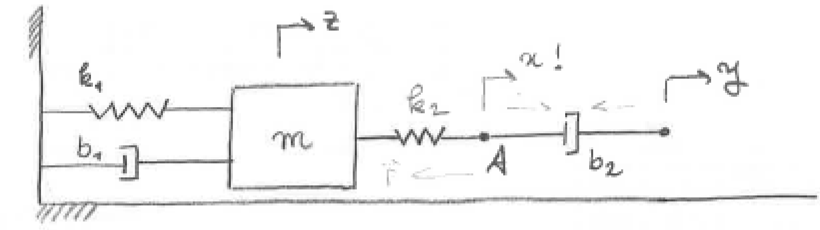
#1) Consider the below given system m=2, $b_1=b_2=16$ and $k_1=k_2=8$ whose transfer function (input y, output z) is presented as follows



$$\frac{Z}{Y} = \frac{4s}{s^3 + 8{,}5 s^2 + 12 s + 2}$$

It is desired that the electro-hydraulic actuator whose equations are presented below is attached to the y-port of the above system. The transfer function of the electro-hydraulic actuator is given by Eq(7.93) (The letter “D” is the Laplace transform variable “s”).

a) Draw the complete closed-loop system block diagram.
b) Select the parameters of the electro-hydraulic actuator such that its transfer function has the natural frequency w_n that is at least two times faster than poles of the above system and it has critical damping, where the disturbance load F_L=1=constant and its coefficient is ($1/k_s=1/10$) in Eq.(7.93) (note that the coeeficient of z_d should be unity). Prepare the Simlink block of the system and find the step response of the closed-loop system. Repeat it for F_L=0 (no disturbance force). Note that due to the feedback loop of the valve shown in Figure 7.8, the step response will go to zero; thus, cut the feedback loop of the valve dynamics in Figure 7.8 and repeat the step response calculation for the modified system.
c) Repeat it for the P-control (modify Eq.(7.92). What is the difference between PD and P-control laws ?

7.4 ELECTRO-HYDRAULIC POSITION SERVO

With the explosion of the use of electronics, electronic controls, and digital computer controls, there has been a corresponding explosion of the combination of electronics with hydraulic controls. The well known strength of electronics is its ability to perform computation and control functions; hydraulics is well known for its ability to operate at high power levels and to provide responsive systems; therefore, the combination is natural.

Shown in Figure 7.8 is an electro-hydraulic position servo system. An electro-hydraulic servovalve is connected to an actuator that drives an inertial load. The position of the actuator is measured with a position sensor, and the signal is fed back to an electronic amplifier that compares the actual position to the desired position and amplifies the error between the two.

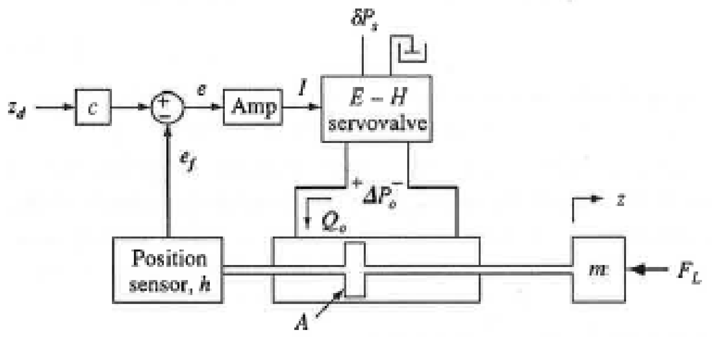


Figure 7.8 Electro-hydraulic position servo control system.

An electro-hydraulic servovalve consists of an electromagnetic torque motor that converts current into a deflection of the input of the valve. Quite often, the valve has two stages: The first stage is a pilot valve that drives the second-stage main spool valve. The pilot valve is usually a flapper-nozzle valve. The reason a two-stage valve is used is to reduce the force required for the electromagnet to move the main valve. The small flapper-nozzle valve requires a very small force to move the flapper. The output pressure from the flapper-nozzle valve is used to position the main spool valve as if it were a ram-type actuator and thereby proportions the output flow from the valve.

The pressure-flow characteristics of the spool valve are almost linear for small-signal operation around the null, since the spool valve is slightly underlapped and can be represented by a linear model using the valve pressure gain G_p to the input current I and output resistance R. The model expresses the loss of pressure as a function of the flow delivered to the actuator:

$$\Delta P_o = G_p I - R Q_o \tag{7.88}$$

If we assume that the compressibility of the hydraulic fluid is small (i.e., the bulk modulus is large when considering the output resistance of the valve), the actuator has equal areas on both sides of the piston, and the flow entering one side of the actuator is instantaneously equal to the flow being pushed by the other side of the piston back into the valve, then the continuity equation for either side of the actuator yields

$$Q_o = \dot{V} = A\dot{z} \tag{7.89}$$

The force balance on the actuator considers the inertia, linear viscous friction, pressure forces, and load disturbance; in this example, there are no springs. The force balance equation is

$$M\ddot{z} + B\dot{z} = A\,\Delta P_o - F_L \tag{7.90}$$

The feedback is from a position sensor, which could be a potentiometer in which the slider is connected to the actuator motion. However, this approach has reliability problems with the sliding contact in the potentiometer. A more popular approach is to use a linear variable differential transformer (LVDT) with an iron core moving inside two transformers that results in a DC voltage E_f which is linear with position after the AC voltages are converted to DC. The gain of the sensor is h (volt/in), and the voltage is

$$E_f = hz \tag{7.91}$$

The amplifier is a summing junction that subtracts the feedback position signal E_f from the input command signal to form an error signal which is then amplified to the current I that drives the valve. In addition to the summation and amplification, a dynamic compensation is added to the circuit to improve the overall dynamics of the system. This dynamic compensation is a proportional-plus-derivative (P-D) controller or filter that causes the error signal to be summed with its derivative and then amplified. The P-D control is accomplished by the amp in Figure 7.8 (See Chapter 4). The mathematics for the circuit can be stated as follows:

$$I = G_a\left(1 + \frac{D}{\omega_d}\right)(cz_d - E_f) \tag{7.92}$$

Starting with the force balance equation and substituting the continuity equation, valve equation, and feedback-and-summing-junction equation yields the following normalized transfer function:

$$z = \frac{\left(\dfrac{D}{\omega_d} + 1\right)\dfrac{c}{h}z_d - \dfrac{1}{k_s}F_L}{\dfrac{M}{k_s}D^2 + \left(\dfrac{B + RA^2}{k_s} + \dfrac{1}{\omega_d}\right)D + 1} \tag{7.93}$$

In this transfer function, the static stiffness k_s is the relation between force and displacement in the steady state with no input (the negative sign comes from the fact that the disturbance force is defined in the direction opposite that of the sign convention):

$$k_s = G_a G_p A h \tag{7.94}$$

The static gain of this system G_s is the variation of the output z with respect to variations in the input u, with no disturbances and in the steady state. It is very interesting that the static gain is a function of the input gain and the feedback gain and is independent of the characteristics of all of the other components:

$$G_s = \frac{c}{h} \tag{7.95}$$

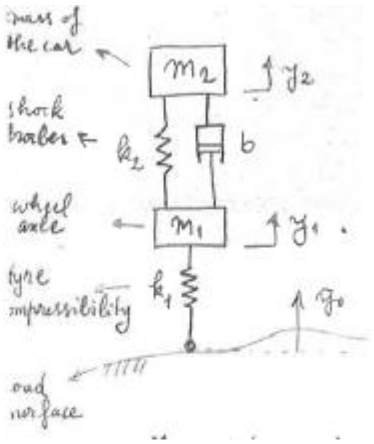
The dynamic characteristics are:

$$\omega_n = \sqrt{\frac{k_s}{M}} \tag{7.96}$$

$$\zeta = \frac{B + RA^2 + k_s/\omega_d}{2\sqrt{k_s M}} \tag{7.97}$$

The proper sizing of the amplifier gain, valve, actuator, and feedback can result in a very responsive, extremely stiff servo system. With this high stiffness in mind, one can visualize the advantage of the use of the P-D controller to enhance the dynamics of the system. Notice that if k_s is very high—say, infinity—then the numerator and denominator terms, $D/\omega_d + 1$, in the command transfer function cancel, so the system has no dynamics to the input command and therefore would be extremely fast. Of course, we wouldn't set the gain to infinity, but we could enhance the system dynamics; this is always a goal of good servo design.

#2) Consider the quarter car model having the following representation where $u = y_0$.



$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1+k_2}{m_1} & -\frac{b}{m_1} & \frac{k_2}{m_1} & \frac{b}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & \frac{b}{m_2} & -\frac{k_2}{m_2} & -\frac{b}{m_2} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{k_1}{m_1} \\ 0 \\ 0 \end{pmatrix} \cdot u(t)$$

$\rightsquigarrow$ state eq.

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

$$x_1 = y_1, \quad x_2 = \dot{y}_1, \quad x_3 = y_2, \quad x_4 = \dot{y}_2$$

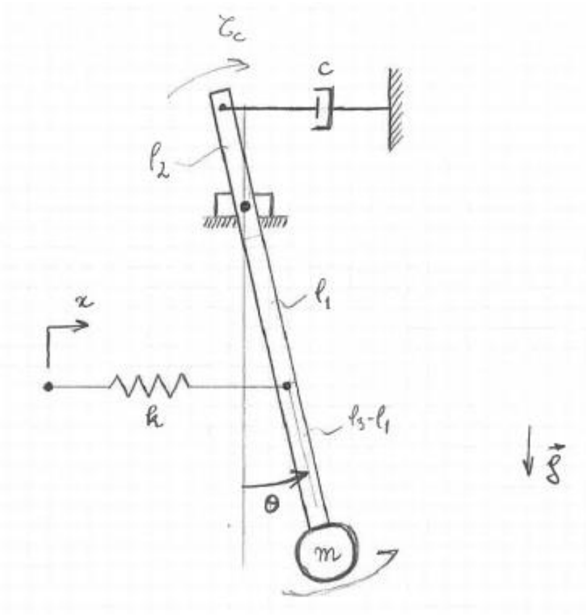
It is desired that an electro-hydraulic actuator whose equations are presented below is attached between the masses m_1 and m

$$\frac{Piston\ force}{Voltage\ input\ to\ the\ valve} = \frac{F}{V_e} = \frac{4s}{1 + 0.008s + 0.0004s^2}$$

a) Modify the governing equations
b) Calculate the transfer function such that V_e is the input and y_2 is the output.
c) Draw the complete system block diagram in Simulink
d) Calculate the system response to unit V_e input with and without the impulsive road disturbance.

Note that the piston force will be applied to both of the upper and lower masses.

#3) Consider the below given system having the following representation where u=x.



$$x_1 = \theta, \quad x_2 = \dot{\theta}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{k l_1^2}{m l_3^2} & -\frac{c l_2^2}{m l_3^2} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{k . l_1}{m l_3^2} \end{pmatrix} \cdot u$$

Suppose that the input displacement is generated by a hydroulic piston having the following transfer function

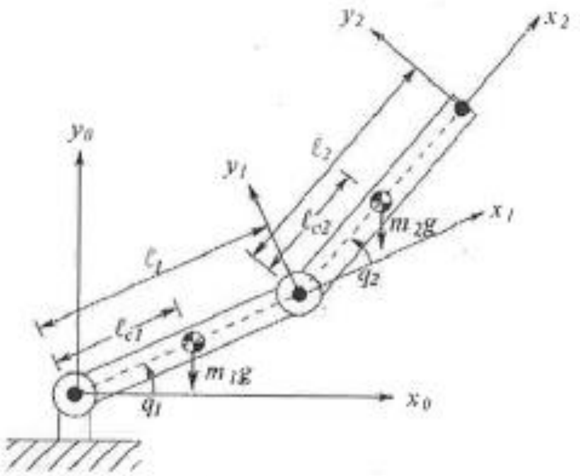
$$\frac{Piston\ displacement}{Voltage\ input\ to\ the\ valve} = \frac{y}{V_e} = \frac{2}{s(1 + 0.008s + 0.0004s^2)}$$

a) Consider that m=10, c=3, k=5, $l_1 = l_2/2 = l_3/2 = 1$. Calculate the transfer function such that V_e is the input and θ is the output.

b) Draw the Simulink block diagram of the complete system

c) Calculate the system response to unit V_e input

#4) Consider the state space respresentation of the cartesian elbow manipulator given below.



Planar Elbow Manipulator

$$\begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix} \cdot \begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{pmatrix} + \begin{pmatrix} c.\dot{q}_2 & c.\dot{q}_2 + c.\dot{q}_1 \\ -c.\dot{q}_1 & 0 \end{pmatrix} \cdot \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} + \begin{pmatrix} g_1 \\ g_2 \end{pmatrix} = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

$$d_{11} = m_1.l_{c_1}{}^2 + m_2.(l_1{}^2 + l_{c_2}{}^2 + 2.l_1.l_{c_2}.c2) + I_1 + I_2$$

$$d_{12} = d_{21} = m_2.(l_{c_2}{}^2 + l_1.l_{c_2}.c2) + I_2 \quad , \quad d_{22} = m_2.l_{c_2}{}^2 + I_2$$

$$c = m_2.l_{c_2}{}^2 + I_2$$

$$g_1 = (m_1.l_{c_1} + m_2.l_1).g.c1 + m_2.l_{c_2}.g.c12 \quad , \quad g_2 = m_2.l_{c_2}.g.c12$$

a) Consider that the joints are attached to armature controlled DC motors having the following transfer functions

$$G(s) = \frac{\Theta(s)}{U(s)} = \frac{k_t}{s(JR_a s + k_t k_b + fR_a)} = \frac{k_m}{s(\tau_m s + 1)}$$

Where $R_a = 20\ \Omega \qquad k_t = 1\ \text{N·m/A} \qquad k_b = 3\ \text{V·s/rad}$

If the motor shaft has the moment of inertia of 2 [N m s^2/rad] and with viscous friction coefficient of 0.1 N m s /rad, find the DC motor transfer function

b) Construct the full nonlinear Simulink model of the manipulator and DC motors, apply a step voltage input to the DC motors and plot the joint angles. Assume unit values for all constants, plot the q_1 and q_2 angles for 5 seconds for the nonzero initial conditions $(q_1)_0 = (q_2)_0 = 30^\circ$.

Note that for the motor torque, use the following equations

$$R_a i_a(t) + L_a \frac{di_a(t)}{dt} + v_b(t) = v_a(t) = u(t)$$

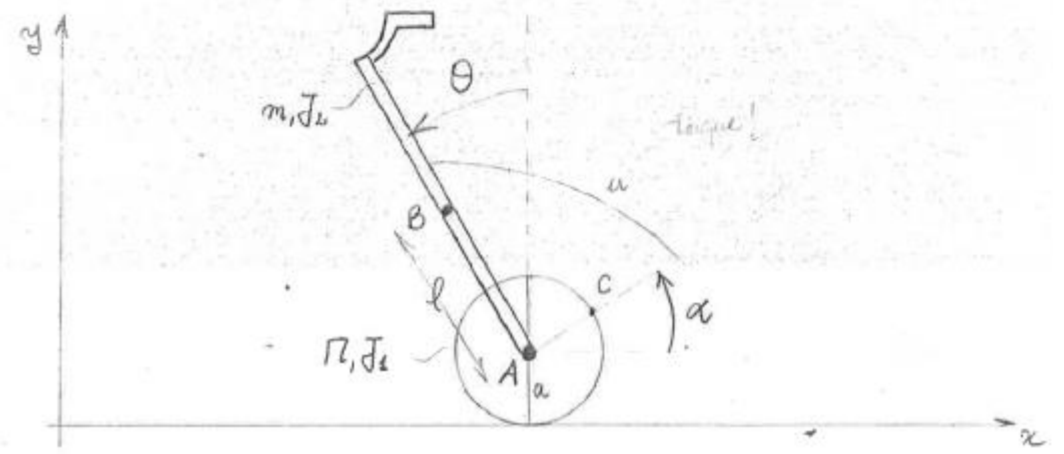
$$R_a i_a(t) + L_a \frac{di_a(t)}{dt} + v_b(t) = v_a(t) = u(t)$$

$$v_b(t) = k_b \frac{d\theta(t)}{dt}$$

$$R_a I_a(s) + L_a s I_a(s) + k_b s \Theta(s) = U(s) \qquad T(t) = k_t i_a(t)$$

#5) Consider the state space respresentation of the segway given below.

SEGWAY



$$x_1 = \alpha\,,\quad x_2 = \theta\,,\quad x_3 = \dot{\alpha}\,,\quad x_4 = \dot{\theta}$$

$$\dot{x}_3 = \frac{k_2 k_3\, s x_2}{k_1 k_2 - k_3^2 c^2 x_2}\cdot x_4^2 - \frac{k_3 k_4\, s x_2\, c x_2}{k_1 k_2 - k_3^2 c^2 x_2} + \frac{k_2 + k_3\, c x_2}{k_1 k_2 - k_3^2 c^2 x_2}\cdot u$$

$$\dot{x}_4 = -\frac{k_3^2\, s x_2\, c x_2}{k_1 k_2 - k_3^2 c^2 x_2}\cdot x_4^2 + \frac{k_1 k_4\, s x_2}{k_1 k_2 - k_3^2 c^2 x_2} - \frac{k_1 + k_3\, c x_2}{k_1 k_2 - k_3^2 c^2 x_2}\cdot u$$

where

$$J_1 + (M + m)a^2 = k_1\,,\quad J_2 + m l^2 = k_2\,,\quad m a l = k_3\,,\quad m g l = k_4$$

Construct a full nonlinear Simulink model for the above system where the the same DC motor given in Problem#4 creates the torque u. Apply a unit voltage to the DC motor, assume unit values for all constants, plot the θ and α angles for 5 seconds for the nonzero initial conditions $(\theta)_0 = (\alpha)_0 = 10^{o}$ without any control torque u. Note that for the motor torque, use the following equations

$$R_a i_a(t) + L_a \frac{di_a(t)}{dt} + v_b(t) = v_a(t) = u(t)$$

$$R_a i_a(t) + L_a \frac{di_a(t)}{dt} + v_b(t) = v_a(t) = u(t)$$

$$v_b(t) = k_b \frac{d\theta(t)}{dt}$$

$$R_a I_a(s) + L_a s I_a(s) + k_b s\Theta(s) = U(s) \qquad\qquad T(t) = k_t i_a(t)$$