Lecture 5: Risk, return, derivatives

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5-risk-return

QTM 385 Quantitative Finance

Lecture 5: Derivative, risk and return

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Suggested reading: Investments Ch 2 and 5

Question from Google form

- I still don't understand how the return of the index is affected after a split when including divisor adjustment

- *Answer: Without split*
- *Time 1 (initial), index value* $= (25 + 100)/2 = 62.5$
- *Time 2 (final), index value* $= (30 + 90)/2 = 60$
- *Percentage change in index* $= -2.5/62.5 = -.04 = -4\%$

Stock	Initial Price	Final Price	Shares (million)	Initial Value of Outstanding Stock ($ million)	Final Value of Outstanding Stock ($ million)
ABC	$ 25	$30	20	$500	$600
XYZ	100	90	1	100	90
Total				$600	$690

Question from Google form

- I still don't understand how the return of the index is affected after a split when including divisor adjustment

- *Answer: With split*
- *Time 1 (initial), index value* $= (25 + 100)/2 = 62.5$
- *Time 1 (XYZ splits), divisor is changed to* 1.2
  - $(25 + 50)/1.2 = (25 + 100)/2 = 62.5$
- *Time 2 (final), index value* $= (30 + 45)/1.2 = 62.5$
- *Percentage change in index* $= 0$

Stock	Initial Price	Final Price	Shares (million)	Initial Value of Outstanding Stock ($ million)	Final Value of Outstanding Stock ($ million)
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Question from Google form

- Are there any "rule of thumb" reasons why in particular trading days certain type of indices such as NDX grow while other types of indices such as Dow and S&P 500 fall other than sampling errors? For example, can we conclude anything if a price-weighted index like Dow falls while market-value-weighted index like NASDAQ 100 grows? If not, what can we say about the traits of each type of market proxy?

- *Answer: The constitutions of Dow, S&P500, and NASDAQ 100 are different. If the Dow falls while NASDAQ 100 increases, then conceptually it means the price of stocks in NASDAQ 100 but not in Dow increases*

Question from Google form

- When should someone look into purchasing index funds vs. ETFs? What are the advantages and drawbacks of each?

- *Answer: Usually there are no shareholder transaction costs for index mutual funds. The taxation and management fees for ETFs are lower. ETFs can be bought and sold on an open exchange, just like regular stocks. Index mutual funds are priced at the end of the day. ETFs are more flexible and convenient. However, in certain 401k and 403b accounts, ETFs are not feasible.*

Lecture plan

- Derivatives

- Measuring returns over different holding periods

Derivatives

- Derivative contracts, e.g., futures and options, provide payoffs that depend on the values of other variables such as commodity prices, bond and stock prices, interest rates, or market index values

- Their values *derive from* the values of other assets

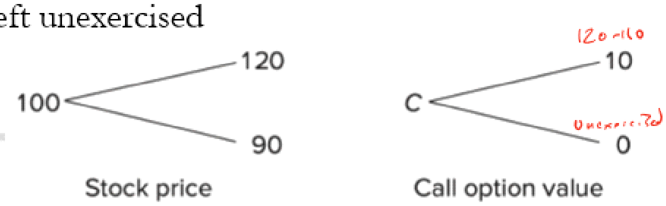
- Also called contingent claims because their payoffs are contingent on the value of other values

Call options

- A call option gives its holder the right to *purchase* an asset for a specified price, called the exercise or strike price, on or before a specified expiration date
  - The holder of the call only exercises the option when the market value of the asset exceeds the exercise price

Will focus on Euro → European: Can only exercise on day of expiration   American: Can exercise at any point before expiration

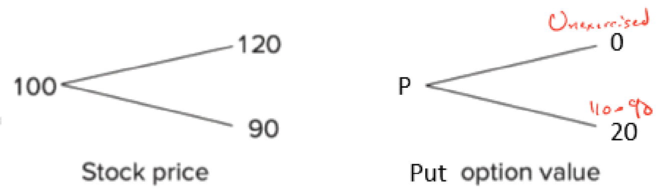
- Example: a February expiration call option on Microsoft stock with an exercise (or strike) price of $110
  - The current stock price is $100
  - If the stock price rises to $120, the holder exercises and earns $10
  - If the stock price falls to $90, the call is left unexercised



Put options

- A put option gives its holder the right to *sell* an asset for a specified exercise price on or before a specified expiration date
  - The holder of the put only exercises the option when the market value of the asset falls below the exercise price

- Example: a February expiration put option on Microsoft stock with an exercise (or strike) price of $110
  - The current stock price is $100
  - If the stock price rises to $120, the put is left unexercised
  - If the stock price falls to $90, the holder exercises and earns $20



Question

- What would be the profit or loss to an investor who bought the January 2019 expiration Apple call option with exercise price $105 if the stock price at the expiration date is $109?

109 − 105 = $4

- What about a purchaser of the put option with the same exercise price and expiration?

105 − 109 = −$4 if exercised, but would not be ∴ $0

Valuation of options

- The payoff of the options is always non-negative, so there is a cost or *premium* to own the options
- We will learn option pricing at the end of this semester
- For both call and put options, each option contract is for the purchase of 100 shares. However, quotations are made on a per-share basis

To hold an option, cost $7.65 per share

Table 2.6 Prices of stock options on Microsoft, January 2, 2019

Expiration	Strike	Call	Put
18-Jan-2019	95	7.65	0.98
18-Jan-2019	100	3.81	2.20
18-Jan-2019	105	1.45	4.79
8-Feb-2019	95	9.50	2.86
8-Feb-2019	100	5.60	3.92
8-Feb-2019	105	3.08	6.35

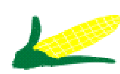
Note: Microsoft stock on this day was $101.51.
Source: Compiled from data downloaded from Yahoo! Finance.

Futures

- A futures contract calls for delivery of an asset at a specified maturity date for an agreed-upon price, called the futures price, to be paid at contract maturity
- Two parties:
  - *Long position* commits to purchasing the asset on the delivery date
  - *Short position* commits to delivering the asset at contract maturity

Example

- A future contract calls for delivery of 5,000 bushels of corn in March 2019 at the price \$3.8025 per bushel



- Suppose at contract maturity, corn is selling for \$3.8225 per bushel
- The profit to the long position: $5{,}000 \times (\$3.8225 - \$3.8025) = \$100$ The short can buy corn @ a cheaper price if it occurs to folfill the future.
- What is the loss to the short position? -\$100

Options vs futures

- The long position of a futures contract: *obliges* to purchase the asset at the futures price
- The long position (owner) of a call option: *conveys the right* to purchase the asset at the exercise price
- With the same futures price and option's exercise price, the call holder has a better position
  - Call options must be purchased
  - Futures contracts can be entered into without cost

Lecture plan

- Derivatives

Done w/ Asset classes (e.g. stocks, bonds, equities, derivatives)

- Measuring returns over different holding periods

Rate of return

- Rate of return: the percentage change in the value of an investment

- Example: for a zero-coupon bond with maturity date T and par value $100 with the current price $P(T)$, rate of return over the holding period is

expected return

$$r(T) = \frac{100}{P(T)} - 1$$

current price of asset

- Rate of return generally increases with T

Horizon, T	Price, $P(T)$	$r(T) = \frac{100}{P(T)} - 1$
Half-year	\$97.36	100/97.36 − 1 = 0.0271 = 2.71%
1 year	\$95.52	100/95.52 − 1 = 0.0469 = 4.69%
25 years	\$23.30	100/23.30 − 1 = 3.2918 = 329.18%

Rate of return over a common period

- Effective annual rate (EAR): the percentage increase in funds *per year*
  - Compare returns on investments with differing horizons
  - Accounts for compound return/interest: the cumulative effect that a series of gains or losses has on an amount of capital over time

assumes reinvestment of gains

$$EAR = \left(\frac{\text{Return}}{\text{price}}\right)^{1/\#\text{ of years}} - 1$$

Year 0 — 6 mons — Year 1

\$97.36 — \$100 — \$102.71

$$EAR = \frac{102.71}{97.36} - 1 = 0.0549$$

Horizon, T	Price, $P(T)$	$r(T) = \frac{100}{P(T)} - 1$	EAR over Given Horizon
Half-year	\$97.36	100/97.36 − 1 = 0.0271 = 2.71%	$(1 + .0271)^2 - 1 = .0549$
1 year	\$95.52	100/95.52 − 1 = 0.0469 = 4.69%	$(1 + .0469) - 1 = 0.0469$
25 years	\$23.30	100/23.30 − 1 = 3.2918 = 329.18%	$(1 + 3.2918)^{1/25} - 1 = .060$

Annual percentage rates

- For short-term investments (with holding periods less than a year), rates of return are often annualized using simple interest that ignores compounding

APR = gains w/o reinvesting

EAR = gains w/ reinvesting interest

- These are called annual percentage rates (APR)
- With n compounding periods per year, we can find EAR from APR by

$$1 + EAR = \left(1 + \frac{APR}{n}\right)^n$$

- Example: If $APR = 18\%$ and a compounding period is a month, then

$$EAR = \left(1 + \frac{18\%}{12}\right)^{12} - 1 = 19.56\%$$

EAR vs APR

- The difference between APR and EAR grows with the frequency of compounding
- Suppose $APR = 18\%$.
  - If the compounding period is a month, then
    $$EAR = \left(1 + \frac{18\%}{12}\right)^{12} - 1 = 19.56\%$$
  - If the compounding period is a week, then
    $$EAR = \left(1 + \frac{18\%}{52}\right)^{52} - 1 = 19.68\%$$
  - If the compounding period is a day, then
    $$EAR = \left(1 + \frac{18\%}{365}\right)^{365} - 1 = 19.72\%$$

The faster the compounding the more the interest can compound => more yield

EAR vs APR

- The difference between APR and EAR grows with the interest rate per period (e.g., month)

- Suppose the compounding period is a month.
  - If $APR = 6\%$, then
  $$EAR = \left(1 + \frac{6\%}{12}\right)^{12} - 1 = 6.17\%$$
  - If $APR = 18\%$, then
  $$EAR = \left(1 + \frac{18\%}{12}\right)^{12} - 1 = 19.56\%$$
  - If $APR = 48\%$, then
  $$EAR = \left(1 + \frac{48\%}{12}\right)^{12} - 1 = 60.10\%$$

diff between EAR and APR grows w/ APR exponentially

Continuous compounding

- As the number of compounding periods n gets larger, we effectively approach *continuous compounding (CC)*
- The continuous compounding rate r_{cc} is defined as the rate that satisfies

$$e^{r_{cc}} = \lim_{n\to\infty}\left(1 + \frac{APR}{n}\right)^{n}$$

where $e = 2.71828$ is the Euler's number

- Using the property $\lim_{x\to\infty}(1+x)^{1/x} = e$, we can solve r_{cc}

$$r_{cc} = \lim_{n\to\infty}\log\left(1 + \frac{APR}{n}\right)^{n} = \lim_{n\to\infty}\log\left(\left(1 + \frac{APR}{n}\right)^{n/APR}\right)^{APR} = \lim_{n\to\infty}\log e^{APR} = APR$$

Total return given continuous compounding

- Given a continuously compounded rate r_{cc}, the total return for any period T is

$$\exp(T \times r_{cc})$$

  - Simplify the calculation
  - For example, $APR = 18\%$ and the investment period is 2 years. Then

$$\lim_{n\to\infty}\left(1+\frac{APR}{n}\right)^{2n} = (\exp(r_{cc}))^2 = \exp(2r_{cc}) = \exp 0.36 = 1.4333$$

Rate of return is 43.33%

Question

- Question: A bank offers two alternative interest schedules for a savings account of $\$100{,}000$ locked in for 3 years: (a) a monthly rate of 1% and (b) an annually, continuously compounded rate, r_{cc} of 12%. Which alternative should you choose?