Modeling and Control of Mechatronics Systems
Spring 2018-2019
HW II Solution

Deniz Ekin Canbay

Contents

1 Problem # 1

- **Section -a**

From the force balance equation of the mechanical components

$$M\ddot{z} + B\dot{z} = A\Delta P_o - F_L$$
$$\Delta P_o = G_p I - RQ_o \qquad Q_o = A\dot{z}$$
$$\Delta P_o = G_p I - RA\dot{z}$$
$$M\ddot{z} + B\dot{z} = A[G_p I - RA\dot{z}] - F_L$$

$$[Ms^2 + (B + RA^2)s]Z(s) = AG_p I(s) - F_L \quad I(s) = G_a[1 + s/\omega_d](cz_d - E_f)$$
$$[Ms^2 + (B + RA^2)s]Z(s) = AG_p G_a[1 + s/\omega_d](cz_d - hZ(s)) - F_L$$

$$[Ms^2 + (B + RA^2 + AG_p G_a h/\omega_d)s + hAG_p G_a]Z(s) = AG_p G_a[1 + s/\omega_d]cz_d - F_L$$
$$k_s = G_a G_p A h \implies [Ms^2 + (B + RA^2 + k_s/\omega_d)s + k_s]Z(s) = k_s/h[1 + s/\omega_d]cz_d - F_L$$

$$Z(s) = \frac{(\frac{s}{\omega_d} + 1)\frac{cz_d}{h} - \frac{F_L}{k_s}}{\frac{M}{k_s}s^2 + (\frac{B+RA^2}{k_s} + \frac{1}{\omega_d})s + 1}$$

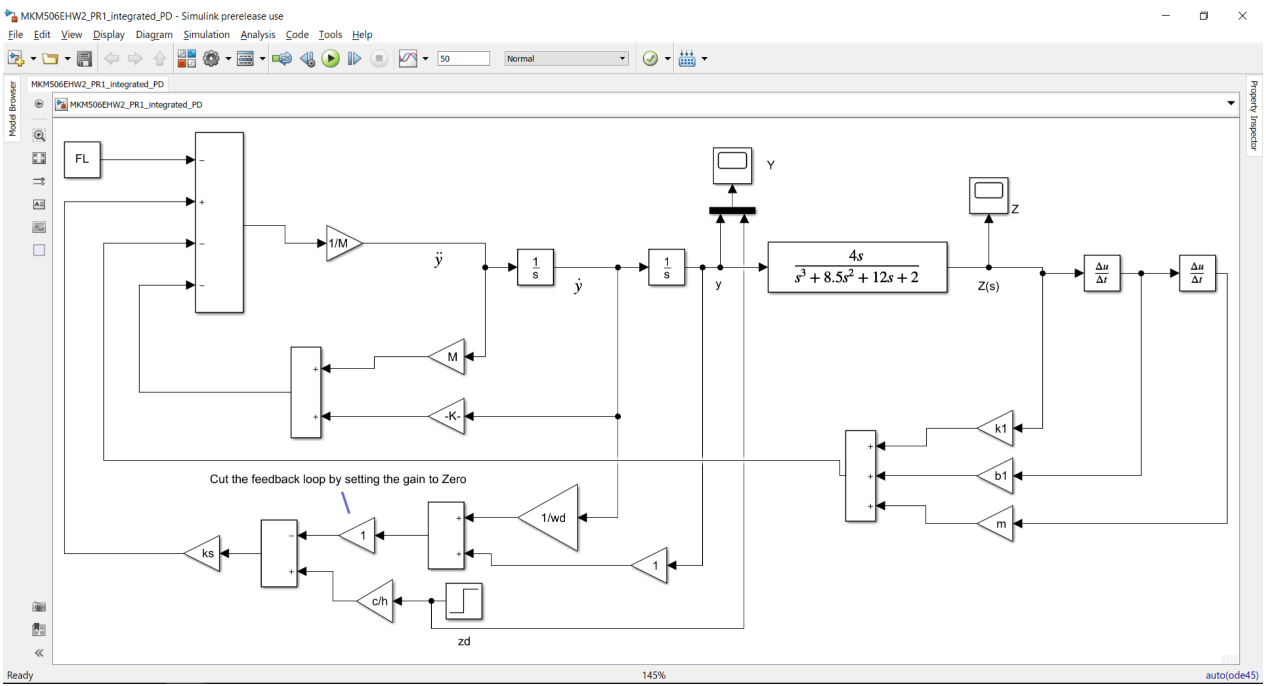

Figure 1: Simulink Model of the System (PD Control)

- **Section -b**

$$\omega_n$$

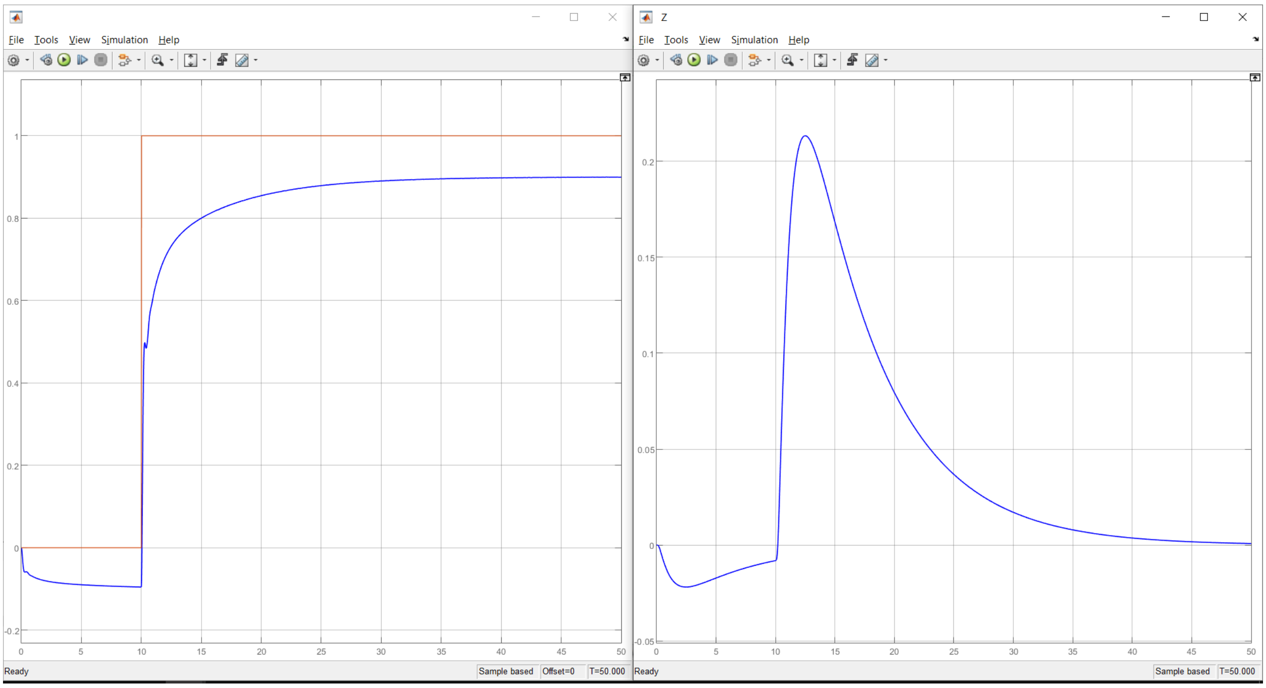

Figure 2: System Response $F_L = 1$ PD Control

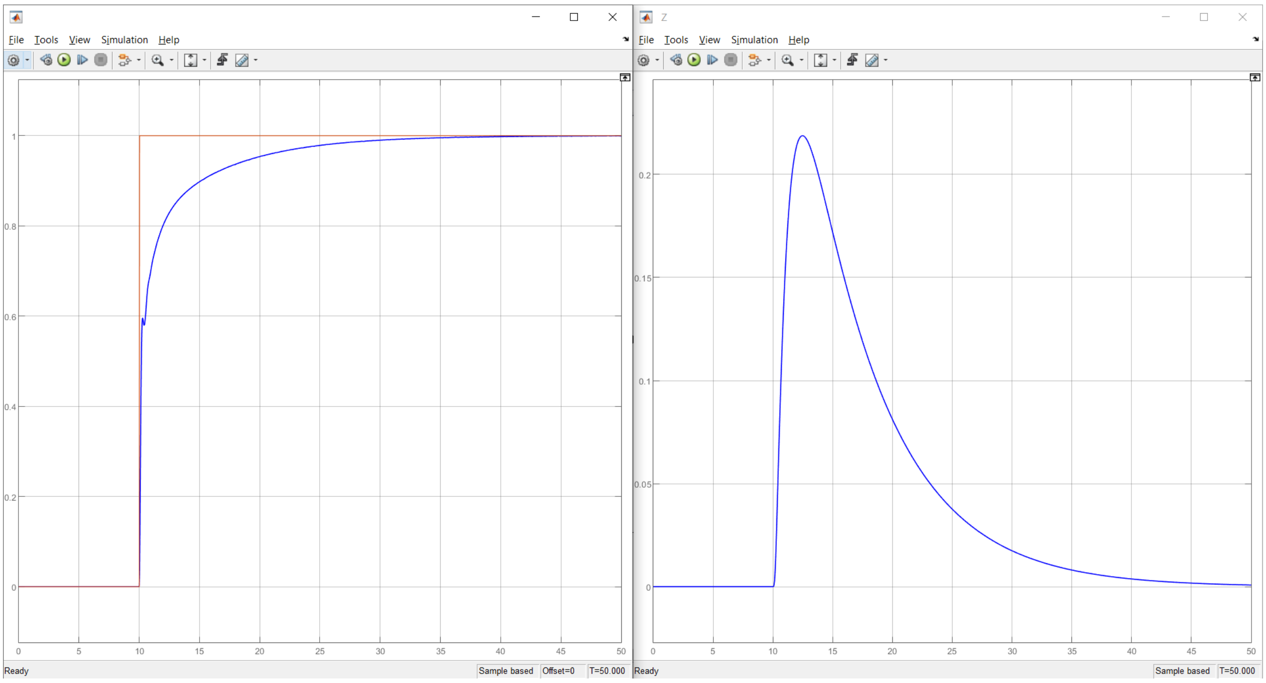

Figure 3: System Response $F_L = 0$ PD Control

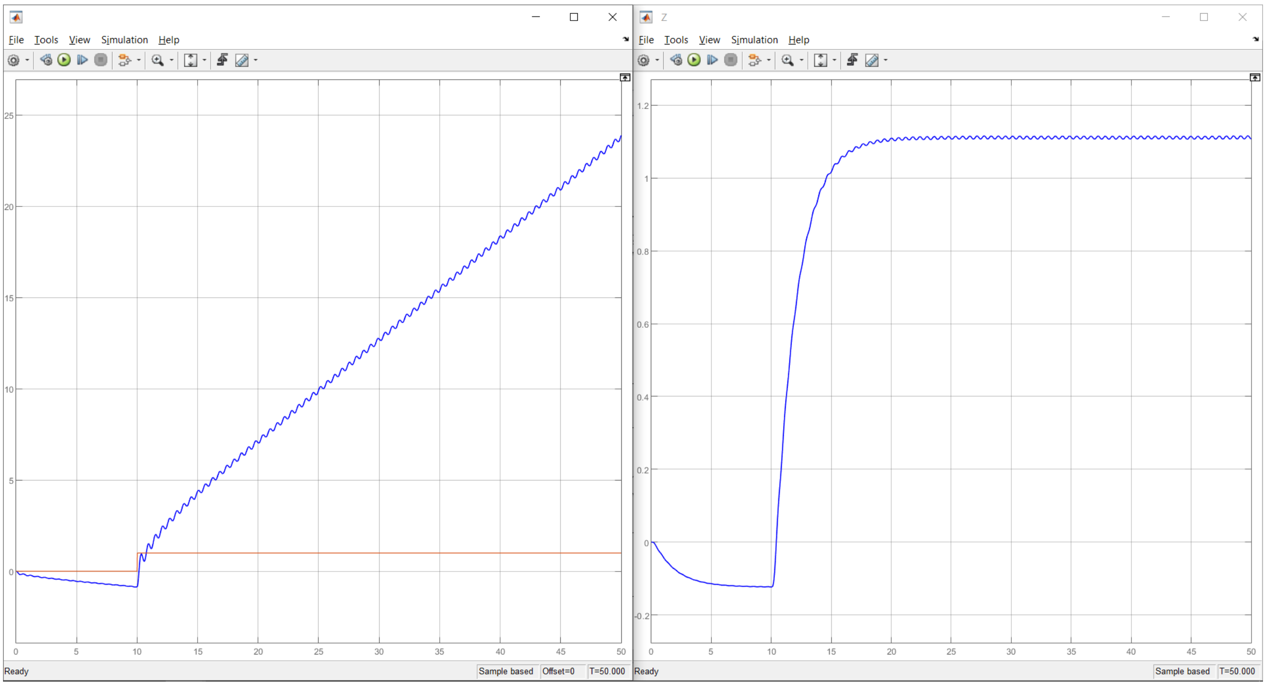

Figure 4: Open Loop System Response $F_L = 1$

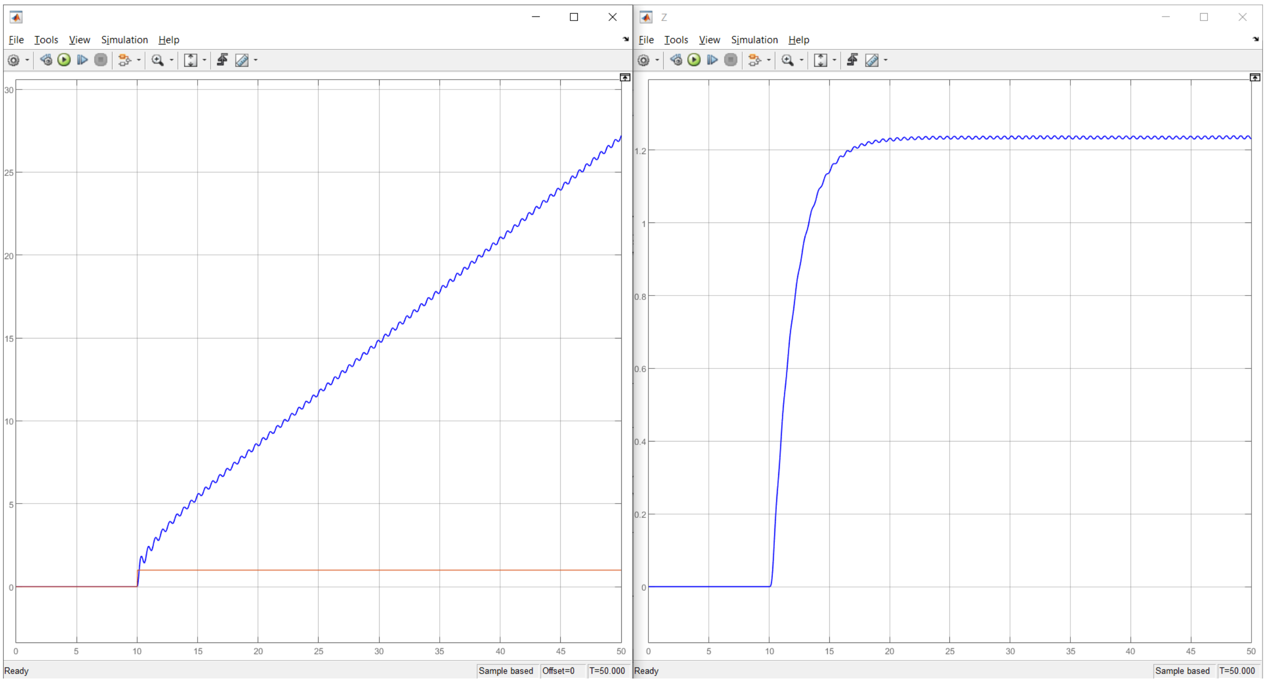

Figure 5: Open Loop System Response $F_L = 0$

- **Section -c**

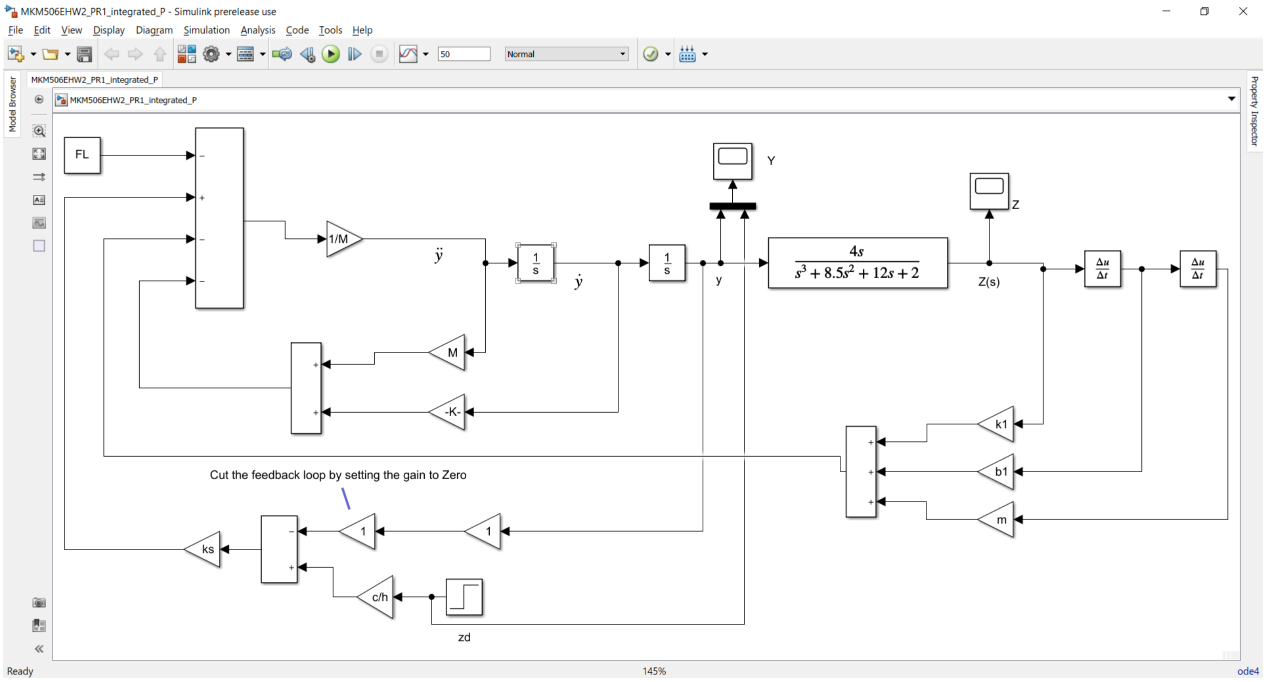

Figure 6: Simulink Model of the System (P Control)

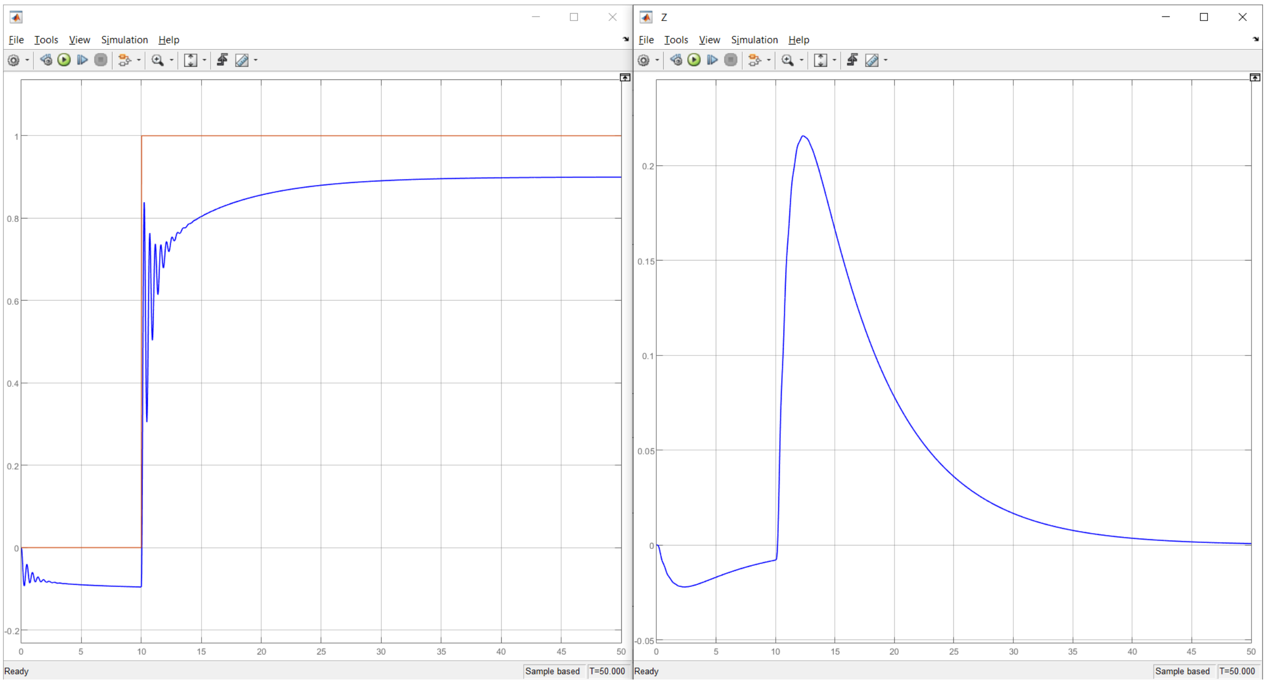

Figure 7: P Control System Response $F_L = 1$

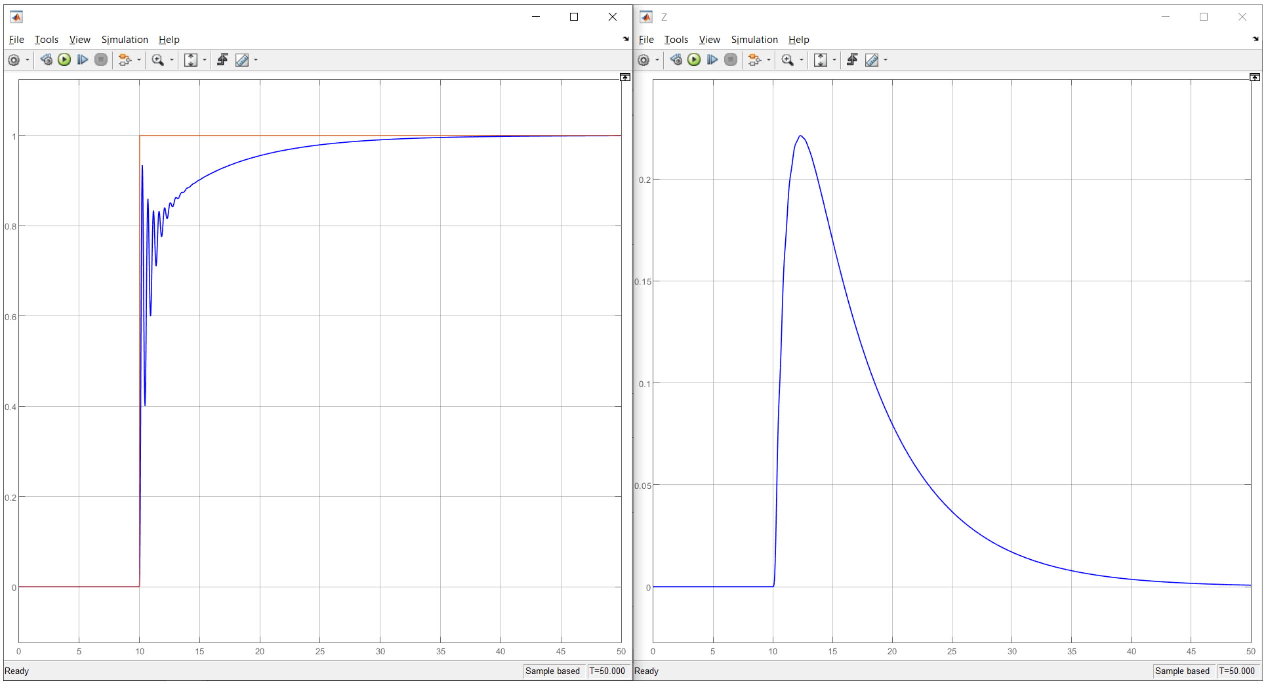

Figure 8: P Control System Response $F_L = 0$

2 Problem # 2

- **Section -a**

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q_1}}\right) - \frac{\partial \mathcal{L}}{\partial q_1} + \frac{\partial \mathcal{D}}{\partial \dot{q_1}} = m_1\ddot{y_1} + (k_1 + k_2)y_1 - k_2y_2 - k_1y_0 - b(\dot{y_2} - \dot{y_1}) = -f(t)$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q_2}}\right) - \frac{\partial \mathcal{L}}{\partial q_2} + \frac{\partial \mathcal{D}}{\partial \dot{q_2}} = m_2\ddot{y_2} + k_2(y_2 - y_1) + b(\dot{y_2} - \dot{y_1}) = f(t)$$

$$x_1 = y_1 \quad x_2 = \dot{y_1} \quad x_3 = y_2 \quad x_4 = \dot{y_2}$$

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{bmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1+k_2}{m_1} & -\frac{b}{m_1} & \frac{k_2}{m_1} & \frac{b}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & \frac{b}{m_2} & -\frac{k_2}{m_2} & -\frac{b}{m_2} \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{k_1}{m_1} & \frac{-1}{m_1} \\ 0 & 0 \\ 0 & \frac{1}{m_2} \end{bmatrix} \begin{bmatrix} u(t) \\ f(f) \end{bmatrix}$$

- **Section -b**

$$(s^2m_1 + bs + k_1 + k_2)Y_1(s) - (bs + k_2)Y_2(s) = k_1U(s) - F(s)$$

$$(s^2m_2 + bs + k_2)Y_2(s) - (bs + k_2)Y_1(s) = F(s) \implies$$

$$m_1 = 100 \quad m_2 = 500 \quad b = 20 \quad k_1 = 25 \quad k_2 = 12$$

$$\frac{Y_1(s)}{F(s)}|_{U=0} = \frac{-5s^2}{500s^4 + 120s^3 + 197s^2 + 5s + 3}$$

$$\frac{Y_2(s)}{F(s)}|_{U=0} = \frac{s^2 + 0.25}{500s^4 + 120s^3 + 197s^2 + 5s + 3}$$

$$\frac{Y_1(s)}{U(s)}|_{F=0} = \frac{125s^2 + 5s + 3}{500s^4 + 120s^3 + 197s^2 + 5s + 3}$$

$$\frac{Y_2(s)}{U(s)}|_{F=0} = \frac{5s + 3}{500s^4 + 120s^3 + 197s^2 + 5s + 3}$$

$$\frac{F(s)}{V_e(s)} = \frac{4s}{0.0004s^2 + 0.008s + 1} \implies$$

$$\frac{Y_2(s)}{V_e(s)} = \frac{Y_2(s)}{F(s)}\frac{F(s)}{V_e(s)} = \frac{s^2 + 0.25}{500s^4 + 120s^3 + 197s^2 + 5s + 3} \quad \frac{4s}{0.0004s^2 + 0.008s + 1}$$

$$\frac{Y_2(s)}{V_e(s)} = \frac{2500s(4s^2 + 1)}{(s^2 + 20s + 2500)(500s^4 + 120s^3 + 197s^2 + 5s + 3)}$$

- **Section -c**

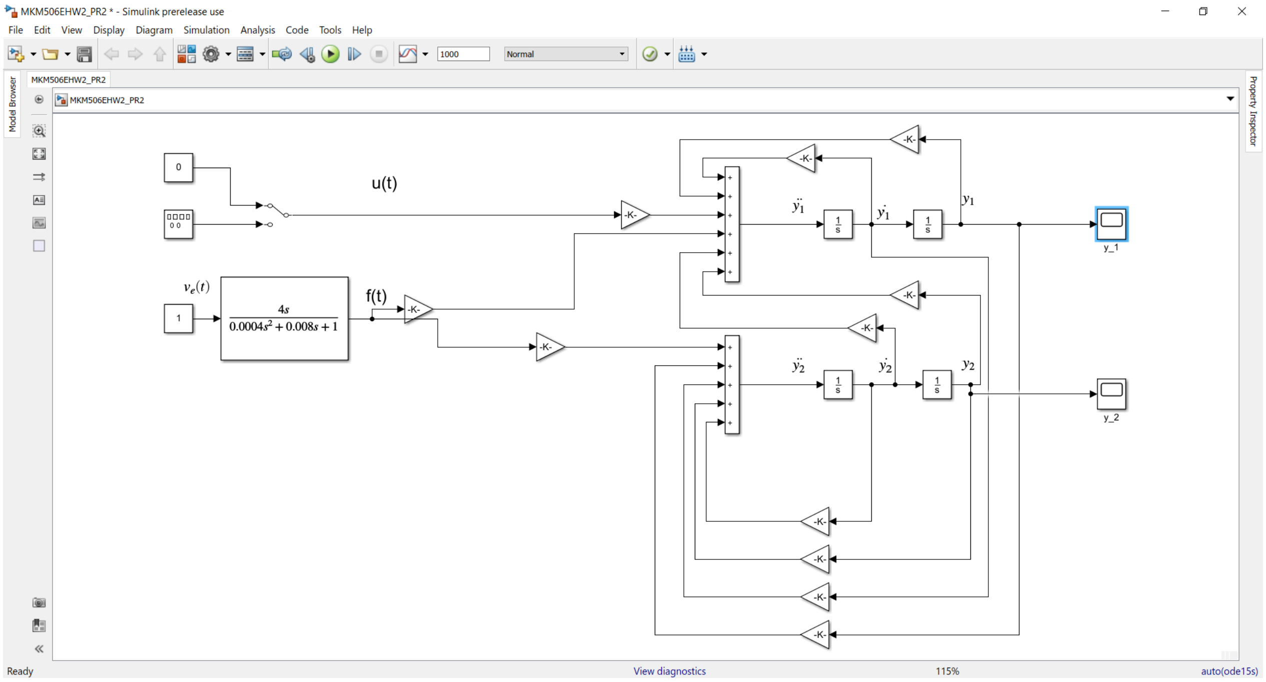

Figure 9: Simulink Model of the System

- **Section -d**

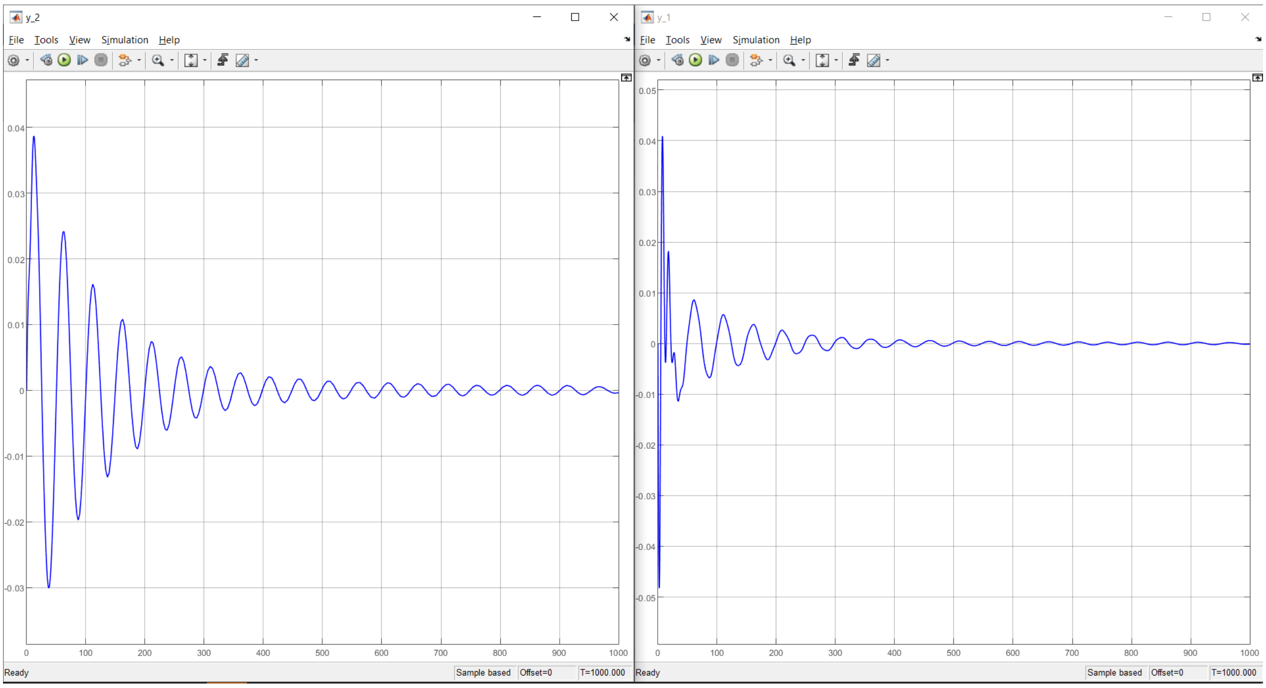

Figure 10: System Responses without Impulsive Road Disturbance: left-y_2 right-y_1

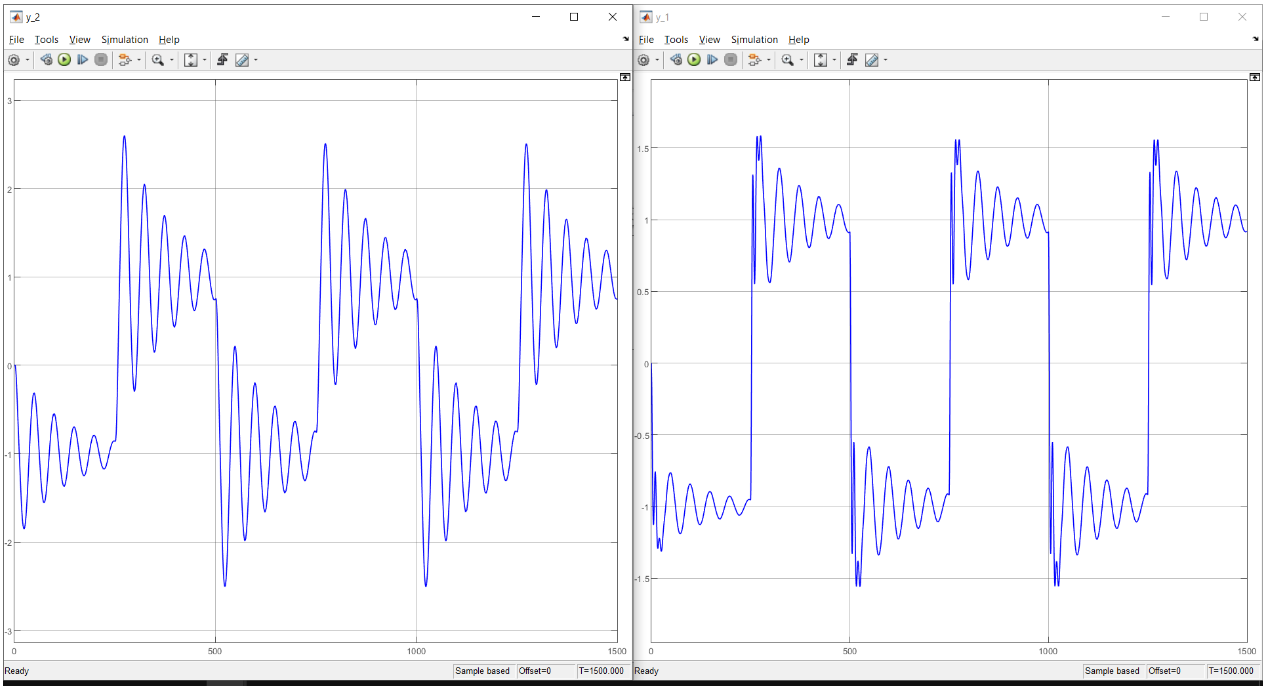

Figure 11: System Responses with Impulsive Road Disturbance: left-y_2 right-y_1

3 Problem # 3

- **Section -a**

From the 1^{st} HW we know that the equation of motion is

$$ml_3^2\ddot{\theta} + kl_1^2\theta + cl_2^2\dot{\theta} = kl_1 x$$

Taking the Laplace Transform

$$[s^2ml_3^2 + scl_2^2 + kl_1^2]\theta(s) = kl_1 X(s)$$

$$\frac{X(s)}{V_e(s)} = \frac{2}{0.0004s^2 + 0.008s + 1}$$

$$\frac{\theta(s)}{V_e(s)} = \frac{\theta(s)}{X(s)}\frac{X(s)}{V_e(s)} = \frac{kl_1}{s^2ml_3^2 + scl_2^2 + kl_1^2}\,\frac{2}{0.0004s^2 + 0.008s + 1}$$

$$\frac{\theta(s)}{V_e(s)} = \frac{5}{40s^2 + 12s + 5}\,\frac{2}{0.0004s^2 + 0.008s + 1} = \frac{10}{0.016s^4 + 0.3248s^3 + 40.1s^2 + 12.04s + 5}$$

- **Section -b**

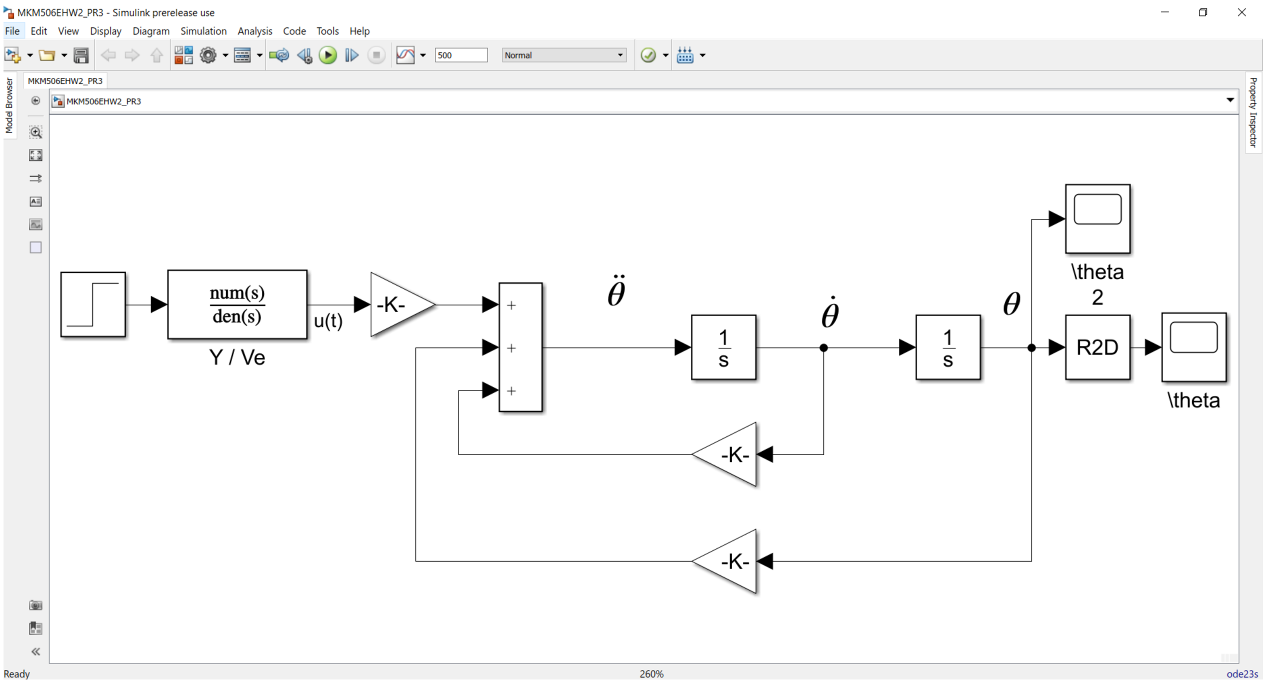

Figure 12: Simulink Model of the System

- **Section -c**

  Unit Ve input causes system to rotate to infinity as there is a free integrator in the transfer function from Valve voltage input to piston displacement. Removing this by multiplying the transfer function with ”s” gives the following response.

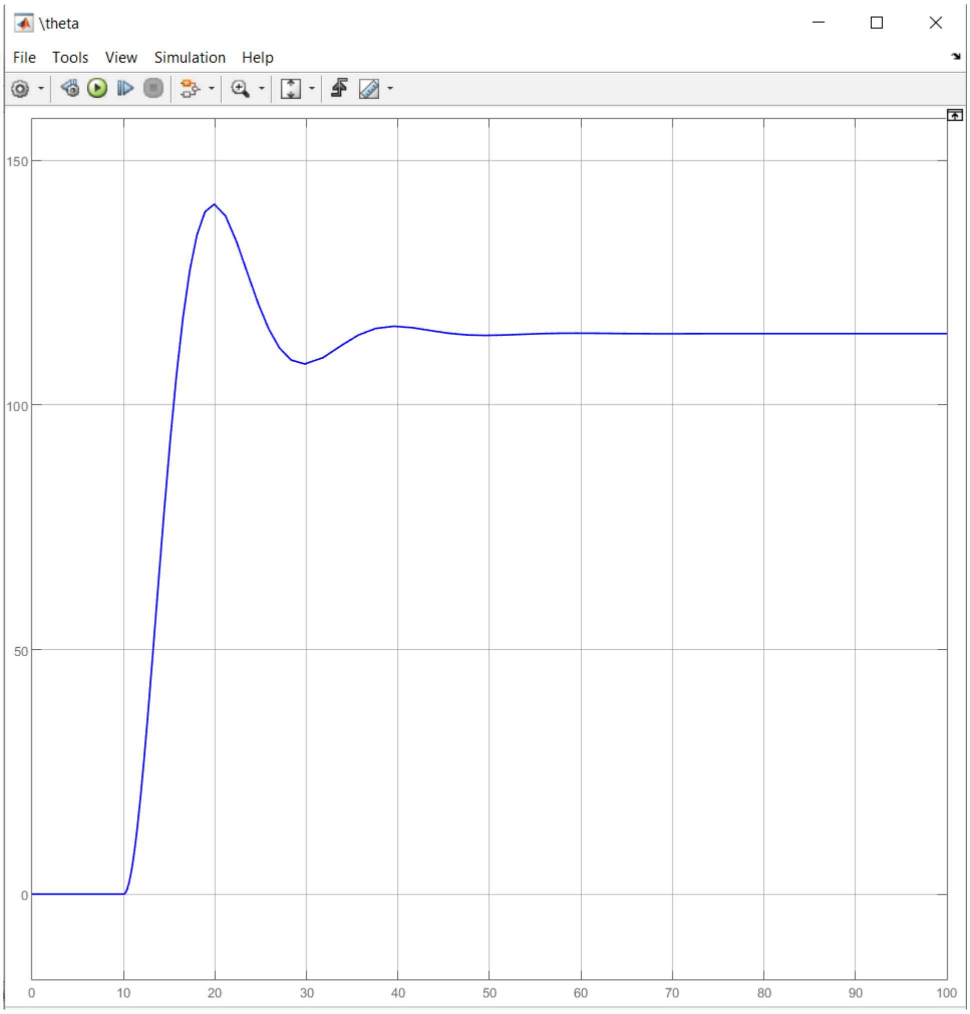

Figure 13: Unit Step Response of the System(in degrees)

4 Problem # 4

- **Section -a**

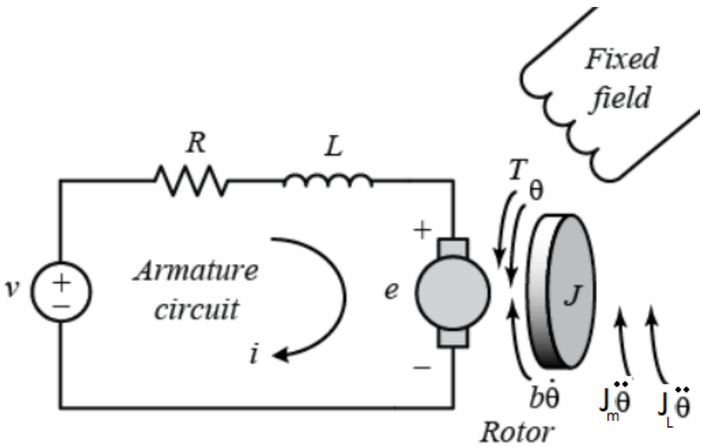

Figure 14: Armature Controlled DC Motor Model

$$R_a i_a(t) + L_a \frac{di_a(t)}{dt} + k_b\dot{\theta}(t) = u(t), \;\; L_a \approx 0 \implies R_a i_a(t) + k_b\dot{\theta}(t) = u(t)$$

$$T_m(t) = k_t i_a(t) \implies i_a(t) = \frac{T_m(t)}{k_t} \implies \frac{R_a}{k_t}T_m(t) + k_b\dot{\theta}(t) = u(t)$$

$$T_m(t) = \frac{k_t}{R_a}u(t) - \frac{k_b k_t}{R_a}\dot{\theta}(t)$$

By taking the Laplace transform

$$T_m(s) = \frac{k_t}{R_a}U(s) - \frac{k_b k_t}{R_a}s\theta(s)$$

- **Section -b**

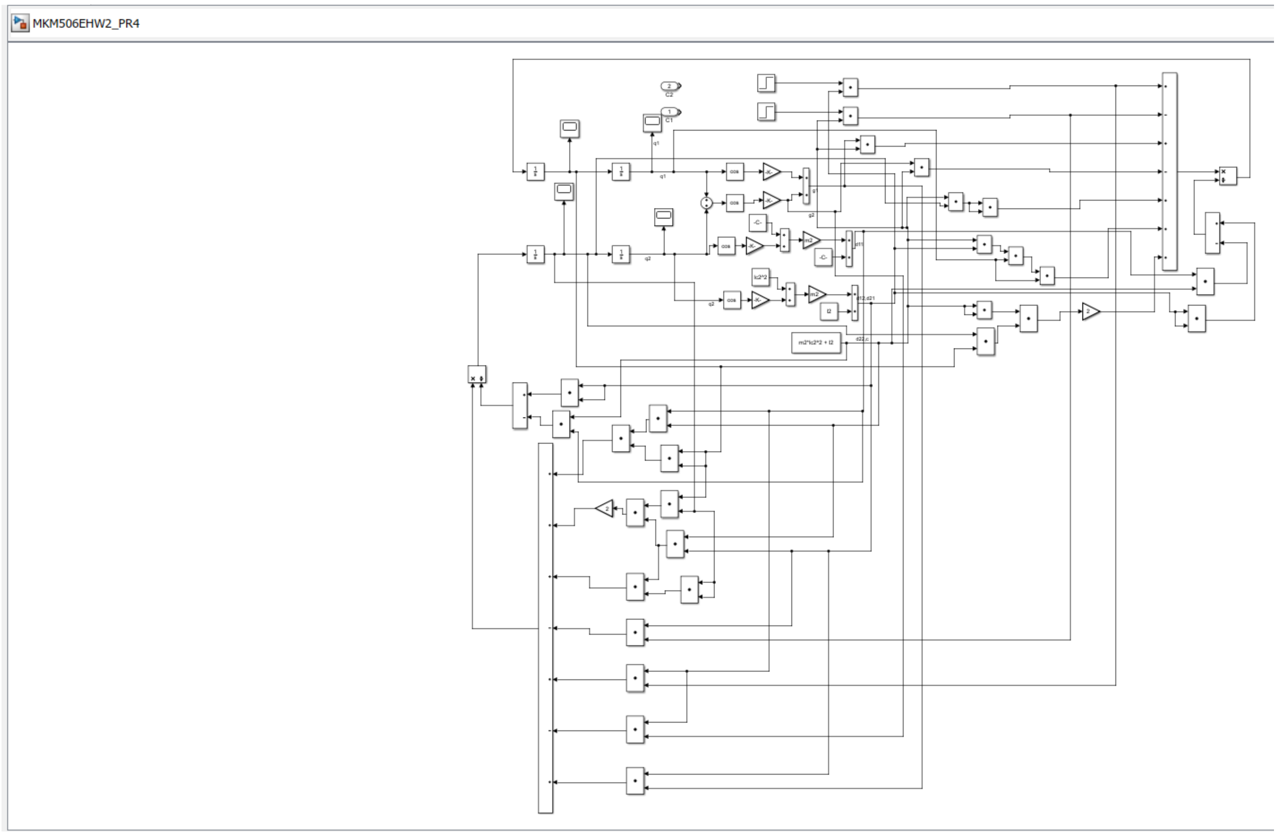

Figure 15: Full Nonlinear Simulink Model of the 2-R Robot arm and DC motors

5 Problem # 5

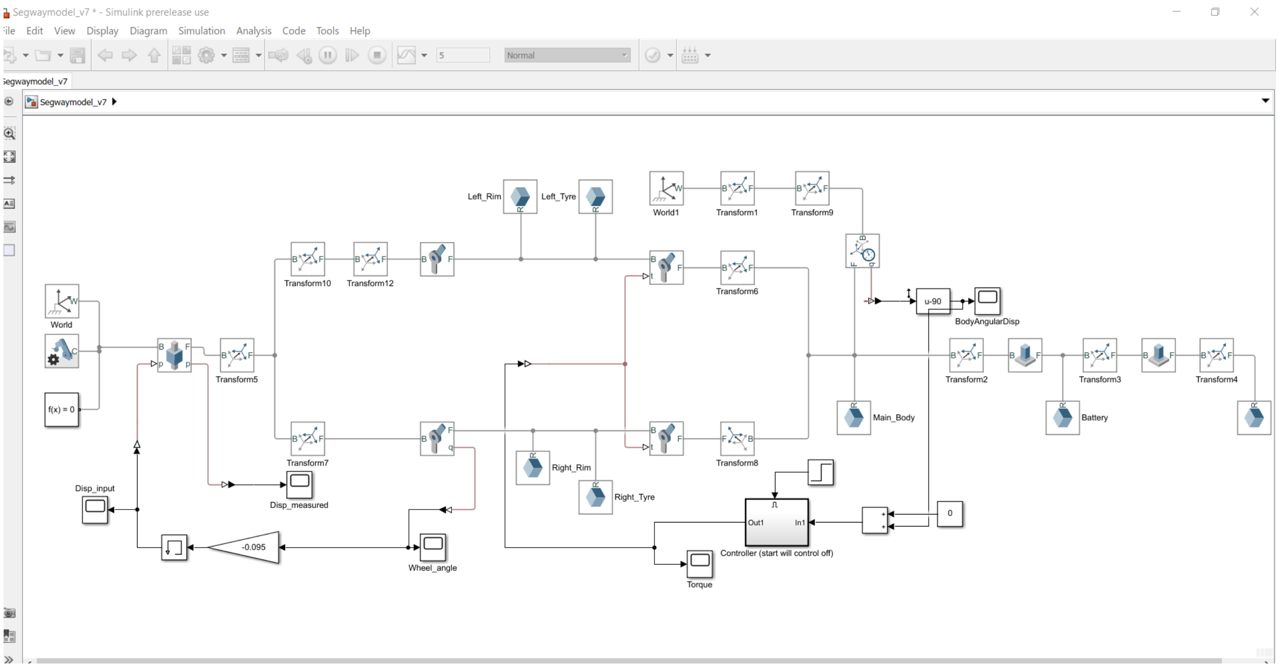

Figure 16: Full Nonlinear Simulink Model of the Segway and DC motors