FINAL SUMMARY NOTES (Glossary)

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Lecture 0: Overview

- Quantitative finance uses mathematical models and large datasets to analyze financial markets and securities
- Modern portfolio theory risk, return, portfolio optimization, and capital asset pricing model
- Security valuation how bonds and derivatives are priced
- Securities fixed-income or debt securities, common stock or equity, derivatives
- **Fixed-income or debt securities** promise either a fixed stream of income or a stream of income determined by a specified formula
- **Common stock or equity** represents an ownership share in the corporation. Equityholders are not promised any particular payment
- **Derivatives** such as options and futures contracts provide payoffs that are determined by the prices of other assets such as bond or stock prices
- **Derivative asset** Claim whose value is directly dependent on the value of other assets.
- **Financial institutions** Connect between the security issuer (e.g., firm) and owner of the security (e.g., individual investor)
- **Financial intermediaries** Banks, investment companies, insurance companies, credit unions
- Investment bankers Issue securities to raise capital for firms
- Venture capital and private equity Invest in smaller and younger firms
- **Risk-return tradeoff** High risk high return, low risk low return
- Capital allocation between risk-free (e.g., Treasury bills) and risky assets (e.g., stocks)
- Asset pricing model A prediction of the relationship between risk and return
- Capital asset pricing models is a set of predictions concerning equilibrium expected returns on risky assets
- **Arbitrage pricing theory** An asset pricing theory that is derived from a factor model, using diversification and arbitrage arguments. The theory describes the relationship between expected return and factor exposure that follows from the absence of risk-free arbitrage opportunities.
- **Bond** A security issued by a borrower that obligates the issuer to make specified payments to the holder over a specific period. A coupon bond obligates the issuer to make interest payments called coupon payments over the life of the bond, then to repay the face value at maturity
- Bond characteristics time to maturity, coupon payments and par value
- **Current yield** A bond's annual coupon payment divided by its price. Differs from yield to maturity.
- **Diversification** Spreading a portfolio over many investments to avoid excessive exposure to any one source of risk.

Lecture 1: The investment environments

- **Real assets** the land, buildings, machines, and knowledge that can be used to produce goods and services
- **Financial assets** stocks, bonds, derivatives, and other claims to the income generated by real assets
- Investor's portfolio collection of investment assets
- Asset allocation choose among broad asset classes; Allocating a portfolio across broad asset classes such as stocks versus bonds

- Security selection choose which securities to hold within each asset class
- **Security analysis** involves the valuation of particular securities that might be included in the portfolio.
- "Top-down" strategy first asset allocation, then security selection
- "Bottom-up" strategy first security selection (may heavy represent one industry)
- **Passive management** holds highly diversified portfolios without spending effort or other resources attempting to improve investment performance through security analysis.
- Active management improves performance by identifying mispriced securities or by timing the performance of broad asset classes
- Firms net demanders of capital
- Households net suppliers of capital
- **Governments** borrowers or lenders
- **Financial intermediaries** stand between security issuer (e.g., firm) and security owner (e.g., individual investor)
- **Investment companies** pool and manage the money of many investors; originally separated from commercial banks last century by law, and then combined after 2008
- Mutual funds have the advantage of large-scale trading and portfolio management
- Hedge funds pursue complex and higher risk strategies
- **Primary market** new issues of securities are offered to the public
- Secondary market investors trade previously issued securities among themselves
- **Peer-to-peer lending** link lenders and borrowers directly, without need of an intermediary like a commercial bank, e.g., LendingClub
- **Cryptocurrencies** payment systems that bypass traditional channels such as credit cards, debit cards, or checks, e.g, bitcoin
- Robo advisors utilize algorithms to automate investment advice

Lecture 2: Asset classes and financial instruments

- Financial markets are segmented into money markets and capital markets
- Money market short-term, marketable, liquid, low-risk debt securities
 - Money market instruments are also called cash equivalents
- Capital market longer term, riskier securities
 - Longer term bond markets
 - o Equity markets
 - Derivative markets for options and futures
- Money market funds are accessible to individual investors
- **Mutual funds** that invest in money market instruments; average maturity of less than 3 months
- **Treasury bills** The most marketable of all money market instruments
 - The government raises money by selling bills to the public
 - o **Investors** buy the bills at a discount from the stated maturity (or face) value
 - o **At maturity**, the government pays the investor
 - **Investor's earnings** Difference between purchase price and maturity value
 - **Issued with initial maturities** of 4, 13, 26, or 52 weeks
- Ask price The price at which a dealer will sell a security
- Bid price The price at which a dealer is willing to purchase a security
- **Bid-ask spread** The difference between a dealer's bid and ask price; Bid yield is higher than asked yield

· Two issues of bank-discount method

- o Assumes that a year has only 360 days
- o Computes the yield as a fraction of par value, but not the price the investor paid
- A certificate of deposit, or CD, is a time deposit with a bank. Time deposits may not be withdrawn on demand; treated as bank deposits by the Federal Deposit Insurance Corporation
- Commercial paper Large, well-known companies issue their own short-term unsecured debt notes; backed by a bank line of credit, which gives the borrower access to cash that can be used to pay off the paper at maturity
- Money market funds Mutual funds invest in money market instruments
- Government money market fund invest in Treasury securities
- Prime money market fund invest in floating-rate commercial paper
- **Municipal money market fund** invests in municipal bonds (free from federal and state tax)
- **Bond market** Longer term borrowing or debt instruments than those trade in the money market
- **Fixed-income capital market** Most of them promise either a fixed stream of income or a stream of income determined by a specific formula
- Coupon payments Interest payments before maturity, commonly made semiannually
- Treasury notes and Treasury bonds Issued by the U.S. government
- Treasury notes maturities ranging up to 10 years
- Treasury bonds maturities ranging from 10 to 30 years

Lecture 3: Bonds and Equities

- Municipal bonds issued by state and local governments
 - o Their interest income is exempt from federal income taxation
 - o Also exempt from state and local taxation in the issuing state
- Cutoff tax bracket

$$1 - \frac{r_{muni}}{r_{taxable}}$$

- Corporate bonds Private firms borrow money from the public
 - Semiannual coupons over their lives and return the face value at maturity
 - Riskier than Treasury bonds
- Secured bonds Secured by a specific asset owned by the issuer
- Unsecured bonds, called debentures Backed by general credit rather than by specified assets
- Callable bonds The firm has the option to repurchase the bond from the holder at a defined call price before maturity
 - Call price is usually higher than the par value
 - Usually called when interest rate is low as the firm can refinance at a lower rate
- Common stocks, also known as equity securities or equities, represent ownership shares in a corporation
- Owner of common stock
 - **Vote** on any matters of corporate governance (can vote by proxy)
 - **Share** of the financial benefits of ownership
- Shareholders elect a board of directors
- The board selects managers
 - The board meets a few times a year
 - o Managers run the firm on a day-to-day basis
- Two characteristics of common stock residual claims and limited liability

- **Residual claim** Stockholders are the last to claim assets and income
- **Limited liability** means that the most shareholders can lose in the event of failure of the corporation is their original investment
- In a **liquidation** of the firm's assets, claim what is left after tax authorities, employees, suppliers, bondholders, and other creditors have been paid
- Not in liquidation, claim to the part of operating income left over after interest and taxes have been paid (as cash dividends or reinvest to increase the value of shares)
- The dividend yield is only part of the return on a stock investment
 - o **Another part** is prospective capital gains (i.e., price increases) or losses
 - Low-dividend firms presumably offer greater prospects for capital gains, or investors would not be willing to hold these stocks in their portfolios
- The P/E ratio or price-earnings ratio The ratio of stock price to last year's earnings per share
 - **Indicates** how much stock purchasers must pay per dollar of earnings that the firm generates
 - When dividend yield and P/E ratio are not reported, the firms have zero dividends, or zero or negative earnings
- **P/E effect** That low P/E stocks have exhibited higher average risk-adjusted returns than high P/E stocks.
- Preferred stock promises to pay a fixed amount of income each year
 - Compare to common stock
 - □ No voting power
 - □ **Preferred dividends** are usually cumulative: unpaid dividends cumulate and need to be paid in full before being paid to holders of common stock
 - Compare to bond
 - □ **Similar** to an infinite-maturity bond
 - □ **No contractual obligation** to pay preferred dividends, but contractual obligation to make the interest payments on the debt
- Well known stock market indexes
 - o **Dow Jones Industrial Average** 30 large, "blue-chip" corporations
 - o Standard & Poor's Composite 500 (S&P 500) about 500 firms
 - NASDAQ more than 3,000 firms traded on the NASDAQ market
 - NASDAO 100 is a subset of the larger firms in NASDAO
- Stock weights in the index
 - o **Price-weighted average** e.g. Dow Jones Industrial Average
 - o Market-value-weighted average e.g. S&P 500, NASDAQ, NASDAQ100
 - Equally weighted average of the returns of each stock in an index; invests equal dollar values in each stock
- Invest in market indexes
 - One way is to purchase index funds (shares in mutual funds that hold shares in proportion to their representation in the S&P 500 or another index); Purchase directly from the fund or through brokers or financial advisers
 - Another approach is to purchase an exchange-traded fund, or ETF, which is a
 portfolio of shares that can be bought or sold as a unit, just as one can buy or sell a
 single share of stock
 - ☐ Trade index portfolios as stocks
 - □ Lower management cost

Lecture 4: Asset classes, financial instruments, risk and return

- **Derivatives** Futures, options, and related derivatives contracts provide payoffs that depend on the values of other variables such as commodity prices, bond and stock prices, interest rates, or market index values
- Contingent claims their payoffs are contingent on the value of other values

- Call option The right to buy an asset at a specified exercise price on or before a specified expiration date
- **Put option** The right to sell an asset at a specified exercise price on or before a specified expiration date
- **Futures option** The right to enter a futures contract at a futures price equal to the stipulated exercise price
- **Futures price** The price at which a futures trader commits to make or take delivery of the underlying asset
- Long position The futures trader who commits to purchasing the underlying asset
- Short position The futures trader committing to deliver the underlying asset
- Futures contract vs call option
 - Futures contract obliges to purchase the asset at the futures price
 - Call option conveys the right to purchase the asset at the exercise price
- **Zero-coupon bond** A bond paying no coupons that sells at a discount and provides only payment of face value at maturity
- Rate of return over the holding period

$$r(T) = \frac{100}{P(T)} - 1$$

- Effective annual rate (EAR) Interest rate annualized using compound rather than simple interest
- Annual percentage rate (APR) Interest rate is annualized using simple rather than compound interest

$$APR = n \times \left[(1 + EAR)^{1/n} - 1 \right]$$

• Continuous compounding (CC)

$$r_{cc} = \log(1 + EAR)$$

$$r_{cc} = \lim_{n \to \infty} \log \left(1 + \frac{APR}{n} \right)^n = \lim_{n \to \infty} \log \left(\left(1 + \frac{APR}{n} \right)^{n/APR} \right)^{APR} = \lim_{n \to \infty} \log e^{APR} = APR$$

Lecture 5: Risk and return

- Interest rate The number of dollars earned per dollar invested per period
- **Nominal interest rate** The interest rate in terms of nominal (not adjusted for purchasing power) dollars
- **Real interest rate** The excess of the interest rate over the inflation rate. The growth rate of purchasing power derived from an investment
- Inflation The rate at which the general level of prices for goods and services is rising
- The consumer price index (CPI) measures purchasing power by averaging the prices of

goods and services in the consumption basket of an average urban family of four

$$r_{real} = \frac{r_{nom} - i}{1 + i}$$

$$r_{real} \approx r_{nom} - i$$

- Supply curve The higher the real interest rate, the greater the supply of household savings
- **Demand curve** The lower the real interest rate, the more businesses will want to invest in physical capital
- Fisher hypothesis

$$r_{nom} = r_{real} + E(i)$$

· Taxes and the real rate of interest

$$r_{nom}(1-t) - i = (r_{real} + i)(1-t) - i = r_{real}(1-t) - it$$

- **Risk** Any investment involves some uncertainty about future holding-period returns, and in many cases that uncertainty is considerable
- Sources of investment risk
 - o macroeconomic fluctuations
 - o changing fortunes of various industries
 - o firm-specific unexpected developments
- Holding-period returns

$$HPR = \frac{Ending price of a share - Beginning price + Cash dividend}{Beginning price}$$

• Dividend yield The annual dividend payment expressed as a percent of the stock price

Lecture 6: Risk and return

• Mean return E(r) probability-weighted average of the rates of return in each scenario

$$E(r) = \sum_{s} p(s)r(s)$$

- p(s): the probability of scenario s
- r(s): the HPR in scenario s

- "Surprise" difference between the actual return and the expected return
- **Variance** the expected value of the squared deviation from the mean; the expected squared "surprise" across scenarios)

$$Var(r) = \sigma^2 = \sum_s p(s)[r(s) - E(r)]^2$$

• **Risk premium** An expected return in excess of that on risk-free securities. The premium provides compensation for the risk of an investment; the difference between the *expected* **HPR** on the mutual fund and the **risk-free rate**, that is, the rate you would earn in risk-free assets such as T-bills, money market funds, or the bank.

$$E(r) - r_f$$

• Excess return Rate of return in excess of the risk-free rate; difference in any particular period between the actual rate of return on a risky asset and the risk-free rate

$$r(s) - r_f$$

• **Sharpe ratio** Reward-to-volatility ratio; ratio of excess return to portfolio standard deviation.

Sharpe ratio =
$$\frac{\text{Risk premium}}{\text{SD of excess return}}$$
$$= \frac{E(r) - r_f}{\sigma}$$

- **Risk averse** An investor who will consider risky portfolios only if they provide compensation for risk via a risk premium
- **Risk lover** An investor who is willing to accept lower expected returns on prospects with higher amounts of risk
- **Risk neutral** An investor who finds the level of risk irrelevant and considers only the expected return of risk prospects
- **Risk-free asset** An asset with a certain rate of return; often taken to be short-term T-bills.
- Risk-free rate The interest rate that can be earned with certainty, commonly taken to be the rate on short-term Treasury bills
- **Risky asset** An asset with an uncertain rate of return

Lecture 7: Capital allocation to risky assets

- Normal distribution
 - Symmetricity Standard deviation is sufficient to capture the risk
 - Scenario analysis is simpler Only mean and variance are sufficient to estimate scenario probability
 - Easy to model Statistical dependence of returns across assets: correlation is sufficient
- First deviation Asymmetry in the probability distribution of returns
- **Second deviation** Likelihood of extreme values on either side of the mean
- **Skew** Measure of the asymmetry of a probability distribution

Skew = Average
$$\left[\frac{(r - \bar{r})^3}{\hat{\sigma}^3} \right]$$

- **Negative skew** Extreme bad outcomes are more frequent than extreme positive ones (skew to the left, fatter left tail, underestimate risk)
- **Positive skew** Opposite case (skew to the right)
- **Kurtosis** Measure of the fatness of the tails of a probability distribution. Indicates probability of observing extreme high or low values.
- Value at risk (VaR) q% value at risk (q% VaR)
- Expected shortfall (ES) The expected loss on a security conditional on returns being in the left tail of the probability distribution
- Conditional tail expectation (CTE) Expectation of a random variable conditional on its falling below some threshold value. Often used as a measure of downside risk
- Utility score to compare competing portfolios based on expected return and risk of those portfolios

$$U = E(r) - \frac{1}{2}A\sigma^2$$

- **Certainty equivalent** is the rate that risk-free investment would need to offer to provide the same utility as the risky portfolio
- Asset allocation of a complete portfolio y in the risky assets and 1 y in the risk-free assets

Lecture 8: Capital allocation to risky assets

- Capital allocation line (CAL) A graph showing all feasible risk—return combinations of a risky and risk-free asset
- **Indifference curve** A curve connecting all portfolios with the same utility according to their means and standard deviations
 - More risk-averse investors (i.e., larger A) have steeper indifference curves than less risk-averse investors
 - Higher indifference curves correspond to higher levels of utility
- Portfolios of one risky asset and a risk-free asset
 - Allocation decision of a complete portfolio C proportion y in the risky portfolio P and 1 − y in the risk-free asset
 - o Rate of return

$$r_C = yr_P + (1 - y)r_f$$

o Expected return

$$E[r_C] = r_f + y[E(r_P) - r_f]$$

Standard error

$$\sigma_C = y\sigma_P$$

Sharpe ratio

$$S = \frac{E(r_P) - r_f}{\sigma_P} \text{ (does not depend on } y)$$

Capital allocation line (CAL)

$$E[r_C] = r_f + S \cdot \sigma_C$$

Lecture 9: Efficient diversification

- Capital allocation line (CAL) A graph showing all feasible risk—return combinations of a risky and risk-free asset
- **Systematic risk** Risks from the condition of the general economy, e.g., business cycle, inflation, interest rates, exchange rates
- Firm-specific risk e.g., research and development and personnel changes
- **Portfolio opportunity set** all combinations of portfolio expected return and standard deviation that can be constructed from the two available assets/portfolios
- **Minimum-variance frontier** Graph of the lowest possible portfolio standard deviation corresponding to each value of portfolio expected return
- Minimum-variance portfolio The portfolio of risky assets with lowest possible variance.
- Risk tolerance and asset allocation
 - o maximize the utility given risk aversion A

$$\max_{y} U = E(r_C) - \frac{1}{2} A \sigma_C^2 = r_f + y [E(r_P) - r_f] - \frac{1}{2} A y^2 \sigma_P^2$$

o Optimal

$$y = \frac{E(r_P) - r_f}{A\sigma_P^2}$$

- **Indifference curve** E(r) vs sigma given A such that utility level is the same
- · Portfolios of two risky assets
 - Bond portfolio

 w_D

o Stock portfolio

$$w_E = 1 - w_D$$

o Return

$$r_P = w_D r_D + w_E r_E$$

Expected return

$$E(r_P) = w_D E(r_D) + w_E E(r_E)$$

Variance

$$\sigma_P^2 = w_D^2 \cdot \sigma_D^2 + w_E^2 \cdot \sigma_E^2 + 2w_D w_E Cov(r_D, r_E)$$

Minimum-variance portfolio

$$w_D^* = \frac{\sigma_E^2 - Cov(r_D, r_E)}{\sigma_D^2 + \sigma_E^2 - 2Cov(r_D, r_E)}$$

Utility maximization portfolio (given A)

$$w_D^* = \frac{E(r_D) - E(r_E) + A(\sigma_E^2 - Cov(r_D, r_E))}{A(\sigma_D^2 + \sigma_E^2 - 2Cov(r_D, r_E))}$$

Sharpe ratio maximization portfolio

$$w_D^* = \frac{E(R_D)\sigma_E^2 - E(R_E)Cov(R_D, R_E)}{E(R_D)\sigma_E^2 + E(R_E)\sigma_D^2 - [E(R_D) + E(R_E)]Cov(R_D, R_E)}$$

Lecture 10: Efficient diversification

- **Efficient diversification** The organizing principle of modern portfolio theory, which describes how investors can devise the best possible risk-return trade-off
- Efficient frontier Graph representing a set of portfolios that maximize expected return at each level of portfolio risk
- Efficient frontier of risky assets The portion of the minimum-variance frontier that lies above the global minimum-variance portfolio
- **Optimal risky portfolio** An investor's best combination of risky assets; the combination that maximizes the Sharpe ratio
- **Separation property** The property that portfolio choice can be separated into two independent tasks: (1) determination of the optimal risky portfolio, which is a purely technical problem, and (2) the personal choice of the best mix of the risky portfolio and the risk-free asset
- Market portfolio The portfolio encompassing all assets in which each asset is held in proportion to its market value
- Mutual fund An investment company pooling and managing funds of investors
- **Mutual fund theorem** A result associated with the CAPM, asserting that investors will choose to invest their entire risky portfolio in a market-index mutual fund
- Capital market line (CML) The capital allocation line that results when using the market index as the risky portfolio

Lecture 11: Markowitz model and CAPM

- The capital asset pricing model (CAPM) gives us a prediction of the relationship that we should observe between the risk of an asset and its expected return
- **Security market line (SML)** Graphical representation of the expected return—beta relationship
- **Alpha value** The abnormal rate of return on a security in excess of what would be predicted by an equilibrium model like the CAPM
- Markowitz model portfolio optimization model, also called mean-variance model
 - A mathematical framework for assembling a portfolio of assets such that the expected return is maximized for a given level of risk
- **Minimum variance frontier** A graph of the lowest possible variance that can be attained for a given portfolio expected return
- **Efficient frontier** is the portion of the minimum-variance frontier that lies above the global minimum-variance portfolio
 - The bottom part of the minimum-variance frontier is inefficient
 - There is always a portfolio with the same standard deviation and a greater expected return positioned directly above i
- Two equivalent approaches to construct efficient frontier
 - Approach 1 Minimize variance for any target expected return \mu (e.g., points marked by squares)

$$\min_{w} \sum_{i=1}^{n} \sum_{j=1}^{n} w_{i} w_{j} Cov(r_{i}, r_{j})$$
s. t. $\sum_{i=1}^{n} w_{i} E(r_{i}) = \mu$

$$\sum_{i=1}^{n} w_{i} = 1$$

• Approach 2 Maximize expected return for any target risk level \sigma^2 (e.g., points marked by circles)

$$\max_{w} \sum_{i=1}^{n} w_i E(r_i)$$

$$s.t. \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j Cov(r_i, r_j) = \sigma^2$$

$$\sum_{i=1}^{n} w_i = 1$$

Lecture 12: CAPM

- Mutual fund theorem investing in a market-index portfolio is efficient (passive strategy)
- Capital allocation line (CAL) A graph showing all feasible risk—return combinations of a risky and risk-free asset
- **Price adjustment process** If investors do not include the stock of some company in the portfolio, the price of this stock drops and then becomes attractive to be included in the portfolio
- A basic principle of equilibrium all investments should offer the same reward-to-risk ratio

$$\frac{E(R_{GE})}{Cov(R_M, R_{GE})} = \frac{E(R_M)}{\sigma_M^2}$$

• Expected return-beta relationship for any asset i

$$E(r_i) = r_f + \beta_i [E(r_M) - r_f]$$

• Expected return-beta relationship for portfolio P with weight w_i in asset i

$$E(r_P) = w_1 E(r_1) + w_2 E(r_2) + \dots + w_n E(r_n)$$

$$= \sum_{i} w_i (r_f + \beta_i [E(r_M) - r_f]) = r_f + \beta_P [E(r_M) - r_f]$$

- **Security market line (SML)** Graphical representation of the expected return—beta relationship
 - If assets are "fairly priced", then they are exactly on the SML. All securities must lie on the SML in market equilibrium
 - If a stock is underpriced, then it will plot above the SML
- **Single-index model** A model of stock returns that decomposes influences on returns into a systematic factor, as measured by the return on a broad market index, and firm specific factors

Lecture 13: CAPM and APT

• Multifactor model Model of security returns positing that returns respond to several

systematic risk factors as well as firm-specific influences

- **Arbitrage** A zero-risk, zero-net investment strategy that still generates profits
- Arbitrage pricing theory (APT) An asset pricing theory that is derived from a factor model, using diversification and arbitrage arguments. The theory describes the relationship between expected return and factor exposure that follows from the absence of risk-free arbitrage opportunities
 - o Security returns can be described by a factor model
 - o There are sufficient securities to diversify away idiosyncratic risk
 - Well-functioning security markets do not allow arbitrage opportunities to persis
- Law of one price If two assets are equivalent in all economically relevant respects, then they should have the same market price

Lecture 14: APT

• Fama-French (FF) Three-Factor Model

$$R_{it} = \alpha_i + \beta_{iM}R_{Mt} + \beta_{iSMB}SMB_t + \beta_{iHML}HML_t + e_i$$

- Small Minus Big(SMB) the return of a portfolio of small stocks in excess of the return on a portfolio of large stocks
 - o Small stocks may be more sensitive to changes in business conditions
- **High Minus Low(HML)** the return of a portfolio of stocks with a high book-to-market ratio in excess of the return on a portfolio of stocks with a low book-to-market ratio
 - o Firms with high book-to-market ratios are more likely to be in financial distress
- Momentum (WML, for Winners Minus Losers) the return on a portfolio that buys recent well-performing stocks and sells poorly performing ones
- Volatility the standard deviation of stock returns
- Quality (profitability) the difference in returns of stocks with high versus low return on assets or similar measures of profitability
- **Investment** the difference between returns on firms with high versus low rates of asset growth; and dividend yield
- Latent factor model

$$R_i = E(R_i) + \beta_{i1}F_1 + \beta_{i2}F_2 + \dots + \beta_{ik}F_k + e_i$$

Lecture 16: Bond prices

- **Bond** The issuer agrees to make specified payments to the bondholder on specified dates
- **Coupon payment** The issuer makes semiannual payments of interest for the life of the bond, in a typical coupon bond
 - Annual payment is the coupon rate times the bond's par value
 - When the bond matures, the issuer repays the debt by paying the bond's par value (equivalently, its face value)
- Zero-coupon bonds no coupon payments, issued at prices considerably below par value
- **Treasury notes** are issued with original maturities ranging between 1 and 10 years
- **Treasury bonds** are issued with maturities ranging from 10 to 30 years
- Flat price/clean price does not reflect the accrued interest
- Invoice price/dirty bond price flat price plus accrued interest
- Nominal risk-free interest rate equals the sum of
 - o a real risk-free rate of return

- o premium above the real rate to compensate for expected inflation
- **Bond value** = Present value of coupons + Present value of par value

Bond value =
$$\sum_{t=1}^{T} \frac{\text{Coupon}}{(1+r)^{t}} + \frac{\text{Par value}}{(1+r)^{T}}$$

$$= \text{Coupon} \times \frac{1}{r} \left[1 - \frac{1}{(1+r)^{T}} \right] + \text{Par value} \times \frac{1}{(1+r)^{T}}$$

$$= \text{Coupon} \times \text{Annuity factor}(r, T) + \text{Par value} \times \text{PV factor}(r, T)$$

- Inverse relationship between prices and yields An increase in the interest rate results in a price decline
 - o the convex shape of the bond price curve
 - bond duration is a way of measuring how much bond prices are likely to change if and when interest rates move
- The main source of risk in the fixed-income market is interest rate fluctuations
- An important factor the maturity of the bond
 - The longer the maturity of the bond, the greater the sensitivity of price to fluctuations in the interest rate
 - o Short-term Treasury securities such as T-bills are considered to be the safest

Lecture 17: Bond prices

- **Yield to maturity** A measure of the average rate of return that will be earned on a bond if held to maturity
- Callable bonds allow the issuer to repurchase the bond at a specified call price before the maturity date
- Call protection an initial time during which the bonds are not callable
- **Realized compound return** Compound rate of return assuming that coupon payments are reinvested until maturity
- Yield curve A graph of yield to maturity as a function of time to maturity

Lecture 18: Term structures

- The upward-sloping yield curve short-term rates are going to be higher next year than they are now
- **Spot rate** yield to maturity on zero-coupon bonds, meaning the rate that prevails today for a time period corresponding to the zero's maturity
- **Short rate** refers to the interest rate for a given time interval (e.g., one year) available at different points in time
- **Forward interest rate** Rate of interest for a future period that would equate the total return of a long-term bond with that of a strategy of rolling over shorter-term bonds
- Liquidity premium Forward rate minus expected future short interest rate
- Callable bonds allow the issuer to repurchase the bond at a specified call price before the maturity date

Lecture 19: Term structures

- Theories of the term structure
 - The expectations hypothesis The forward rate equals the market consensus expectation of the future short interest rate
 - **Liquidity preference theory** Short-term investors dominate the market so that the forward rate will generally exceed the expected short rate

Lecture 20: Bond risk and duration

- Bond prices and yields are inversely related As yields increase, bond prices fall; as yields fall, bond prices rise
- **Interest rate sensitivity** The sensitivity of bond prices to changes in market interest rates and yields
- Macaulay's duration Effective maturity of bond, equal to weighted average of the times until each payment, with weights proportional to the present value of the payment
- The sensitivity and duration is affected by
 - Yield to maturity
 - o Time to maturity
 - o Coupon rate

• Interest rate sensitivity vs YTM

- **Property 1** The sensitivity of a bond's price to a change in its yield is inversely related to the yield to maturity at which the bond currently is selling
- **Property 2** An increase in a bond's yield to maturity results in a smaller price change than a decrease in yield of equal magnitude
- **Property 3** Prices of long-term bonds tend to be more sensitive to interest rate changes than prices of short-term bonds
- Property 4 The sensitivity of bond prices to changes in yields increases at a
 decreasing rate as maturity increases. In other words, interest rate risk is less than
 proportional to bond maturity
- Property 5 Interest rate risk is inversely related to the bond's coupon rate. Prices of low-coupon bonds are more sensitive to changes in interest rates than prices of highcoupon bonds