QTM 385 Midterm Cathy Zhuang

Saturday, March 18, 2023 10:47

Table 1: Listing of stocks on 2023/03/14

symbol	close	net chg	shares outstanding	52 wk high	52 wk low	div	yield%	P/E
Apple	152.74	2.12	15.82B	179.61	124.17	0.23	0.60	25.93
Coca-Cola	59.99	0.18	4.33B	54.02	67.20	0.46	3.07	27.37
Exxon Mobil	106.66	0.40	4.12B	119.63	76.25	0.91	3.40	8.05

"div" is the dividend payment from last quarter. "P/E" is the ratio of a company's share (stock) price to the company's annual earnings per share.

(a) [2pts] How many shares of Coca-Cola could you buy for \$2,000? Truncate your solution to an integer.

(b) [2pts] What would be your annual dividend income from those shares of Coca-Cola? (You can assume the dividend payments are the same for all quarters last year)

(c) [5pts] Which of Apple, Coca-Cola and Exxon Mobil has the highest earnings per share? Which has the lowest? Compare the dividend payout ratio (ratio of the dividend payment over quarterly earnings per share) for the three stocks, and comment on your findings.

a. 2000/59.99=33.3389 = 33 shares
b. (2000/59.99) * (0.46*4) = 61.3436 = 61.344
c. Earnings per share = close divided by p/e
   - Apple: 152.74/25.93 = 5.8905 = 5.891
   - Coca-Cola: 59.99/27.37 = 2.1918 = 2.192
   - Exxon Mobil: 106.66/8.05 = 13.2497 =13.250

   Highest earnings per share is Exxon Mobil.
   Lowest earnings per share is Coca-Cola.

   Dividend payout ratios:
   - Apple: 0.23/(5.8905/4) = 0.1562 = 0.156
   - Coca-Cola: 0.46/(2.1918/4) = 0.8395 = 0.840
   - Exxon Mobil: 0.91/(13.2497/4) = 0.2747 = 0.275

   Highest dividend payout ratio is Coca-Cola.
   Lowest dividend payout ratio is Apple.

   So while Exxon Mobil has the highest earnings per share, it does not have the highest dividend payout ratio. Additionally, while Coca-Cola has the lowest earnings per share, it has the highest dividend payout ratio. Apple has the lowest dividend payout ratio but not the lowest earnings per share.

(d) [3pts] What were Apple, Coca-Cola and Exxon Mobil's closing price on 2023/03/13 (i.e., the day before the listing)?

Using the closing price on 2023/03/13 and 2023/03/14,

(e) [4pts] Calculate the rate of return on a price-weighted index of the three stocks on 2023/03/14.

(f) [4pts] Calculate the rate of return on a market-value-weighted index of the three stocks on 2023/03/14.

(g) [4pts] Calculate the rate of return on an equally weighted index of the three stocks on 2023/03/14.

d.	Closing price 3/13	Close price 3/14
Apple	152.74 - 2.12 = 150.62	152.74
Coca-Cola	59.99 - 0.18 = 59.81	59.99
Exxon Mobil	106.66 - 0.40 = 106.26	106.66

e. Price weighted index

   Price weighted index on 3/13: (150.62 + 59.81 + 106.26)/3=105.5633
   Price weighted index on 3/14: (152.74 + 59.99 + 106.66)/3=106.4633

   The rate of return is (106.4633/105.5633) - 1 = 0.0085 -> 0.85%

f. Market value weighted index

   Assume MVWI on 3/13 -> 1
   MVWI on 3/14:
   ((152.74 * 15,820,000,000) + (59.99 * 4,330,000,000) + (106.66 * 4,120,000,000)) / ((150.62 * 15,820,000,000) + (59.81 * 4,330,000,000) + (106.26 * 4,120,000,000)) = 1.0117

   The rate of return is (1.0117/1) - 1 = 0.0117 -> 1.17%

g. Equally weighted index

   Assume investing in one each of Apple, Coca-Cola, Exxon Mobil.
   EWI on 3/13 -> 3
   EWI on 3/14: (152.74/150.62) + (59.99/59.81) + (106.66/106.26) = 3.0208

   The rate of return is (3.0208/3) - 1 = 0.0069 -> 0.69%

Problem 2 [27pts]

Visit Professor Kenneth French's data library Web site: **https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html** and download the *value-weighted monthly returns* of "Portfolios Formed on Size" from January 1981–December 2022.

(a) For the portfolio of stocks whose firm size is at the bottom 30% and the portfolio of stocks whose firm size is at the top 30%, compute

2

- [2pts] mean
- [2pts] SD
- [2pts] skew
- [2pts] kurtosis
- [3pts] 5% value at risk (VaR)
- [3pts] 5% expected shortfall (ES)
- [4pts] Sharpe ratio (Hint: You can download the monthly risk-free rate from "Fama/French 3 factors")

```
{'Firm size: Bottom 30%': {'avg': 1.02,
                           'es': -13.077,
                           'kurtosis': 2.546,
                           'sd': 5.989,
                           'sharpe ratio': 0.119,
                           'skew': -0.48,
                           'value at risk': -8.66},
 'Firm size: Top 30%': {'avg': 0.978,
                        'es': -9.604,
                        'kurtosis': 1.7,
                        'sd': 4.395,
                        'sharpe ratio': 0.152,
                        'skew': -0.56,
                        'value at risk': -6.906}}
```

Relevant code:

```
# Reading in cleaned up data
# Prior to this, I took out all data before Jan 1981 and after Dec 2022 (so I also took out Jan 2023)

datacleaned = pd.read_csv("C:\\Users\\Cathy\\Downloads\\midterm_data.csv")
fama = pd.read_csv('C:\\Users\\Cathy\\Downloads\\fama_clean_editedformidterm.csv')

datacleaned['Riskfree'] = fama['RF']

df = datacleaned
```

```
# converting date number to date object
df['Date'] = pd.to_datetime(df['Date'], format="%Y%m")
df = df.dropna() # drop NA's
df.head() # just checking
```

```
riskfreerate = np.mean(df['Riskfree'])

def avg(x):
    return round(np.mean(x), 3)
def sd(x):
    return round(np.std(x), 3)
def skew(x):
    return round (sp.stats.skew(x), 3)
def kurtosis(x):
    return round (sp.stats.kurtosis(x), 3)
def varisk(x, quantile=0.05):
    return round (np.quantile(x, quantile), 3)
def es(x, quantile=0.05):
    q = varisk(x, quantile)
    return round(x[x < q].mean(), 3)
def sharpe(x):
    return round((np.mean(x) - riskfreerate) / np.std(x), 3)


dicts = {}
for i, data in enumerate([df['Lo 30'], df['Hi 30']]):
    d = {}
    d['avg'] = avg(data)
    d['sd'] = sd(data)
    d['skew'] = skew(data)
    d['kurtosis'] = kurtosis(data)
    d['value at risk'] = varisk(data)
    d['es'] = es(data)
    d['sharpe ratio'] = sharpe(data)
    if i == 0:
        dicts['Firm size: Bottom 30%'] = d
    else:
        dicts['Firm size: Top 30%'] = d
pprint(dicts)
```

(b) [3pts] Which portfolio has a higher Sharpe ratio? Explain why.

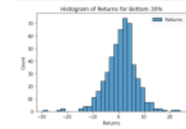
(c) [3pts] Comment on the distribution of the returns (e.g., left or right skewed, heavy-tailed or light-tailed). Do you suggest that the returns come from the normal distribution? Explain why.

(d) [3pts] Compute both the sample covariance and sample correlation between the two portfolios.

b. The top 30% portfolio has a higher Sharpe ratio. This means that the top 30% portfolio has a higher reward-to-volatility ratio. Firm sizes who are the largest are more established and thus may be less volatile, and smaller firms are more volatile, which could explain why there is a higher reward-to-volatility ratio for the top 30% portfolio.

c.

```
# distribution of returns
sns.histplot(data=df['Lo 30'], label = 'Returns')
plt.title("Histogram of Returns for Bottom 30%")
plt.legend()
plt.xlabel('Returns')
plt.show()
```




```
# distribution of returns
sns.histplot(data=df['Hi 30'], label = 'Returns')
plt.title("Histogram of Returns for Top 30%")
plt.legend()
plt.xlabel('Returns')
plt.show()
```

c. For the bottom 30%, the distribution is slightly left skewed with high kurtosis. This is confirmed in the histogram as we can see that the density is very high in the middle and at the tail. I do not suggest that the returns come from the normal distribution due to high kurtosis. Compared to the normal distribution, this distribution is more likely to have returns around zero and extreme gain/loss.
   For the top 30%, the distribution slightly left skewed with lower kurtosis than the bottom 30%, which means it is relatively lighter-tailed. However, kurtosis is still larger than what I would like, as there is still higher density at zero and extreme gain/loss. I probably would not suggest a normal distribution.

d. (covariance on top, correlation on bottom)

```
# sample covariances and correlations
covariances = np.cov(df['Lo 30'], df['Hi 30'])
correlations = np.corrcoef(df['Lo 30'], df['Hi 30'])

print(round(covariances[0,1], 3))
print(round(correlations[0,1], 3))
```

```
20.531
0.781
```

Problem 3 [49pts]

Consider a mean-variance in
C, and a risk-free fund. The ex
$E[r_C] = 0.2$. The variance and
$Var(r_C) = 4$, and $Cov(r_B, r_C) =$

(a) [8pts] What are the invest
and C, and what are the e

(b) [5pts] Solve for the propor
deviation of the optimal ri

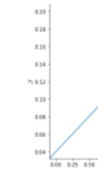
(c) [4pts] Draw the investment
to the opportunity set.

(d) [2pts] What is the Sharpe

a. The variance minimizing portf
   The return and standard dev

b. The optimal risky portfolio allo
   The return and standard dev

c.



d. The Sharpe ratio is 0.0812

Relevant code:

vestor forming a portfolio out of two risky portfolios B and
pected returns of two risky portfolios are $E[r_B] = 0.1$ and
ovariance between two risky portfolios are $Var(r_B) = 2.25$,
1. The risk-free return is $r_f = 0.04$.

tment proportions in the minimum-variance portfolio of B
xpected value and standard deviation of its rate of return?

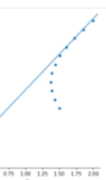
tions of B and C, and for the expected return and standard
isky portfolio P.

opportunity set of B and C. Draw the CAL that is tangent

ratio of the best feasible CAL?

olio allocation is 70.588% in B and 29.412% in C
viation are: 12.941% and 1.372

ocation is 21.053% in B and 78.947% in C
viation are: 17.895% and 1.710



(e) Suppose your client's degree of risk aversion is $A = 1$, and you would like to maximize the utility score of your client. You can use the risk-free fund and the optimal risky portfolio P.

1) [3pts] What is the optimal proportion, y, of the total investment that should be invested in P?
2) [4pts] What is the proportion invested in B, C and the risk-free fund, respectively? Comment on how the proportion varies if the risk aversion A varies.

3

3) [5pts] What are the expected value and standard deviation of the rate of return on your client's optimized portfolio? What is the Sharpe ratio of this optimized portfolio? Compare this Sharpe ratio with the Sharpe ratio in (d). Comment on what you find. Moreover, comment on how the Sharpe ratio varies if the risk aversion A varies.
4) [4pts] What is your client's utility score of this optimized portfolio? Comment on how the utility score varies if the risk aversion A varies.
5) [4pts] Draw the indifference curve for your client corresponding to the utility score of this optimized portfolio. Overlay the indifference curve with best feasible CAL and opportunity set in (c).

1. 4.750%

2.

	Proportions
Portfolio B	4.75 * .21053 = 1.00002 = 1.000%
Portfolio C	4.75 * 0.78947 = 3.74998 = 3.750 %
Risk Free	100-4.75= 95.25 %

As risk aversion A increases, investors are more risk averse and therefore would allocate relatively more assets to risk free assets than someone who is less risk averse. As risk aversion A decreases, investors are less risk averse and therefore would allocate relatively more assets to risky assets than someone who is more risk averse.

3.

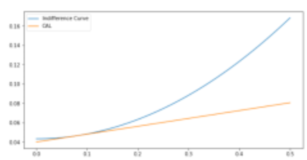
Expected value: 0.0466 = 4.66%
Standard deviation: 0.0812

Sharpe ratio: 0.0812

The Sharpe ratio should stay the same with the ratio in d.
If the risk aversion A varies, the Sharpe ratio should still stay the same since the Sharpe ratio of any complete portfolio in this case should be 0.0812.

4. Utility score is : 0.043
As A increases (investors are more risk averse), the utility score decreases.
As A decreases (investors are less risk averse), the utility score increases.

5.



Relevant code:

```
# optimal y
A = 1
y = (opt_ret - rf)/ (A * opt_var)
y*100 # optimal allocation to risky asset
```

4.750000000000001

```
rf = .04
s_ret = .1
s_sd = np.sqrt(2.25)
b_ret = .2
b_sd = np.sqrt(4)
cov = 1
```

```
U = expectedreturn - (0.5 * A * (np.sqrt(opt_var) * y)**2)
U
```

0.0433

```
# does it change
A = 5
U = expectedreturn - (0.5 * A * (np.sqrt(opt_var) * y)**2)
U
```

```
A = 1
U = expectedreturn - (0.5 * A * (np.sqrt(opt_var) * y)**2)
U
```

0.0433

```
x = np.linspace(0, 0.5, )
y = U + (0.5 * A * x**2)
y1 = ((opt_ret - rf) / (np.sqrt(opt_var)))* x + rf
fig = plt.figure(figsize = (10, 5))
plt.plot(x, y, label = "Indifference Curve")
plt.plot(x, y1, label = "CAL")
plt.legend()
```

(f) Suppose your client's degree of risk aversion is $A = 1$ and you would like to maximize the utility score of your client using only two risky portfolios B and C.

1) [4pts] What is the optimal proportion invested in B and C?
2) [4pts] Is the Sharpe ratio of this optimized portfolio higher or lower than the Sharpe ratio of the optimized portfolio in (e)? How about the utility score? Explain why.

(g) [3pts] Suppose there are k additional risky portfolios ($k \geq 0$) besides B and C that can be used to construct the optimal risky portfolio P. Describe how the opportunity set, the Sharpe ratio of the best feasible CAL, and investors' utility score change with k and explain why.

(h) [3pts] Following (g), suppose there are ℓ constraints about the weights of each portfolio in constructing the optimal risky portfolio P. Describe how the opportunity set, the Sharpe ratio of the best feasible CAL, and investors' utility score change with ℓ and explain why.

F.
1. 68.235% in B
31.765% in C
2. Sharpe ratio is 0.09598 = 0.096
Sharpe ratio is higher than the Sharpe ratio of the optimized portfolio in e. This is because we don't have the risk free asset anymore.
Utility score is -0.8105882352941176 = -0.0811 which is much lower than before. This is because the variance is very high AND the investor is not very risk averse. It is better not to invest at this point.

g. As we increase the number k additional risky portfolios:
The opportunity set shifts left because now there is lower risk or variance which also gives the chance for higher return.
The Sharpe ratio of the best feasible CAL increases for this same reason -> the slope of the CAL is the Sharpe ratio, and since we are having less variance and more return for a given value, the slope has increased.
The utility score will also increase for this reason, since expected return is higher and variance is lower.

h. As we increase the number l constraints:
The opportunity set shifts right. This is because we will have more variance or risk and also lower return.
The Sharpe ratio decreases for this same reason -> the slope of the CAL is the Sharpe ratio, and since we are having more variance and less return for a given value, the slope has decreased.
The utility score decreases, since expected return is lower and variance is higher.

Relevant code:

```
# Defining portfolio characteristics
A = 1
s_ret = .1
s_sd = np.sqrt(2.25)
b_ret = .2
b_sd = np.sqrt(4)
cov = 1
rf = .04

# portfolio based on risk aversion
w_d = (s_ret - b_ret + (A * ((b_sd**2) - cov))) / (A * ((s_sd**2)+(b_sd**2) - (2*cov)))
w_e = 1 - w_d

opt_ret = (w_d*0.1) + (w_e*0.2)
opt_var= ((w_d**2) * (s_sd**2)) + ((w_e**2) * (b_sd**2)) + (2 * w_d * w_e * cov)


w_d, w_e, opt_ret, opt_var
```

```
sharpe = (opt_ret) / (np.sqrt(opt_var))
print(f"(e) The sharpe ratio is {sharpe:.5f}")
```

(e) The sharpe ratio is 0.09598