# Lecture 16: Bond pricing

Monday, March 27, 2023 2:33 PM

16-bond-pricing

## QTM 385 Quantitative Finance

## Lecture 16: Bond prices

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Suggested reading: Investments Ch 14

# Question from Google form

- How does is the variable value for "A" in the utility equation get determined for an investor. I know it indicates the degree of risk aversion but how is that measured to give a value?

- *Answer: Usually through questionnaire. Ask some questions about whether investors are indifferent between two investment opportunities. Use the answers to calculate **A***

# Question from Google from

- What's the intuitive difference between minimum variance and optimal portfolio?

- *Answer: "Optimal" is defined as the solution to an optimization problem (with a certain objective function)*
  - *Minimum variance portfolio is optimal when the objective function is to minimize variance*
  - *The objective function can also be to maximize Sharpe ratio or maximize the utility score*

# Bond value

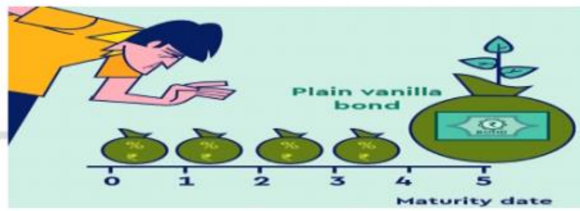
- For simplify, assume there is one interest rate $r$ for discounting cash flows of any maturity

- Bond value = Present value of coupons + Present value of par value
  - For a bond with $T$ periods until maturity

$$Bond\ value = \sum_{t=1}^{T} \frac{Coupon}{(1+r)^t} + \frac{Par\ value}{(1+r)^T}$$

$$= Coupon \times \frac{1}{r}\left[1 - \frac{1}{(1+r)^T}\right] + Par\ value \times \frac{1}{(1+r)^T}$$

$$= Coupon \times Annuity\ factor(r, T) + Par\ value \times PV\ factor(r, T)$$

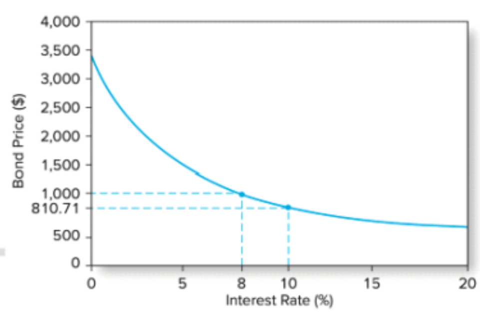



# Example

- 30-year maturity bond with 8% coupon, par value of \$1,000 paying 60 semiannual coupon payments of \$40 each

$$Price = \sum_{t=1}^{60} \frac{40}{(1+r)^t} + \frac{1000}{(1+r)^{60}}$$

- If the interest rate is 8% annually ($r = 4\%$), then $Price = \$1000$
- If the interest rate is 10% annually ($r = 5\%$), then $Price = \$810.71$



# Solving bond price in excel

- See bond-pricing.xlsx

| | A | B | C |
|---|---|---|---|
| 1 | | 8% coupon, semi-annual bond | |
| 2 | | maturing Nov 2045 | Formula in column B |
| 3 | | | |
| 4 | Settlement date | 11/15/15 | =DATE(2018,11,15) |
| 5 | Maturity date | 11/15/45 | =DATE(2045,11,15) |
| 6 | Annual coupon rate | 0.08 | |
| 7 | Yield to maturity | 0.1 | |
| 8 | Redemption value (% of face value) | 100 | |
| 9 | Coupon payments per year | 2 | |
| 10 | | | |
| 11 | | = PRICE(settlement date, maturity date, annual coupon rate, yield to | |
| 12 | Flat price (% of par) | 81.0707 | =PRICE(B4,B5,B6,B7,B8,B9) |

# Yield to maturity

- Yield to maturity: Interest rate that makes the present value of a bond's payments equal to its price
- Suppose an 8% coupon, 30-year semiannual bond is selling at $1,276.76
- The yield to maturity, denoted by $r$, is the value that satisfy the equation

$$1276.76 = \sum_{t=1}^{60} \frac{40}{(1+r)^t} + \frac{1000}{(1+r)^{60}}$$

# Solving yield to maturity in excel

- See bond-pricing.xlsx

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | | Semiannual coupons | | annual coupons | | | | |
| 2 | | | | | | | | |
| 3 | Settlement date | 1/1/15 | | 1/1/15 | | | | |
| 4 | Maturity date | 1/1/45 | | 1/1/45 | | | | |
| 5 | Annual coupon rate | 0.08 | | 0.08 | | | | |
| 6 | Flat price (% of par) | 127.676 | | 127.676 | | | | |
| 7 | Redemption value (% of face value) | 100 | | 100 | | | | |
| 8 | Coupon payments per year | 2 | | 1 | | | | |
| 9 | | | | | | | | |
| 10 | | = YIELD(settlement date, maturity date, annual coupon rate, bond price, redemption value as percent of par value, number of coupon payments per year) | | | | | | |
| 11 | Yield to maturity (decimal) | 0.0600 | =YIELD(B3,B4,B5,B6,B7,B8) | 0.0599 | | | | |
| 12 | | | | | | | | |

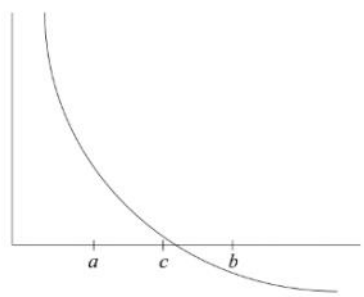
EMORY

# Solving yield to maturity using bisection method

- Solve $r$ that satisfies $f(r) = 0$ for $r \geq -1$, where $f(r)$ is defined as

$$f(r) = \sum_{t=1}^{60} \frac{40}{(1+r)^t} + \frac{1000}{(1+r)^{60}} - 1276.76$$

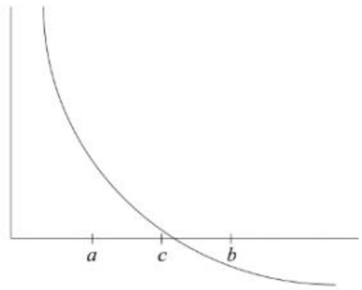
- $f(r)$ is monotonically decreasing in $r$
- A unique solution exists that satisfies $f(r) = 0$
- Bonus question for HW 4




EMORY

# Solving yield to maturity using bisection method

- Start with $a$ and $b$ where $a < b$, $f(a) > 0$, and $f(b) < 0$
- Then $f(\xi)$ must be zero for some $\xi \in [a, b]$
- If we evaluate $f$ at the midpoint $c = (a + b)/2$, either
  - $f(c) = 0$
    - We are done
  - $f(a)f(c) < 0$
    - We continue the process with the new bracket $[a, c]$
  - $f(c)f(b) < 0$
    - We continue the process with the new bracket $[c, b]$
- The bracket is halved in the latter two cases

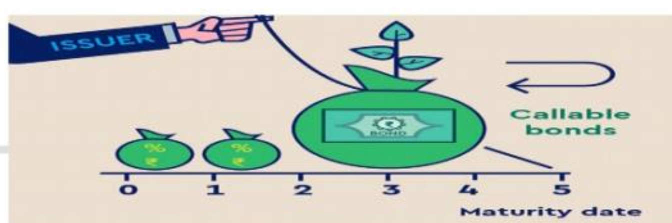



- After $n$ steps, we will have confined $\xi$ within a bracket of length $\frac{(b-a)}{2^n}$

# Yield to call

- **Callable bonds** allow the issuer to repurchase the bond at a specified call price before the maturity date
- Call protection: an initial time during which the bonds are not callable
- A callable bond with par value $1,000, an 8% coupon rate, and a 30-year to maturity, but callable at 110% of par value after 3 years
- Suppose the bond calls in $n$ periods, the yield to call, denoted by $r$, is the value that satisfies

$$Bond\ value = \sum_{t=1}^{n} \frac{Coupon}{(1+r)^t} + \frac{Call\ price}{(1+r)^n}$$



# Solving yield to call in excel

- A callable bond with par value $1,000, an 8% coupon rate, and a 30-year to maturity. This bond currently sells at $1,150 and is called at 110% of par value after 3 years

| | A | B |
|---|---|---|
| 1 | | Yield to call |
| 2 | | |
| 3 | Settlement date | 1/1/15 |
| 4 | Maturity/Call date | 1/1/18 |
| 5 | Coupon rate | 0.08 |
| 6 | Final payment | 110 |
| 7 | Price | 115 |
| 8 | Coupon payments per year | 2 |
| 9 | | |
| 10 | | = YIELD(settlement date, matur |
| 11 | Yield | 0.056055224 |

# Question 1

- Question: For a 10-year, 8% coupon semi-annual bonds with call price $1,100, the yield to maturity is 7%. For simplicity, assume that bonds are called as soon as the present value of remaining payments exceeds the call price. If the market interest rate suddenly falls to 6%, what will be the capital gain the bond?

- Hint:
  - Step 1: calculate the current value of bond with yield to maturity 7%
  - Step 2: calculate the value of bond when interest rate falls to 6%

# Question 2

- Question: A 10-year maturity 9% semi-annual coupon bond is callable in five years at a call price of \$1,050. The bond currently sells at a yield to maturity of 8%. What is the yield to call?

- Hint:
  - Step 1: calculate the current value of bond with yield to maturity 8%
  - Step 2: suppose the bond is called in five years, calculate the yield to call

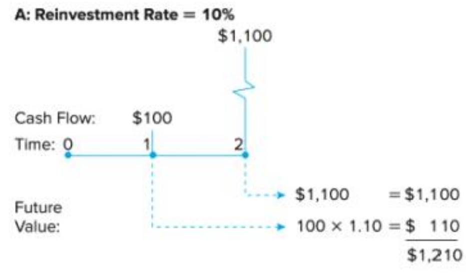
# Realized compound return

- After an investment period $T$, we can calculate the realized compound return. Suppose the initial value is $V_0$, final value is $V_T$ and realized compound return $r$ is

$$V_0(1+r)^T = V_T$$

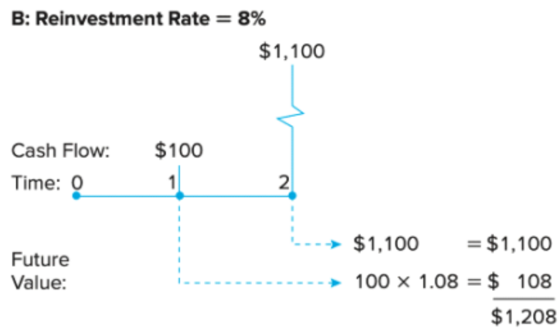
$$\Rightarrow r = \left(\frac{V_T}{V_0}\right)^{1/T} - 1$$

- Example: Consider, a 2-year bond selling at par value, paying a 10% coupon once a year and YTM is 10%. The coupon payment is reinvested at rate 10%
  - $r = \left(\frac{1210}{1000}\right)^{1/2} - 1 = 10\% = YTM$



# Realized compound return vs YTM

- Coupons may not be reinvested to earn the bond's yield to maturity. Then realized compound return does not equal to YTM
- Initial value of the investment is $V_0 = 1000$. Final value of the investment is $V_2 = 1208$. The compound rate of return
  - $r = \left(\frac{1208}{1000}\right)^{1/2} - 1 = 9.91\%$




# Calculation of realized compound return

- Realized compound return can be calculated either after the horizon or using a forecast of future reinvestment rates