# 7_Meta+Network+Analysis

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## Advanced Network Analysis
## 7. Meta Network Analysis

Weihua An

EMORY UNIVERSITY



## Outline

1. Fit a random graph model on each network.
2. Combine and compare the results across networks through meta analysis.
   - Univariate Meta Regression
     - Fixed effects model
     - Random effects model
   - Multivariate Meta Regression
     - Fixed effects model
     - Random effects model



ERGM (SAOM)

classroom: fit the same model (reciprocity, transitivity, .... etc.)

| | reciprocity | | transitivity |
|---|---|---|---|
| 1 | 3 | (SE) | ⋮ |
| 2 | 2 | (SE) | ⋮ |
| ⋮ | ⋮ | ⋮ | ⋮ |
| 100 | 5 | (SE) | etc. |

Why can't we take a simple average? Assumes weight is the same

## Univariate Meta Regression

criticize bc assumes $\theta_i$ is constant thru all the networks

If intercept only, intercept equal to simple average?

Dependent variable: estimated coefficients on one variable (e.g., reciprocity).

- Fixed effects model

$$\hat{\theta}_{ki} = \theta_i + \mathbf{x}_k'\boldsymbol{\beta}_i + e_{ki} \quad (1)$$

where $\hat{\theta}_{ki}$ denotes the $i$th estimated coefficient in the $k$th network, $\theta_i$ a common effect to be estimated, $\mathbf{x}_k$ the characteristics of the $k$th network (ecological factors, measured at the network level or higher levels), and $e_{ki}$ an error term with a zero mean and a variance $\hat{\sigma}^2_{ki}$ (which is known). Assuming independence and normality of the error terms:

— need to put some structure on error term. The variance of each error term is given by the square of the std. error

$$\hat{\theta}_{ki} \sim \text{Normal}(\underbrace{\theta_i + \mathbf{x}_k'\boldsymbol{\beta}_i}_{\text{mean}}, \underbrace{\hat{\sigma}^2_{ki}}_{\text{variance}}). \quad (2)$$

Assume normality of error terms can be written as

- Random effects model

$$\hat{\theta}_{ki} \sim \text{Normal}(\theta_i + \mathbf{x}_k'\boldsymbol{\beta}_i, \hat{\sigma}^2_{ki}) \quad (3)$$

$$\hat{\theta}_{ki} \sim \text{Normal}(\underbrace{\theta_i + \mathbf{x}_k'\beta_i}_{\text{mean}}, \underbrace{\hat{\sigma}_{ki}^2}_{\text{variance}}). \quad (2)$$

Assume normality of error terms can be written as

- Random effects model

$$\hat{\theta}_{ki} \sim \text{Normal}(\theta_i + \mathbf{x}_k'\beta_i, \hat{\sigma}_{ki}^2) \quad (3)$$

$$\theta_i \sim \text{Normal}(\mu_i, v_i^2) \quad (4)$$

where θi is now a number w/ a distribution

This is like a random intercept model, where $\mu_i$ is the mean effect of the $i$th variable. $v_i^2$ (to be estimated) measures the between-network variation of the effects. If it equals to zero, the model falls back to the FE model.

Assumption: Independence across coefficients of different variables.

Vector as dependent variable



## Multivariate Meta Regression

model 2 or more vectors at the same time

Matrix as dependent variable

Dependent variable: coefficients of all (or selected multiple) variables

- Fixed effects model

all coefficients

$$\hat{\boldsymbol{\theta}}_k \sim \text{Normal}_{\boldsymbol{I}}(\boldsymbol{\theta} + \boldsymbol{X}_k'\beta, \boldsymbol{\Sigma}_k), \quad (5)$$

The coefficients in the $k$th network are assumed to follow a multivariate normal distribution. $\hat{\boldsymbol{\theta}}_k$ represents the vector of coefficients in the $k$th network. $\boldsymbol{\Sigma}_k$ is the variance-covariance matrix of the coefficients in the $k$th network.

Std. error comes from √ of diagonals of var-covar matrix

- Random effects model

$$\hat{\boldsymbol{\theta}}_k \sim \text{Normal}_{\boldsymbol{I}}(\boldsymbol{\theta} + \boldsymbol{X}_k'\beta, \boldsymbol{\Sigma}_k), \text{ where } \boldsymbol{\theta} \sim \text{Normal}_{\boldsymbol{I}}(\boldsymbol{\mu}, \boldsymbol{\Omega}) \quad (6)$$

where $\boldsymbol{\theta}$ represents the mean effect and $\boldsymbol{\Omega}$ the between-network covariation. If $\boldsymbol{\Omega}$ equals to zero, the model falls back to the FE model.

These models can account for correlations among the coefficients of different variables, but are difficult to estimate when the number of networks is small.



## Model Comparison

- Overall model fitness measure, like Akaike's Information Criterion (AIC) or Bayesian Information Criterion (BIC)
- Cochran $Q$ test (Gasparrini et al., 2012). A large $Q$ value indicates significant heterogeneity and random effects models are preferred.

  G lots of differences in coefficients

$$Q = \sum_k \hat{e}_k' \boldsymbol{\Sigma}_k^{-1} \hat{e}_k$$

  where $\hat{e}_k$ is the vector of residuals and $\boldsymbol{\Sigma}_k$ the variance-covariance matrix of the coefficients in the fixed effects model for the $k$th network (Gasparrini et al., 2012). Q-statistic follows a Chi-square distribution with $K - 1$ degree of freedom. A small $P$ value indicates significant heterogeneity.

- Information squared: The $I^2$ statistic shows the proportion of variation in the coefficients across networks that is attributable to heterogeneity rather than sampling error. A larger $I^2$ indicates more heterogeneity.

$$I^2 = (1 - \frac{K - M - 1}{Q}),$$

  where $M$ is the number of coefficients in the model.

## Example

An (2022) fitted an ERGM on the friendship network in each of the six schools and used the univariate random effects meta regression to combine the results.

**Table 3.** Aggregated ERGM Results on the Friendship Networks across Schools

| Variables | Est. | SE | Q |
|---|---|---|---|
| ***Receiver Effects*** | | | |
| Gender (Boy) | 0.16 | 0.04 *** | 0.06 |
| Age (Older) | -0.03 | 0.02 | 0.06 |
| Height | 0.07 | 0.02 *** | 0.12 |
| Ranking | 0.18 | 0.03 *** | 0.00 |
| Smoking | 0.40 | 0.05 *** | 0.00 |
| Personality (Optimistic) | 0.01 | 0.02 | 0.09 |
| Family Economic Condition (Good) | 0.04 | 0.02 | 0.86 |
| ***Sender Effects*** | | | |
| Gender (Boy) | -0.26 | 0.03 *** | 0.12 |
| Age (Older) | 0.01 | 0.02 | 0.17 |
| Height | 0.02 | 0.02 | 0.69 |
| Ranking | 0.04 | 0.02 * | 0.55 |
| Smoking | 0.05 | 0.03 | 0.37 |
| Personality (Optimistic) | 0.02 | 0.02 | 0.84 |
| Family Economic Condition (Good) | -0.02 | 0.02 | 0.53 |
| ***Homophily*** | | | |
| Gender | 1.18 | 0.06 *** | 0.00 |
| Age | 0.11 | 0.02 *** | 0.60 |
| Height | 0.08 | 0.02 *** | 0.10 |
| Ranking | 0.21 | 0.02 *** | 0.14 |
| Smoking | 0.25 | 0.03 *** | 0.17 |
| Personality | 0.02 | 0.01 | 0.42 |
| Family Economic Condition | 0.05 | 0.02 * | 0.83 |
| ***Propinquity*** | | | |
| Same Grade | 1.32 | 0.13 *** | 0.00 |
| Same Classroom | 1.07 | 0.04 *** | 0.00 |
| ***Endogenous Tie Formation Processes*** | | | |
| Mutuality (Reciprocity) | 2.16 | 0.12 *** | 0.00 |
| GWESP (Transitivity) | 1.13 | 0.03 *** | 0.00 |
| GWDSP (Two-path) | -0.17 | 0.01 *** | 0.00 |
| GWIDEGREE (Preferential attachment) | -1.11 | 0.23 *** | 0.00 |
| GWODEGREE (Differential sociability) | -1.21 | 0.28 *** | 0.00 |
| ***Edges*** | -6.32 | 0.25 *** | 0.00 |

Q score

overall across all schools

pvalue of Q score

gender receive effect don't differ across schools

smoking effects differ significantly across schools



## Example

McFarland et al. (2014) fitted ERGMs on the friendship networks of 129 schools in the U.S. and studied how school features moderate the network formation patterns.

**Table 4.** Moderator Results

| | | | Development | | | | Size | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | High School (vs. Middle School) | | | | Larger | | | |
| ***Schools*** | Main | | Bivar. | | Full | | Bivar. | | Full | |
| Edges | −6.689 | *** | −.135 | * | .248 | ** | −.524 | *** | −.504 | *** |
| Mutuality | 3.393 | *** | .282 | *** | .294 | *** | .630 | *** | .529 | *** |
| Closure | .954 | *** | .023 | * | .016 | | .090 | *** | .074 | *** |
| Hierarchy | .123 | *** | .008 | *** | .010 | ** | .010 | *** | .006 | ** |
| Club Ties | .411 | *** | .127 | *** | .158 | *** | .141 | *** | .087 | *** |
| Same Race | .499 | *** | .064 | * | .059 | | .099 | *** | .045 | ~ |
| Same Gender | .184 | *** | −.037 | *** | −.059 | *** | −.005 | | .021 | ** |
| Same Age | 1.231 | *** | −.056 | | −.351 | *** | .079 | * | .116 | *** |
| GPA Diff. | −.207 | *** | .010 | | .006 | | .002 | | −.010 | |
| SES Diff. | −.035 | *** | .001 | | −.005 | | −.004 | * | −.001 | |

networks in HS have more mutual ties than middle schools

In larger schools, ties more likely to be mutual

more gender homophily in large schools

HS has less gender homophily than middle school