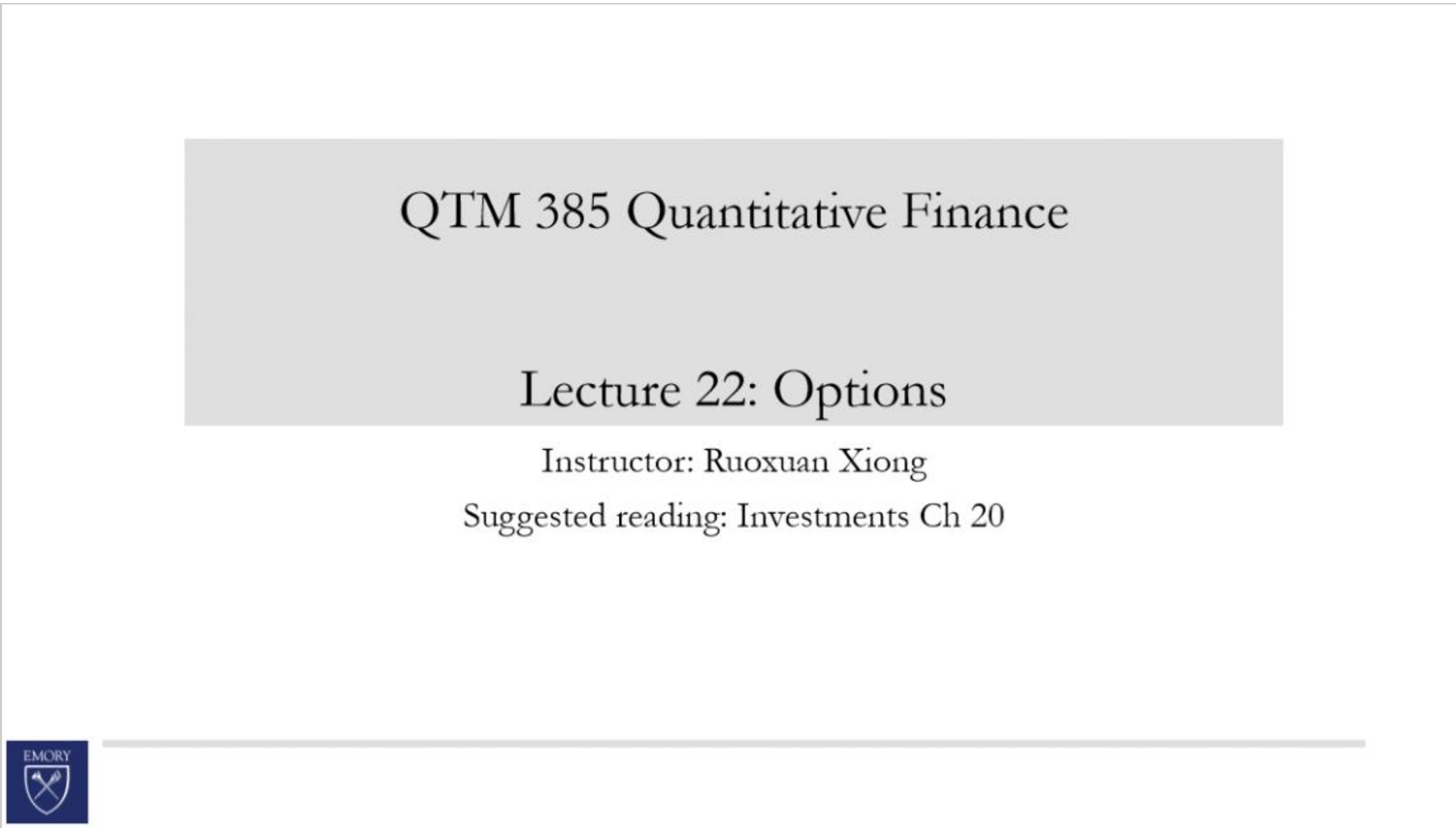
Lecture 22: Options

Wednesday, April 12, 2023 23:47

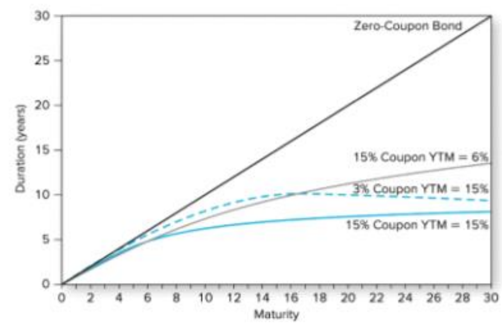
22-option



Question from Google form

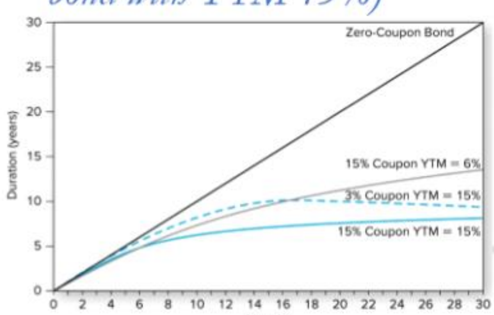
- Question: Could you explain more about why duration need not always increase with time to maturity?

- Answer: Duration need not always increase with time to maturity. Example, 3% coupon bond with YTM 15% (deep discount bond)



Question from Google form

- Question: Could you explain more about why duration need not always increase with time to maturity?
- *Answer: This happens for deep discount bond (3% bond with YTM 15%)*



	Period	Time until payment (Years)	1.5% coupon Cash Flow	PV of CF (Discount rate = 7.5% per per	Weight*	Column (C) times Column (F)
8% coupon b	1	0.5	15	13.0435	0.1148	0.0574
	2	1	15	11.3422	0.0998	0.0998
	3	1.5	15	9.8627	0.0868	0.1302
	4	2	15	8.5763	0.0755	0.1510
Sum	5	2.5	15	7.4577	0.0657	0.1641
	6	3	15	6.4849	0.0571	0.1713
	7	3.5	15	5.6391	0.0496	0.1737
	8	4	15	4.9035	0.0432	0.1727
	9	4.5	15	4.2639	0.0375	0.1689
	10	5	15	3.7078	0.0326	0.1632
	11	5.5	15	3.2241	0.0284	0.1561
	12	6	15	2.8036	0.0247	0.1481
	13	6.5	15	2.4379	0.0215	0.1395
	14	7	15	2.1199	0.0187	0.1306
	15	7.5	15	1.8434	0.0162	0.1217
	16	8	15	1.6030	0.0141	0.1129
	17	8.5	15	1.3939	0.0123	0.1043
	18	9	15	1.2121	0.0107	0.0960
	19	9.5	15	1.0540	0.0093	0.0881
	20	10	15	0.9165	0.0081	0.0807
	21	10.5	15	0.7970	0.0070	0.0737
	22	11	15	0.6930	0.0061	0.0671
	23	11.5	15	0.6026	0.0053	0.0610
	24	12	15	0.5240	0.0046	0.0554
	25	12.5	15	0.4557	0.0040	0.0501
	26	13	15	0.3962	0.0035	0.0453
	27	13.5	15	0.3445	0.0030	0.0409
	28	14	15	0.2996	0.0026	0.0369
	29	14.5	15	0.2605	0.0023	0.0333
	30	15	1015	15.3296	0.1350	2.0243
				113.5927	1.0000	5.1186
annual YTM	0.15					

15 year bond

	Period	Time until payment (Years)	1.5% coupon Cash Flow	PV of CF (Discount rate = 7.5% per per	Weight*	Column (C) times Column (F)
8% coupon b	1	0.5	15	13.0435	0.1106	0.0553
	2	1	15	11.3422	0.0961	0.0961
	3	1.5	15	9.8627	0.0836	0.1254
	4	2	15	8.5763	0.0727	0.1454
Sum	5	2.5	15	7.4577	0.0632	0.1580
	6	3	15	6.4849	0.0550	0.1649
	7	3.5	15	5.6391	0.0478	0.1673
	8	4	15	4.9035	0.0416	0.1663
	9	4.5	15	4.2639	0.0361	0.1626
	10	5	15	3.7078	0.0314	0.1571
	11	5.5	15	3.2241	0.0273	0.1503
	12	6	15	2.8036	0.0238	0.1426
	13	6.5	15	2.4379	0.0207	0.1343
	14	7	15	2.1199	0.0180	0.1258
	15	7.5	15	1.8434	0.0156	0.1172
	16	8	15	1.6030	0.0136	0.1087
	17	8.5	15	1.3939	0.0118	0.1004
	18	9	15	1.2121	0.0103	0.0925
	19	9.5	15	1.0540	0.0089	0.0849
	20	10	15	0.9165	0.0078	0.0777
	21	10.5	15	0.7970	0.0068	0.0709
	22	11	15	0.6930	0.0059	0.0646
	23	11.5	15	0.6026	0.0051	0.0587
	24	12	15	0.5240	0.0044	0.0533
	25	12.5	15	0.4557	0.0039	0.0483
	26	13	15	0.3962	0.0034	0.0437
	27	13.5	15	0.3445	0.0029	0.0394
	28	14	1015	20.2734	0.1718	2.4058
	29	14.5	0	0.0000	0.0000	0.0000
	30	15	0	0.0000	0.0000	0.0000
				117.9764	1.0000	5.3176
annual YTM	0.15					

14 year bond

American option vs European options

- **American option:** gives its holder the right to **purchase/sell** an asset for a specified price, called the exercise or strike price, *on or before* some specified expiration date
  - Most traded options in the United States
- **European call:** allows for exercise *only on* the expiration date
- American options are generally more valuable than European options

- **Premium**: Purchase price of the option
- **Holders** of call options: Pays premium to hold the option now
- **Sellers/Writers** of call options: Receives premium income now

Call option

- **Call option:** gives its holder the right to **purchase** an asset for a specified price, called the exercise or strike price, *on or before (or only on)* some specified expiration date

- Stock price at expiration time T: S_T
- Exercise price: X

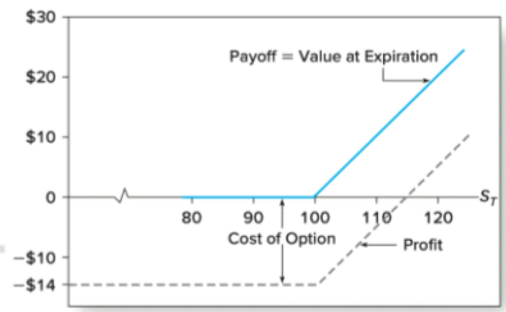
Values of call option at expiration

- The value of the call at expiration to call holder equals

$$payoff\ to\ call\ holder = \begin{cases} S_T - X & if\ S_T > X \\ 0 & if\ S_T \le X \end{cases}$$

- Example: You hold a call option on FinCorp stock with an exercise price of $X = \$100$. At expiration is
  - $S_T = \$90$: Option value $= 0$, profit $= 0 - 14 = -14$
  - $S_T = \$100$: Option value $= 0$, profit $= -14$
  - $S_T = \$110$: Option value $= 10$, profit $= -4$
  - $S_T = \$120$: Option value $= 20$, profit $= 6$
- Purchasing the call is a bullish strategy

(bet on stock price increase)



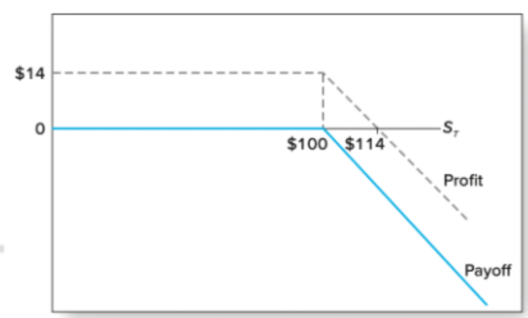
Values of call option at expiration

- Conversely, the value to the call writer is

$$payoff\ to\ call\ writer = \begin{cases} -(S_T - X) & if\ S_T > X \\ 0 & if\ S_T \le X \end{cases}$$

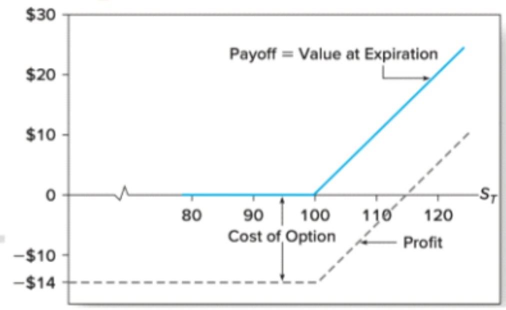
- Example: You write a call on FinCorp stock with an exercise price of $X = \$100$. At expiration is
  - $S_T = \$90$: Option value $= 0$, profit $= 14$
  - $S_T = \$100$: Option value $= 0$, profit $= 14$
  - $S_T = \$110$: Option value $= 10$, profit $= 4$
  - $S_T = \$120$: Option value $= 20$, profit $= -6$
- Selling the call is a bearish strategy

(bet on stock price decrease)



Terminology for option values

- **In the money**: The exercise of an option produces a *positive* cash flow
  - E.g., exercise price of a call (e.g., $100) lower than the market price (e.g., $120)
- **Out of the money**: The exercise of an option produces a *negative* cash flow
  - E.g., exercise price of a call (e.g., $100) higher than the market price (e.g., $90)
- **At the money**: The exercise price and asset price are equal

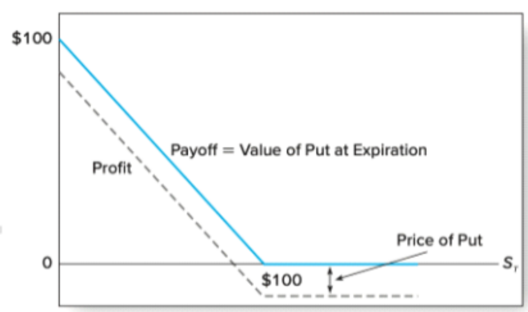


Put option

- **Put option:** gives its holder the right to **sell** an asset for a specified price, called the exercise or strike price, *on or before* (*or only on*) some specified expiration date
- The value of the put at expiration to put holder equals

$$payoff\ to\ put\ holder = \begin{cases} 0 & if\ S_T > X \\ X - S_T & if\ S_T \leq X \end{cases}$$

- Example: You hold a put option on FinCorp stock with an exercise price of $X = \$100$. At expiration is
  - $S_T = \$80$: Option value = 20
  - $S_T = \$90$: Option value = 10
  - $S_T = \$100$: Option value = 0
  - $S_T = \$110$: Option value = 0



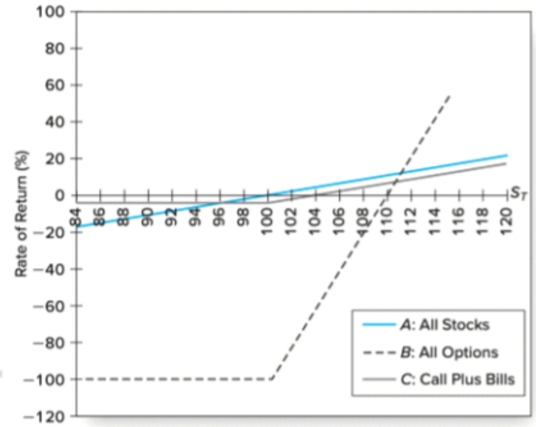
Investments with options

- Consider three strategies with cost $10,000
  - **Strategy A**: Invest entirely in stock. Buy 100 shares, each selling for $100
  - **Strategy B**: Invest entirely in at-the-money call options. Buy 1,000 calls, each selling for $10
  - **Strategy C**: Purchase 100 call options for $1,000. Invest your remaining $9,000 in 1-year T-bills, to earn 3% interest

Value of three strategies

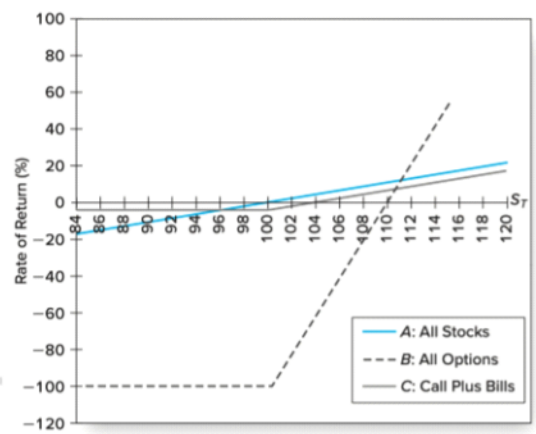
Portfolio	Stock Price $95	$100	$105	$110	$115	$120
Portfolio *A*: All stock	$9,500	$10,000	$10,500	$11,000	$11,500	$12,000
Portfolio *B*: All options	0	0	5,000	10,000	15,000	20,000
Portfolio *C*: Call plus T-bills	9,270	9,270	9,770	10,270	10,770	11,270

Portfolio	Stock Price $95	$100	$105	$110	$115	$120
Portfolio *A*: All stock	−5.0%	0.0%	5.0%	10.0%	15.0%	20.0%
Portfolio *B*: All options	−100.0	−100.0	−50.0	0.0	50.0	100.0
Portfolio *C*: Call plus T-bills	−7.3	−7.3	−2.3	2.7	7.7	12.7



Options for speculation and risk hedging

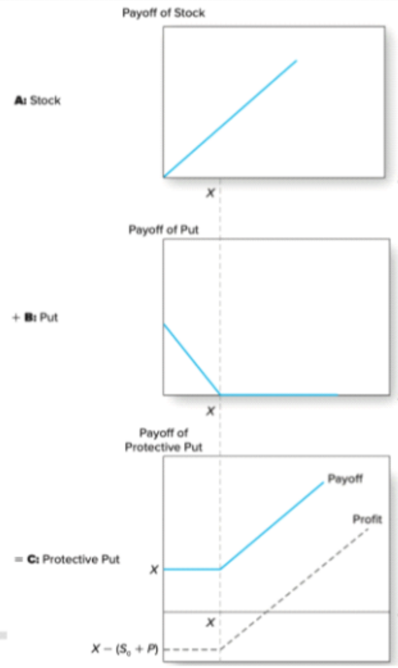
- Options can be used by speculators as effectively leveraged stock positions (like portfolio B, large negative or positive returns)
- Options can be used by investors who desire to tailor their risk exposures in creative ways, e.g., limitation on downside risk (like portfolio C)



Protective put

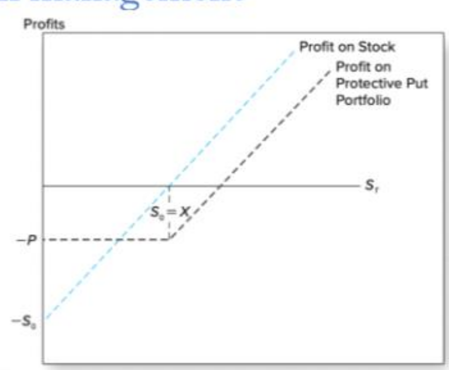
- An unlimited variety of payoff patterns can be achieved by combining puts and calls with various exercise prices
- **Protective put**: *Invest* in stock (stock price at time 0: S_0) and *purchase a put* option on the stock (cost P)
- Suppose the strike of the put is $\$100$
- At expiration, if $X = S_T = \$97$, then the put is worth $X - S_T = \$3$ and portfolio value is $S_T + (X - S_T) = \$97 + \$3 = \$100$
- If stock is worth $S_T = \$104$, the put is worth $\$0$. the portfolio value is $S_T + 0 = \$104$

	$S_T \leq X$	$S_T > X$
Stock	S_T	S_T
+ Put	$X - S_T$	0
Total	X	S_T



Protective put as portfolio insurance

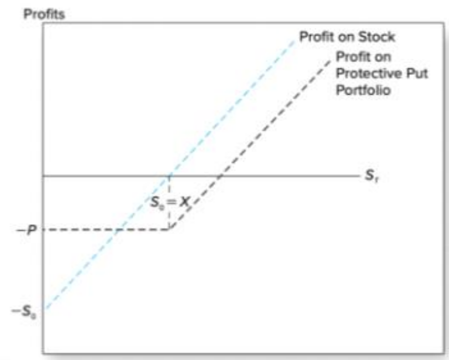
- Protective put provides a form of *portfolio insurance*
- The cost of the protection is that if the stock price increases, your profit is reduced by the amount you spent on the put, which turned out to be unneeded
- Derivative securities can be used effectively for risk management



The put-call parity relationship

- **Protective put**: *invest* in stock and *purchase a put* with exercise price X
- **Call-plus-bills portfolio**: purchase a risk-free zero-coupon bond with face value X and hold a call with exercise price X
- Two portfolios have the same payoff: provide a guaranteed minimum payoff and unlimited upside potential
- Their cost should be the same

	$S_T \le X$	$S_T > X$
Value of call option	0	$S_T - X$
Value of zero-coupon bond	X	X
Total	X	S_T

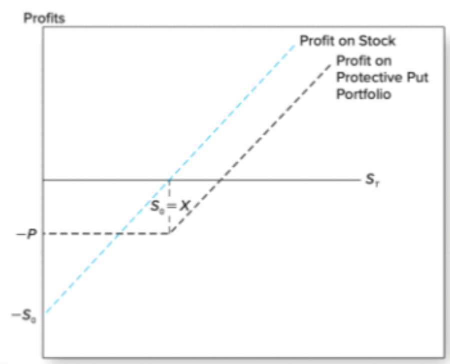


The put-call parity relationship

- The cost of the protective put is $S_0 + P$
- The cost of the call-plus-bills is $C + X/(1 + r_f)^T$
- Their cost should be the same, so the **put-call parity theorem** states

$$C + \frac{X}{(1 + r_f)^T} = S_0 + P$$

	$S_T \le X$	$S_T > X$
Value of call option	0	$S_T - X$
Value of zero-coupon bond	X	X
Total	X	S_T



Question

- Question: Suppose the current stock price is $S_0 = \$110$, 1-year expiration call with exercise price $X = \$105$ sells at $C = \$17$ and 1-year expiration put with exercise price $X = \$105$ sells at $P = \$5$. Suppose the risk-free interest rate is 5%. Is there an arbitrage opportunity?