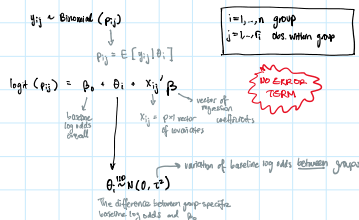


GLMMs Summary

Wednesday, November 22, 2023 16:12

Take GLME to the next level by introducing random intercepts.

Random Intercept Logistic Regression Model



Likelihood

y is data, θ is vector of ALL RANDOM EFFECTS, $[y, \theta]$ is joint density

$$[y, \theta] = \prod_{i=1}^n [y_i, \theta_i] \rightarrow \text{measuring joint density of } [y_1, \dots, y_n, \theta_1, \dots, \theta_n]$$

$$= \prod_{i=1}^n \left(\prod_{j=1}^{J_i} \frac{e^{y_{ij} \theta_i}}{1 + e^{\theta_i}} \right) \prod_{j=1}^{J_i} \frac{1}{1 + e^{\theta_i}} \quad (\text{all } y_{ij} \text{ conditionally independent given random effects})$$

$$= \prod_{i=1}^n \left(\prod_{j=1}^{J_i} \frac{e^{y_{ij} \theta_i}}{1 + e^{\theta_i}} \right) (2\pi\tau^2)^{-n/2} \exp\left(-\frac{1}{2\tau^2} \theta_i^2\right)$$

$$L(\beta, \beta_0, \tau^2 | y) = \prod_{i=1}^n \int \left(\prod_{j=1}^{J_i} \frac{e^{y_{ij} \theta_i}}{1 + e^{\theta_i}} \right) \cdot (2\pi\tau^2)^{-n/2} \exp\left(-\frac{1}{2\tau^2} \theta_i^2\right) d\theta_i$$

$$= \prod_{i=1}^n \left(\prod_{j=1}^{J_i} \frac{e^{y_{ij} \theta_i}}{1 + e^{\theta_i}} \right) \cdot (2\pi\tau^2)^{-n/2} \exp\left(-\frac{1}{2\tau^2} \theta_i^2\right)$$

$$= \prod_{i=1}^n \left(\prod_{j=1}^{J_i} \frac{e^{y_{ij} \theta_i}}{1 + e^{\theta_i}} \right) \cdot (2\pi\tau^2)^{-n/2} \exp\left(-\frac{1}{2\tau^2} \theta_i^2\right)$$

→ for GLMM, convergence becomes an issue here.

→ increase niter in R to help it converge?

→ in spreadsheets not comparable

Baseline log odds even depends on intercepts ± 1.96 ($\sqrt{\tau^2}$)

Random Intercept Poisson Model

observed y_{ijk} count $= y_{ijk} \sim \text{Poisson}(\lambda_{ijk})$
 $\theta_i \sim N(0, \tau^2)$
 group-specific deviation in baseline log expected [outcome]
 variation between groups

$$\log(E[y_{ijk} | \theta_i]) = \log \lambda_{ijk} = \beta_0 + \beta_1 \cdot I[\text{sex}_i = \text{F}] + \beta_2 I[\text{sex}_i = \text{M}] + \beta_3 I[\text{sex}_i = \text{F}] I[\text{age}_i = 10] + \beta_4 I[\text{sex}_i = \text{F}] I[\text{age}_i = 20] + \beta_5 I[\text{sex}_i = \text{F}] I[\text{age}_i = 30]$$

log expected count at baseline for the age group (non-white whites in 1970)
 θ_i will be baseline expected count.

θ_i = ratio of deaths for non-white vs. non-white of 10 model assuming the random intercepts...
 θ_i = within-race modification of deaths for whites vs.
 θ_i = odds ratio of... for a 1-unit increase in a model controlling for...

CI for baseline counts: θ
 conf. var estimate → value used in computing by Hmisc for [year]
 $\theta_0 + \theta_1 + \dots = \text{expected count for } \dots \text{ in 1970...}$

GOF test: H_0 : Model adequately fits the data
 H_1 : Lack of fit

Random Intercept Poisson Model WITH POPULATION EFFECT

→ Same model but add log popsize which is an offset variable

$$\lambda_{ijk} = e^{\beta_0 + \beta_1 \cdot \text{sex}_i + \beta_2 \cdot \text{age}_i + \dots}$$

$$= e^{\beta_0 + \beta_1 \cdot \text{sex}_i + \beta_2 \cdot \text{age}_i + \dots}$$

Then,

$$\frac{y_{ijk}}{\text{popsize}} = e^{\beta_0 + \beta_1 \cdot \text{sex}_i + \beta_2 \cdot \text{age}_i + \dots}$$

θ_0 is per capita deaths instead of expected counts

$$\rightarrow \theta_0 = \lambda_i / \text{pop}_i$$

Quick derivation:

$$\log \lambda_i = \beta_0 + \log \text{pop}_i$$

$$\log \lambda_i - \log \text{pop}_i = \beta_0$$

$$\log \left(\frac{\lambda_i}{\text{pop}_i} \right) = \beta_0$$

$$\theta_0 = \frac{\lambda_i}{\text{pop}_i}$$

3.3 What is the odds ratio and 95% confidence interval for treatment=intervention in age group 25-34 versus Control in age group 25-34?

1.72 (0.378, 7.78)

```
control <- 2.0624
intervention <- -1.5228
```

```
q3_3 <- exp(2.0624 + -1.5228)
q3_3
```

```
## [1] 1.72
```

```
q3_3var <- vcov(q3_2)
q3_3var2 <- q3_3var["treatmentIntervention", "treatmentIntervention"] +
  q3_3var["treatmentIntervention:agegroup25-34", "treatmentIntervention:agegroup25-34"] +
  2 * q3_3var["treatmentIntervention", "treatmentIntervention:agegroup25-34"]
```

```
q3_3 * exp(- 1.96 * sqrt(q3_3var2))
```

```
## [1] 0.378
```

```
q3_3 * exp(1.96 * sqrt(q3_3var2))
```

```
## [1] 7.78
```