Lecture 13: CAPM and APT

Wednesday, March 1, 2023 2:34 PM

13-capm-apt

All assets are available to investors
Short positions are allowed?
No risk or tax
Investors can borrow and lend at risk free interest rate

QTM 385 Quantitative Finance

Lecture 13: CAPM and APT

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Suggested reading: Investments Ch 9 and 10

Expected return-beta relationship

- Expected return-beta relationship for any asset i

$$E(r_i) = r_f + \beta_i[E(r_M) - r_f]$$

  - Sum of risk-free rate r_f (compensation for "waiting") and a risk premium $\beta_i[E(r_M) - r_f]$ (compensation for "worrying")
  - Firm-specific risk is not priced by the market

- Expected return-beta relationship for portfolio P with weight w_i in asset i (with $\sum_i w_i = 1$ and $\sum_i w_i \beta_i = \beta_P$)

$$E(r_P) = w_1 E(r_1) + w_2 E(r_2) + \cdots + w_n E(r_n)$$
$$= \sum_{i=1}^{n} w_i(r_f + \beta_i[E(r_M) - r_f]) = r_f + \beta_P [E(r_M) - r_f]$$

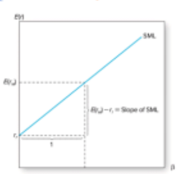
Portfolio is a weighted average of the assets, weights sum up to 1.
And beta_p quantifies the covariance of portfolio P with market portfolio?
Beta_p is weighted average of betas

Security market line

- Security's risk premium is proportional to both beta and risk premium of the market portfolio, i.e.,

$$\beta[E(r_M) - r_f]$$

- Security market line (SML): expected return-beta relationship $E(r)$ vs β
  - Slope is $E(r_M) - r_f$
  - Graphs *individual asset* risk premiums as a function of asset risk (held as parts of well-diversified portfolios, i.e., contribution to portfolio variance)



How does expected return vary with the beta

Individual asset risk premiums only depends on beta and not firm specific risk

Interpretation of β in security market line

- In the security market line,

$$\beta[E(r_M) - r_f]$$

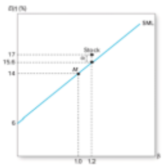
  - If $\beta = 0$, then $E(r_i) = r_f$ and the risk is diversifiable
  - If $\beta < 0$, then $E(r_i) < r_f$ and payoffs in bad times (insurance)
  - If $\beta > 0$, then $E(r_i) > r_f$ and the risk is compensated

Beta can be positive, zero, or negative

e.g. negative: Return of individual asset will move in different direction than market portfolio, making positive returns during a bad time during the market

Security market line

- If assets are "fairly priced", then they are exactly on the SML. All securities must lie on the SML in market equilibrium
- If a stock is underpriced, then it will plot above the SML
- Stock's **alpha** α: Difference between the fair and actually expected rate of return
- For example, $\alpha = 1.4\%$ for the stock in the figure



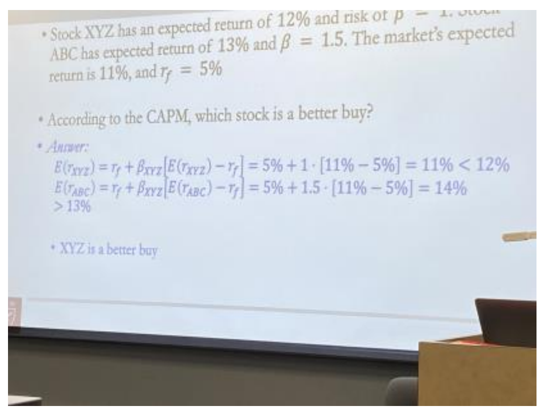
Question

- Stock XYZ has an expected return of 12% and risk of $\beta = 1$. Stock ABC has expected return of 13% and $\beta = 1.5$. The market's expected return is 11%, and $r_f = 5\%$
- According to the CAPM, which stock is a better buy?

RIsk free rate of 5%

Stock XYZ is a better buy

Market risk premium is 11% - 5% she has a typo



Alpha for ABC is –1%
THe larger the alpha the better the buy is because we get a higher expected return than predicted by the SML

The CAPM and single-index market model

- The single index model states

$$R_i = \alpha_i + \beta_i R_M + e_i$$

  - The realized excess return on any stock is the sum of three parts
    - Marketwide factors: $\beta_i R_M$
    - Nonmarket premium: α_i
    - Firm-specific outcomes: e_i
  - The risk premium has $E(R_i) = \alpha_i + \beta_i E(R_M) + e_i$
  - Based on CAPM, the equilibrium value of α_i is 0. Otherwise, if $\alpha_i > 0$, investors will buy the stocks, bid up the prices, and expected return will be lower

Linear model with just one regressor is single index model, and the regressor is the index

Alpha I cannot be captured by the market return BiRM
No randomness in alpha I and is the same as the one before in the previous slides

Ei is firm specific return

So the risk premium in this case will have this relationship and ei should have mean 0 because the nonzero parts will be predicted by the alphas

Factor models of security returns

Arbitrage pricing theory: slightly more general than CAPM because we rely on weaker assumptions and

Factor models of security returns

Arbitrage pricing theory: slightly more general than CAPM because we rely on weaker assumptions and can use more factors or more indices to explain why different assets have different risk premiums/expected returns

- Uncertainty in asset returns has two sources
  - A common or macroeconomic factor
  - Firm-specific events

Factors in most cases are macroeconomic factors
Or characteristics of the firm like firm size

- Single factor model

$$R_i = E(R_i) + \beta_i F + e_i$$

  - $E(R_i)$: the expected excess return on stock i
  - F: deviation of the common factor from its expected value. Therefore, $E(F) = 0$. It measures new information concerning the macroeconomy
  - e_i: nonsystematic components of returns (related to firm-specific events, uncorrelated with F)

IF market is good, then return is larger -> E(Ri) it is a systematic risk factor?????

Example of the single factor model

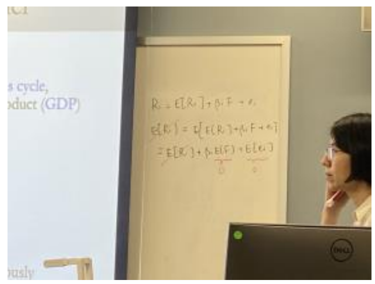
***************INSERT THE PICTURE HERE ****************

- Single factor model: $R_i = E(R_i) + \beta_i F + e_i$

Ri = E[Ri] + BiF + ei
E[Ri] = E[E[Ri] + BiF + ei]
= E[Ri] + BiE[F] + E[ei]

E[F] = 0 and E[ei] = 0

  - For example, F is macro factor (news about the state of the business cycle, measure by the unexpected percentage change in gross domestic product (GDP)
  - The consensus is that GDP will increase by 4% this year
  - Suppose also that a stock's $\beta_i = 1.2$
  - If GDP increases by only 3%, then $F = 3\% - 4\% = -1\%$
  - The stock is 1.2% lower ($\beta_i F = 1.2 \times (-1\%) = -1.2\%$) than previously expected



Factor models

- Confining systematic risk to a single factor is restrictive
  - Extra-market sources of risk may arise from interest rates, inflation, and other sources of uncertainty
- Multifactor models can better describe security returns

E.g. russia ukraine war can also drive individual return to deviate from the mean

- For example, a two-factor model

Basically MLR

$$R_i = E(R_i) + \beta_{i,GDP} \cdot GDP + \beta_{i,IR} \cdot IR + e_i$$

  - IR: Unexpected change in interest rates

The two-factor model

- $R_i = E(R_i) + \beta_{i,GDP} \cdot GDP + \beta_{i,IR} \cdot IR + e_i$

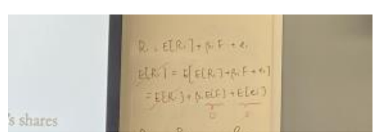
For different firms, the betas can be different

- Regulated electric-power utility firm in a mostly residential area
  - $\beta_{i,GDP}$ is low
  - $\beta_{i,IR}$ is high

Intuitively, beta gdp will be low while the beta interest rate will be high for electric power firm
Because people's usage of electric power doesn't really depend on the general market condition (so revenue will be pretty stable regardless of market condition) but the beta interest rate might be high because the revenue of the utility firm is stable over time, so if you invest in an electropower company, it's pretty much investing in a bond, you get a fixed stream of dividends, so if you're getting a fixed stream of income, more sensitive to interest rate
Discount factor? Future money is less valuable, drives current price down and affects return?

For airlines, revenue will largely depend on market condition because people travel more if market is good
Interst rate is low because investment is not like a bond

- An airline firm
  - $\beta_{i,GDP}$ is high
  - $\beta_{i,IR}$ is low

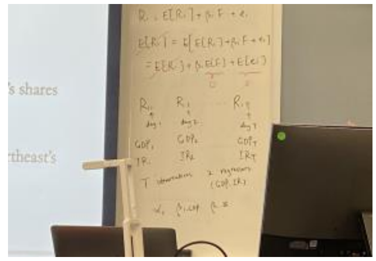


Example

- Suppose we estimate the two-factor model for Northeast Airlines and find

$$R_i = .133 + 1.2 \cdot GDP - .3 \cdot IR + e_i$$

  - Expected excess return $E(R_i)$ is 13.3%
  - If GDP increases 1% beyond current expectation, return on Northeast's shares increases on average by 1.2%
  - If increase rates increase 1% beyond current expectation, return on Northeast's shares decreases on average by .3%



Just like a regression problem, but instead of observations it's the return for t days or weeks and the two variables are gdp and interest rate?

TIME SERIES REGRESSION

Multi-factor model

- A description of the factors that affect security returns

$$R_i = E(R_i) + \beta_{i,GDP} \cdot GDP + \beta_{i,IR} \cdot IR + e_i$$

- There is no "theory" in the equation
- Where $E(R_i)$ comes from? What determines a security's expected excess rate of return?
- Arbitrage pricing theory helps to determine $E(R_i)$ in equilibrium

In this setting, how do we explain E(Ri)? That is what arbitrage pricing theory is about

Arbitrage pricing theory

- Stephen Ross developed the arbitrage pricing theory (APT) in 1976
- Like the CAPM, the APT predicts a security market line linking expected returns to risk
- Three key propositions
  1. Security returns can be described by a factor model
  2. There are sufficient securities to diversify away idiosyncratic risk
  3. Well-functioning security markets do not allow arbitrage opportunities to persist

CAPM first introduced in 1960s
APT is similar, explains E(Ri)
Basically why different assets have different expected returns

Three assumptions

1. Basically we can write the return in the equation above. Return = expected value plus deviation from the mean which is split into two parts: systematic and nonsystematic parts
2. If this is not about asset I but it's about the portfolio instead, we can write that R_p = E[R_p] + Beta_p,gdp * GDP + beta_p,IR * IR. We can kill the e_i term.
3. No arbitrage opportunity

Law of one price

- If two assets are equivalent in all economically relevant respects, then they should have the same market price
- If they observe a violation of the law, they will engage in arbitrage activity—simultaneously buying the asset where it is cheap and selling where it is expensive
  - E.g., different prices of a stock on two different exchanges
  - Involve long–short positions

If no arbitrage opportunity, then we can use the Law of One Price

Diversification in a single-factor security market

- If a portfolio is well diversified, its firm-specific or nonfactor risk becomes negligible
- A portfolio with n stocks, each with weight w_i and $\sum_i w_i = 1$

$$R_P = E(R_P) + \beta_P F + e_P$$

  - $\beta_P = \sum w_i \beta_i$
  - $E(R_P) = \sum w_i E(R_i)$
  - Nonsystematic return $e_P = \sum w_i e_i$
- Portfolio variance can be decomposed into two parts

$$\sigma_P^2 = Var(\beta_P F + e_P) = \beta_P^2 Var(F) + Var(e_P)$$

  - If the portfolio is equally weighted $w_i = 1/n$, then

$$Var(e_P) = Var(\sum w_i e_i) = \sum w_i^2 Var(e_i) = \frac{1}{n}\sum \frac{Var(e_i)}{n} = \frac{1}{n}\overline{Var(e_i)} \to 0 \Rightarrow e_P \to 0$$

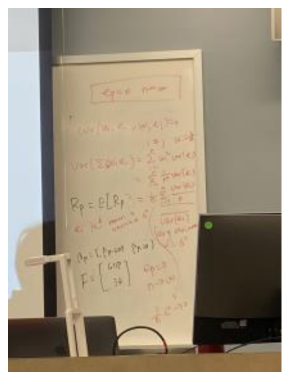
Work with portfolio level data

Beta_p = [Beta_p,gdp , Beta_p,IR]

F = [GDP
IR]

Through later analysis we can show that e_p is zero -> THIS IS THE POINT OF THIS SLIDE

As n increases to infinity, e_p is zero



Then take out 1/n to get the values on the slide

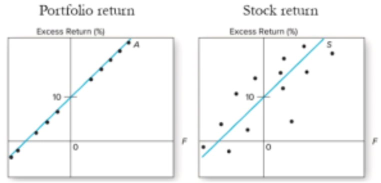
If ei is iid with mean 0 then the average variance will be equal to sigma squared

Well-diversified portfolios

- For a well-diversified portfolio

$$R_P = E(R_P) + \beta_P F$$

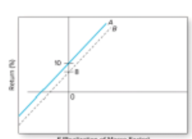
Portfolio return — Stock return



$R_A = 10\% + 1.0 \times F$ — $R_S = 10\% + 1.0 \times F + e_S$

The security market line of the APT

- Only the systematic or factor risk of a portfolio of securities should be related to its expected returns
- All well-diversified portfolios with the same beta must have the same expected return
- Otherwise, an arbitrage opportunity exists
  - sell short \$1 million of B
  - buy \$1 million of A
  - a zero-net-investment strategy



The security market line of the APT

The security market line of the APT

- Their risk premiums must be proportional to beta
  - Otherwise, an arbitrage opportunity exists
- If the excess return on a well-diversified portfolio R follows

$$R_P = E(R_P) + \beta_P F$$

- Then the risk premium is

$$E(R_P) = \beta_P E(F)$$

The APT and the CAPM

- The APT serves many of the same functions as the CAPM
- APT gives us a benchmark for rates of return
- APT highlights the crucial distinction between nondiversifiable risk (factor risk), which requires a reward in the form of a risk premium, and diversifiable risk, which does not
- APT does not require that almost all investors be mean-variance optimizers. Rely on a highly plausible assumption that precludes arbitrage opportunities

The APT and the CAPM

- The CAPM provides a statement on the expected return–beta relationship for all securities
- APT implies that this relationship holds for all but perhaps a small number of securities, as APT is built on well-diversified portfolios