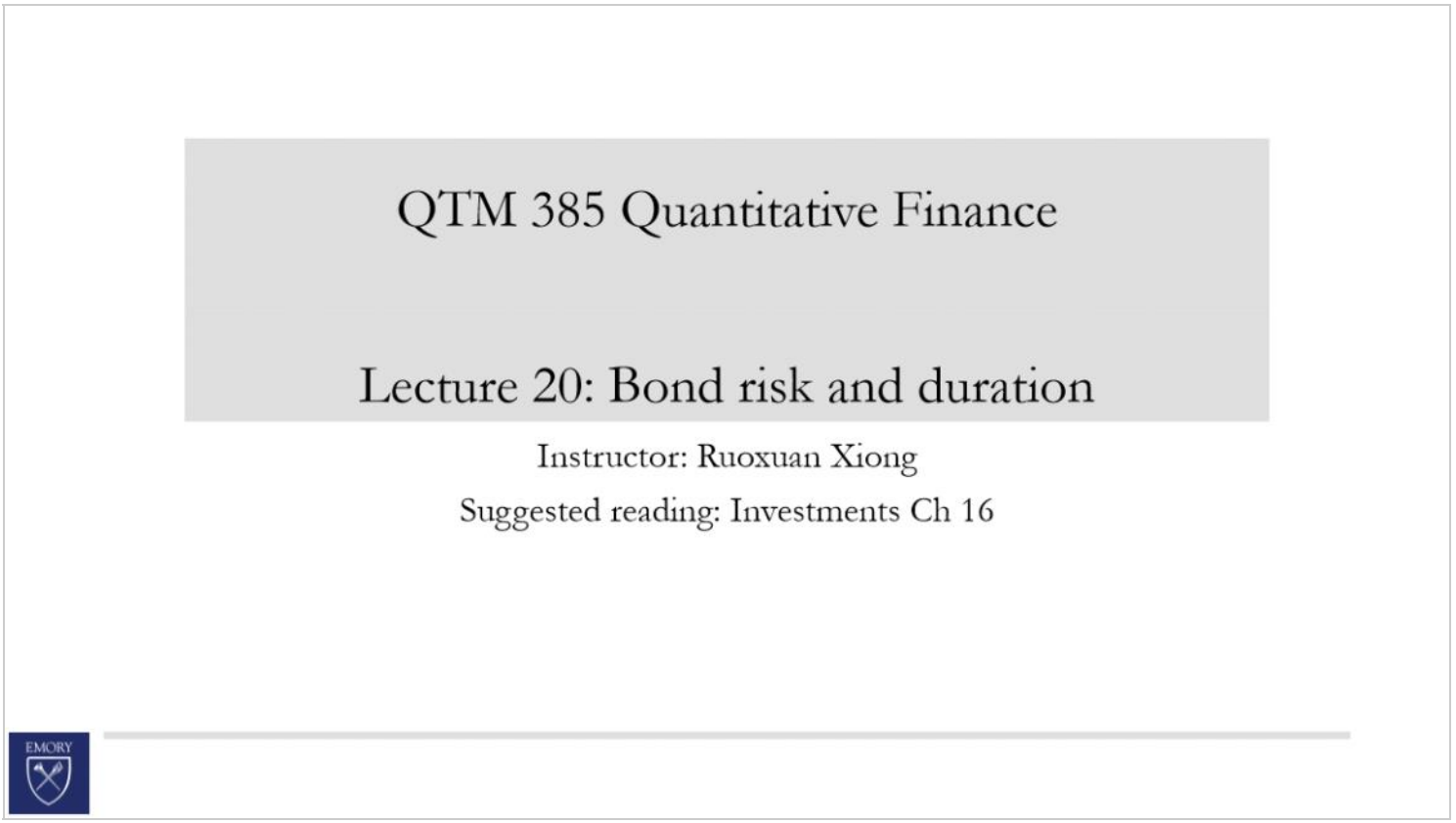
Lecture 20: Bond Risk

Monday, April 10, 2023 14:41

20-bond-risk



Interest rate sensitivity

- Bond *prices* and *yields* are *inversely related*: As yields increase, bond prices fall; as yields fall, bond prices rise
- **Interest rate sensitivity**: The sensitivity of bond prices to changes in market interest rates and yields
- When YTM is y, the bond price is $P = \sum_{t=1}^{T} \frac{Coupon}{(1+y)^t} + \frac{Par\ value}{(1+y)^T}$
- Suppose the YTM changes by Δy. The new YTM is $y + \Delta y$ and the new price is $P + \Delta P$
- The sensitivity can be measure by $\Delta P/P$

Measuring interest rate sensitivity

- **Interest rate sensitivity**: The sensitivity of bond prices to changes in market interest rates and yields
- The sensitivity can be measured by how change in its yield to maturity Δy results in the proportional change in a bond's price $\Delta P/P$
- We can show

$$\frac{\Delta P}{P} = -D \times \frac{\Delta(1+y)}{1+y}$$

where D is the Macaulay's duration (defined in the next slide)

Macaulay's duration

- **Macaulay's duration**: the average of t weighted by the "importance" each coupon or principal payment w_t

$$D = \sum_{t=1}^{T} t \times w_t$$

  - w_t is defined as

$$w_t = \frac{CF_t/(1+y)^t}{P}$$

  - $\sum_{t=1}^{T} w_t = 1$, y is the bond's yield to maturity, and CF_t is the cash flow at time t
  - D is a measure of the average maturity of the bond's promised cash flows

Modified duration and bond sensitivity

- **Modified duration:** $D^* = \frac{D}{1+y}$
- We have

$$\frac{\Delta P}{P} = -D \times \frac{\Delta(1+y)}{1+y} = -D^* \Delta y$$

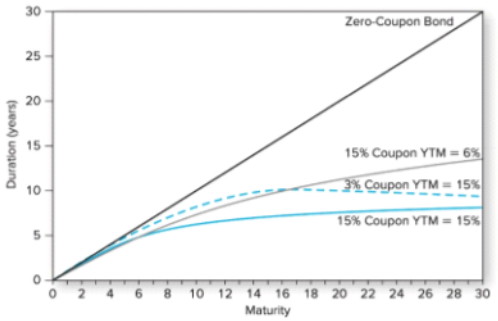
  - by using $\Delta(1+y) = \Delta y$
  - Bond's price sensitivity to interest rate changes generally *increases with duration*

Interest rate sensitivity

- The sensitivity and duration is affected by
  - Yield to maturity
  - Time to maturity
  - Coupon rate

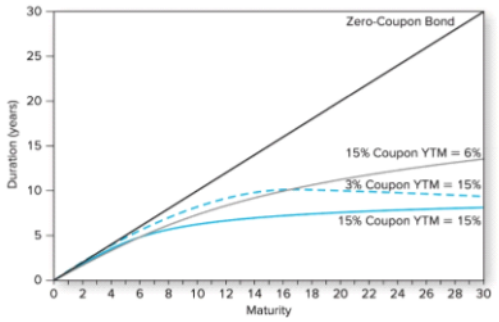
Duration for zero coupon bond

- The duration of a zero-coupon bond equals its time to maturity
  - A coupon bond has a lower duration than a zero with equal maturity
  - $D = \sum_{t=1}^{T} t \times w_t = \sum_{t=1}^{T} t \times \frac{CF_t/(1+y)^t}{Bond\ price} = T \times \frac{CF_T/(1+y)^T}{Bond\ price} = T$



Duration vs YTM

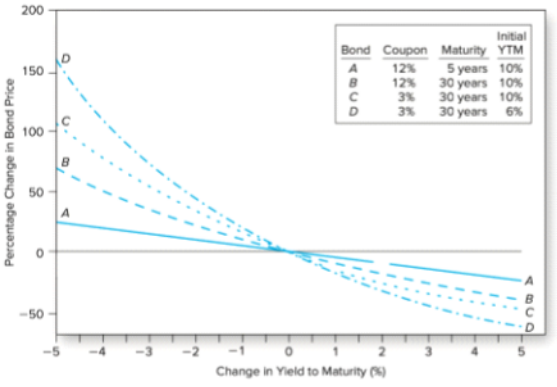
- Holding other factors constant, the duration of a coupon bond is higher when the bond's *yield to maturity* is lower
  - Comparing two 15% coupon bonds, the lower-yield bond has longer duration
  - At lower yields, the more distant payments have relatively greater present values and a greater share of the bond's total value (duration is higher)
  - $D = \sum_{t=1}^{T} t \times \frac{CF_t/(1+y)^t}{Bond\ price}$



Interest rate sensitivity vs YTM

- **Property 1**: The *sensitivity* of a bond's price to a change in its yield is *inversely related* to the *yield to maturity* at which the bond currently is selling
  - Bond C has a higher yield to maturity than bond D
  - Bond C is less sensitive to changes in yields

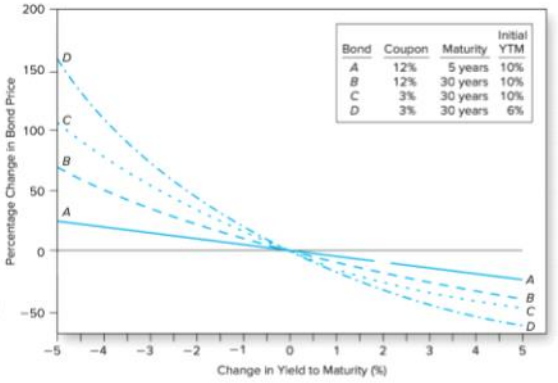
Bond C	changes in YTM	percentage changes in bond pri	Bond D	changes in YTM	percentage changes in bond pri
	-0.05	1.04730338		-0.05	1.594191137
	-0.045	0.880782208		-0.045	1.32754004
	-0.04	0.733066971		-0.04	1.094110533
	-0.035	0.601781555		-0.035	0.889467758
	-0.03	0.484867074		-0.03	0.709791761
	-0.025	0.380538041		-0.025	0.551789966
	-0.02	0.287244751		-0.02	0.412622373
	-0.015	0.203640966		-0.015	0.289837588
	-0.01	0.128556158		-0.01	0.181318099
	-0.005	0.060971633		-0.005	0.085233427
	0	0		0	0
	0.005	-0.055132508		0.005	-0.075753227
	0.01	-0.105101258		0.01	-0.143214256
	0.015	-0.150496341		0.015	-0.203413334
	0.02	-0.191833881		0.02	-0.257244658



Interest rate sensitivity vs YTM

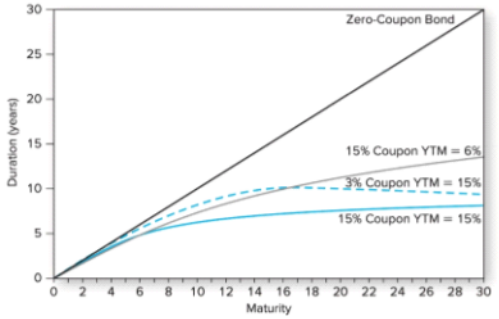
- **Property 2**: An *increase* in a bond's *yield to maturity* results in a *smaller* price change than a *decrease* in *yield* of equal magnitude
  - Intuition: change by increase in interest rate $\propto \frac{1}{1+r} - \frac{1}{1+r+\Delta r} = \frac{\Delta r}{(1+r)(1+r+\Delta r)}$
  - change by decrease in interest rate $\propto \frac{1}{1+r-\Delta r} - \frac{1}{1+r} = \frac{\Delta r}{(1+r)(1+r-\Delta r)}$
  - $\frac{\Delta r}{(1+r)(1+r-\Delta r)} > \frac{\Delta r}{(1+r)(1+r+\Delta r)}$

Bond A	12% coupon, semi-annual bond	Bond A	changes in YTM	percentage changes in bond pri
	5 year maturity bond			
	10% YTM			
Settlement date	11/15/20		-0.05	0.212682137
Maturity date	11/15/25		-0.045	0.188991689
Annual coupon date	0.12		-0.04	0.165879928
Yield to maturity	0.1		-0.035	0.143330884
Redemption value (% of face value)	100		-0.03	0.121329074
Coupon payments per year	2		-0.025	0.09985948
			-0.02	0.078907536
= PRICE(settlement date, maturity date, annual coupon rate, yield to			-0.015	0.058459115
Flat price (% of par)	107.7217		-0.01	0.038500516
			-0.005	0.019018443
			0	0
			0.005	-0.018567326
			0.01	-0.036695677
			0.015	-0.054396836
			0.02	-0.071682237



Duration vs time to maturity

- Holding other factors constant, a bond's *duration generally increases* with its *time to maturity*
  - Duration need not always increase with time to maturity. Example, 3% coupon bond with YTM 15% (deep discount bond)
  - For coupon bonds, duration increases by less than a year with a year's increase in maturity
  - $D = \sum_{t=1}^{T} t \times \frac{CF_t/(1+y)^t}{Bond\ price}$

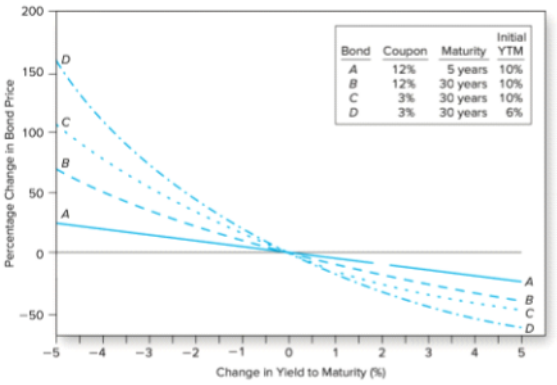


Interest rate sensitivity vs time to maturity

- **Property 3**: Prices of *long-term* bonds tend to be *more sensitive* to interest rate changes than prices of *short-term* bonds
  - Bond B (longer maturity) exhibits greater sensitivity to interest rate changes than bond A (shorter maturity)
  - Intuition: If rate increases, the bond price decreases; the impact is higher for more distant cash flows

Bond A	changes in YTM	percentage changes in bond pri
	-0.05	0.212682137
	-0.045	0.188991689
	-0.04	0.165879928
	-0.035	0.143330884
	-0.03	0.121329074
	-0.025	0.09985948
	-0.02	0.078907536
	-0.015	0.058459115
	-0.01	0.038500516
	-0.005	0.019018443
	0	0
	0.005	-0.018567326
	0.01	-0.036695677
	0.015	-0.054396836
	0.02	-0.071682237

Bond B	changes in YTM	percentage changes in bond pri
	-0.05	0.750454396
	-0.045	0.639408192
	-0.04	0.538953875
	-0.035	0.447898623
	-0.03	0.365196378
	-0.025	0.289928759
	-0.02	0.221288553
	-0.015	0.15856543
	-0.01	0.101133569
	-0.005	0.04844094
	0	0
	0.005	-0.04462038
	0.01	-0.085801929
	0.015	-0.123883071
	0.02	-0.159164236

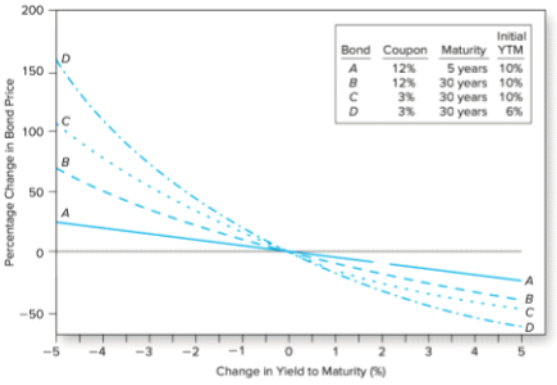


Interest rate sensitivity vs time to maturity

- **Property 4**: The sensitivity of bond prices to changes in yields increases at a decreasing rate as maturity increases. In other words, interest rate risk is *less than proportional* to bond maturity
  - Bond B has six times the maturity of bond A
  - Bond B has less than six times the interest rate sensitivity

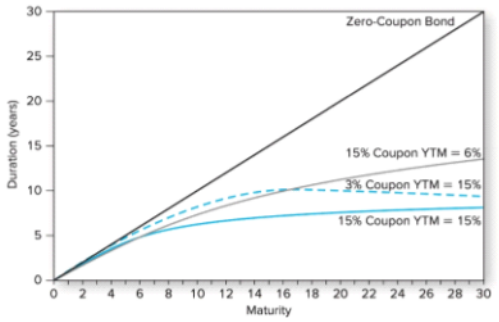
Bond A	changes in YTM	percentage changes in bond pri
	-0.05	0.212682137
	-0.045	0.188991689
	-0.04	0.165879928
	-0.035	0.143330884
	-0.03	0.121329074
	-0.025	0.09985948
	-0.02	0.078907536
	-0.015	0.058459115
	-0.01	0.038500516
	-0.005	0.019018443
	0	0
	0.005	-0.018567326
	0.01	-0.036695677
	0.015	-0.054396836
	0.02	-0.071682237

Bond B	changes in YTM	percentage changes in bond pri
	-0.05	0.750454396
	-0.045	0.639408192
	-0.04	0.538953875
	-0.035	0.447898623
	-0.03	0.365196378
	-0.025	0.289928759
	-0.02	0.221288553
	-0.015	0.15856543
	-0.01	0.101133569
	-0.005	0.04844094
	0	0
	0.005	-0.04462038
	0.01	-0.085801929
	0.015	-0.123883071
	0.02	-0.159164236



Duration vs coupon rate

- Holding maturity constant, a bond's duration is *lower* when the *coupon rate* is *higher*
  - Higher coupon rate bond: A higher fraction of total value is tied up in the earlier coupon payments (less sensitive to yields)
  - Comparing 3% and 15% coupon bonds with YTM 15%



Interest rate sensitivity vs coupon rate

- **Property 5**: Interest rate risk is inversely related to the bond's coupon rate. Prices of *low-coupon* bonds are *more sensitive* to changes in interest rates than prices of high-coupon bonds
  - Bond B has a higher coupon rate than bond C
  - Bond C is more sensitive to changes in interest rate
  - Intuition: price of low coupon bond depends more on distant cash flows (that are more sensitive to interest rate changes)

Bond B	changes in YTM	percentage changes in bond pri	Bond C	changes in YTM	percentage changes in bond pri
	-0.05	0.750454396		-0.05	1.04730338
	-0.045	0.639408192		-0.045	0.880782208
	-0.04	0.538953875		-0.04	0.733066971
	-0.035	0.447898623		-0.035	0.601781555
	-0.03	0.365196378		-0.03	0.484867074
	-0.025	0.289928759		-0.025	0.380538041
	-0.02	0.221288553		-0.02	0.287244751
	-0.015	0.15856543		-0.015	0.203640966
	-0.01	0.101133569		-0.01	0.128556158
	-0.005	0.04844094		-0.005	0.060971633
	0	0		0	0
	0.005	-0.04462038		0.005	-0.055132508
	0.01	-0.085801929		0.01	-0.105101258
	0.015	-0.123883071		0.015	-0.150496341
	0.02	-0.159164236		0.02	-0.191833881