QTM 385 Final Cathy Zhuang

Monday, May 1, 2023 12:40

Problem 1 (CAPM and APT) [35pts]

Assume the CAPM and its assumptions hold. For every security i, the expected return-beta relationship holds

$$E(r_i) - r_f = \beta_i [E(r_M) - r_f],$$

where

- $\beta_i = Cov(r_i, r_M)/\sigma^2_M$
- r_i is the return of security i
- r_M is the return of the market portfolio
- σ_M is the standard deviation of the return on the market portfolio
- r_f is the risk-free rate.

Additionally, assume there exist 2 securities, A and B. You are provided with the following information concerning the market, the risk-free asset, and securities A and B:

$$E(r_M) = 0.15, \sigma_M = 0.4$$
$$r_f = 0.05$$
$$\sigma_A = 0.2, \rho_{A,M} = 0.5$$
$$\sigma_B = 0.5, \rho_{B,M} = 0.1$$

($\rho_{A,M}$ is the correlation of return of A, r_A, with the return of market portfolio, r_M.) Show all your calculations and justify your answers.

(a) [5pts] What are β_A and β_B?

(b) [5pts] What are risk premium $E(R_A)$ and $E(R_B)$?

(c) [5pts] What are expected return $E(r_A)$ and $E(r_B)$?

(d) [5pts] What is the idiosyncratic risk of securities A and B in terms of variance?

(e) [5pts] Of securities A and B, which one has the highest Sharpe ratio?

(f) [5pts] Assume there exists another additional security C with mean $E[R_C] = 0.1$ and standard deviation $\sigma_C = 0.1$. Is this consistent with the CAPM? Explain your answer.

(g) [5pts] Assume that the CAPM is actually wrong and APT (arbitrage pricing theory) with a two-factor model holds. The second factor is a size factor with expected return $E[r_S] = 0.08$ and standard deviation $\sigma_S = 0.5$. Assume that the market betas are still the same as in part (a), but securities A and B have the following loadings with respect to the value factor $\beta_{S,A} = 0.1$ and $\beta_{S,B} = -0.05$. What are $E[r_A]$ and $E[r_B]$ predicted by APT?

A. $\beta_A = cov(r_M, r_A) / \sigma^2_M$
$= \rho_{A,M} * \sigma_A\sigma_M / \sigma^2_M$
$= 0.5 * 0.2 * 0.4 / (0.4)^2$
$= 0.25 = 25\%$

$\beta_B = cov(r_M, r_B) / \sigma^2_M$
$= \rho_{B,M} * \sigma_B\sigma_M / \sigma^2_M$
$= 0.1 * 0.5 * 0.4 / (0.4)^2$
$= 0.125 = 12.5\%$

B. $E(R_A) = \beta_A[E(r_M) - r_f]$
$= 0.25(0.15 - 0.05)$
$= 0.025 = 2.5\%$

$E(R_B) = \beta_B[E(r_M) - r_f]$
$= 0.125(0.15 - 0.05)$
$= 0.0125 = 1.25\%$

C. $E(r_A) = r_f + \beta_A[E(r_M) - r_f]$
$= 0.05 + 0.25(0.15 - 0.05)$
$= 0.075 = 7.5\%$

$E(r_B) = r_f + \beta_B[E(r_M) - r_f]$
$= 0.05 + 0.125(0.15 - 0.05)$
$= 0.0625 = 6.25\%$

D. $\sigma^2_{(eA)} = \sigma^2_A - \beta^2_A\sigma^2_M$
$= (0.2)^2 - 0.25^2*(0.4)^2$
$= 0.03$

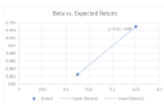
$\sigma^2_{(eB)} = \sigma^2_B - \beta^2_B\sigma^2_M$
$= (0.5)^2 - 0.125^2*(0.4)^2$
$= 0.2475$

E. Sharpe ratio A = risk premium / σ_A
$= 0.025 / 0.2$
$= 0.125$

Sharpe ratio B = risk premium / σ_B
$= 0.0125 / 0.5$
$= 0.025$

Security A has the highest Sharpe ratio.

F. If CAPM holds, then the beta and expected return should fall on this same line. So if our risk premium is 0.1, that means our expected return is the r_f + risk premium, which is 0.05 + 0.1 = 0.15.



This means that our beta must equal 0.5 if CAPM still holds (so it is on the same line).
Then, we also know that the variance must equal to $\beta^2_C\sigma^2_M = (0.5)^2(0.4)^2 = 0.04$
The standard deviation is then the square root of the variance, which is 0.2. Since our given standard deviation was 0.1, not 0.2, we can infer that it is not consistent with CAPM.

G. $E(r_A) = r_f + \beta_A(E(r_M) - r_f) + \beta_{S,A}*E(r_S)$
$E(r_A) = 0.05 + 0.25(0.15 - 0.05) + 0.1*0.08 = 0.083$
$= 8.3\%$

$E(r_B) = r_f + \beta_B(E(r_M) - r_f) + \beta_{S,B}*E(r_S) +$
$E(r_B) = 0.05 + 0.125(0.15 - 0.05) - 0.05*0.08 = 0.0585$
$= 5.85\%$

Problem 2 (Bond Pricing, YTM, YTC) [10pts]

Suppose Berkshire Hathaway Energy Co issues a 30-year maturity 4.600% coupon bond paying coupons semi-annually with par value \$1,000. The bond currently sells at a yield to maturity of 5.005%.

(a) [5pts] What is the price of the bond currently selling?

(b) [5pts] Suppose this bond is callable in 20 years with call price \$1,000. Calculate the yield to call.

A. The price is 93.7446% of the par, which is 0.937446* 1000 = 937.446.

	varying maturity date
Settlement date	1/1/2000
Maturity date	1/1/2030
Annual coupon rate	0.046
Yield to maturity	0.05005
Redemption value (% of face value)	100
Coupon payments per year	2
Flat price (% of par)	93.7446
Days since last coupon	0
Days in coupon period	182
Accrued interest	0.0000
Invoice price	93.7446

B. Yield to call is 0.051.

	Semiannual coupons
Settlement date	1/1/2000
Maturity date	1/1/2020
Annual coupon rate	0.046
Flat price (% of par)	93.7446
Redemption value (% of face value)	100
Coupon payments per year	2
	=YIELD(settlement date, matur
Yield to maturity (decimal)	0.0510

Problem 3 (Term structure and sensitivity) [30pts]

The following table gives the term structure of interest rates in USD. All rates for this question are annually compounded.

Year	1/2	1	3/2	2	3	4	5
USD spot rate	4.94%	4.63%	4.31%	3.97%	3.64%	3.53%	3.45%

Table 1: US \$ spot rates as of 04/25/2023

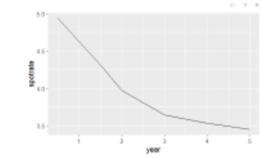
(a) [5pts] Sketch the spot rate curve. What does this shape imply about the expected dynamics of rates in the next a few years?

(b) [5pts] What is the implied forward rate f_2 between 1 and 2 years?

(c) [5pts] Suppose the liquidity premium is 1%. What is the holding period return if the investor holds the 3-year zero for one year?

(d) [15pts] Suppose the yield-to-maturity for the 3-year zero with par value \$1,000 falls by 1%. Calculate the change (in dollars) in the price this bond currently sells by the duration rule and by the modified duration with convexity rule. Compare the changes calculated by the rules with the actual change. Which rule is more accurate and explain why?

a. The downwards sloping shape implies that people expect that the future interest rate will fall. This means that for longer term bonds, you don't need to offer a high coupon rate in the future to make it appealing for people.



b. $f_2 = [(1+y_2)^2 / (1 + y_1)^1] -1$
$f_2 = [(1+0.0397)^2 / (1 + 0.0463)^1] -1$
$= 0.03314163242$
$= 0.033$

c. Let's say we are looking at years 1-4. The forward rate year 1 to year 2 is 3.3%. Then the expected short rate is 3.3 - 1 =2.3%.
Then the price at the end of year 1 is 100/(1+0.023)= 97.7517

Let's say the bond sells at 100/(1+0.0353)^2 = 93.297
The HPR is then (97.7517/ 93.297) - 1 = 0.0477 so 4.77%

d.

Inputs	
Settlement date	1/1/2000
Maturity date	1/1/2003
Annual coupon rate	0
Yield to maturity	0.0364
Coupon per year	1
Outputs	
Macaulay duration	3.0000
Modified duration	2.8946

Initial Duration = 3.000
Initial modified duration = 2.8946
Convexity = 11.00377

Then the prices are:
ΔP/P = - D * Δy
ΔP/P = -D * Δy + 1/2 * convexity * Δy^2
P` = P * (1+ ΔP/P)

Duration price:
ΔP/P = - 3 * -0.01 = 0.03 --> 3% price increase
P` = 89.8293 * (1+ 0.03) = 92.5242

Modified duration with convexity price:
ΔP/P = -3 * -0.01 + 1/2 * 11.00377 * (-0.01)^2 = 0.03055 --> 3.055% price increase
P` = 89.8293 * (1+ 0.03055) = 92.57359

Settlement date	1/1/2000
Maturity date	1/1/2003
Annual coupon rate	0
Yield to maturity	0.0364
Redemption value (% of face value)	100
Coupon payments per year	1
Flat price (% of par)	89.8293

The actual price increase is 2.95%.

	1% change in YTM
Settlement date	1/1/2000
Maturity date	1/1/2003
Annual coupon rate	0
Yield to maturity	0.02640
Redemption value (% of face value)	100
Coupon payments per year	1
Flat price (% of par)	92.4805
Percentage change in bond price	0.029514[illegible]

Both rules resulted in an overestimate, but the original duration estimate was a little closer. This is because the modified duration with convexity tends to underestimate the risk and overestimate the price. However, in most cases, the modified duration with convexity may be closer to the actual than the original duration estimate because we have accounted for convexity.

Problem 4 (Option) [25pts]

Suppose a stock is currently selling at $S_0 = 50$. In one period, the stock price is equally likely to be 60 and 40. We would like to price a call option with an exercise price of 55 that expires in one period. The risk-free interest rate is $r = 0.05$.

(a) [5pts] What is the hedge ratio of the call?

(b) [5pts] Form a hedged portfolio that consists of one share of stock and has (nonrandom) payoff in one period. What is the (nonrandom) payoff to this portfolio in one period?

(c) [5pts] What is the present value of this hedged portfolio?

(d) [5pts] Solve for the value of the call.

(e) [5pts] Use the put-call parity theorem to calculate the price of the put with the same exercise price and expiration date.

A. The range of S is 60 - 40 = 20
The range of C is 60 - 55 = 5

The hedge ratio is 5/20=0.25

B. The payoff when the stock is down is 0.25 * 40 - 0 = 10
The payoff when the stock is up is 0.25 * 60 - 5 = 10

C. The present value of this hedged portfolio is 10/(1+0.05)=9.524

D. The value of the call is 0.25 * 50 - 9.5238 = 2.976

E. -S0 + C + X/(1+rf)^T = P
-50 + 2.976 + 55/(1+0.05) = 5.357