Lecture 14: APT

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QTM 385 Quantitative Finance

Lecture 14: APT

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Suggested reading: Investments Ch 10

Multi-factor model

- A description of the factors that affect security returns

$$R_i = E(R_i) + \beta_{i,GDP} \cdot GDP + \beta_{i,IR} \cdot IR + e_i$$

- There is no "theory" in the equation
- Where $E(R_i)$ comes from? What determines a security's expected excess rate of return?
- Arbitrage pricing theory helps to determine $E(R_i)$ in equilibrium

CAPM relied on a lot of assumptions that might not hold in reality

Two factors that may affect returns of assets or portfolios
- Expected return of security i
- Then GDP
- IR is interest rate (realized interest rate minus people's expectations = how much interest rate deviates from the mean)

Arbitrage pricing theory

- Stephen Ross developed the arbitrage pricing theory (APT) in 1976
- Like the CAPM, the APT predicts a security market line linking expected returns to risk
- Three key propositions
  1. Security returns can be described by a factor model
  2. There are sufficient securities to diversify away idiosyncratic risk
  3. Well-functioning security markets do not allow arbitrage opportunities to persist

APT = arbitrage pricing theory

APT WORKS FOR PORTFOLIO RETURN BUT NOT FOR INDIVIDUAL ASSETS
CAPM WORKS FOR BOTH

APT - results are weaker because only work for portfolio

NO ARBITRAGE OPPORTNITY

Law of one price

- If two assets are equivalent in all economically relevant respects, then they should have the same market price

E.g. Apple is traded on both NASDAQ and NYSE

Law of one price

- If two assets are equivalent in all economically relevant respects, then they should have the same market price
- If they observe a violation of the law, they will engage in arbitrage activity—simultaneously buying the asset where it is cheap and selling where it is expensive
  - E.g., different prices of a stock on two different exchanges
  - Involve long–short positions

E.g. Apple is traded on both NASDAQ and NYSE
Apple stock is identical on both
If the prices are different, then arbitrage opportunity exists -> violation of the law of one price

Diversification in a single-factor security market

- If a portfolio is well diversified, its firm-specific or nonfactor risk becomes negligible
- A portfolio with n stocks, each with weight w_i and $\sum_i w_i = 1$

$$R_P = E(R_P) + \beta_P F + e_P$$

  - $\beta_P = \sum w_i \beta_i$
  - $E(R_P) = \sum w_i E(R_i)$
  - Nonsystematic return $e_P = \sum w_i e_i$
- Portfolio variance can be decomposed into two parts

$$\sigma_P^2 = Var(\beta_P F + e_P) = \beta_P^2 Var(F) + Var(e_P)$$

  - If the portfolio is equally weighted $w_i = 1/n$, then

$$Var(e_P) = Var(\sum w_i e_i) = \sum w_i^2 Var(e_i) = \frac{1}{n}\sum \frac{Var(e_i)}{n} = \frac{1}{n}\overline{Var(e_i)} \to 0 \Rightarrow e_P \to 0$$

Capital R is excess return
E(Rp) is risk premium

F is the factor? It is a concatenation of GDP and IR

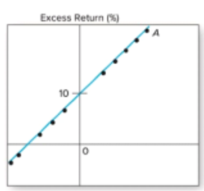
Residual is orthogonal with X
X is uncorrelated with E

Well-diversified portfolios

- For a well-diversified portfolio

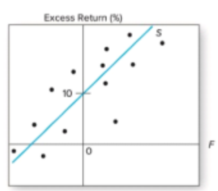
$$R_P = E(R_P) + \beta_P F$$

Portfolio return



$R_A = 10\% + 1.0 \times F$

Stock return



$R_S = 10\% + 1.0 \times F + e_S$

IF we plot all the portfolio returns versus the F and the beta, we have such a graph

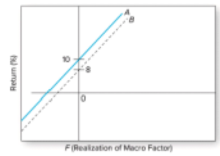
Intercept term is 10 meaning E(Rp) is 10
Beta p = 1
If factor has different values, then return is proportional to that and + 10

The security market line of the APT

- Only the systematic or factor risk of a portfolio of securities should be related to its expected returns

- All well-diversified portfolios with the same beta must have the same expected return

- Otherwise, an arbitrage opportunity exists
  - sell short $1 million of B
  - buy $1 million of A
  - a zero-net-investment strategy



The security market line of the APT

- Their risk premiums must be proportional to beta
  - Otherwise, an arbitrage opportunity exists

- If the excess return on a well-diversified portfolio R follows

$$R_P = E(R_P) + \beta_P F$$

- Then the risk premium is

$$E(R_P) = \beta_P E(F)$$

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E(Rp) = Beta_p * E(Rm)

This beta in CAPM is equal to Cov(Rp, Rm)/Var(Rm)

Single-factor APT vs CAPM

- The APT serves many of the same functions as the CAPM

- APT gives us a benchmark for rates of return

- APT highlights the crucial distinction between nondiversifiable risk (factor risk), which requires a reward in the form of a risk premium, and diversifiable risk, which does not

- APT does not require that almost all investors be mean-variance optimizers. Rely on a highly plausible assumption that precludes arbitrage opportunities

In CAPM, we all assume investors are rational, lol

APT relies on no arbitrage opportunity

Single-factor APT and CAPM

- The CAPM provides a statement on the expected return–beta relationship for all securities

- APT implies that this relationship holds for all but perhaps a small number of securities, as APT is built on well-diversified portfolios

Multifactor APT and risk premium

- Consider a two-factor model for portfolio P

$$R_P = E(R_P) + \beta_P F = E(R_P) + \beta_{P1} F_1 + \beta_{P2} F_2$$

Prof opinion: this is nicer because we have a richer model to explain why different assets have different returns

  - $F = [F_1, F_2]'$ and $\beta_P = [\beta_{P1}, \beta_{P2}]$
  - For example
    - F_1: the departure of GDP growth from expectations
    - F_2: the unanticipated change in interest rates
  - Factor portfolios: benchmark (well-diversified) portfolios (or tracking portfolios) in the APT, have a beta of one on one of the factors and a beta of zero on any other factor

Beta of one of the factors is 1, and all the other betas is zero

Risk premium for multifactor APT

- $E(R_1) = E(r_1) - r_f$: risk premium of the first factor portfolio
- $E(R_2) = E(r_2) - r_f$: risk premium of the second factor portfolio

- The risk premium of any portfolio P

$$E(R_P) = \beta_P E(F) = \beta_{P1} \cdot E(R_1) + \beta_{P2} \cdot E(R_2)$$
$$\Rightarrow E(r_P) = r_f + \beta_{P1} \cdot \left(E(r_1) - r_f\right) + \beta_{P2} \cdot \left(E(r_2) - r_f\right)$$

Replace capital R with little r and risk free rate
1:1 mapping between risk premium and expected return

Expected return of P being derived from E(Rp)

Example

- Suppose that the two factor portfolios have expected returns $E(r_1) = 10\%$ and $E(r_2) = 12\%$ respectively and $r_f = 4\%$
- A well-diversified portfolio P has $\beta_{P1} = .5$ and $\beta_{P2} = .75$. The expected return of P equals

$$E(r_P) = r_f + \beta_{P1} \cdot (E(r_1) - r_f) + \beta_{P2} \cdot (E(r_2) - r_f)$$
$$= 4 + .5 \times (10 - 4) + .75 \times (12 - 4) = 13\%$$

Fama-French (FF) Three-Factor Model

- Identify the most likely sources of systematic risk: Use firm characteristics for security returns

$$R_{it} = \alpha_i + \beta_{iM} R_{Mt} + \beta_{iSMB} SMB_t + \beta_{iHML} HML_t + e_i$$

  - SMB = Small Minus Big (i.e., the return of a portfolio of small stocks in excess of the return on a portfolio of large stocks)
  - HML = High Minus Low (i.e., the return of a portfolio of stocks with a high book-to-market ratio in excess of the return on a portfolio of stocks with a low book-to-market ratio)

Fama-French (FF) Three-Factor Model

$$R_{it} = \alpha_i + \beta_{iM} R_{Mt} + \beta_{iSMB} SMB_t + \beta_{iHML} HML_t + e_i$$

Book to market ratio is high vs. low

- These variables may proxy for hard-to-measure more-fundamental variables
- HML: Firms with high book-to-market ratios are more likely to be in financial distress
- SMB: Small stocks may be more sensitive to changes in business conditions

Estimating and implementing a three-factor SML

- The equilibrium expected rate of return on Amazon stock

$$r_{amazon,t} - r_{ft} = \alpha_{amazon} + \beta_M(r_{M,t} - r_{ft}) + \beta_{HML}HML_t + \beta_{SMB}SMB_t + e_{amazon,t}$$

  - Risk-free rate $r_{ft} = 1\%$
  - Market risk premium $E(r_M) - r_f = 6\%$
  - Risk premium on SMB $E(R_{SMB}) = E(r_{SMB}) - r_f = 2\%$
  - Risk premium on HML $E(R_{HML}) = E(r_{HML}) - r_f = 2\%$

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Estimating and implementing a three-factor SML

- Fitted three-factor model

$$E(r_{amazon}) = r_f + \beta_M(E(r_M) - r_f) + \beta_{SMB}(E(r_{SMB}) - r_f) + \beta_{HML}(E(r_{HML}) - r_f)$$
$$= 1\% + (1.612\times6\%) + (-.689\times2\%) + (-1.133\times2\%) = 7.028\%$$

  - the considerable hedging value it offers against the size and value risk factors (risk premium lower than the single factor model)

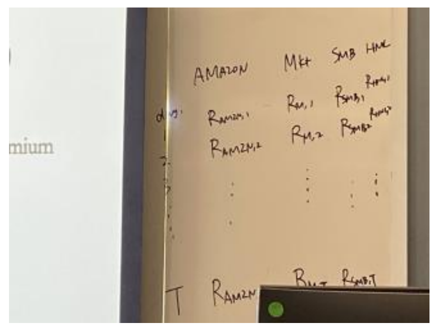
- Fitted single-factor model

$$E(r_{amazon}) = r_f + \beta_M(E(r_M) - r_f)$$
$$= 1\% + (1.533\times6\%) = 10.198\%$$

	Single-Factor Model		Three-Factor Model	
	Regression Coefficient	t-Statistic	Regression Coefficient	t-Statistic
Intercept (alpha)	1.916%	2.065	1.494%	1.790
$r_M - r_f$	1.533	4.865	1.612	5.866
SMB			−0.689	−2.126
HML			−1.133	−3.304
R-square	.286		.455	
Residual std. dev.	6.864%		6.101%	

If we included fewer factors, we would have overestimated amazon returns

Need to run a regression to calculate expected return
Run a time series regression by day



Other factors and smart-beta ETF

- Other factors and smart-beta ETFs: exposure to specific characteristics such as value, growth, or volatility. E.g.,
  - Size (SMB)
  - Value (HML)
  - Momentum (WML, for Winners Minus Losers): the return on a portfolio that buys recent well-performing stocks and sells poorly performing ones
  - Volatility: the standard deviation of stock returns
  - Quality (profitability): the difference in returns of stocks with high versus low return on assets or similar measures of profitability
  - Investment: the difference between returns on firms with high versus low rates of asset growth; and dividend yield

Latent factor model

- Consider a latent multifactor model

$$R_i = E(R_i) + \beta_{i1}F_1 + \beta_{i2}F_2 + \cdots + \beta_{ik}F_k + e_i$$

  - $F_1, F_2, \cdots, F_k$ are unobserved
  - Estimate $F_1, F_2, \cdots, F_k$ from the data (e.g., using principal component analysis)
  - F_1 is strongly correlated with the market factor