Lecture 17: Bond pricing continued

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17-bond-pricing

QTM 385 Quantitative Finance

Lecture 17: Bond prices

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Suggested reading: Investments Ch 14

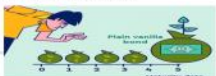
Bond value

- For simplify, assume there is one interest rate r for discounting cash flows of any maturity
- Bond value = Present value of coupons + Present value of par value
  - For a bond with T periods until maturity

$$Bond\ value = \sum_{t=1}^{T} \frac{Coupon}{(1+r)^t} + \frac{Par\ value}{(1+r)^T}$$

$$= Coupon \times \frac{1}{r}\left[1 - \frac{1}{(1+r)^T}\right] + Par\ value \times \frac{1}{(1+r)^T}$$

$$= Coupon \times Annuity\ factor(r,T) + Par\ value \times PV\ factor(r,T)$$



For every bond, it pays coupon rate and then par value

Bond value = What is the value of the bond today?

Yield to maturity

- Yield to maturity: Interest rate that makes the present value of a bond's payments equal to its price
- Suppose an 8% coupon, 30-year semiannual bond is selling at \$1,276.76
- The yield to maturity, denoted by r, is the value that satisfy the equation

$$1276.76 = \sum_{t=1}^{60} \frac{40}{(1+r)^t} + \frac{1000}{(1+r)^{60}}$$

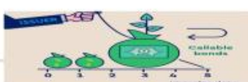
Two methods to solve:

1. Use excel, use YIELD function
2. Numerical approach (bisection method): the key idea is that there is a unique solution such that the left hand side is equal to the right hand side
   - Start with two possible interest rates
   - Very low interest rate -> know right side > left side
   - High interest rate -> know right < left
   - Use an iterative approach, partition intervals by half in every iteration, and evaluate the half point, and continue. After a number of iterations, we can narrow down a tight interval such that the true interest rate lies in that interval.

Yield to call

- Callable bonds allow the issuer to repurchase the bond at a specified call price before the maturity date
- Call protection: an initial time during which the bonds are not callable
- A callable bond with par value \$1,000, an 8% coupon rate, and a 30-year to maturity, but callable at 110% of par value after 3 years
- Suppose the bond calls in n periods, the yield to call, denoted by r, is the value that satisfies

$$Bond\ value = \sum_{t=1}^{n} \frac{Coupon}{(1+r)^t} + \frac{Call\ price}{(1+r)^n}$$



Yield to call = Rate of return for callable bond

Callable bond usually issued by corporations

Because it has the callable feature, the yield is often higher than non-callable bond

Solving yield to call in excel

- A callable bond with par value \$1,000, an 8% coupon rate, and a 30-year to maturity. This bond currently sells at \$1,150 and is called at 110% of par value after 3 years

	A	B
1		Yield to call
2		
3	Settlement date	1/1/15
4	Maturity/Call date	1/1/18
5	Coupon rate	0.08
6	Final payment	110
7	Price	115
8	Coupon payments per year	2
9		
10		= YIELD(settlement date, matu
11	Yield	0.056055224

Settlement date is arbitrary

Price is quoted as a percentage of the par, so 115%

Identical for the way to solve yield to maturity, except replace yield to maturity with n

Question 1

- Question: For a 10-year, 8% coupon semi-annual bonds with call price \$1,100, the yield to maturity is 7%. For simplicity, assume that bonds are called as soon as the present value of remaining payments exceeds the call price. If the market interest rate suddenly falls to 6%, what will be the capital gain the bond?
- Hint:
  - Step 1: calculate the current value of bond with yield to maturity 7%
  - Step 2: calculate the value of bond when interest rate falls to 6%

Use this formula:

$$Bond\ value = \sum_{t=1}^{T} \frac{Coupon}{(1+r)^t} + \frac{Par\ value}{(1+r)^T}$$

Where r is yield to maturity, there are T periods until maturity

Bond value =

107.1062

CORRECT

Then just change yield to maturity to 0.06

varying maturity date
10/24/2022
10/24/2032
0.08
0.07
100
2
r value, number of coupon payments p
107.1062
0
182
0.0000
107.1062

varying maturity date
10/24/2022
10/24/2032
0.08
0.06
100
2
value, number of coupon payments
114.8775
0
182
0.0000
114.8775

Here, the price is higher than the call price, so it will be called and the

Follow up: what if coupon rate is not 8% but 6% instead?

F
varying maturity date
10/24/2022
10/24/2032
0.06
0.07
100
2
alue, number of coupon payments

varying maturity date
10/24/2022
10/24/2032
0.06
0.06
100
2
value, number of coupon paymen

- Step 1: calculate the current value of bond with yield to maturity 7%
- Step 2: calculate the value of bond when interest rate falls to 6%

Then just change yield to maturity to 0.06

	0
	182
	0.0000
	107.1062

HERE, the bond will not be called, because it is only callable at 110% ($1100)

	182
	0.0000
	114.8775

Here, the price is higher than the call price, so it will be called and the issue will pay 110% of the par to the investor

So this is the new value of the bond with the call option, it will stay at 110, not go up to 114

So to calculate the capital gain of the INVESTOR, we calculate the difference between 110 and 107.1062 = 2.8938 %

For the capital gain of the bond issuer, we subtract the 114.8775 by 107.1062 t
will ask about capital gain of the investor

2.8938% * 1000 = $28.9

Question 2

2

- Question: A 10-year maturity 9% semi-annual coupon bond is callable in five years at a call price of $1,050. The bond currently sells at a yield to maturity of 8%. What is the yield to call?

- Hint:
  - Step 1: calculate the current value of bond with yield to maturity 8%
  - Step 2: suppose the bond is called in five years, calculate the yield to call

What is the bond price today?
And then assume that the bond WILL be called in 5 years. Then use the spreadsheet where you solve for yield to call

Yield to call means the yield IF the bond is called (assume called immediately when callable)

F
varying maturity date
10/24/2022
10/24/2042
0.09
0.08
100
2
value, number of coupon payments
109.8964
0
182
0.0000
109.8964

A	B
	Semiannual coupons
Settlement date	1/1/2015
Maturity date	1/1/2020
Annual coupon rate	0.09
Flat price (% of par)	109.8964
Redemption value (% of face value)	105
Coupon payments per year	2
	= YIELD(settlement date, matu
Yield to maturity (decimal)	0.0744

Then you need to divide annual yield by HALF because it is semiannual so the yield to call is 0.037188

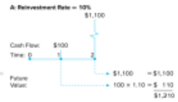
Realized compound return

- After an investment period T, we can calculate the realized compound return. Suppose the initial value is V_0, final value is V_T and realized compound return r is

$$V_0(1+r)^T = V_T$$

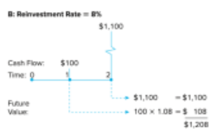
$$\Rightarrow r = \left(\frac{V_T}{V_0}\right)^{1/T} - 1$$

- Example: Consider, a 2-year bond selling at par value, paying a 10% coupon once a year and YTM is 10%. The coupon payment is reinvested at rate 10%
  - $r = \left(\frac{1210}{1000}\right)^{1/2} - 1 = 10\% = YTM$



Realized compound return vs YTM

- Coupons may not be reinvested to earn the bond's yield to maturity. Then realized compound return does not equal to YTM
- Initial value of the investment is $V_0 = 1000$. Final value of the investment is $V_2 = 1208$. The compound rate of return
  - $r = \left(\frac{1208}{1000}\right)^{1/2} - 1 = 9.91\%$



Calculation of realized compound return

- Realized compound return can be calculated either after the horizon or using a forecast of future reinvestment rates

Bond prices over time

- A bond sells at par value when its coupon rate equals the market interest rate
- When coupon rate is lower than the market interest rate, price should be lower than the par value; vice versa

Figure 14.6 Price path of two 30-year maturity bonds, each selling at a yield to maturity of 8%. Bond price approaches par value as maturity date approaches.

but usually the question

10/24/2032	10/24/2032
0.06	0.06
0.07	0.06
100	100
2	2

alue, number of coupon payments	value, number of coupon paymen
92.8938	100.0000
0	0
182	182
0.0000	0.0000
92.8938	100.0000

Bond will not be called

Technically capital gain is 1000 – 928.938 = $7.1062

YTM versus holding-period return

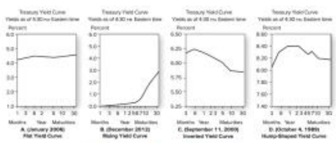
- If YTM is unchanged over the holding period, the HPR on the bond equals YTM; Otherwise, a bond's HPR fluctuates with yields
  - If YTM rises, then the bond price falls and HPR falls
  - If YTM falls, then the bond price rises and HPR rises
- *Example*: Consider an 8% coupon, 30-year annual bond selling at par $1,000
  - If you hold the bond for a year and YTM remains 8%, then the price will remain at par and HPR is 8%
  - If YTM falls and the price rises to $1,050, then the HPR is greater than 8%

$$HPR = \frac{80 + (1050 - 1000)}{1000} = 13\%$$

The yield curve

- **Yield curve:** Yield to maturity as a function of time to maturity
  - Central to bond valuation
  - Gauge their expectations for future interest rates against those of the market

on-the-run yield curve: the plot of yield as a function of maturity for recently issued coupon bonds selling at or near par value

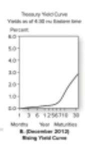


Future interest rate increases

Future interest rate falls

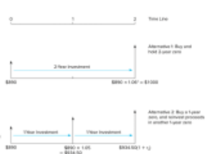
The yield curve and future interest rates

- The upward-sloping yield curve: short-term rates are going to be higher next year than they are now
- **Spot rate:** yield to maturity on zero-coupon bonds, meaning the rate that prevails today for a time period corresponding to the zero's maturity
- **Short rate:** refers to the interest rate for a given time interval (e.g., one year) available at different points in time



Spot rate and short rate

- Consider two 2-year investment strategies with equal rate of returns
  - buying and holding a 2-year zero-coupon bond
  - buying a 1-year zero and rolling over the proceeds into a 1-year bond
- 2-year spot rate is an "average" of today's and next year's short rates
  - This year's short rate r_1. Next year's short rate r_2. Two year's spot rate y_2
  - $(1 + y_2)^2 = (1 + r_1)(1 + r_2)$
  - Then $r_2 = \frac{(1+y_2)^2}{1+r_1} - 1 = 7.01\%$



Spot rate and short rate

- Consider two 3-year investment strategies with equal rate of returns
  - buying and holding a 3-year zero-coupon bond
  - buying a 2-year zero and rolling over the proceeds into a 1-year bond
- Finding the short rate r_3 in year 3
  - $(1 + y_3)^3 = (1 + y_2)^2(1 + r_3)$
  - Then $r_3 = \frac{(1+y_3)^3}{(1+y_2)^2} - 1 = \frac{1.07^3}{1.06^2} - 1 = 9.025\%$

Table 15.1

Prices and yields to maturity on zero-coupon bonds ($1,000 face value)

Maturity (years)	Yield to Maturity (%)	Price
1	5%	$952.38 = $1,000/1.05
2	6	$890.00 = $1,000/1.06^2
3	7	$816.30 = $1,000/1.07^3
4	8	$735.03 = $1,000/1.08^4

Forward rate

- Consider two n-year investment strategies with equal rate of returns
  - buying and holding an n -year zero-coupon bond
  - buying an $(n-1)$-year zero and rolling over the proceeds into a 1-year bond
  - $(1 + y_n)^n = (1 + y_{n-1})^{n-1}(1 + r_n)$
  - $r_n = \frac{(1+y_n)^n}{(1+y_{n-1})^{n-1}} - 1$
  - We also denote r_n as f_n to refer to **forward interest rate**