$$

which is 70% of the Earth's radius (6,400 km).

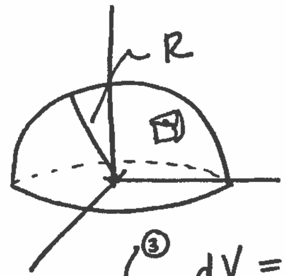
3.22

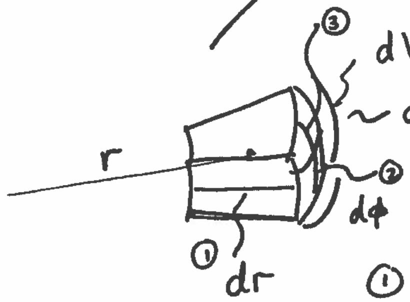
In [5]:

```
Image(filename='Ch. 3, problem 3.22.png', width=600)
```

Out[5]:

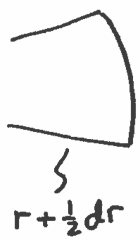
3.22

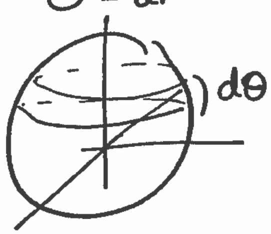




$dV = dx\,dy\,dz$

① = dr



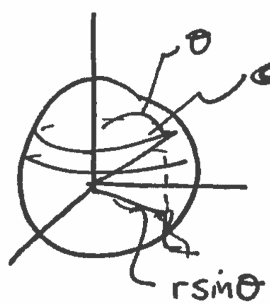


② $= (r+\tfrac{1}{2}dr)\cdot d\theta \simeq r\,d\theta$ ✓

Define angle!

$(\text{angle}) \equiv \dfrac{(\text{arc})}{(\text{tot. arc})}$

③



$(\text{circum.}) = 2\pi(r\sin\theta)$

So $(\text{arc length}) = d\phi(r\sin\theta)$

③ $= r\sin\theta\cdot d\phi$

$$\Rightarrow dV = (dr)\cdot(r\,d\theta)\cdot(r\sin\theta\,d\phi)$$

$$= r^2 dr\,\sin\theta\,d\theta\cdot d\phi \ ✓$$

The x and y positions of the center of mass are both zero by symmetry, so we only need to calculate the z-coordinate of the center of mass $\langle z_{\text{CM}} \rangle$. Intuitively, we expect $\langle z_{\text{CM}} \rangle$ is a bit less than R since more mass is located near the x-y plane than near the top of the hemisphere.

$$\begin{aligned}\langle z_{\text{CM}} \rangle = \int_{\text{V}} z\,\rho\,dV &= \rho \int_{r=0}^{R} \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} r\cos\theta\, r^2 dr\;\sin\theta\, d\theta\, d\phi \\ &= 2\pi\rho \int_{r=0}^{R} r^3 dr \int_{\theta=0}^{\pi/2} \cos\theta \sin\theta d\theta \\ &= \frac{1}{2}\pi\rho R^4 \int_{\theta=0}^{\pi/2} \cos\theta\sin\theta d\theta.\end{aligned}$$

Take $u \equiv \sin\theta$, which means $du = \cos\theta\, d\theta$.

$$\langle z_{\text{CM}} \rangle = \frac{1}{2}\pi\rho R^4 \int_{u=0}^{1} u\,du = \frac{1}{4}\pi\rho R^4.$$

The whole mass of the hemisphere is $M = \frac{1}{2}\frac{4\pi}{3}R^3\rho = \frac{2\pi}{3}R^3\rho$, so

$$\boxed{\langle z_{\text{CM}} \rangle = \frac{3}{8}\rho R.}$$

3.25

Since the string exerts a radial tension, changing the string's length does not change the mass's angular momentum.

$\vec{\text{l}} = \vec{r} \times \vec{p}$.

$\frac{d\vec{\text{l}}}{dt} = \vec{r} \times \vec{F}$, but $\vec{F} \,||\, \vec{r}$, so $\vec{r} \times \vec{F} = 0$. Thus, $\vec{\text{l}} = \text{const}$.

$\mathrm{l}_0 = r_0 m v_{\mathrm{T}}^{(0)}$, where $v_{\mathrm{T}}^{(0)}$ is the initial tangential velocity (which, in this case, is all of the velocity since there is no radial component).

$v_{\mathrm{T}}^{(0)} = r_0 \omega_0 \Rightarrow \mathrm{l}_0 = m r_0^2 \omega_0 = m r^2 \omega.$

Thus, $\boxed{\omega = \left(\frac{r_0}{r}\right)^2 \omega_0}$.

3.29

Angular momentum is conserved by assumption.

$L_0 = I_0 \omega_0 = I\omega.$

The mass of the sphere $M = \rho \frac{4\pi}{3} R^3$, where ρ is the mass density. So $I = \frac{2}{5} M R^2 = \frac{2}{5}\rho \frac{4\pi}{3} R^5$.

Conservation of angular momentum means $L_0 = \frac{2}{5}\rho\frac{4\pi}{3} R_0^5 \omega_0 = \frac{2}{5}\rho\frac{4\pi}{3} R^5 \omega \Rightarrow \omega = \left(\frac{R_0}{R}\right)^5 \omega_0$.

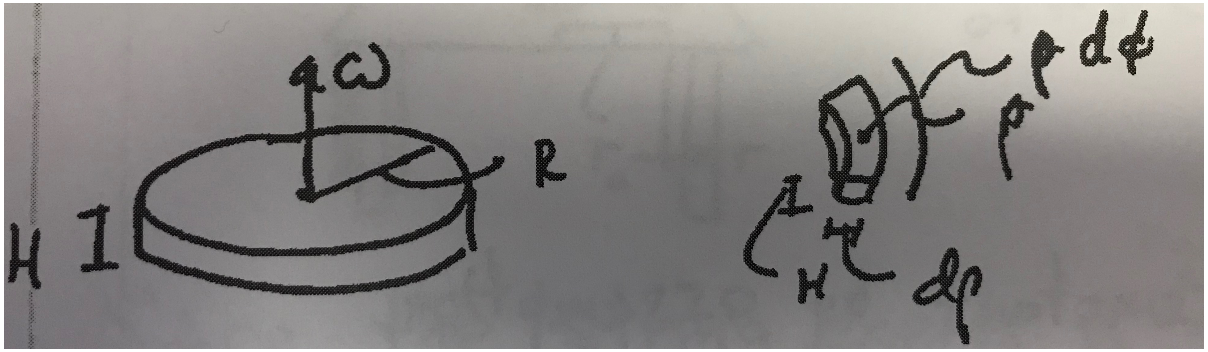
So if $R = 2R_0$, $\omega = \left(\frac{1}{2}\right)^5 \omega_0 = \boxed{\frac{1}{32}\omega_0}$.

3.31

$I = \sum m\, \rho^2 \rightarrow \int dm\, \rho^2.$

In [3]:
```
Image(filename='IMG_9003.jpg', width=600)
```

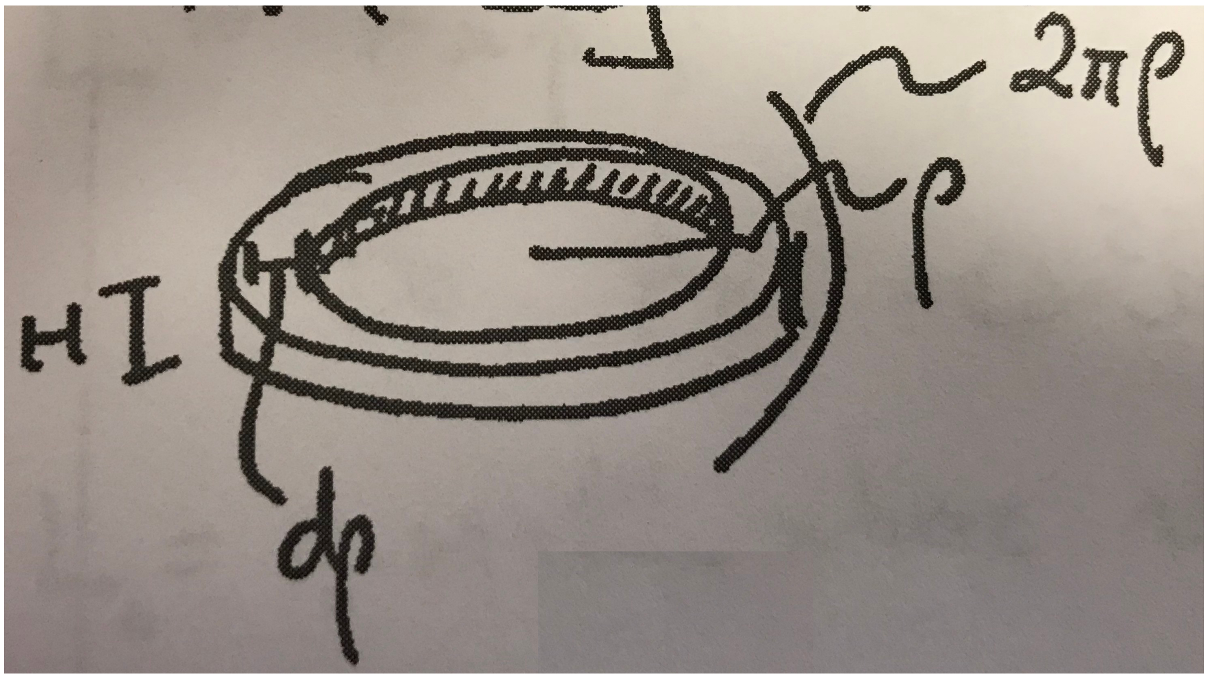
Out[3]:



The radial distance from the rotation axis ρ is the same for all mass elements, no matter their value of ϕ, so we can divide the disk up into infinitesimal washers of height H, circumference $2\pi\rho$, and radial thickness $d\rho$:

In [4]:
```
Image(filename='IMG_9004.jpg', width=600)
```

Out[4]:



The mass of this infinitesimal washer is $dm = \lambda\, 2\pi\, \rho H\, d\rho$, where λ is the mass density.

$$I = \int_{\rho=0}^{R} \lambda\, 2\pi\, \rho H\, d\rho\, \rho^2 = 2\pi\, \lambda\, H \int_{\rho=0}^{R} \rho^3 d\rho = 2\pi\, \lambda\, H\, \frac{1}{4} R^4.$$

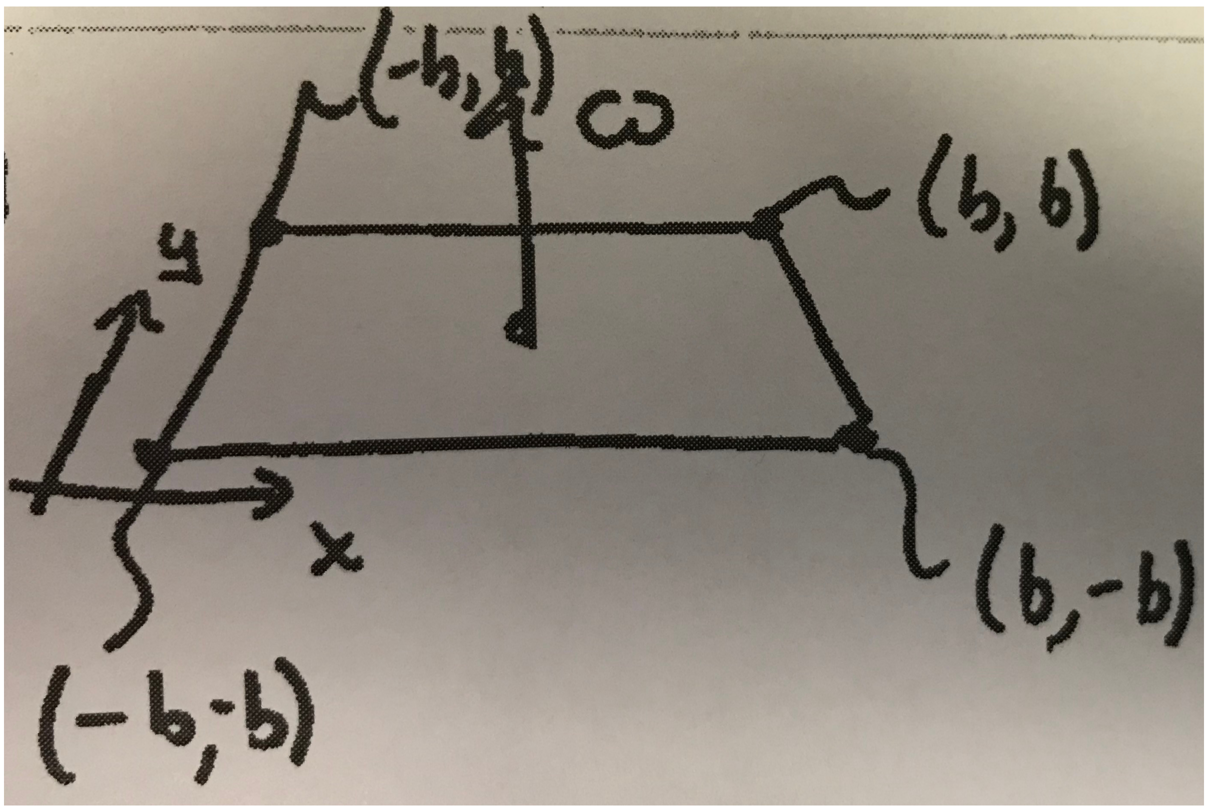
We know that the total mass of the washer $M = \lambda\, \pi R^2 H$, so

$$I = \left(\pi R^2 H \lambda\right)\, \frac{1}{2} R^2 = \boxed{\frac{1}{2} M R^2}.$$

3.33

In [5]:
```
Image(filename='IMG_9005.jpg', width=600)
```

Out[5]:



$I = \int dm\, r^2$, $dm = \rho\, dx\, dy$, where ρ is the mass density - $\rho = \dfrac{M}{4\,b^2}$.

$$I = \int_{x=-b}^{b} \int_{y=-b}^{b} \rho\, dx\, dy \left(x^2 + y^2\right) = \rho \int_{x=-b}^{b} \left[yx^2 + \frac{1}{3}y^3\right]_{y=-b}^{b} dx = \rho \int_{x=-b}^{b} \left(2bx^2 + \frac{2}{3}b^3\right) dx = \rho \left[2b\frac{1}{3}x^3 + \frac{2}{3}b^3 x\right]_{x=\cdot}^{b}$$