7_Meta+Network+Analysis

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Advanced Network Analysis 7. Meta Network Analysis Weihua An **EMORY**

Outline

- 1. Fit a random graph model on each network.
- 2. Combine and compare the results across networks through meta analysis.

 $\hat{\theta}_{ki} \sim \text{Normal}(\theta_i + \mathbf{x}_k' \boldsymbol{\beta}_i, \hat{\sigma}_{ki}^2)$

- Univariate Meta Regression
 - Fixed effects model
 Random effects model
- Multivariate Meta Regression

 - Fixed effects model
 Random effects model

ERGM (SAON) classroom. fit the same model (reciprocity, transitivity, ... etc.) reuprovity 3 (SE) 0 0 2 (5%) 5 (56)

etc Why contine take a simple awage? Assumes weight is the same

(AHUZE BL assumes Univariate Meta Regression & contrare se accourses

B: is another thin all the networks It suppressly outs. interest equal to average? Dependent variable: estimated coefficients on one variable (e.g., reciprocity). ► Fixed effects model < $\hat{\theta}_{ki} = \theta_i + \mathbf{x}'_k \boldsymbol{\beta}_i + e_{ki}$ med to put some structure on error term. where $\hat{\theta}_{ki}$ denotes the *i*th estimated coefficient in the *k*th network, θ_i a common effect to be estimated, \mathbf{x}_k the characteristics of the kth network (ecological factors, measured at the network level or higher levels), and e_{ki} an error term with a zero mean and a variance $\hat{\sigma}_{ki}^2$ (which is known). Assuming independence The various of each error team to and normality of the error terms: Assume normality of error terms $\hat{\theta}_{ki} \sim \text{Normal}\big(\underbrace{\theta_i + \mathbf{x}_k' \boldsymbol{\beta}_i}_{\text{Normal}}, \underbrace{\hat{\sigma}_{ki}^2}_{\text{Normal}}\big).$ Random effects model can be written as

$$\hat{\theta}_{ki} \sim \mathsf{Normal}(\underbrace{\theta_i + \mathbf{x}_k' \beta_i}_{\mathsf{mg,K}}, \underbrace{\hat{\sigma}_{ki}^2}_{\mathsf{Volime}}).$$

Random effects model

$$\hat{\theta}_{ki} \sim \text{Normal}(\theta_i + \mathbf{x}_k' \boldsymbol{\beta}_i, \hat{\sigma}_{ki}^2)$$
 (3)

 $\theta_i \sim \text{Normal}(\mu_i, v_i^2)$

This is like a random intercept model, where μ_i is the mean effect of the ith variable. v_i^2 (to be estimated) measures the between-network variation of the effects. If it equals to zero, the model falls back to the FE model.

Assumption: Independence across coefficients of different variables.

Assume normality of error terms can be written as)

rector as dependent variable

Multivariate Meta Regression

Dependent variable: coefficients of all (or selected multiple) variables

Fixed effects model مراه المحافظة ال

Fixed effects model

$$\hat{\theta}_k \sim \text{Normal}_I(\theta + X'_k \beta, \Sigma_k),$$
 (5)

The coefficients in the kth network are assumed to follow a multivariate normal distribution. $\hat{\theta}_k$ represents the vector of coefficients in the kth network. Σ_k is the <u>variance-covariance matrix</u> of the coefficients in the kth network.

Random effects model

$$\hat{\theta}_k \sim \text{Normal}_I(\theta + X_k'\beta, \Sigma_k)$$
, where $\theta \sim \text{Normal}_I(\mu, \Omega)$ (6)

where θ represents the mean effect and $\underline{\Omega}$ the between-network covariation. If $\underline{\Omega}$ equals to zero, the model falls back to the FE model.

These models can account for correlations among the coefficients of different variables, but are difficult to estimate when the number of networks is small.

Model Comparison

- Overall model fitness measure, like Akaike's Information Criterion (AIC) or Bayesian Information Criterion (BIC)
- Cochran Q test (Gasparrini et al., 2012). A large Q value indicates significant heterogeneity and random effects models are preferred.

$$Q = \sum_k \hat{e}_k' \mathbf{\Sigma}_k^{-1} \hat{e}_k$$

where \hat{e}_k is the vector of residuals and $\mathbf{\Sigma}_k$ the variance-covariance matrix of the coefficients in the fixed effects model for the kth network (Gasparrini et al., 2012). Q-statistic follows a Chi-square distribution with K-1 degree of freedom. A small P value indicates significant heterogeneity.



The I^2 statistic shows the proportion of variation in the coefficients across networks that is attributable to heterogeneity rather than sampling error. A larger I2 indicates more heterogeneity.

$$I^2 = (1 - \frac{K - M - 1}{Q}),$$

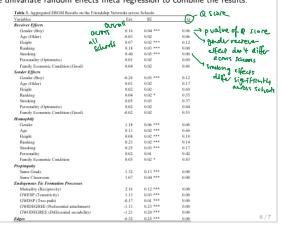
where M is the number of coefficients in the model

matrix as dependent variable

Std. emr comes from I of diagonals of var-covar matrix

Example

An (2022) fitted an ERGM on the friendship network in each of the six schools and used the univariate random effects meta regression to combine the results.



Example

McFarland et al. (2014) fitted ERGMs on the friendship networks of 129 schools in the U.S. and studied how school features moderate the network formation patterns.

Table 4. Moderator Results

