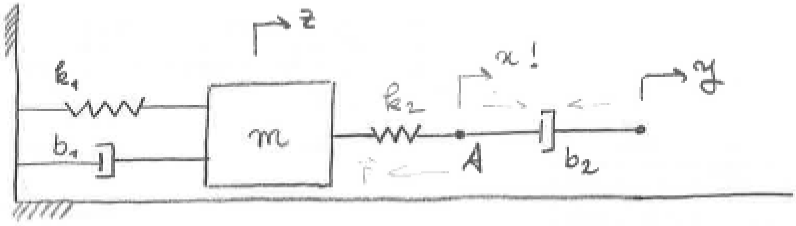
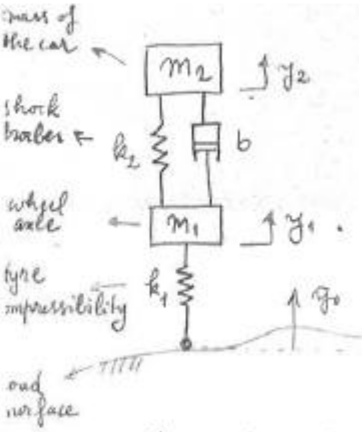
#1) a) Find the transfer function for the below given system (input z, output y).



b) Plot the step response of the system for $m=2$, $b_1=b_2=16$, $k_1=k_2=8$.

c) Find the poles and zeros of the system. Is the system stable and minimum phase ?

#2) a) Show that the quarter car model has the following representation where $u=y_0$.



$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1+k_2}{m_1} & -\frac{b}{m_1} & \frac{k_2}{m_1} & \frac{b}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & \frac{b}{m_2} & -\frac{k_2}{m_2} & -\frac{b}{m_2} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{k_1}{m_1} \\ 0 \\ 0 \end{pmatrix} \cdot u(t)$$

state eq.

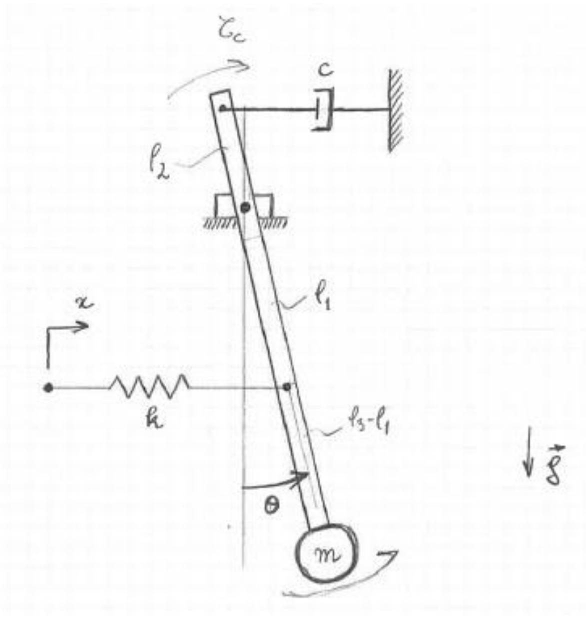
$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

$$x_1 = y_1, \quad x_2 = \dot{y}_1, \quad x_3 = y_2, \quad x_4 = \dot{y}_2$$

b) Plot the impulse and step responses of the system for $m_1=100$, $m_2=500$, $b=20$, $k_1=25$, $k_2=12$.

c) Find the transfer functions, poles and zeros of the system. Is the system stable and minimum phase ?

#3) a) Show that the below given system has the following representation where $u=x$.



$$x_1 = \theta, \quad x_2 = \dot{\theta}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{k l_1^2}{m l_3^2} & -\frac{c l_2^2}{m l_3^2} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{k . l_1}{m l_3^2} \end{pmatrix} \cdot u$$

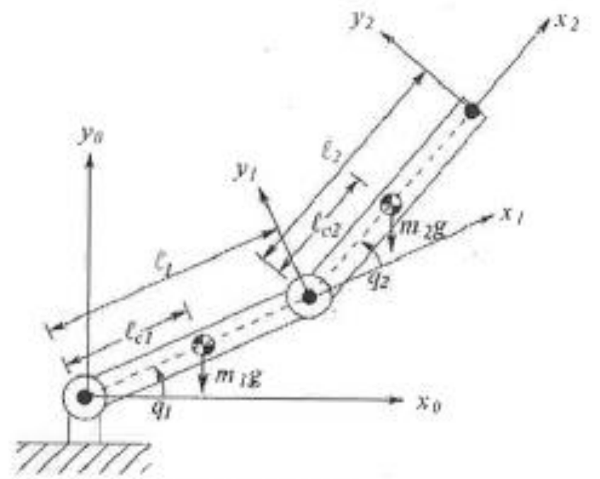
b) Plot the impulse response of the system for m=10, c=3, k=5,$l_1=l_2/2=l_3/2=1$.

c) Find the transfer functions, poles and zeros of the system.

#4) a) Show that the state space respresentation of the cartesian elbow manipulator is as given below.

b) Assuming unit values for all constants, plot the q_1 and q_2 angles for 5 seconds for the nonzero initial conditions $(q_1)_0= (q_1)_0=30^\circ$.

c) Find the transfer functions, poles and zeros of the system.



$$\begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix} \cdot \begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{pmatrix} + \begin{pmatrix} c.\dot{q}_2 & c.\dot{q}_2 + c.\dot{q}_1 \\ -c.\dot{q}_1 & 0 \end{pmatrix} \cdot \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} + \begin{pmatrix} g_1 \\ g_2 \end{pmatrix} = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

$$d_{11} = m_1.l_{c_1}{}^2 + m_2.(l_1{}^2 + l_{c_2}{}^2 + 2.l_1.l_{c_2}.c2) + I_1 + I_2$$

$$d_{12} = d_{21} = m_2.(l_{c_2}{}^2 + l_1.l_{c_2}.c2) + I_2 \quad , \quad d_{22} = m_2.l_{c_2}{}^2 + I_2$$

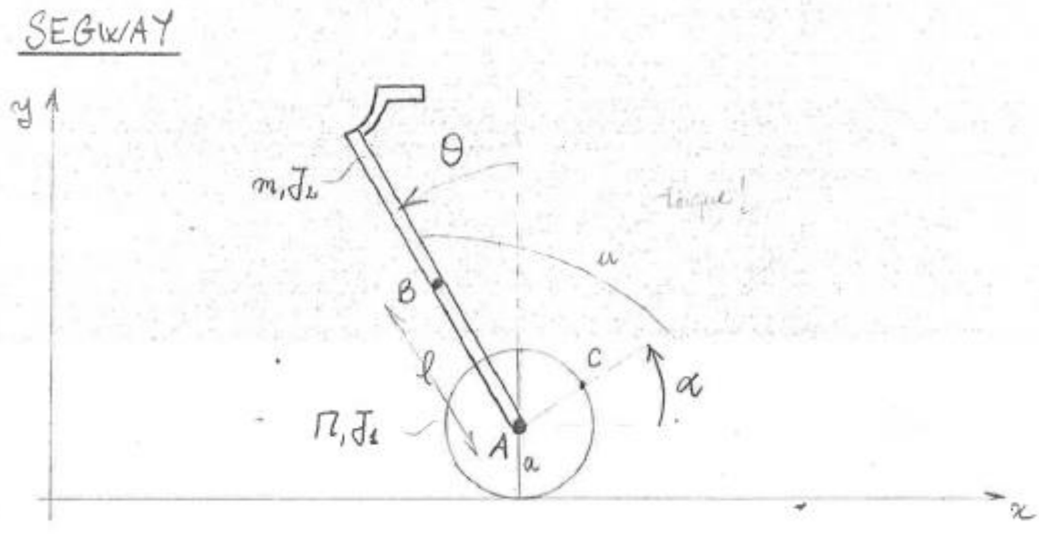
$$c = m_2.l_{c_2}{}^2 + I_2$$

$$g_1 = (m_1.l_{c_1} + m_2.l_1).g.c1 + m_2.l_{c_2}.g.c12 \quad , \quad g_2 = m_2.l_{c_2}.g.c12$$

#5) a) Show that the state space respresentation of the segway is as given below.

b) Assuming unit values for all constants, plot the θ and α angles for 5 seconds for the nonzero initial conditions $(\theta)_0= (\alpha)_0=10^\circ$ without any control torque u.

c) Find the transfer functions, poles and zeros of the system.



$$x_1 = \alpha\,,\ x_2 = \theta\,,\ x_3 = \dot{\alpha}\,,\ x_4 = \dot{\theta}$$

$$\dot{x}_3 = \frac{k_2 k_3 s x_2}{k_1 k_2 - k_3^2 c^2 x_2} \cdot x_4^2 - \frac{k_3 k_4 s x_2 c x_2}{k_1 k_2 - k_3^2 c^2 x_2} + \frac{k_2 + k_3 c x_2}{k_1 k_2 - k_3^2 c^2 x_2} \cdot u$$

$$\dot{x}_4 = -\frac{k_3^2 s x_2 c x_2}{k_1 k_2 - k_3^2 c^2 x_2} \cdot x_4^2 + \frac{k_1 k_4 s x_2}{k_1 k_2 - k_3^2 c^2 x_2} - \frac{k_1 + k_3 c x_2}{k_1 k_2 - k_3^2 c^2 x_2} \cdot u$$

where

$$J_1 + (M + m)a^2 = k_1\,,\quad J_2 + m\ell^2 = k_2\,,\ ma\ell = k_3\,,\ mg\ell = k_4$$