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BIOL 450 Dr. Prinz

6 April 2022

Assignment 7: Dynamical Systems

Part A

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# Morris-Lecar model
dv/dt = ( I - gca*minf(V)*(V-Vca)-gk*w*(V-VK)-gl*(V-Vl)+s(t))/c
dw/dt = phi*(winf(V)-w)/tauw(V)
v(0)=-16
w(0)=0.014915
minf(v)=.5*(1+tanh((v-v1)/v2))
winf(v)=.5*(1+tanh((v-v3)/v4))
tauw(v)=1/cosh((v-v3)/(2*v4))
param vk=-84,vl=-60,vca=120
param i=40,gk=8,gl=2,c=20
param v1=-1.2,v2=18

# Uncomment the ones you like!!
#par1-3 v3=2,v4=30,phi=.04,gca=4.4
par4-6 v3=12,v4=17.4,phi=.06666667,gca=4
param s1=0,s2=0,t1=50,t2=55,t3=500,t4=550

# double pulse stimulus
s(t)=s1*heav(t-t1)*heav(t2-t)+s2*heav(t-t3)*heav(t4-t)
@ total=4000,dt=1,xlo=-75,xhi=75,ylo=-.25,yhi=.5,xp=v,yp=w
done
```

Part B

In the phase plane, for our intial parameters, we observe an unstable fixed point that is surrounded by an ellipse-like limit cycle (the solid black line). Since we are observing a limit cycle, this means that our W (the activation variable for potassium current) and V(membrane potential) are cycling. In other words, dw/dt and dv/dt are changing; the activation variable for potassium current and the membrane potential are cycling or changing over time. Should we initialize our variables near the unstable fixed point, W and V will go into the limit cycle; these trajectories suggest a stable limit cycle. In other words, when the initial conditions are near the limit cycle but not necessarily near an action potential's threshold, the trajectories will be pulled into the limit cycle.

Because our variables are cycling in the stable limit cycle, we should see large changes in the potassium current and the membrane potential, which occur during action potentials. Thus, we predict that the voltage vs. time graph will look like there are many repetitive action potentials. Additionally, we are examining the voltage over a large time interval, so we should see multiple spikes. Per our voltage and time table, we also see large increases and decreases in voltage, which also indicates spiking activity.

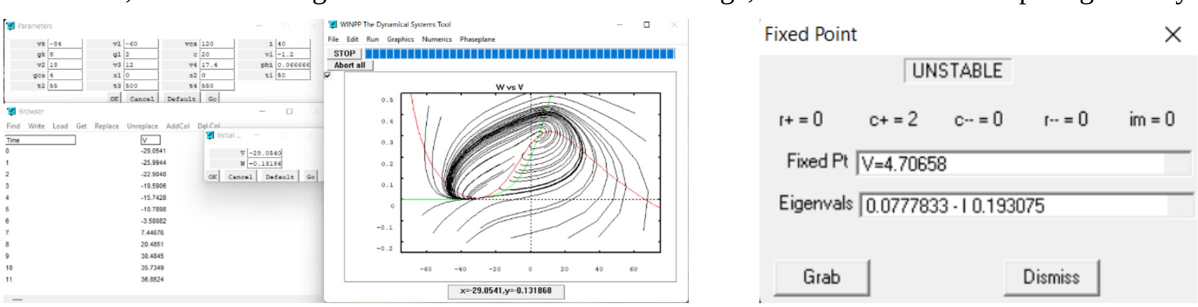


Figure 1. Phase plane with nullclines and parameters (left), with fixed point (right).

Part C

The voltage vs. time plot matches our expectations, since we can see that there are many spikes. Since we know that our voltage and potassium current activation variables are actively cycling, we should expect the repetitive spikes that we see. Additionally, since we are plotting over 4000 ms, a large time interval, we should see at least multiple spikes.

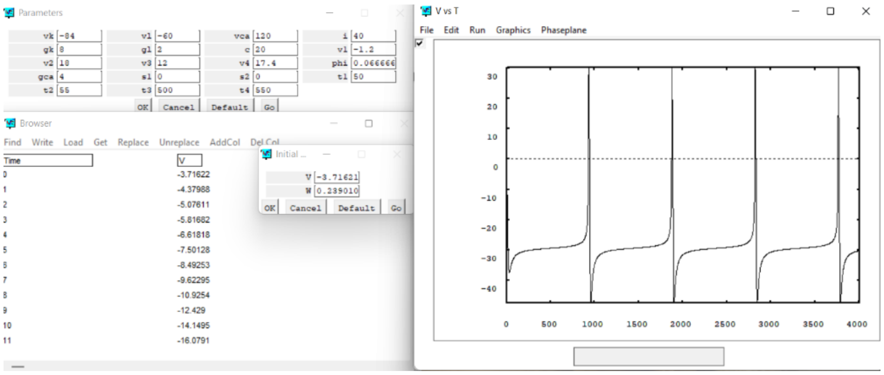


Figure 2. Voltage vs. time plot of the neuron's activity at phi = 0.0666.

Part D

We see that once we increase phi, our phase plane looks very different. First, the panel shows that the W and V nullclines are such that there is a small, ellipse-like limit cycle with a stable fixed point, with the limit cycle much smaller than our previous plots (phi = 0.0666). This means that should we initialize our variables near the stable fixed point, they will eventually arrive at that stable fixed point, where neither voltage nor the potassium activation variables are changing. This may correspond to arriving at equilibrium, or a reversal potential, since neither the net current flow nor membrane potential would not be changing. This stable fixed point also contrasts with the unstable fixed point that we observed when phi = 0.0666. Now, instead of going into a limit cycle, variables may also reach equilibrium. If we examine the MorrisLecar ODE file, we can see that dw/dt = phi*(winf(V)-w)/tauw(V). We know that winf(V) is our steady state activation variable as a function of voltage, and our tauw(V) is our time constant as a function of voltage. We observe that when we increase phi, the effect of the time constant (the denominator) decreases, meaning that there is a larger change in potassium current over time, and the neuron can spike over a shorter time. Thus, phi may be a deinactivation variable, where increasing a deinactivation variable means that there is less of a refractory period and the potassium channels take less time to recover from inactivation, resulting in more spikes in a shorter amount of time, which is indeed what we observe.

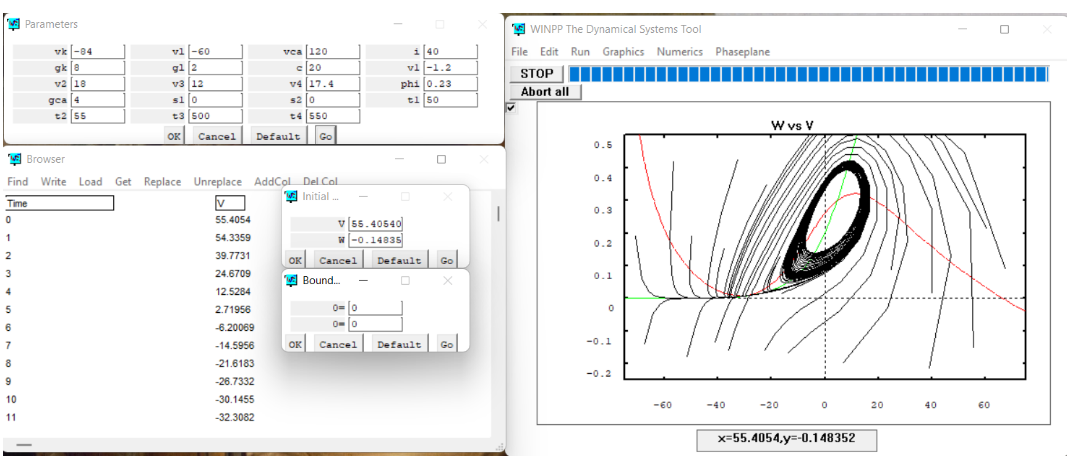
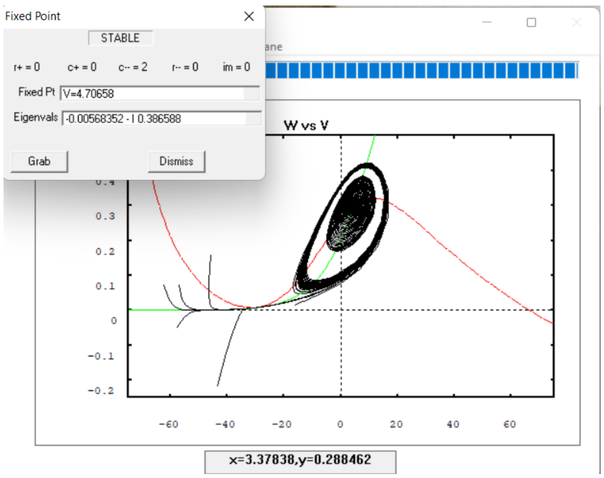


Figure 3. Top: Phase plane with nullclines when phi = 0.23. Bottom: an example of a fixed point in the system.



Part E

Compared to our previous voltage vs. time plot where phi = 0.0666, we see that when phi = 0.23 there are many more spikes observed in the same time frame. In other words, more spikes are occurring with a shorter time interval in between spikes. Additionally, it seems like the peak voltages seen in the limit cycle decrease from 30 mV to 20 mV, and the lowest voltages increase from -50 mV to approximately -15 mV.

These changes match the expectations we formulated in the previous question. If an increase in phi means an increase in the deactivation variable, then we would be able to enable a spike during shorter time intervals, leading to more spikes in less time. We can observe this in the voltage vs. time graph.

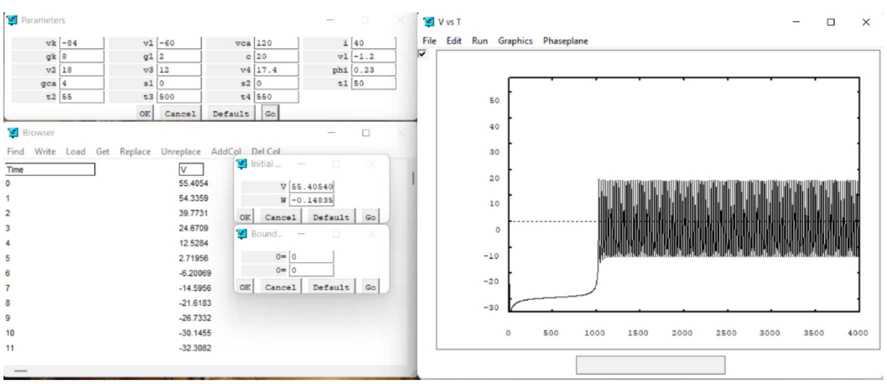


Figure 4. Voltage vs. time plot of the neuron's activity at phi = 0.23.

Part F

After 40 nA, there is only one fixed point, and the nullclines no longer intersect at two or more places—only one. The bifurcation table and diagram are as follows:

Table 1. Bifurcation table with fixed points.

Current (nA)	Voltage at fixed point	Stable or Unstable
30	3.87151	Unstable
30	-41.8452	Stable
30	-19.5632	Unstable

31	3.95989	Unstable
31	-41.1023	Stable
31	-20.0404	Unstable
32	4.04708	Unstable
32	-40.3309	Stable
32	-20.5464	Unstable
33	-39.525	Stable
33	4.13312	Unstable
33	-21.0872	Unstable
34	-38.6761	Stable
34	4.21805	Unstable
34	-21.6714	Unstable
35	4.30192	Unstable
35	-37.7721	Stable
35	-22.3112	Unstable
36	4.38477	Unstable
36	-36.7944	Stable
36	-23.0251	Unstable
37	4.46662	Stable
37	-35.7111	Stable
37	-23.8451	Unstable
38	4.54752	Stable
38	-34.4597	Stable
38	-24.8338	Unstable
39	4.62749	Stable
39	-32.8756	Stable
39	-26.1558	Unstable
40	4.70658	Stable
41	4.78479	Stable
42	4.86217	Stable
43	4.93875	Stable
44	5.01453	Stable
45	5.08955	Stable

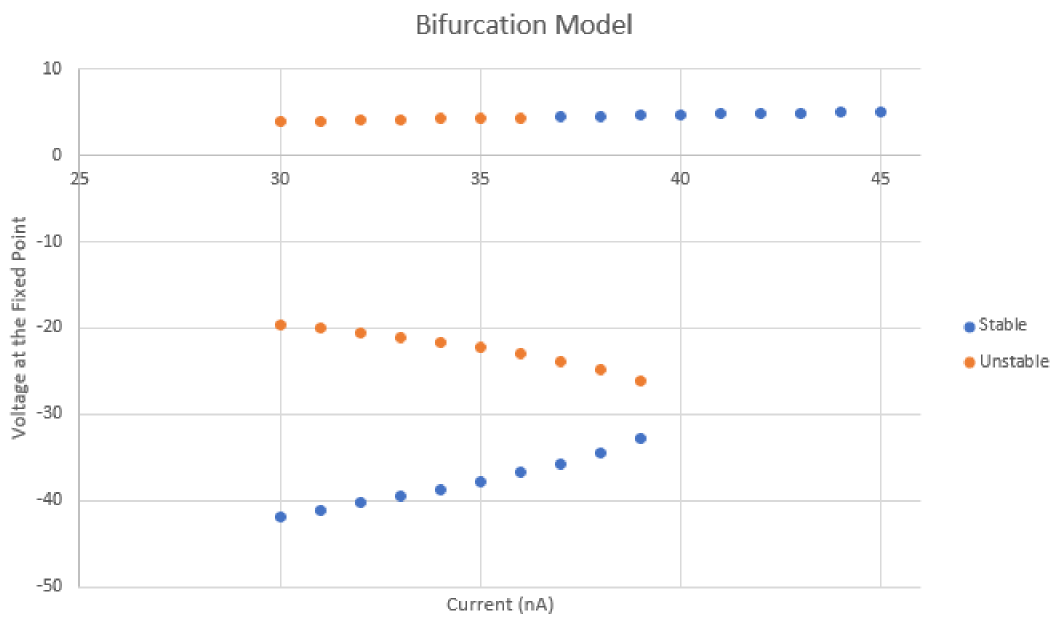


Figure 5. Bifurcation model of voltage at fixed points and current when phi = 0.23. The color indicates whether the fixed point is stable or unstable.

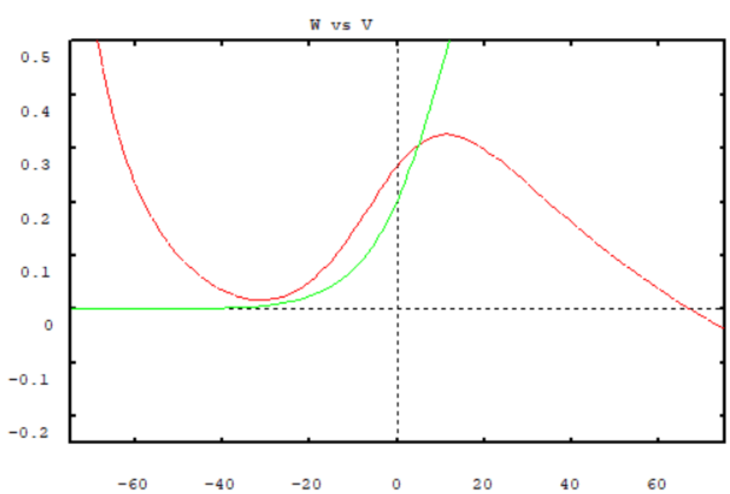


Figure 6. An example of the nullclines only intersecting once, at a current of 44 nA. The nullclines are very close but not intersecting at around -30 mV.

Part G

```
# Morris-Lecar model
dv/dt = ( I - gca*minf(V)*(V-Vca)-gk*w*(V-VK)-gl*(V-Vl)+s(t))/c
dw/dt = phi*(winf(V)-w)/tauw(V)
di/dt = e *(v0 - V)
v(0)=-16
w(0)=0.014915
minf(v)=.5*(1+tanh((v-v1)/v2))
winf(v)=.5*(1+tanh((v-v3)/v4))
tauw(v)=1/cosh((v-v3)/(2*v4))
param vk=-84,vl=-60,vca=120
```

```
param gk=8,gl=2,c=20
param v1=-1.2,v2=18
param v0=-26,e=0.001

# Uncomment the ones you like!!
#par1-3 v3=2,v4=30,phi=.04,gca=4.4
par4-6 v3=12,v4=17.4,phi=.06666667,gca=4
param s1=0,s2=0,t1=50,t2=55,t3=500,t4=550

# double pulse stimulus
s(t)=s1*heav(t-t1)*heav(t2-t)+s2*heav(t-t3)*heav(t4-t)
@ total=4000,dt=1,xlo=-75,xhi=75,ylo=-.25,yhi=.5,xp=v,yp=w
done
```

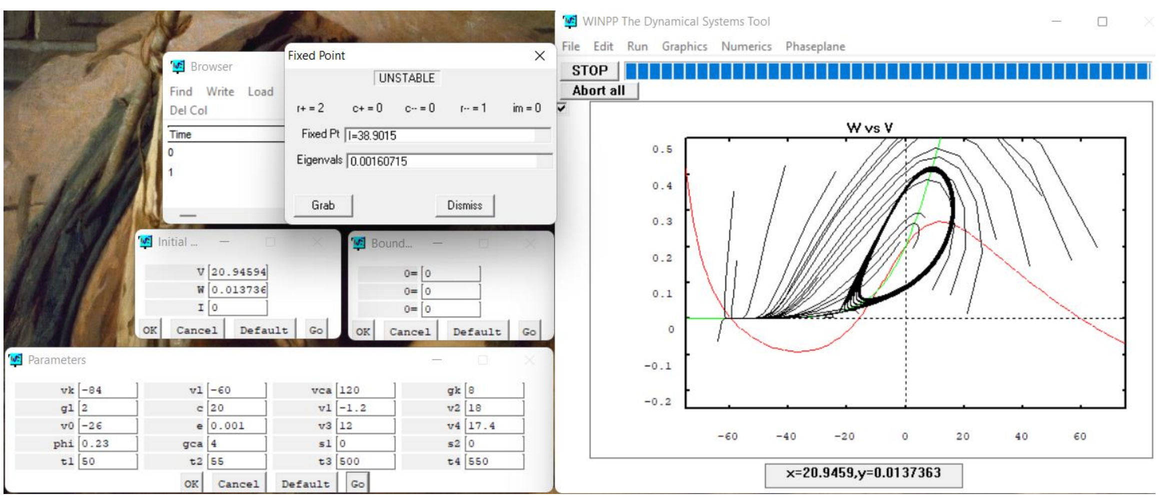
Part H

We see that dI/dt = ε * (v0 - v), so the change in current over time depends on some parameter ε and how different the voltage is from the initial value. So, we know that this current somehow depends on voltage, and the larger the difference between the membrane potentials, the greater the change in current over time. We also know that there is no net change in current over time if v is equal to -26 mV. This may be a voltage-dependent ionic current, and the ion is such that its reversal potential is at -26 mV, which would explain why there is not net current at that point.

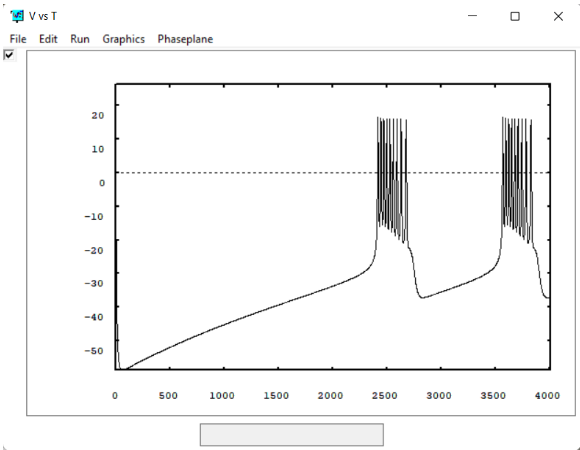
We also know that our dI/dt counts for the slow part of a fast-slow dynamical system, where W and V are the fast components of the system. Thus, the physiological equivalent of this membrane current may be a calcium-gated slow channel, such as a potassium channel. Slow buildup of calcium results in an active phase, while recovery of calcium results in a quiescent period.

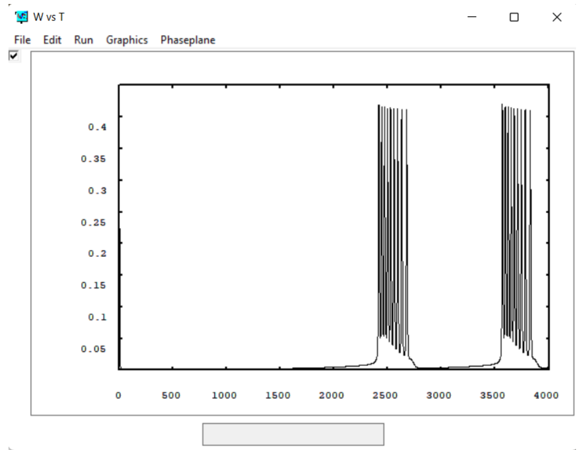
Part I

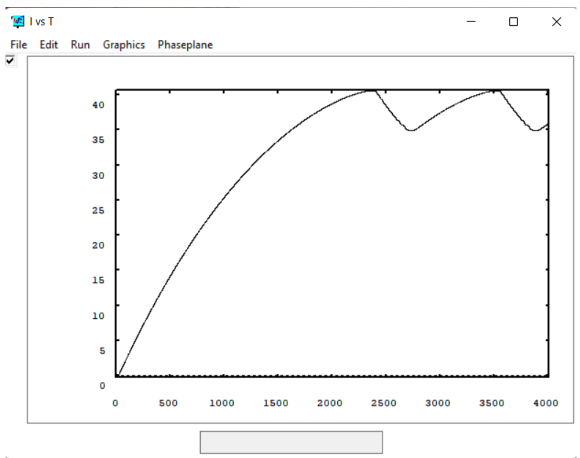
We observe an unstable point and an ellipse-like limit cycle.



Figures 7-10. Top: Phase plane with nullclines when phi = 0.23, with the unstable fixed point shown. Note that the unstable fixed point is at a value of current (I). Bottom (left to right): Voltage (V) vs. Time plot, Activation Variable of Potassium (W) vs. Time plot, Current (I) vs. Time plot.

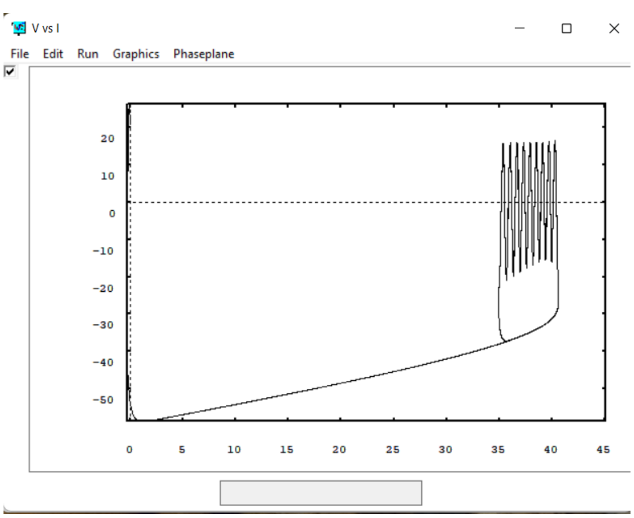


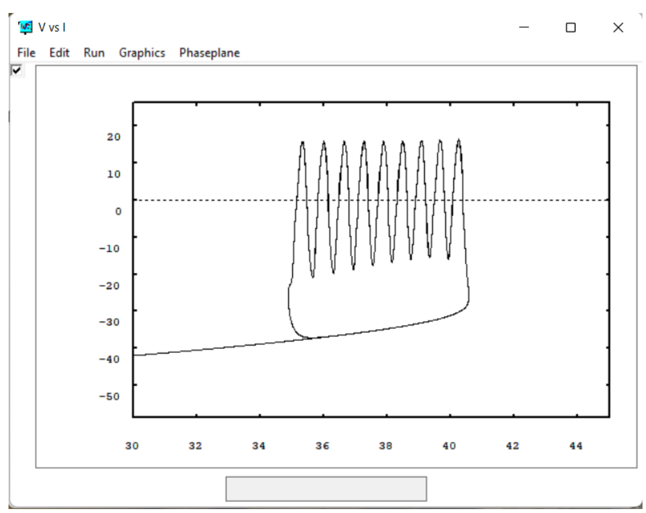




Part J

The dynamic variable that changes least during the dynamics of the model – the "slow variable" – is the current (I). As shown in the plots above, it changes the least in terms of relative frequency and magnitude;





the other variables have many peaks and troughs that occur in a very short period, while the current only really has two peaks and does not change as much in magnitude, relatively. When plotting the current on the x axis and the membrane voltage on the y axis, we can see a bursting pattern or active phase. It seems that bursting only occurs when the current is around 35 nA; before then, the model exhibits an inactive period. The I vs. Time plot above also shows that there only is an active phase once the current reaches above 35 nA, further suggesting an active phase above 35 nA and an inactive period when the current is smaller than 35 nA.

Figure 11. Left: A plot of with the current (I) on the x axis and the membrane voltage (V) on the y axis. Right: A zoom-in of the same plot.

Part K

Unlike the previous bifurcation diagram, which was in 2D and only accounted for V and W over time, this diagram accounts for V, W and I changing over time.

The plot shows the continuous dynamics of the system compressed from 3D space to 2D space of V, W, and I. That is, if we were to take slices or cross-sections of this in 3D space, we would see the phase portrait for the fast V-W system for different values of the slow variable I as the system moves through its burst cycle. Compressing these “slices” together results in this sort of shape that we see.

The previous bifurcation diagram, then, is what we can observe when dw/dt and dv/dt are both zero for a given current value. That is, if we take the fixed points from each “slice” (each I) and then plot it, we will get the previous bifurcation diagram.